Gregory-Laflamme as the confinement/deconfinement transition in holographic QCD

Takeshi Morita

University of Crete

based on arXiv:1107.4048 (JHEP09 (2011) 073)

with G. Mandal (Tata Institute in India)

1. Introduction

◆ Holographic construction of "4d pure Yang-Mills" Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism

4 dim SU(N) YM ⟺ IIA SUGRA

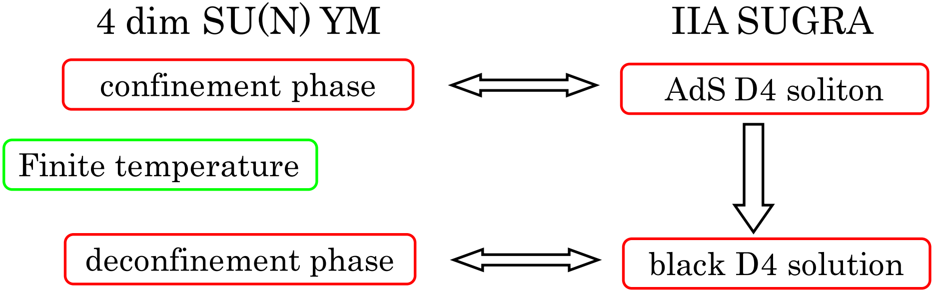
confinement phase ⟺ AdS D4 soliton

1. Introduction

◆ Holographic construction of “4d pure Yang-Mills” Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

confinement phase ⟺ AdS D4 soliton

Finite temperature

⇓

deconfinement phase ⟺ black D4 solution

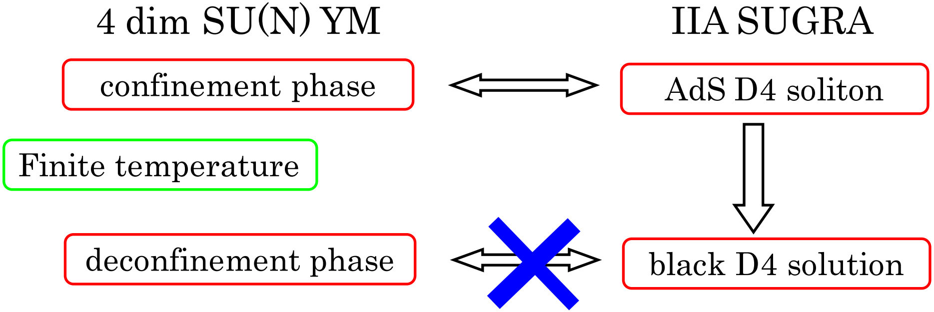
This identification was widely believed and employed in many studies.

1. Introduction

◆ Holographic construction of “4d pure Yang-Mills” Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



Several problems have been reported in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

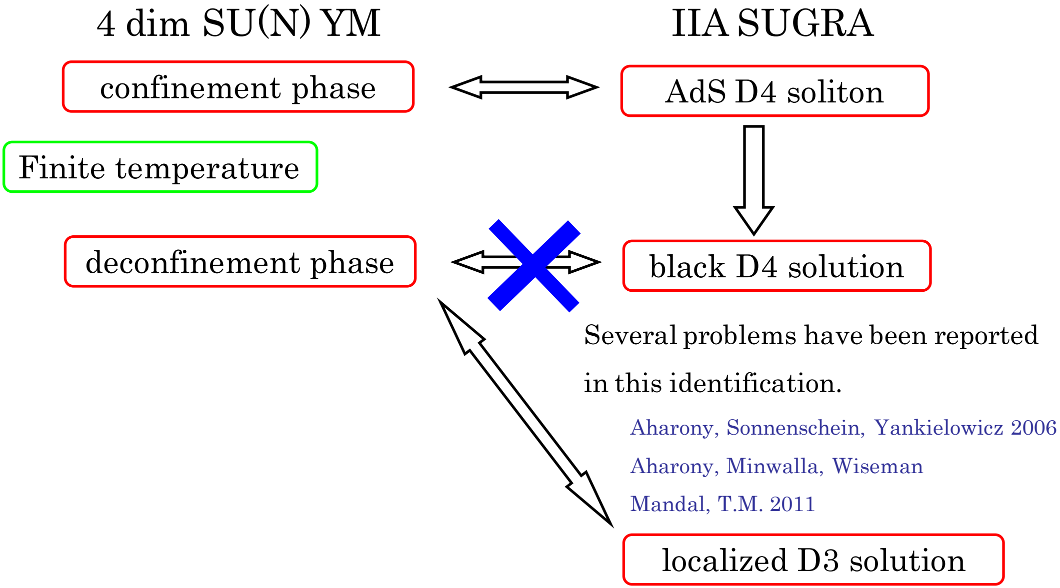
Mandal, T.M. 2011

1. Introduction

◆ Holographic construction of "4d pure Yang-Mills" Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

confinement phase

AdS D4 soliton

Finite temperature

deconfinement phase

black D4 solution

Several problems have been reported in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

Mandal, T.M. 2011

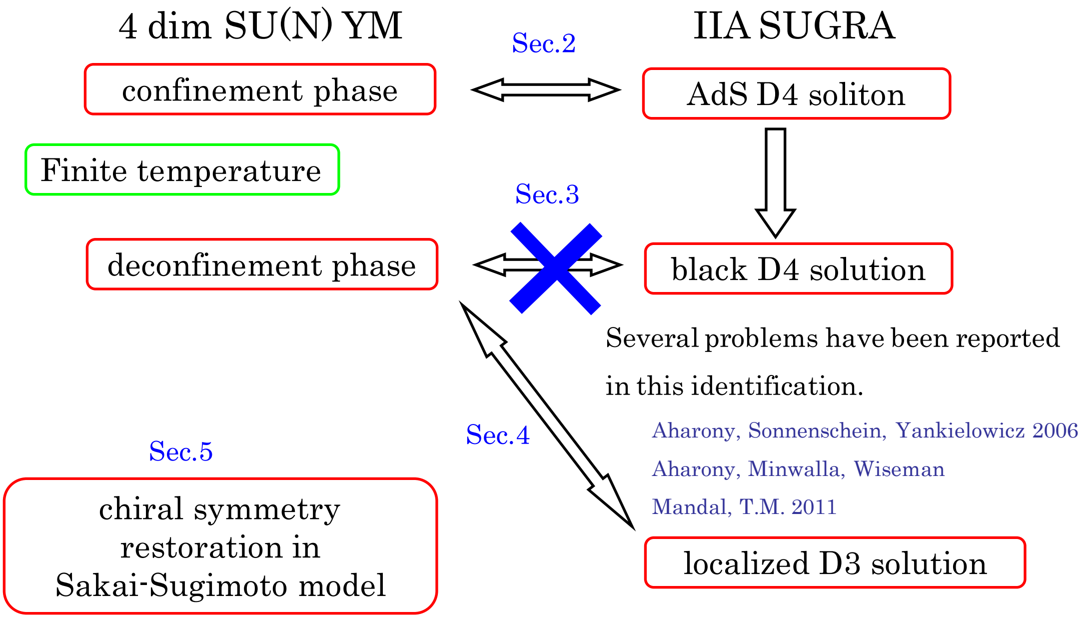
localized D3 solution

Plan of my talk

◆ Holographic construction of "4d pure Yang-Mills" Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

confinement phase

Sec.2

AdS D4 soliton

Finite temperature

Sec.3

deconfinement phase

black D4 solution

Several problems have been reported

in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

Mandal, T.M. 2011

Sec.4

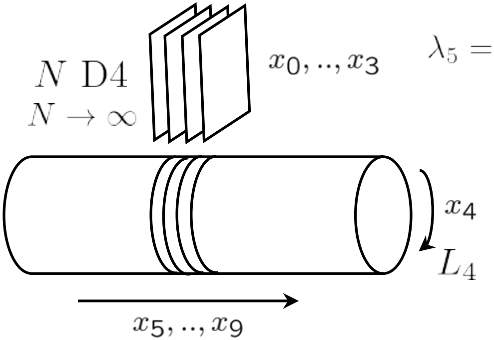
Sec.5

chiral symmetry
restoration in
Sakai-Sugimoto model

localized D3 solution

2. Review of holographic QCD

Witten 1998



$\lambda_5 = \lambda_4 L_4$: 't Hooft coupling of 5dSYM and 4dYM

x_4: anti-periodic (AP) boundary condition for fermions (Scherk-Schwarz circle)
→ mass $1/L_4$ → breaks supersymmetry

	(0)	1	2	3	(4)	5	6	7	8	9
D4	-	-	-	-	-					
	↑ β				↑ L_4					

0 and 4 are periodic coordinates.

5d SYM on $S^1_\beta \times S^1_{L_4}$ 4d limit **4d pure YM**

$$\begin{cases} A_\mu \quad (\mu = 0, .., 3) \to \underline{\text{massless}} \text{ (zero mode)} \\ A_4 \quad \to \lambda_4 / L_4 \\ Y_i \quad (i = 5, .., 9) \to \lambda_4 / L_4 \\ \psi \quad \to 1 / L_4 \end{cases}$$

A_4 and Y_i: one-loop

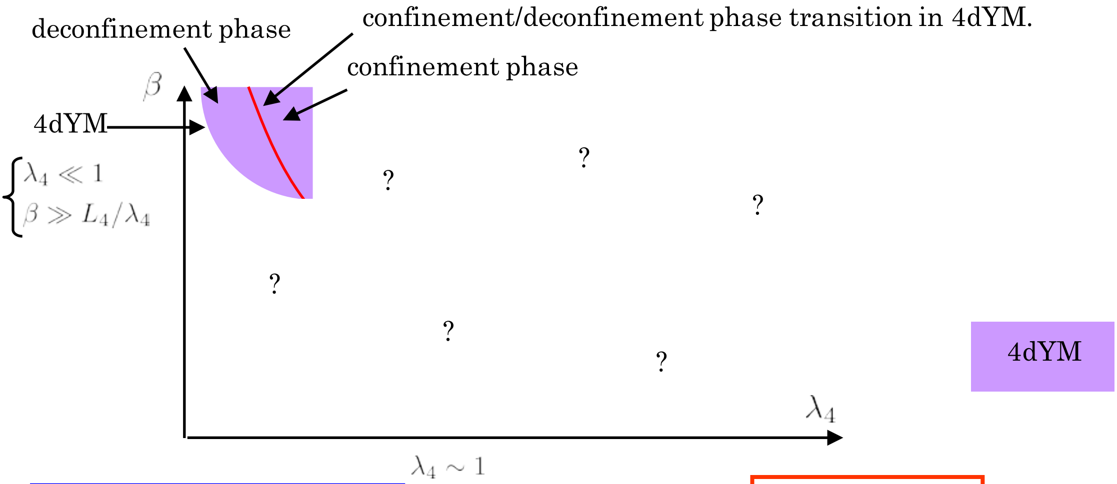
⇨

$$\begin{cases} \lambda_4 \ll 1 & \text{: suppression of the loops from KK modes} \\ \beta \gg L_4 / \lambda_4 & \text{: temperature} \ll \text{KK scale} \end{cases}$$

2. Review of holographic QCD

Witten 1998

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$



deconfinement phase

confinement/deconfinement phase transition in 4dYM.

confinement phase

β

4dYM

$\left\{\begin{array}{l} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{array}\right.$

? ? ? ? ? ?

4dYM

λ_4

$\lambda_4 \sim 1$

5d SYM on $S^1_\beta \times S^1_{L_4}$

4d limit

4d pure YM

$$\left\{\begin{array}{l} A_\mu \quad (\mu=0,..,3) \rightarrow \underline{\text{massless}}\ (\text{zero mode}) \\ \left.\begin{array}{l} A_4 \rightarrow \lambda_4/L_4 \\ Y_i \quad (i=5,..,9) \rightarrow \lambda_4/L_4 \end{array}\right\} \text{one-loop} \\ \psi \rightarrow 1/L_4 \end{array}\right.$$

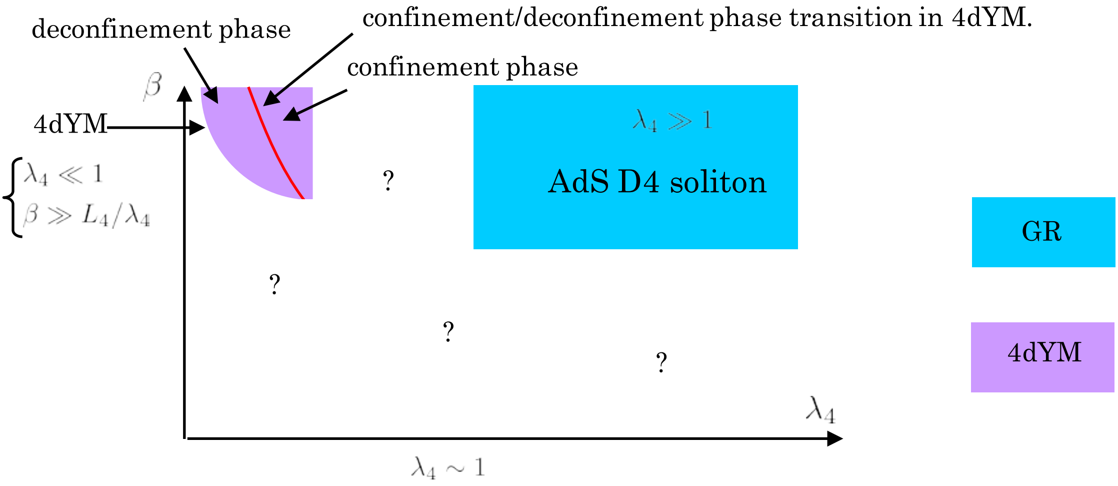
⇨

$$\left\{\begin{array}{ll} \lambda_4 \ll 1 & : \text{suppression of the loops from KK modes} \\ \beta \gg L_4/\lambda_4 & : \text{temperature} \ll \text{KK scale} \end{array}\right.$$

2. Review of holographic QCD

Witten 1998

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$



deconfinement phase

confinement/deconfinement phase transition in 4dYM.

confinement phase

4dYM

$\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

$\lambda_4 \gg 1$

AdS D4 soliton

GR

4dYM

?

?

?

?

β

λ_4

$\lambda_4 \sim 1$

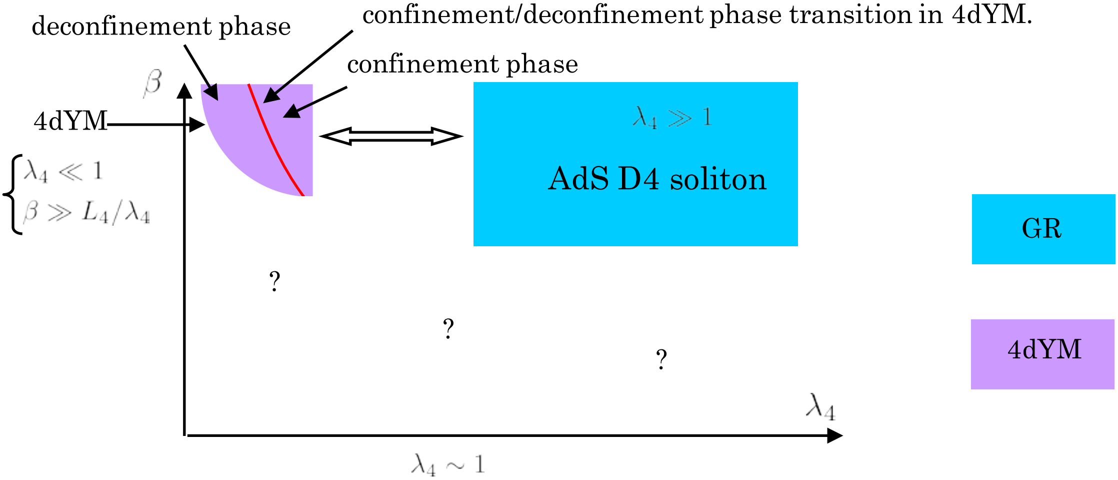
AdS D4 soliton

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(dt^2 + \sum_{i=1}^{3} dx_i^2 + f_4(u) dx_4^2 \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right] \qquad f_4(u) = 1 - \left(\frac{u_0}{u} \right)^3$$

This geometry describes the strong coupling 5d SYM in the low temperature regime.

Witten 1998

◆ Holographic pure YM (Holographic QCD)



deconfinement phase

confinement/deconfinement phase transition in 4dYM.

confinement phase

β

4dYM

$\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

$\lambda_4 \gg 1$

AdS D4 soliton

?

?

?

λ_4

$\lambda_4 \sim 1$

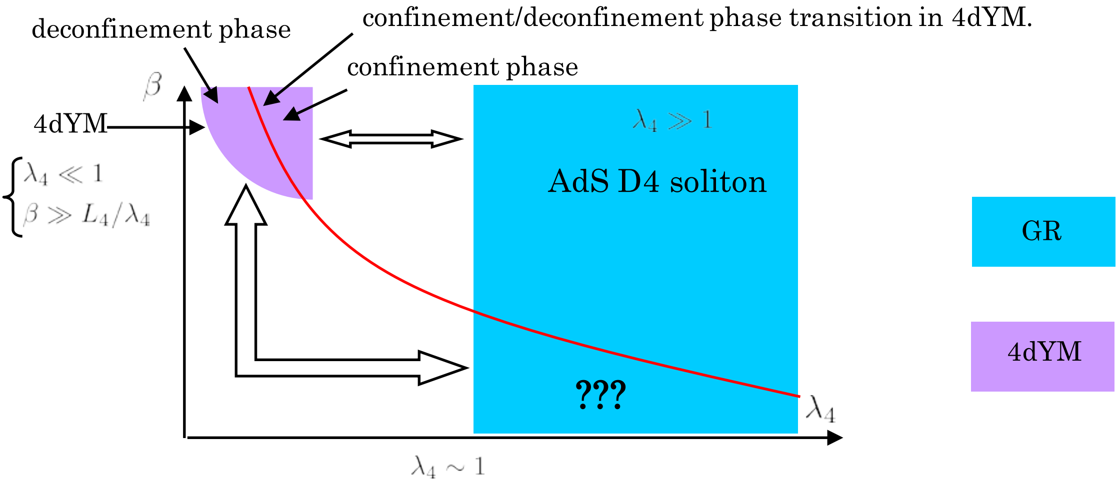
GR

4dYM

- Although there is no over wrap, we can extrapolate the information of the confinement phase of the 4d YM through the AdS D4 soliton. The results obtained from this extrapolation agree with known properties of pure Yang-Mills qualitatively.
- Sakai-Sugimoto model which is an application of this set up reproduces some experimantal QCD results.

Witten 1998

◆ Holographic pure YM (Holographic QCD)



deconfinement phase

confinement/deconfinement phase transition in 4dYM.

confinement phase

β

4dYM

$\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

$\lambda_4 \gg 1$

AdS D4 soliton

???

λ_4

$\lambda_4 \sim 1$

GR

4dYM

It is natural to explore a higher temperature regime to answer;

- What is the gravity dual of the deconfinement phase?
- What is the gravity dual of the confinement/deconfinement transition?

2. Review of holographic QCD

◆ Two ways for obtaining a thermal partition function of 4d pure Yang-Mills

	(0)	1	2	3	(4)	5	6	7	8	9
D4	-	-	-	-	-					
	AP or P				AP					

We need to fix the boundary condition of fermions.

4d limit $\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

$$\begin{tikzcd} \text{AP b.c.:} & Z = \mathrm{Tr}\left(e^{-\beta H_{5dSYM}}\right) \arrow[dr] & \\ & & Z = \mathrm{Tr}\left(e^{-\beta H_{4dYM}}\right) \\ \text{P b.c.:} & \tilde{Z} = \mathrm{Tr}\left((-1)^F e^{-\beta H_{5dSYM}}\right) \arrow[ur] & \end{tikzcd}$$

Since fermions decouple under the weak and low temperature limit (4d limit) and F=0 dominates, the temporal boundary condition is irrelevant.

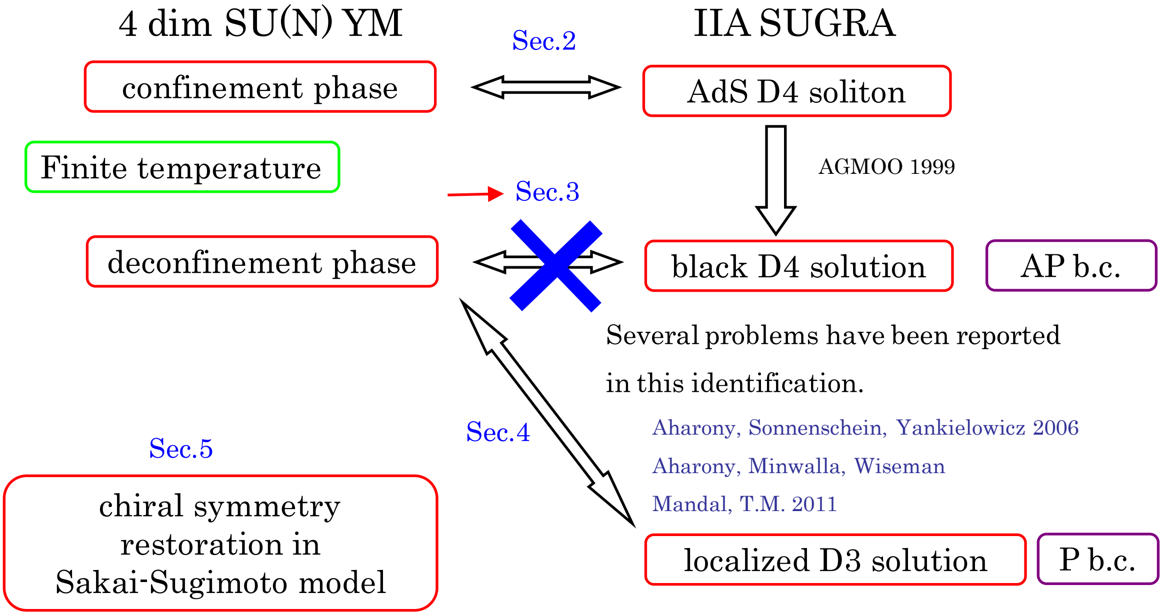
We will compare the phase structures of the SYM in these two boundary conditions.

Plan of my talk

◆ Holographic construction of “4d pure Yang-Mills” Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

Sec.2

confinement phase

AdS D4 soliton

AGMOO 1999

Finite temperature

Sec.3

deconfinement phase

black D4 solution

AP b.c.

Several problems have been reported in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

Mandal, T.M. 2011

Sec.4

Sec.5

chiral symmetry
restoration in
Sakai-Sugimoto model

localized D3 solution

P b.c.

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

⟶ This boundary condition has been used in the studies of holographic QCD.

	(0)	1	2	3	(4)	5	6	7	8	9
D4	-	-	-	-	-					
	↑ AP				↑ AP					

The system has a Z_2 symmetry: $t \leftrightarrow x_4$.

- By using this Z_2 symmetry, we obtain a new solution.

AdS D4 soliton (confinement geometry)

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(\underline{dt^2} + \sum_{i=1}^{3} dx_i^2 + \underline{f_4(u)dx_4^2} \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

$$\updownarrow \quad Z_2\colon\ t \leftrightarrow x_4$$

$$f_4(u) = 1 - \left(\frac{u_0}{u} \right)^3$$

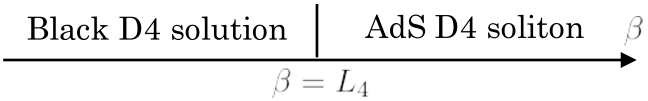
Black D4 solution

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(\underline{f_4(u)dt^2} + \sum_{i=1}^{3} dx_i^2 + \underline{dx_4^2} \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

- A phase transition happens at the self-dual point $\beta = L_4$.



- By using this Z_2 symmetry, we obtain a new solution.

AdS D4 soliton (confinement geometry)

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(\underline{dt^2} + \sum_{i=1}^{3} dx_i^2 + \underline{f_4(u) dx_4^2} \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

$$f_4(u) = 1 - \left(\frac{u_0}{u} \right)^3$$

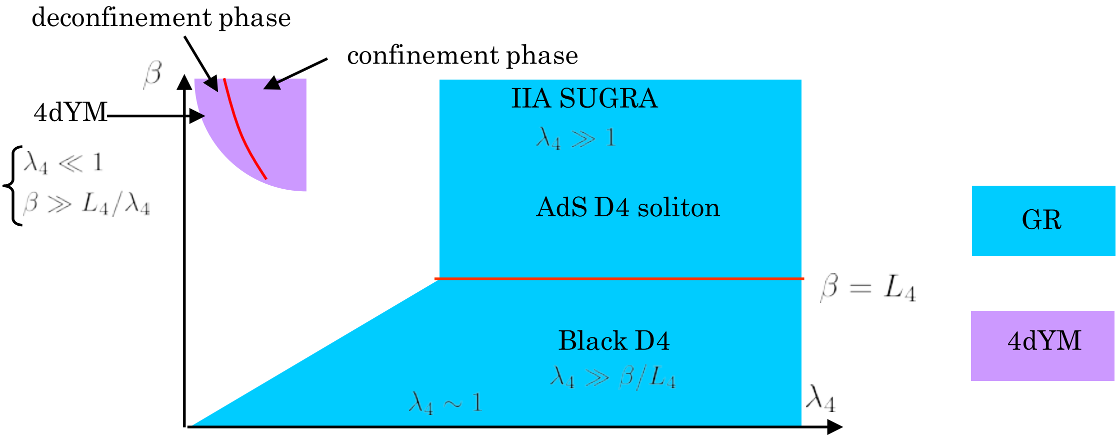
$$\Big\updownarrow \quad Z_2 \colon \; t \leftrightarrow x_4$$

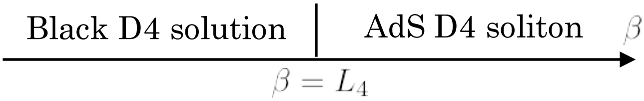
Black D4 solution

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(\underline{f_4(u) dt^2} + \sum_{i=1}^{3} dx_i^2 + \underline{dx_4^2} \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

3. Problem of the previous interpretation of the C/D transition

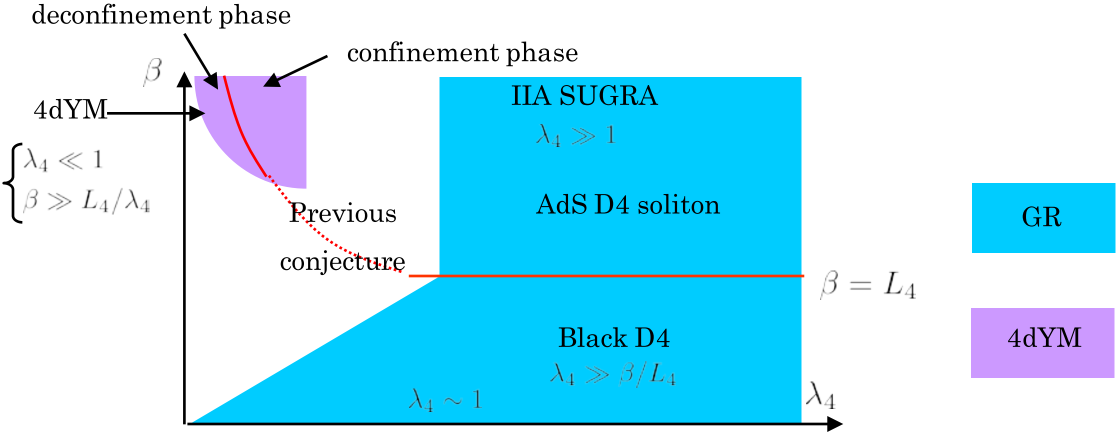
◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.





3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.



deconfinement phase

confinement phase

4dYM

$\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

β

IIA SUGRA

$\lambda_4 \gg 1$

AdS D4 soliton

Previous

conjecture

$\beta = L_4$

Black D4

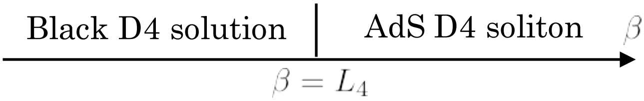
$\lambda_4 \gg \beta/L_4$

$\lambda_4 \sim 1$

λ_4

GR

4dYM



Black D4 solution

AdS D4 soliton

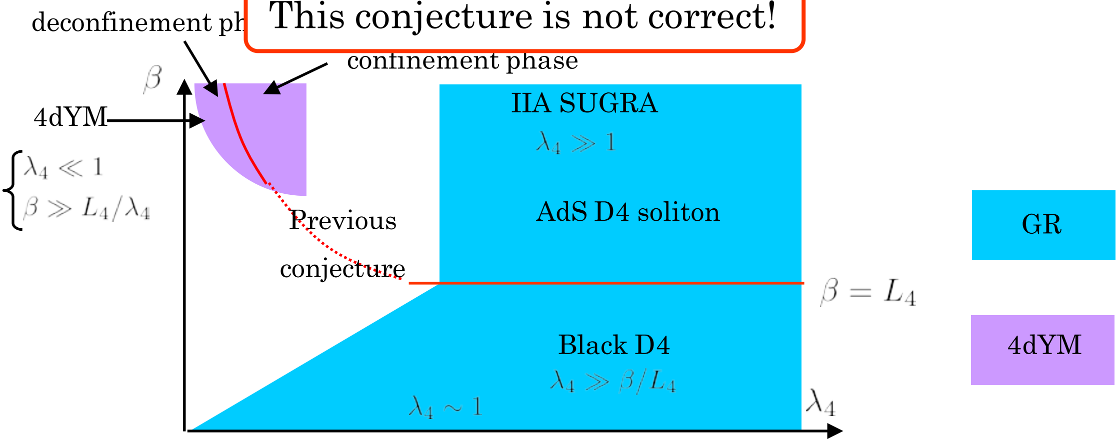
β

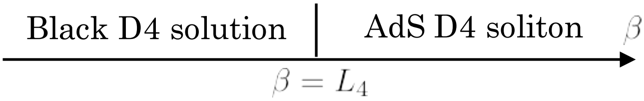
$\beta = L_4$

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

This conjecture is not correct!





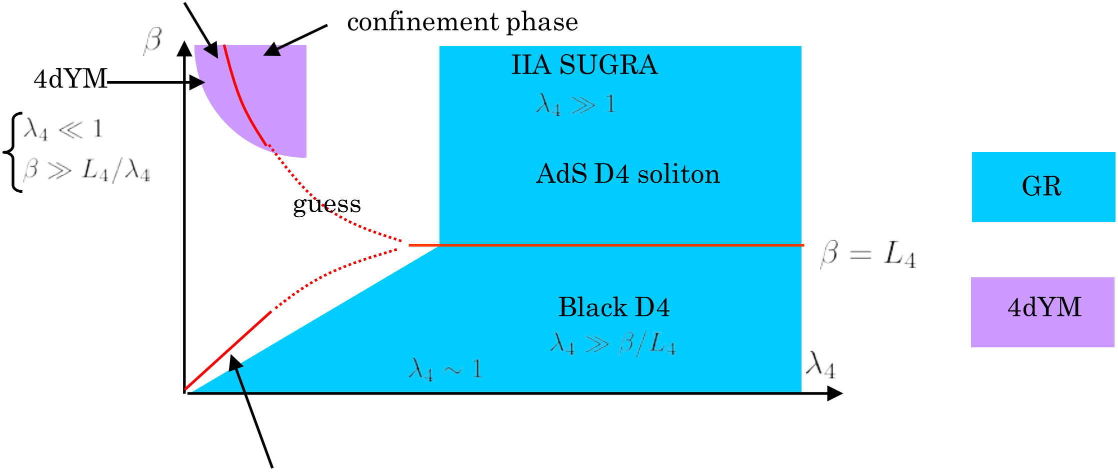
3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

Aharony, Sonnenschein, Yankielowicz 2006

Mandal, T.M. 2011

deconfinement phase



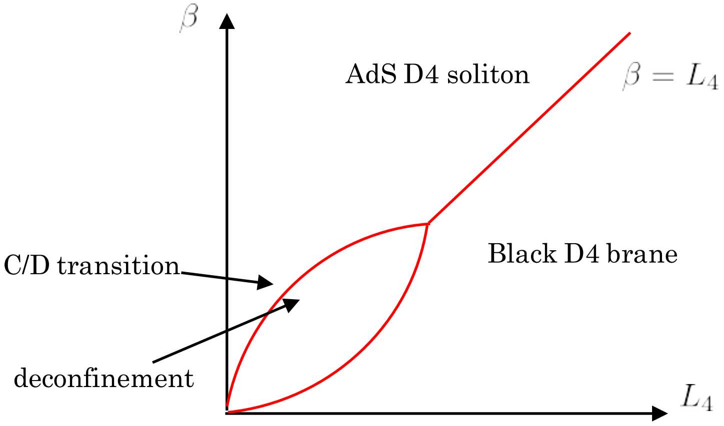
A transition obtained from the $Z_2(\beta \leftrightarrow L_4)$

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

Aharony, Sonnenschein, Yankielowicz 2006

Mandal, T.M. 2011



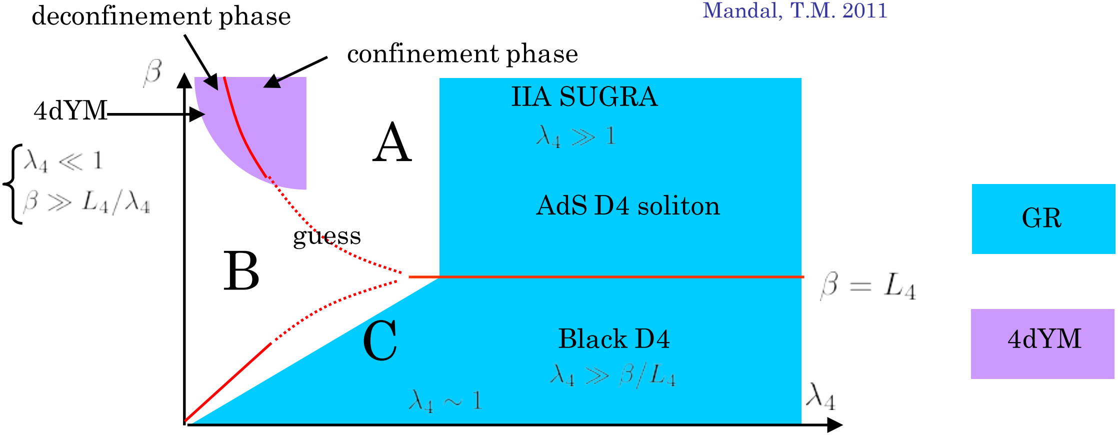
- The Z_2($\beta \leftrightarrow L_4$) symmetry requires at least three phases.

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

Aharony, Sonnenschein, Yankielowicz 2006

Mandal, T.M. 2011



- Z_N symmetries of the deconfinement phase and black D4 are different too.

A: $\begin{cases} Z_N \text{ along } S^1_\beta \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{cases}$

- confinement phase
- AdS D4 soliton

B: $\begin{cases} \cancel{Z_N \text{ along } S^1_\beta} \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{cases}$

- deconfinement phase

C: $\begin{cases} \cancel{Z_N \text{ along } S^1_\beta} \\ Z_N \text{ along } S^1_{L_4} \end{cases}$

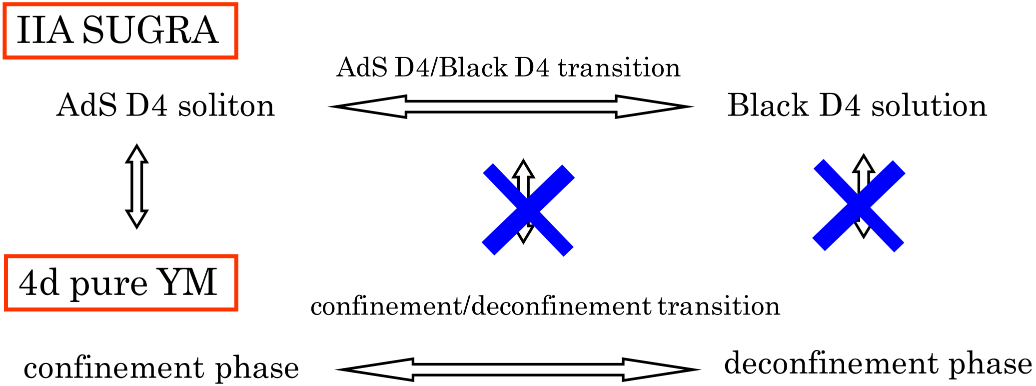
- Black D4

3. Problem of the previous interpretation of the C/D transition

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (AP,AP) B.C.

Aharony, Sonnenschein, Yankielowicz 2006

Mandal, T.M. 2011



IIA SUGRA

AdS D4/Black D4 transition

AdS D4 soliton ⟺ Black D4 solution

4d pure YM

confinement/deconfinement transition

confinement phase ⟺ deconfinement phase

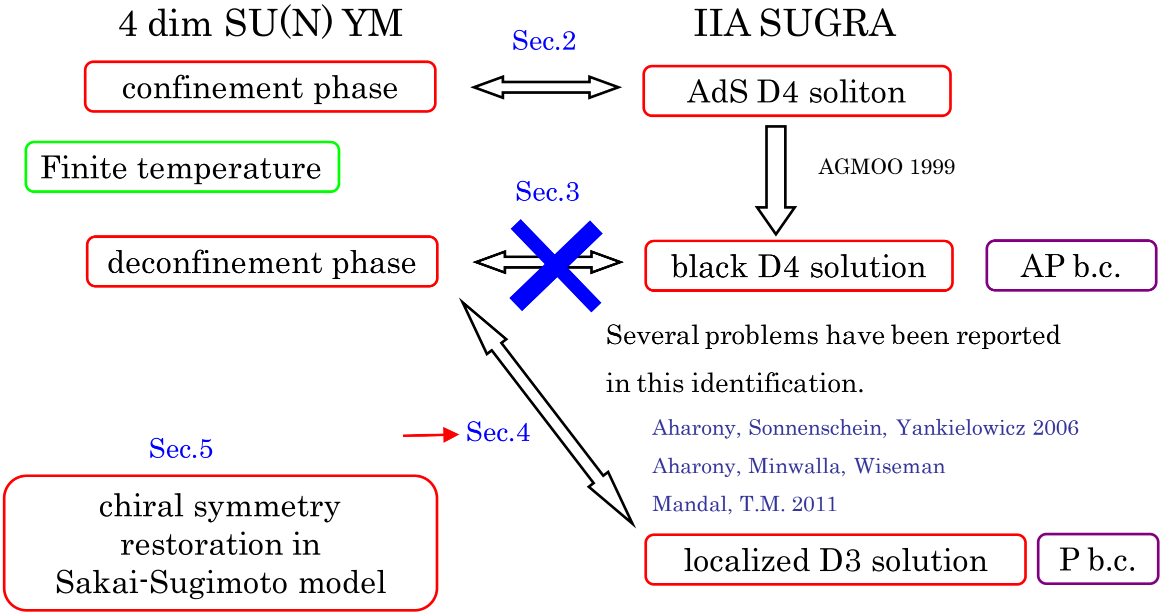
Although the holography predicts the phase diagram of 5d SYM with the (AP,AP) boundary condition consistently, the above identifications to the 4d YM are not correct.

Plan of my talk

◆ Holographic construction of "4d pure Yang-Mills" Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

Sec.2

confinement phase

AdS D4 soliton

AGMOO 1999

Finite temperature

Sec.3

deconfinement phase

black D4 solution

AP b.c.

Several problems have been reported

in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

Mandal, T.M. 2011

Sec.4

Sec.5

chiral symmetry
restoration in
Sakai-Sugimoto model

localized D3 solution

P b.c.

4. Our proposal

Mandal-T.M. 2011

- Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (P,AP) B.C.

$$\tilde{Z} = \mathrm{Tr}\left((-1)^F e^{-\beta H_{5dSYM}}\right) \xrightarrow[\begin{cases}\lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4\end{cases}]{} Z = \mathrm{Tr}\left(e^{-\beta H_{4dYM}}\right)$$

- Possible solution in the P boundary condition along S^1_β.

AdS D4 soliton OK

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(dt^2 + \sum_{i=1}^{3} dx_i^2 + \underline{f_4(u) dx_4^2} \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

$$f_4(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

Black D4 solution NG

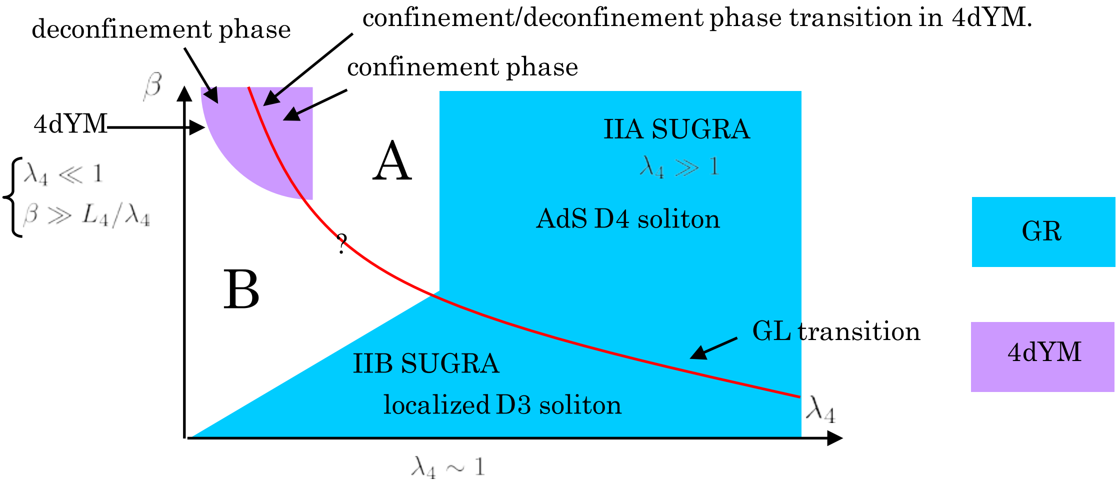
$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4\lambda_5}} \left(\underline{f_4(u) dt^2} + \sum_{i=1}^{3} dx_i^2 + dx_4^2 \right) + \frac{\sqrt{d_4\lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

fermion has to be anti-periodic around the cigar geometry.

4. Our proposal

Mandal-T.M. 2011

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (P,AP) B.C.



A: $\left\{\begin{array}{l} Z_N \text{ along } S^1_\beta \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{array}\right.$

- confinement phase
- AdS D4 soliton

B: $\left\{\begin{array}{l} \cancel{Z_N \text{ along } S^1_\beta} \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{array}\right.$

- deconfinement phase
- Localized D3 soliton ← New solution!

cf. Aharony, Marsano, Minwalla, Wiseman 2004,

Harmark, Obers 2004

4. Our proposal

Mandal-T.M. 2011

◆ Gregory-Laflamme (GL) transition in the P boundary condition.

AdS D4 soliton in IIA SUGRA

↕

$\beta \sim L_4/\lambda_4$: Winding modes along S^1_β are excited.

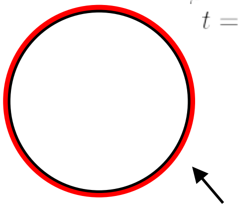
IIA Gravity description is invalid.

T-dual on S^1_β

$(\beta \to \beta' \equiv \alpha'/\beta)$

S^1_β

$t = t + \beta$



N D4 brane

$\beta = cL_4/\lambda_4$

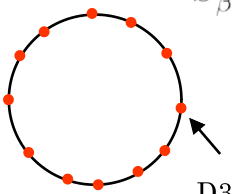
smeared D3 soliton in IIB SUGRA ⟷ localized D3 soliton

Gregory-Laflamm (GL)

transition

$S^1_{\beta'}$

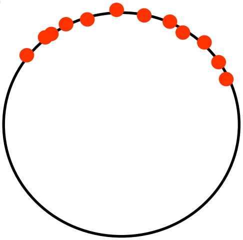
$t' = t' + 1/\beta$



D3 brane

N D3 branes are uniformly distributed.

$\beta > cL_4/\lambda_4$



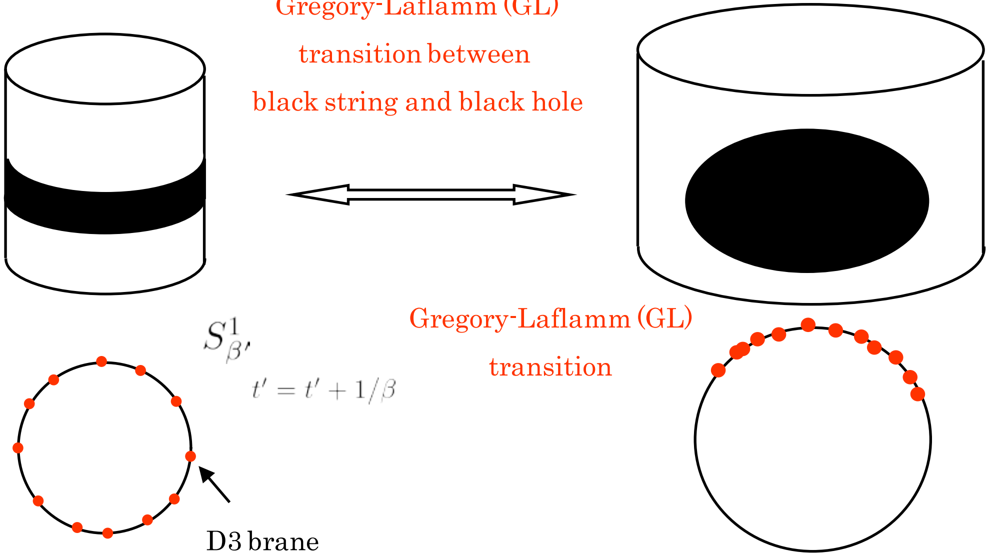
N D3 branes are localized.

$\beta < cL_4/\lambda_4$

4. Our proposal

Mandal-T.M. 2011

◆ Gregory-Laflamme (GL) transition in the P boundary condition.



Gregory-Laflamm (GL) transition between black string and black hole

$S^1_{\beta'}$

$t' = t' + 1/\beta$

Gregory-Laflamm (GL) transition

D3 brane

N D3 branes are uniformly distributed.

$\beta > cL_4/\lambda_4$

N D3 branes are localized.

$\beta < cL_4/\lambda_4$

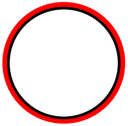
4. Our proposal

Mandal-T.M. 2011

◆ Metrics of gravity solutions

AdS D4 soliton in IIA SUGRA

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4 \lambda_5}} \left(\underline{dt^2} + \sum_{i=1}^{3} dx_i^2 + f_4(u) dx_4^2 \right) + \frac{\sqrt{d_4 \lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + u^2 d\Omega_4^2 \right) \right]$$

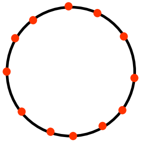


T-dual on S^1_β

$(\beta \to \beta' \equiv \alpha'/\beta)$

$$t = t + \beta \qquad f_4(u) = 1 - \left(\frac{u_0}{u} \right)^3$$

smeared D3 soliton in IIB SUGRA



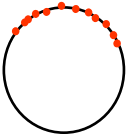
$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4 \lambda_5}} \left(\sum_{i=1}^{3} dx_i^2 + f_4(u) dx_4^2 \right) + \frac{\sqrt{d_4 \lambda_5}}{u^{3/2}} \left(\frac{du^2}{f_4(u)} + \underline{dt'^2} + u^2 d\Omega_4^2 \right) \right].$$

$$t' = t' + 1/\beta$$

GL transition

localized D3 soliton in IIB SUGRA (high temperature & near horizon approximation)

≒ AdS D3 soliton in a flat space (ignore the size of the $S^1_{\beta'}$.)



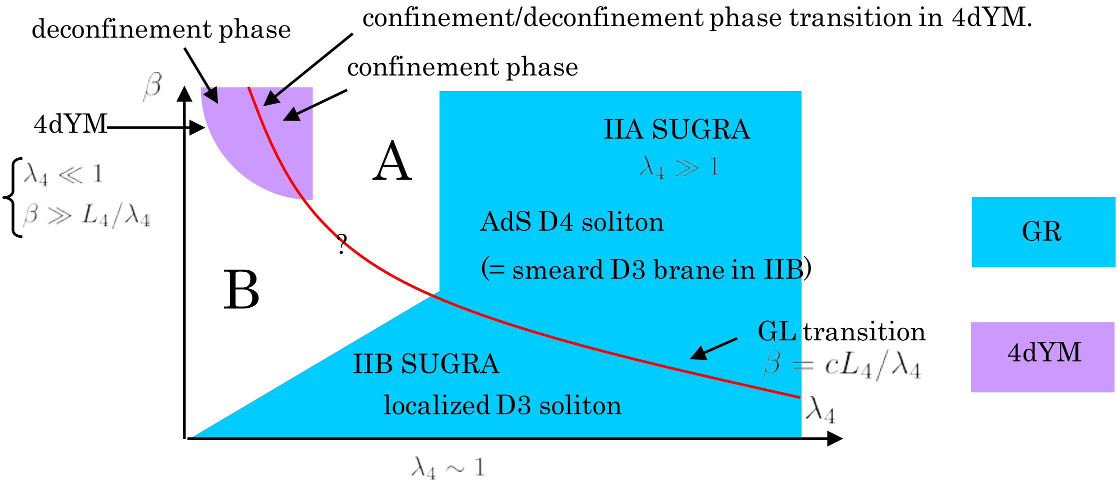
$$ds^2 = \alpha' \left[\frac{\tilde{u}^2}{\sqrt{d_3 \lambda_5/\beta}} \left(\sum_{i=1}^{3} dx_i^2 + f_3(\tilde{u}) dx_4^2 \right) + \frac{\sqrt{d_3 \lambda_5/\beta}}{\tilde{u}^2} \left(\frac{d\tilde{u}^2}{f_3(\tilde{u})} + \tilde{u}^2 d\Omega_5^2 \right) \right].$$

$$f_3(\tilde{u}) = 1 - \left(\frac{\tilde{u}_0}{\tilde{u}} \right)^4, \quad \tilde{u}_0 = \sqrt{d_3 \lambda_5/\beta} \frac{\pi}{2L_4}. \qquad \tilde{u}^2 \sim u^2 + t'^2$$

4. Our proposal

Mandal-T.M. 2011

◆ Phase structure of 5dSYM on $S^1_\beta \times S^1_{L_4}$ with (P,AP) B.C.



deconfinement phase

confinement/deconfinement phase transition in 4dYM.

confinement phase

4dYM

$\begin{cases} \lambda_4 \ll 1 \\ \beta \gg L_4/\lambda_4 \end{cases}$

IIA SUGRA

$\lambda_4 \gg 1$

AdS D4 soliton

(= smeard D3 brane in IIB)

IIB SUGRA

localized D3 soliton

GL transition

$\beta = cL_4/\lambda_4$

GR

4dYM

$$\text{A: } \begin{cases} Z_N \text{ along } S^1_\beta \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{cases} \qquad \text{B: } \begin{cases} \cancel{Z_N \text{ along } S^1_\beta} \\ \cancel{Z_N \text{ along } S^1_{L_4}} \end{cases}$$

We find that the GL transition may continue to
the confinement/deconfinement transition in 4dYM.

4. Our proposal

Mandal-T.M. 2011

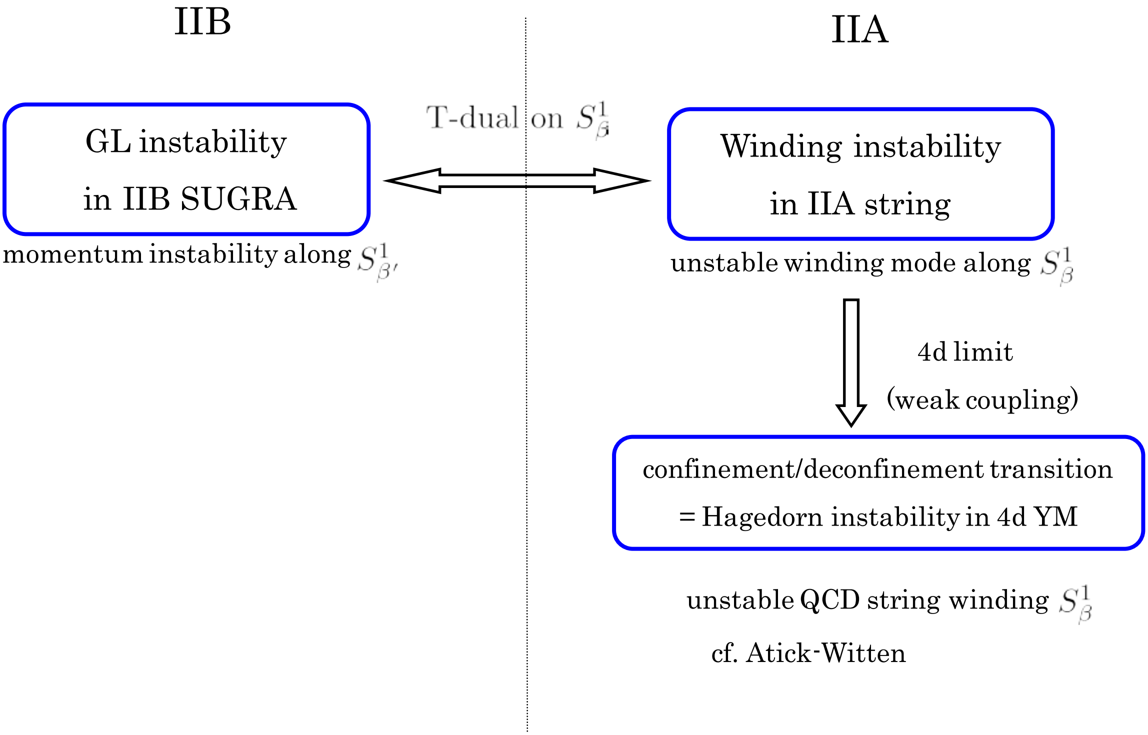
◆ Why is the GL transition related to the C/D transition?

- Natural relation between the GL instability and Hagedorn instability in gauge theory.
- Natural relation between the D3 brane distribution in gravity and the Polyakov loop in guage theory.

4. Our proposal

Mandal-T.M. 2011

◆ GL and Hagedorn transition.



IIB

IIA

T-dual on S^1_{β}

GL instability
in IIB SUGRA

momentum instability along $S^1_{\beta'}$

Winding instability
in IIA string

unstable winding mode along S^1_{β}

4d limit
(weak coupling)

confinement/deconfinement transition
= Hagedorn instability in 4d YM

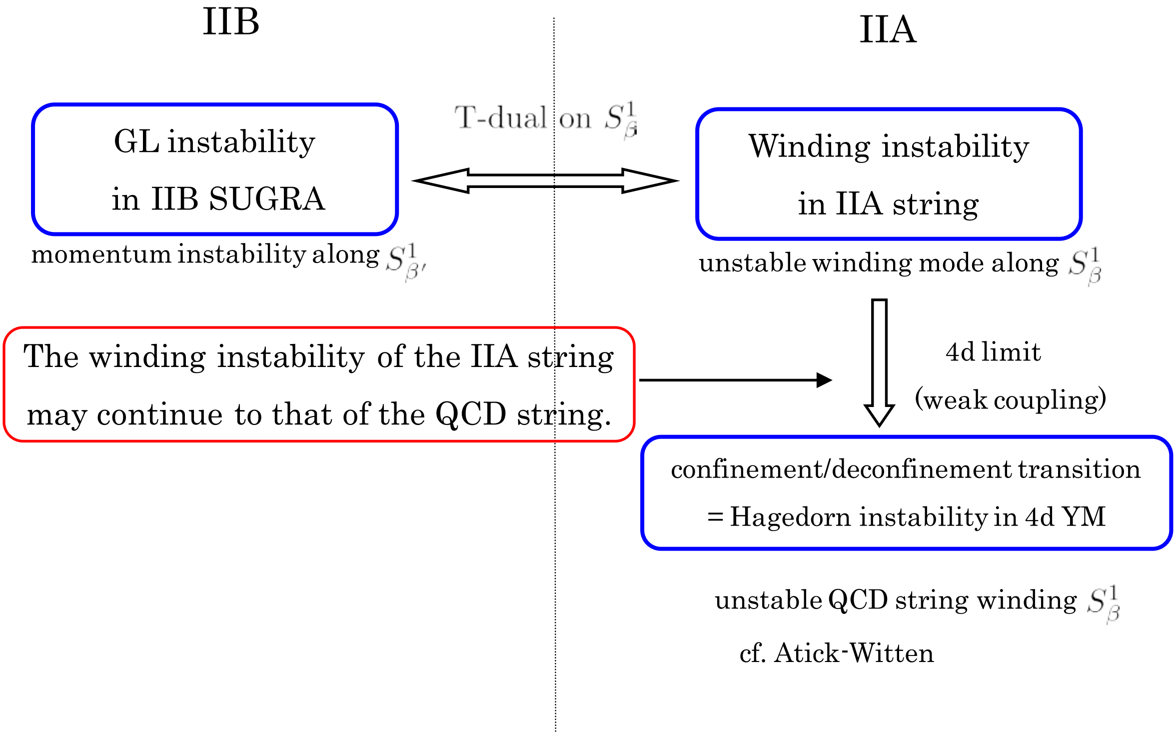
unstable QCD string winding S^1_{β}

cf. Atick-Witten

4. Our proposal

Mandal-T.M. 2011

◆ GL and Hagedorn transition.



IIB

IIA

GL instability
in IIB SUGRA

momentum instability along $S^1_{\beta'}$

T-dual on S^1_β

Winding instability
in IIA string

unstable winding mode along S^1_β

4d limit
(weak coupling)

The winding instability of the IIA string
may continue to that of the QCD string.

confinement/deconfinement transition
= Hagedorn instability in 4d YM

unstable QCD string winding S^1_β

cf. Atick-Witten

4. Our proposal

Mandal-T.M. 2011

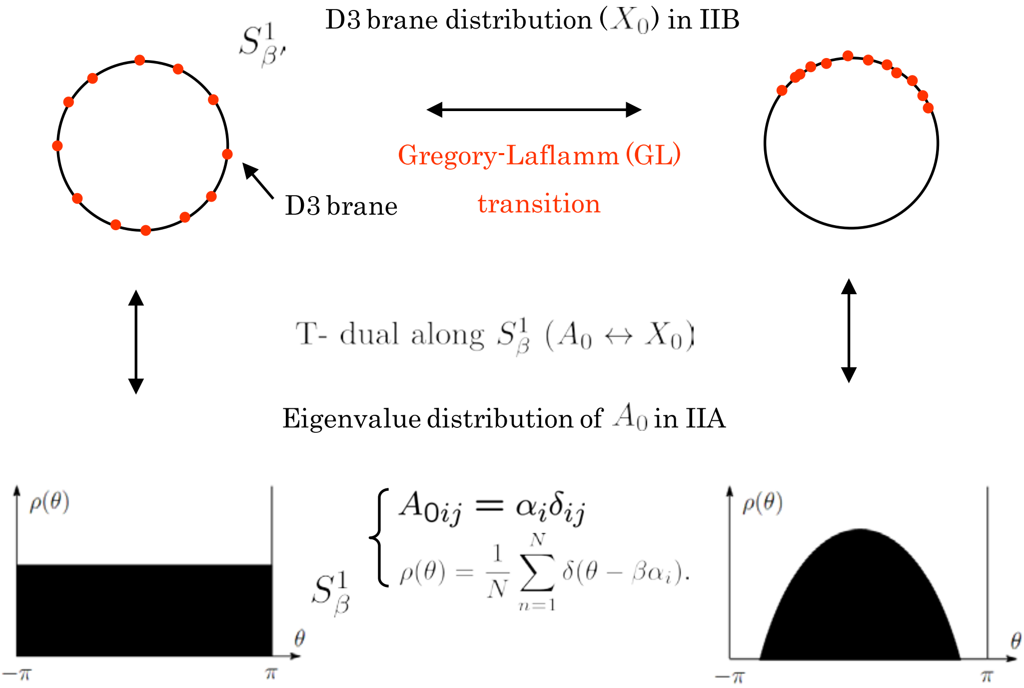
◆ Why is the GL transition related to the C/D transition?

- Natural relation between the GL instability and Hagedorn instability in gauge theory.
- Natural relation between the D3 brane distribution in gravity and the Polyakov loop in guage theory.

4. Our proposal

Mandal-T.M. 2011

◆ D3 brane distribution and Polyakov loop in guage theory



D3 brane distribution (X_0) in IIB

Gregory-Laflamm (GL)

transition

T- dual along S^1_β ($A_0 \leftrightarrow X_0$)

Eigenvalue distribution of A_0 in IIA

$$\begin{cases} A_{0ij} = \alpha_i \delta_{ij} \\ \rho(\theta) = \dfrac{1}{N} \displaystyle\sum_{n=1}^{N} \delta(\theta - \beta\alpha_i). \end{cases}$$

S^1_β

4. Our proposal

Mandal-T.M. 2011

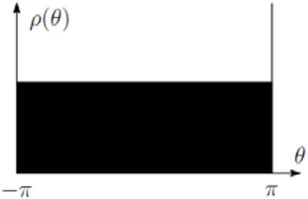
◆ D3 brane distribution and Polyakov loop in guage theory

We can evaluate the Polyakov loop operator which is the order parameter of the confinement/deconfinement transition from the A_0 distribution

$$W_0 = \frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^\beta A_0 dt\right]\right)$$

↑

Eigenvalue distribution of A_0 in IIA



$W_0 = 0$ confinement

S^1_β

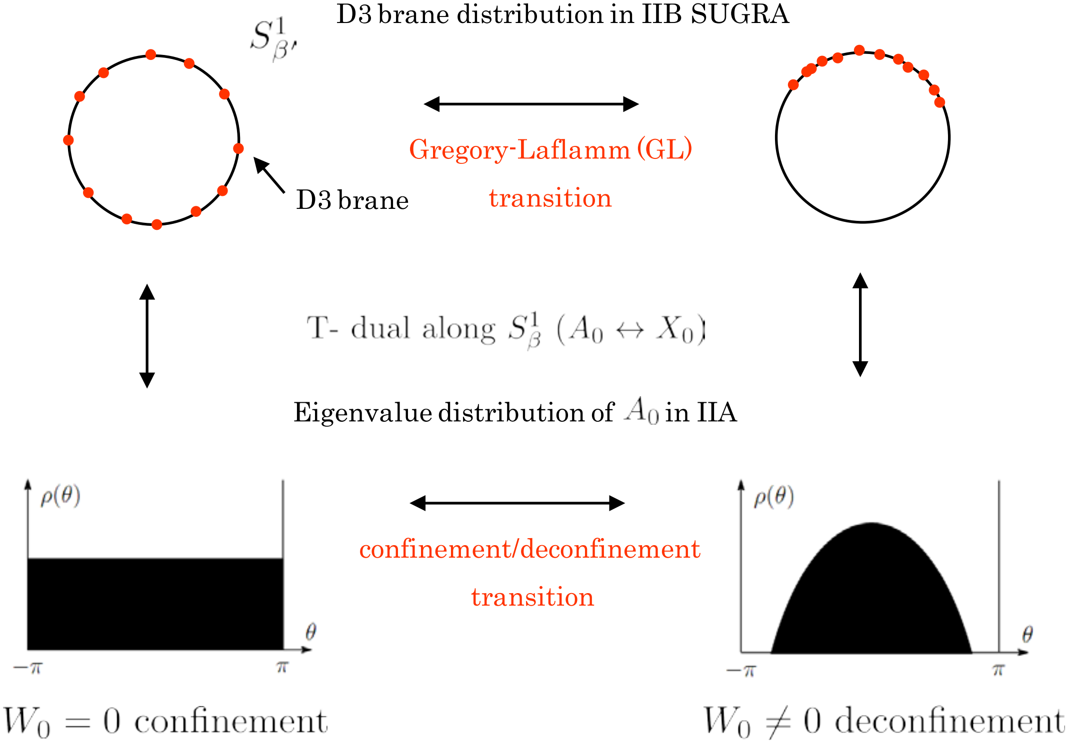
$$\begin{cases} A_{0ij} = \alpha_i \delta_{ij} \\ \rho(\theta) = \frac{1}{N}\sum_{n=1}^{N}\delta(\theta - \beta\alpha_i). \end{cases}$$

$W_0 \neq 0$ deconfinement

4. Our proposal

Mandal-T.M. 2011

◆ D3 brane distribution and Polyakov loop in guage theory



D3 brane distribution in IIB SUGRA

$S^1_{\beta'}$

D3 brane

Gregory-Laflamm (GL) transition

T- dual along S^1_β $(A_0 \leftrightarrow X_0)$

Eigenvalue distribution of A_0 in IIA

confinement/deconfinement transition

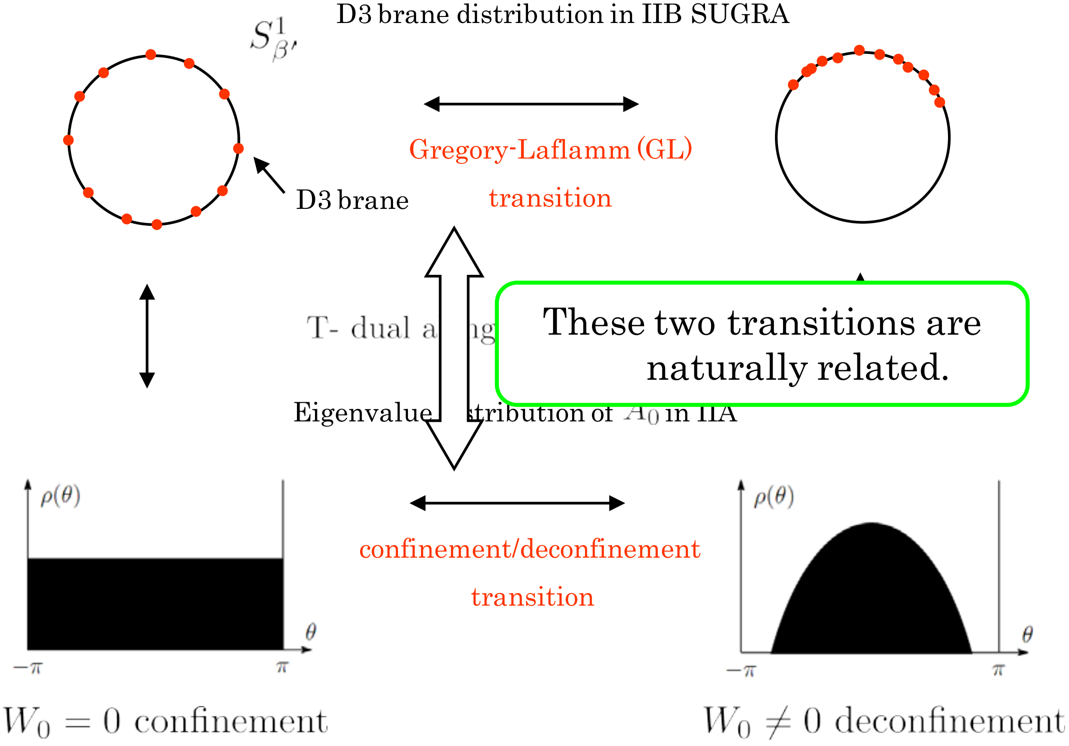
$W_0 = 0$ confinement

$W_0 \neq 0$ deconfinement

4. Our proposal

Mandal-T.M. 2011

◆ D3 brane distribution and Polyakov loop in guage theory



D3 brane distribution in IIB SUGRA

D3 brane

Gregory-Laflamm (GL) transition

T- dual a ng

Eigenvalue tribution of A_0 in IIA

These two transitions are naturally related.

confinement/deconfinement transition

$W_0 = 0$ confinement

$W_0 \neq 0$ deconfinement

4. Our proposal

Mandal-T.M. 2011

◆ Why is the GL transition related to the C/D transition?

- Natural relation between the GL instability and Hagedorn instability in gauge theory.
- Natural relation between the D3 brane distribution in gravity and the Polyakov loop in guage theory.

⇓

We can explicitly confirm these agreements in a 2d gauge theory;

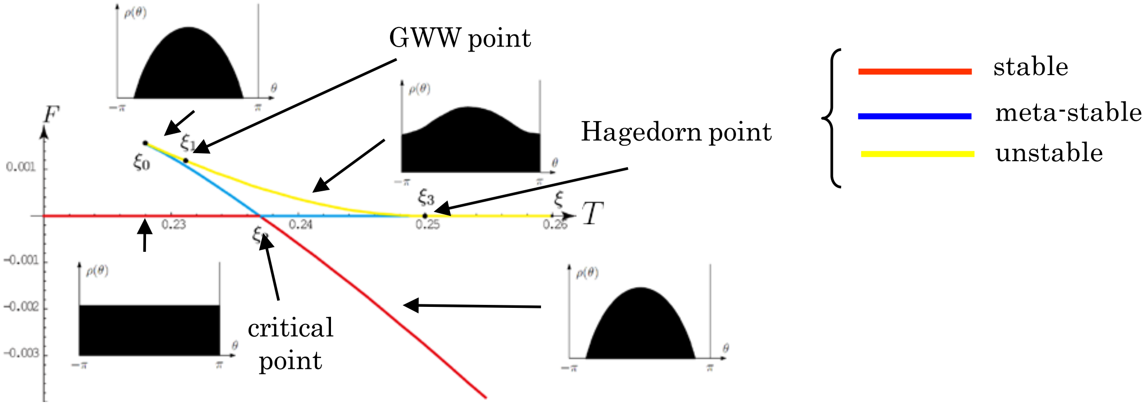
$$S=\int_0^\beta dt\int dx\,Tr\left(\frac{1}{2g^2}F_{01}^2+\sum_{I=1}^{8}\frac{1}{2}\left(D_\mu Y^I\right)^2-\sum_{I,J}^{8}\frac{g^2}{4}[Y^I,Y^J][Y^I,Y^J]\right)$$

We can construct the gravity dual of this theory by using D2 brane and obtain the same phase structure to the 4d YM case. On the other hand, a gauge theory analysis is possible by using a 1/D expansion. Mandal-T.M. 2011

4. Our proposal

Mandal-T.M. 2011

◆ Confinement/deconfinement transition in the 2d gauge theory.



GWW point

Hagedorn point

critical point

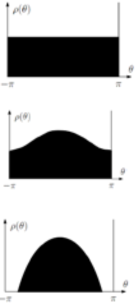
- stable
- meta-stable
- unstable

Fig. Free energy vs. temperature in confinement/deconfinement transition through a 1/D expansion.

Mandal-T.M. 2011

3 Polyakov loop distributions appear:

- uniform distribution: (confinement)
- non-uniform distribution: (deconfinement)
- gapped distribution: (deconfinement)



$$\begin{cases} A_{0ij} = \alpha_i \delta_{ij} \\ \rho(\theta) = \frac{1}{N} \sum_{n=1}^{N} \delta(\theta - \beta\alpha_i). \end{cases}$$

4. Our proposal

Mandal-T.M. 2011

◆ The GL transition in gravity

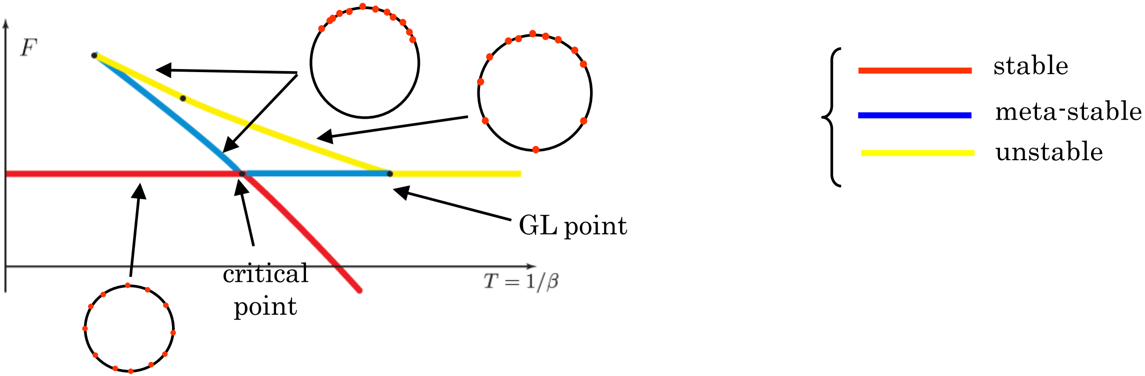
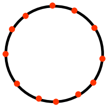


Fig. Free energy vs. temperature in a typical 1st order GL transition

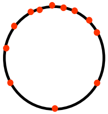
Kudoh-Wiseman 2004

3 solutions appear:

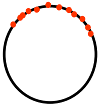
- AdS D4 (= uniformly smeard solitonic D3):

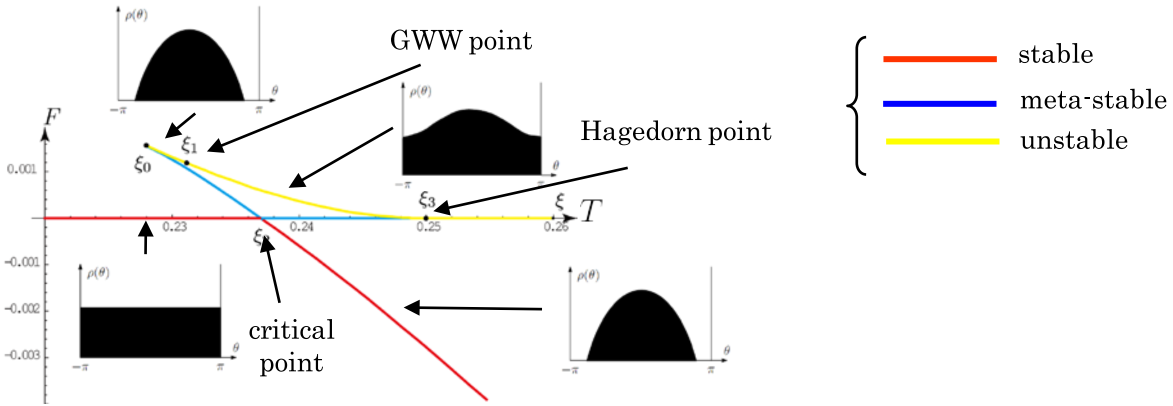


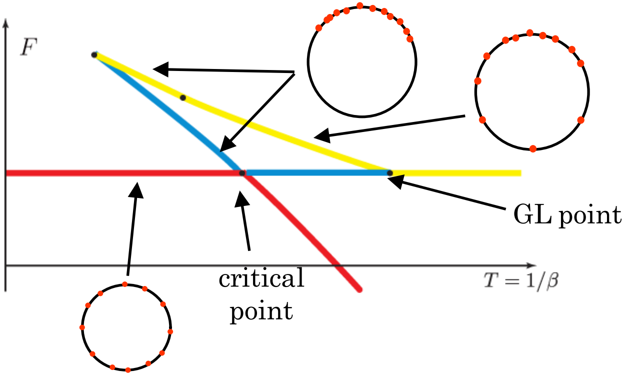
- non-uniform smeared solitonic D3:

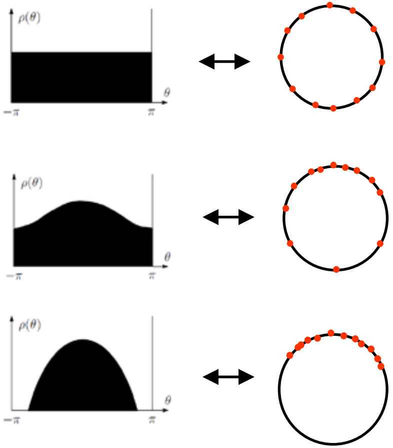


- localized solitonic D3:









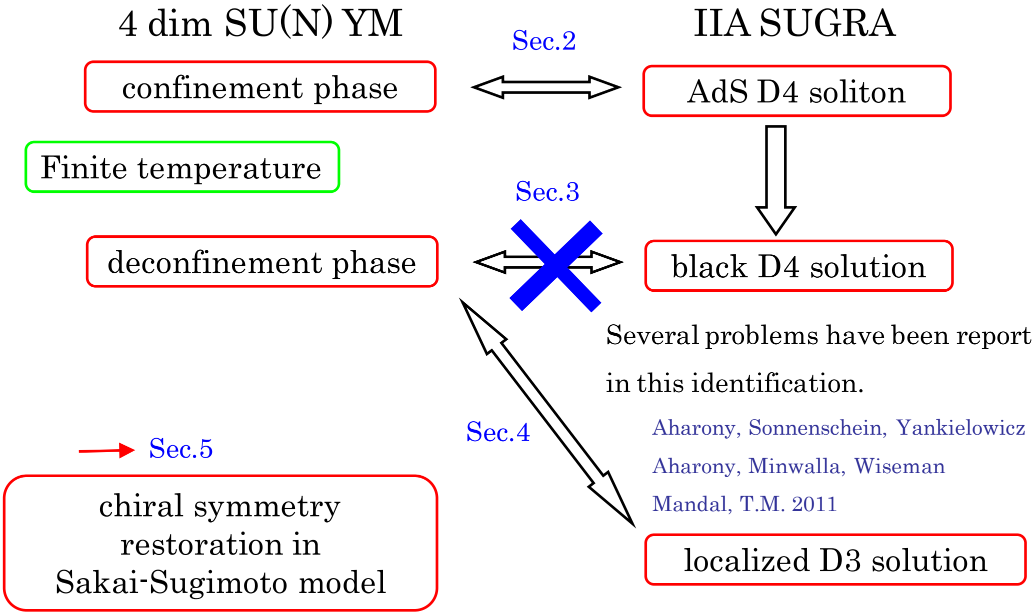
These two transitions share similar nature!

Plan of my talk

◆ Holographic construction of "4d pure Yang-Mills" Witten 1998

4 dim SU(N) YM ← 5dim SU(N) SYM = N D4 brane

Scherk-Schwarz mechanism



4 dim SU(N) YM

IIA SUGRA

Sec.2

confinement phase

AdS D4 soliton

Finite temperature

Sec.3

deconfinement phase

black D4 solution

Several problems have been reported in this identification.

Aharony, Sonnenschein, Yankielowicz 2006

Aharony, Minwalla, Wiseman

Mandal, T.M. 2011

Sec.4

localized D3 solution

Sec.5

chiral symmetry restoration in Sakai-Sugimoto model

5. chiral symmetry restoration in Sakai-Sugimoto model

◆ Sakai-Sugimoto model

Sakai-Sugimoto 2004

Put $\mathrm{D8}/\overline{\mathrm{D8}}$ branes on the D4 brane geometry and ignore their backreaction

(probe approximation $N_c \gg N_f$)

	(0)	1	2	3	(4)	5	6	7	8	9
N_c D4	—	—	—	—	—					
N_f $\mathrm{D8}/\overline{\mathrm{D8}}$	—	—	—	—		—	—	—	—	—

← Same to Witten's D4 geometry

⇓

SU(N) gluon

⇓

massless quarks

relevant massless fields: gluon+quarks (realistic QCD model)

This system has $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

It is shown that this chiral symmetry is broken in low temperature.

→ We can expect that the chiral symmetry would be restored in a higher temperature as in the real QCD.

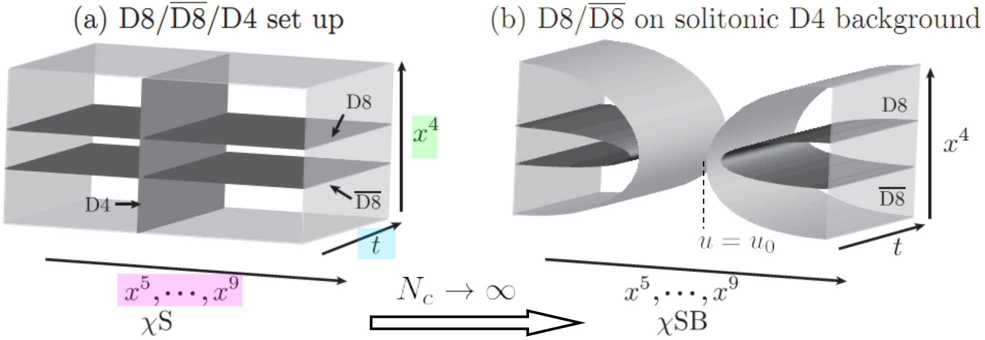
c.f. Aharony, Sonnenschein, Yankielowicz 2006 showed it by using the black D4 geometry, which is not related to the deconfinement phase…

5. chiral symmetry restoration in Sakai-Sugimoto model

◆ Chiral symmetry breaking in a low temperature phase

Sakai-Sugimoto 2004

	(0)	1	2	3	(4)	5	6	7	8	9
N_c D4	—	—	—	—	—					
N_f D8/$\overline{\text{D8}}$	—	—	—	—		—	—	—	—	—



D8/$\overline{\text{D8}}$ are separated.

$U(N_f)_L \times U(N_f)_R$ symmetry

D8/$\overline{\text{D8}}$ are connected.

$U(N_f)_V$ symmetry

AdS D4 soliton

$$ds^2 = \alpha' \left[\frac{u^{3/2}}{\sqrt{d_4 \lambda_5}} \left(dt^2 + \sum_{i=1}^{3} dx_i^2 + \underline{f_4(u) dx_4^2} \right) + \frac{\sqrt{d_4 \lambda_5}}{u^{3/2}} \left(\underline{\frac{du^2}{f_4(u)}} + u^2 d\Omega_4^2 \right) \right]$$

$$f_4(u) = 1 - \left(\frac{u_0}{u} \right)^3$$

$$u^2 = x_5^2 + \cdots + x_9^2$$

5. chiral symmetry restoration in Sakai-Sugimoto model

Mandal-T.M. 2011

◆ Chiral symmetry restoration in high temperature

Strategy

	(0)	1	2	3	(4)	5	6	7	8	9
N_c D4	—	—	—	—	—					
N_f D8/$\overline{\text{D8}}$	—	—	—	—		—	—	—	—	—

Sakai-Sugimoto model

$\Downarrow$ T-dual along the t-circle

	(0′)	1	2	3	(4)	5	6	7	8	9
N_c D3		—	—	—	—					
N_f D7/$\overline{\text{D7}}$		—	—	—		—	—	—	—	—

Evaluate the possible D7 brane configurations after taking the T-duality.

- subtle issue: We will use the P boundary condition although the model involves the quarks.

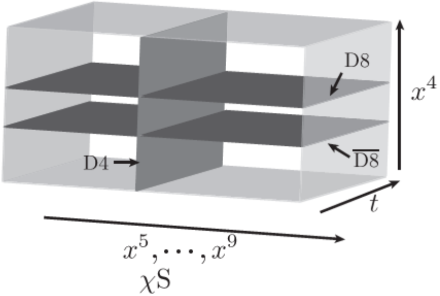
⟶ As far as we use the probe approximation, it may not be relevant.

5. chiral symmetry restoration in Sakai-Sugimoto model

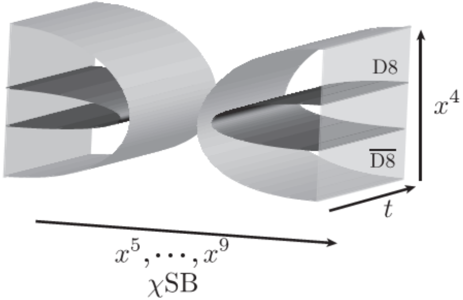
Mandal-T.M. 2011

◆ Chiral symmetry restoration in high temperature

(a) D8/$\overline{\text{D8}}$/D4 set up

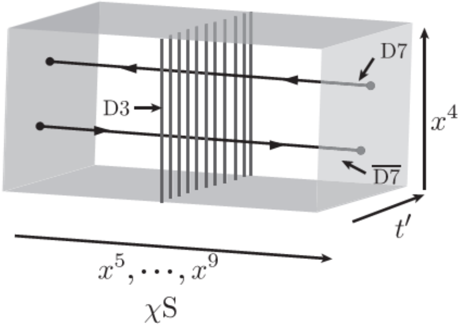


(b) D8/$\overline{\text{D8}}$ on solitonic D4 background

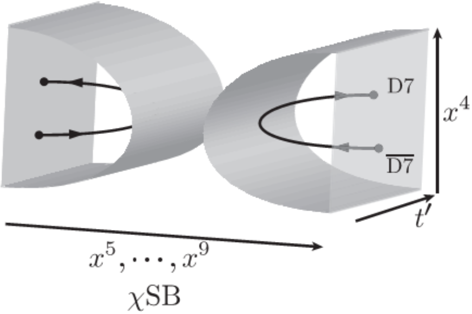


⇩ T-dual along the t-circle

(e) D7/$\overline{\text{D7}}$/D3 set up



(f) D7/$\overline{\text{D7}}$ on uniform SD3 background

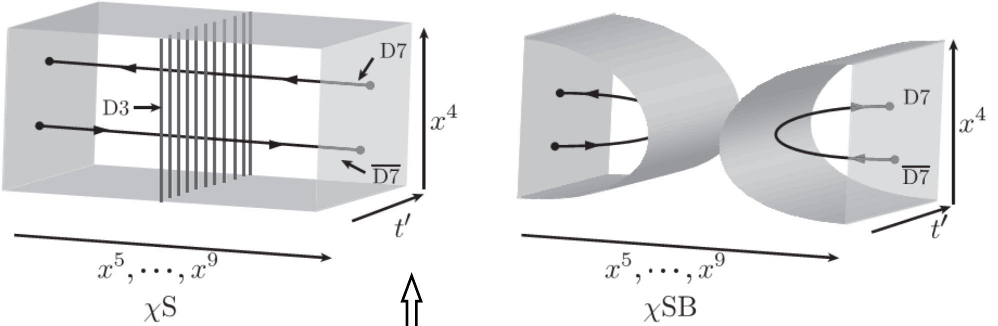


5. chiral symmetry restoration in Sakai-Sugimoto model

Mandal-T.M. 2011

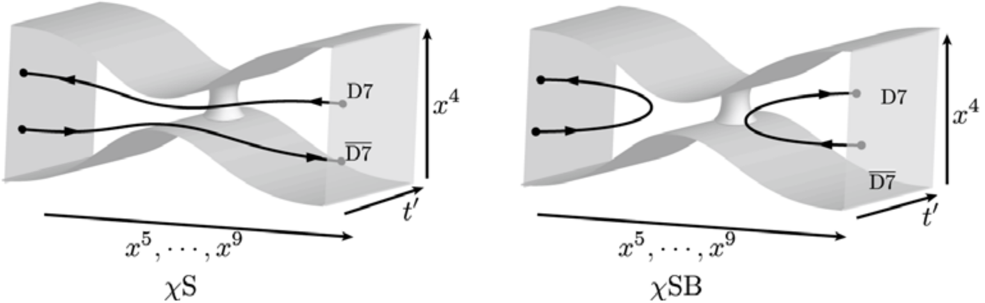
◆ Chiral symmetry restoration in high temperature

(e) D7/$\overline{\text{D7}}$/D3 set up (f) D7/$\overline{\text{D7}}$ on uniform SD3 background



⇕ the GL transition in the IIB

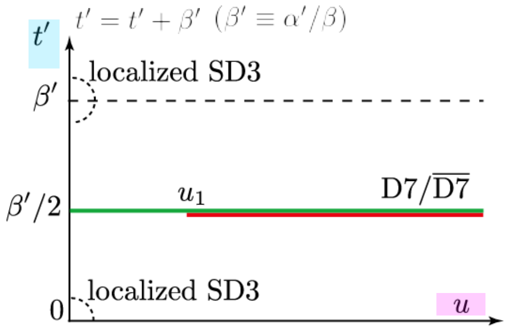
(g) D7/$\overline{\text{D7}}$ on localized SD3 background (h) D7/$\overline{\text{D7}}$ on localized SD3 background

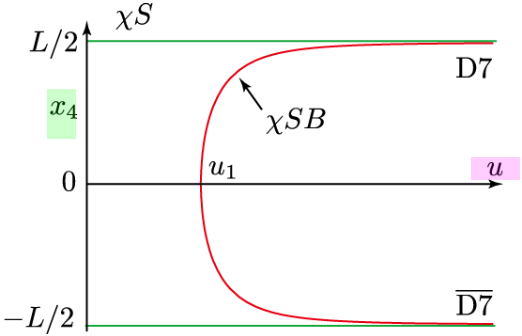


5. chiral symmetry restoration in Sakai-Sugimoto model

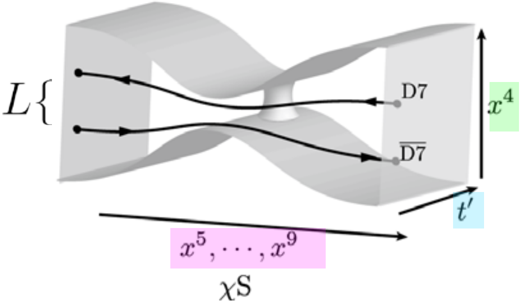
Mandal-T.M. 2011

◆ Chiral symmetry restoration in high temperature

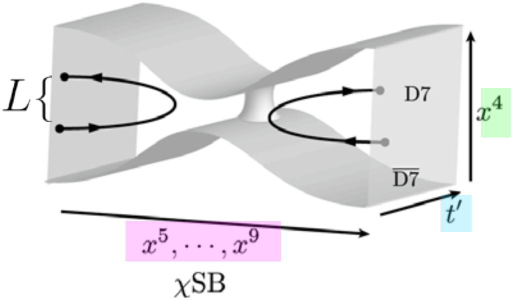




(g) $\mathrm{D7}/\overline{\mathrm{D7}}$ on localized SD3 background



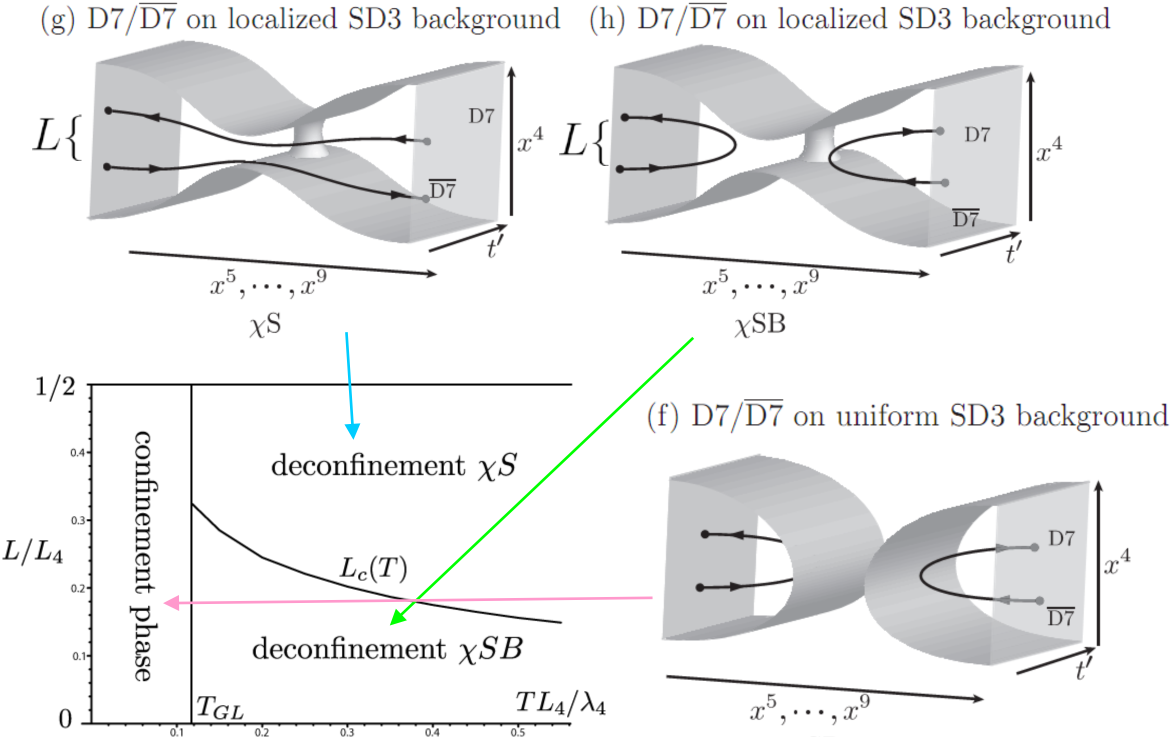
(h) $\mathrm{D7}/\overline{\mathrm{D7}}$ on localized SD3 background



5. chiral symmetry restoration in Sakai-Sugimoto model

Mandal-T.M. 2011

◆ Chiral symmetry restoration in high temperature



(g) $D7/\overline{D7}$ on localized SD3 background

(h) $D7/\overline{D7}$ on localized SD3 background

(f) $D7/\overline{D7}$ on uniform SD3 background

All transitions are 1st order.

Conclusions

- We proposed the following new correspondence in the holographic QCD.

4 dim SU(N) YM		IIA/IIB SUGRA
• confinement phase		• AdS D4 soliton (= smeard D3 soliton)
• deconfinement phase	⟺	• localized D3 soliton in IIB
• C/D transition		• Gregory-Laflamm transition in IIB
• unstable QCD string wrapping S^1_β		• unstable IIA string wrapping S^1_β (= unstable KK mode along $S^1_{\beta'}$)

- Gauge theory analysis of the C/D transition in the 2d gauge theory agree with our conjecture.
- Chiral symmetry restoration/breaking transition in the Sakai-Sugimoto model

Future Direction

- What is the T-dual of the localized D3 soliton?
- Real time formalism. Does the universal viscosity ratio change?
  (No black brane appears in our proposal!)
- Derivation of YM from (AP,AP).

Ignore the black D4 brane solution as a artifact of holographic QCD (c.f. doubler in lattice)