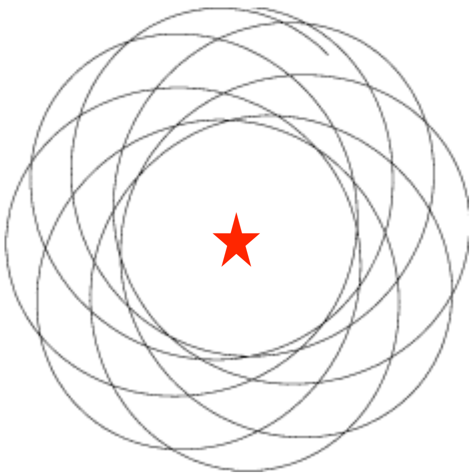
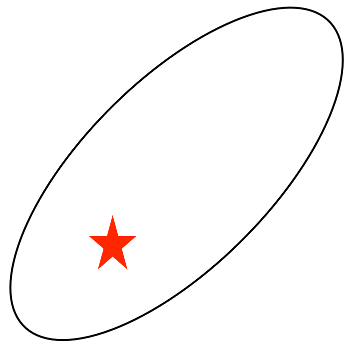
Kozai-Lidov oscillations

- Kozai (1962 - asteroids); Lidov (1962 - artificial satellites)
- arise most simply in restricted three-body problem (two massive bodies on a Kepler orbit + a test particle)
- e.g., wide binary star + planet orbiting one member of the binary
- in Kepler potential $\Phi = -GM/r$, eccentric orbits have a fixed orientation



generic axisymmetric potential



Kepler potential

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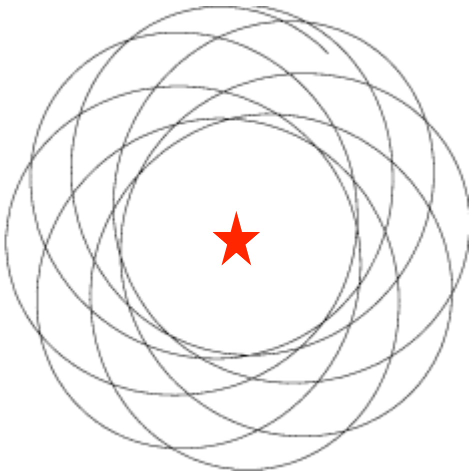

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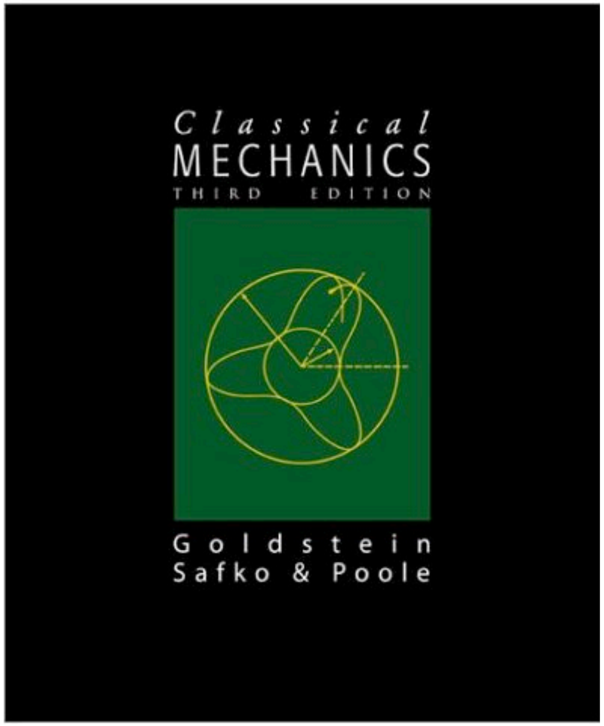
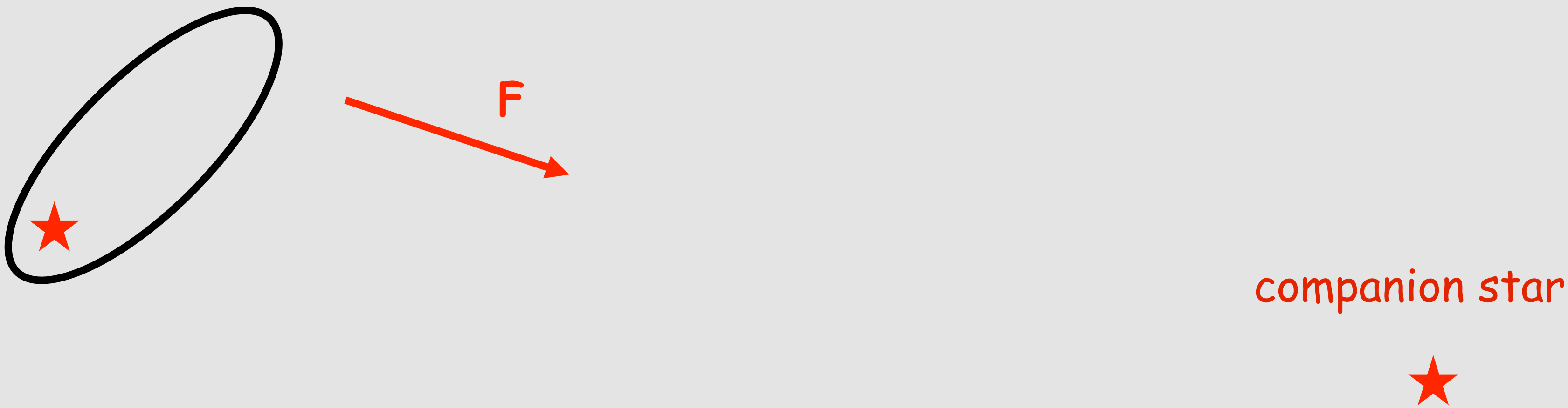


Kozai-Lidov oscillations

• now subject the Kepler orbit to a weak, time-independent external force **F** from the companion star

• because the orbit orientation is fixed even weak external forces act for a long time in a fixed direction relative to the orbit and therefore change the angular momentum or eccentricity

• if **F ~ ε** then **timescale** for evolution **~ 1/ε** but **nature** of evolution is independent of **ε**



Kozai-Lidov oscillations

Consider a planet orbiting one member of a binary star system:

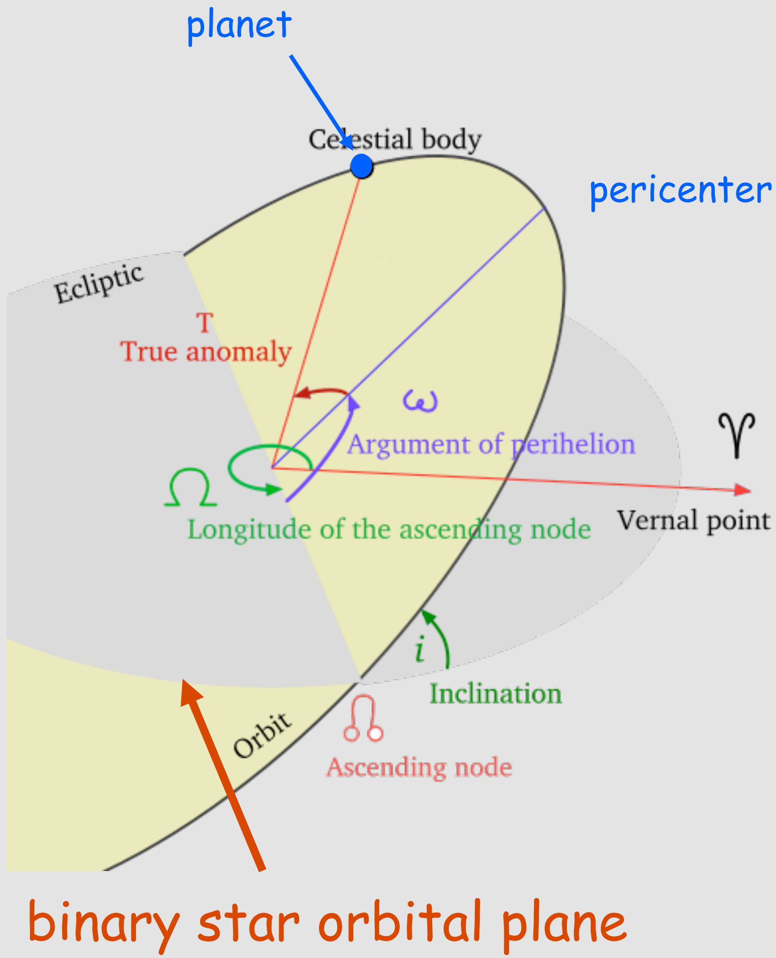
• because the force from the companion star is weak we can average over both planetary and binary star orbits

• keep only the quadrupole term from the companion

• because of averaging the gravitational potential from the companion is fixed, so energy E is conserved ($E=-GM_*/2a$ so semi-major axis a is conserved)

• for circular companion orbit the potential is axisymmetric so J_z is conserved

• accidentally, it turns out that J_z is conserved even if companion orbit is eccentric



Averaged Hamiltonian is

$$H = \epsilon[5e^2 \sin^2 i \sin^2 \omega - (1 - e^2) \cos^2 i - 2e^2]$$

where

$$\epsilon \equiv \frac{3GM_c a^2}{8(1 - e_c^2)^{3/2} a_c^3}.$$

Action-angle variables are

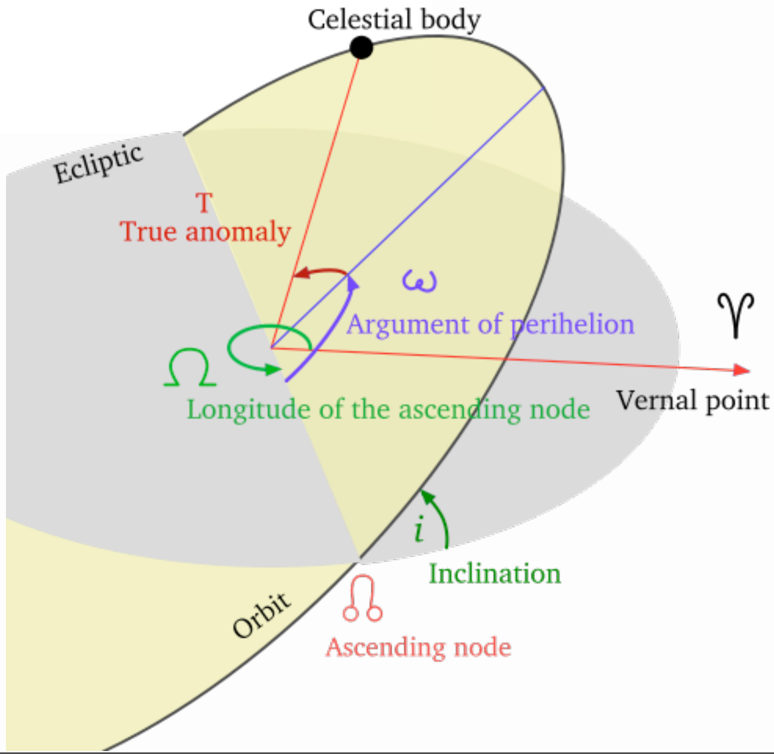
z-angular momentum longitude of node

$$J_1 = [GM_\star a(1 - e^2)]^{1/2}, \quad J_2 = J_1 \cos i, \quad \theta_1 = \omega, \quad \theta_2 = \Omega.$$

angular momentum argument of pericenter

Hamiltonian is independent of Ω so J_2 is conserved. Remaining motion has one degree of freedom and follows $H =$ constant contours.

$$\frac{dJ_1}{dt} = -\frac{\partial H}{\partial \omega}, \quad \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1}.$$



Let $\mathbf{j}$ point in the direction of the angular momentum vector with magnitude $|\mathbf{j}| = (1 - e^2)^{1/2}$. Let $\mathbf{e}$ point towards pericenter with magnitude e. Then

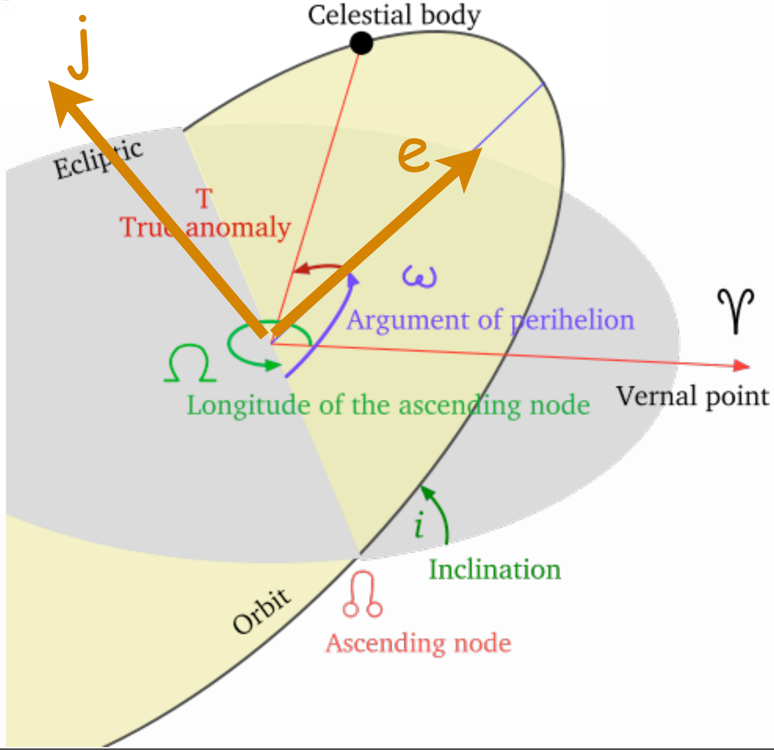
$$H = \epsilon[5(\mathbf{e} \cdot \mathbf{n})^2 - (\mathbf{j} \cdot \mathbf{n})^2 - 2e^2]$$

where $\mathbf{n}$ is the normal to the companion orbit. The equations of motion are

$$\frac{d\mathbf{j}}{d\tau} = \mathbf{e} \times \nabla_{\mathbf{e}} H + \mathbf{j} \times \nabla_{\mathbf{j}} H$$

$$\frac{d\mathbf{e}}{d\tau} = \mathbf{j} \times \nabla_{\mathbf{e}} H + \mathbf{e} \times \nabla_{\mathbf{j}} H$$

where $\tau = t/(GM_\star a)^{1/2}$.



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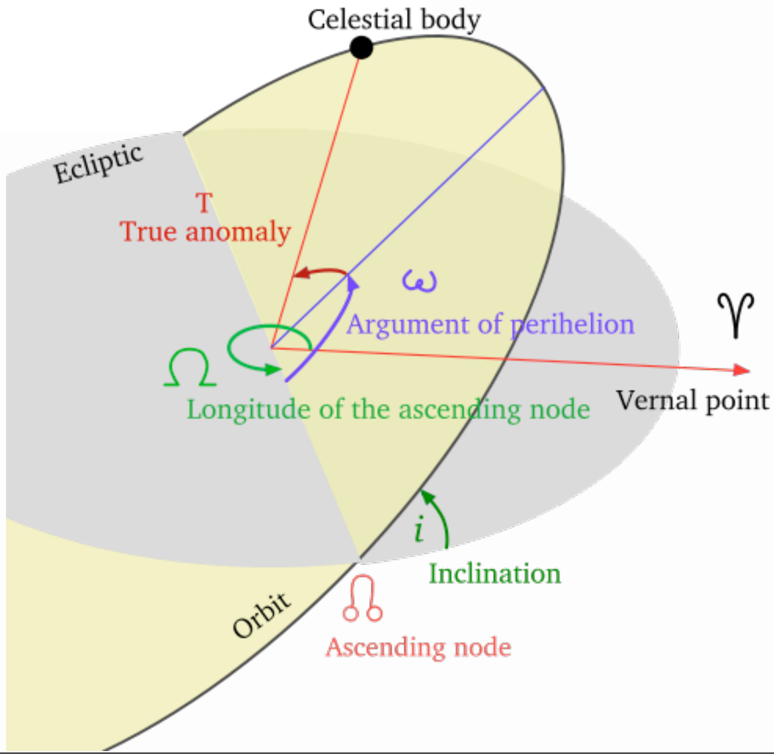
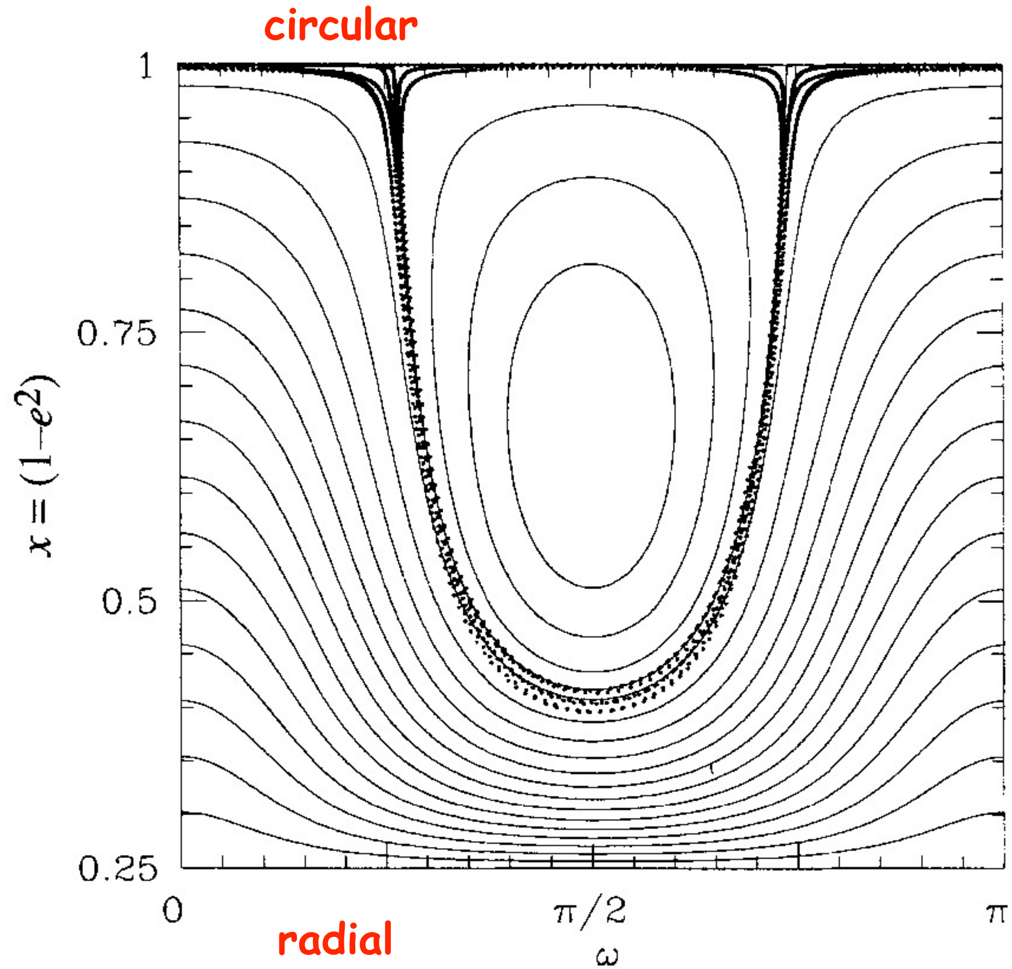


Kozai-Lidov oscillations

- initially circular orbits remain circular if and only if the initial inclination is < $39^o = \cos^{-1}(3/5)^{1/2}$
- for larger initial inclinations the phase plane contains a separatrix
- circular orbits cannot remain circular, and are excited to high inclination and eccentricity -- not a rigid hoop (surprise # 1)
- circular orbits are chaotic (surprise # 2)



Kozai-Lidov oscillations

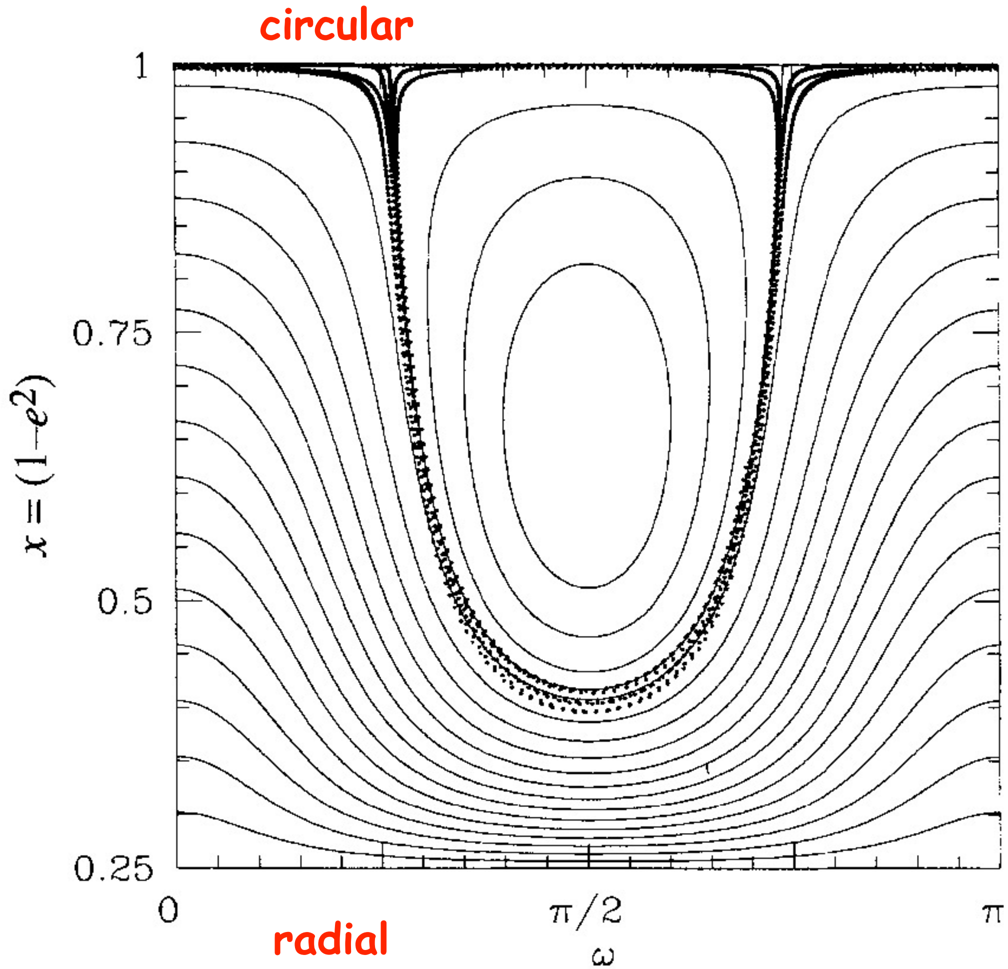
• circular orbits cannot remain circular, and are excited to high inclination and eccentricity (surprise # 1)

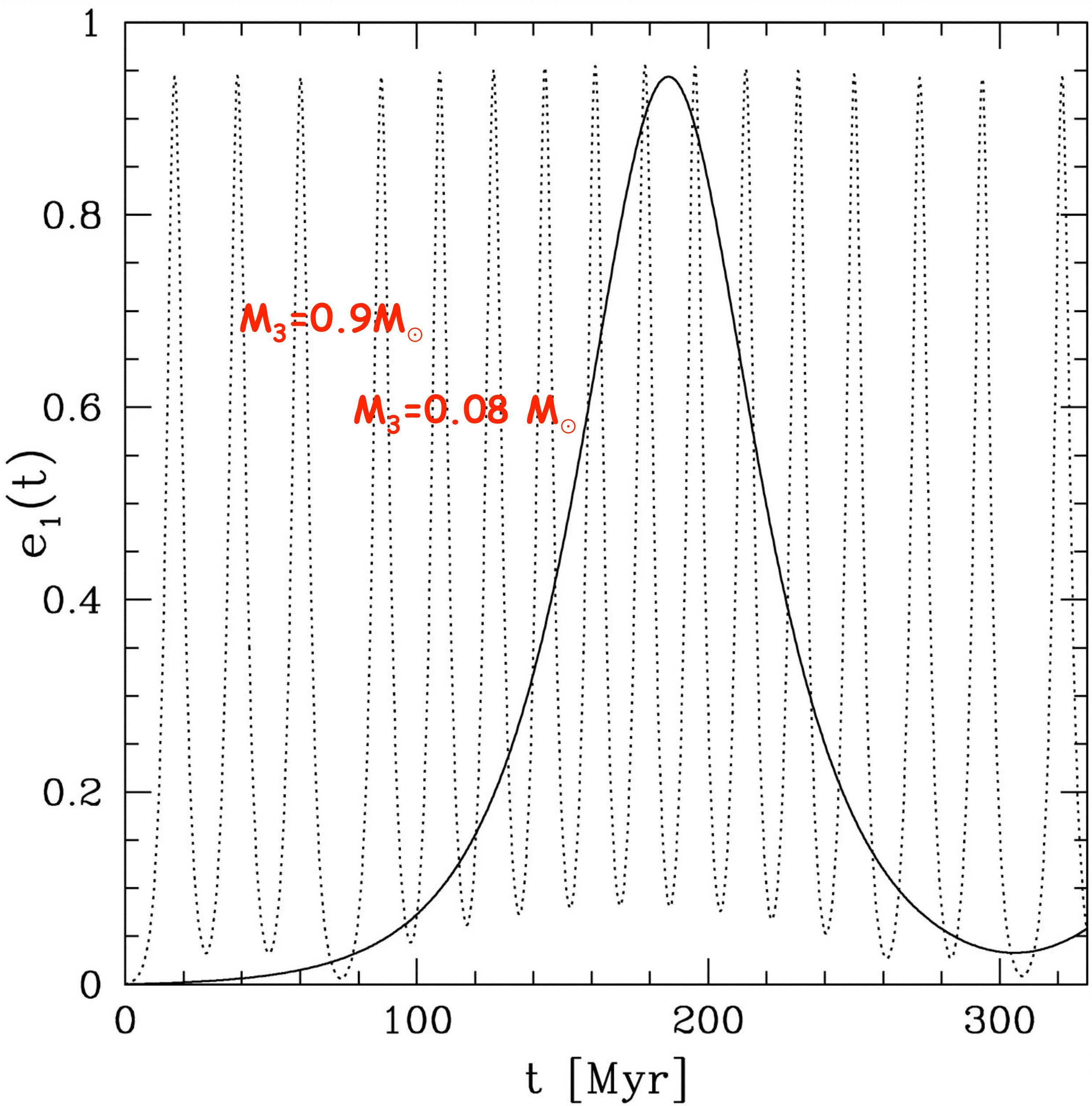
•circular orbits are chaotic (surprise # 2)

• as the initial inclination approaches 90°, the maximum eccentricity achieved in a Kozai oscillation approaches unity ⇒ tidal dissipation

or collision (surprise # 3)

• mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)





eccentricity oscillations of a planet in a binary star system

- a_{planet} = 2.5 AU
- companion has inclination 75°, semi-major axis 750 AU, mass 0.08 $M_\odot$ (solid) or 0.9 $M_\odot$ (dotted)

(Takeda & Rasio 2005)

Kozai-Lidov oscillations

• circular orbits are excited to high inclination and eccentricity (surprise # 1)

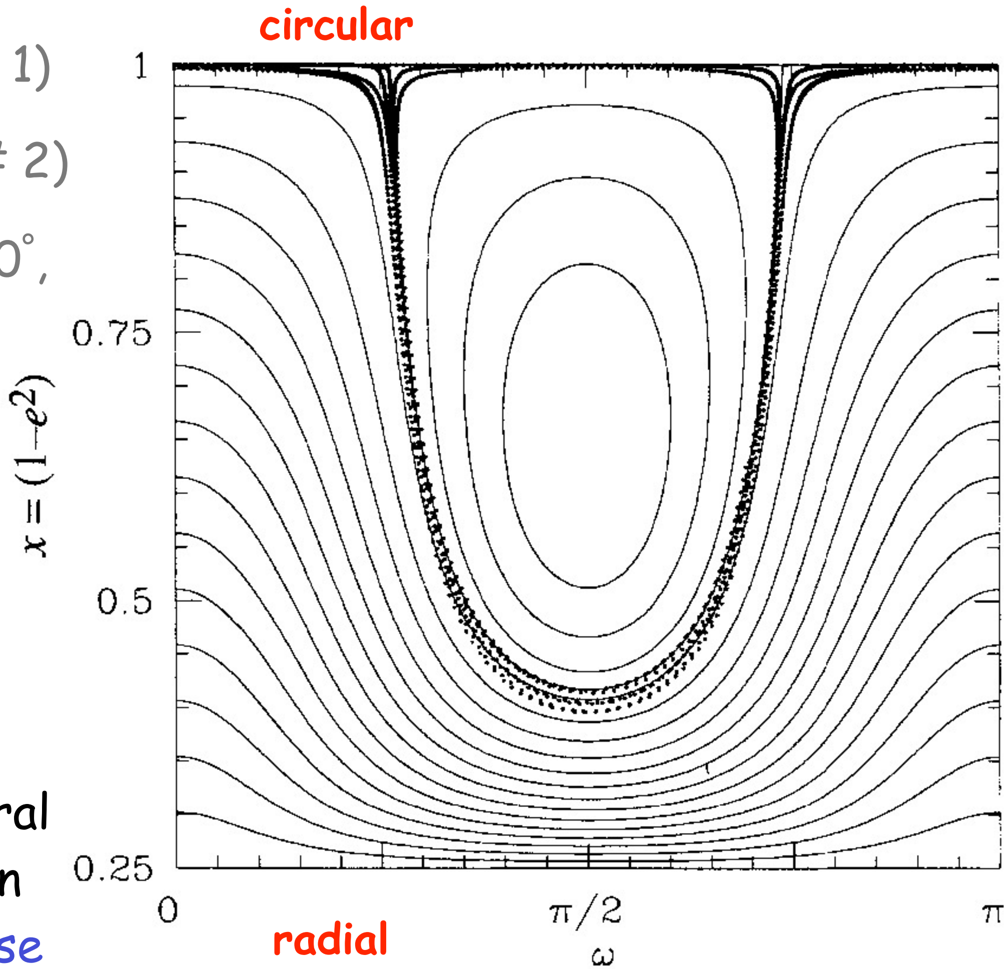
•circular orbits are chaotic (surprise # 2)

• as the initial inclination approaches 90°, the maximum eccentricity approaches unity ⇒ tidal dissipation or collision

(surprise # 3)

• mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)

• small additional effects such as general relativity or octupole tidal potential can strongly affect the oscillations (surprise # 5)



1. Irregular satellites of the giant planets

Hill (or tidal, or Roche) radius

$$r_H = a_p\,(m/3M_\odot)^{1/3}$$

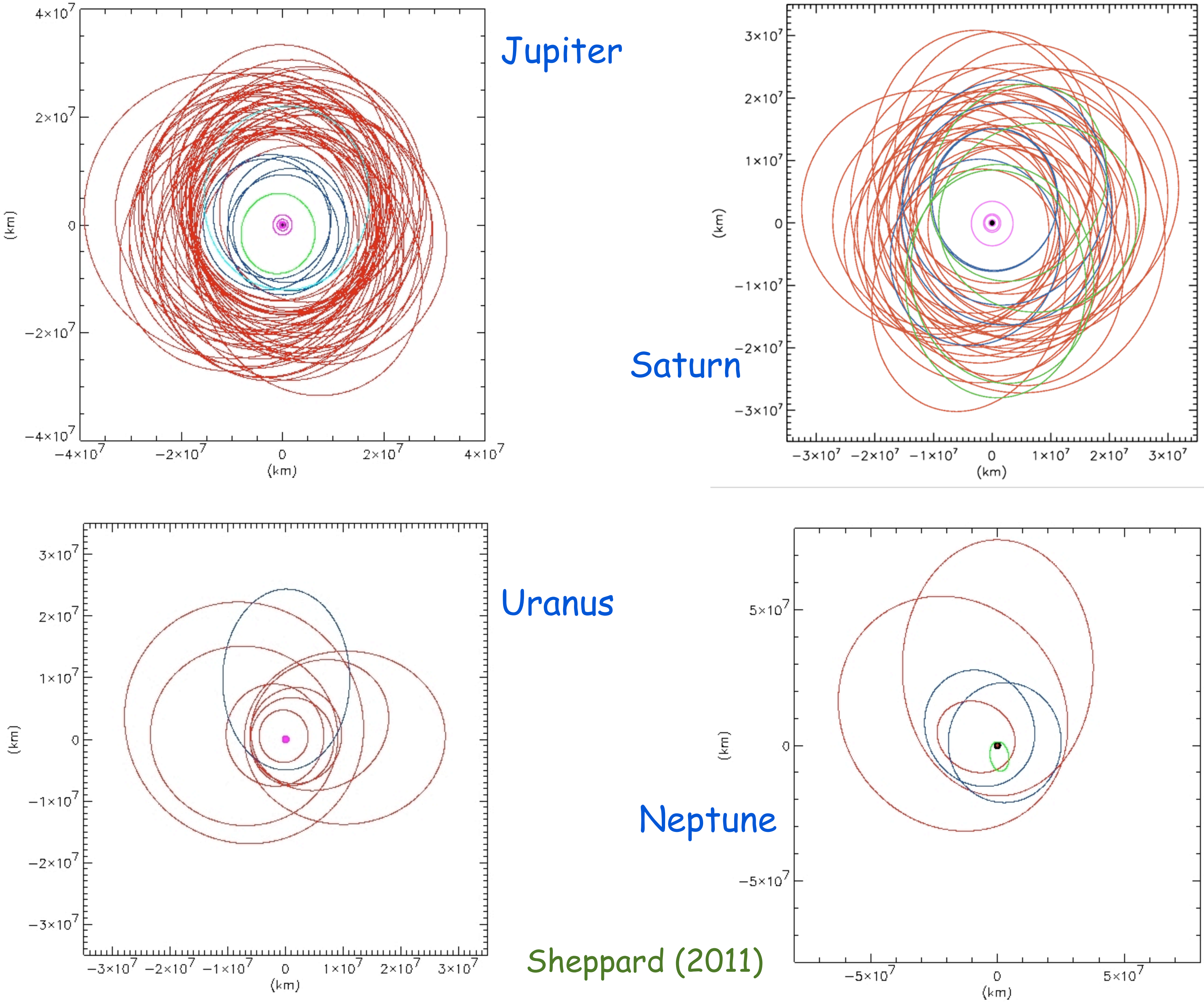
represents approximately the maximum radius at which an orbit stays bound to the planet

• at $r < 0.05r_H$, satellites of the giant planets tend to be on nearly circular, prograde orbits near the planetary equator ("regular" satellites). Probably formed from a protoplanetary disk

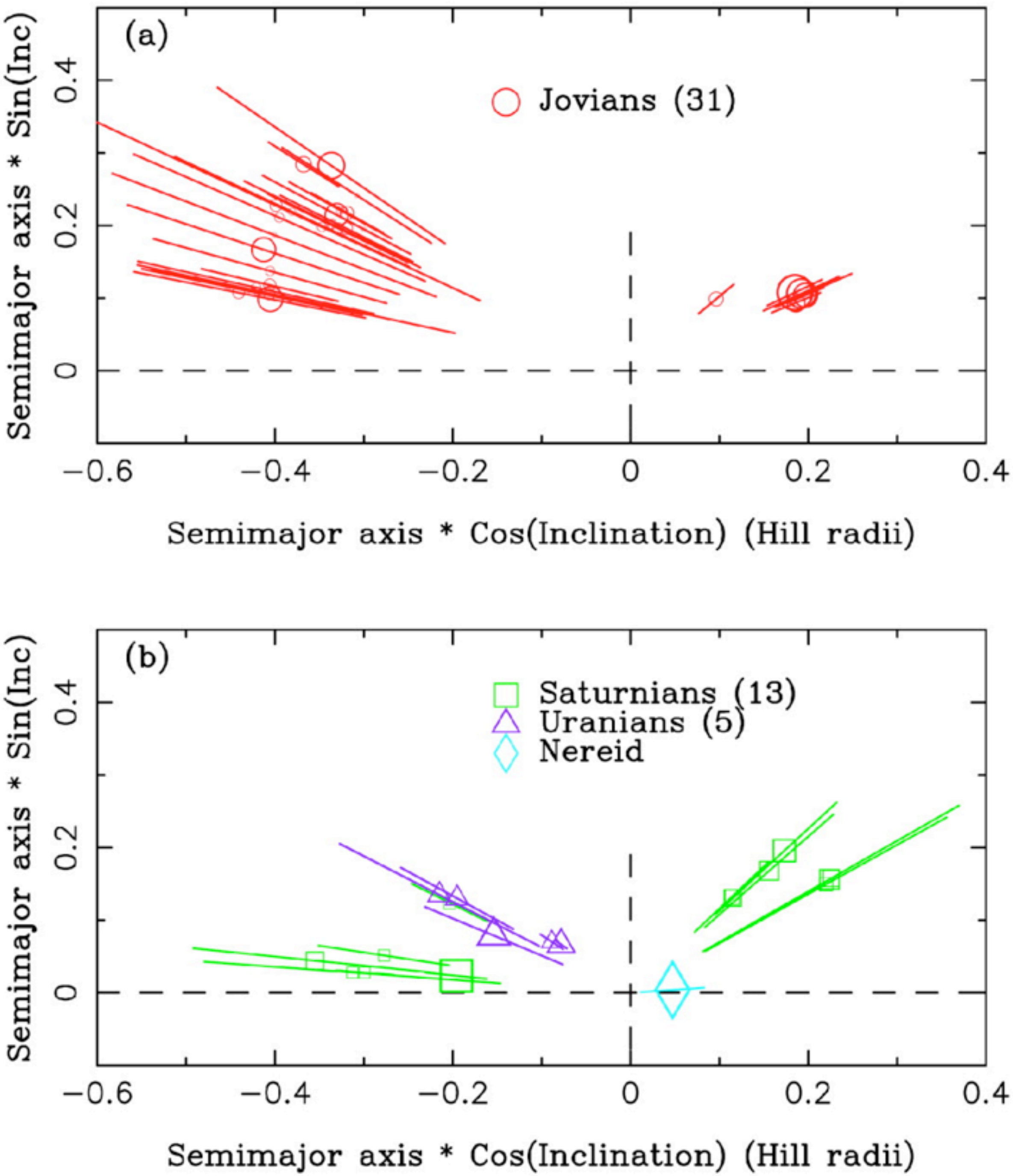
• at $r > 0.05r_H$ the satellites have large eccentricities and inclinations, including retrograde orbits (irregular" satellites). Probably captured from heliocentric orbits

• irregular satellites are much smaller than regular ones but there are a lot more of them (97). Total satellite count:

Jupiter: 65 Saturn: 62 Uranus: 27 Neptune: 13

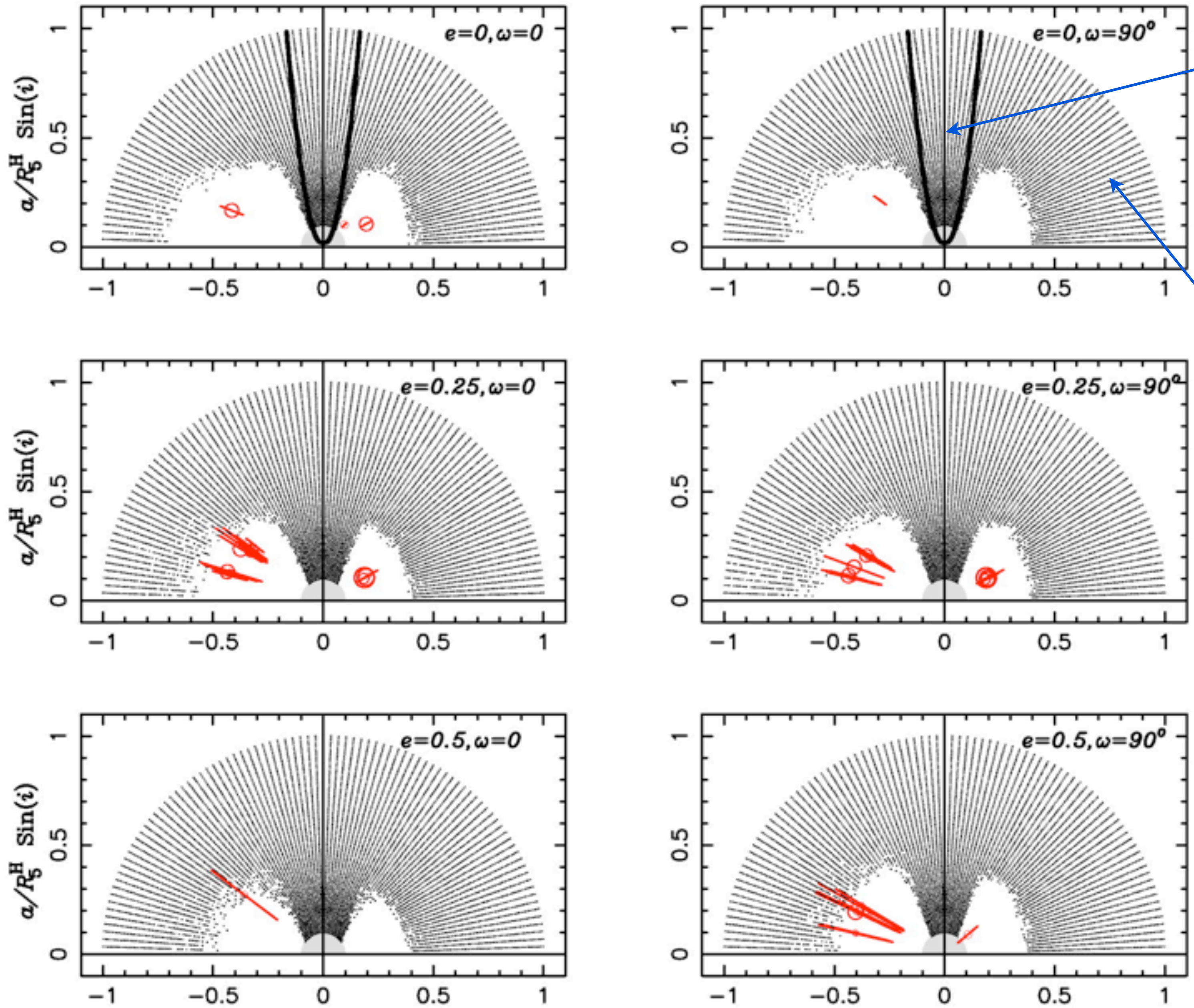


Sheppard (2011)



there are no irregular satellites with inclinations between 40° and 140°

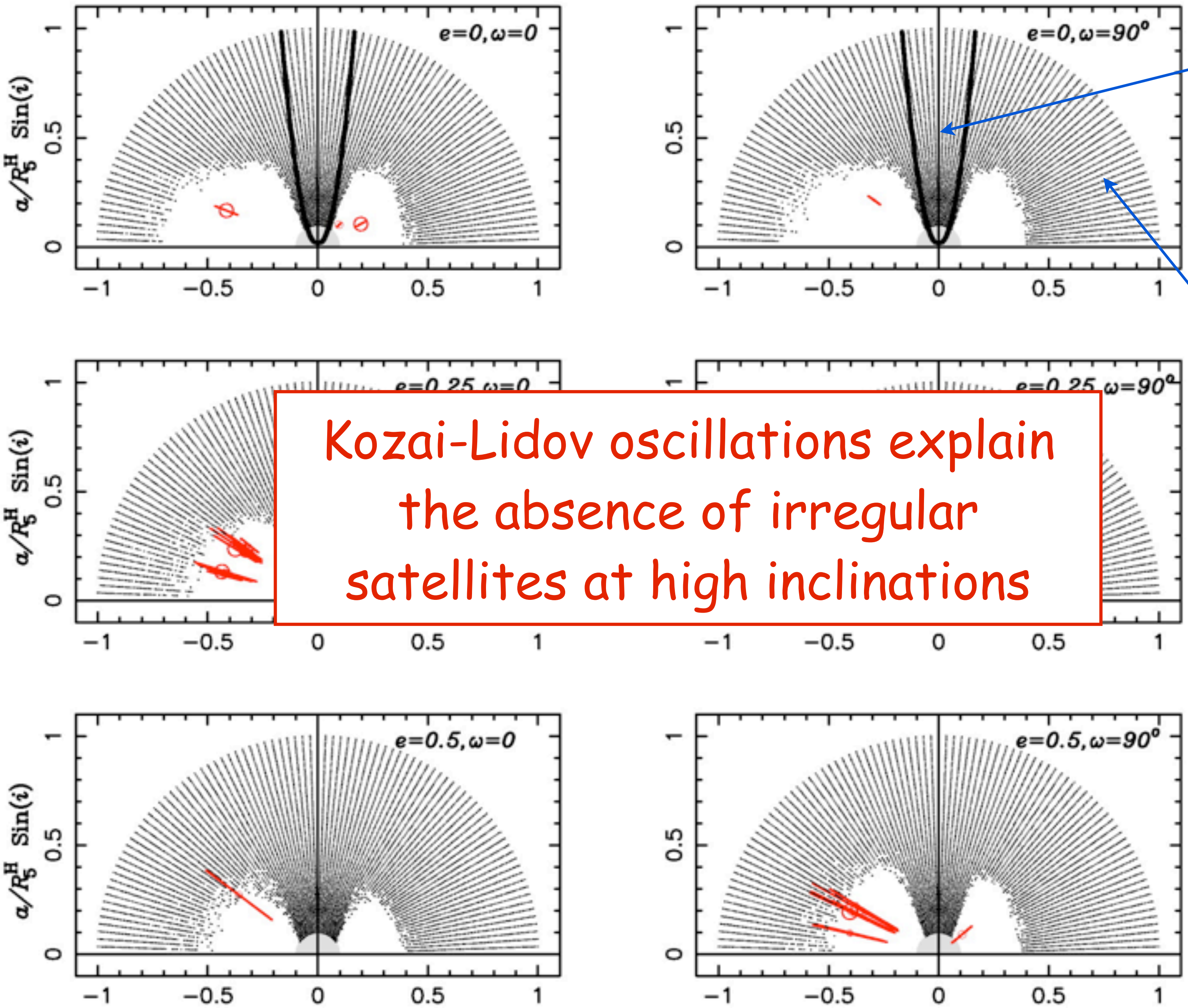
Nesvorny et al. (2003)



unstable due to K-L oscillations

unstable because too close to Hill radius

Nesvorny et al. (2003)



unstable due to K-L oscillations

unstable because too close to Hill radius

Kozai-Lidov oscillations explain the absence of irregular satellites at high inclinations

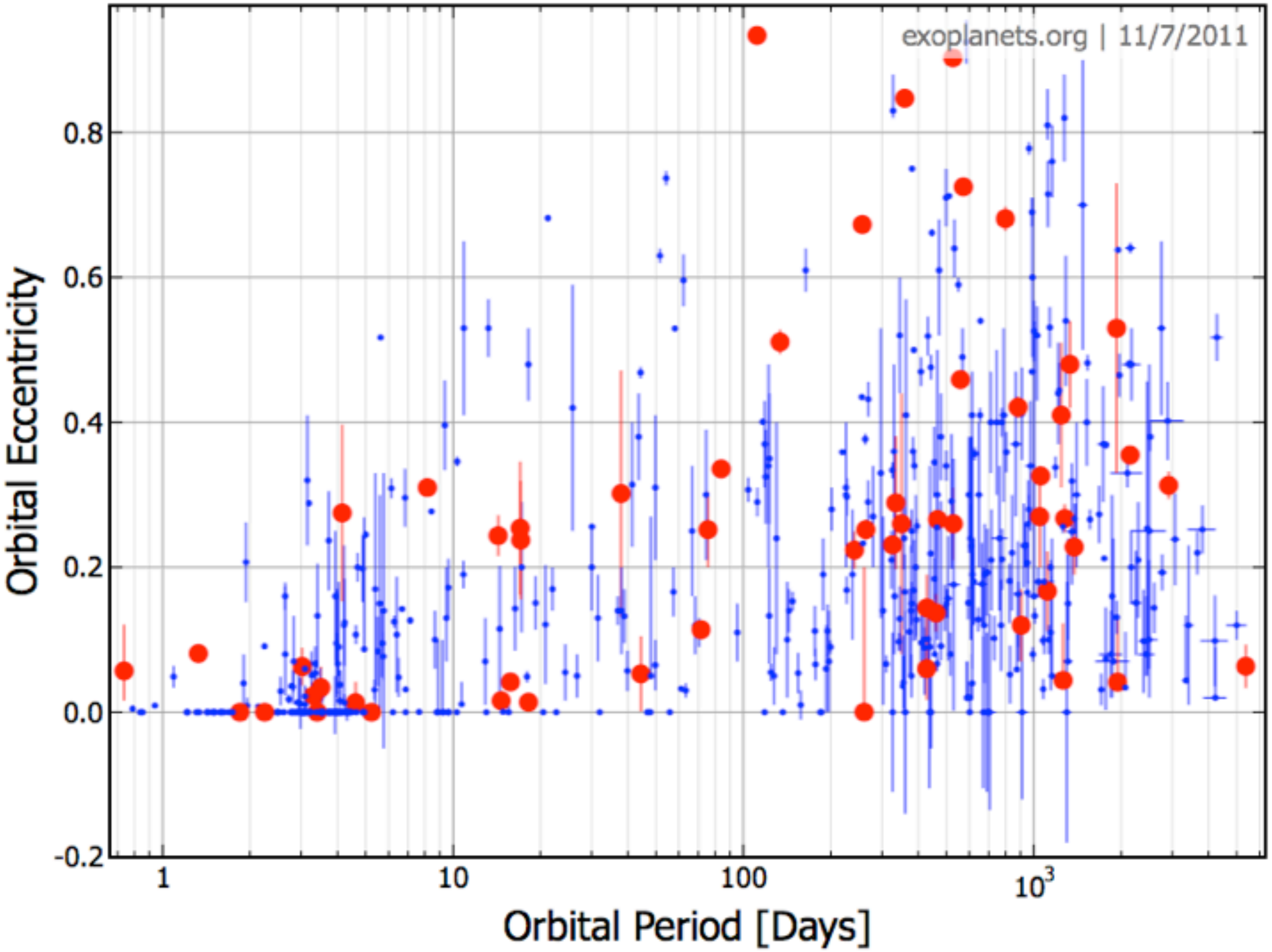
Nesvorny et al. (2003)

2. Exoplanet eccentricities

Kozai-Lidov oscillations may excite eccentricities of planets in **some** binary star systems, but probably not all planet eccentricities:

- not all have stellar companion stars (so far as we know)
- suppressed by additional planets
- suppressed by general relativity (!)

red = binary

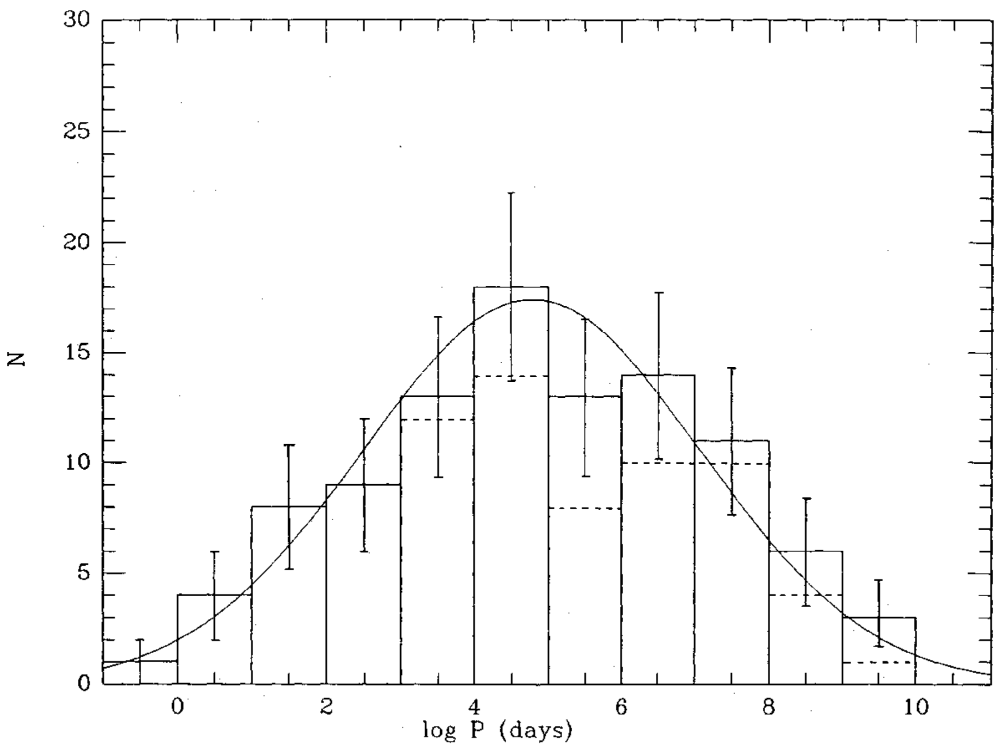


3. Formation of close binary stars

Binary stars are common: roughly 2/3 of nearby stars are in binaries, with a wide distribution of periods:

$$dn \propto \exp\left[-\frac{(\log P/P_0)^2}{2\sigma_P^2}\right] \qquad P_0 = 170\,\mathrm{yr},\ \sigma_P = 2.3$$

(Duquennoy & Mayor 1991)



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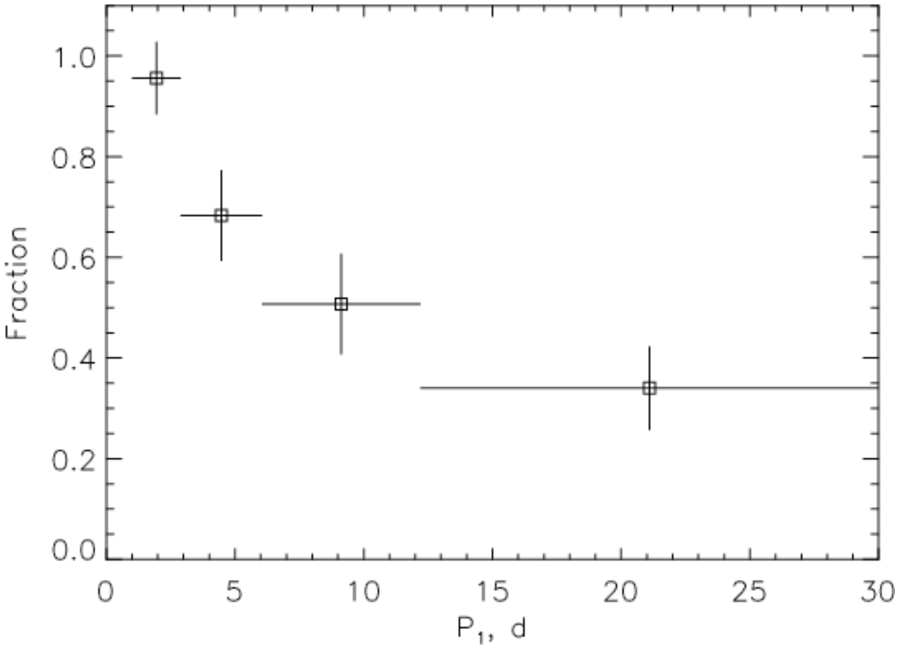
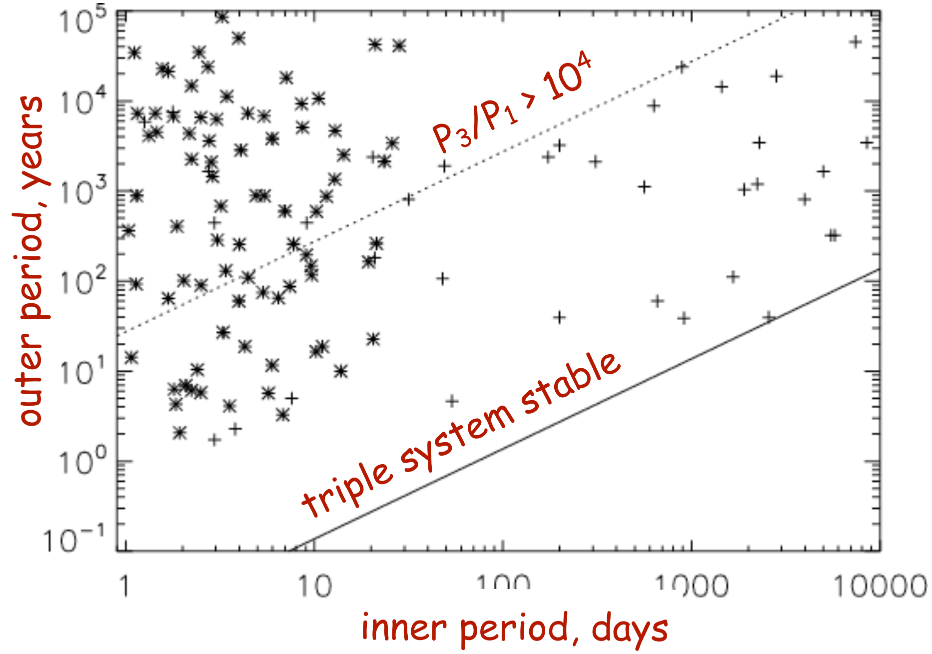
If formation of inner and outer binary in a hierarchical triple star is independent we expect (1) about (2/3)X(2/3)~0.5 of all systems to be triple and (2) characteristics of inner and outer binary to be independent





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This is not true: 96% of binaries with P < 3 d are in triples, but only 34% of binaries with P > 12 d are in triples (Tokovinin et al. 2006)

How can a tertiary companion that is 1000 X further away affect the formation of a binary star?

How do you form a binary with a separation of a few stellar radii when stars shrink by orders of magnitude during their formation?

Formation of close binary stars

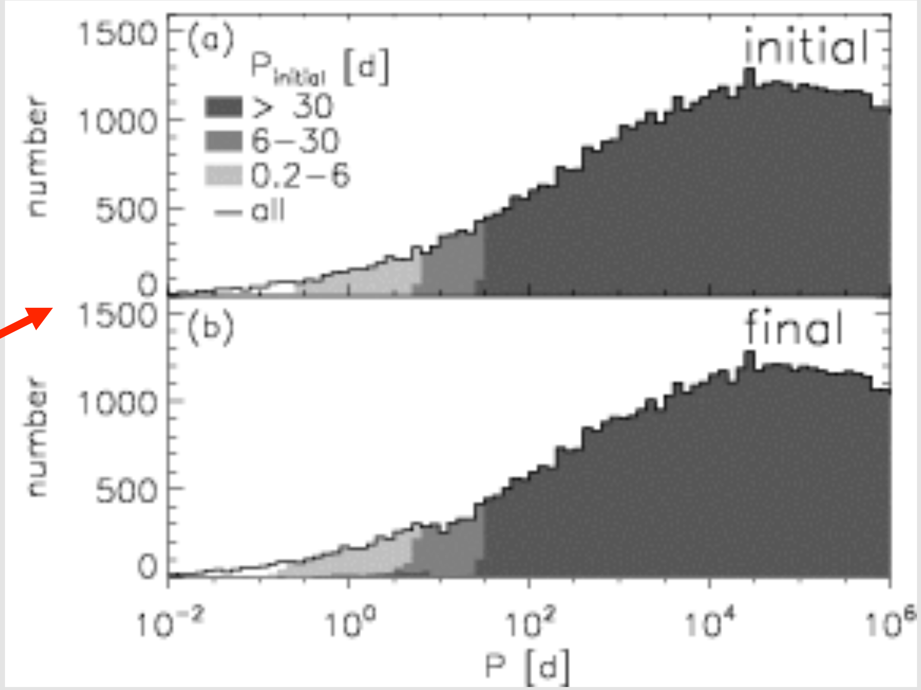
follow orbit evolution of binary or triple star systems, including:

- secular evolution of orbit due to quadrupole tidal field from a tertiary
- apsidal precession due to rotational distortion of stars in the inner binary
- apsidal precession due to mutual tidal distortion of stars in the inner binary
- stellar spins
- tidal friction (Eggleton & Kiseleva-Eggleton 2001)
- relativistic precession

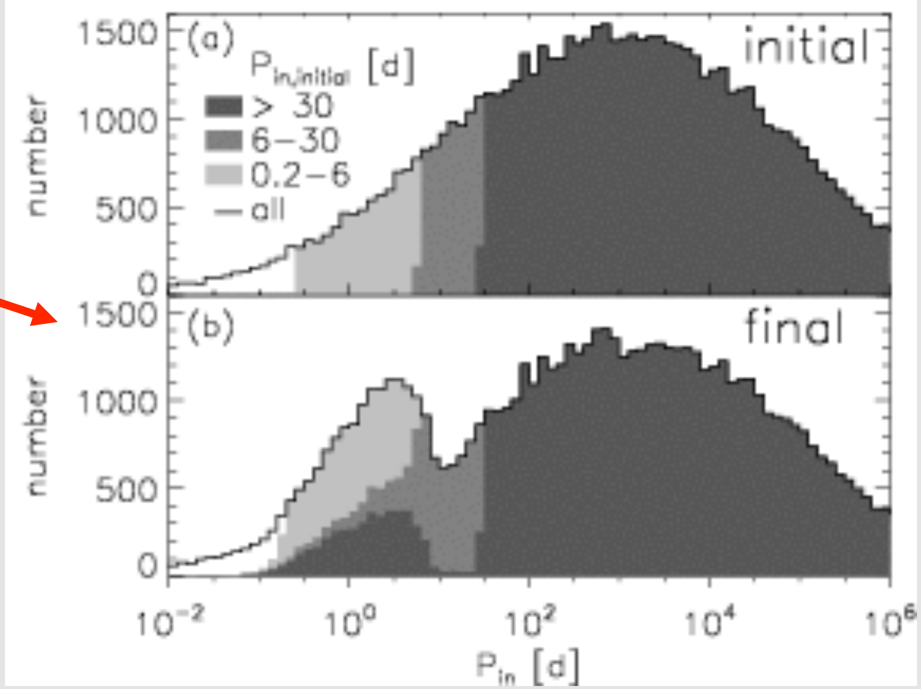
Fabrycky & Tremaine (2007)

Formation of close binary stars

• choose binary stars at random from the Duquennoy & Mayor (1991) distribution, then evolve under tidal friction



• choose triple stars by sampling twice from the binary-star distribution and discard if unstable, then evolve under tidal friction



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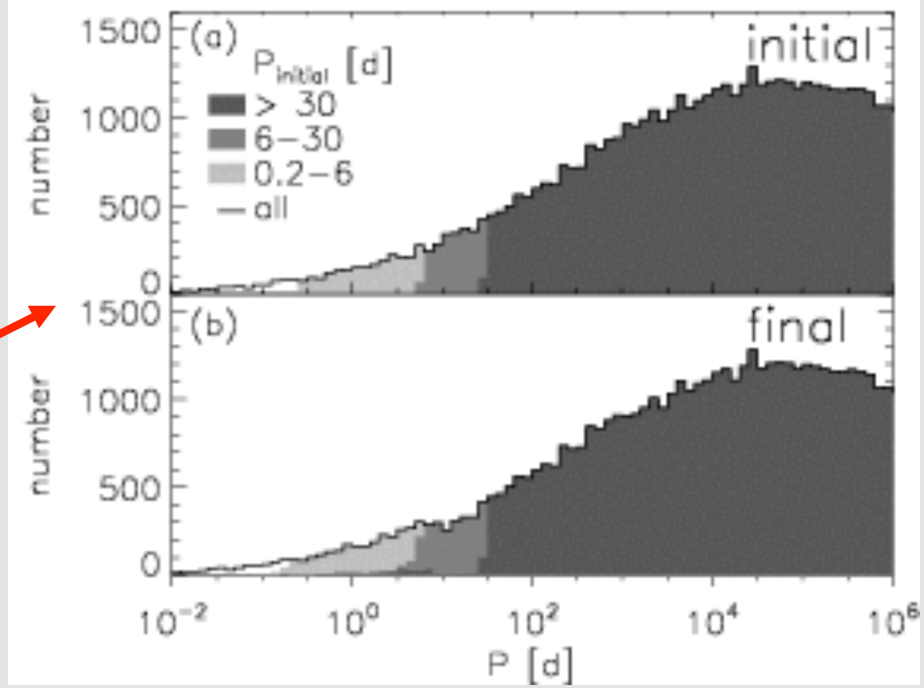

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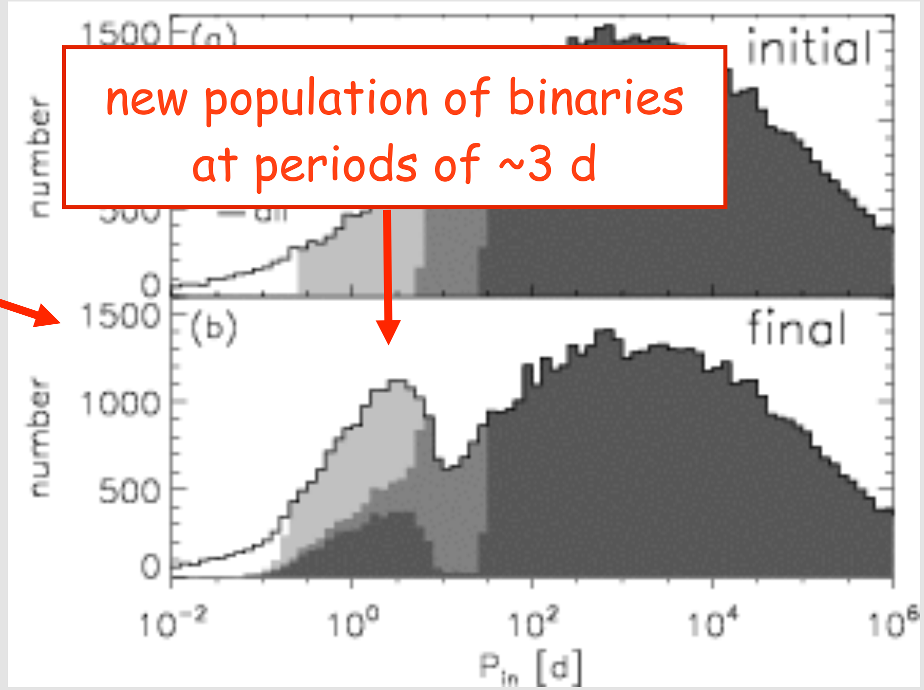
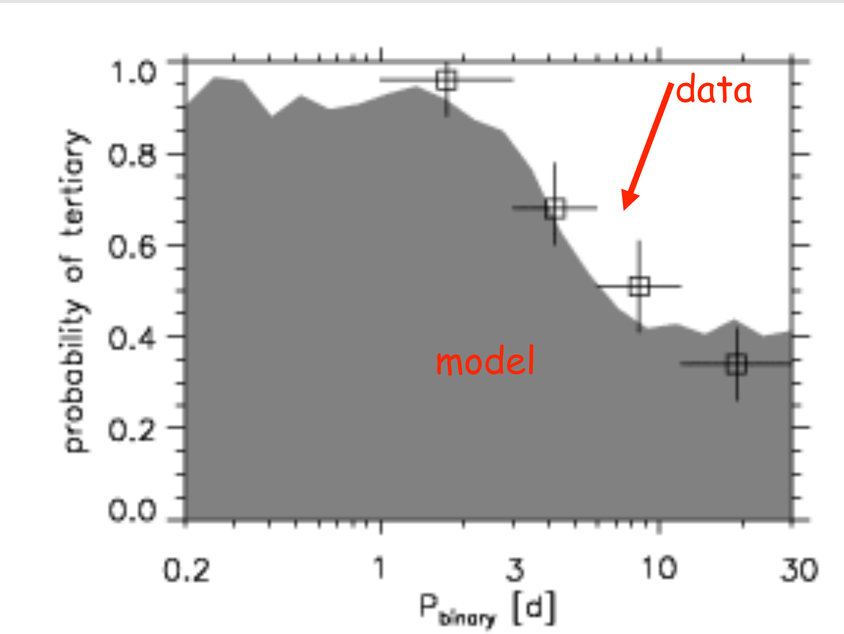
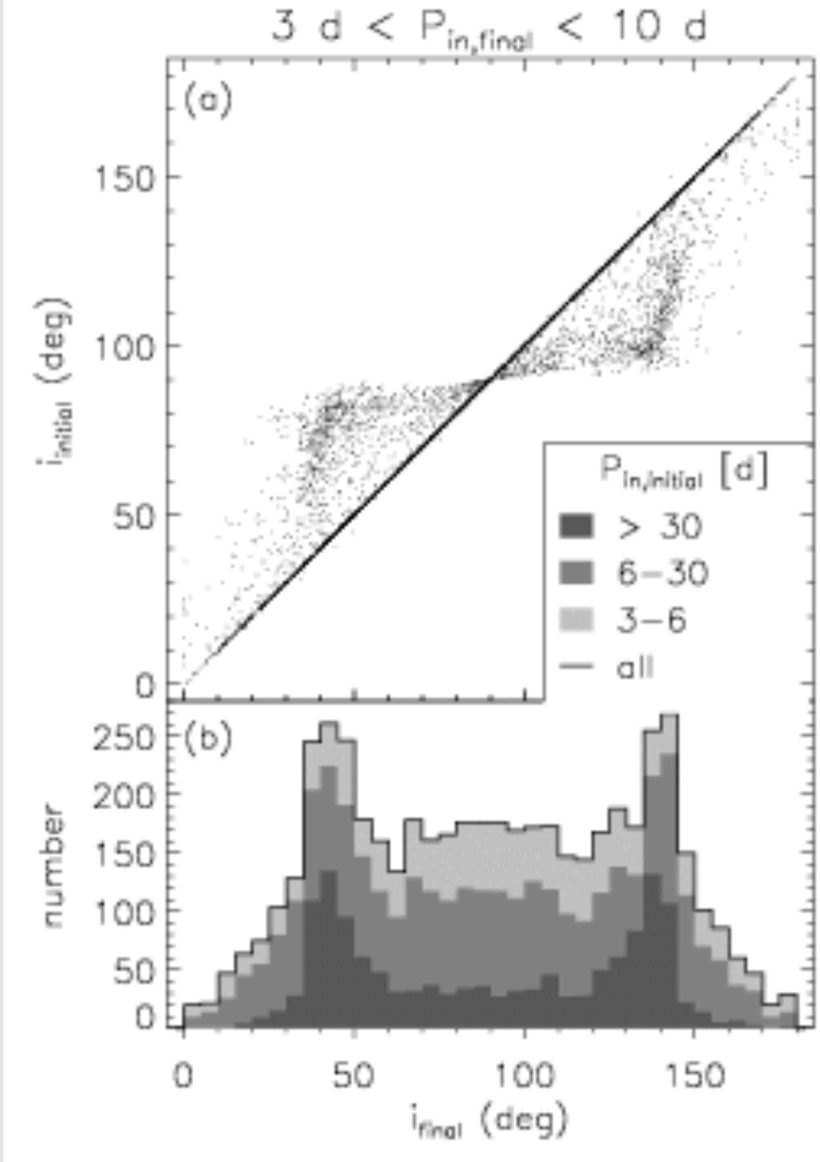


• combine the distributions assuming (a) 25% of systems are triple; (b) period distribution is cut off at 6 d (radius of dynamically stable protostars)

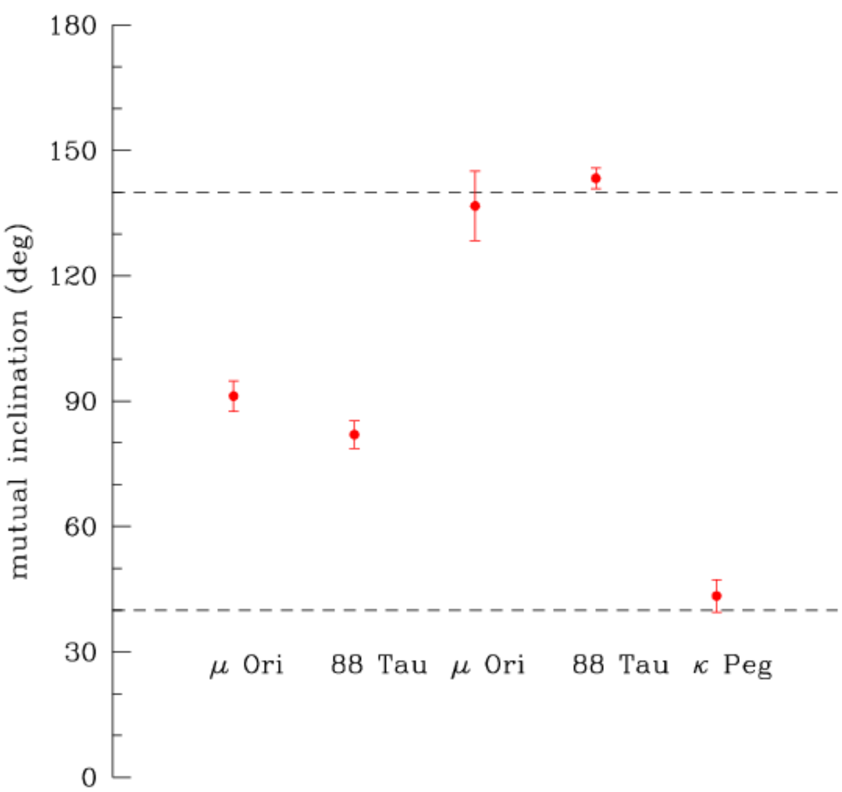
• Kozai-Lidov cycles may be responsible for almost **all** close binary stars

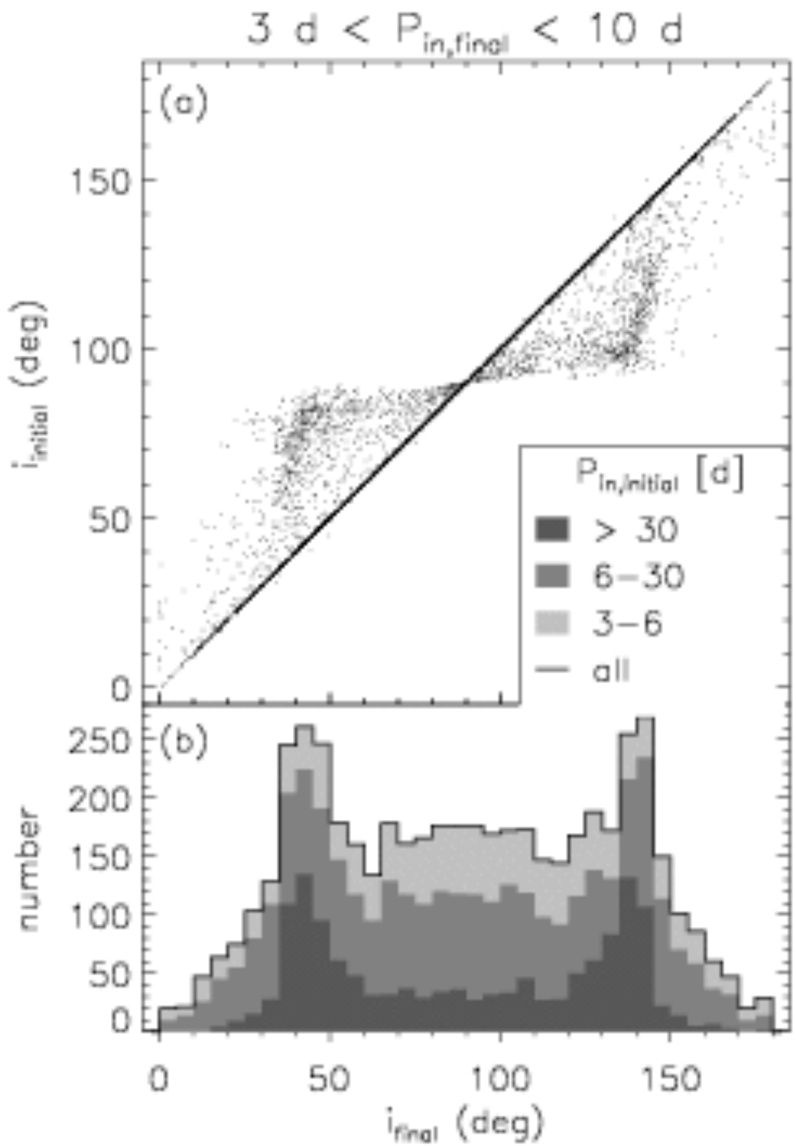




• in this simple model, there is a strong peak near 40 and 140 degrees in the mutual inclinations of systems with 3 d < P_{in} < 10 d

Muterspaugh et al (2007) list five triple systems in this period range



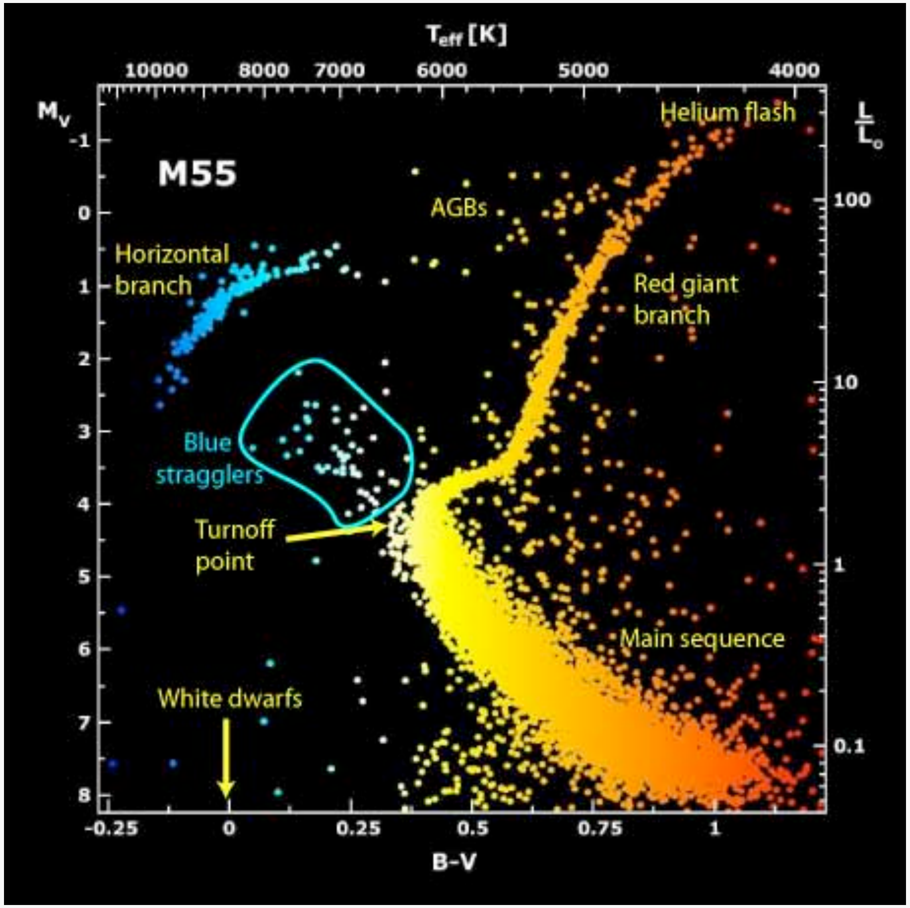


4. Blue stragglers

Blue stragglers are stars in globular clusters that appear to be anomalously young

Possible origins:

- stellar collision and merger
- mass transfer or coalescence in a primordial binary system



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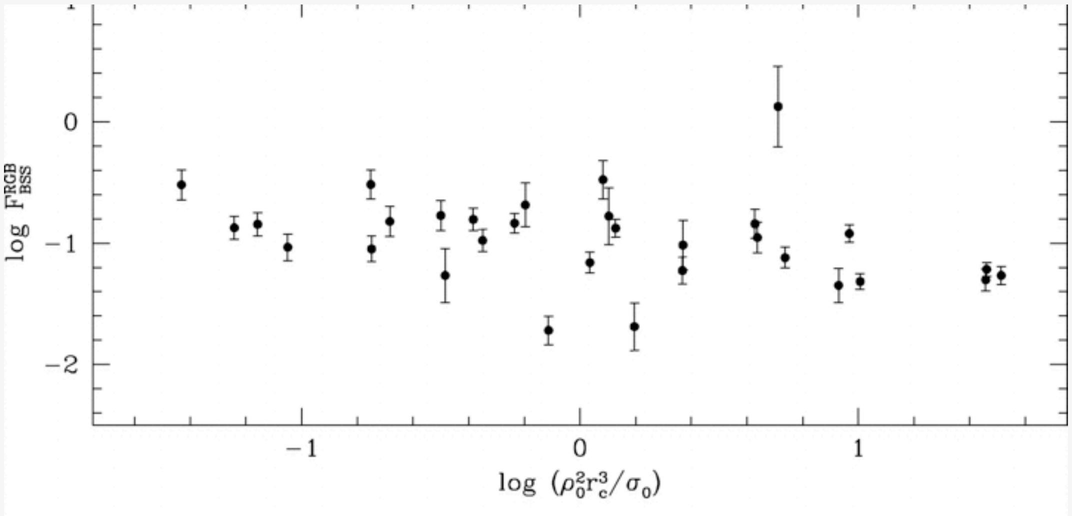
Problems:

• frequency is not correlated with expected collision rate (or any other cluster properties)



Leigh et al. (2007)

4. Blue stragglers

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Problems:

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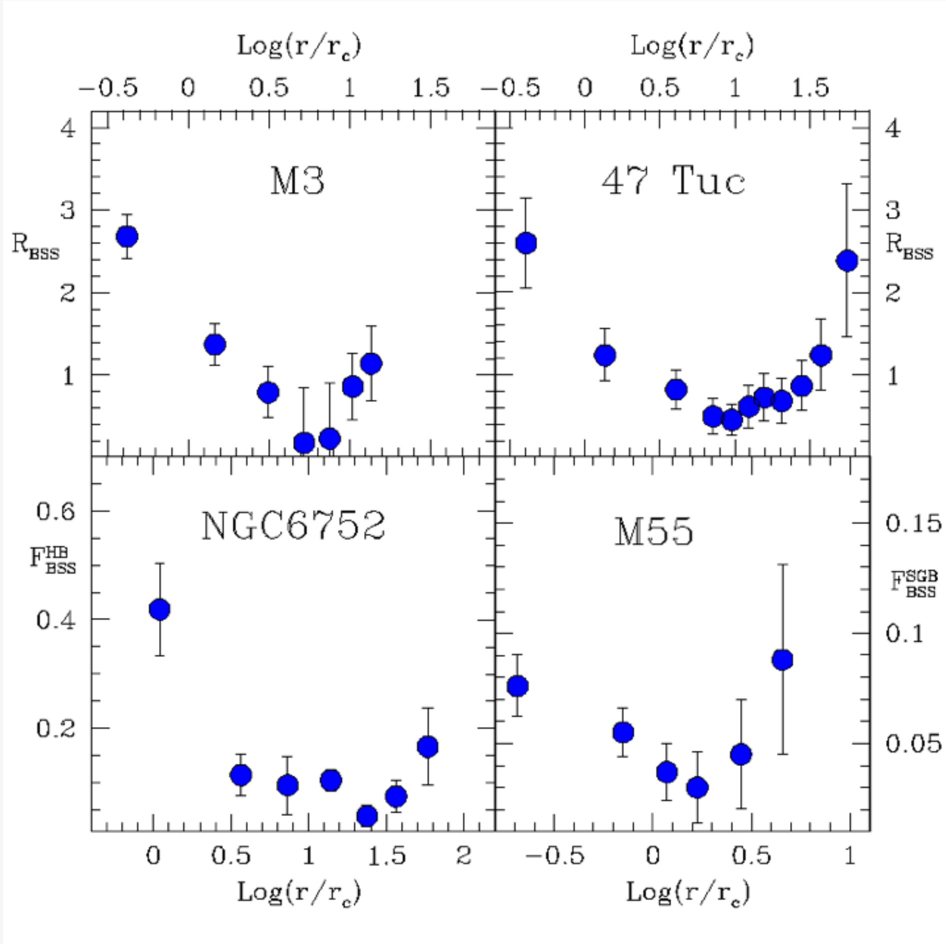
• radial distribution is difficult to interpret (maybe both mechanisms operate?)



Ferraro (2005)

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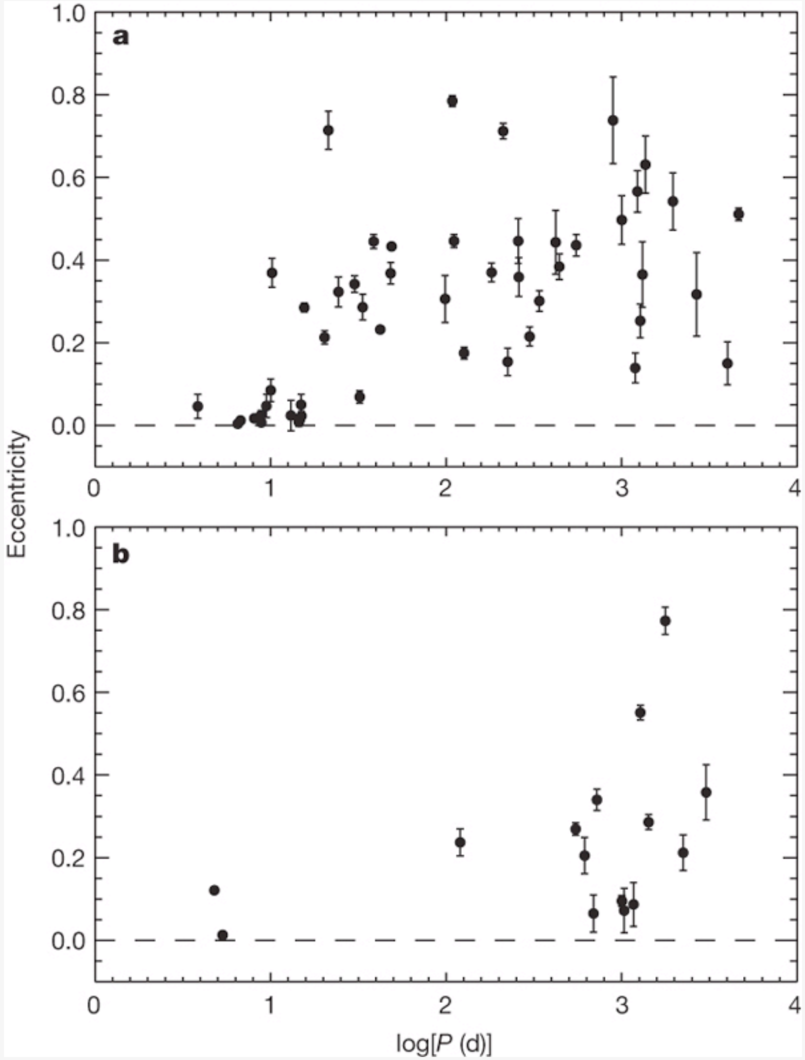
• radial distribution is difficult to interpret

• binary fraction of blue stragglers in NGC 188 is three times that in solar neighborhood



Mathieu & Geller (2009)

4. Blue stragglers

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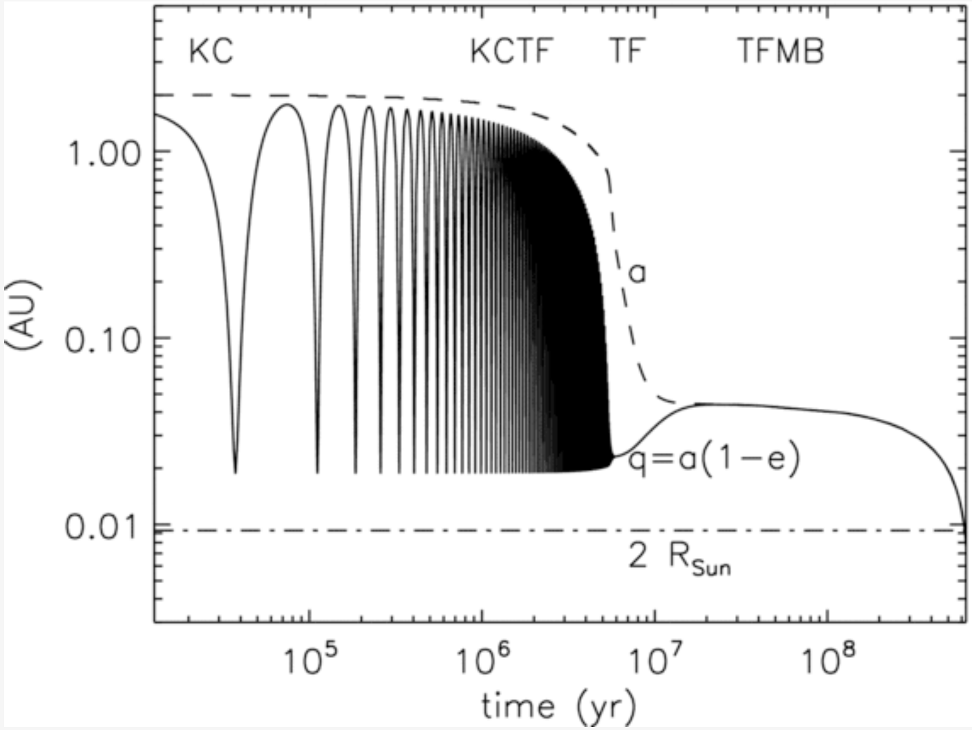
- stellar collision and merger
- mass transfer or coalescence in a primordial binary system
- Kozai-Lidov oscillations in a triple system leading to merger (Perets & Fabrycky 2009)



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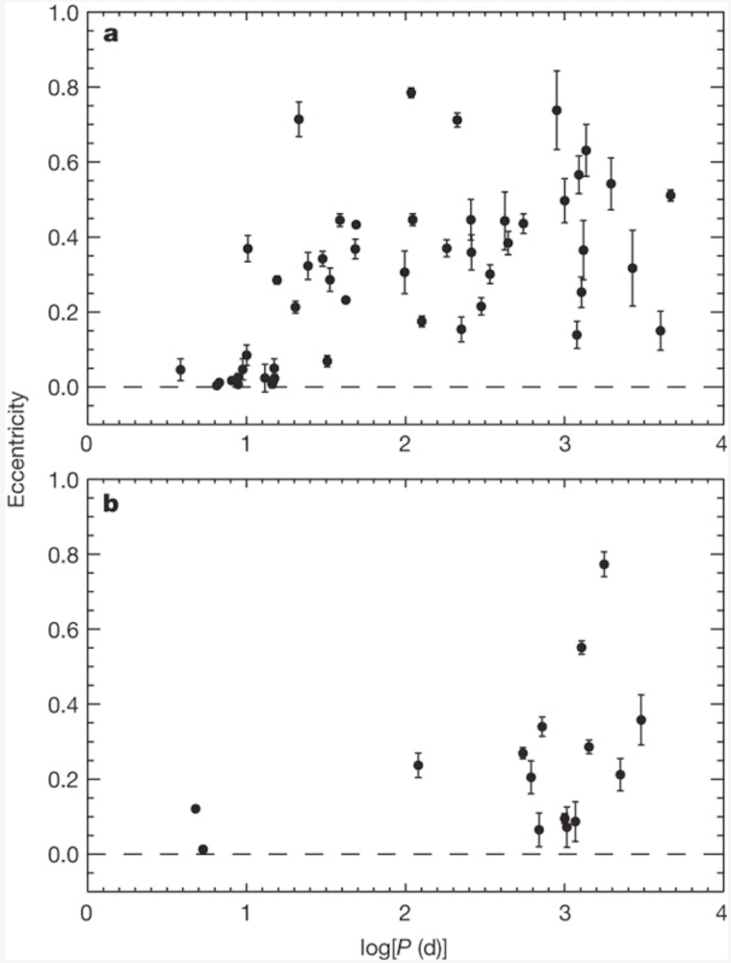


5. Type Ia supernovae

These arise from white dwarfs that exceed the Chandrasekhar limit, either through

- mass accretion from a main-sequence companion star
- mergers of white dwarf-white dwarf binaries

- if most close binaries are in triples then most SN Ia progenitors are in triples so Kozai-Lidov oscillations will strongly affect rate (Thompson 2011)

- may explain "prompt" Ia supernovae

- predicts periodic gravitational pulses (Gould 2011)

- why have we not found nearby WD-WD binaries? Possible color contamination by main-sequence third body

6. Planetary migration

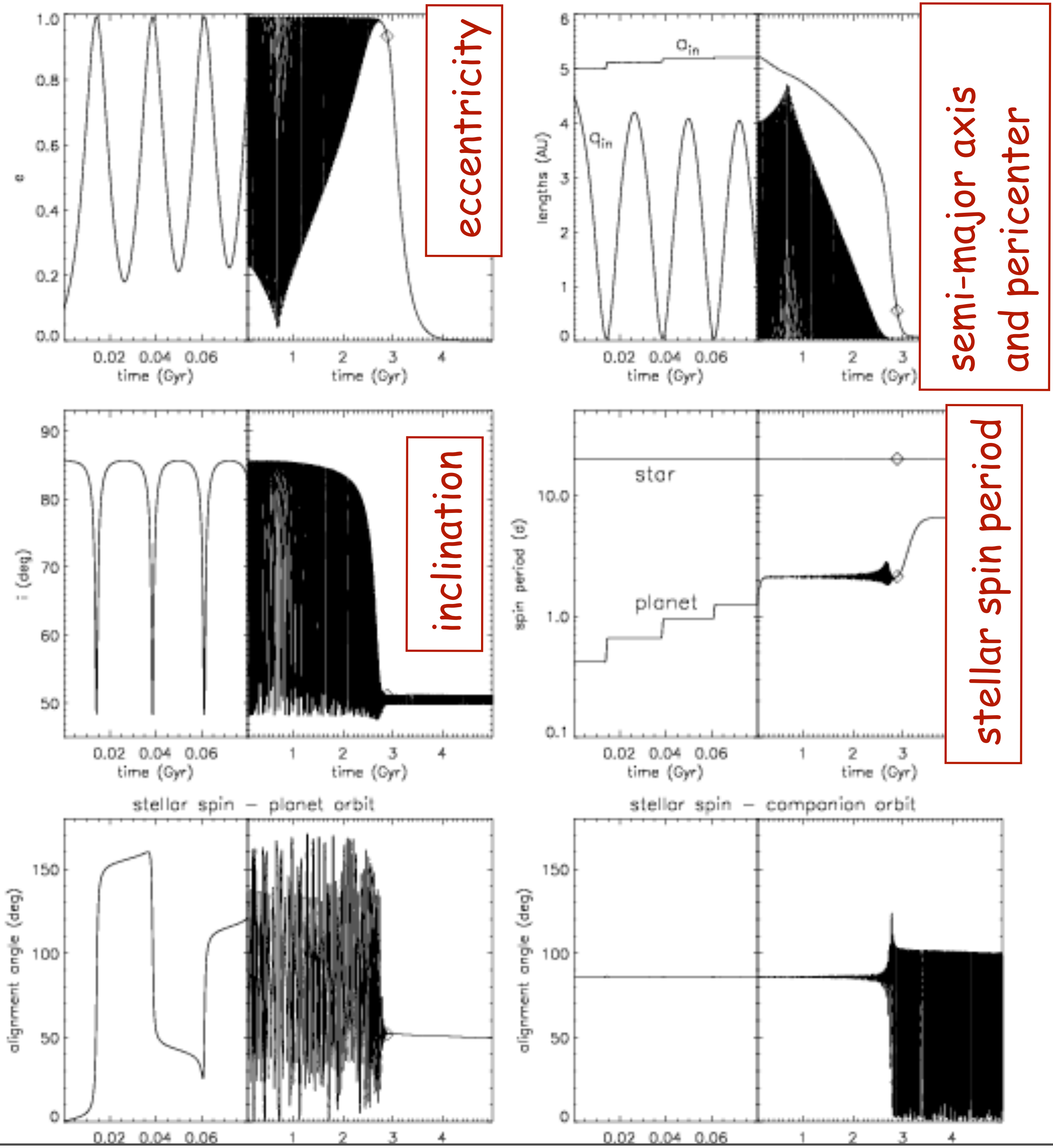
Planet-planet scattering + tidal friction may form hot Jupiters

- suppose scattering leads to an isotropic distribution of velocities

- tidal friction is only important for pericenter $q < 0.02$ AU, so must scatter onto nearly radial orbit. Probability $\sim q/a$

- if Kozai-Lidov oscillations are present angular momentum oscillates but L_z is conserved. Probability of $q < 0.02$ AU at some point in the cycle is $\sim (q/a)^{1/2}$

- Kozai-Lidov oscillations due to outer planets are a critical part of all high-eccentricity migration scenarios

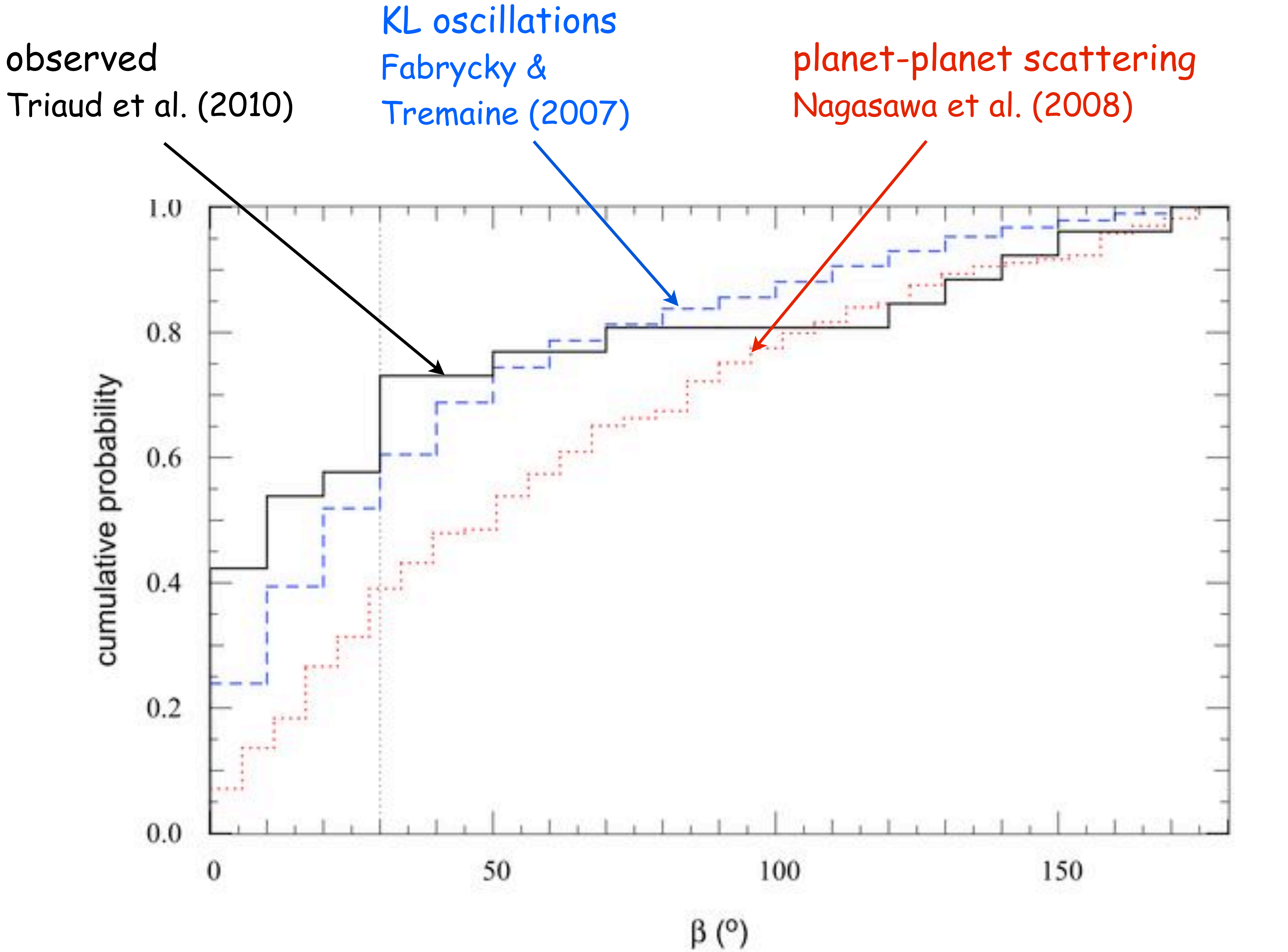


Kozai oscillations with tidal friction in a model of HD 80606b

initial conditions:

- a = 5 AU
- i = 86 ±
- a_{out} = 1000 AU

(Wu & Murray 2003)



distribution of projected obliquities

7. Black-hole mergers

Kozai-Lidov oscillations may accelerate the merger of binary black holes (the "final parsec problem") where external field may come from triaxial galaxy potential or a third black hole (Blaes et al. 2002, Yu 2002, Tanikawa & Umemura 2011)

8. Comets

Kozai-Lidov oscillations induced by the Galactic tidal field drive comets onto orbits that intersect the planetary system

Kozai-Lidov oscillations

- distant satellites of the giant planets have inclinations near 0 or 180° but not near 90°
- may excite eccentricities of planets in binary star systems, but probably not all planet eccentricities
- may enhance merger rate of binary black holes in the centers of galaxies
- source of long-period comets
- formation of close binary stars
- formation of blue stragglers
- formation of hot Jupiters
- obliquities of host stars of transiting exoplanets
- Type Ia supernovae, gamma-ray bursts, gravitational wave sources
- **homework:** why do Earth satellites stay up?