Systematic Errors in Weak Lensing Measurements

Jun Zhang, Eiichiro Komatsu
(The University of Texas at Austin)

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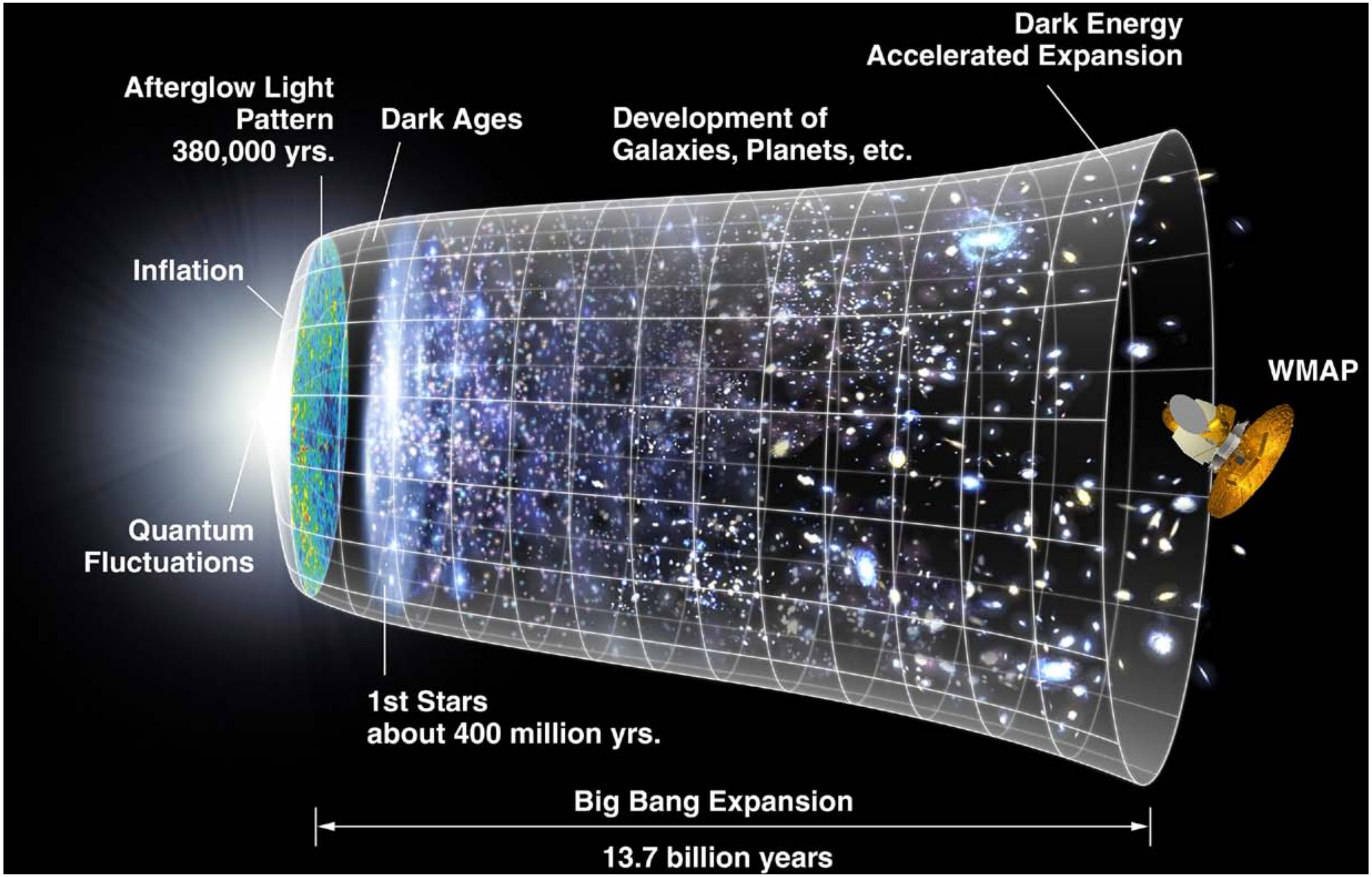
Astro-ph/0612146.

IPMU, Nov 16, 2011

Outline:

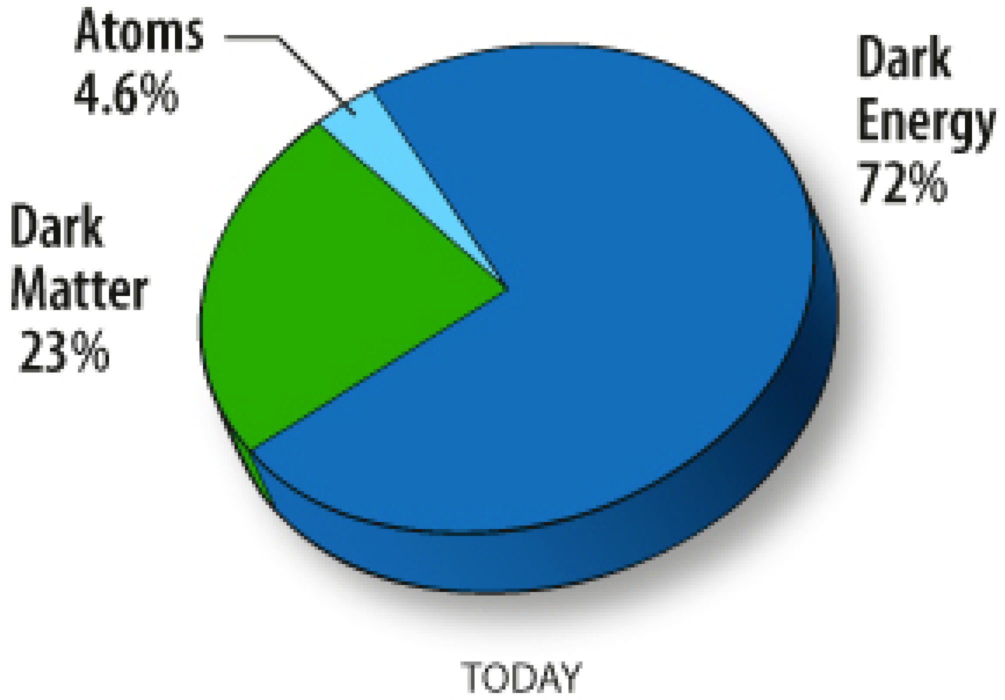
- Introduction of Weak Lensing
- Problems in Weak Lensing Measurement
  - The Form of Cosmic Shear Estimator (general)
  - The Reduced Shear (general)
  - The Pixelation Effect (specific)
  - The Photon Noise (specific)
- Summary

Introduction



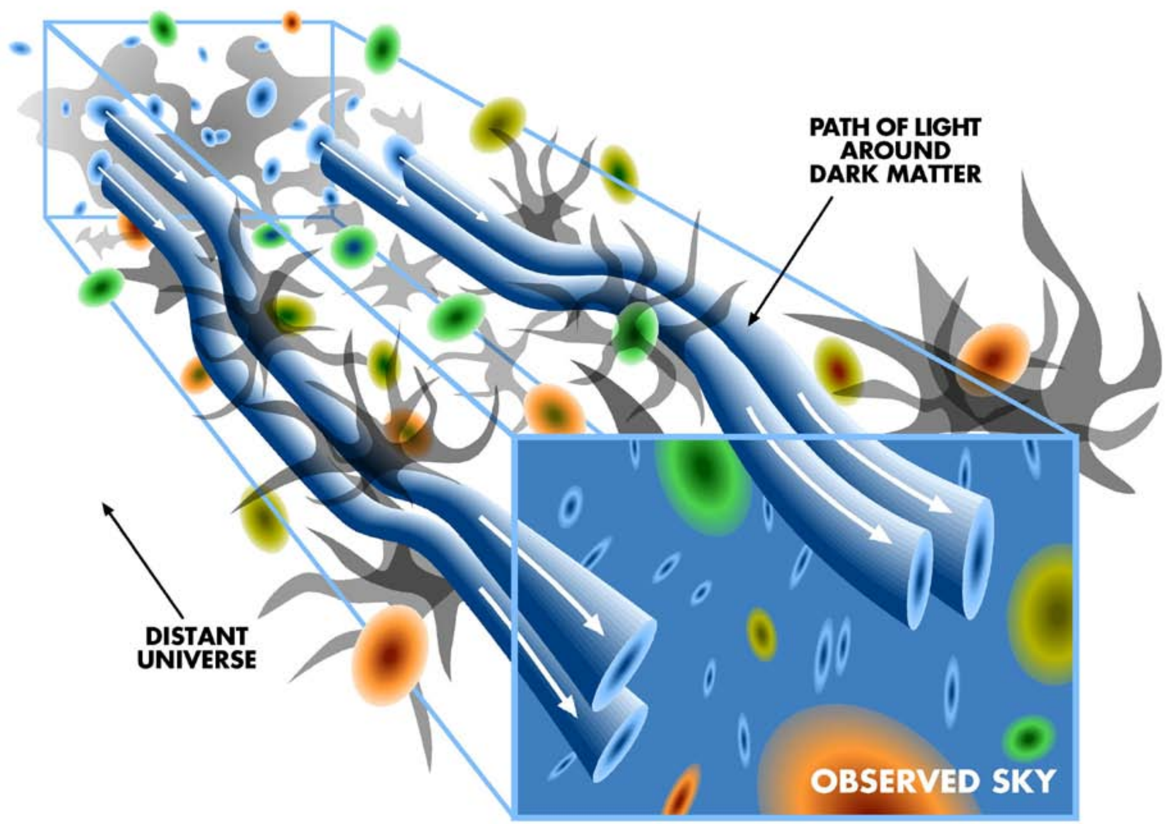
Credit: NASA / WMAP Science Team

Introduction



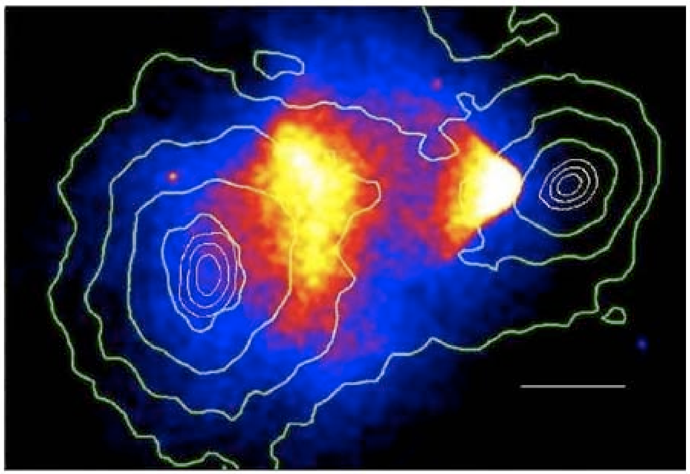
Credit: NASA/WMAP

Introduction



Credit: Wittman et al. , (LSST team)

Introduction



Credit: Clowe et al. (2006)

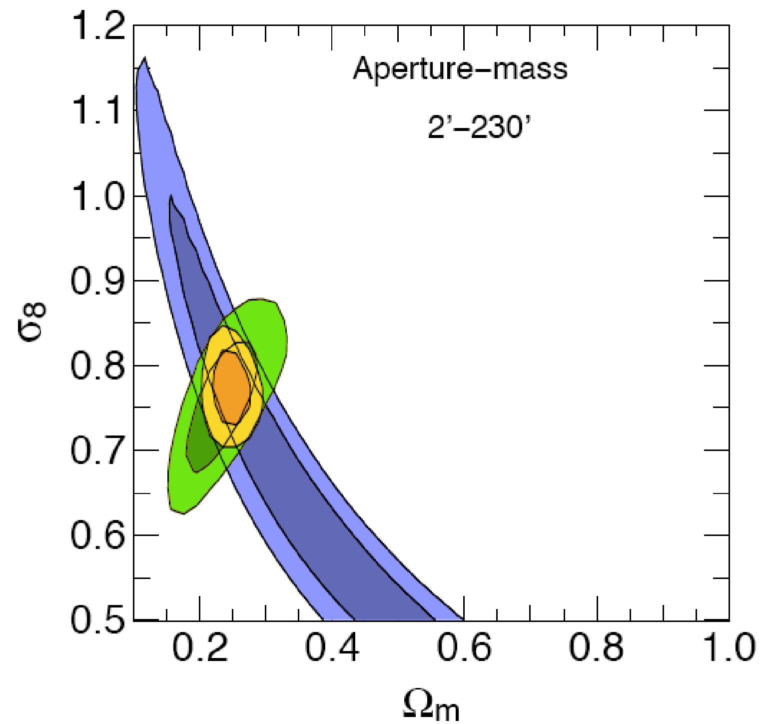
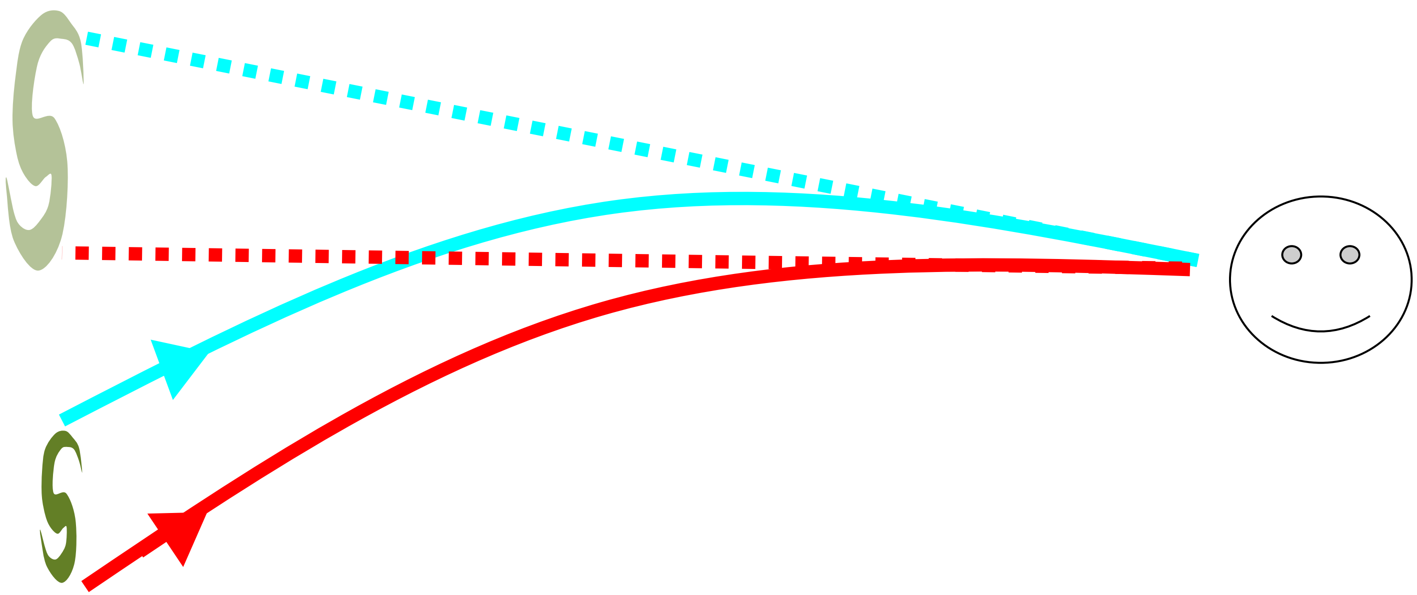
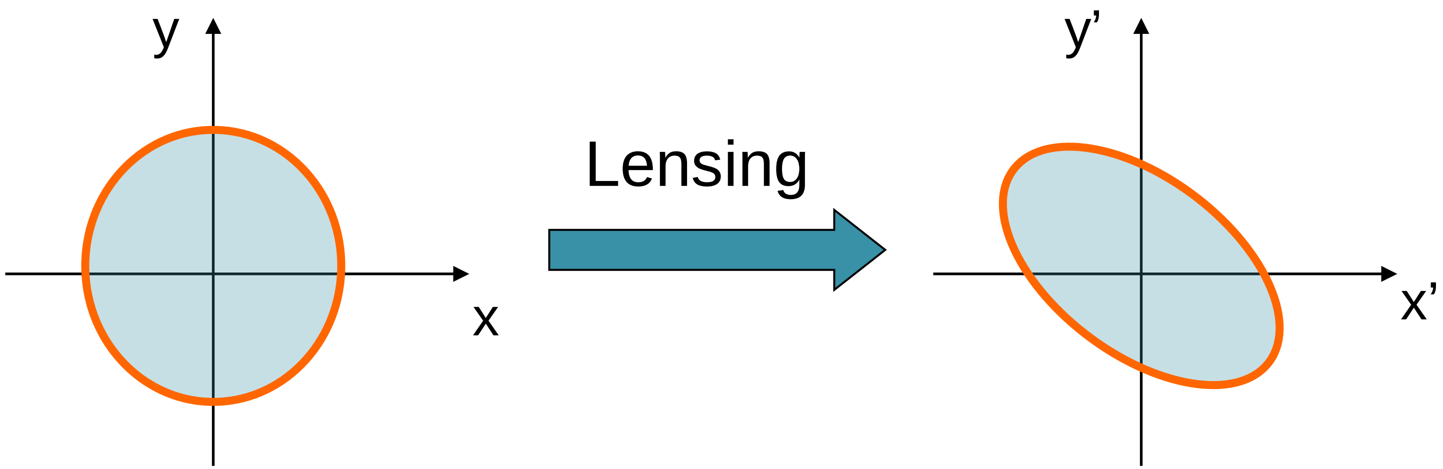


Image Credit: Hoekstra & Jain (2008)

Introduction

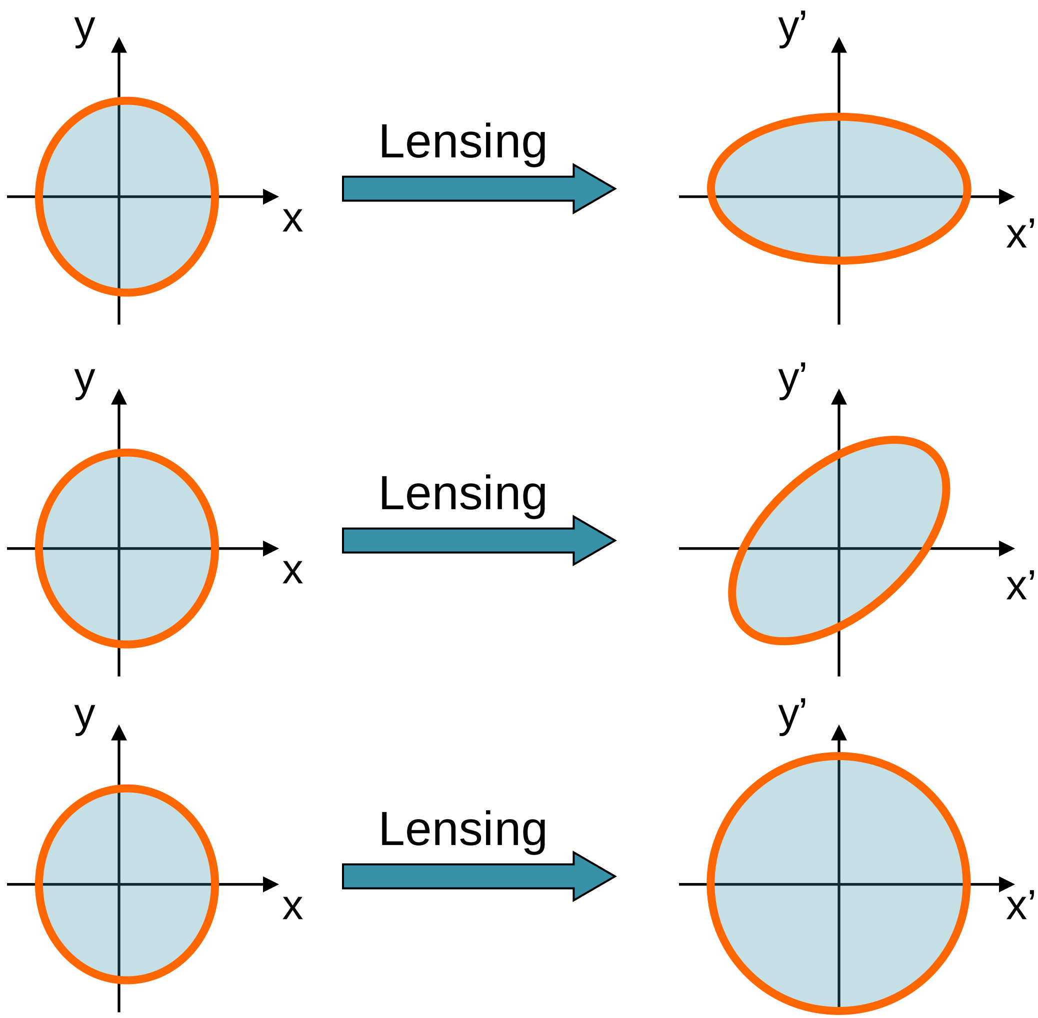


Introduction



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1+\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1+\kappa-\gamma_1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Introduction



$$\kappa = \gamma_2 = 0, \; \gamma_1 \neq 0$$

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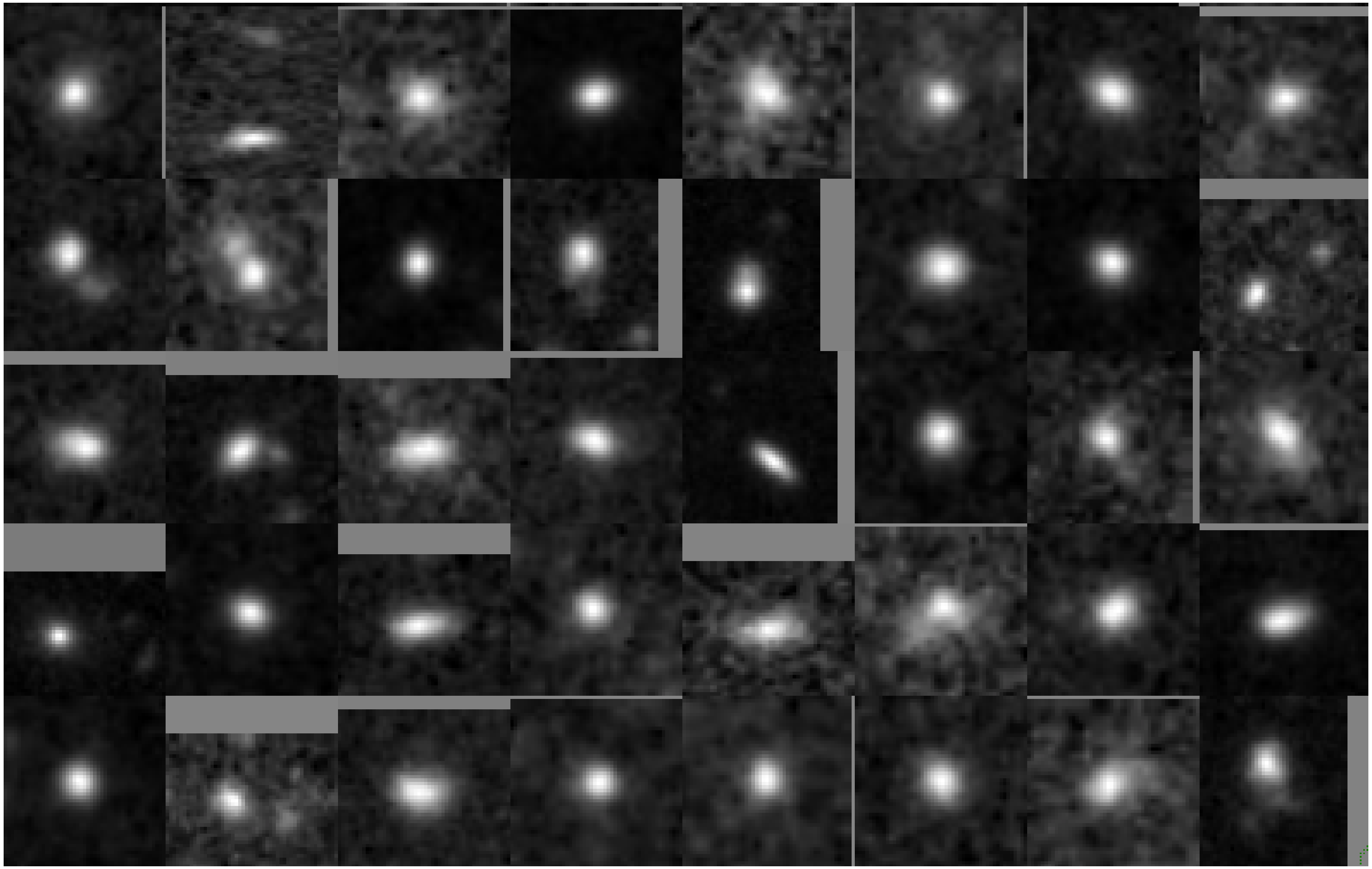
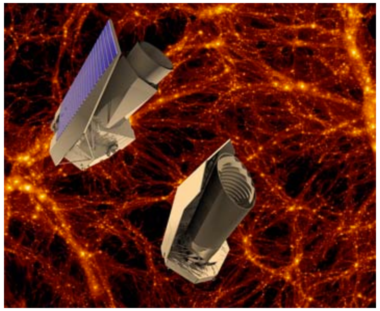
Introduction



Introduction

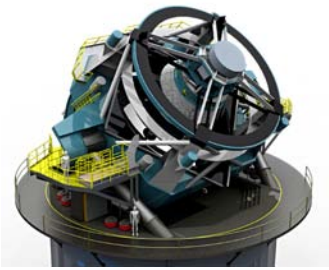
DES

KDUST

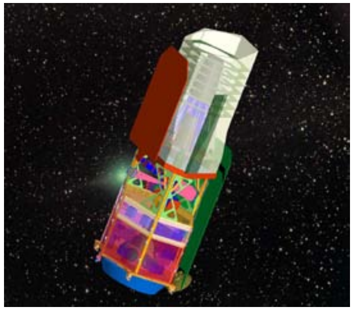


EUCLID

HSC



LSST



WFIRST

Problems in Weak Lensing Measurements

How to accurately measure the cosmic shear?

The shear signal: a few percent

V.S.

Intrinsic variance of the galaxy ellipticity

(shape noise): of order one

Systematic errors are not tolerable!!!

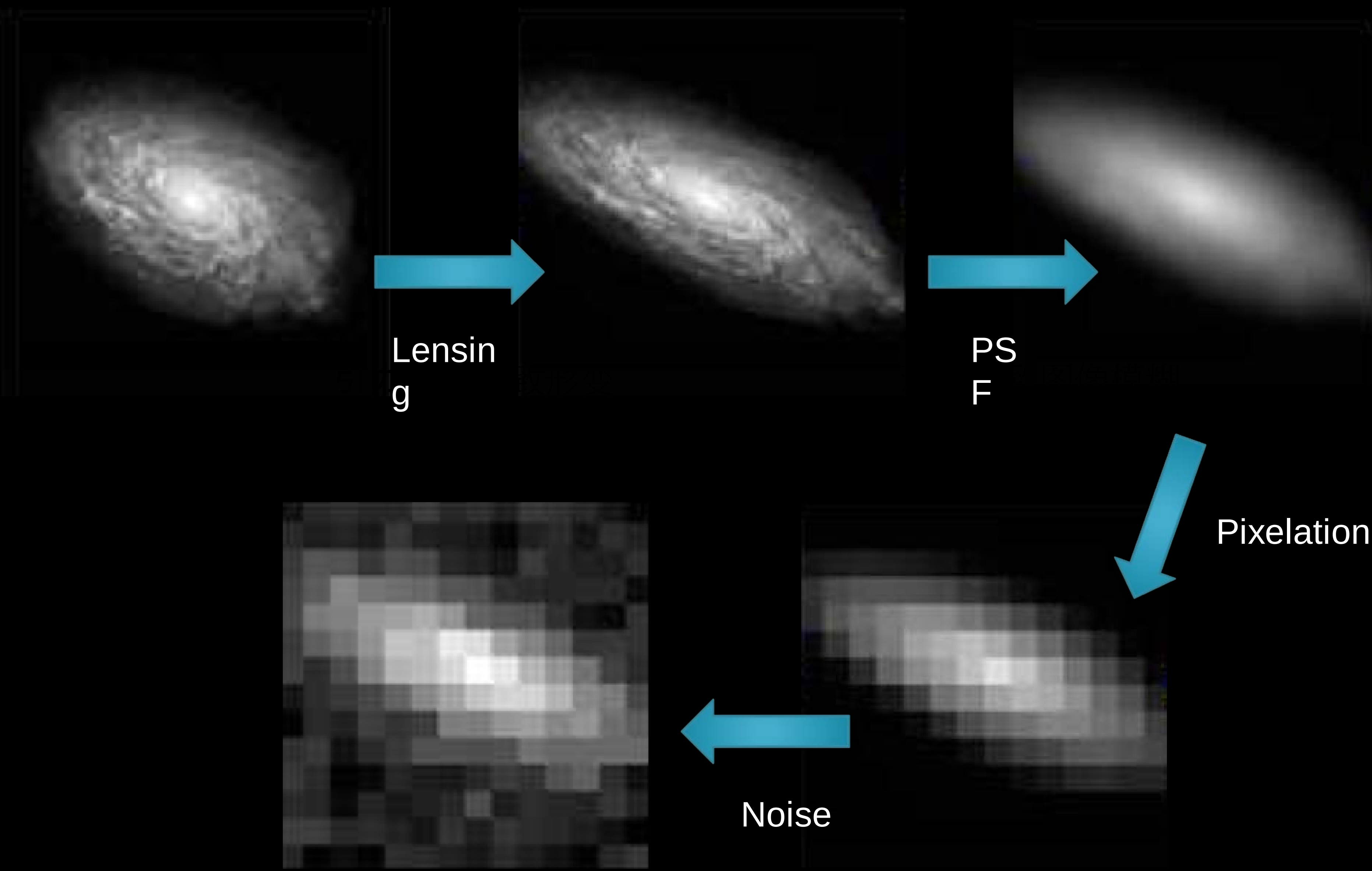
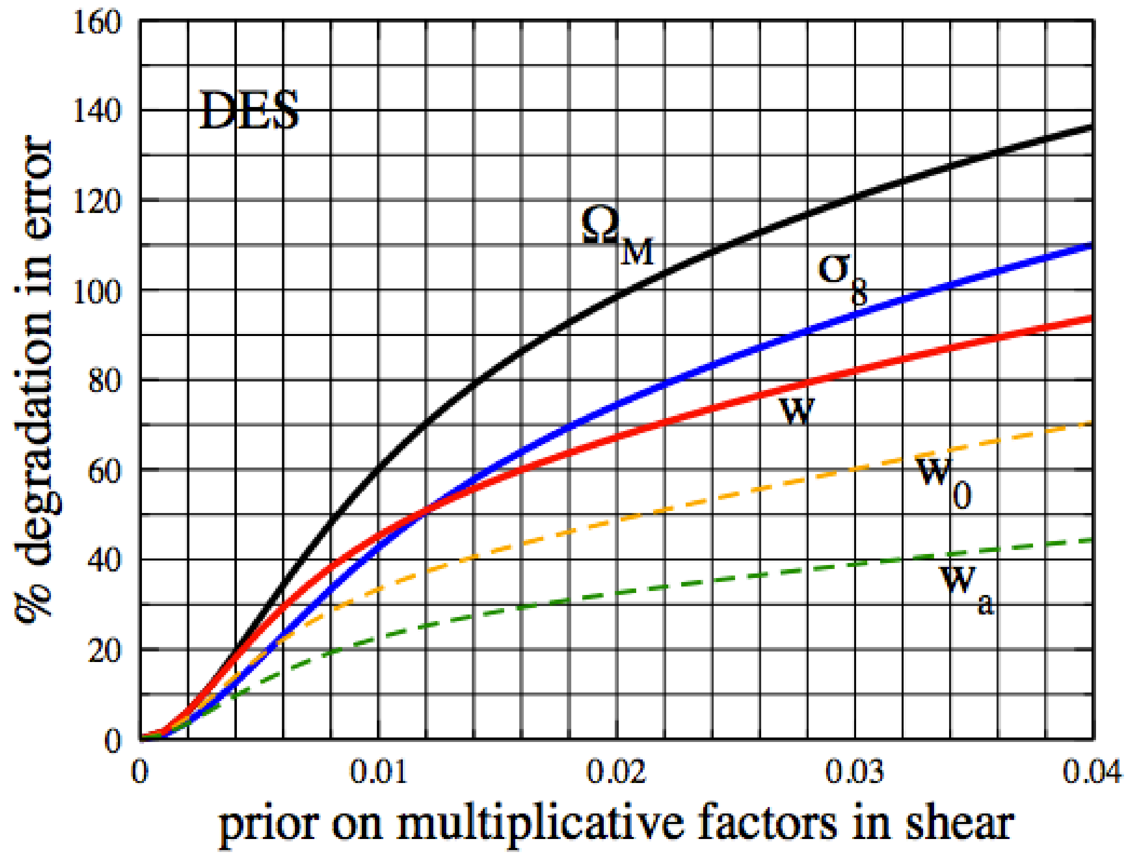


Image Credit: GREAT08

Problems in Weak Lensing Measurements

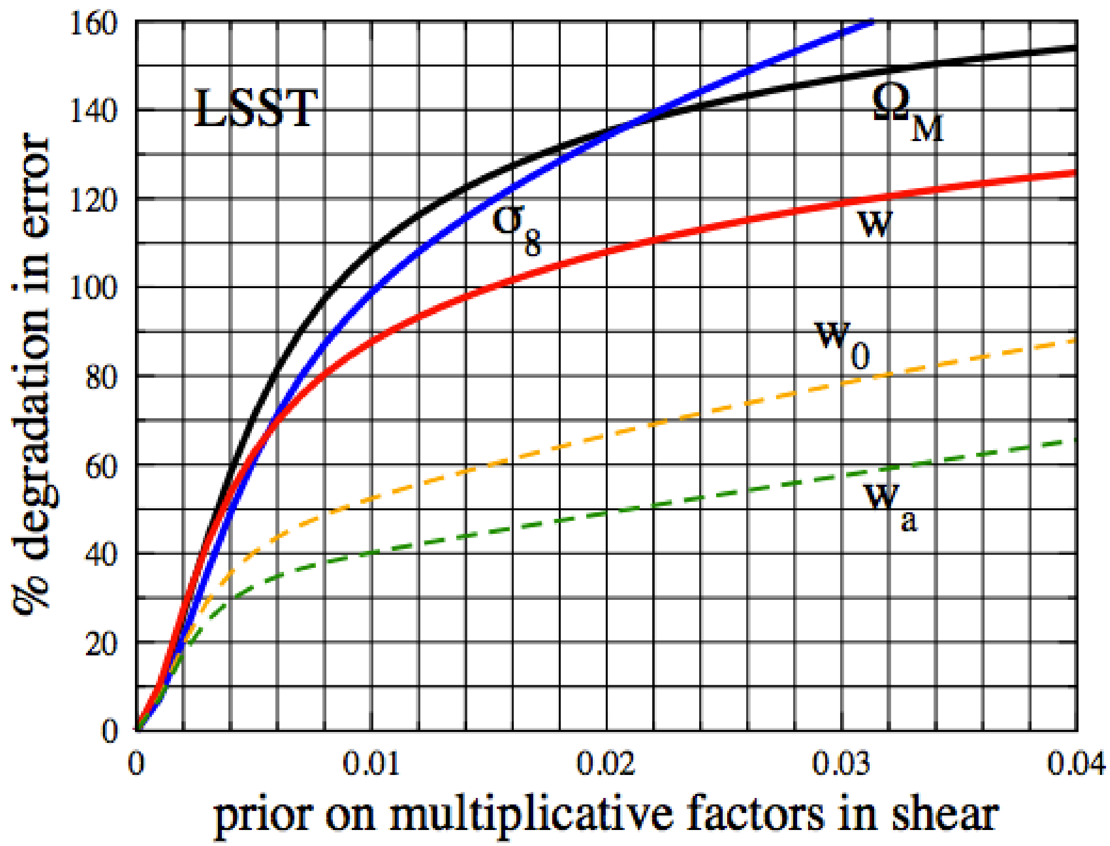
$$\gamma^{measured} = \gamma^{real}(1+m) + c$$

Problems in Weak Lensing Measurements



Credit: Huterer et al. 2006, MNRAS, 366, 101

Problems
in Weak Lensing Measurements



Credit: Huterer et al. 2006, MNRAS, 366, 101

Problems
in Weak Lensing Measurements

Sources of Systematic Errors:

• Corrections due to the Point Spread Function (PSF);
• Photon Noise;
• Pixelation Effect;
• High Order Terms in Shear/Convergence;
• Charge Transfer Inefficiency;
• PSF determination from Star Shapes;
• Source Selection Bias;
• Photometric Redshift Errors;
• Galaxy Intrinsic Alignment.

Problems
in Weak Lensing Measurements

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Problems
in Weak Lensing Measurements

- The Form of Cosmic Shear Estimator (general)
- The Reduced Shear (general)
- The Pixelation Effect (specific)
- The Photon Noise (specific)

The Form of Shear Estimator

?

$$\langle \Gamma \rangle = \gamma$$

$$\langle \Gamma(\vec{x}_1)\Gamma(\vec{x}_2)\ldots\Gamma(\vec{x}_N)\rangle = \langle \gamma(\vec{x}_1)\gamma(\vec{x}_2)\ldots\gamma(\vec{x}_N)\rangle$$

STEP I, STEP II, GREAT 08,
GREAT 10

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma$$

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma$$ ✗

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma$$ ✗

$$\frac{\langle A \rangle}{\langle B \rangle} = \gamma$$ ✔

The Form of Shear Estimator

Warming Up Exercise

$$Q_{ij} = \int d^2\vec{x} x_i x_j f_L(\vec{x})$$

$$\begin{aligned} \epsilon_1 &= \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} \\ \epsilon_2 &= \frac{2Q_{12}}{Q_{11} + Q_{22}} \end{aligned}$$

The Form of Shear Estimator

Warming Up Exercise

$$\frac{1}{2}\langle\epsilon_1\rangle = \gamma_1(1 - \langle\epsilon_1^2\rangle)$$

$$\frac{1}{2}\langle\epsilon_2\rangle = \gamma_2(1 - \langle\epsilon_2^2\rangle)$$

$$\begin{aligned}
\frac{1}{2}\frac{\langle Q_{11} - Q_{22}\rangle}{\langle Q_{11} + Q_{22}\rangle} &= \gamma_1 \\
\frac{\langle Q_{12}\rangle}{\langle Q_{11} + Q_{22}\rangle} &= \gamma_2
\end{aligned}$$

The Form of Shear Estimator

Are there ideal/conventional shear estimators existing in convenient forms?

$$\langle \Gamma \rangle \overset{?}{=} \gamma$$

The Form of Shear Estimator

Coordinate Rotation:

$$\gamma_1^\theta + \mathrm{i}\gamma_2^\theta = (\gamma_1 + \mathrm{i}\gamma_2)\exp(\mathrm{i}2\theta)$$

The Form of Shear Estimator

Lemma Based on any pair of (Γ_1, Γ_2), one can build a new pair (Γ_1', Γ_2') to form a spin-2 quantity through the following procedure:

$$\Gamma_1' + \mathrm{i}\Gamma_2' = \frac{1}{2\pi}\int_0^{2\pi} d\theta \exp(-\mathrm{i}2\theta)\left(\Gamma_1^\theta + \mathrm{i}\Gamma_2^\theta\right)$$

The Form of Shear Estimator

Proof. Firstly, under a clockwise coordinate rotation by angle θ_0, we have:

$$
\begin{aligned}
& \Gamma_1'^{\theta_0} + \mathrm{i}\Gamma_2'^{\theta_0} \\
= \quad & \frac{1}{2\pi}\int_0^{2\pi} d\theta \exp(-\mathrm{i}2\theta)\left(\Gamma_1^{\theta_0+\theta} + \mathrm{i}\Gamma_2^{\theta_0+\theta}\right) \\
= \quad & \frac{1}{2\pi}\int_0^{2\pi} d\theta' \exp[-\mathrm{i}2(\theta' - \theta_0)]\left(\Gamma_1^{\theta'} + \mathrm{i}\Gamma_2^{\theta'}\right) \\
= \quad & \left(\Gamma_1' + \mathrm{i}\Gamma_2'\right)\exp(\mathrm{i}2\theta_0)
\end{aligned}
$$

Therefore, $\Gamma_1' + \mathrm{i}\Gamma_2'$ form a spin-2 quantity.

The Form of Shear Estimator

$$\left\langle \Gamma_1' + \mathrm{i}\Gamma_2' \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta \exp(-\mathrm{i}2\theta) \left\langle \Gamma_1^\theta + \mathrm{i}\Gamma_2^\theta \right\rangle$$

$$\left\langle \Gamma_1^\theta \right\rangle = \gamma_1^\theta = \gamma_1 \cos 2\theta - \gamma_2 \sin 2\theta$$

$$\left\langle \Gamma_2^\theta \right\rangle = \gamma_2^\theta = \gamma_1 \sin 2\theta + \gamma_2 \cos 2\theta$$

$$\left\langle \Gamma_1^\theta + \mathrm{i}\Gamma_2^\theta \right\rangle = (\gamma_1 + \mathrm{i}\gamma_2) \exp(\mathrm{i}2\theta)$$

$$\left\langle \Gamma_1' + \mathrm{i}\Gamma_2' \right\rangle = \gamma_1 + \mathrm{i}\gamma_2$$

The Form of Shear Estimator

Indeed, we only need to consider spin-2 shear estimators because of the following:

$$
\begin{aligned}
& \frac{1}{2\pi}\int_0^{2\pi} d\theta \exp(-\mathrm{i}n\theta)\left\langle \Gamma_1^\theta + \mathrm{i}\Gamma_2^\theta \right\rangle \\
= \quad & \frac{1}{2\pi}\int_0^{2\pi} d\theta \exp(-\mathrm{i}n\theta)\left(\gamma_1 + \mathrm{i}\gamma_2\right)\exp(\mathrm{i}2\theta) \\
= \quad & 0 \ (\text{ if } n \neq 2 \ )
\end{aligned}
$$

The Form of Shear Estimator

On spin-2 shear estimators:

$$\Gamma_1(\gamma_1,\gamma_2,\kappa) = (\Gamma_1)_0 + \gamma_1\left(\frac{\partial\Gamma_1}{\partial\gamma_1}\right)_0 + \gamma_2\left(\frac{\partial\Gamma_1}{\partial\gamma_2}\right)_0 + \kappa\left(\frac{\partial\Gamma_1}{\partial\kappa}\right)_0$$

$$\Gamma_2(\gamma_1,\gamma_2,\kappa) = (\Gamma_2)_0 + \gamma_1\left(\frac{\partial\Gamma_2}{\partial\gamma_1}\right)_0 + \gamma_2\left(\frac{\partial\Gamma_2}{\partial\gamma_2}\right)_0 + \kappa\left(\frac{\partial\Gamma_2}{\partial\kappa}\right)_0$$

The Form of Shear Estimator

On spin-2 shear estimators:

$$\langle \Gamma_1(\gamma_1, \gamma_2, \kappa) \rangle = \gamma_1 \langle A + B_1 \rangle + \gamma_2 \langle C + B_2 \rangle$$
$$\langle \Gamma_2(\gamma_1, \gamma_2, \kappa) \rangle = \gamma_1 \langle B_2 - C \rangle + \gamma_2 \langle A - B_1 \rangle$$

$$A = \frac{1}{2}\left(\partial_{\gamma_1}\Gamma_1 + \partial_{\gamma_2}\Gamma_2\right)$$ Scalar

$$C = \frac{1}{2}\left(\partial_{\gamma_2}\Gamma_1 - \partial_{\gamma_1}\Gamma_2\right)$$ Pseudo-Scalar

$$B_1 = \frac{1}{2}\left(\partial_{\gamma_1}\Gamma_1 - \partial_{\gamma_2}\Gamma_2\right)$$ Spin-4

$$B_2 = \frac{1}{2}\left(\partial_{\gamma_2}\Gamma_1 + \partial_{\gamma_1}\Gamma_2\right)$$ Spin-4

The Form of Shear Estimator

On spin-2 shear estimators:

$$\frac{\partial \Gamma_1}{\partial \gamma_1} + \frac{\partial \Gamma_2}{\partial \gamma_2} = 2$$

The Form of Shear Estimator

Consider a special type of galaxies:

$$f_S\left[a(x^2+y^2)+b(x^2-y^2)+2cxy\right]$$

$$(a+b>0,\ a^2-b^2>c^2)$$

For example, if $f_S(R) = H(R_c - R)$ (H is the step function) and (a, b, c) satisfy the above conditions, the galaxy surface brightness is then distributed evenly inside the ellipse defined by $a(x^2+y^2)+b(x^2-y^2)+2cxy \leqslant R_c$.

The Form of Shear Estimator

Consider a special type of galaxies:

$$\epsilon_1 = \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} = -\frac{b}{a}$$

$$\epsilon_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}} = -\frac{c}{a}$$

The Form of Shear Estimator

Consider a special type of galaxies:

$\Gamma_1 + i\Gamma_2$ as power series of $\epsilon_1 + i\epsilon_2$ and $\epsilon_1 - i\epsilon_2$

The Form of Shear Estimator

Consider a special type of galaxies:

$\Gamma_1 + \mathrm{i}\Gamma_2$ as power series of $\epsilon_1 + \mathrm{i}\epsilon_2$ and $\epsilon_1 - \mathrm{i}\epsilon_2$

$$\Gamma_1 + \mathrm{i}\Gamma_2 = (\epsilon_1 + \mathrm{i}\epsilon_2) g(u)$$

$$u = \epsilon_1^2 + \epsilon_2^2$$

The Form of Shear Estimator

Consider a special type of galaxies:

$$\begin{aligned} 2 &= \frac{\partial \Gamma_1}{\partial \gamma_1} + \frac{\partial \Gamma_2}{\partial \gamma_2} \\ &= 2(2-u)g(u) + 4u(1-u)\frac{dg}{du} \end{aligned}$$

The Form of Shear Estimator

Consider a special type of galaxies:

$$
\begin{aligned}
2 &= \frac{\partial \Gamma_1}{\partial \gamma_1} + \frac{\partial \Gamma_2}{\partial \gamma_2} \\
&= 2(2-u)g(u) + 4u(1-u)\frac{dg}{du}
\end{aligned}
$$

$$
\boxed{\Gamma_1 + \mathrm{i}\Gamma_2 = (\epsilon_1 + \mathrm{i}\epsilon_2)\frac{1 - \sqrt{1-\epsilon_1^2-\epsilon_2^2}}{\epsilon_1^2+\epsilon_2^2}}
$$

Bartelmann & Schneider 2001, Phys. Rept., 340, 291

The Form of Shear Estimator

In the Presence of PSF:

? $$\frac{\partial \Gamma_1}{\partial \gamma_1} + \frac{\partial \Gamma_2}{\partial \gamma_2} = 2$$

✗ $$\frac{\partial \Gamma_1}{\partial \gamma_1} = \frac{\partial \Gamma_2}{\partial \gamma_2} = 1$$

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma \qquad ✗$$

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma$$ ✗

$$\frac{\langle A \rangle}{\langle B \rangle} = \gamma$$ ✔

The Form of Shear Estimator

$$\langle \Gamma \rangle = \gamma$$ ✗

$$\frac{\langle W\Gamma \rangle}{\langle W \rangle} = \gamma$$ ✔

The Form of Shear Estimator

$$\langle \Gamma(\vec{x}_1)\Gamma(\vec{x}_2)\ldots\Gamma(\vec{x}_N)\rangle = \langle \gamma(\vec{x}_1)\gamma(\vec{x}_2)\ldots\gamma(\vec{x}_N)\rangle \quad ✗$$

$$\frac{\langle A(\vec{x}_1)A(\vec{x}_2)\ldots A(\vec{x}_N)\rangle}{\langle B(\vec{x}_1)B(\vec{x}_2)\ldots B(\vec{x}_N)\rangle} = \langle \gamma(\vec{x}_1)\gamma(\vec{x}_2)\ldots\gamma(\vec{x}_N)\rangle \quad ✔$$

The Form of Shear Estimator

$$\frac{1}{2}\frac{\langle\langle(\partial_1 f_O)^2 - (\partial_2 f_O)^2\rangle_g\rangle_{en}}{\langle\langle(\partial_1 f_O)^2 + (\partial_2 f_O)^2 + \Delta\rangle_g\rangle_{en}} = -\gamma_1$$

$$\frac{\langle\langle\partial_1 f_O \partial_2 f_O\rangle_g\rangle_{en}}{\langle\langle(\partial_1 f_O)^2 + (\partial_2 f_O)^2 + \Delta\rangle_g\rangle_{en}} = -\gamma_2$$

$$\Delta = \frac{\beta^2}{2}\vec{\nabla} f_O \cdot \vec{\nabla}(\nabla^2 f_O)$$

Ref: Astro-ph/0612146, arXiv: 0901.4781, arXiv: 1002.3614

The Form of Shear Estimator

$$\frac{1}{2}\frac{\langle\langle(\partial_1 f_O)^2-(\partial_2 f_O)^2\rangle_g\rangle_{en}}{\langle\langle(\partial_1 f_O)^2+(\partial_2 f_O)^2+\Delta\rangle_g\rangle_{en}} = -\frac{\gamma_1}{1-\kappa}$$

$$\frac{\langle\langle\partial_1 f_O\partial_2 f_O\rangle_g\rangle_{en}}{\langle\langle(\partial_1 f_O)^2+(\partial_2 f_O)^2+\Delta\rangle_g\rangle_{en}} = -\frac{\gamma_2}{1-\kappa}$$

$$\Delta = \frac{\beta^2}{2}\vec{\nabla} f_O \cdot \vec{\nabla}(\nabla^2 f_O)$$

Ref: Astro-ph/0612146, arXiv: 0901.4781, arXiv: 1002.3614

The Form of Shear Estimator

✧ **Correct for the Point Spread Function;**

✧ **No Assumptions on the Morphologies of the Galaxies or the PSF;**

✧ **Exact & Simple Math;**

✧ **Well Defined Treatment of Background Noise;**

✧ **Accurate to the 2nd Order in Shear/Convergence;**

✧**Immune to Misidentification of Sources (faint/small galaxies, stars)**

✧ **< 10^{-2} seconds/Galaxy**

✧ **S / N per galaxy Increased.**

The Reduced Shear

$$\mathbf{M} = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

$$g_{1,2} = \frac{\gamma_{1,2}}{1-\kappa}$$

The Reduced Shear

$$g_{1,2}\left[a + b\kappa + c(g_1^2 + g_2^2) + \cdots\right]$$

If one plans to calibrate the multiplicative factor “a+bκ”, one should pay attention to its dependence on “κ”, which, though, is not known a priori in observations.

The Reduced Shear

$$g_{1,2}\left[a + b\kappa + c(g_1^2 + g_2^2) + \cdots\right]$$

Theorem: $\mathit{if}\ a \equiv 1$, $\mathit{we\ must\ have}\ b \equiv 0.$

Proof If $a \equiv 1$, the statistical expectations of the shear estimators can be written as:

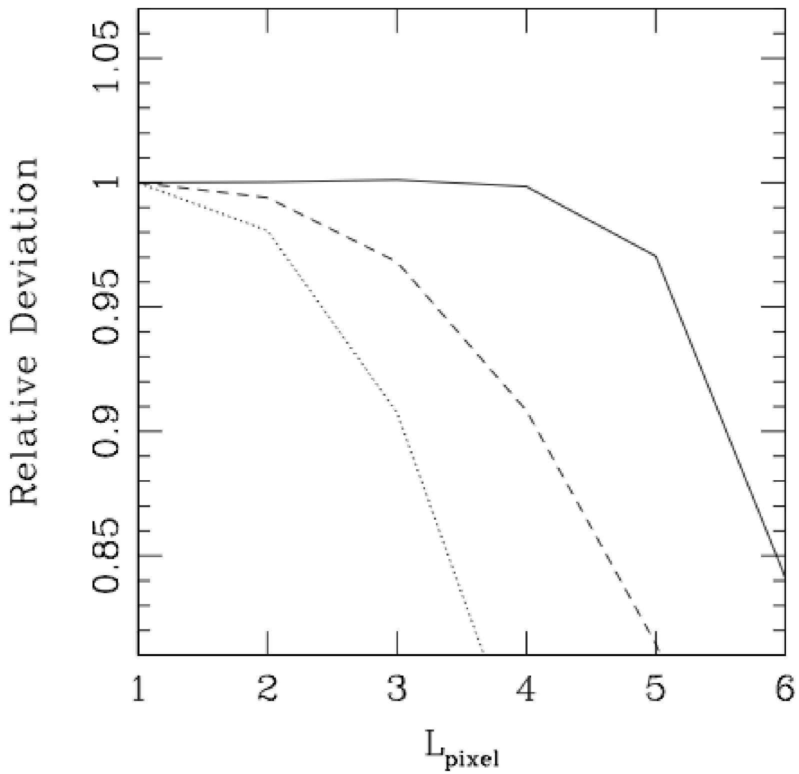
$$g_{1,2}\left[1 + b\kappa + c(g_1^2 + g_2^2) + \cdots\right], \tag{3.1}$$

For a given set of observed galaxies, let us consider the following two cases with fixed g_1 and g_2:

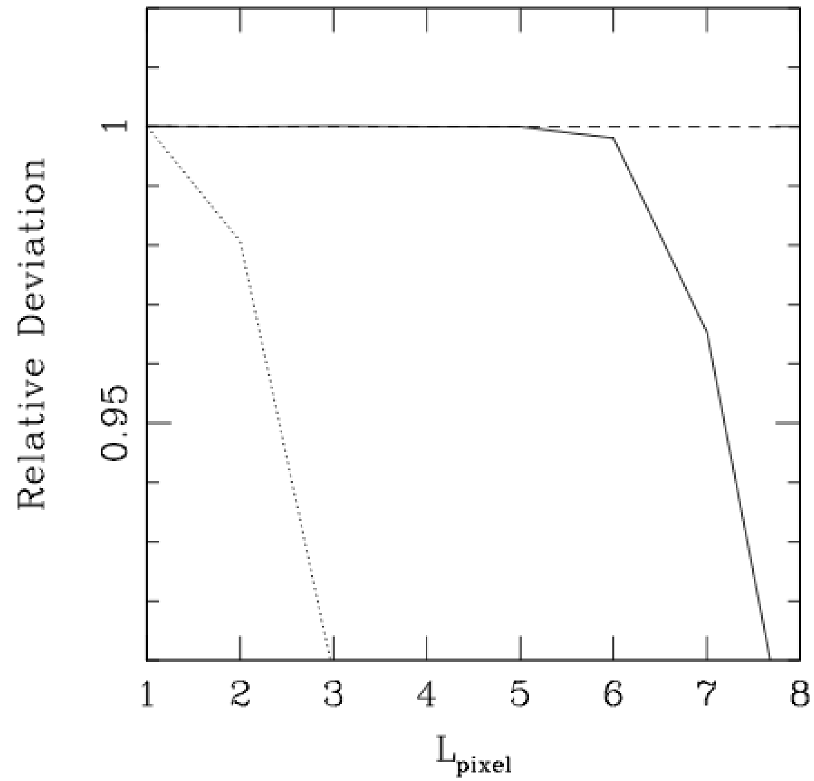
1. $\kappa = 0$;
2. $\kappa \neq 0$, and the galaxies are intrinsically larger than those in the first case by a factor of $(1 - \kappa)$.

Since the observed galaxy images in the two cases are identical, the shear measurement method should yield the same results. On the other hand, according to eq.(3.1), we should get (if accurate to the second order) $g_{1,2}$ in the first case, and $g_{1,2}(1 + b\kappa)$ in the second case. Therefore, we have $b \equiv 0$. ∎

The Pixelation Effect (specific)



Moffat PSF



Gaussian PSF

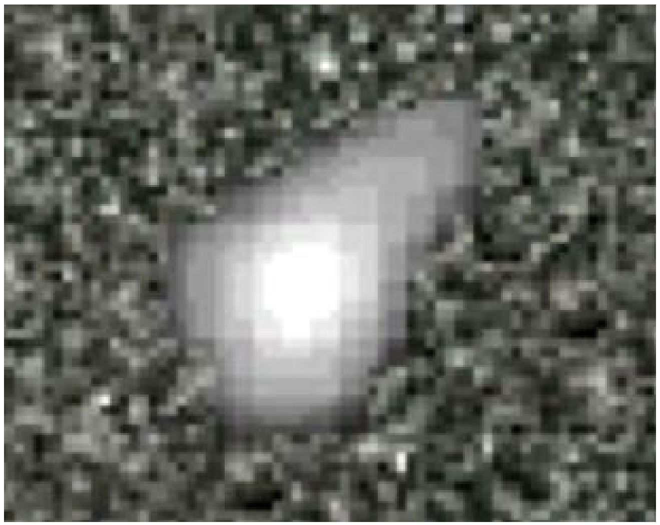
FWHM of both PSF = 12

The Pixelation Effect (specific)

	Small Gal.	Medium Gal.	Large Gal.	Largest Gal.
$L_{pixel} = 1$	$m_1(10^{-3}) : 2.0 \pm 1.5$	-0.8 ± 1.5	3.8 ± 1.5	2.8 ± 1.4
	$c_1(10^{-5}) : 9.6 \pm 4.8$	4.3 ± 4.8	-2.1 ± 4.7	-4.8 ± 4.7
$L_{pixel} = 2$	3.2 ± 1.5	-0.5 ± 1.5	3.9 ± 1.5	2.9 ± 1.4
	9.4 ± 4.8	4.2 ± 4.8	-2.1 ± 4.7	-4.8 ± 4.7
$L_{pixel} = 3$	7.2 ± 1.5	0.6 ± 1.5	4.2 ± 1.5	3.0 ± 1.4
	10.2 ± 4.8	4.3 ± 4.8	-2.1 ± 4.7	-4.8 ± 4.7
$L_{pixel} = 4$	-2.5 ± 1.5	-1.9 ± 1.5	3.5 ± 1.5	2.8 ± 1.4
	8.9 ± 4.8	4.1 ± 4.8	-2.1 ± 4.7	-4.8 ± 4.7
$L_{pixel} = 5$	-91.2 ± 1.3	-30.6 ± 1.4	-4.6 ± 1.4	0.4 ± 1.4
	9.8 ± 4.3	4.1 ± 4.6	-2.1 ± 4.7	-4.8 ± 4.6
$L_{pixel} = 6$	-372 ± 1	-174.5 ± 1.2	-58.4 ± 1.4	-16.2 ± 1.4
	7.5 ± 3.1	3.2 ± 3.9	-2.0 ± 4.4	-4.7 ± 4.6

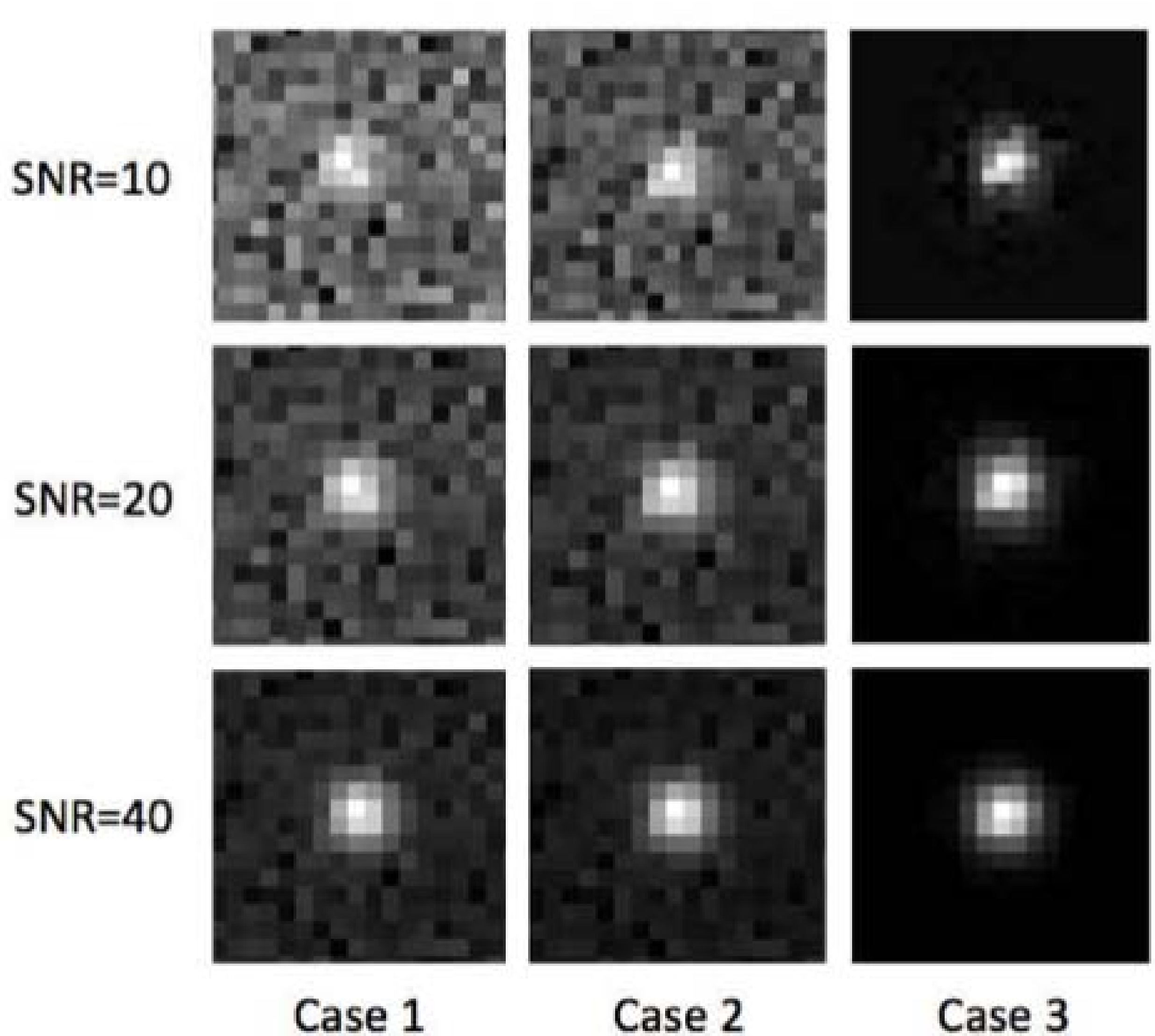
FWHM (PSF) = 12

The Photon Noise (specific)



$$
\begin{aligned}
\langle(\partial_x f^s)^2\rangle &= \langle(\partial_x f_O)^2\rangle - \langle(\partial_x f^n)^2\rangle \\
\langle(\partial_y f^s)^2\rangle &= \langle(\partial_y f_O)^2\rangle - \langle(\partial_y f^n)^2\rangle \\
\langle\partial_x f^s \partial_y f^s\rangle &= \langle\partial_x f_O \partial_y f_O\rangle - \langle\partial_x f^n \partial_y f^n\rangle \\
\langle\vec{\nabla} f^s \cdot \vec{\nabla}(\nabla^2 f^s)\rangle &= \langle\vec{\nabla} f_O \cdot \vec{\nabla}(\nabla^2 f_O)\rangle - \langle\vec{\nabla} f^n \cdot \vec{\nabla}(\nabla^2 f^n)\rangle
\end{aligned}
$$

The Photon Noise (specific)



The Photon Noise (specific)

	Case 1	Case 2	Case 3
SNR= 80	$m_1(10^{-3}) : 0.2 \pm 1.4$	5.0 ± 1.2	8.8 ± 0.8
	$c_1(10^{-5}) : -7.1 \pm 4.7$	-6.9 ± 3.8	-4.6 ± 2.6
SNR= 60	0.7 ± 1.8	9.3 ± 1.4	16.6 ± 0.9
	-8.4 ± 5.9	-8.2 ± 4.6	-5.1 ± 2.8
SNR= 40	1.9 ± 2.6	21.3 ± 2.0	39.2 ± 1.0
	-11.1 ± 8.6	-10.9 ± 6.5	-6.1 ± 3.3
SNR= 30	2.9 ± 3.5	38.0 ± 2.7	72.5 ± 1.2
	-13.9 ± 11.4	-13.8 ± 8.6	-7.2 ± 3.9
SNR= 20	5.0 ± 5.5	87.5 ± 4.2	179.5 ± 1.8
	-19.9 ± 17.8	-20.5 ± 13.6	-10.0 ± 5.7
SNR= 10	10.3 ± 13.3	451.9 ± 12.3	1529.5 ± 6.8
	-40.8 ± 43.4	-56.0 ± 40.1	-34.8 ± 22.2

The Photon Noise (specific)

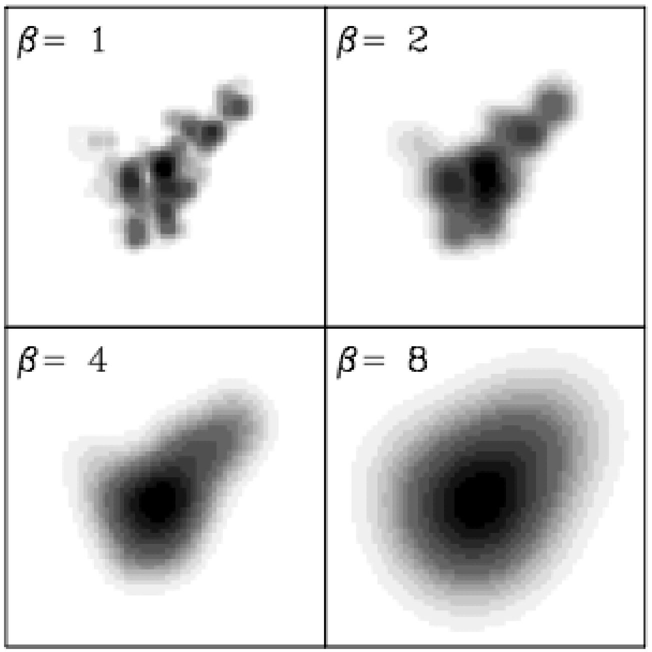
$$m \approx \frac{\delta g}{g} \propto \frac{\langle \delta(f^2) \rangle}{\langle f^2 \rangle}$$

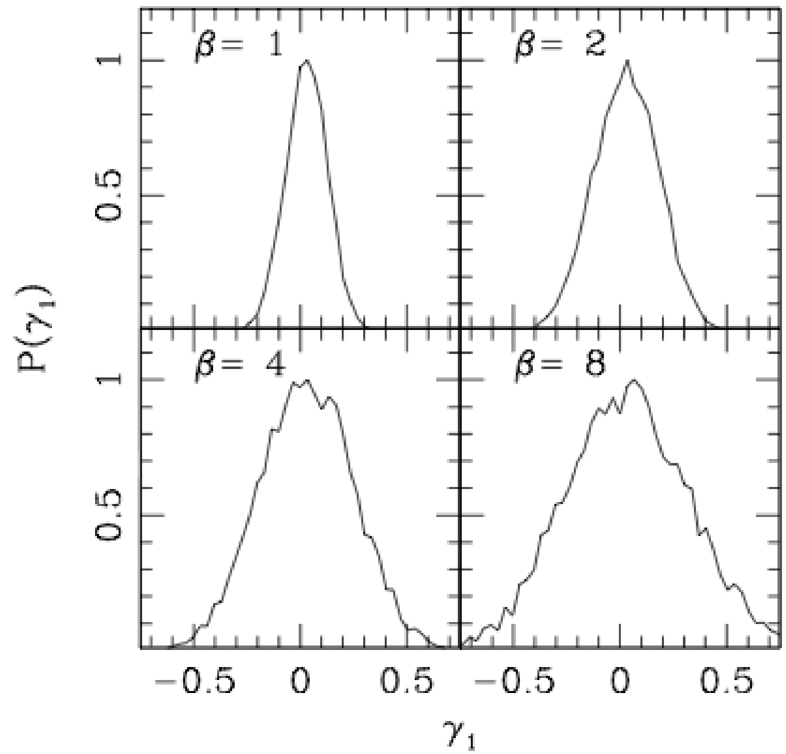
$$\langle \delta(f^2) \rangle = \langle (f_S + f_N)^2 - f_S^2 \rangle = \langle 2 f_S f_N + f_N^2 \rangle = \langle f_N^2 \rangle$$

$$\Rightarrow m \propto \frac{\langle f_N^2 \rangle}{\langle f_S^2 \rangle} = \frac{1}{\mathrm{SNR}_S^2}$$

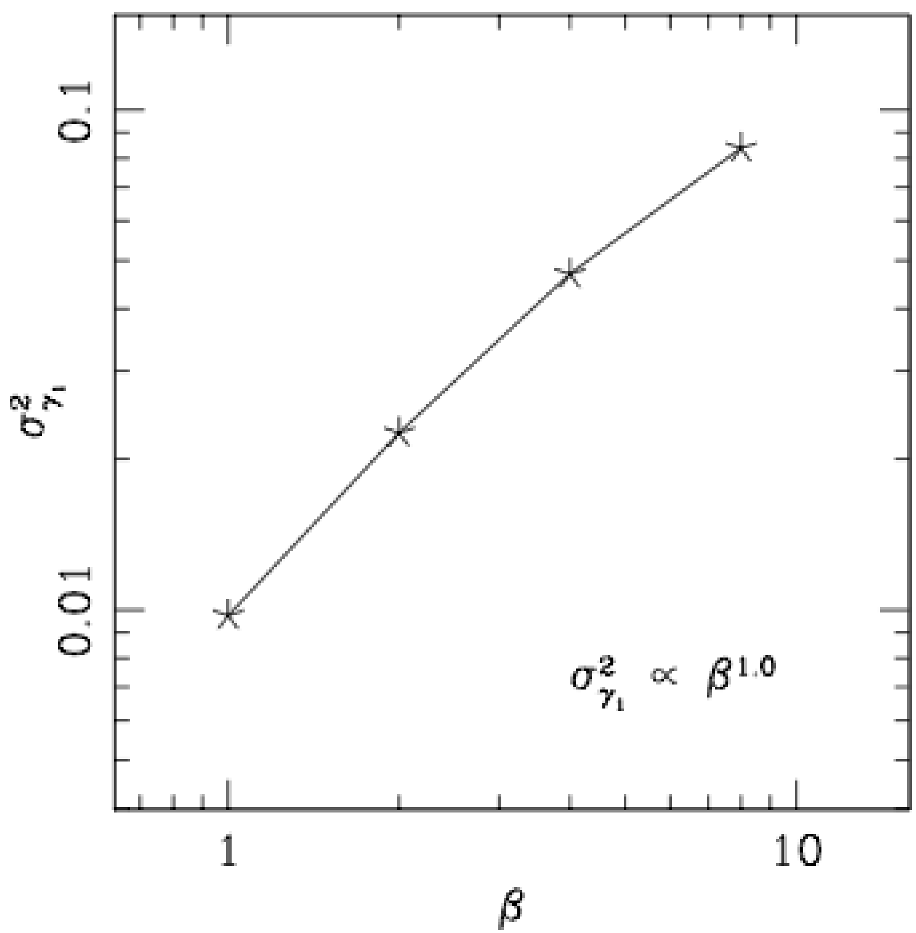
In fact
: $m \approx 60/\mathrm{SNR}_S^2$, except when $\mathrm{SNR}_S \lesssim 10$.

A Fun Thing to Share





A Fun Thing to Share



Summary

- The Form of Shear Estimator (general)
- The Reduced Shear (general)
- The Pixelation Effect (specific)
- The Photon Noise (specific)