# Beyond the standard cosmological model: neutrinos and non-Gaussianity
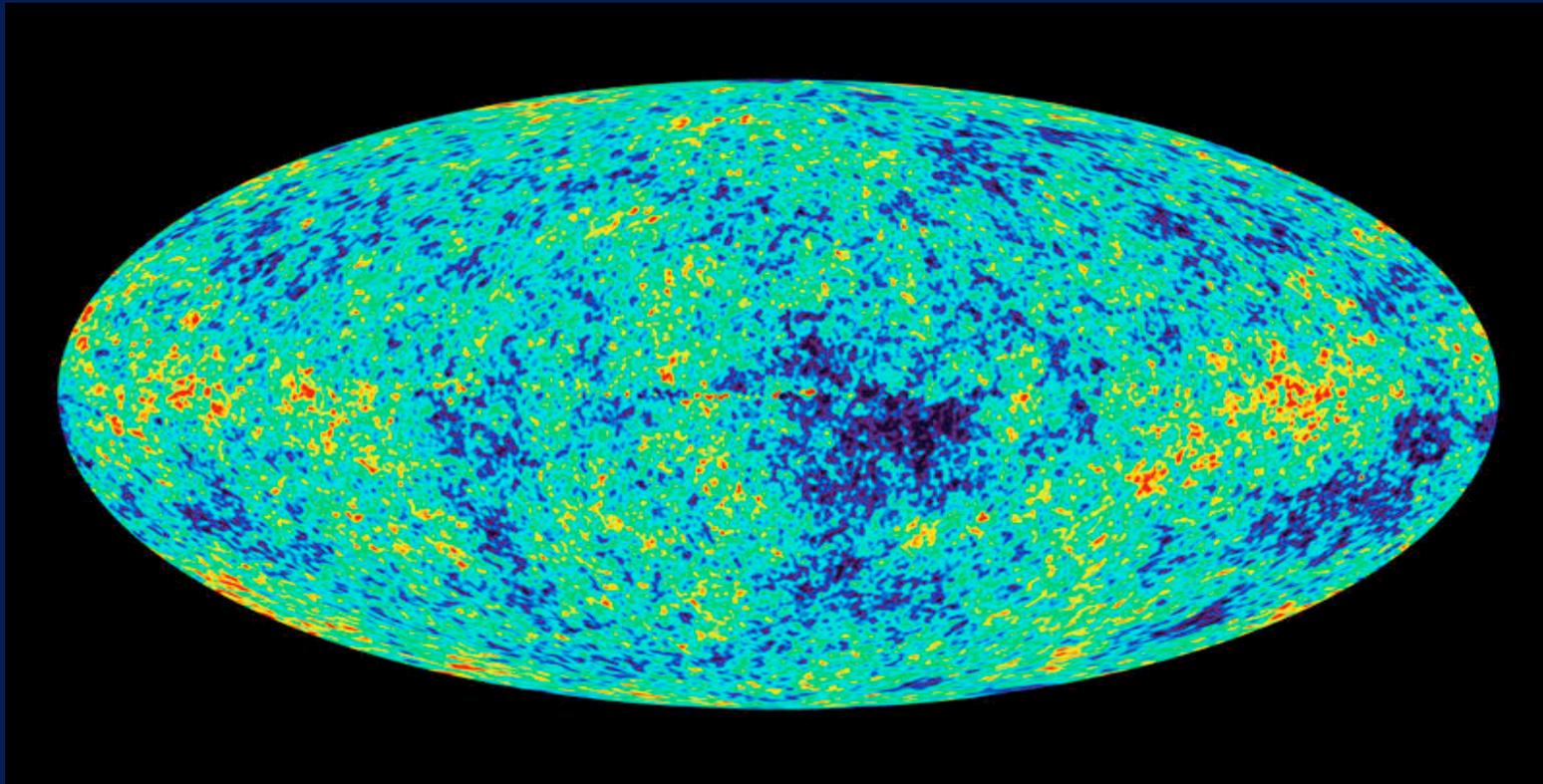
WMAP team

Tristan L. Smith (UC Berkeley)

Outline

* What does the CMB/LSS tell us so far?

* Large-scale observations largely confirm standard cosmological picture

* However, there are some small holes being poked in the standard model

* With future data, either these holes will be fixed or…

u11867304 fotosearch.com

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Outline

* Concentrate on two extensions of the standard cosmological model:

* Constraints on the effective number of neutrino species

* CMB constraints to the primordial non-Gaussian amplitude

* Before going into the extensions, let us review the standard model
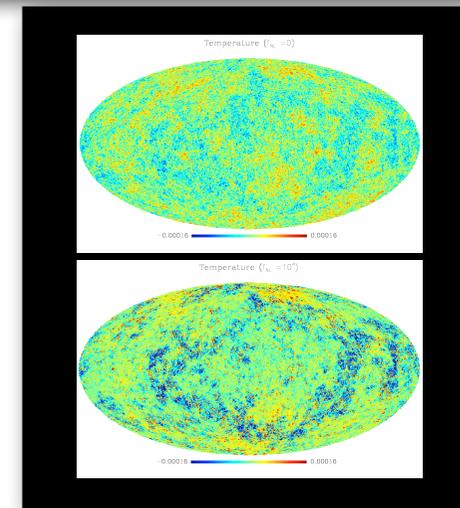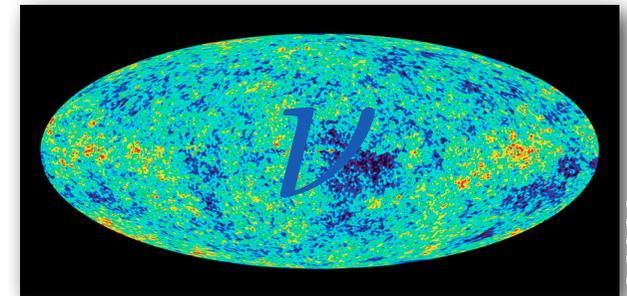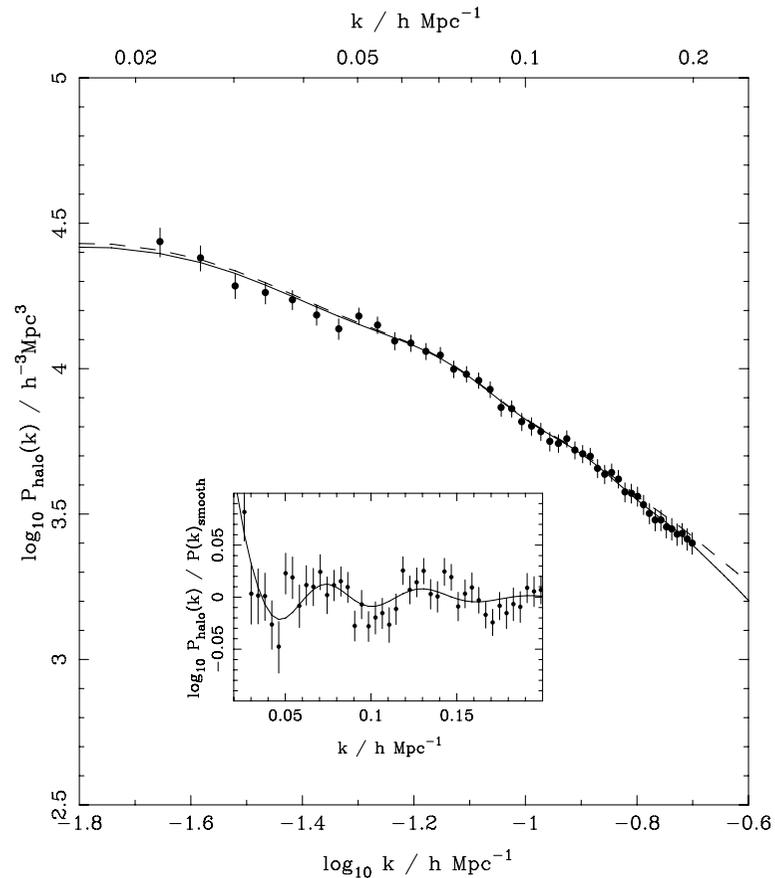
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Reid et al. (2010)

Keisler et al. (2011)

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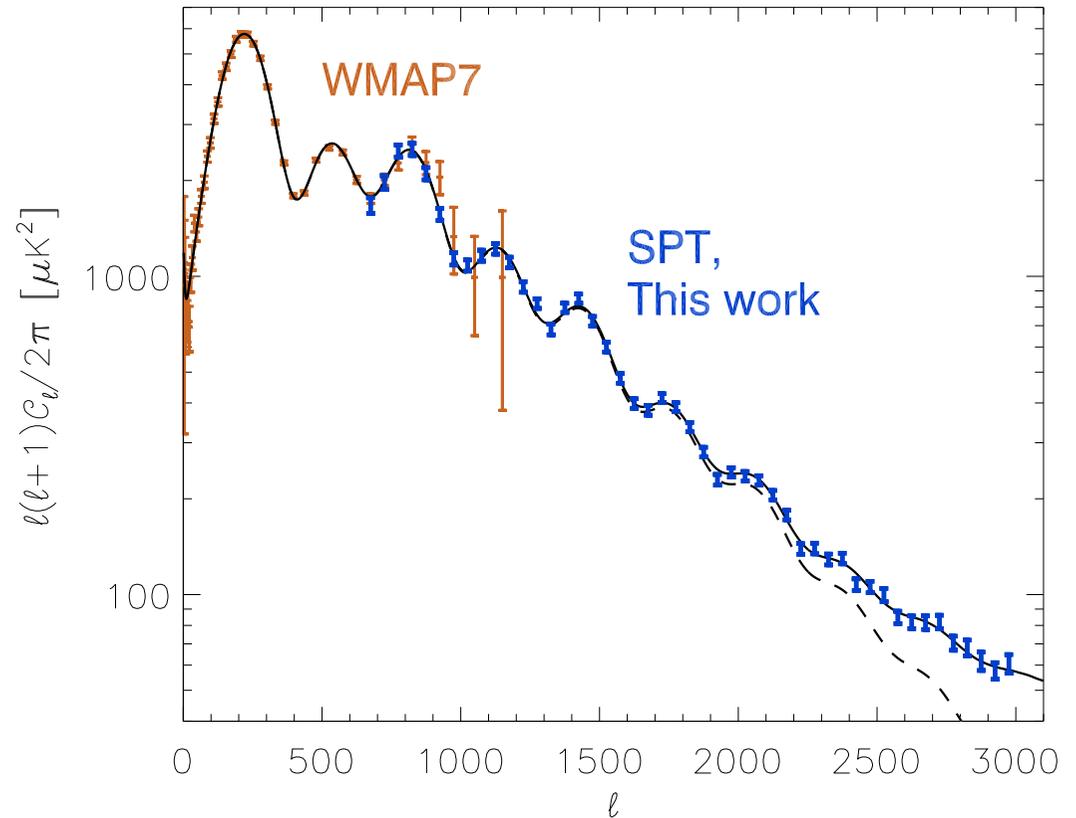
Extensions of the standard cosmological model

* There are some extensions which are 'expected' at some level:

    * Non-zero gravitational-wave contribution

    * Running of the spectral index

    * Non-zero neutrino mass

    * Time varying dark energy equation of state

* Others would seriously challenge the standard cosmological model:

    * Effective number of neutrino species

    * Non-Gaussianity

    * Parity violating interactions

    * Anisotropic processes

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Constraints on neutrino interactions using cosmological observations
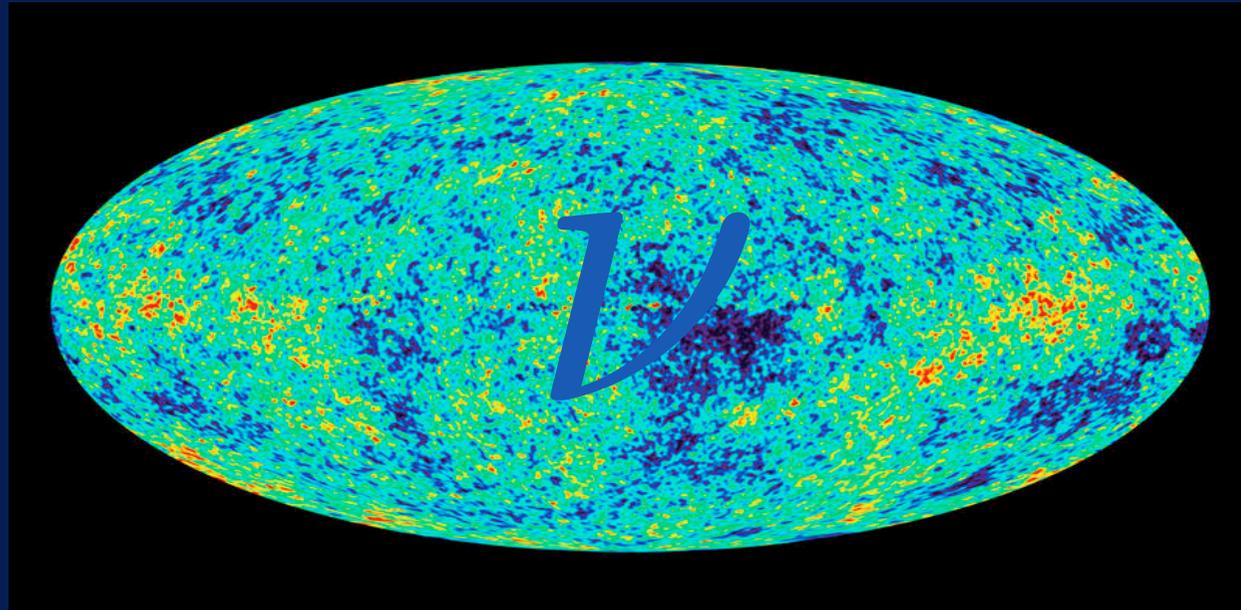
WMAP

Tristan L. Smith
Sudeep Das
Oliver Zahn
(BCCP, UC Berkeley)
arXiv:1105.3246 [astro-ph.CO]

Status of the standard model of cosmology from the CMB
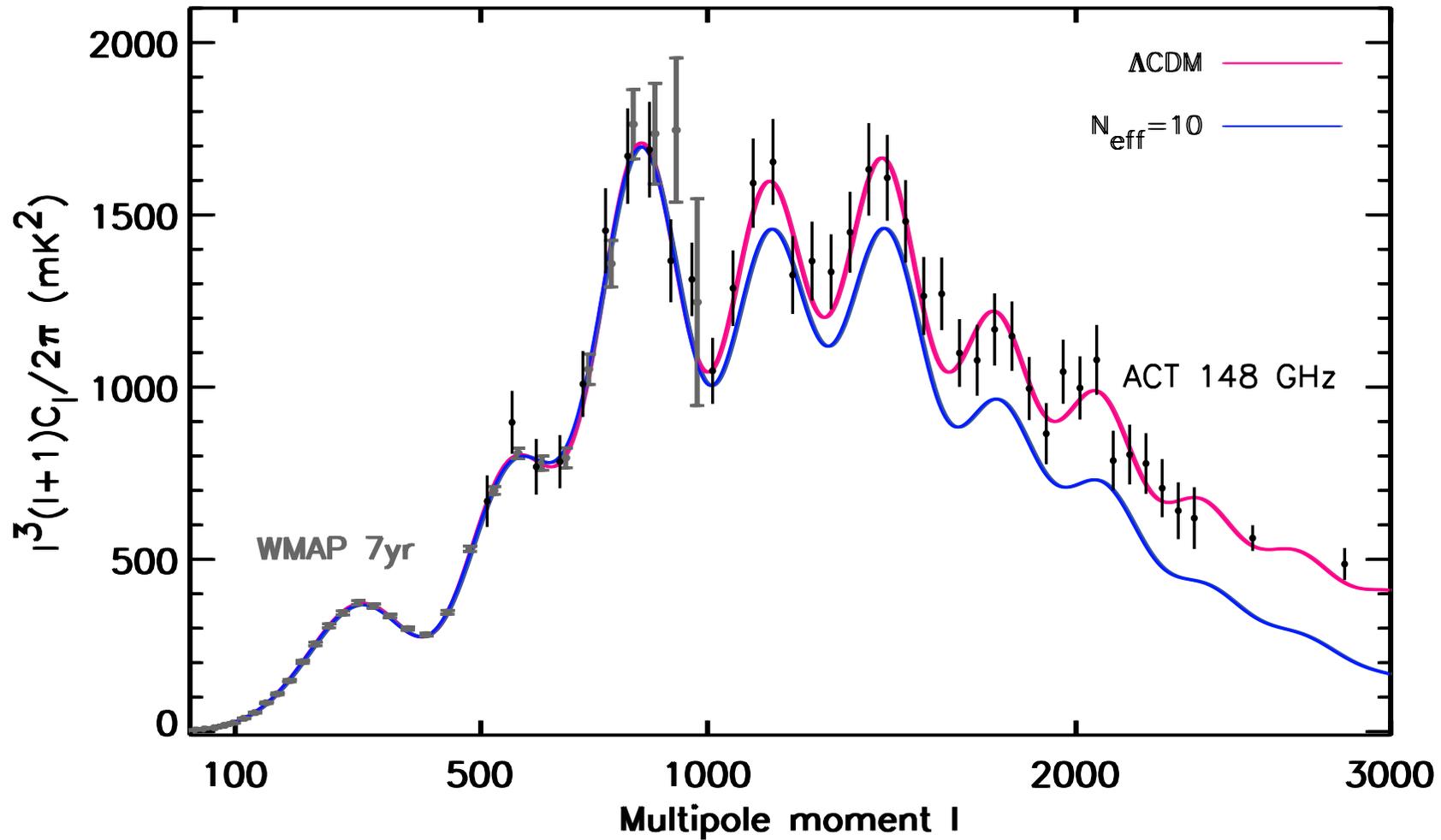
ACT, Dunkley et al. (2011)

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H0+WMAP+ACT+SPT+SDSS

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H0+WMAP+ACT+SPT+SDSS

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Status of the standard model of cosmology from the CMB

95% CL

Today

Time

2003

N_{eff}

SPT+WMAP+H_0+BAO
SPT+WMAP
WMAP

Likelihood $p(N_{eff})$

N_{eff}

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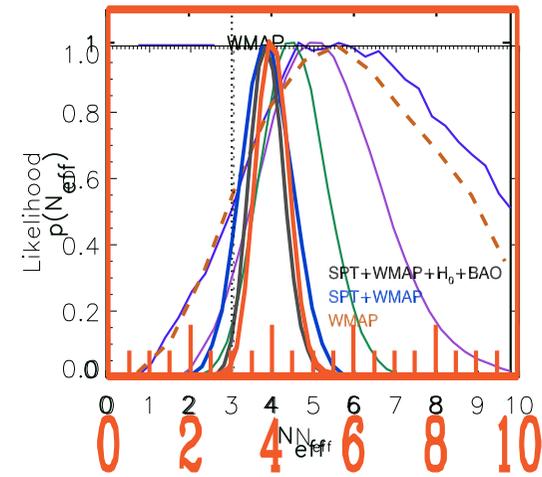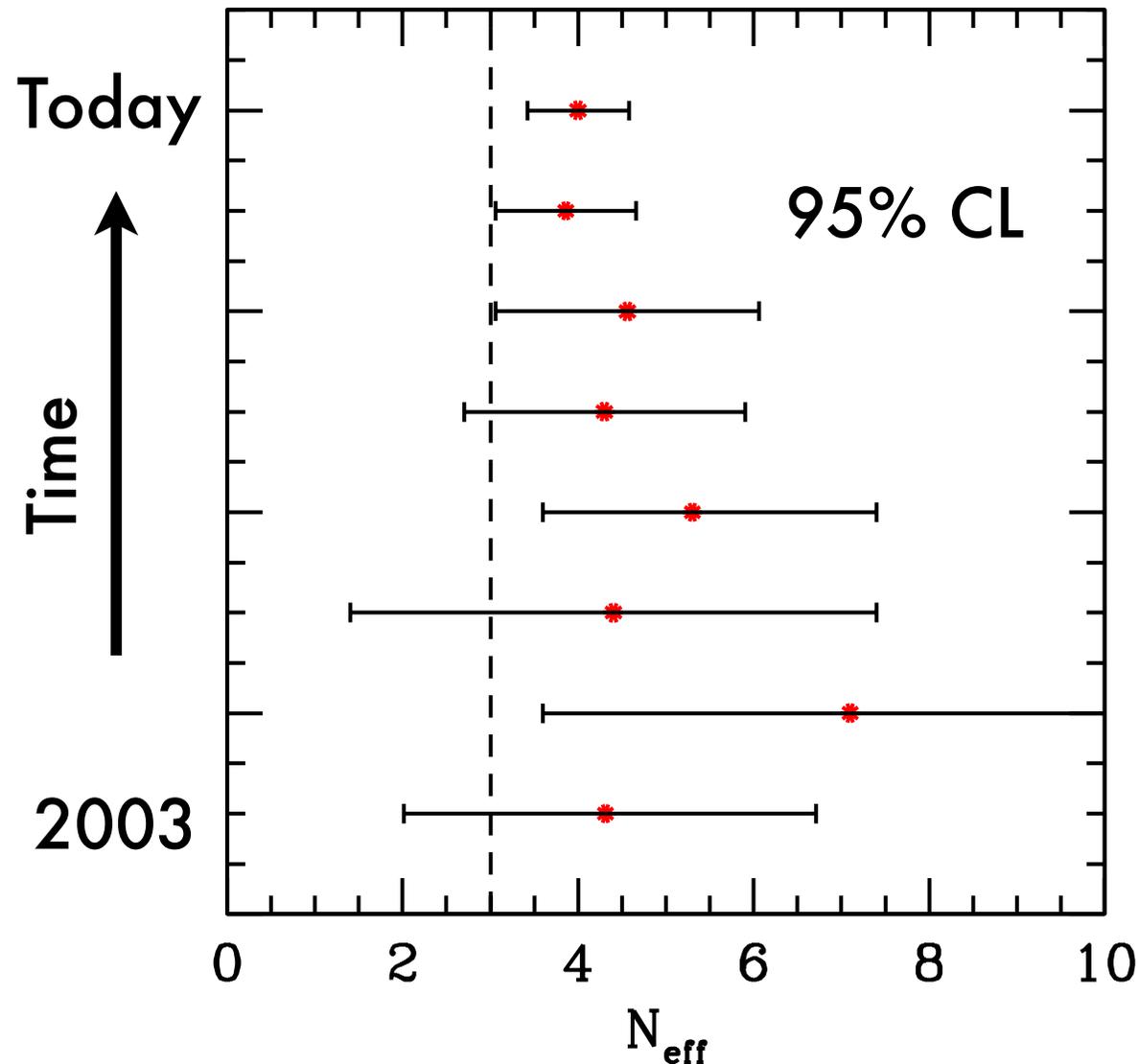
Interpreting extra relativistic energy density

* So far we have a hint that there exists an anomalous radiative background… but no other information

* What can this be? Most explanations (such as sterile neutrinos) suppose that this background will be non-interacting

* We were interested in exploring to what extent the observations can show that this background is non-interacting

* Following Hu (1998) we modify the evolution equations by introducing two new parameters $c_{\rm eff}^2$ and $c_{\rm vis}^2$

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Interpreting extra relativistic energy density

* Changes to the effective sound-speed modifies the pressure support

$$c^2_{\rm eff} = 1/3 \text{ ——}$$

$$0.8 \text{ — —}$$

$$0.1 \text{ —·—}$$

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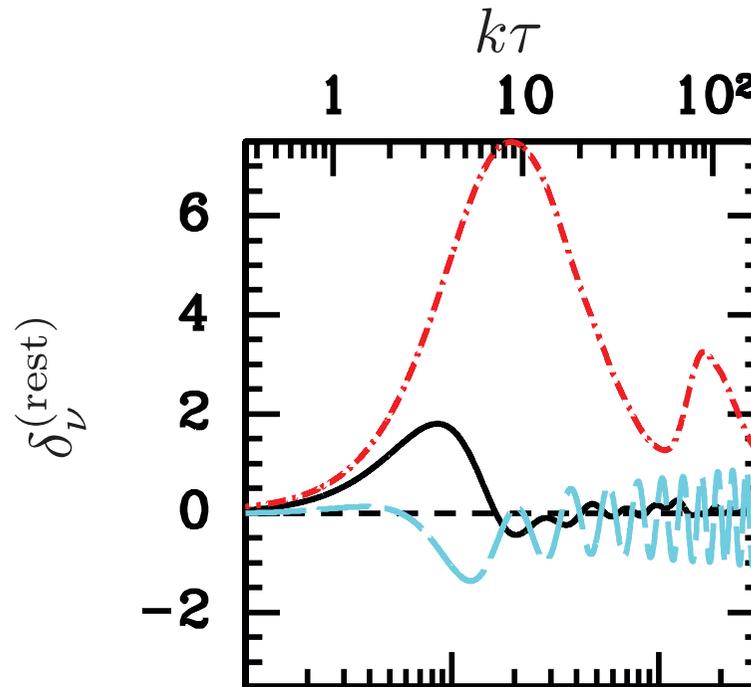
Interpreting extra relativistic energy density

* Changes to the viscosity parameter controls to what extent the fluid is imperfect (i.e., anisotropic stress)

$$c^2_{\mathrm{vis}} = 1/3 \quad \text{———}$$

$$1 \quad \text{– – –}$$

$$0 \quad \text{–·–·–}$$

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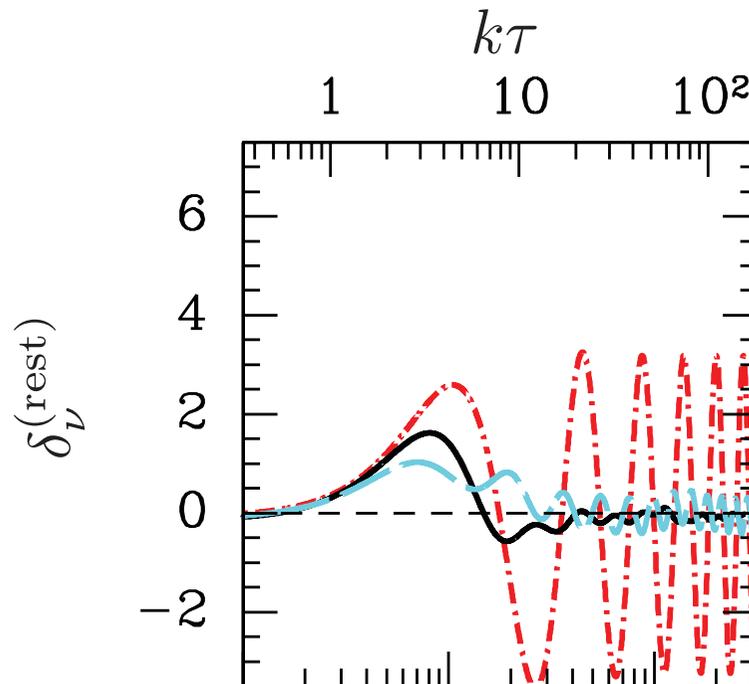
What do these new parameters physically mean?

* These parameters give some measure of the interactions this anomalous background may have

* An analogy with the tightly coupled photon-baryon fluid

* Bell et al. (2006) considered a model where a sterile neutrino is tightly coupled to a scalar field

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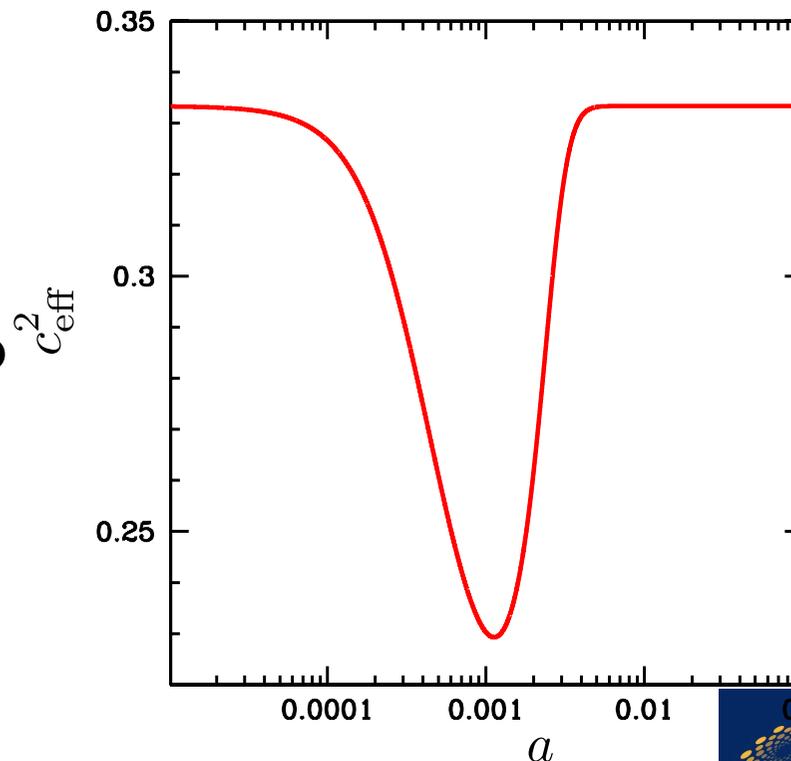
Interpreting extra relativistic energy density

* The effects on the CMB spectrum

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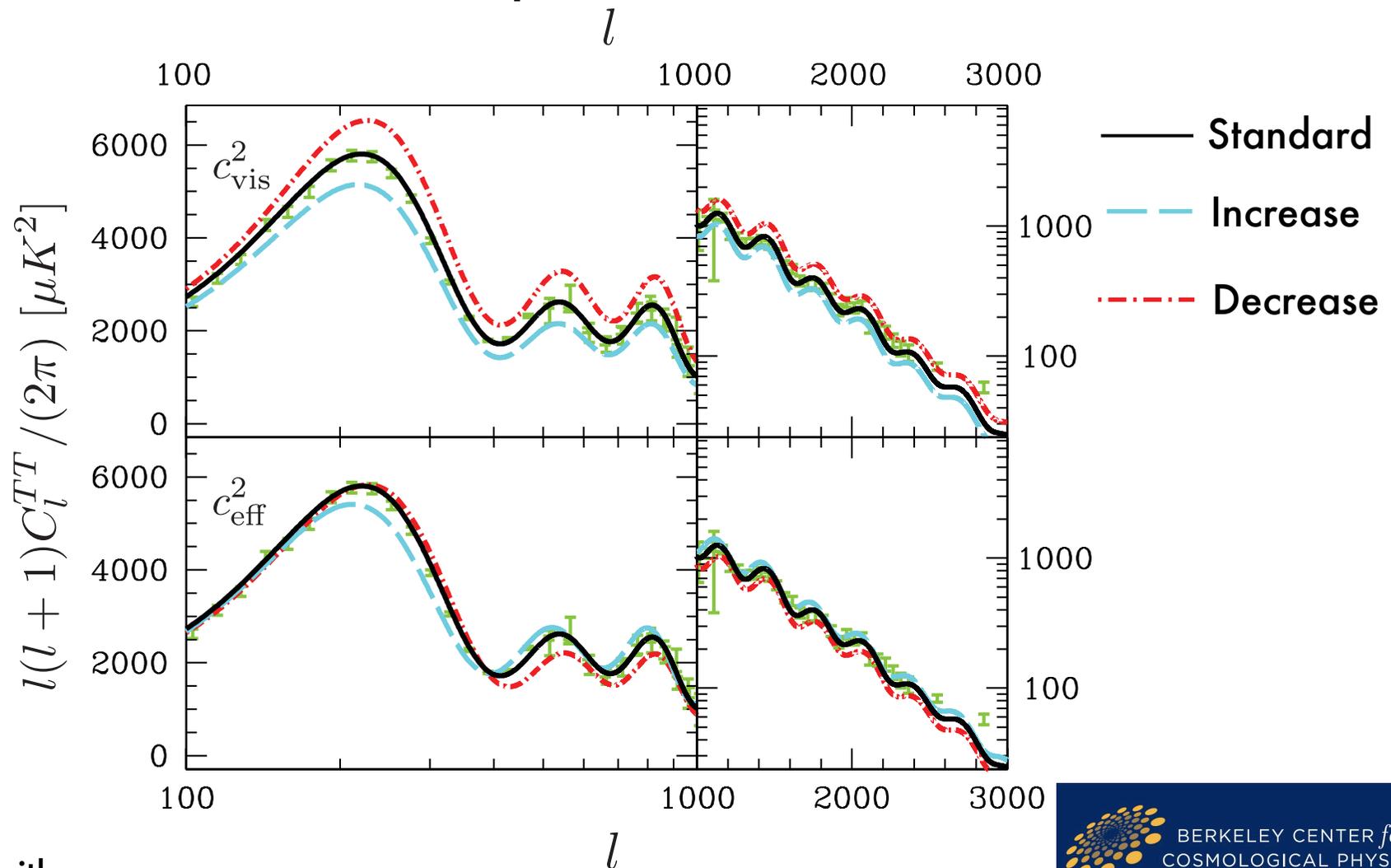
The data

* We used various combinations of CMB and large-scale structure data:
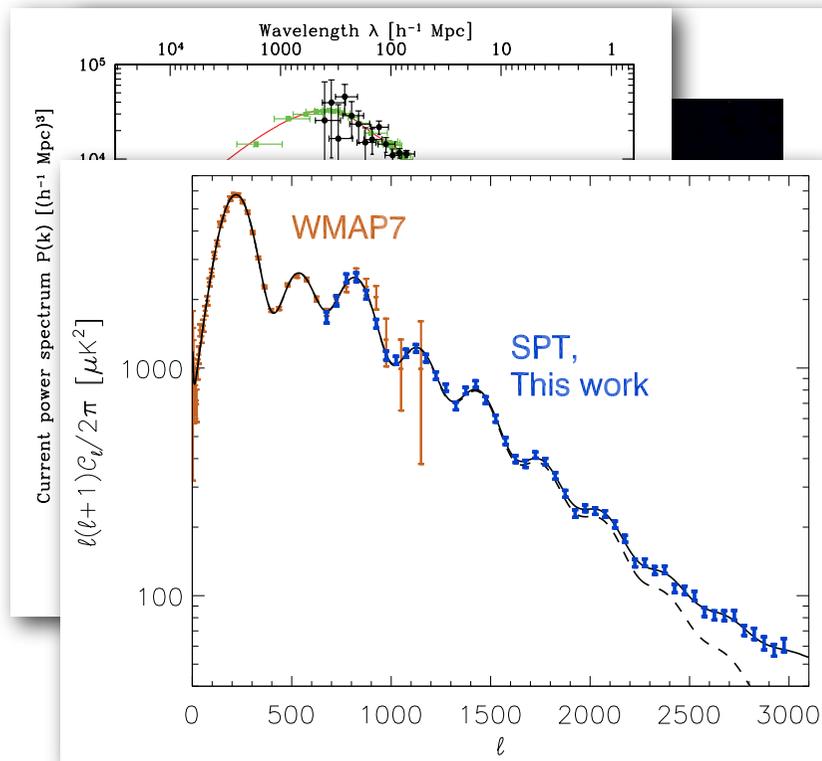
* H_0 from HST

* SDSS matter power spectrum

* Lya forest

* CMB
  * WMAP7
  * ACBAR
  * ACT
  * SPT

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The Results

Marginalized 1D constraints

$N_{\rm eff}$	$c^2_{\rm vis}$	$c^2_{\rm eff}$
$4.0^{+0.17+0.58}_{-0.18-0.57}$	$1/3$	$1/3$

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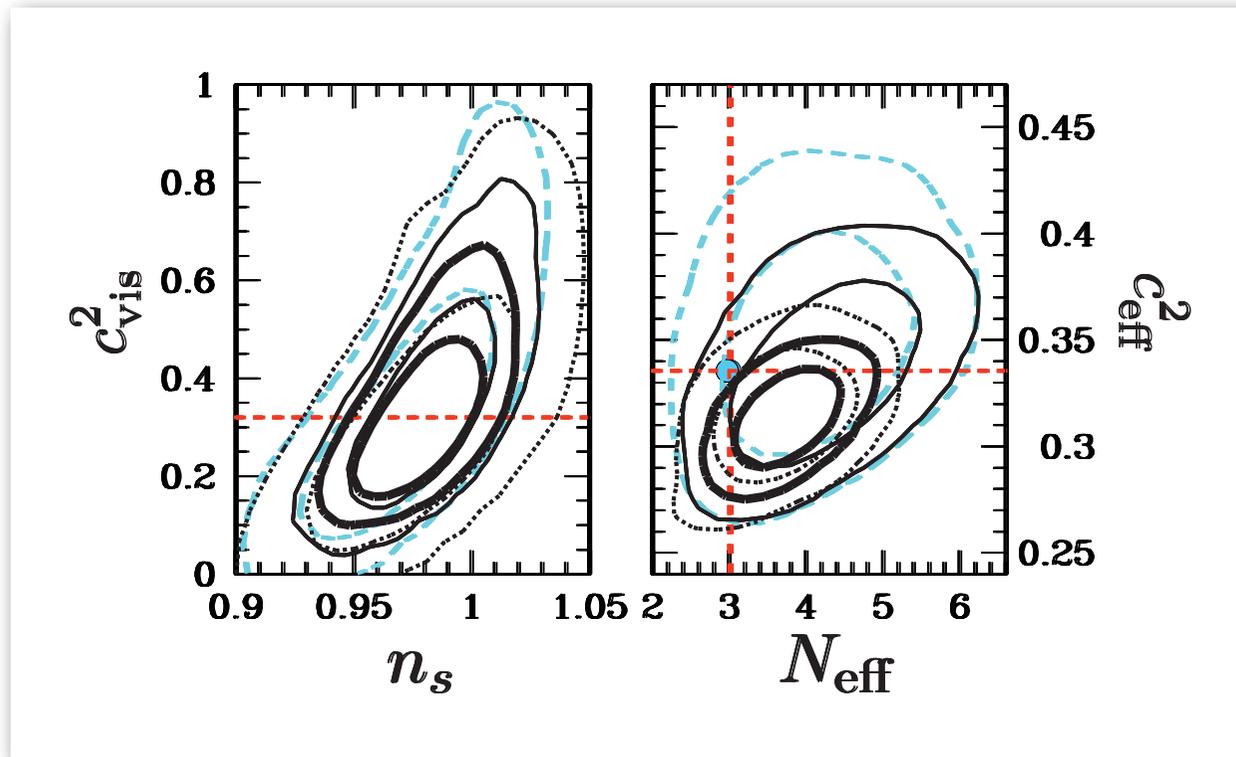
WMAP+SDSS

WMAP+SDSS + Lya

CMB-only

All

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Conlusions and future directions

* These constraints provide further evidence that there may be extra non-interacting neutrino-like degrees of freedom

* Planck will be able to constrain:

$$N_{\rm eff} = 3.0 \pm 0.17$$
$$c^2_{\rm eff} = 0.333 \pm 0.004$$
$$c^2_{\rm vis} = 0.333 \pm 0.026$$

* Extend parameterization for neutrino mass

* Explore to what extent the data is able to constrain a time evolving $c_{\rm eff}(a)$ and $c_{\rm vis}(a)$

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Temperature ($f_{NL} = 0$)

-0.00016 0.00016

Temperature ($f_{NL} = 10^4$)

-0.00016 0.00016
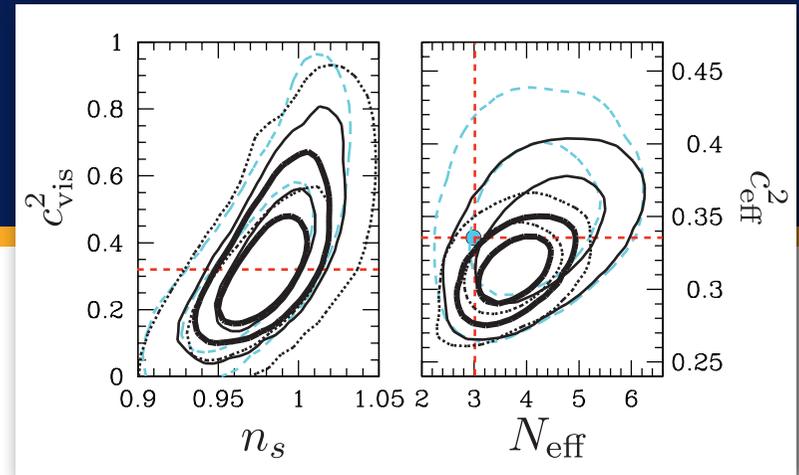
B. Wandelt

The PDFs of non-Gaussianity estimators

In collaboration with Marc Kamionkowski, Alan Heavens, and Benjamin Wandelt

Phys. Rev. D 83, 023007 (2011)
Phys. Rev. D 84, 063013 (2011)
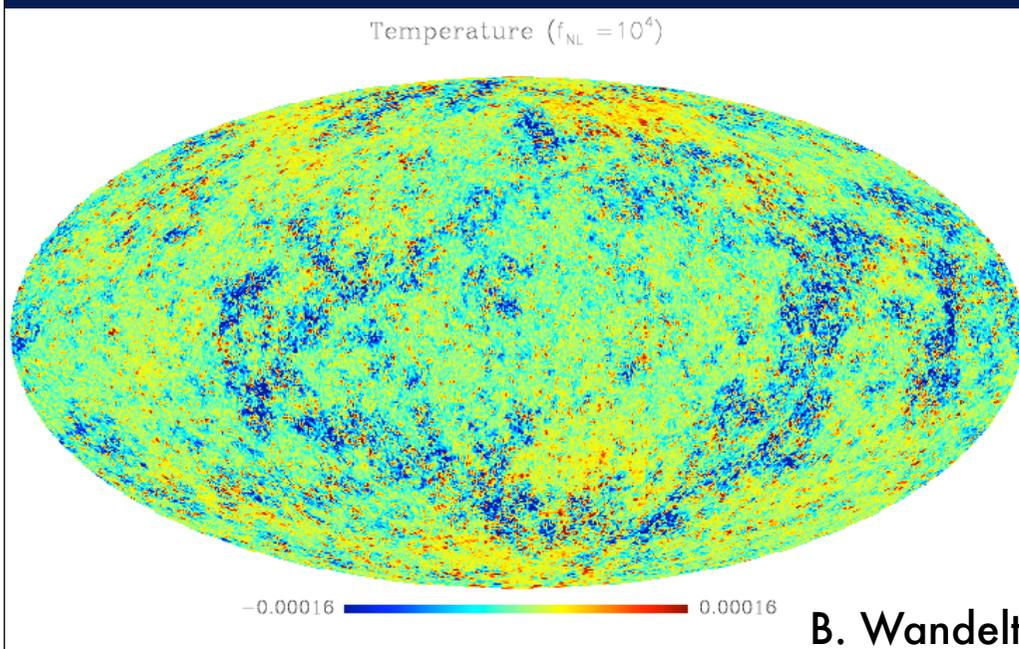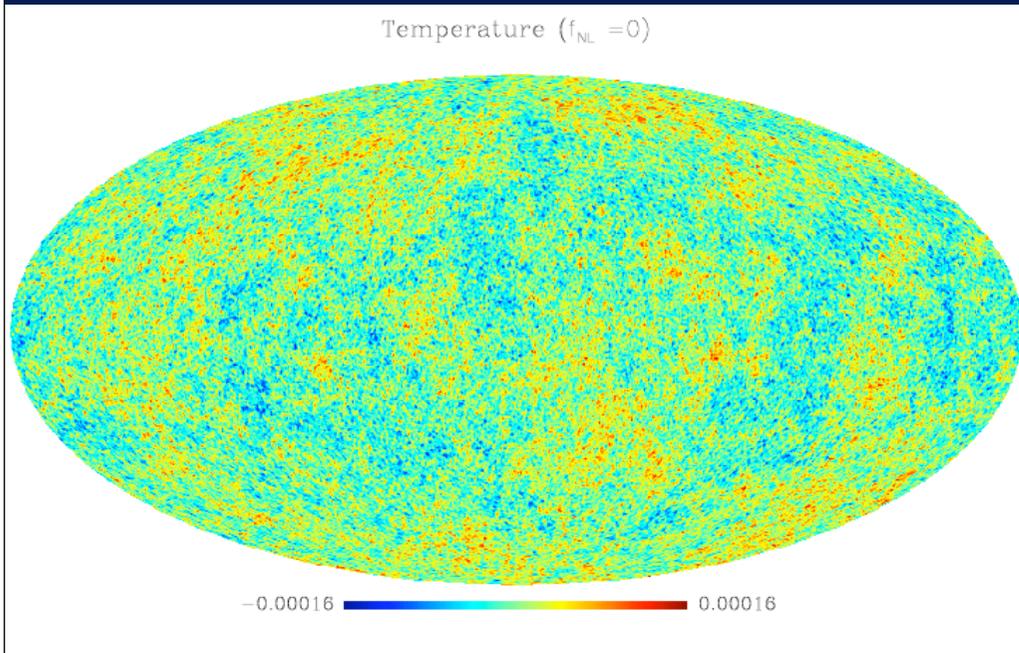
Non-Gaussian estimation from the CMB

* The standard cosmological model predicts that the primordial fluctuations obey Gaussian statistics

* It is simple to think of a few basic ways to test this prediction

* One way is to look at the PDF of the temperature fluctuations in the CMB

* It turns out that the signal-to-noise using the PDF is sub-optimal

Komatsu (2002)

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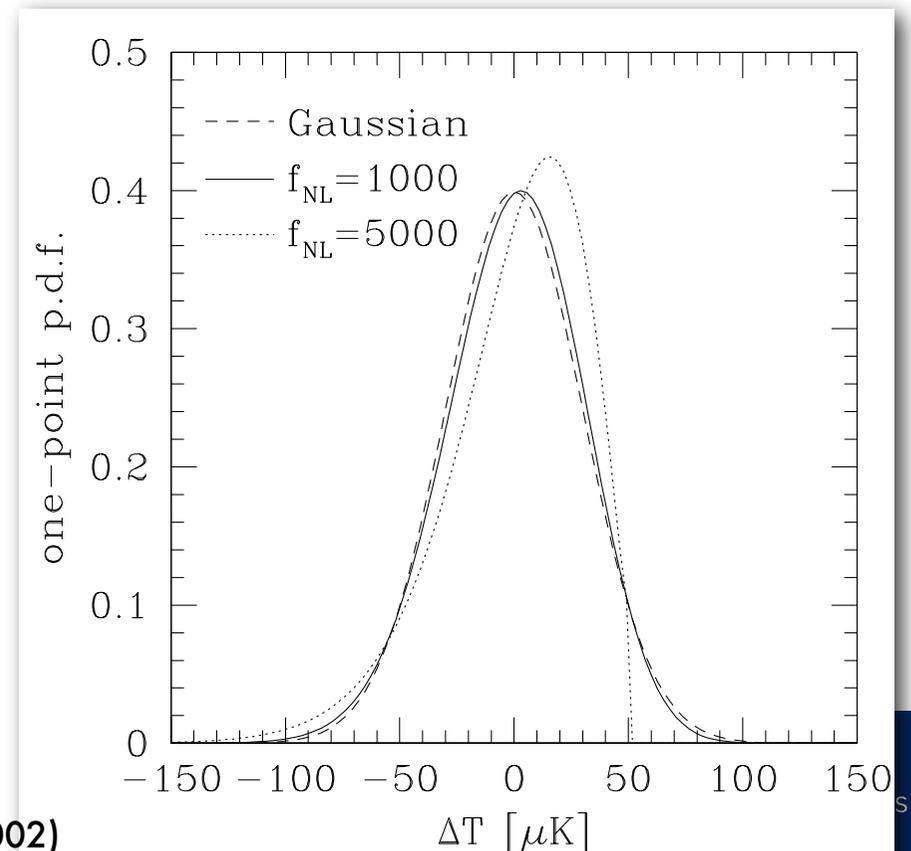
Non-Gaussian estimation from the CMB

✱ Instead, we want to use the fact that any process which is Gaussian is uniquely determined by its mean, μ, and variance, σ

$$\langle (x - \mu)^1 \rangle = 0$$

$$\langle (x - \mu)^2 \rangle = \sigma^2$$

$$\langle (x - \mu)^3 \rangle = 0$$

$$\langle (x - \mu)^4 \rangle = 3\sigma^4$$

✱ An obvious test of Gaussianity then asks: is the third moment zero? and is the fourth moment just given by the Gaussian piece?

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Non-Gaussian estimation from the CMB

* To make progress we use a parameterization for the level of non-Gaussianity in the CMB maps

$$\Phi(\vec{x}) = \phi(\vec{x}) + f_{\rm nl}\left[\phi(\vec{x})^2 - \langle\phi(\vec{x})^2\rangle\right]$$

where ϕ is a Gaussian random field and Φ is the primordial curvature potential

* We can see that, for instance, the three-point function is now non-zero:

$$\langle\Phi(\vec{x}_1)\Phi(\vec{x}_2)\Phi(\vec{x}_3)\rangle \sim f_{\rm nl}\langle\phi(\vec{x}_1)\phi(\vec{x}_2)\rangle\langle\phi(\vec{x}_3)\phi(\vec{x}_4)\rangle$$

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Non-Gaussian estimation from the CMB

✳ The harmonic coefficients of the temperature field on the sky are related to the primordial curvature potential

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Phi(\mathbf{k}) g_{Tl}(k) Y_{lm}^*(\hat{\mathbf{k}})$$

so that correlations in harmonic space are also non-Gaussian

$$\langle a_{l_1 m_1} a_{l_2 m_2} \rangle = C_{l_1} \delta_{l_1,l_2} \delta_{m_1 m_2}$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \sim f_{\mathrm{nl}} B_{l_1,l_2,l_3}$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} \rangle_c \sim \tau_{\mathrm{nl}} T_{l_1,l_2,l_3,l_4}$$

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Why do we want to do this?

* Any constraint to primordial non-Gaussianity probes the physics of the very early universe

* In particular, assuming that inflation was driven by a single field one can show [Creminelli and Zaldarriaga (2004)]

$$f_{\rm nl} \approx \frac{5}{12}(1 - n_s) \longrightarrow f_{\rm nl} \approx 0.02$$

* So that if we find $f_{\rm nl} > 0.02$ then *all single field inflationary models will be ruled out*

* Measurement of the amplitude of the trispectrum, $\tau_{\rm nl}$, would further allow us to test multi-field inflationary models

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$$T_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_3) = 6 g_{\mathrm{NL}} \left[P_\phi(k_1) P_\phi(k_2) P_\phi(k_3) + (3 \text{ cyclic}) \right]$$

$$+ \frac{25}{18} \tau_{\mathrm{NL}} \left[P_\phi(k_1) P_\phi(k_2) \{ P_\phi(k_{13} + P_\phi(k_{14}) \} + (11 \text{ cyclic}) \right]$$

* Most multi-field inflationary models predict the inequality:

$$\tau_{\mathrm{nl}} \geq \frac{1}{2} f_{\mathrm{nl}}^2$$

(Suyama & Yamaguchi, 2008; Komatsu, 2010, Sugiyama, Komatsu & Futamase, 2011)

* Therefore if Planck measures a non-zero value for f_{nl} then *all single field inflation models are ruled out* and...

$$\tau_{NL} \geq \frac{1}{2} \left(\frac{6}{5} f_{NL} \right)^2$$

* Next, we can use a constraint on τ_{nl} to *test the consistency of most multi-field models of inflation*

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Non-Gaussian estimation

* Constraints to non-Gaussianity

* WMAP constraint on bispectrum:

$$f_{\mathrm{nl}} = 32 \pm 21 \ (68\%)$$

Komatsu et al. (2010)

* WMAP constraint on trispectrum:

$$\tau_{\mathrm{nl}} = (0.96 \pm 0.68) \times 10^4$$

Smidt et al. (2010)

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Non-Gaussian estimation

 * For the rest of this talk, we will work in a simplified limit: flat-sky, Sachs-Wolfe limit:

$$T(\hat{n}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \Phi(\vec{k}) \int_0^{\tau_0} d\tau e^{i\vec{k}\cdot\hat{n}(\tau_0-\tau)} S(k,\tau)$$

$$a_{lm} = \int T(\hat{n}) Y_{lm}(\hat{n}) d^2\hat{n}$$

flat-sky: $\quad a(\vec{l}) = \int T(\hat{n}) e^{i\vec{l}\cdot\hat{n}} d^2\hat{n}$

Sachs-Wolfe: $\quad S_{SW}(k,\tau) = \frac{1}{3}\delta(\tau-\tau_D)$

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Non-Gaussian estimation

* These approximations will reproduce the correct scalings and give order of magnitude estimates:

Babich and Zaldarriaga (2004)

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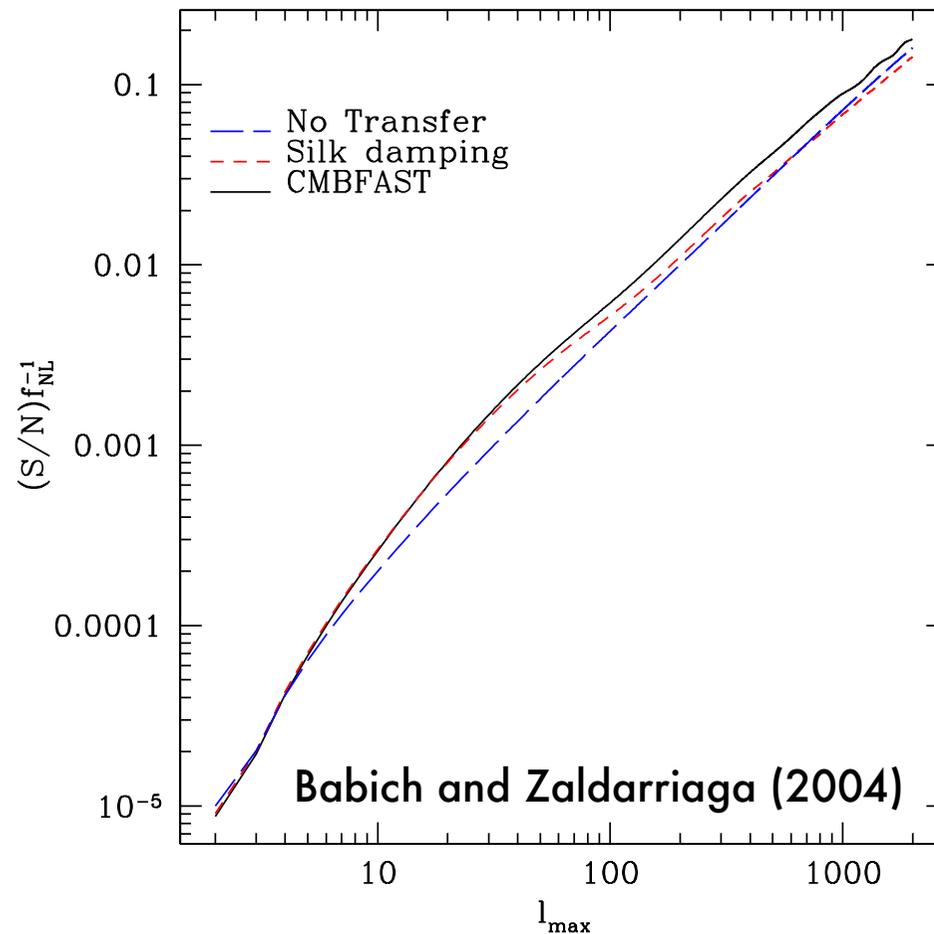
Estimators for $f_{\rm nl}$ and $\tau_{\rm nl}$

* Expectation values of the harmonic coefficients are given by

$$\langle a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)\rangle = f_{\rm nl}B(l_1,l_2,l_3)\delta_{\vec{l}_1+\vec{l}_2+\vec{l}_3}$$

$$\langle a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)a(\vec{l}_4)\rangle = \tau_{\rm nl}T(\vec{l}_1,\vec{l}_2,\vec{l}_3,\vec{l}_4)\delta_{\vec{l}_1+\vec{l}_2+\vec{l}_3+\vec{l}_4}$$

* We can construct an estimator as a weighted sum, i.e.

$$\widehat{f_{\rm nl}} = N^{-1}\sum_{\vec{l}_1,\vec{l}_2,\vec{l}_3} a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)W(\vec{l}_1,\vec{l}_2,\vec{l}_3)$$

* We optimize this estimator by requiring the signal-to-noise to be maximized

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Estimators for $f_{\rm nl}$ and $\tau_{\rm nl}$

* Maximizing the S/N gives an inverse-variance weighted sum:

$$\widehat{f_{\rm nl}} = \frac{1}{N} \sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3=0} \frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)B(l_1,l_2,l_3)}{C_{l_1}C_{l_2}C_{l_3}}$$

$$\widehat{\tau_{\rm nl}} = \frac{1}{N} \sum_{\vec{l}_1+\vec{l}_2+\vec{l}_3+\vec{l}_4=0} \frac{a(\vec{l}_1)a(\vec{l}_2)a(\vec{l}_3)a(\vec{l}_4)T(\vec{l}_1,\vec{l}_2,\vec{l}_3,\vec{l}_4)}{C_{l_1}C_{l_2}C_{l_3}C_{l_4}}$$

* What are the statistics of these estimators? Usually we assume the central limit theorem applies…

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Statistics of the estimators

$$\widehat{f}_{\mathrm{nl}} = \frac{1}{N} \sum_{\vec{l}_1 + \vec{l}_2 + \vec{l}_3 = 0} \frac{a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3) B(l_1, l_2, l_3)}{C_{l_1} C_{l_2} C_{l_3}}$$

$$\widehat{\tau}_{\mathrm{nl}} = \frac{1}{N} \sum_{\vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \vec{l}_4 = 0} \frac{a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3) a(\vec{l}_4) T(\ldots, \vec{l}_3, \vec{l}_4)}{C_{\ldots} C_{\ldots}}$$

Central limit theorem does not apply!

* An observation gives .... measurements
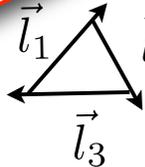
* For $\widehat{f}_{\mathrm{nl}}$ we .... gles , N_{pix}^2 of these

* For $\widehat{\tau}_{\mathrm{nl}}$ we ..... over all quadrilaterals , N_{pix}^3 of these

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Examples of non-Gaussian PDFs

✳ At its core, the bispectrum $f_{\rm nl}$ estimator is a weighted sum of the product of three Gaussian random variables:

$$\widehat{A} = \sum_{i,j,k} W_{i,j,k} a_i a_j a_k$$

✳ The simplest case is $W_{i,j,k} = 1$

$$\widehat{A} = \sum_{i,j,k} a_i a_j a_k$$

$$= \left(\sum_i a_i \right)^3$$

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$$= \left(\sum_i a_i \right)^3$$

$$99.7\% \text{ C.L.} \rightarrow \quad \widehat{A} \gtrsim 7\sigma_{\widehat{A}}$$

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Computationally intensive

* Simulations of a non-Gaussian CMB sky are computationally intensive

* An efficient realization of a non-Gaussian CMB sky takes 20 minutes (Elsner and Wandelt) on a GHz processor

* 'Fast estimators' (Komatsu, Spergel, and Wandelt) allow for the estimator to be computed in $\sim 100 \times \ell_{\max}^3$ operations

* Which means a single realization takes at least $30 \left(\dfrac{\ell_{\max}}{1000} \right)^3$ m

* 1000 realizations takes 500 CPU-hours!

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Computationally intensive

* 1000 realizations takes at least 500 CPU-hours!

* This is good enough to determine the variance of the estimator, but not to determine the shape of the PDF

* For a non-Gaussian process, 1000 realizations gives the following histogram:

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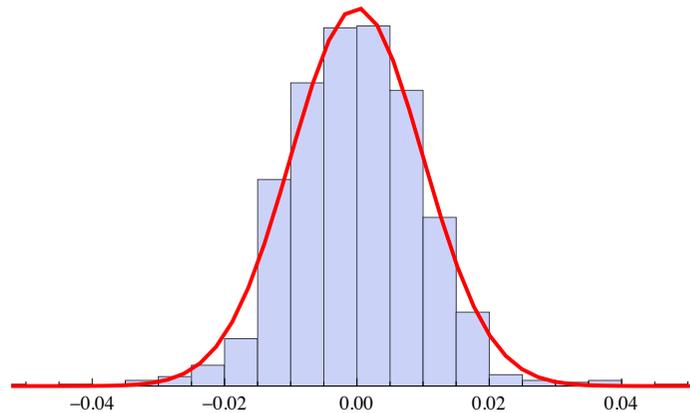
# Computationally intensive

* 1000 realizations takes at least 500 CPU-hours!

* This is good enough to determine the variance of the estimator, but not to determine the shape of the PDF

* For a non-Gaussian process, 1000 realizations gives the following histogram; for 10^5 realizations we find this (!!):

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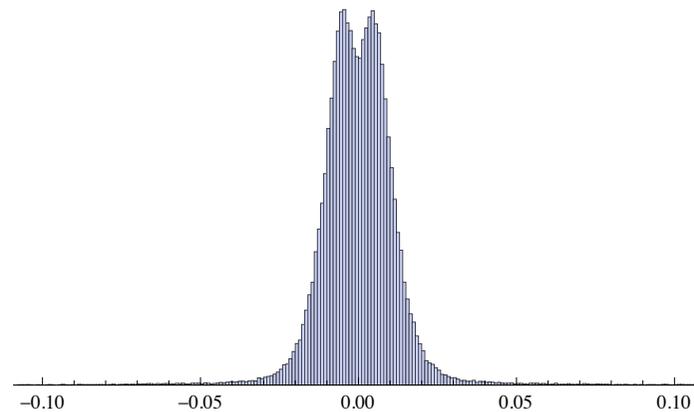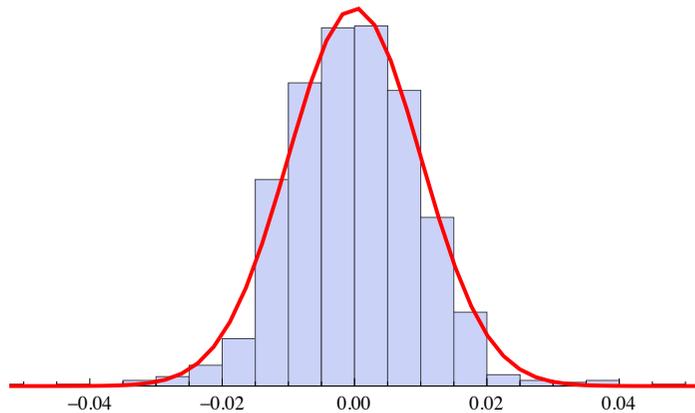
Full shape of the PDF

* First investigate the shape of the PDF for $P_f\left(\widehat{f}_{\mathrm{nl}}; f_{\mathrm{nl}} = 0, l_{\max}\right)$

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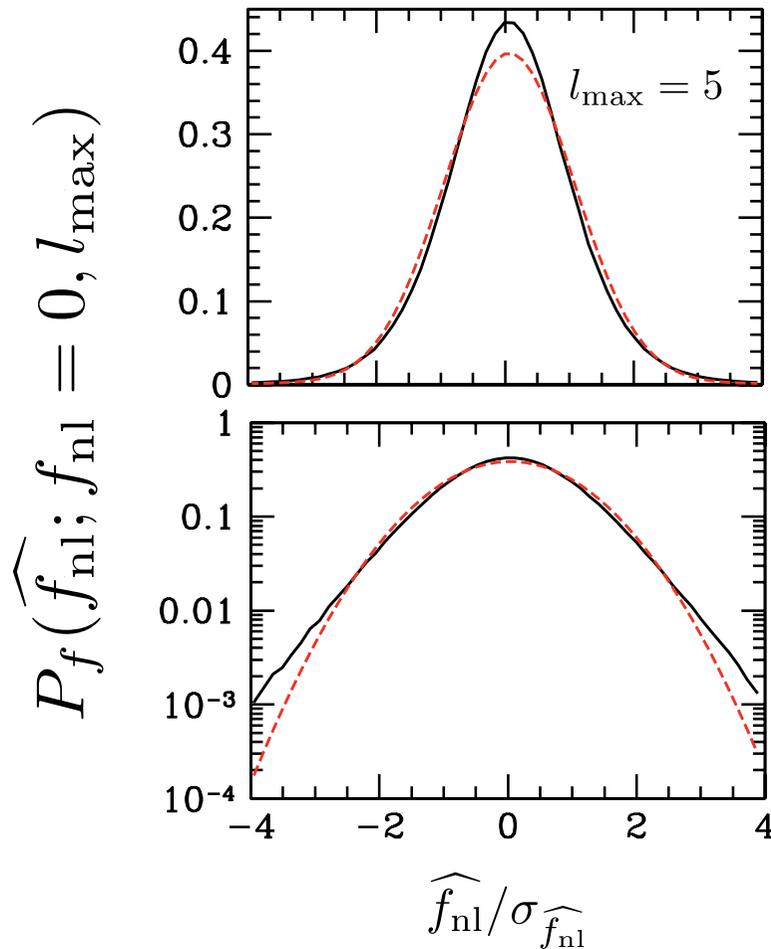
* **Doing the same for** $P_\tau(\widehat{\tau_{\mathrm{nl}}}; f_{\mathrm{nl}} = 0, l_{\max})$**, we find it is** *highly non-Gaussian*:
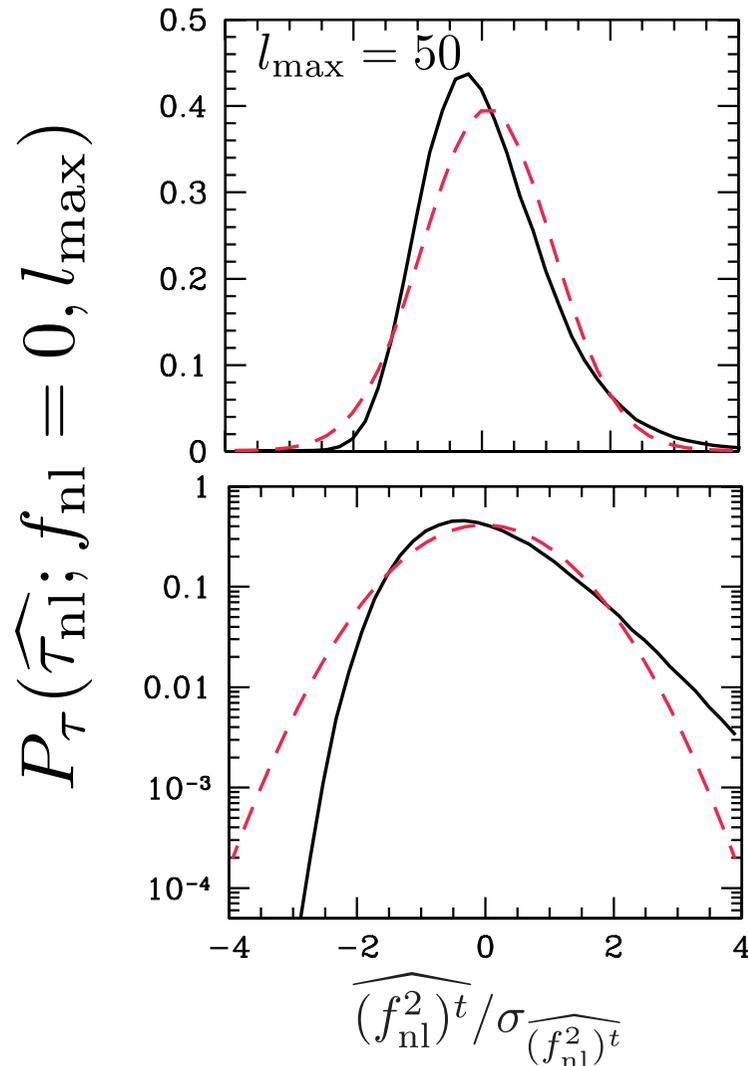
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Full shape of the PDF

✱ For $f_{\rm nl} \neq 0$ the non-Gaussianity in the map imparts *additional* non-Gaussianity to the bispectrum PDF and trispectrum PDF

✱ For $l_{\rm max} \simeq 3000$, $f_{\rm nl} = 30$, and $\tau_{\rm nl} = (f_{\rm nl})^2 = 900$
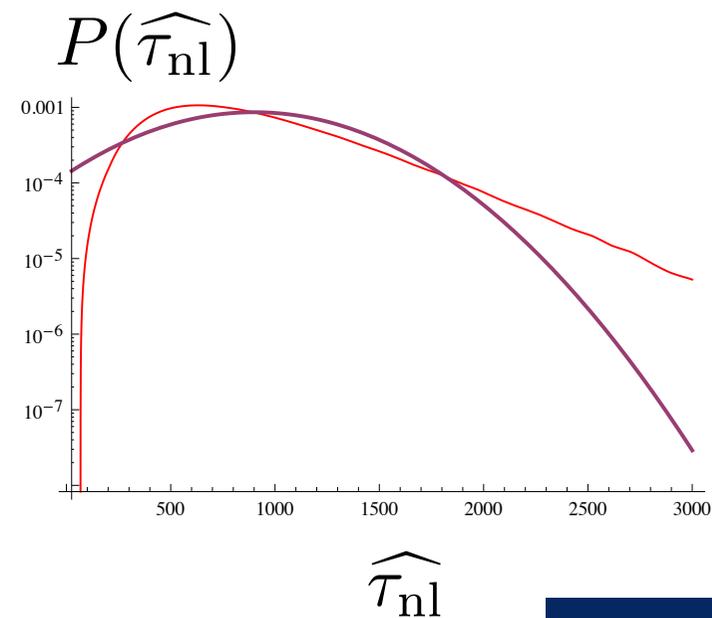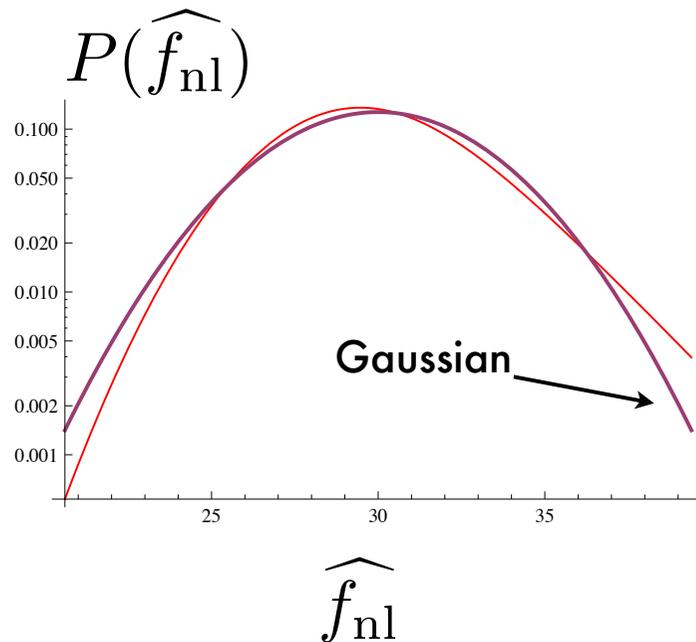
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Full shape of the PDF

✳ For $f_{\rm nl} \neq 0$ the non-Gaussianity in the map imparts *additional* non-Gaussianity to the bispectrum PDF and trispectrum PDF

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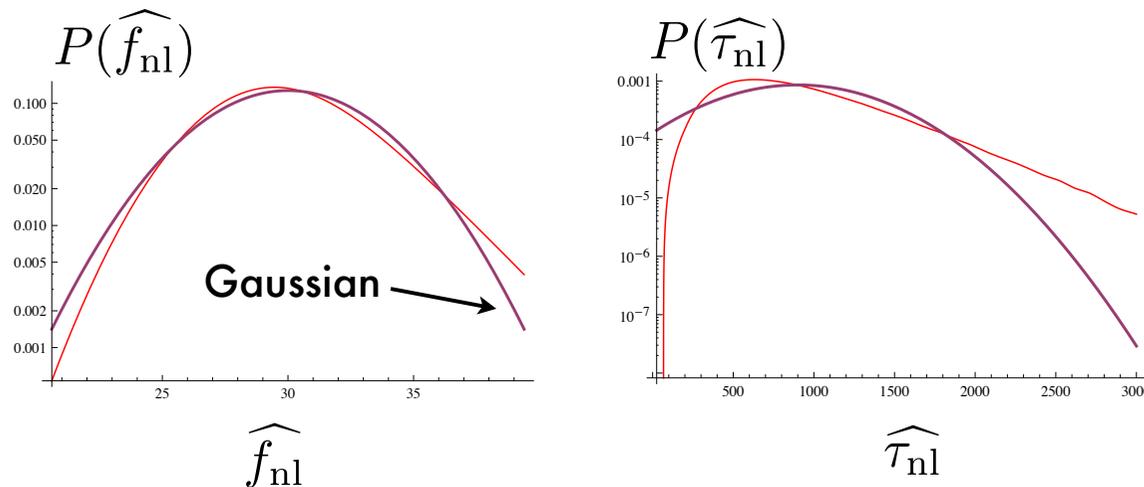
✳ In addition, the variance of these estimators depends on the value of $f_{\rm nl}$ and $\tau_{\rm nl}$ ...

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Evolution of the variance

✳ The fact that the variance depends on $f_{\rm nl}$ and $\tau_{\rm nl}$ is easy to see:

$$\widehat{f_{\rm nl}} = \frac{1}{N} \sum_{\vec{l}_1 + \vec{l}_2 + \vec{l}_3 = 0} \frac{a(\vec{l}_1) a(\vec{l}_2) a(\vec{l}_3) B(l_1, l_2, l_3)}{C_{l_1} C_{l_2} C_{l_3}}$$

$$a(\vec{l}) = \bar{a}(\vec{l}) + f_{\rm nl} \delta a^2(\vec{l})$$

$$\widehat{f_{\rm nl}} = F_0 + f_{\rm nl} F_1 + f_{\rm nl}^2 F_2 + f_{\rm nl}^3 F_3$$

$$\langle \widehat{f_{\rm nl}}^2 \rangle = \sum_{i,j} \langle F_i F_j \rangle (f_{\rm nl})^{i+j}$$

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Evolution of the variance

* Our calculations show that the variances of these estimators scale with $l_{\max}$ as:

$$\sigma^2_{f_{\mathrm{nl}}} = \frac{1}{72 A l^2_{\max} \ln(l_{\max})} + \frac{f^2_{\mathrm{nl}}}{2 \ln^3(l_{\max})}$$

$$\sigma^2_{\tau_{\mathrm{nl}}} = \frac{1.74 \times 10^{-2}}{A^2 l^4_{\max}} + \frac{0.028 \tau_{\mathrm{nl}}}{A l^2_{\max}} + 0.23 \tau^2_{\mathrm{nl}}$$

* Now we have everything we need to evaluate the significance of a hypothetical detection...

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Hypothetical detection:

* Suppose we have an experiment with $l_{\max} = 3000$ and measure $\langle \widehat{f_{\mathrm{nl}}} \rangle = 30$ then we have

$$f_{\mathrm{nl}} = 30^{+7.25}_{-5.5} \ (95\% \ \mathrm{C.L.})$$

* We now have strong evidence that single-field inflation is wrong– now can we use the inequality, $\tau_{\mathrm{nl}} < 1/2(f_{\mathrm{nl}})^2$ to test multi-filed inflation?

* Assume we measure $\langle \widehat{\tau_{\mathrm{nl}}} \rangle = 0$ then the 95% C.L. upper limit is

$$\tau_{\mathrm{nl}} < 250 \ (95\% \ \mathrm{C.L.})$$

* So the inequality is holds at > 95% C.L. and we could conclude that most multi-field inflation models are at odds with observations!

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What could have gone wrong?

✳ Our full calculations give

$$f_{\rm nl} = 30^{+7.25}_{-5.5} \ (95\% \ {\rm C.L.})$$

$$\boxed{\tau_{\rm nl} < 250 \ (95\% \ {\rm C.L.})}$$

✳ If we did not take into account the non-Gaussian shape of the PDF then we would have concluded

$$f_{\rm nl} = 30^{+7.5}_{-5.4} \ (95\% \ {\rm C.L.})$$

$$\boxed{\tau_{\rm nl} < 1000 \ (95\% \ {\rm C.L.})}$$

✳ If, in addition, we did not take into account how the variance depends on the amplitudes we would have concluded

$$f_{\rm nl} = 30 \pm 2.8 \ (95\% \ {\rm C.L.})$$

$$\boxed{\tau_{\rm nl} < 90 \ (95\% \ {\rm C.L.})}$$

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Conclusions

* Small-scale CMB observations show an anomalously large value for $N_{\rm eff}$

* Explored how observations probe the interaction/clustering properties of this anomalous radiative energy density

* We found that with an expanded parameterization the data is still at odds with the standard neutrino sector at > 95 % CL *and* consistent with a non-interacting fluid

* The central limit theorem does not apply to non-Gaussian estimators– PDFs of these estimators may, themselves, be non-Gaussian

* The effect on the bispectrum estimator is small; the effect on the trispectrum estimator is large– must be included when calculating the significance of a measurement

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