A Chiral Magnetic Effect from AdS/CFT with Flavor

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Introduction

- The AdS/CFT correspondence is a powerful tool to study strongly-coupled gauge theories in terms of classical gravity theory
- Especially, it is expected to be applicable to the low energy QCD that is hard to be explained by other analytical means
- The gravity calculation almost reproduces the physical quantities of the dual field theory, and gives the same answer if they are independent of the gauge coupling

Introduction

- A chiral magnetic effect is a counterintuitive phenomenon in QCD such that an electric current is parallel to a magnetic field

$$J^z \propto \mu_5 B_z$$

  in the presence of an axial chemical potential

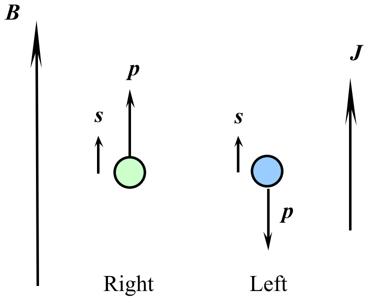
- We would like to see if there is a counterpart of the CME in a holographic setup!

Chiral magnetic effect

- The axial chemical potential causes the imbalance of the number of left and right fermions

$$\mu_5 \quad \Longleftrightarrow \quad N_L \neq N_R$$

- That produces the net current proportional to $N_R - N_L$ along the magnetic field



Holographic setup

- The near horizon limit of a stack of D3-branes has two different aspects
  - $\mathcal{N} = 4$ super Yang-Mills theory (open)
  - $AdS_5 \times S^5$ geometry (closed)

- We would like to have additional degrees of freedom to realize the CME

Field theory	Gravity dual
fundamental quarks	probe D7-branes
an axial chemical potential μ_5	an angular velocity ω in S^5
a magnetic field B	a gauge field B on D7-branes
the quark mass m	the length between D3 and D7

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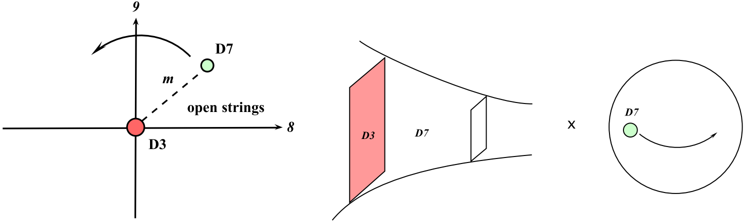
D3/D7 configuration

	0	1	2	3	4	5	6	7	8	9
D3	$\times$	$\times$	$\times$	$\times$						
D7	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		



- D3/D7 system $\quad\Leftrightarrow\quad$ $\mathcal{N}=2$ SYM with fund hypermultiplets

Dual CFT with time-dependent mass

- One can immediately read off

Field theory	Gravity dual
the quark mass m	the length between D3 and D7
the phase of the mass term ϕ	the angle in 8-9 plane

- The phase of the quark mass is converted to the axial chemical potential by $U(1)_R$ symmetry $\psi \to e^{-\frac{i}{2}\phi\gamma^5}\psi$

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}\partial\!\!\!/\psi + |m|\bar{\psi}e^{i\phi\gamma^5}\psi \\ &\to i\bar{\psi}\partial\!\!\!/\psi + \frac{\partial_\mu\phi}{2}\bar{\psi}\gamma^\mu\gamma^5\psi + |m|\bar{\psi}\psi\end{aligned}$$

- In our setup, ϕ is time-dependent

$$\mu_5 \equiv \frac{\partial_t\phi}{2} = \frac{\omega}{2}$$

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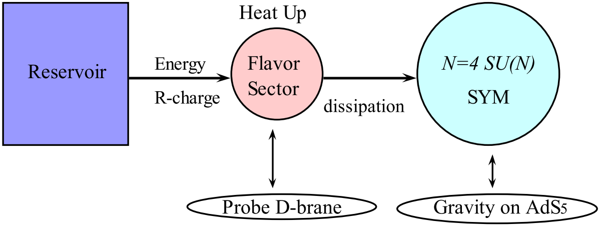
Non-equilibrium system in AdS/CFT



- The dual field theory is regarded as an open system with time-dependent masses
- The probe brane has an emergent horizon which enables us to define a new temperature

Plan

1. Chiral magnetic effect in QCD
2. Holographic model of the CME
3. Thermal properties of rotating D-branes

I: Chiral magnetic effect in QCD

Derivation of the chiral magnetic effect

- A θ term in the electromagnetism changes the electric current

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma}$$
$$\rightarrow \quad j^\mu = \partial_\nu F^{\nu\mu} - \frac{1}{8\pi^2}\epsilon^{\mu\nu\sigma\rho}\partial_\nu\theta F_{\rho\sigma}$$

- When $\theta = \theta(t)$ and $F_{xy} = B_z$, we obtain the CME

$$J^z = \frac{1}{4\pi^2}(\partial_t\theta)B_z$$

- In the confined phase of QCD, the chiral symmetry is broken and the pseudoscalar plays a role of θ angle

$$\mathcal{L}_{eff} = \phi(t,x)\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma}$$

Deconfined phase in QCD

- Suppose the fermions are coupled to an axial chemical potential

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} + \mu_5 \gamma^0 \gamma^5)\psi$$

- One can remove the c.p. by the $U(1)_A$ symmetry $\psi \to e^{i\mu_5 t \gamma^5}\psi$, but the axial anomaly gives rise to an additional term

$$\mathcal{L}_{QCD} + \mathcal{L}_{anomaly} = i\bar{\psi}\not{D}\psi + \frac{\mu_5 t}{16\pi^2}\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\rho\sigma}$$

- Then, the chiral magnetic current is produced in the presence of the magnetic field

$$J^z = \frac{1}{2\pi^2}\mu_5 B_z$$

II: Holographic model of the CME

Probe D7-branes in $AdS_5 \times S^5$

- Now we consider a probe D7-brane wrapped on S^3 in S^5

$$\begin{aligned} ds^2 &= -|g_{tt}|\, dt^2 + g_{xx}\, d\vec{x}^2 + g_{rr}\, dr^2 + g_{SS}\, ds^2_{S^3} + g_{RR}\, dR^2 + g_{\phi\phi}\, d\phi^2 \\ &= \frac{\rho^2}{L^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{L^2}{\rho^2}\left(dr^2 + r^2 ds^2_{S^3} + dR^2 + R^2 d\phi^2\right) \end{aligned}$$

- The world volume spans $(t, \vec{x}, r, S^3)$
- In this coordinates, the four-form potential is given by

$$C_4 = g_{xx}^2\, \mathrm{vol}_{\mathbb{R}^{3,1}} - g_{SS}^2\, d\phi \wedge \mathrm{vol}_{S^3}\,.$$

Probe D7-branes in $AdS_5 \times S^5$

- The action that describes the dynamics of D7-branes is

$$S_{D7} = S_{DBI} + S_{WZ},$$

$$S_{DBI} = -N_f T_{D7} \int d^8\xi \sqrt{-\det\left(g^{D7}_{ab} + (2\pi\alpha')\,\tilde{F}_{ab}\right)},$$

$$S_{WZ} = +\frac{1}{2} N_f T_{D7} \left(2\pi\alpha'\right)^2 \int P[C_4] \wedge \tilde{F} \wedge \tilde{F},$$

- The ansatz for rotating D7-branes is

$$R = R(r)\ , \qquad \phi(t,r) = \omega t + \varphi(r)$$
$$A_y = Bx\ , \qquad A_z = A_z(r)$$

Quick look at the origin of the CME

- The Wess-Zumino term would be at the boundary

$$
\begin{aligned}
S_{WZ} &\sim \int_{AdS_5} \frac{r^2}{\rho^2} d\phi \wedge F \wedge F \\
&\sim \int_{\mathbb{R}^{1,3}} \phi F \wedge F
\end{aligned}
$$

- In our setup, the angle plays a role of the time-dependent θ term

$$
\phi \quad \xrightarrow{r \to \infty} \quad \omega t
$$

- The same effective action for the CME!

Two types of solutions

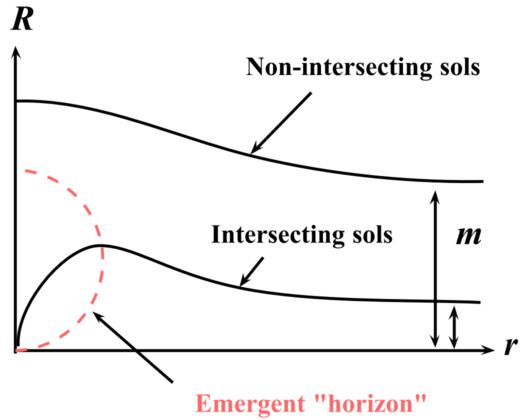
- According to the AdS/CFT, one can make the following dictionary

Bulk field	Source	Operator
R	the quark mass m	$\langle \mathcal{O}_m \rangle \sim \bar{\psi}\psi$
ϕ	the phase of the mass $\phi = \omega t$	$\langle \mathcal{O}_\phi \rangle \sim \|m\| i\bar{\psi} e^{i\phi\gamma^5}\gamma^5\psi$
A_z	the boundary gauge field A_z	$\langle J^z \rangle$

- There are two types of solutions
  - Intersecting solutions: $\langle \mathcal{O}_\phi \rangle, \langle J^z \rangle \neq 0$ (CME phase)
  - Non-intersecting solutions: $\langle \mathcal{O}_\phi \rangle = \langle J^z \rangle = 0$
- This means that the pseudo-scalar meson always condenses in the CME phase!

Shapes of D7-branes

- The solutions for rotating D7-branes are characterized by an emergent "horizon"

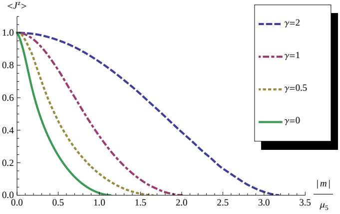


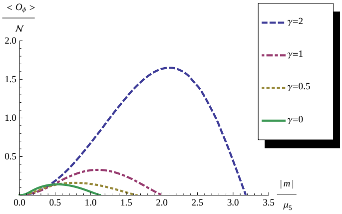
The chiral magnetic current

- One can analytically evaluate the value of the electric current in the massless limit

$$\langle J^z \rangle = \frac{N_c N_f}{2\pi^2} \mu_5 B$$

- This is exactly the same as the value of the dual field theory!
- Our model makes it possible to compute the mass dependence of the current





III: Thermal properties of rotating D-brane

Emergence of black hole on rotating D-brane

- The induced metric on the D7-branes is

$$
\begin{aligned}
ds^2 &= -|g_{tt}^{D7}|dt^2 + 2g_{tr}^{D7}dtdr + g_{rr}^{D7}dr^2 + gxxd\vec{x}^2 + g_{ss}d\Omega_3^2 \\
&= -|g_{tt}^{D7}|\Big(\underbrace{dt - \frac{g_{tr}^{D7}}{|g_{tt}^{D7}|}dr}_{d\tau}\Big)^2 + \left(g_{rr}^{D7} + \frac{(g_{tr}^{D7})^2}{|g_{tt}^{D7}|}\right)dr^2 + \cdots
\end{aligned}
$$

$$
g_{tt}^{D7} = -|g_{tt}| + g_{\phi\phi}(\partial_t\phi)^2
$$

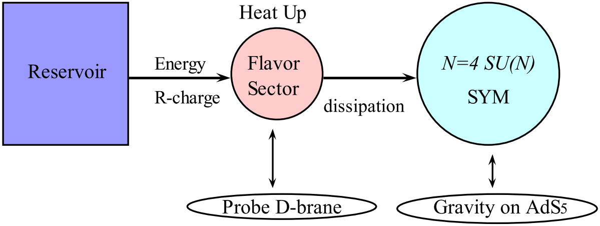
- This is an AdS black hole with the horizon at $g_{tt}^{D7} = 0$, and the Hawking temperature is given by [Das-TN-Takayanagi 10]

$$
T_H = \frac{|g_{tt}^{D7}|'}{4\pi\sqrt{|g_{tt}^{D7}|g_{rr}^{D7} + (g_{tr}^{D7})^2}}
$$

- Notice that the bulk of AdS is still at zero temperature under a probe approximation

Non-equilibrium state and charge/energy dissipations

- Our model describes a non-equilibrium state and there must be charge/energy dissipations from the flavor sector (the probe D7) to the adjoint sector (the bulk AdS)
- The source of the rotation of the D7-branes should be interpreted as a reservoir to heat up the flavor sector



Charge/energy dissipations in field theory

- The rate of change of the R-charge is

$$\partial_t \langle J_R^t \rangle = \langle \mathcal{O}_\phi \rangle$$

- The rate of change of the energy is given by

$$\partial_t E = \partial_t V = \langle \mathcal{O}_\phi \rangle \partial_t \phi = \omega \langle \mathcal{O}_\phi \rangle$$

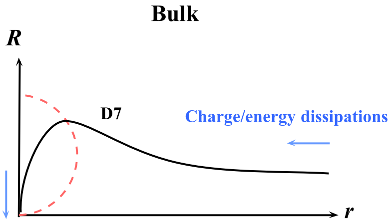
- Then the two rates of changes are related

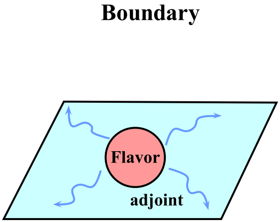
$$\partial_t E = \omega \partial_t \langle J_R^t \rangle$$

as expected since the leak of R-charge to the adjoint sector takes energy with it

Charge/energy dissipations

- In the boundary theory, the charge and energy leak from the flavor sector to the adjoint sector
- In the bulk, they flow from the boundary into the bulk





Charge/energy dissipations in the bulk

- The R-charge is given by the angular momentum of the D7-brane

$$\langle J_R^t \rangle = \int_{r_H}^{\infty} dr \pi_\phi^t$$

- To obtain the energy flux, consider the energy-momentum tensor in the bulk Θ^M_N (not on the brane!)
- The boundary energy is obtained as

$$E = \langle T_{tt} \rangle = \int_{r_H}^{\infty} dr \Theta_{tt}$$

Charge/energy dissipations in the bulk

- These two charges are conserved

$$\partial_t \langle J_R^t \rangle = \int_{r_H}^{\infty} dr \partial_t \pi_\phi^t = - \int_{r_H}^{\infty} dr \partial_r \pi_\phi^r = - \pi_\phi^r \Big|_{r=r_H}^{\infty} = 0$$

$$\partial_t \langle T_{tt} \rangle = - \partial_t \langle T_t^t \rangle = - \int_{r_H}^{\infty} dr \partial_t \Theta_t^t = \int_{r_H}^{\infty} dr \partial_r \Theta_t^r = {\Theta^r}_t \Big|_{r_H}^{\infty} = 0$$

- But the rate of the R-charge and energy dissipations are non-zero!

$$\pi_\phi^r = \langle \mathcal{O}_\phi \rangle \ , \qquad {\Theta^r}_t = - \omega \langle \mathcal{O}_\phi \rangle$$

- The holographic calculation reproduces the relation

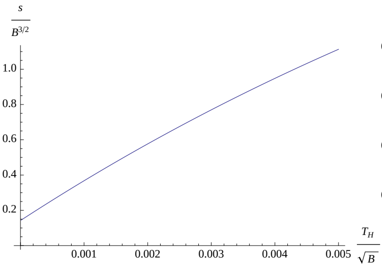
$$\partial_t E \big|_\infty = \omega \partial_t \langle J_R^t \rangle \big|_\infty$$

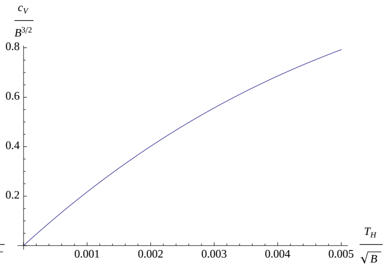
Summary

- We constructed the holographic model of the CME by rotating D7-branes in $AdS_5 \times S^5$ spacetime
- The mass dependence of the CME was easily studied and the result was reeasonable
- The probe brane has the emergent horizon and the system is in non-equilibrium state
- The charge/energy dissipations were computed holographically, and it reproduced the same relation

Future direction

- Partial evidence for Fermi surface?





- There are no fermionic degrees of freedom in the bulk!