Spinoptics in a Stationary Spacetime

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(Based on V.F. & A.Shoom, Phys.Rev. D84, 044026 (2011))

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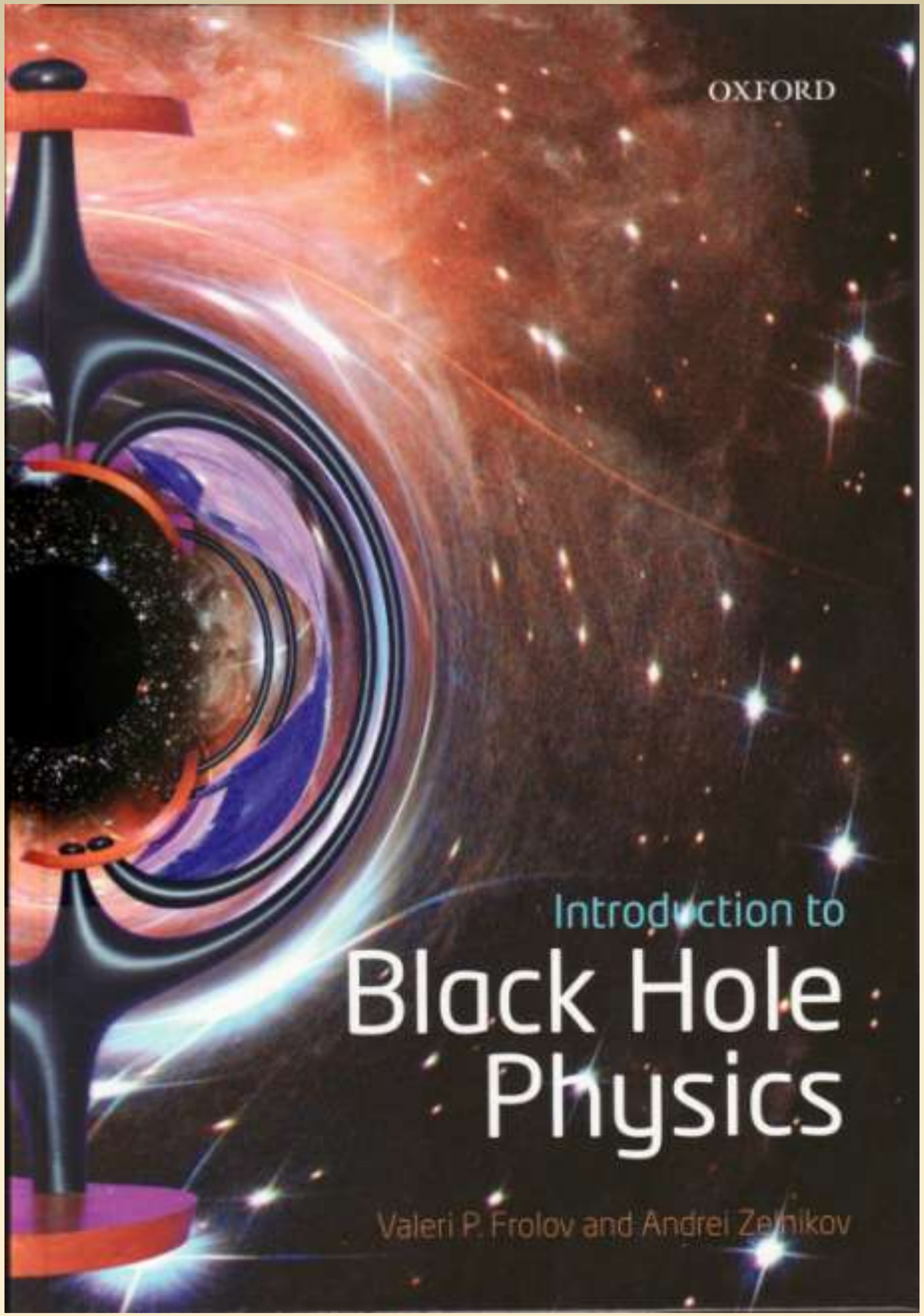


http://www.amazon.ca/Introduction-Black-Physics-Valeri-Frolov/dp/0199692297

Main goal is to study how the spin of a photon affects its motion in the gravitational field

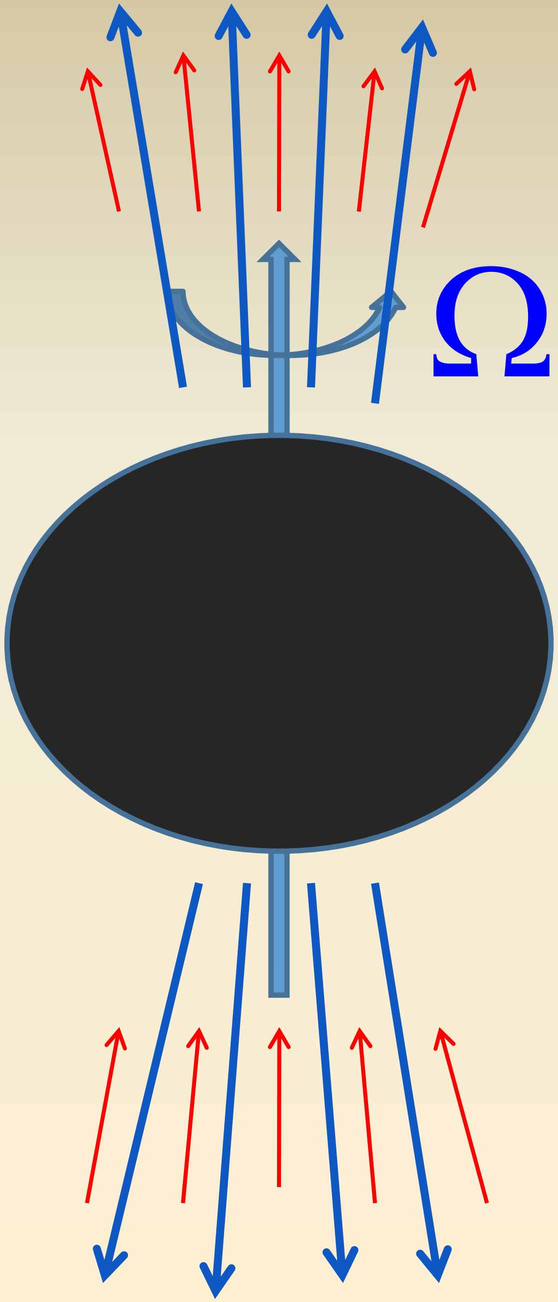
Spin dependence of the kick effect for BH-BH coalescence

Hawking effect is a well known example of gravitational spin-spin interaction

Hawking radiation of a rotating BH:

$$\dot{\mathrm{E}} = \sum_{\mathrm{J}} \int \mathrm{d}\omega \frac{\omega \Gamma_J}{\exp[-T^{-1}(\omega - m\Omega)]}$$

Anisotropy of emission of particles with spin


Ω

Gravito-electromagnetism

Weak field limit:

$$ds^2 = -c^2(1-2\frac{\Phi}{c^2})dt^2 - \frac{4}{c}(\vec{A}\cdot d\vec{x})dt + (1+2\frac{\Phi}{c^2})d\vec{x}^2,$$

$$\Phi \propto \frac{GM}{r}, \quad \vec{A} \propto \frac{G}{c}\frac{\vec{J}\times\vec{x}}{r^3},$$

Transverse gauge condition: $\frac{1}{c}\frac{\partial\Phi}{\partial t} + \nabla\left(\tfrac{1}{2}\vec{A}\right) = 0$

Define: $\vec{E} = \nabla\Phi + \frac{1}{c}\frac{\partial}{\partial t}\left(\tfrac{1}{2}\vec{A}\right), \quad \vec{B} = -\nabla\times\vec{A}$

Then one has:

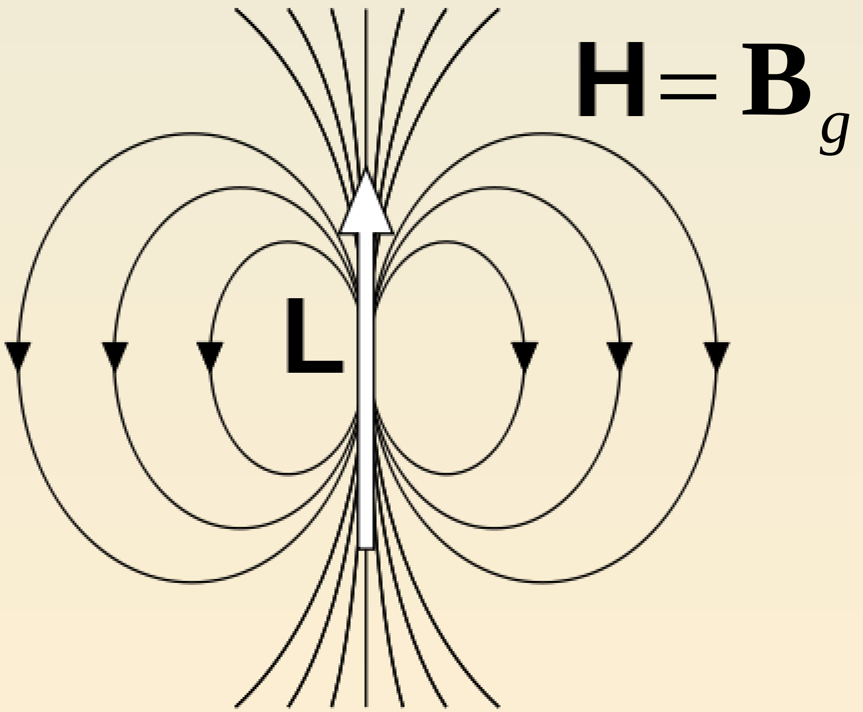
$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\tfrac{1}{2}\vec{B}\right), \quad \nabla\cdot\left(\tfrac{1}{2}\vec{B}\right) = 0,$$

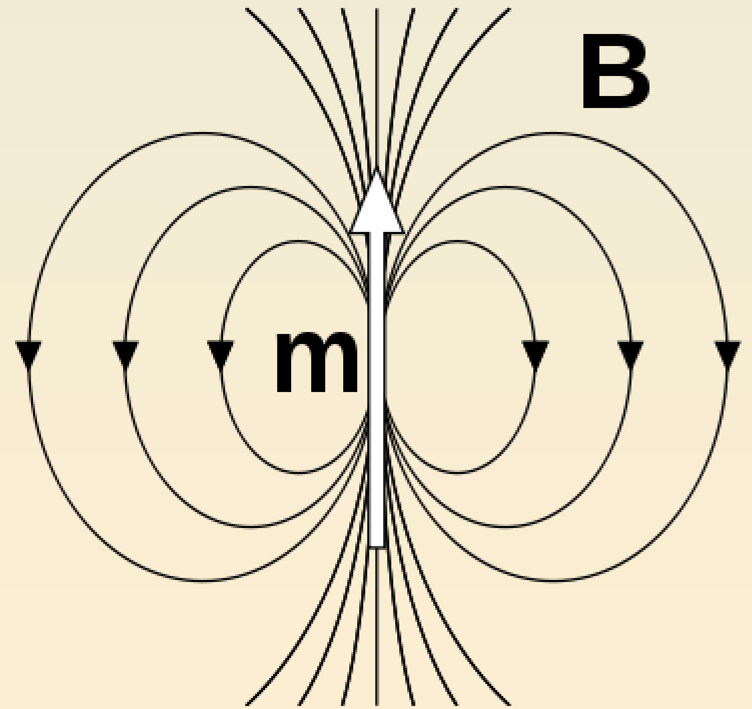
$$\nabla\cdot\vec{E} = -4\pi G\rho, \quad \nabla\times\left(\tfrac{1}{2}\vec{B}\right) = \frac{1}{c}\frac{\partial}{\partial t}\left(\tfrac{1}{2}\vec{E}\right) - \frac{4\pi G}{c}\vec{j},$$

$$\nabla\cdot\vec{j} + \frac{\partial}{\partial t}\rho = 0$$

For a particle motion:

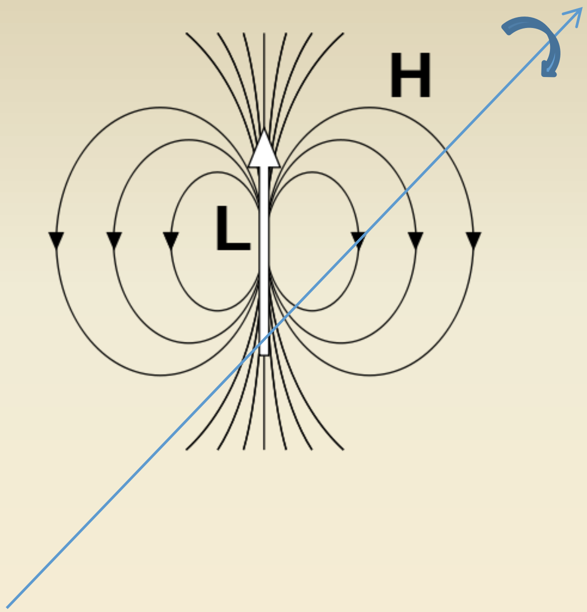
$$\frac{d\vec{p}}{dt} = \vec{F}, \quad \vec{F} = \mu\vec{E} + 2\mu\left[\frac{\vec{v}}{c} \times \vec{B}\right]$$

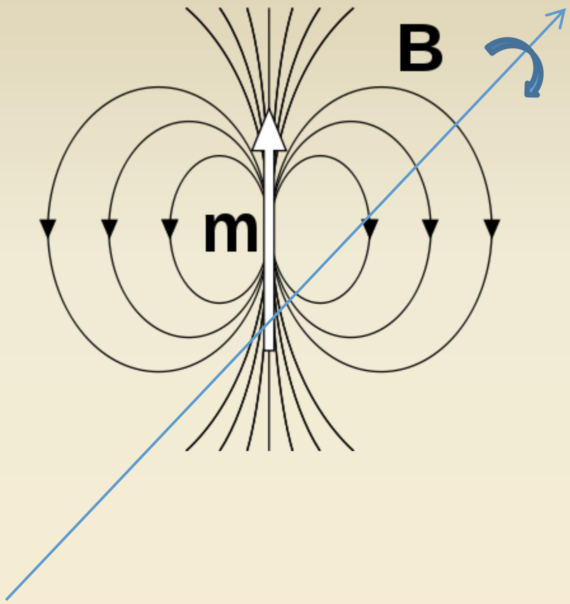




GRAVITY

Electromagnetism





Particle with spin

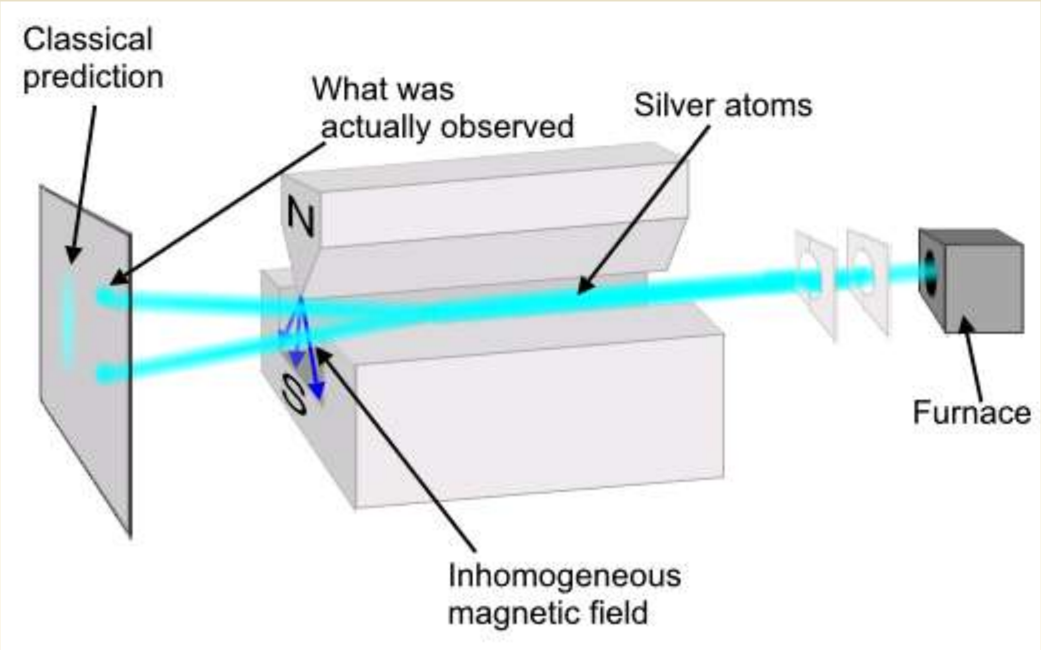
Particle with magnetic dipole moment

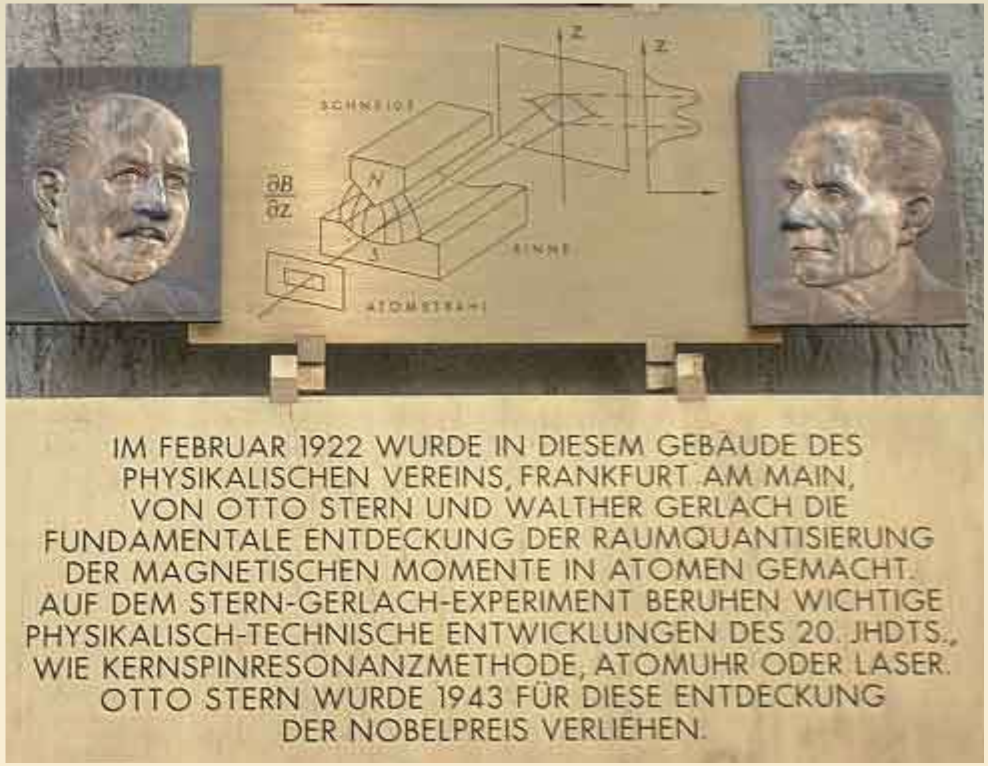
Maxwell equations

Dirac (Pauli) equation

Geometric optics (WKB) approximation

Stern-Gerlach Experiment




IM FEBRUAR 1922 WURDE IN DIESEM GEBÄUDE DES
PHYSIKALISCHEN VEREINS, FRANKFURT AM MAIN,
VON OTTO STERN UND WALTHER GERLACH DIE
FUNDAMENTALE ENTDECKUNG DER RAUMQUANTISIERUNG
DER MAGNETISCHEN MOMENTE IN ATOMEN GEMACHT.
AUF DEM STERN-GERLACH-EXPERIMENT BERUHEN WICHTIGE
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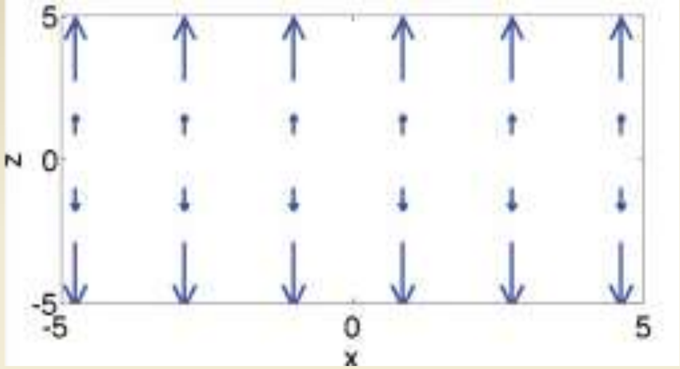
WIE KERNSPINRESONANZMETHODE, ATOMUHR ODER LASER.
OTTO STERN WURDE 1943 FÜR DIESE ENTDECKUNG
DER NOBELPREIS VERLIEHEN.

Hsu, Berrondo, Van Huele “Stern-Gerlach dynamics of quantum propagators”
Phys. Rev. A 83, 012109 (2011)



$$H = \frac{p_x^2 + p_z^2}{2m} - \mu B_1 z \sigma_z$$

$$K(\vec{x}, \vec{x}_0;\ t\ ) = K(x, x_0;\ t\ )K(z, z_0;\ t\ )$$

$$K(x, x_0;\ t\ ) = \sqrt{\frac{m}{2\pi i\hbar t}} \exp\left(-\frac{m(x - x_0)^2}{2i\hbar t} \right)$$

$$K(z, z_0;\ t\ ) = \sqrt{\frac{m}{2\pi i\hbar t}} \exp\left(-\frac{m(z - z_0)^2}{2i\hbar t} - \frac{\mu B_1 \sigma_z (z + z_0) t}{2i\hbar} + \frac{\mu^2 B_1^2 t^3}{24 i\hbar m} \right)$$

$$H = \frac{p_z^2}{2m} - \lambda z, \quad \lambda = \mu B_1 \sigma_z,$$

$$\dot{z} = p_z / m, \quad \dot{p}_z = \lambda,$$

$$S = \int_{z_0}^{z} (p_z dz - H dt) = \frac{m(z - z_0)^2}{2t} + \frac{\mu B_1 \sigma_z (z + z_0) t}{2} + \frac{\mu^2 B_1^2 t^3}{12m},$$

$$K(z, z_0; t) \propto \exp(iS / \hbar)$$

Magnetic moment of the electron: $\mu = \frac{g}{2} \mu_B, \quad \mu_B = \frac{e\hbar}{2m_e}$

By applying formal WKB to the Pauli equation with this μ, one would get

$$K(z, z_0; t) \propto \exp(iS_0 / \hbar), \quad S_0 = \frac{m(z - z_0)^2}{2t}$$

Lessons

(i) In the exact solution for a wave packet there exists correlation between orientation of spin and spatial trajectory of electron;

(ii) At late time the up and down spin wave packets are moving along classical trajectories;

(iii) Formal WKB solution represents the motion of the`center of mass’ of two packets

$$L = z - z_0 = Vt; \quad \Delta x = \frac{\mu B_1}{m}\frac{t^2}{2};$$

Condition when terms with μ become important can be written as

$$\Delta x \sim L \Rightarrow 2m(z - z_0) \sim \mu B_1 t^2$$

or, equivalently, $L \sim mV^2/(\mu B_1)$

To obtain a correct long time asymptotic behavior of the wave packet one needs:

(i) to `diagonalize' the field equations;
(ii) to `enhance' spin-dependent term
(iii) include it in the eikonal function

$$\begin{pmatrix}\text{Dirac}\\ \text{equation}\end{pmatrix} \Rightarrow \begin{pmatrix}\text{Two component}\\ \text{positive freq. eqn.}\end{pmatrix} \Rightarrow$$

$$\begin{pmatrix}\text{WKB approxi-}\\ \text{mation}\end{pmatrix} \Rightarrow \begin{pmatrix}\text{Particle with}\\ \text{classical spin}\end{pmatrix} \Rightarrow$$

$$\begin{pmatrix}\text{Enhanced}\\ \text{phase space}\end{pmatrix}$$

Geometric optics (WKB) - `big picture'

(i) Field equation: $\hat{D}\psi = 0, \;\; \hat{D} = D(x, -i\varepsilon\partial_x)$

(ii) WKB ansatz: $\psi(x) = A(x)\exp(iS(x)/\varepsilon)$

(iii) The `leading order' term gives: $D(x,k) = 0, \; k = \partial_x S$

(iv) To solve the first order PDE for $S(x)$ one uses the method of integrating along the characteristics

(v) Lagrangian manifold is N dimensional surface in $2N$ dimensional phase space upon which the canonical symplectic 2-form $dk \wedge dx$ vanishes

(vi) One has on this surface $\partial_i k_j = \partial_j k_i$

(vii) Action function $S(x) = \int k(x)dx$ (independent of path)

$(viii)$ If the Lagrangian manifold is a subset of the $(2N-1)$ dimensional surface $D(x,k) = 0,$ then $S(x)$ satisfies the Hamilton-Jacobi equation

(ix) The transport equation for the amplitude is $\partial_i (A^2 \partial_{k_i} D(x,k)) = 0$

Phase Space

Phase space: $\{\mathrm{P}^{2m}, \Omega, H\}$

Symplectic form Ω is a closed non-degenerate 2-form

$d\Omega=0$ ($\Omega = d\alpha$)

Hamiltonian H is a scalar function on the symplectic manifold P^{2m}

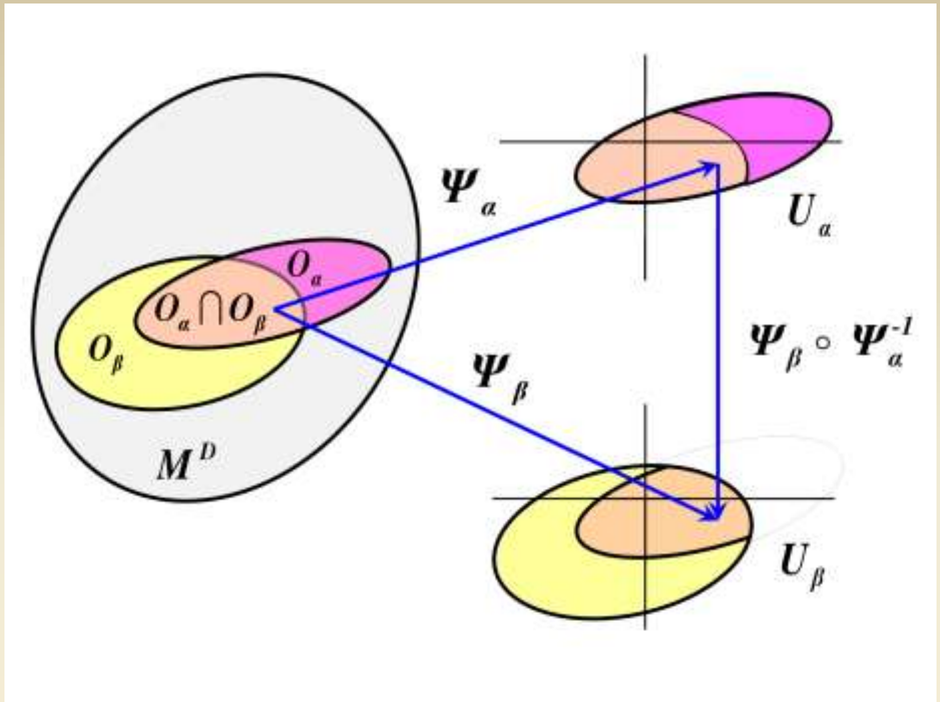
z^A are coordinates on P^{2m}

Poisson bracket $\{F,G\}=\Omega^{AB}F_{,A}G_{,B}$

$\eta^{A}=H_{,B}\Omega^{BA}$ is a generator of the Hamiltonian flow

Equation of motion is $\dot{z}^{A}=\eta^{A}$

One has $\dot{F}=\{H,F\}$



Darboux theorem:

In the vicinity of any point it is always possible to choose canonical coordinates

$z^A = (p_1, \ldots, p_m, q_1, \ldots, q_m)$ in which $\Omega = \sum_{i=1}^{m} dp_i \wedge dq_i$

Lagrangian manifold $M^n \subset P^{2n}$:

$$F_i(p,q) = f_i \quad \Rightarrow \quad p_i = p_i(f,q)$$

$$\Omega = \sum_i dp_i \wedge dq_i = \sum_{i,j} (\partial p_i / \partial q_k)\, dq_k \wedge dq_i = 0$$

On M^n one has: $\Omega = d\alpha, \quad \alpha = \sum_i p_i dq_i$,

$$S(q) = \int_{q^0}^{q} \sum_i p_i(f,q)\, dq_i$$

Illustration: Solving eqn.

$$\tfrac{1}{2}(\nabla S)^2 = 1 \quad (*)$$

$(q_1,\ldots,q_n) \in Q^n; \quad (p_1,\ldots,p_n) \in P^n;$

Initial position of the wave front:

$\tilde{S}(\vec{q}) = 0 \Rightarrow \vec{q}^{\,0} \in \Gamma^{n-1} \subset Q^n; \quad F(\vec{q})\tilde{S}(\vec{q}) = 0;$

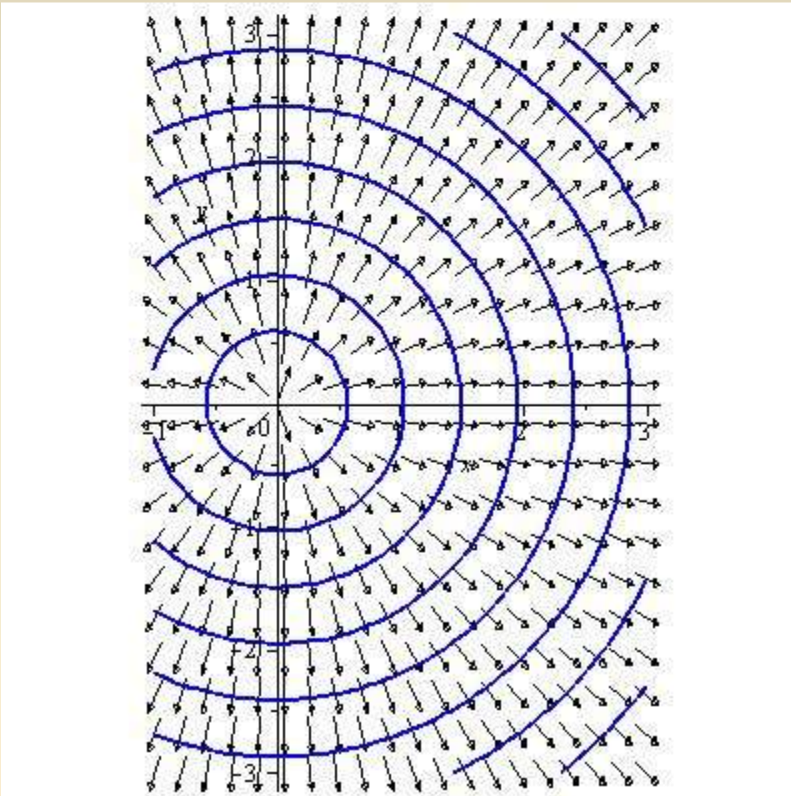
Condition (*) is satisfied on Γ^{n-1} for a special choice

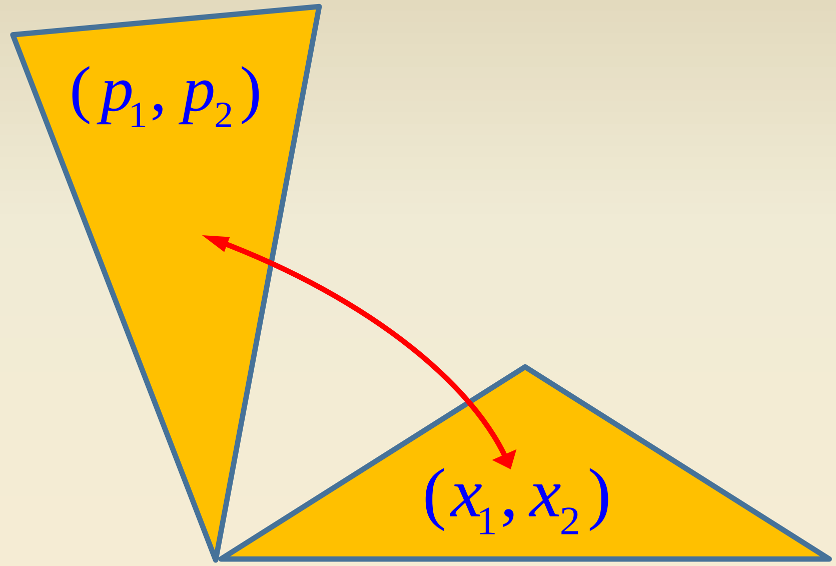
$S^0 = \tilde{S}/(\nabla\tilde{S}\nabla\tilde{S})^{1/2}; \Rightarrow p_i^0 = \nabla_i S^0$

For (q_i^0, p_i^0) solve dynamical eqn. with Hamiltonian $\mathrm{H} = \vec{\mathrm{p}}^2/2$.

This gives $(n-1)$ – parameter family of curves, i.e. n – dim

Lagrangian surface and $S(\vec{q}) = \int \vec{p}\, d\vec{q}$ is a required solution of (*).





$$\vec{p} = \Gamma(\vec{x}) : \quad M^2 \subset R^4$$

Spinoptics in gravitational field

(i) Spin induced effects
(ii) Many-component field
(iii) Helicity states
(iv) Massless field
(v) Gauge invariance

Maxwell equations in a Stationary ST

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -h(dt - g_i \, dx^i)^2 + h\gamma_{ij} \, dx^i \, dx^j$$

$$t \to \tilde{t} = t + q(x^i), \quad g_i \to \tilde{g}_i = g_i + q_{,i}$$

$$h = -g_{00} \, , g_i = \frac{g_{0i}}{h}, -g = h^4 \gamma$$

$$\gamma_{ij} = -\frac{g_{ij}}{g_{00}} + \frac{g_{0i} g_{0j}}{g_{00}^2} = \frac{g_{ij}}{h} + g_i g_j$$

3+1 form of Maxwell equations

$$E_i \equiv F_{i0}\,, \quad B_{ij} \equiv F_{ij}\,, \quad D^i \equiv h^2 F^{0i}\,, \quad H^{ij} \equiv h^2 F^{ij}\,.$$

$$D_i = E_i - H_{ij} g^j\,, \quad B^{ij} = H^{ij} - E^i g^j + E^j g^i\,.$$

$$B_{ij} = e_{ijk} B^k\,, \quad H^{ij} = e^{ijk} H_k\,.$$

$$C = [A \times B],\, C^i = e^{ijk} A_j B_k \quad \Rightarrow \quad D = E - [g \times H],\, B = H + [g \times E].$$

$$\mathrm{div}\vec{B} = 0,\, \mathrm{curl}\vec{E} = -\dot{\vec{B}}, \quad \mathrm{div}\vec{D} = 0,\, \mathrm{curl}\vec{H} = \dot{\vec{D}}.$$

$$\mathrm{E} \equiv \frac{1}{8\pi}[(\vec{E},\vec{D}) + (\vec{B},\vec{H})], \quad \vec{V} \equiv \frac{1}{4\pi}[\vec{E} \times \vec{H}], \qquad \dot{\mathrm{E}} + \mathrm{div}\vec{V} = 0$$

Master equation for c-polarized light

$$\vec{F}^{\pm} \equiv \vec{E} \pm i\vec{H}, \quad \vec{G}^{\pm} \equiv \vec{D} \pm i\vec{B}$$

$$\vec{E} = e^{-i\omega t}\mathcal{E} + e^{i\omega t}\mathcal{E}^*, \quad \vec{H} = e^{-i\omega t}\mathcal{H} + e^{i\omega t}\mathcal{H}^*$$

$$\vec{D} = e^{-i\omega t}\mathcal{D} + e^{i\omega t}\mathcal{D}^*, \quad \vec{B} = e^{-i\omega t}\mathcal{B} + e^{i\omega t}\mathcal{B}^*,$$

$$\mathcal{F}^{\pm} = \mathcal{E} \pm i\mathcal{H}, \quad \mathcal{G}^{\pm} = \mathcal{D} \pm i\mathcal{B}.$$

$$\mathrm{div}\mathcal{G}^{\pm} = 0, \quad \mathrm{curl}\mathcal{F}^{\pm} = \pm\omega\mathcal{G}^{\pm}, \quad \mathcal{G}^{\pm} = \mathcal{F}^{\pm} \pm i[\vec{g} \times \mathcal{F}^{\pm}]$$

$$\mathrm{curl}\mathcal{F}^{\pm} = \pm\omega\mathcal{F}^{\pm} + i\omega[\vec{g} \times \mathcal{F}^{\pm}]$$

Summary of Step 1

- Monochromatic waves of frequency ω
- Two helicity states $\sigma = \pm$
- For a single photon its helicity is a conserved quantity
- For a classical em beam of light its circular polarization is conserved if an external field does not create photons
- Right (left) circularly polarized modes for $-$ $(+)$
- Master equation covers states with given circular polarization
- Complex vector functions $\mathcal{F}^{\pm}(x^i)$ of 3 variables describe monochromatic em modes with given helicity

“Standard” geometric optics

Small parameter: $\varepsilon=(\omega\ell)^{-1}$

ℓ is characteristic length scale of the problem

GO ansatz $\mathscr{F}^{\sigma} = \vec{f}^{\,\sigma} e^{i\omega S}$

$$L\vec{f} \equiv \vec{f} - i\sigma[\vec{n}\times\vec{f}\,],$$

$$\vec{n} \equiv \vec{p} - \vec{g}, \quad \vec{p} \equiv \nabla S,$$

Exact equation: $L\vec{f} = \sigma\omega^{-1}\mathrm{curl}\,\vec{f}$

GO expansion: $\vec{f} = \vec{f}_0 + \omega^{-1}\vec{f}_1 + \omega^{-2}\vec{f}_2 + \ldots$

$$
\begin{aligned}
& L\vec{f}_0 \\
& \quad + \omega^{-1}[L\vec{f}_1 - \sigma \operatorname{curl}\vec{f}_0] + \ldots \\
& \quad + \omega^{-2}[L\vec{f}_2 - \sigma \operatorname{curl}\vec{f}_1] + \ldots = 0
\end{aligned}
$$

$$L\vec{f}_0 = 0, \quad L\vec{f}_{i+1} = \sigma \operatorname{curl}\vec{f}_i$$

Properties of the operator L

$$\det L = 0 \quad \Rightarrow \quad (n,n) = 1,$$

$$\det(L - \lambda I) = 0 \quad \Rightarrow$$

$$(1-\lambda)[(1-\lambda)^2 - (n,n)] = 0;$$

Eigenvectors of $L: \quad \lambda = (0,1,2)$

Eigenvectors $(\vec{e}_1, \vec{n}, \vec{e}_2)$ are orthonormal

$$\vec{m} \equiv \frac{1}{\sqrt{2}}(\vec{e}_1 + i\sigma\vec{e}_2), \vec{m}^* \equiv \frac{1}{\sqrt{2}}(\vec{e}_1 - i\sigma\vec{e}_2)$$

In the basis $(\vec{n}, \vec{m}, \vec{m}^*)$ one has $L = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{Vmatrix}.$

Since $L\vec{f}_0 = 0 \Rightarrow \vec{f}_0 = A\vec{m},$

A is a complex function of x^i

$$(\vec{n}, \vec{f}_0) = 0,$$

$$\text{curl}\vec{f} = \vec{n}(\vec{n}, \text{curl}\vec{f}) - i\sigma[\vec{n} \times \text{curl}\vec{f}]$$

Ray trajectories

Eikonal equation: $(\nabla S - \vec{g})^2 = 1$

Effective Hamiltonian: $H(x^i, p_i) \equiv \frac{1}{2}(\vec{p} - \vec{g})^2 = \frac{1}{2}\gamma^{ij}(p_i - g_i)(p_j - g_j),$

Canonical symplectic form: $\Omega = \sum_{i=1}^{3} dp_i \wedge dx^i$

$$H = \frac{1}{2} \quad \Rightarrow \quad \left(\frac{d\vec{x}}{d\ell}\right)^2 = 1$$

$$\frac{D^2\vec{x}}{d\ell^2} = \left[\frac{d\vec{x}}{d\ell} \times \operatorname{curl} \vec{g}\right] \qquad S(x) = \int_{x_0}^{x} (\vec{p}, d\vec{x}) = \int_{x_0}^{x} \left[1 + \left(\vec{g}, \frac{d\vec{x}}{d\ell}\right)\right] d\ell$$

Transport equation

We determine vectors of the basis $(\vec{n}, \vec{m}, \vec{m}^*)$ along rays by requiring that they are Fermi transported;

$$\mathbf{F}_n\, \vec{a} = \nabla_n \vec{a} - (\vec{n}, \vec{a})\, \vec{w} + (\vec{w}, \vec{a})\, \vec{n}, \quad \vec{w} = \nabla_n \vec{n}$$

$$\mathscr{F}^{\sigma} \approx f_0^{\sigma}\, \vec{m}^{\sigma} e^{i\omega \tilde{S}(\vec{x})},$$

$$\tilde{S}(\vec{x}) = \int_{\vec{x}_0}^{\vec{x}} \left[1 + \left(\vec{\tilde{g}}, \frac{d\vec{x}}{d\ell} \right) \right] d\ell,$$

$$\vec{\tilde{g}} = \vec{g} + \frac{\sigma}{2\omega} \operatorname{curl} \vec{g}$$

Faraday rotation

Linearly polarized light: $\mathscr{F} = \mathscr{F}^{+} + \mathscr{F}^{-},$

$\mathscr{F} \approx \sqrt{2}\, f_0 \vec{k}_0\, e^{i\omega S}$

$\vec{k}_0 \equiv \vec{e}_1 \cos\varphi - \vec{e}_2 \sin\varphi, \quad (\vec{k}_0, \vec{k}_0) = 1$

$$\frac{d\varphi}{d\ell} = \frac{1}{2}(\mathrm{curl}\,\vec{g}, \vec{n})$$

The vector of polarization $\vec{k}_0$ rotates with respect to the Fermi propagated frame with the angular velocity

$$\Omega = \frac{1}{2}(\mathrm{curl}\,\vec{g}, \vec{n})$$

Characteristic scale $L_F : \Delta\phi = L_F\, |\nabla \vec{g}\,| \propto 2\pi$

$L_F \propto 4\pi / |\nabla \vec{g}\,|$

4-D point of view:

(i) Light ray is a 4D null geodesic

(ii) Vector of linear polarization is 4D parallel transported

Modified geometric optics

$$\vec{\tilde{n}} \equiv \vec{p} - \vec{\tilde{g}}, \quad \vec{p} \equiv \nabla \tilde{S}, \quad \vec{\tilde{g}} \equiv \vec{g} + \frac{\sigma}{2\omega} \mathrm{curl}\, \vec{g},$$

$$\tilde{L}\, \vec{f} \equiv \vec{f} - i\sigma[\vec{\tilde{n}} \times \vec{f}\,] = 0,$$

$$\tilde{L}\, \vec{f} = \frac{\sigma}{\omega} \mathrm{curl}\, \vec{f} + \frac{i}{2\omega}[\mathrm{curl}\, \vec{g} \times \vec{f}\,]$$

$$\tilde{L}\vec{f}_0 = 0, \quad \tilde{L}\vec{f}_{i+1} = \sigma \operatorname{curl} \vec{f}_i$$

$$\det \tilde{L} = 0 \quad \Rightarrow \quad (\tilde{\vec{n}}, \tilde{\vec{n}}) = 1 \quad \Rightarrow \quad (\nabla \tilde{S} - \tilde{\vec{g}}) = 1,$$

$$\tilde{H}(x^i, p_i) = \frac{1}{2}(\vec{p} - \tilde{\vec{g}})^2 \equiv \frac{1}{2}\gamma^{ij}(p_i - \tilde{g}_i)(p_j - \tilde{g}_j),$$

In the basis $(\tilde{\vec{n}}, \tilde{\vec{m}}, \tilde{\vec{m}}^*)$ one has $\tilde{L} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{Vmatrix}$.

$$(\tilde{\vec{n}}, \tilde{\vec{n}}) = 1, \; (\tilde{\vec{n}}, \tilde{\vec{m}}) = (\tilde{\vec{n}}, \tilde{\vec{m}}^*) = 0, \; (\tilde{\vec{m}}, \tilde{\vec{m}}^*) = 1$$

We denote a Fermi transported frame along a modified ray as $(\vec{\tilde{n}}, \vec{\tilde{m}}, \vec{\tilde{m}}*)$. After solving the transport equation we obtain:

$$\boldsymbol{\mathcal{F}}^{\sigma} \propto \tilde{f}_0^{\,\sigma}\, \vec{\tilde{m}}\, e^{i\omega \tilde{S}(\mathbf{x})},$$

$$\tilde{S}(\vec{x}) = \int_{\vec{x}_0}^{\vec{x}} \left[1 + \left(\vec{\tilde{g}}, \frac{d\vec{x}}{d\ell} \right) \right] d\ell, \quad \vec{\tilde{g}} = \vec{g} + \frac{\sigma}{2\omega} \operatorname{curl} \vec{g}$$

$$\frac{D^2 \vec{x}}{d\ell^2} = \left[\frac{d\vec{x}}{d\ell} \times \operatorname{curl} \vec{g} \right] + \frac{\sigma}{2\omega} \left[\frac{d\vec{x}}{d\ell} \times \operatorname{curl} \operatorname{curl} \vec{g} \right],$$

$$\tilde{S}(\vec{x}) = \int_{\vec{x}_0}^{\vec{x}} \left[d\ell + (\vec{g}, d\vec{x}) + \frac{\sigma}{2\omega} (\operatorname{curl} \vec{g}, d\vec{x}) \right]$$

Application to Kerr metric

$$h = (\Delta - a^2 \sin^2\theta)/\Sigma, \quad g_i = -\frac{2aMr}{\Sigma h}\sin^2\theta\delta_i^{\phi},$$

$$d\ell^2 = \gamma_{ij}dx^i dx^j = \frac{\Sigma}{\Delta h}dr^2 + \frac{\Sigma}{h}d\theta^2 + \frac{\Delta\sin^2\theta}{h^2}d\phi^2$$

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 - 2Mr + a^2$$

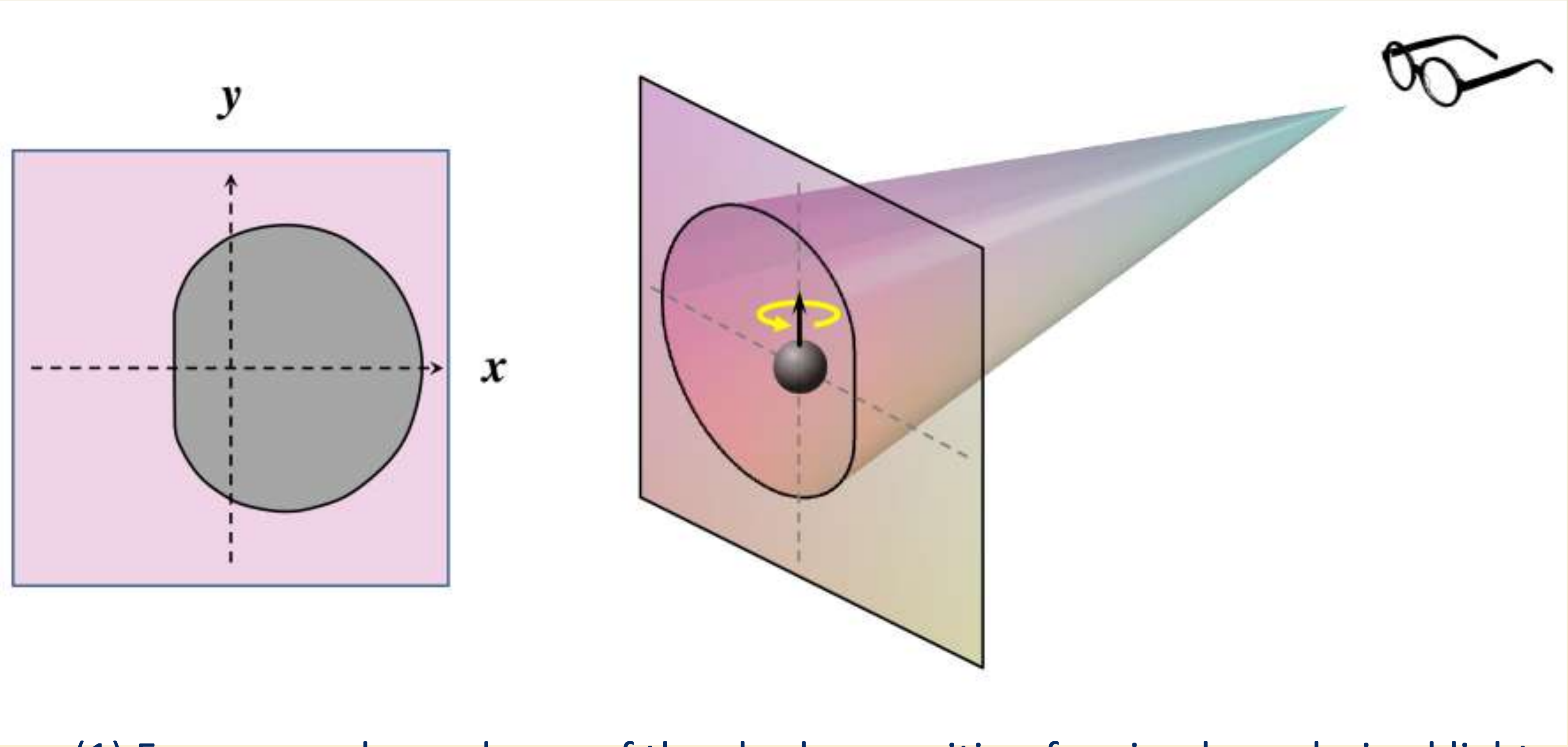
Black hole horizon: $\Delta = 0 \quad \Rightarrow \quad r = r_{\pm} = M + \sqrt{M^2 - a^2}$

Black hole ergosurface: $h = 0 \quad \Rightarrow \quad r = r_e = M + \sqrt{M^2 - a^2\cos^2\theta}$

$$(\mathrm{curl}\, g)^i = -\frac{4aMr\Delta}{\Sigma^3}\cos\theta\,\delta^i_r - \frac{2aM(r^2 - a^2\cos^2\theta)}{\Sigma^3}\sin\theta\,\delta^i_\theta\,,$$

$$(\mathrm{curl}\,\mathrm{curl}\, g)^i = \frac{4aM^2}{\Sigma^3}\delta^i_\phi\,.$$

Rainbow effect for BH shadow



(1) Frequency dependence of the shadow position for circular polarized light;

(2) For given frequency shadow position depends on the polarization

SUMMARY

(1) Standard GO picture: In the Kerr ST a linearly polarized photon moves a null geodesic and its polarization vector is parallel propagated.

(2) Modified GO picture: Linear polarized photon beam splits into two circular polarized beams.

(3) Right and left polarized photons have different trajectories.

(4) In a stationary ST their motion can be obtained by introducing frequency dependent effective metric.