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Monopole-vortex complex
and
dual gauge symmetries from flavor

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Main ideas and themes

- Confinement as a dual superconductivity (dual Higgs phase) - of non-Abelian variety?

monopole condensation

- The subtle interplay between the global flavor symmetry and the strong gauge dynamics (soliton monopoles and vortices)

(cfr. Jackiw-Rebbi, charge fractionalization, Witten)

- Dual gauge symmetry as a new manifestation of the global flavor symmetry

- Many hints and evidences

Seiberg's duality in N=1 SQCD; Kutasov's duality; Seiberg-Witten curves for N=2 gauge theories; Many new N=2 dualities (SCFT); ...

- Physics of the r-vacua

Non-Abelian magnetic monopoles in N=2 SQCD
Seiberg-Witten solutions; Tachikawa-Terashima

Setting

- Softly broken N=2 G theory with N_F matter multiplets, G=SU(N), SO(N),USp(2N)
- Hierarchical gauge symmetry breaking $G \to H \to 1$, at $v_1 \gg v_2$

monopoles of $\prod_2 (G/H)$ $\Leftrightarrow$ vortices of $\prod_1 (H)$

Plan of the talk

I. Non-Abelian monopole-vortex complex

Cipriani, Dorigoni, Gudnason, Konishi, Michelini, PRD (2011)

Konishi, Michelini, Ohashi PRD (2010)

II. Nature and fluctuations
of the vortex zeromodes:
the GNO duality (non-Abelian vortices)

Hanay-Tong, Auzzi-Bolognesi-Evslin-Konishi-Yung (2003)
Shifman, Gorsky, Yung (2004)

Gudnason, Jiang, Konishi
JHEP 2010

Pisa, TiTech, Cambridge, Minnesota groups
2003-2011

III. Non-Abelian monopoles

1974 - 2011

IV. Summary: Flavor to Dual Gauge Symmetry

1. Monopole-vortex complex

Monopole-vortex connection

Hierarchical symmetry breaking

$$G \xrightarrow{v_1} H \xrightarrow{v_2} \mathbb{1} \qquad\qquad v_1 \gg v_2,$$

monopole vortex

- Apparent paradox (no monopoles, no vortices ???!!!) ⇨ monopoles are confined by vortices; vortices end at monopoles
- Topology and symmetry connect monopoles and vortices
- Non-Abelian vortices ⇨ non-Abelian monopoles

A 35-year old problem, possibley relevant to quark confinement

- Study e.g.,

$$SU(N+1) \xrightarrow{v_1} U(N) \xrightarrow{v_2} \mathbb{1}\ , \qquad v_1 \gg v_2$$

Auzzi, Bolognesi, Evslin, Konishi '04

in more detail

Cipriani, Dorigoni, Gudnason, Konishi, Micheline '11

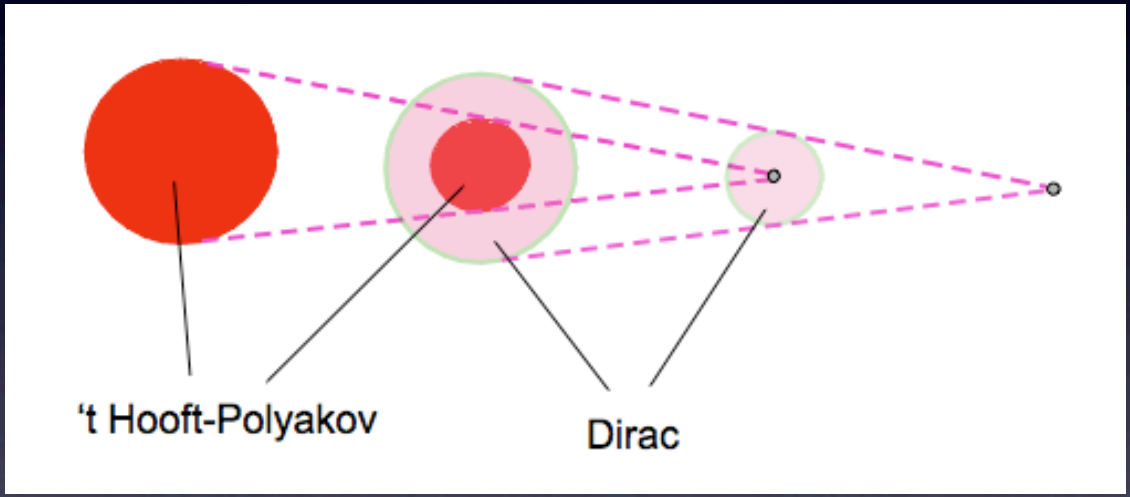
Homotopy-group map

$$G \xrightarrow{v_1} H \xrightarrow{v_2} \mathbb{1} \qquad\qquad v_1 \gg v_2,$$

Vortex ! (but also monopole Wu-Yang)

Exact sequence:

$$\cdots \to \pi_2(G) \to \pi_2(G/H) \to \pi_1(H) \to \pi_1(G) \to \cdots$$



$\pi_1(G) = \pi_1(H) / \pi_2(G/H)$

- $\pi_2(G) = 1 \;\Rightarrow$ Regular monopoles confined by vortices

{
- $\pi_1(G) = 1 \;\Rightarrow$ All vortices "end" at regular monopoles e.g. SU(N)
- $\pi_1(G) = Z_2 \;\Rightarrow$ k=2 vortices "end" at regular monopoles!

't Hooft G=SO(3); H=U(1)

cfr., SO(N) k=1 vortices are there: confine Dirac monopoles

The model (softly broken SU(N+1) $\mathcal{N}$=2 susy QCD with N_F =N quarks)

$$\mathcal{L} = \frac{1}{4g^2}F_{\mu\nu}^2 + \frac{1}{g^2}|\mathcal{D}_\mu\phi|^2 + |\mathcal{D}_\mu Q|^2 + \left|\mathcal{D}_\mu \bar{\tilde{Q}}\right|^2 + \mathcal{L}_1 + \mathcal{L}_2,$$

Φ=adj scalar;
Q=squarks

$$\begin{aligned} \mathcal{L}_2 &= -g^2|\mu\,\phi^A + \sqrt{2}\,\tilde{Q}\,t^A Q|^2 - \tilde{Q}\,[m+\sqrt{2}\phi]\,[m+\sqrt{2}\phi]^\dagger\,\tilde{Q}^\dagger \\ &- \; Q^\dagger\,[m+\sqrt{2}\phi]^\dagger\,[m+\sqrt{2}\phi]\,Q\;, \end{aligned}$$

flavor →

the vacuum
(class. r=N vac)

$$\langle\Phi\rangle = v_1 \begin{pmatrix} 1_{N\times N} & 0_{N\times 1} \\ 0_{1\times N} & -N \end{pmatrix}, \qquad \langle Q\rangle = \langle\tilde{Q}\rangle = v_2 \begin{pmatrix} 1_{N\times N} \\ 0_{1\times N} \end{pmatrix}$$

color ↑

take equal masses
$m_i = m$

$$v_1 \equiv -\frac{m}{\sqrt{2}}, \qquad v_2 \equiv \sqrt{(N+1)\,m\,\mu}\;.$$

$$|m| \gg |\mu| \gg \Lambda, \qquad \therefore \quad |v_1| \gg |v_2|\;.$$

$\Delta L = \mu\, \Phi^2 \;\; |_{\theta\theta}$

- Terms with μ (breaks $\mathcal{N}$=2 susy to $\mathcal{N}$=1)
  - ➪ v_2 : breaks the low-energy gauge symmetry
  - ➪ interaction terms connecting monopole and vortex
  - ➪ known BPS monopole and vortex solns in the limit μ →0,

- Solve the eqns with μ ➪

• Gauge symmetry broken at two hierarchically different scales

$$SU(N+1) \xrightarrow{\langle \Phi \rangle \sim m} SU(N) \times U(1) \xrightarrow{\langle Q \rangle \sim \sqrt{m\mu}} 1$$

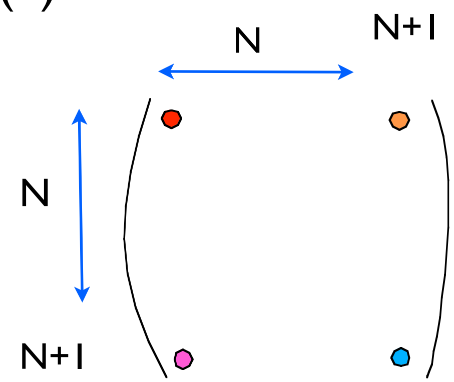
• Global flavor symmetry unbroken

$$SU(N)_{color} \otimes SU(N)_{flavor} \xrightarrow{\langle Q \rangle} SU(N)_{C+F}$$

color-flavor locked phase

⇨ <u>Monopoles</u> in $SU(N+1)/U(N) \supset SU(2)/U(1)$

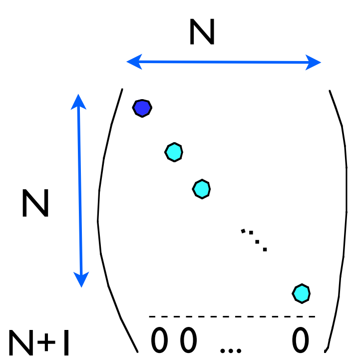
$\Phi,\ A_i \sim$



SU(2) living in
(1, N+1) subplane

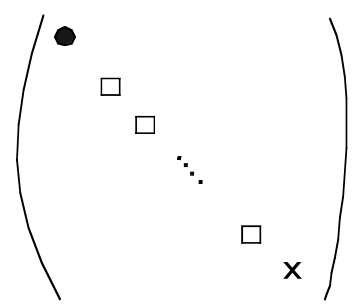
⇨ <u>Vortices</u> in $U(N) \supset U(1)$

$Q \sim$



winding of the squark
in (1,1) corner

$\Phi,\ A_i \sim$



$\{q, A, \phi\}^{MV} =$

$$
\begin{aligned}
q &= \begin{pmatrix} q_1(\rho,z) & \\ & q_2(\rho,z)\mathbf{1}_{N-1} \end{pmatrix} \ ; \\
A_\rho &= \frac{\cos\theta}{r}(S_2\cos\varphi - S_1\sin\varphi)\,\Delta(\rho,z) \ ; \\
A_\varphi &= \frac{1}{\rho}\left[\frac{f(\rho,z)}{N}\begin{pmatrix} 1 & & \\ & \mathbf{1}_N & \\ & & -N \end{pmatrix} + \frac{f_{\mathrm{NA}}(\rho,z)}{N}\begin{pmatrix} N-1 & & \\ & -\mathbf{1}_{N-1} & \\ & & 0 \end{pmatrix} - \sin\theta(S_1\cos\varphi + S_2\sin\varphi)\,\Delta(\rho,z)\right] \\
A_z &= -\frac{\sin\theta}{r}(S_2\cos\varphi - S_1\sin\varphi)\,\Delta(\rho,z) \ ; \\
\phi &= \left(v_1 + \frac{\lambda(\rho,z)}{\sqrt{2N(N+1)}}\right)\begin{pmatrix} 1 & & \\ & \mathbf{1}_N & \\ & & -N \end{pmatrix} + \frac{\lambda_{\mathrm{NA}}(\rho,z)}{\sqrt{2N(N-1)}}\begin{pmatrix} N-1 & & \\ & -\mathbf{1}_{N-1} & \\ & & 0 \end{pmatrix} \ . \qquad (3.48)
\end{aligned}
$$

- appropriate b. c.

  (i.e., reduces to the standard BPS monopole and vortex in the appopriate regions)

- breaks $SU(N)_{C+F} \Rightarrow SU(N-1)\times U(1)$

General remarks

- Smooth monopole-vortex complex needs :

  (i) Monopole and vortex orientation (in color) be correlated

  (ii) None of the fields "wind"

  (must work in the "singular gauge")

- Dirac string of the monopole solution hidden inside the vortex core

  (i) Better said, it simply matches with the gauge field singularity in the vortex core

  (ii) Innocuous as $|D_i q|^2 \sim |A_i|^2 \; |q|^2$ and $q = 0$ along the core

  (iii) Relative spatial orientation fixed

- Search for the minimum energy configuration under the constraint that the monopoles and antimonopole centers are fixed

Numerical solutions:
for SU(2)→U(1)→1 embedded in SU(N+1)→U(N)→1

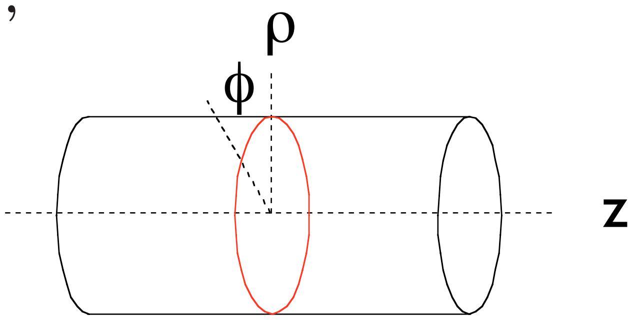
exact for g (SU(N)) = g (U(1))

$$\mathcal{L} = -\frac{1}{4g^2}(F^a_{\mu\nu})^2 + \frac{1}{g^2}|D_\mu \phi^a|^2 + |D_\mu q|^2$$
$$- \frac{g^2}{8}\left|-\xi\delta^{a3} + \mu\lambda^a + q^\dagger \tau^a q\right|^2 - \left|\left[m - m\tau^3 + \frac{1}{\sqrt{2}}\lambda^a\tau^a\right] q\right|^2$$

Equations of motion

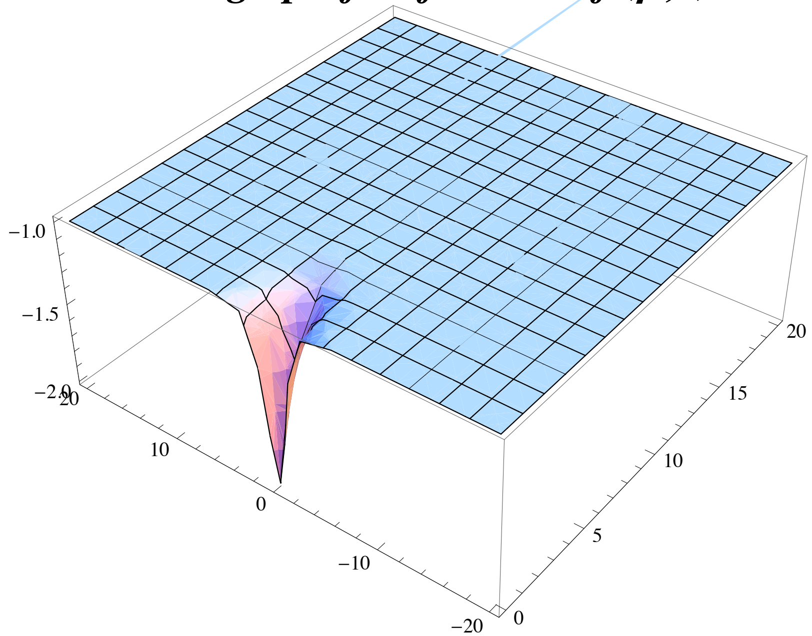
Ansatz

$$A_\rho = \frac{z}{\rho^2+z^2}\,(\tau_2\cos\varphi - \tau_1 \sin\varphi)\,\frac{f(\rho,z)-1}{2}\ ;$$
$$A_z = \frac{\rho}{\rho^2+z^2}\,(\tau_1\sin\varphi - \tau_2 \cos\varphi)\,\frac{f(\rho,z)-1}{2}\ ;$$
$$A_\varphi = -\frac{1}{\sqrt{\rho^2+z^2}}\,(\tau_1\cos\varphi + \tau_2 \sin\varphi)\,\frac{f(\rho,z)-1}{2} + \tau_3\,\frac{1}{2\rho}\ell(\rho,z)\ ;$$
$$\phi^a = -\sqrt{2}\,m\,\delta^{a3} + \lambda^a\ , \qquad \lambda^a = \delta^{a3}\,s(\rho,z)\ ;$$
$$q = \begin{pmatrix} q_1(\rho,z) \\ 0 \end{pmatrix}\ .$$

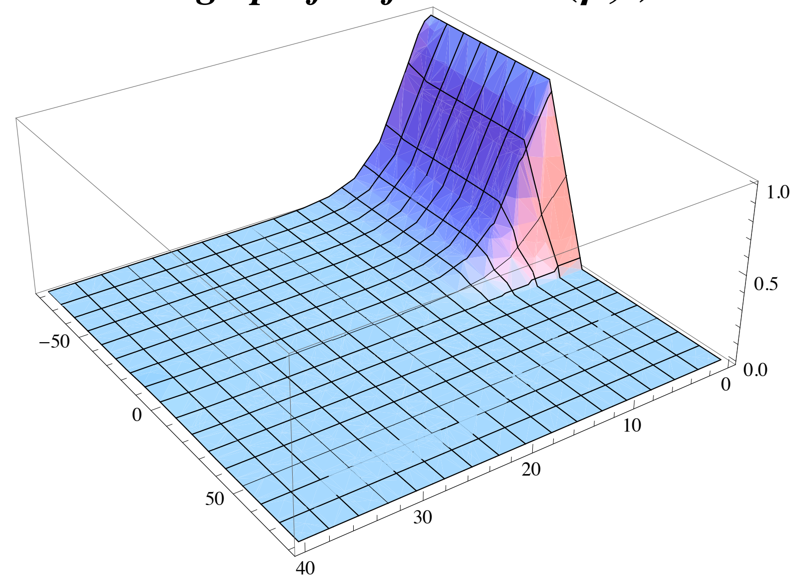


Equations for the profile functions, f, l, s, q

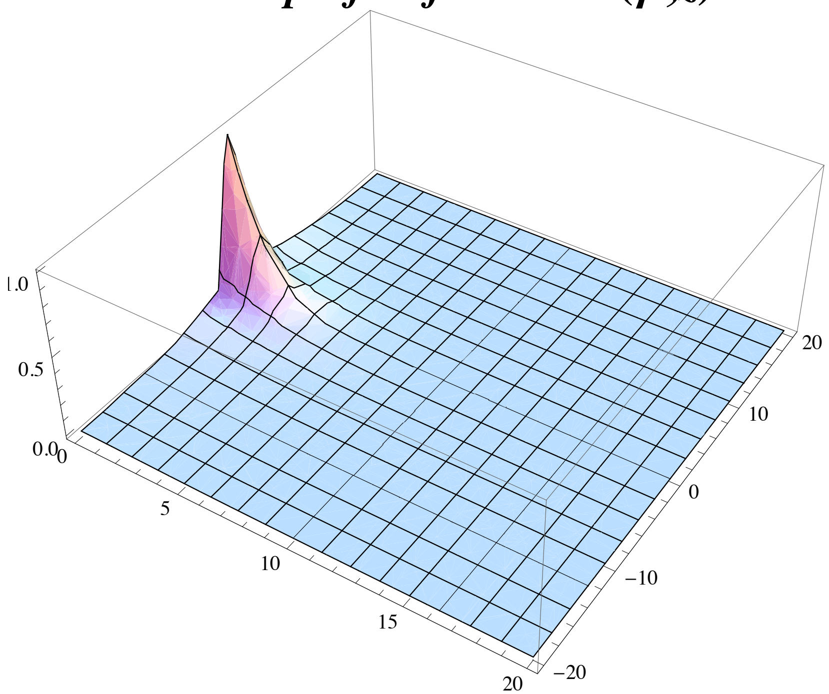
Gauge profile function $f(\rho,z)$



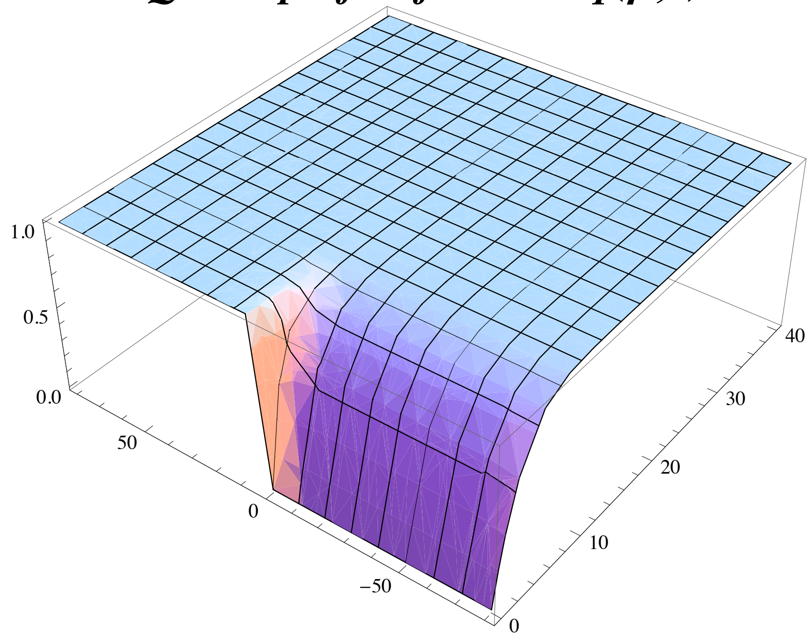
Gauge profile function $l(\rho,z)$



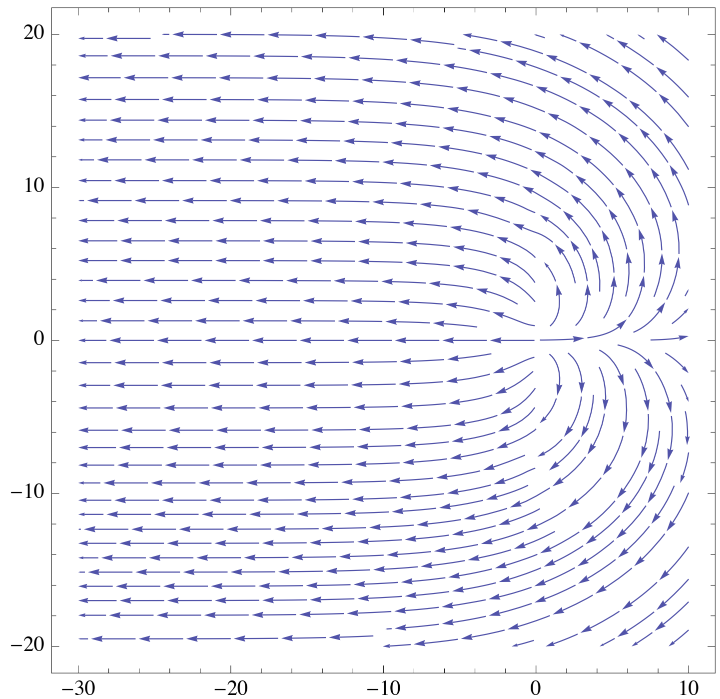
Scalar profile function $s(\rho,z)$



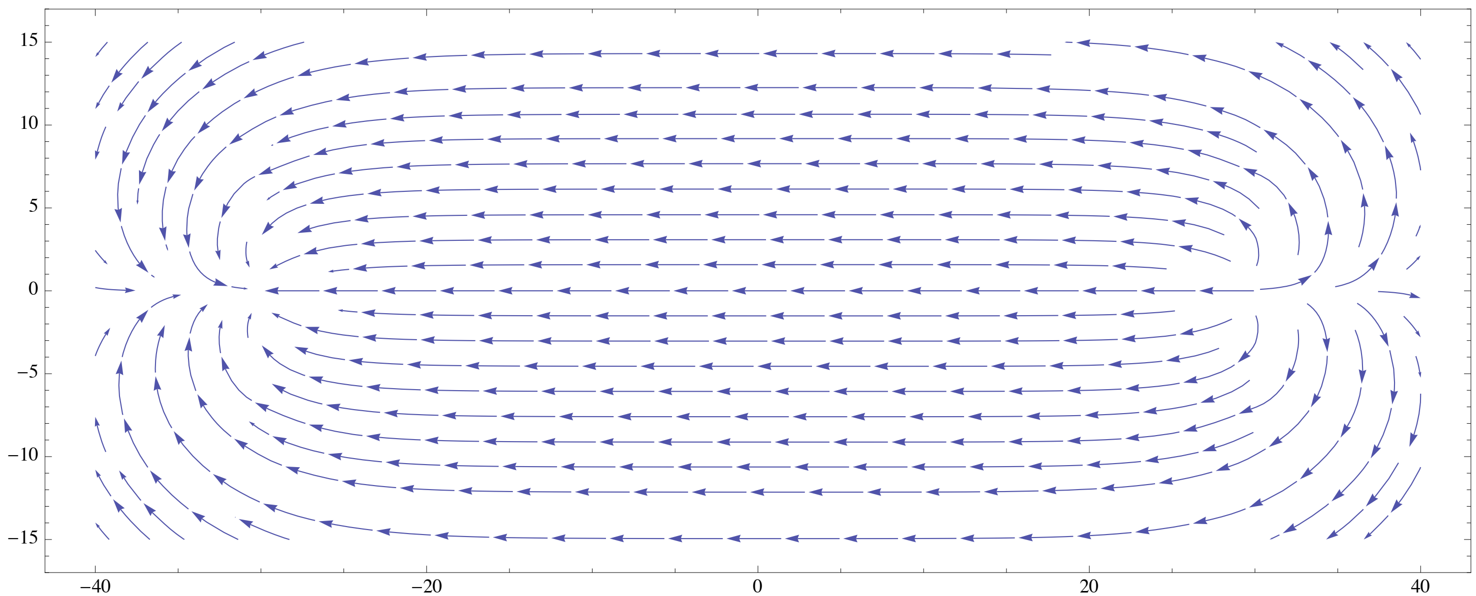
Quark profile function $q(\rho,z)$

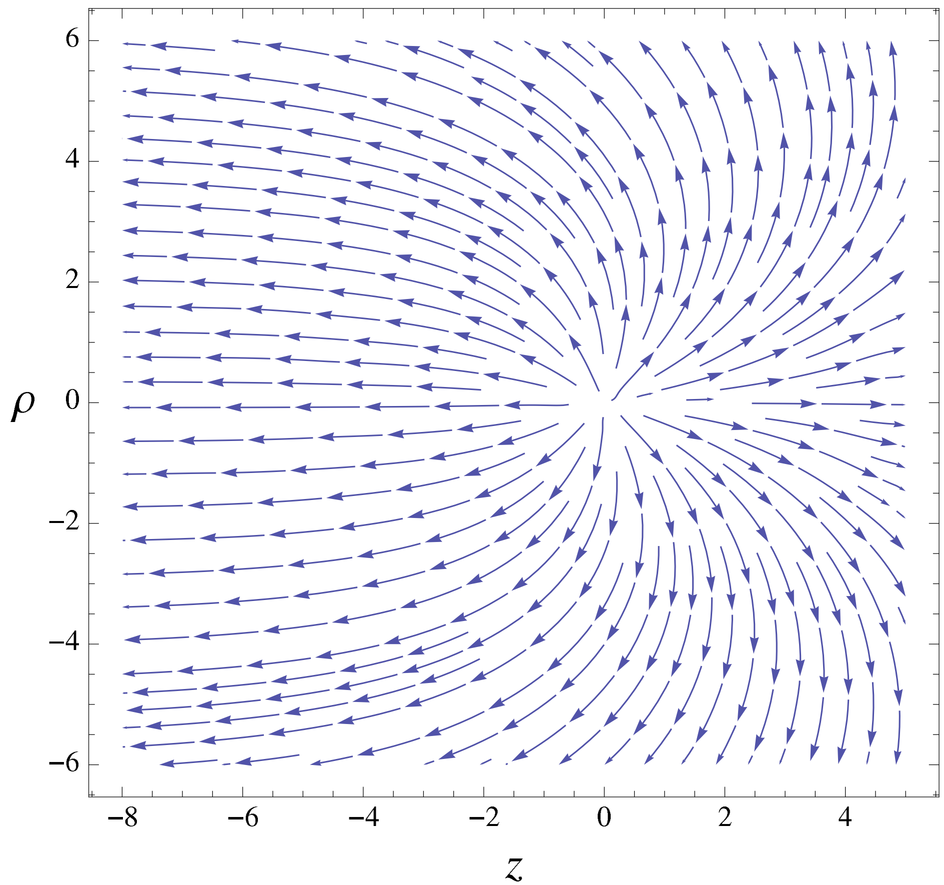


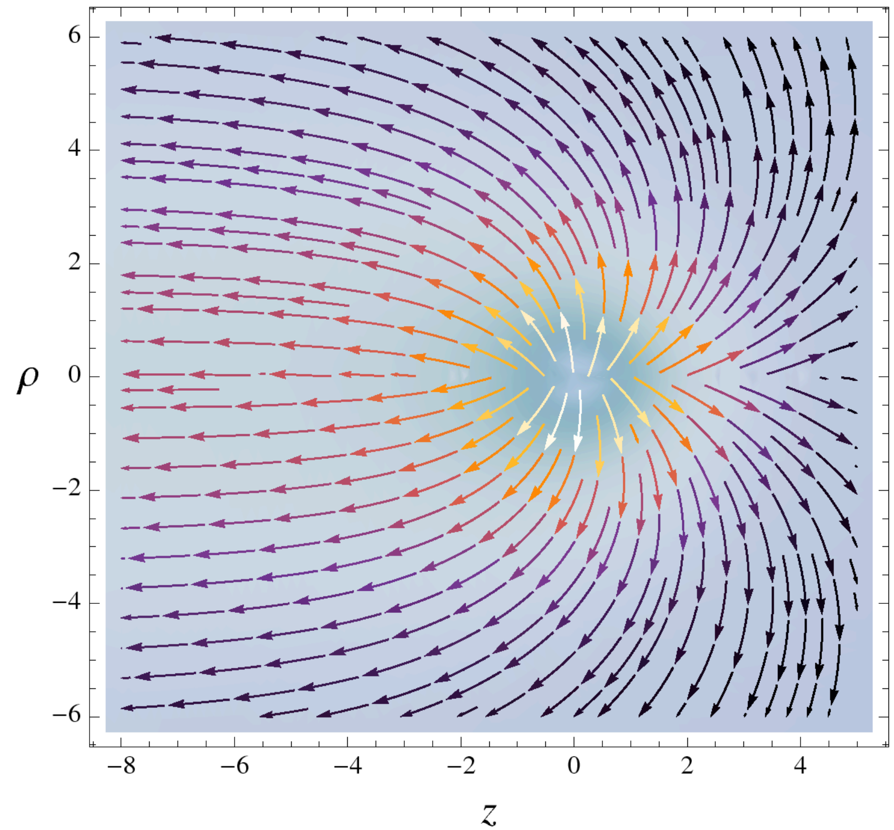
Color-flavor magnetic flux in the monopole-vortex complex



μ=0.1; m=1;
g=1;







direction of the color magnetic fields near the monopole center

the same as the left figure, but with the field intensity also shown

Analytic results

- MV complex in SU(2)→U(1)→1 system
  studied recently in the limit, [monopole=point; vortex = thin line], with duality transformations explicitly performed

Chatterjee-Lahiri JHEP '10

- MV complex in SU(2)→U(1)→1 system
  in a θ vacuum of SU(2);
  Dual system solved in the presence of a static monopole

Konishi-Michelini-Ohashi PR '10

Witten's effect (U(1) elec. charge of the monopole) visible only near the monopole center

$$E_i = F_{0i} = \alpha\, B_i^{(mon)}, \qquad B_i = \frac{1}{2}\epsilon_{ijk}F_{jk} = B_i^{(mon)} + B^{(vor)}\delta_i^3$$

$$B_i^{(mon)} = \frac{n}{g}\partial_i G(\mathrm{r}), \qquad B^{(vor)} = \frac{n}{g}\, m^2 \int_{-\infty}^{0} dz'\, G(x, y, z - z')$$

$$G(\mathrm{r}) = \frac{4\pi}{-\Delta + m^2}\,\delta^3(\mathrm{r}) = \frac{e^{-mr}}{r}$$

$$m \equiv \frac{g\, v_2}{\sqrt{2}}\,. \qquad \alpha \equiv \frac{\theta g^2}{8\pi^2}$$

Remarks:

cfr. real-world mesons !

- Non BPS: Born-Oppenheimer type approximation

- The whole MV complex breaks $SU(N)_{C+F}$ (exact degeneracy under):

$$q^U = \begin{pmatrix} U & \\ & 1 \end{pmatrix} q\, U^{-1}\,, \qquad (\phi^U, A_i^U) = \begin{pmatrix} U & \\ & 1 \end{pmatrix} (\phi, A_i) \begin{pmatrix} U^{-1} & \\ & 1 \end{pmatrix}$$

$\Rightarrow$ orientational zeromodes living in $SU(N)/U(N-1) \sim CP^{N-1}$

- The degeneracy between e.g., (1 N+1) and (2 N+1) monopoles, is broken by the squark vev (cfr. old difficulties of non-Abelian monopoles)
  Demise of the naïve "non-Abelian monopole" (no multiplet of $SU(N) \subset SU(N+1)$)

- Resurrection of an exact $SU(N)_{C+F}$ symmetry (continuous CP^{N-1} degeneracy) under the simultaneous CF rotations of the whole complex $\Rightarrow$

- A new exact (magnetic) continuous symmetry for the monopole; under which monopole ~ <u>N</u> of SU(N) :
the origin of the dual SU(N) group

(cfr. Jackiw-Rebbi effects at $\sim 1 / v_1$)

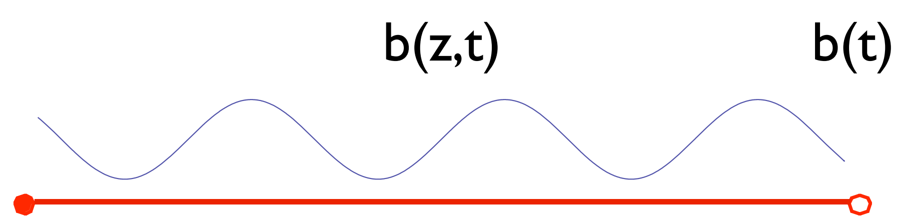
Summary of Part I

- Gauge symmetry completely (hierarchically) broken

$$SU(N+1) \xrightarrow{\langle\Phi\rangle \sim m} SU(N) \times U(1) \xrightarrow{\langle Q\rangle \sim \sqrt{m\mu}} 1$$

- Global flavor SU(N) symmety unbroken (no Nambu-Goldstone bosons in 4D)

- Soliton monopole-vortex complex breaks it to SU(N-1)xU(1)
  ⇨ orientational zeromodes (can fluctuate)



endow the monopole
with fluctuating CP^{N-1} modes

~ <u>N</u> of a new (dual) SU(N) :
Origin of the dual gauge group

- ⇨ Study dynamics of b(z,t) in the low-energy approximation for general gauge group: non-Abelian vortices

II. Vortex zeromodes: Nature of its fluctuation and GNO duality

Non-Abelian vortices

Hanany-Tong, '03
Auzzi-Bolognesi-Evslin-Konishi-Yung.

Def: Vortex solutions with continuous (non-Abelian) moduli

Natural generalizations of <u>ANO vortex</u>

Shifman-Yung, ... (Minnesota).
Eto-Nitta-Ohashi-Sakai- ... (TiTech, Tokyo).
Tong, (Cambridge).
Pisa group, '03-'11

- Global (flavor) symmetry: e.g. U(N) theory with $N_f = N$ "squarks"

- "Color-flavor locked" phase

$$\langle q \rangle = v \, \mathbb{1}_{N \times N}$$

- Local gauge symmetry broken (Higgs)
  ⇨ vortex solutions

flavor

$$(q)^i_\alpha = \begin{pmatrix} q_1^{(1)} & q_1^{(2)} & \cdots & & q_1^{(N)} \\ q_2^{(1)} & q_2^{(2)} & \vdots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \\ \vdots & \vdots & & & \vdots \\ q_N^{(1)} & q_N^{(2)} & \cdots & 0 & q_N^{(N)} \end{pmatrix}$$

color

- Global symmetry $G_F = G_{C+F} = SU(N)$ unbroken
- Individual vortex breaks it
  ⇨ Orientational zeromodes in $SU(N)/U(N-1) = CP^{N-1}$
  ⇨ They can fluctuate in (z,t)

The models (with G' x U(1), G' = SU(N), SO(N), USp(2N),... gauge groups with appprop flavor)

$$\mathcal{L} = -\frac{1}{4e^2} F^0_{\mu\nu} F^{0\mu\nu} - \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + (\mathcal{D}_\mu q_f)^\dagger \mathcal{D}^\mu q_f - \frac{e^2}{2} \left| q_f^\dagger\, t^0 q_f - \frac{v^2}{\sqrt{4N}} \right|^2 - \frac{g^2}{2} \left| q_f^\dagger\, t^a q_f \right|^2$$

- Ignore the massive monopoles of $G \to G' \times U(1)$; $\phi = \langle \phi \rangle$

  ⇨ Vortices are BPS

- G' =SU(N) case studied extensively ⇨ Examples of SU(2)xU(1)

- Rich physics and mathematics (general gauge groups, structure of the vortex moduli space -- non-trivial complex manifold; semi-local vortices; fractional vortices; non BPS vortices; interactions and stability; higher-winding vortices; group theory of NA vortices; vortices in high-density QCD; multi-component superconductors)

'03-'11

- Here:

  Nature of the orientational zero modes

  How they transform (GNO duality !); higher-winding cases subtle

  How they fluctuate (Worldsheet effective action)

Methods of analysis

- The standard field equations of motion

  (most physical, and intuitive; standard differential eqs,   Taubes eqs., existence, stability analysis)

- The moduli-matrix

  (vortex moduli as complex manifolds; transformation properties)

- The Kähler-quotient

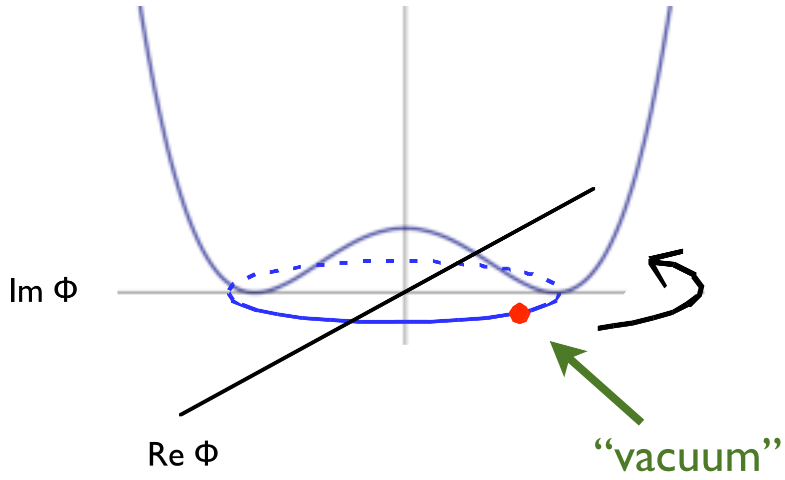
  (group-theoretic aspects)

Effective vortex worldsheet action

A global continuous symmetry broken spontaneously

⇨ a massless (Nambu-Goldstone) particle

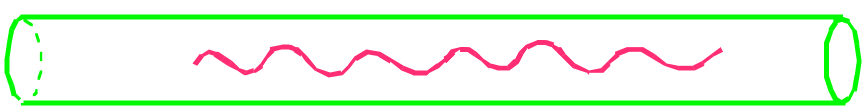
Gudnason, Jiang, Konishi 2010

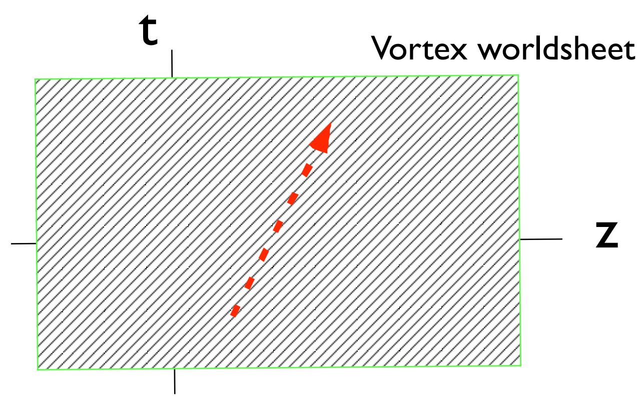


$$V \propto (|\phi|^2 - \xi)^2$$

Nambu-Goldstone modes (Cfr. π mesons of SU(2)×SU(2) /SU(2))

Orientational zeromodes are sort of Nambu-Goldstone modes propagating along the vortex axis only (no symmetry breaking in the bulk)





Need to introduce the gauge field components A_0 , A_3

A naïve guess: $$A_\alpha = -i\,\rho(r)\,U^{-1}\partial_\alpha U$$ → No (massive as well as massless modes)

$$i\,\left(U^{-1}\partial_\alpha U\right) \;\Rightarrow\; i\,\left(U^{-1}\partial_\alpha U\right)_\perp \equiv i\begin{pmatrix} 0 & -X^{-\frac{1}{2}}\partial_\alpha B^\dagger Y^{-\frac{1}{2}} \\ Y^{-\frac{1}{2}}\partial_\alpha B X^{-\frac{1}{2}} & 0 \end{pmatrix}$$

Projection onto the Nambu-Goldstone modes

Delduc, Valent '85

In our case, the scalar q rotates: the correct Ansatz is:

$$A_\alpha = i\,\rho(r)\,U\,\left(U^{-1}\partial_\alpha U\right)_\perp U^{-1}\,, \qquad \alpha = 0, 3 \qquad (※)$$

Gudnason, Jiang, Konishi JHEP 2010

then

$$\mathrm{Tr}\,|\mathcal{D}_\alpha\, q|^2 = -\left[\frac{\rho^2}{2}\left(\phi_1^2+\phi_2^2\right) + (1-\rho)\left(\phi_1-\phi_2\right)^2\right]\mathrm{Tr}\left[\left(U^{-1}\partial_\alpha U\right)_\perp\right]^2$$

$$\frac{1}{g^2}\mathrm{Tr}\,F_{i\alpha}^2 = -\frac{1}{g^2}\left[\left(\partial_r\rho\right)^2 + \frac{1}{r^2}f_{\mathrm{NA}}^2\,(1-\rho)^2\right]\mathrm{Tr}\left[\left(U^{-1}\partial_\alpha U\right)_\perp\right]^2\,,$$

minimiz w.r.t. ρ

$$\Rightarrow \quad S_{1+1} = 2\beta\int dtdz\;\mathrm{tr}\left\{X^{-1}\partial_\alpha B^\dagger Y^{-1}\partial_\alpha B\right\}$$

$$= 2\beta\int dtdz\;\mathrm{tr}\left\{\left(1_N + B^\dagger B\right)^{-1}\partial_\alpha B^\dagger\left(1_N + BB^\dagger\right)^{-1}\partial_\alpha B\right\}$$

where

$$\beta = \frac{2\pi}{g^2}\mathcal{I} \qquad \mathcal{I} = \int_0^\infty dr\;\partial_r\left(f_{\mathrm{NA}}\left[\left(\frac{\phi_1}{\phi_2}\right)^2 - 1\right]\right) = f_{\mathrm{NA}}(0) = 1$$

Construction of $L_{\rm eff}$

$$G' \times U(1)$$

$$\mathcal{L} = -\frac{1}{4e^2}F^0_{\mu\nu}F^{0\mu\nu} - \frac{1}{4g^2}F^a_{\mu\nu}F^{a\mu\nu} + (\mathcal{D}_\mu q_f)^\dagger \mathcal{D}^\mu q_f$$
$$-\frac{e^2}{2}\left|q_f^\dagger t^0 q_f - \frac{v^2}{\sqrt{4N}}\right|^2 - \frac{g^2}{2}\left|q_f^\dagger t^a q_f\right|^2$$

$$G' = SO(2N), USp(2N)$$

f=1,2, ... 2N

(Also for SU(N) / higher-winding vortices)

$$\langle q \rangle = \frac{v}{\sqrt{2N}} 1_{2N}$$

$$q = \begin{pmatrix} e^{i\theta}\phi_1(r)1_N & 0 \\ 0 & \phi_2(r)1_N \end{pmatrix} = \frac{e^{i\theta}\phi_1(r) + \phi_2(r)}{2}1_{2N} + \frac{e^{i\theta}\phi_1(r) - \phi_2(r)}{2}T\ ,$$

$$A_i = \frac{1}{2}\epsilon_{ij}\frac{x^j}{r^2}\left[(1-f(r))\,1_{2N} + (1-f_{\rm NA}(r))\,T\right]\ ,$$

{q,A} leaves U(N) invariant

$$T = \mathrm{diag}\,(1_N, -1_N)$$

boundary conditions

$$\phi_{1,2}(\infty) = \frac{v}{\sqrt{2N}}\ , \quad f(\infty) = f_{\rm NA}(\infty) = 0\ ,$$

$$\phi_1(0) = 0\ , \quad \partial_r\phi_2(0) = 0\ , \quad f(0) = f_{\rm NA}(0) = 1$$

C-S Lin and Y. Yang '10

Vortex of generic orientation (singular gauge)

$$q = U \begin{pmatrix} \phi_1(r) 1_N & 0 \\ 0 & \phi_2(r) 1_N \end{pmatrix} U^{-1}$$

U $\subset$ SO(2N)/U(N), or USp(2N)/UN)

$$A_i = -\frac{1}{2}\epsilon_{ij}\frac{x^j}{r^2}\left[f(r)\, 1_{2N} + f_{\rm NA}(r)\, UTU^{-1}\right]$$

where

$$U = \begin{pmatrix} 1_N & -B^\dagger \\ 0 & 1_N \end{pmatrix} \begin{pmatrix} X^{-\frac{1}{2}} & 0 \\ 0 & Y^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1_N & 0 \\ B & 1_N \end{pmatrix} = \begin{pmatrix} X^{-\frac{1}{2}} & -B^\dagger Y^{-\frac{1}{2}} \\ BX^{-\frac{1}{2}} & Y^{-\frac{1}{2}} \end{pmatrix}$$

U= "reducing matrix" Delduc, Valent '85

and

$$X \equiv 1_N + B^\dagger B\,, \quad Y \equiv 1_N + BB^\dagger$$

B = antisymm NxN for SO(2N);
symm NxN for USp(2N);
1x N-1 for SU(N)

Allow the zeromodes to slowly fluctuate:

$$B = B(x^\alpha)\,, \qquad x^\alpha = (x^3, x^0)$$

but then

$$\sum_{\alpha=0,3}\left[\sum_{f=1}^{2N} |\partial_\alpha q_f|^2 + \sum_{i=1,2}\frac{1}{2g^2}|F_{i\alpha}|^2\right] \quad \rightarrow \infty \ \text{ energy}$$

Introduce A_0 , A_3

A naïve guess: $$A_\alpha = -i\,\rho(r)\,U^{-1}\partial_\alpha U$$ → No (massive as well as massless modes)

$$i\,\left(U^{-1}\partial_\alpha U\right) \Rightarrow i\,\left(U^{-1}\partial_\alpha U\right)_\perp \equiv i\begin{pmatrix} 0 & -X^{-\frac{1}{2}}\partial_\alpha B^\dagger Y^{-\frac{1}{2}} \\ Y^{-\frac{1}{2}}\partial_\alpha B X^{-\frac{1}{2}} & 0 \end{pmatrix}$$

Projection onto the NG modes

In our case, the scalar q rotates: the correct Ansatz is:

Delduc, Valent '85

$$A_\alpha = i\,\rho(r)\,U\,\left(U^{-1}\partial_\alpha U\right)_\perp U^{-1}\,, \qquad \alpha = 0, 3 \qquad (※)$$

Gudnason, Jiang, Konishi JHEP 2010

then

$$\mathrm{Tr}\,|\mathcal{D}_\alpha\, q|^2 = -\left[\frac{\rho^2}{2}\left(\phi_1^2+\phi_2^2\right) + (1-\rho)\left(\phi_1-\phi_2\right)^2\right]\mathrm{Tr}\left[\left(U^{-1}\partial_\alpha U\right)_\perp\right]^2$$

$$\frac{1}{g^2}\mathrm{Tr}\,F_{i\alpha}^2 = -\frac{1}{g^2}\left[\left(\partial_r\rho\right)^2 + \frac{1}{r^2}f_{\mathrm{NA}}^2\,(1-\rho)^2\right]\mathrm{Tr}\left[\left(U^{-1}\partial_\alpha U\right)_\perp\right]^2\,,$$

minimiz w.r.t. ρ

$$\Rightarrow \quad S_{1+1} = 2\beta\int dtdz\;\mathrm{tr}\left\{X^{-1}\partial_\alpha B^\dagger Y^{-1}\partial_\alpha B\right\}$$
$$= 2\beta\int dtdz\;\mathrm{tr}\left\{\left(1_N + B^\dagger B\right)^{-1}\partial_\alpha B^\dagger\left(1_N + BB^\dagger\right)^{-1}\partial_\alpha B\right\}$$

where

$$\beta = \frac{2\pi}{g^2}\mathcal{I} \qquad\qquad \mathcal{I} = \int_0^\infty dr\;\partial_r\left(f_{\mathrm{NA}}\left[\left(\frac{\phi_1}{\phi_2}\right)^2 - 1\right]\right) = f_{\mathrm{NA}}(0) = 1$$

Remarks

- S is a 2D sigma model with target Hermitian symmetric spaces
  SO(2N)/U(N) or USp(2N)/U(N)

Kahler potential

$$K = \mathrm{tr}\log\left(1_N + BB^\dagger\right)$$

- Supersymmetric models ⇨ (2,2) susy sigma models
- Coupling given by $2\pi/g^2$ (calculation of β universal)

Susy 4D ⟶ 2D (2,2) susy sigma models: exact beta fn,
Morozov, Perelomov, Shiman '84

- U(N) model

$$B = \begin{pmatrix} b_1 \\ \vdots \\ b_{N-1} \end{pmatrix} \quad \rightarrow \quad CP^{N-1} \text{ sigma model}$$

Auzzi-Bolognesi-Evslin-Konishi-Yung (2003);,
Gorsky-Shifman-Yung (2004)
(justify/explain the prescription there)

- U(N) model : k-winding vortex in $\underbrace{\boxed{}\boxed{}\cdots\boxed{}}_{k}$

  CP^{N-1} 2D sigma model but with $\beta = \dfrac{2\pi}{g^2}\mathcal{I}$, $\quad \mathcal{I} = f_{\mathrm{NA}}(0) = k$

- U(N) model : k-winding vortex in $\left.\begin{array}{c}\boxed{}\\ \boxed{}\\ \vdots \\ \boxed{}\end{array}\right\} k$: sigma model in

$$Gr_{N,k} = \frac{SU(N)}{SU(k)\times SU(N-k)\times U(1)}$$

- A class of k=2 vortices in SO(2N) model : 2D sigma model in

$$SO(2N)/[SO(2)\times SO(2N-2)]$$

Meaning of the Ansatz (※)

Jiang, 2011 unpublished,
Fujimori et. al. 2011

$$\mathcal{L}^0 = \mathrm{Tr}\Big\{ -\frac{1}{e^2}F_{12}F^{12} - \frac{1}{g^2}\hat{F}_{12}\hat{F}^{12} + \mathcal{D}_i q(\mathcal{D}^i q)^\dagger - e^2\big|X^0 t^0 - \xi t^0\big|^2 - g^2\big|X^a t^a\big|^2\Big\}$$

⇨ Minimum-tension BPS vortex solutions, indep. on the <u>orientations U</u>

i=1,2

$$\mathcal{L}^{(2)} = \mathrm{Tr}\Big\{ -\frac{1}{e^2}F_{\alpha i}F^{\alpha i} - \frac{1}{g^2}\hat{F}_{\alpha i}\hat{F}^{\alpha i} + \mathcal{D}_\alpha q(\mathcal{D}^\alpha q)^\dagger\Big\},$$

α=3,0

will leads to massive excitations once U fluctuates, U= U(z,t)
UNLESS A_α is introduced such that 4D equations of motions

$$\begin{aligned} 0 =& \frac{1}{e^2}\partial^i F^0_{i\alpha} - i\mathrm{Tr}\big[q^\dagger t^0 \mathcal{D}_\alpha q - (\mathcal{D}_\alpha q)^\dagger t^0 q\big], \\ 0 =& \frac{1}{g^2}\mathcal{D}^i F^a_{i\alpha} - i\mathrm{Tr}\big[q^\dagger t^a \mathcal{D}_\alpha q - (\mathcal{D}_\alpha q)^\dagger t^a q\big], \\ 0 =& \mathcal{D}_\alpha \mathcal{D}^\alpha q, \end{aligned}$$

<u>*N.B.*</u>

are obeyed.

Solution ⇨ Ansatz (※) + extremization with respect to ρ(r) !!!

Symmetric criticality

Gorsky-Shifman-Yung, Gudnason-Jiang-Konishi

Goddard-Nuyts-Olive-Weinberg (GNOW) duality

- Infinitesimal transformations of the k=1 vortex (SO(2N) case):

$$U = 1_{2N} + \begin{pmatrix} 0_N & -B^\dagger \\ B & 0_N \end{pmatrix} + \ldots \; , \qquad B^T = -B,$$

- An abstract spinor of SO(2N) group

Σ_{ij} in terms of $$a_k = \frac{1}{2} \underbrace{\tau_3 \otimes \cdots \otimes \tau_3}_{k-1} \otimes \tau_- \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{N-k} \, , \qquad k = 1, 2, \ldots N$$ and $(a_k)^\dagger$

Identify the points on the vortex moduli and spinor states

$$(\pm, \cdots, \pm) \sim |s_1\rangle \otimes |s_2\rangle \otimes \cdots |s_N\rangle \; , \qquad |s_j\rangle = |\downarrow\rangle \quad \text{or} \quad |\uparrow\rangle$$

$$\vec{\mu} = (\pm\tfrac{1}{2}, \cdots, \pm\tfrac{1}{2})$$

Notes

with the origin $$(+ \ldots +) \sim |\downarrow \ldots \downarrow\rangle$$

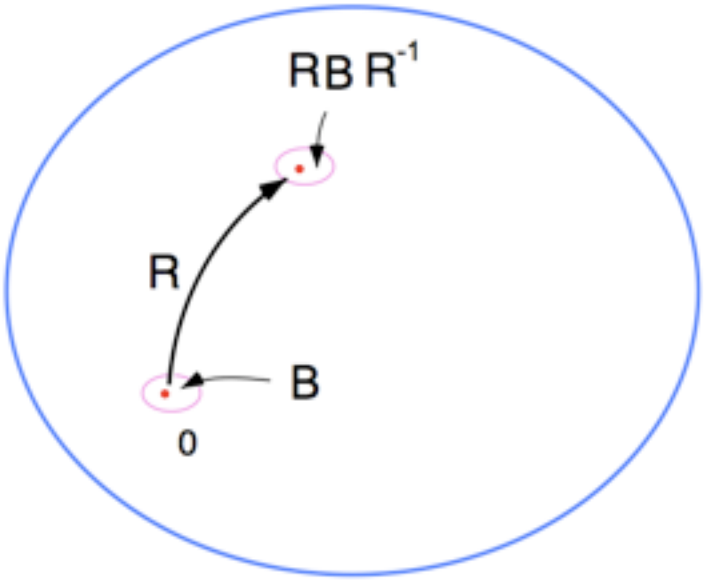
vortex spin

but

$$e^{i\omega_{\alpha\beta}\Sigma_{\alpha\beta}} \simeq \mathbf{1} + \cancel{\alpha_{ij}\, a_i^\dagger a_j} + \beta_{ij}\, a_i^\dagger a_j^\dagger + \cancel{\beta_{ij}^\dagger\, a_i a_j} + \cancel{i\,\omega_{2i,2i-1}} + O(\omega^2)$$

where $$\beta_{ij} \equiv -[\,\omega_{2i,2j} - \omega_{2i-1,2j-1} + i\,\omega_{2i-1,2j} + i\,\omega_{2i,2j-1}\,]$$

⇨ Identify $$\mathbf{B}_{ij} = \beta_{ij}$$ locally



R = finite U transformation

Vortex moduli ~ spinor state moduli = SO(2N)/U(N)$\oplus$ SO(2N)/U(N)

B = local coordinates (2^{N-1} coordinate patches)*

* USp(2N) theory ⇨ 2^{N} coordinate patches; moduli space = USp(2N)/U(N)
= spinor states of SO(2N+1)

Fluctuation of SO(2N) vortex orientations ~
fluctuation of massless Spin(2N) spinor states

The vortex (and kink monopoles) in a mass-deformed theories ($m_i \neq m_j$)

SU(N)

$$\langle \Phi \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m_1 & & & \\ & \ddots & & \\ & & m_N & \\ & & & -m_1 - m_2 - \ldots m_N \end{pmatrix}$$

$SU(N) \Rightarrow U(1)^{N-1}$

Gorsky-Shifman-Yung 2004

- CP^{N-1} vortex moduli replaced by N-1 points

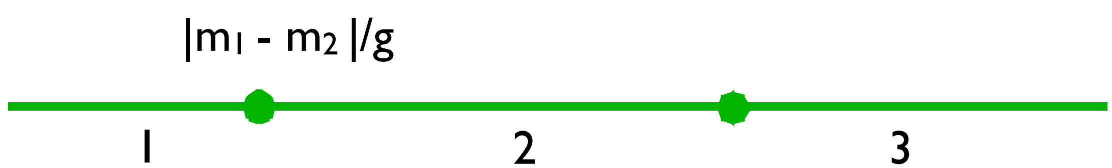
SO(2N), USp(2N) theory

Eto, Fujimori, Gudnason, Jiang, Konishi, Ohashi, Nitta
about to appear 2011

- SO(2N)/U(N) or USp(2N)/U(N) vortex moduli replaced by
  $2^{N-1} \oplus 2^{N-1}$ (or 2^N) points

- SO(2N)/U(N) or USp(2N)/U(N) sigma model replaced by massive sigma models

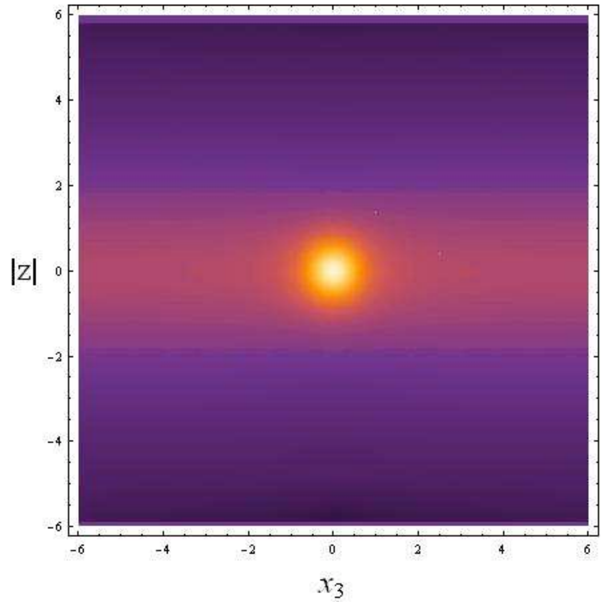
also
Arai, Shin 2011

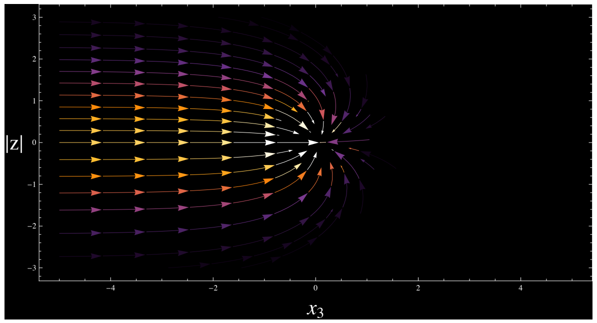


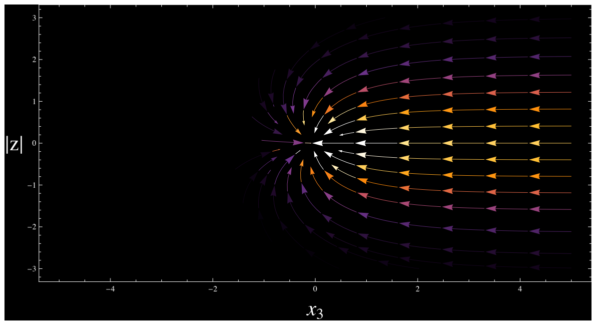
- Kinks along the vortex connecting different Abelian vortices = Abelian monopoles

- Flavor symmetric limit $m_i \to m$ NON SMOOTH (colored clouds!)

E. Weinberg







$$\mathcal{L} = \frac{4\pi}{g^2}\mathrm{Tr}\Big\{ \left(1_n + B^\dagger B\right)^{-1} \partial_\alpha B^\dagger \left(1_n + BB^\dagger\right)^{-1} \partial^\alpha B$$

$$- \left(1_n + B^\dagger B\right)^{-1} \left\{M_n, B^\dagger\right\} \left(1_n + BB^\dagger\right)^{-1} \left\{M_n, B\right\} \Big\} \,.$$

$$M = \left(\begin{array}{c|c} M_n & \\ \hline & -M_n \end{array}\right), \qquad M_n = \mathrm{diag}(m_1, m_2, \cdots, m_n)$$

III. Non-Abelian monopoles

Non-Abelian monopoles

- Embedding of 't Hooft-Polyakov monopole $SU(2) \to U(1)$ in $G \to H$, e.g. $SU(N+1) \to SU(N) \times U(1)$

Goddard-Nuyts-Olive, E.Weinberg, Lee,Yi, Bais, Schroer, ... '77-80, ...

Difficulties
- topological obstructions
- non-normalizable zeromodes
- colored cloud

Abouelsaad et.al. Coleman, et. al., '83-'84

Dorey, Hollowood, et. al.

E. Weinberg

- Degenerate monopoles to transform under the GNO dual of H, not under H itself (non-local field transformations)

In fact,

- Light non-Abelian monopoles in N=2 supersymmetric QCD in the r -vacua, $r \leqq N_F / 2$ (flavor essential)

Seiberg-Witten '94

Argyres,Plesser,Seiberg,'96 Hanany-Oz, '96

Carlino-Konishi-Murayama '00

- N=1 perturbation ⇨ confinement as non-Abelian dual Meissner effect
- Almost SCFT vacua : confinement by condensation of monopole composites

Auzzi,Grena,Konishi

- 2-1 Correspondence between classical $(r, N_F - r)$ and quantum r- vacua

Bolognesi-Konishi-et.al '05

Di Pietro, Giacomell '11

- Many different types of confining vacua in N=1 susy models (confinement index, etc.)

Okouchi-Konishi '10

Making bridge between semi-classical and quantum monopoles and vortices - a highly nontrivial task

- Dynamical Abelianization

- Isomonodromy

Bilal-Ferrari, '96
Cappelli, Valtancoli, Vergnano '97
for SU(2)

Quark singularity (at large m_i) ➠ Monopole singularity (at small m_i)

Higgs vacuum ➠ Confining vacuum

Di Pietro, Giacomell '11
SU(N)

Dynamical Abelianization

dt dz

- U can fluctuate, U= U(z,t) : gapless excitations ---- only along (z,x)
- Vortex worldsheet action (U(2))

Auzzi, Bolognesi,Evslin,Konishi,Yung,
Shifman-Yung

$$S_\sigma^{(1+1)} = \beta \int d^2x \, \frac{1}{2} \, (\partial \, n^a)^2$$ + fermionic terms

N=(2,2) supersymmetric CP[1] sigma model

Hanany-Tong,
Shifman et. al.

$$S_\sigma^{(1+1)} = \int d^2x \left[d^2\theta \, d^2\bar{\theta} \, \frac{1}{\beta} \, \bar{Y} \, Y + \Lambda_\sigma \, d\theta^1 d\bar{\theta}_2 \cosh Y \right]$$

Hori, Vafa

Tong, Gorsky-Shifman-Yung

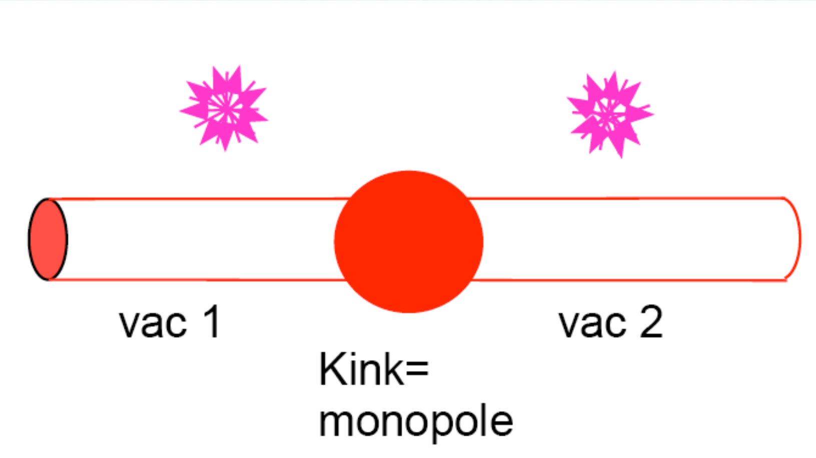
≡ Gauge dynamics in 4D in Coulomb phase

(Seiberg-Witten)

beta function and the spectrum match

2 vacua → kinks = (Abelian) monopoles !

- Realization of 2D - 4D duality Dorey
- Global SU(2) unbroken (Coleman)
- Vortex dynamically Abelianizes



SU(N) SQCD

- U(N), N_f =N model from SU(N+1) ⇒ SU(N) × U(1)/Z_N
- r= N_f vacuum (classical)
- quantum mechanically only r < N_f / 2

$$\langle \Phi \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & \ddots & \vdots & \vdots \\ 0 & \dots & m & 0 \\ 0 & \dots & 0 & -N\,m \end{pmatrix};$$

- classical r (> N_f/2) ⇔ quantum (N_f - r) vacua

Carlino-Murayama-Konishi
Bolognesi-Konishi-Marmorini

m ≫ μ ≫ Λ : ⇔ m ~ μ ≪ Λ :

(Vacuum counting; symmetry)

- U(N) N_f =N : quantum r =0 vacua (Abelian monopoles ! OK with MV)

global symmetry

r	Deg. Freed.	Eff. Gauge Group	Phase	Global Symmetry
0	monopoles	$U(1)^{N-1}$	Confinement	$U(n_f)$
1	monopoles	$U(1)^{N-1}$	Confinement	$U(N_f-1)\times U(1)$
$2, .., [\frac{N_f-1}{2}]$	NA monopoles	$SU(r)\times U(1)^{N-r}$	Confinement	$U(N_f-r)\times U(r)$
$N_f/2$	rel. nonloc.	-	Almost SCFT	$U(N_f/2)\times U(N_f/2)$

But non-Abelian vortices which do not dynamically Abelianize should exist --

in the right vacua

“Truly non-Abelian” Vortices

Dorigoni-KK-Ohashi ‘08

The Model: the same SU(N), N_f =N, softly broken N=2 SQCD, but with appropriately tuned masses*

* select the right quantum vacuum at $m_i \to 0$ (cfr N=1 SQCD)

$$M = \begin{pmatrix} m^{(1)}\, \mathbb{1}_{n\times n} & 0 \\ 0 & m^{(2)}\, \mathbb{1}_{r\times r} \end{pmatrix}$$

$$N = n+r \; ;$$

with $$n\, m^{(1)} + r\, m^{(2)} = 0$$

Adjoint scalar VEV

$$\langle \Phi \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m^{(1)}\, \mathbb{1}_{n\times n} & 0 \\ 0 & m^{(2)}\, \mathbb{1}_{r\times r} \end{pmatrix}$$

$$|m_0| \gg |\mu| \gg \Lambda \, .$$

SU(N) $_{\text{local}}$ ⇒

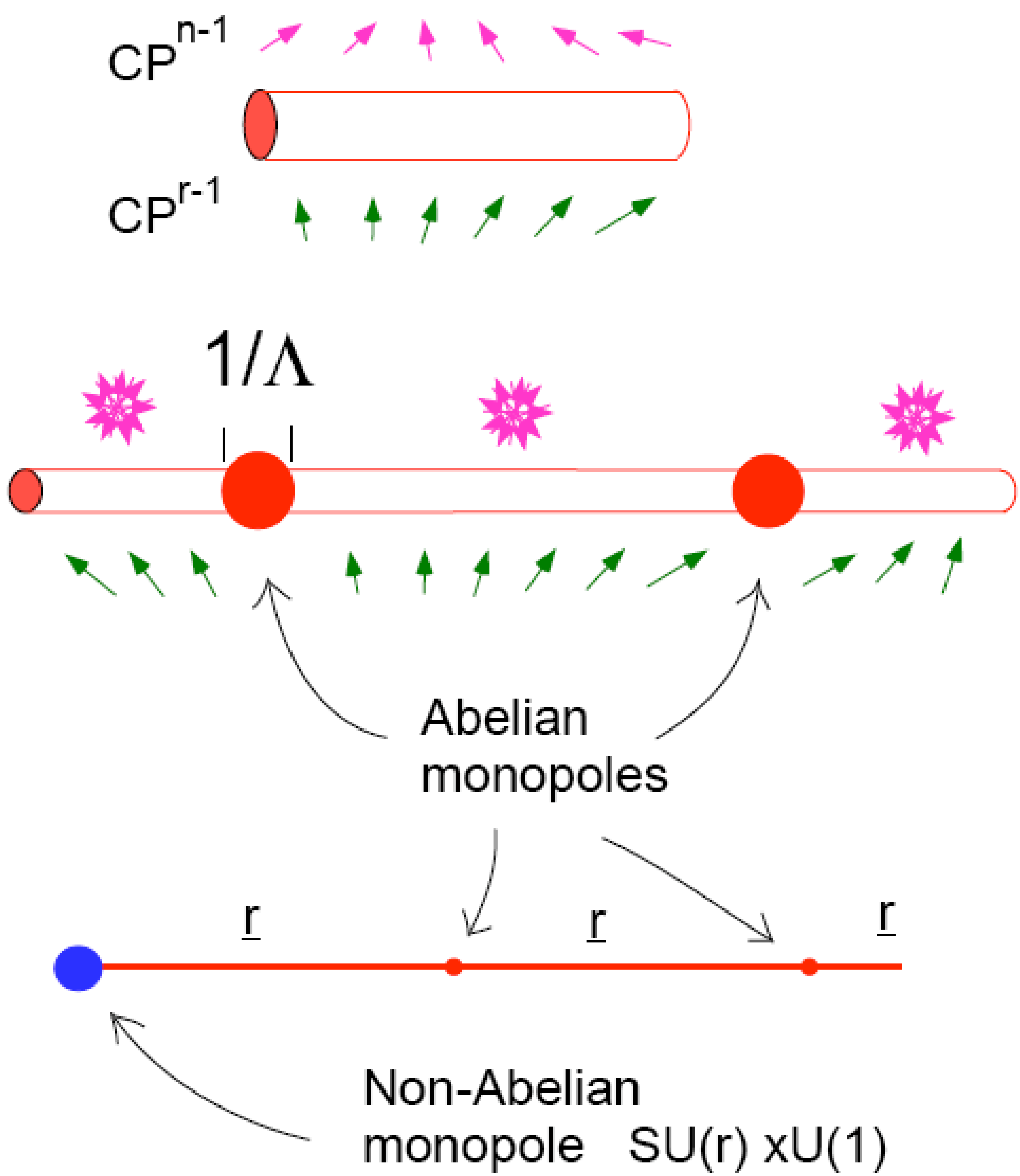
$$G = \frac{SU(n) \times SU(r) \times U(1)}{\mathbb{Z}_K}\, , \quad K = \mathrm{LCM}\,\{n, r\}$$

Global symmetry, “broken” by the vortex

$$[SU(n) \times SU(r) \times U(1)]_{C+F} \to SU(n-1) \times SU(r-1) \times U(1)^3,$$

Vortex moduli ~ $CP^{n-1} \times CP^{r-1}$

Idea: for n > r (r < N_f /2), the CP^{n-1} Abelianizes, leaving weakly fluctuating CP^{r-1}

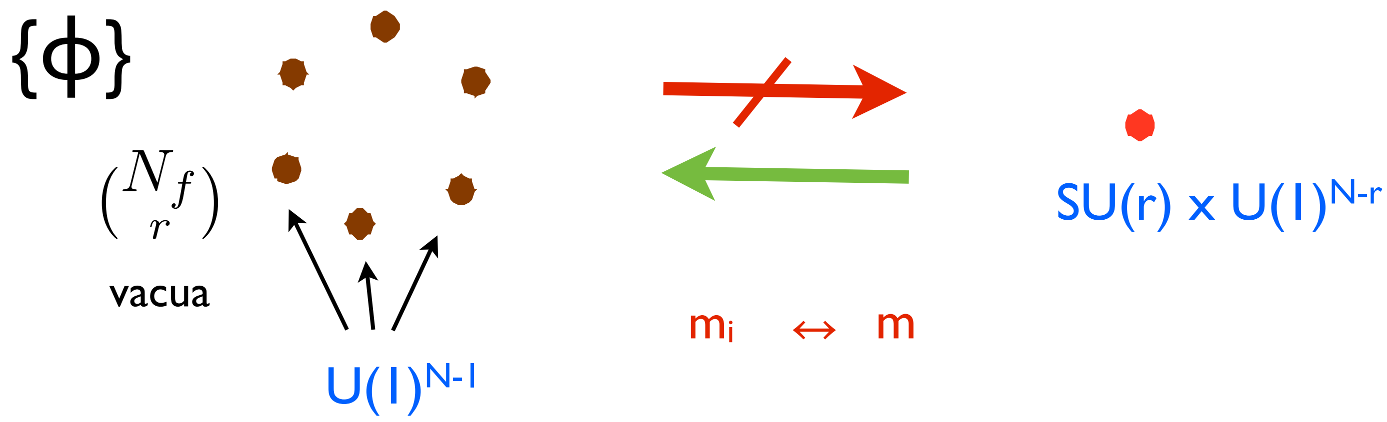

CP^{n-1}
CP^{r-1}
$1/\Lambda$
Abelian
monopoles
r
r
r
Non-Abelian
monopole SU(r) xU(1)

IV. Summary

Flavor to Dual Gauge Symmetry

- Flavor symmetric limit $m_i \to m$ subtle

  Quantum r vacua cannot be reached from the mass-deformed theory



- N_F non-Abelian monopoles in <u>r</u> instead of $\binom{N_f}{r}$ Abelian monopoles

  required by the correct flavor $SU(N_F)$ symmetry realization (WI's)

- Strong indication (both semi-classically and quantum mechanically) that the dual gauge group = a manifestation of the flavor symmetry in conjunction with gauge dynamics (monopoles and vortices)

- Orientational zeromodes and their fluctuation in M-V-M complex the most direct way to see such a connection so far

Real world QCD

- Cannot say much, but if XSB ~ Confinement

Scenario I $\langle M_i^j \rangle = \delta_i^j v \neq 0$ with Abelian monopoles M of $U(1)^2 \subset SU(3)$

Scenario II $\langle M_i^\alpha \tilde{M}_\alpha^j \rangle = \delta_i^j v \neq 0$ with nonAbelian monopoles M, $\tilde{M}$ of $U(2) \subset SU(3)$

$i = SU_L(N_F)$, $j = SU_R(N_F)$

Scenario II preferred from the correct flavor symmetry realization

END

AND...

Thanks to the collaborations ('00 -'11) with:

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Nitta, Ohashi, Yokoi, Eto, Fujimori,
Marmorini, Ferretti, Vinci, Gudnason, Dorigoni, Michelini, Jiang,
Giacomelli, Cipriani, Di Pietro

Armenia-Italy-Japan-USA-Russia-Denmark-China
collaboration