Precision constraints on UED models and implications for the LHC

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TF, C. Pasold, arXiv:1111.7250

TF, arXiv:1112.xxxx

TF, D. Gerstenlauer, 1201.xxxx

Outline

- UED review / Motivation
- Modifying the UED mass spectrum
  - split UED (sUED)
  - non-minimal UED (nUED)
- Electroweak precision constraints on sUED and nUED
- Flavor violation in sUED and nUED
- Conclusions and Outlook

UED: The basic setup

- UED models are models with flat, compact extra dimensions in which *all* fields propagate. [Appelquist, Cheng, Dobrescu,(2001)]
- The Standard Model (SM) particles are identified with the lowest-lying modes of the respective Kaluza-Klein (KK) towers.
- The underlying 5D parameters are fixed by matching the zero mode couplings and masses to the SM values.

A toy example: complex scalar field in 5 dimensions ($\mathcal{M}_4 \times S^1$)

$$S_{5D} = \int_0^{2\pi R} dy \int d^4x \left\{ (\partial_M \Phi)^\dagger \partial^M \Phi - \hat{m}^2 |\Phi|^2 - \hat{\lambda} |\Phi|^4 \right\},$$

where $M = 0, 1, 2, 3, 5$.

- Equations of Motion: $(\partial_\mu \partial^\mu + \partial_5 \partial^5 + \hat{m}^2)\Phi = 0$.
- Separation of variables: $\Phi = \phi^{(n)}(x) f_n(y)$.
- Solve for the y part: $(\partial_5 \partial^5 - M_n^2) f_n = 0$, with the respective boundary conditions,
- to find $M_n^2 = (n/R)^2$, $f_n^c = \mathcal{N}_{c,n} \cos(M_n y)$, where $n = 0, 1, 2, ...$, and $f_n^s = \mathcal{N}_{s,n} \sin(M_n y)$, where $n = 1, 2, ...$

- Entering the solutions back into the action and integrating over y yields

$$\mathcal{L}_{4D} = (\partial_\mu \phi_c^{(0)})^\dagger \partial^\mu \phi_c^{(0)} - \hat{m}^2 |\phi_c^{(0)}|^2$$
$$+ \sum_{n=1}^{\infty} (\partial_\mu \phi_{s,c}^{(n)})^\dagger \partial^\mu \phi_{s,c}^{(n)} - \left((n/R)^2 + \hat{m}^2 \right) |\phi_{s,c}^{(n)}|^2$$
$$- \sum_{p,q,r,s} \lambda^{pqrs} \phi^{(p)\dagger} \phi^{(q)\dagger} \phi^{(r)} \phi^{(s)},$$

where

$$\lambda^{pqrs} = \hat{\lambda} \mathcal{F}^{pqrs} \equiv \hat{\lambda} \int dy f^{*(p)} f^{*(q)} f^{(r)} f^{(s)}.$$

- The masses of the KK states are $m_{(n)}^2 = \hat{m}^2 + (n/R)^2$.
- The 4D effective couplings λ^{pqrs} are determined from the 5D coupling $\hat{\lambda}$ via the overlap integrals $\mathcal{F}^{pqrs}$.
- KK-number conservation
  λ^{pqrs} vanishes unless $p + q = r + s$,
  where $(p, q)/(r, s)$ are the KK-modes of the incoming/outgoing particles.
  (This is a direct consequence of (discrete) 5-momentum conservation.)

Compactifying on S^1/Z_2

Fermions and gauge fields can be KK-decomposed analogously.
The zero mode spectrum is supposed to be identified with the SM.

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Compactifying on S^1/Z_2

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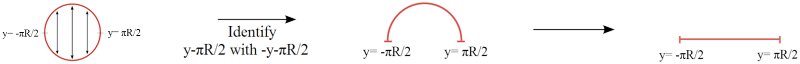
The zero mode spectrum is supposed to be identified with the SM.

BUT:

- The higher dimensional Lorentz group does not contain 4D chiral fermion representations. All KK modes of a higher dimensional fermion appear as a (sets of) Dirac fermions in the effective field theory.
- When decomposing a higher dimensional vector field one obtains a 4D vector A_μ *and* a 4D scalar A_5 at each KK-level *including* massless zero modes.

Solution: (specializing to five dimensional UED)
Compactification on an orbifold (S^1/Z_2)



- Presence of the orbifold fixed points imposes boundary conditions on the 5D fields at $y = 0, \pi R$.
- On S^1/Z_2, the boundary conditions can be chosen such that
  - half of the fermion zero mode is projected out $\Rightarrow$ chiral fermions
  - $A_5^{(0)}$ is projected out $\Rightarrow$ no additional massless scalar
- The presence of orbifold fixed points breaks 5D translational invariance.
  $\Rightarrow$ KK-number conservation is violated, *but* a discrete Z_2 parity (KK-parity) remains.
  $\Rightarrow$ The lightest KK mode (LKP) is (still) stable.

- UED action

$$S_{UED,bulk} = S_g + S_H + S_f,$$

with

$$\begin{aligned} S_g &= \int d^5x \left\{ -\frac{1}{4\hat{g}_3^2} G^A_{MN} G^{AMN} - \frac{1}{4\hat{g}_2^2} W^I_{MN} W^{IMN} - \frac{1}{4\hat{g}_Y^2} B_{MN} B^{MN} \right\}, \\ S_H &= \int d^5x \left\{ (D_M H)^\dagger (D^M H) + \hat{\mu}^2 H^\dagger H - \hat{\lambda} (H^\dagger H)^2 \right\}, \\ S_f &= \int d^5x \left\{ i\overline{\psi}\gamma^M D_M \psi + \left(\hat{\lambda}_E \overline{L} E H + \hat{\lambda}_U \overline{Q} U \tilde{H} + \hat{\lambda}_D \overline{Q} D H + \text{h.c.} \right) \right\}. \end{aligned}$$

UED as an effective field theory

- UED is a five dimensional model
  $\Rightarrow$ non-renormalizable.
- It should be considered as an effective field theory with a cutoff Λ.
- Naive dimensional analysis (NDA) result: $\Lambda \sim 50/R$.
  This cutoff is low!
- Bounds from unitarity imply $\Lambda \sim \mathcal{O}(10)$ [Chivukula, Dicus, He (2001)]
- *Without knowledge of the underlying theory all operators allowed by all symmetries should be considered.*

MUED

The well-studied example: "Minimal" UED [Cheng, Matchev, Schmaltz (2002)]

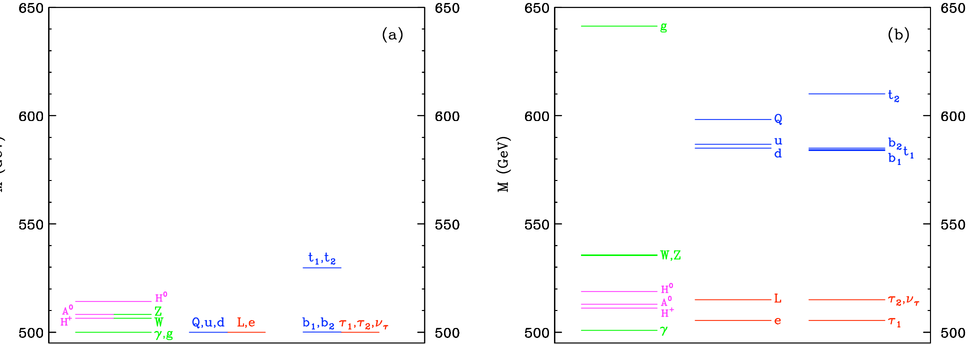
- Consider UED models as effective field theories (EFT) which is valid below a cutoff energy scale Λ.
- Calculate 1-loop corrections.
- Cancel divergencies by adding the respective (boundary localized) counter terms.

Note: The finite counter term part could only be determined when knowing the UV completion above the cut-off Λ.

Definition of MUED:

Assume that ALL boundary localized terms vanish at the cutoff Λ and are only induced at lower energies via RG running.

MUED - the spectrum



[Cheng, Matchev, Schmaltz, PRD **66** (2002) 036005, hep-ph/0204342]

(M)UED pheno review

Phenomenological constraints on the compactification scale R^{-1}

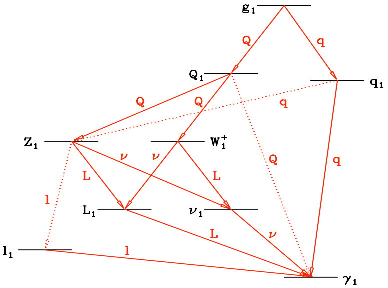
- Lower bounds:
  - FCNCs [Buras, Weiler *et al.* (2003); Weiler, Haisch (2007)]
    $R^{-1} \gtrsim 600(330)\,\mathrm{GeV}$ at 95% (99%) cl.
  - Electroweak Precision Constraints [Appelquist, Yee (2002); Gogoladze, Macesanu (2006); *Gfitter* (2011)]
    $R^{-1} \gtrsim 750(300)\,\mathrm{GeV}$ for $m_H = 115(800)\,\mathrm{GeV}$ at 95% cl.
  - no detection of KK-modes at LHC, yet [Murayama, Nojiri, Tobioka (2011); Bhattacherjee, Gosh (2011)]
    $R^{-1} \gtrsim 600\,\mathrm{GeV}$ at 95% cl.
- Upper bound:
  - preventing over closure of the Universe by $B^{(1)}$ dark matter
    $R^{-1} \lesssim 1.5\mathrm{TeV}$ [Servant, Tait (2002); Matchev, Kong (2005), Burnell, Kribs (2005); Belanger, Kakizaki, Pukhov (2010)]

UED vs. SUSY at LHC:

- Determining the spin of particles [Barr *et al.* (2004) and many follow-ups]
- Studying the influence of 2^{nd} KK mode particles
  [Datta, Kong, Matchev (2005), Kim, Oh, Park (2011), Chang, Lee, Song (2011)]
- Measuring total cross sections [Kane *et al.* (2005)], several follow-ups]
- Using differences of the UED and SUSY Higgs sector.[Oda *et al.* (2011), Kim, Oh (2011)]

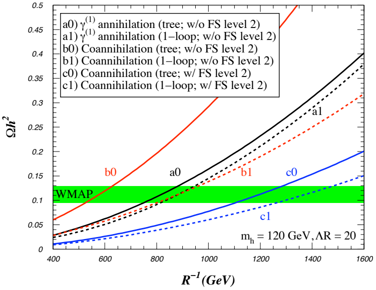
Relevance of the detailed mass spectrum I: LHC phenomenology

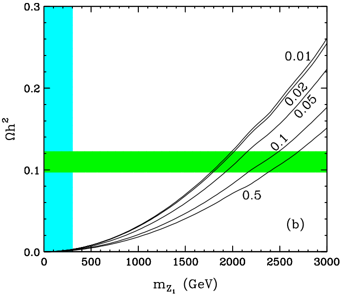
The KK mass spectrum determines decay channels and decay rates of KK particles produced at LHC.



[Cheng, Matchev, Schmaltz, PRD**66** (2002) 056006, hep-ph/0205314]

Relevance of the detailed mass spectrum II: Dark Matter relic density





Left: Relic density for $\gamma^{(1)}$ dark matter in MUED including coannihilation effects of first and second KK modes [Belanger *et al.* (2010)]

Right: Relic density for $W^{3(1)}$ dark matter including coannihilation effects with first KK modes and different mass degeneracies [Arrenberg, Kong (2008)].
Realization of $W^{3(1)}$ dark matter, see [TF, Menon, Phalen (2008)]

split-UED: Bulk mass terms for fermions

[Park, Shu, *et al.* (2009); for earlier work, see Csaki (2003)]

In split UED (sUED), a fermion bulk mass term is introduced.
A plain bulk mass term for fermions of the form

$$S \supset \int d^5x \; - M\overline{\Psi}\Psi$$

is forbidden by KK parity, but
it can be allowed if realized by a KK-parity odd background field

$$S \supset \int d^5x - \lambda\Phi\overline{\Psi}\Psi,$$

where $\Phi(-y) = -\Phi(y)$
The simplest case: $M = \mu\theta(y)$
(similar to the bulk fermion mass term in Randall-Sundrum models)

Variation of the free action leads to the EOMs:

$$i\gamma^\mu \partial_\mu \Psi_R - \gamma^5 \partial_5 \Psi_L - m_5(y)\Psi_L = 0 \quad ,$$
$$i\gamma^\mu \partial_\mu \Psi_L - \gamma^5 \partial_5 \Psi_R - m_5(y)\Psi_R = 0 \quad ,$$

KK decomposition:

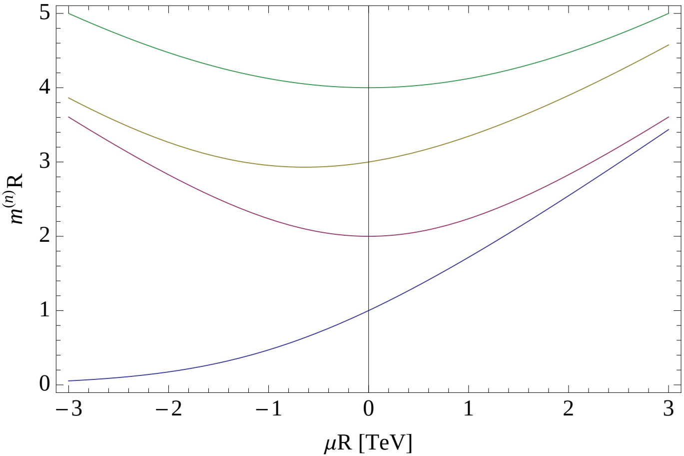
$$\Psi_R(x,y) = \sum_{n=0}^{\infty} \Psi_R^{(n)}(x)\ f_R^{(n)}(y)\ ; \qquad \Psi_L(x,y) = \sum_{n=0}^{\infty} \Psi_L^{(n)}(x)\ f_L^{(n)}(y)\ ,$$

Solutions for a fermion with right-handed zero mode:

KK zero modes	even numbered KK-modes	odd numbered KK-modes
$f_R^{(0)}(y) = \sqrt{\frac{\mu}{1-e^{-\mu\pi R}}}\, e^{-\mu\|y\|}$	$f_R^{(n)}(y) = \mathcal{N}_R^{(n)}\left(\cos(k_n y) - \frac{\mu}{k_n}\sin(k_n\|y\|)\right)$	$f_R^{(n)}(y) = \mathcal{N}_R^{(n)}\sin(m_n y)$
$f_L^{(0)}(y) = 0$	$f_L^{(n)}(y) = \mathcal{N}_L^{(n)}\sin(m_n y)$	$f_L^{(n)}(y) = \mathcal{N}_L^{(n)}\left(\cos(k_n y) + \frac{\mu}{k_n}\sin(k_n\|y\|)\right)$
$k_0 = 0$	$k_n = n/R$	$\cot(\frac{\pi R}{2} k_n) = -\mu$

and $m_n = \sqrt{k_n^2 + \mu^2}$.

sUED Fermion Mass Spectrum



Masses of the first four fermion KK modes in units of $1/R$ as a function of μR.

sUED overlap integrals

To obtain couplings between KK particles, we simply have to integrate over S^1/Z_2.
Compared to MUED, the fermion wave functions altered.
$\Rightarrow$ One obtains non-vanishing interactions of zero mode fermions with non-zero mode gauge bosons of strength

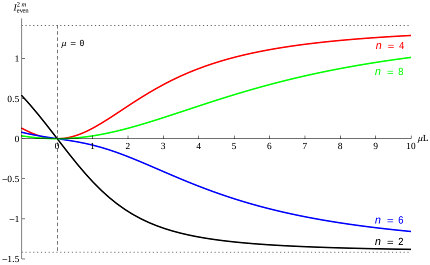
$$g_{eff}^{00n} = g_0 \mathcal{F}_{00n}$$

with the overlap integral given by

$$\mathcal{F}_{00n} \equiv \int_{-\pi R/2}^{\pi R/2} \frac{1}{\pi R} f_\psi^{(0)*} f_A^{(n)} f_\psi^{(0)}$$

$$= \frac{(\mu\pi R)^2(-1+(-1)^n e^{\mu\pi R}(\coth(\mu\pi R/2)-1)}{\sqrt{2(1+\delta_{0n}((\mu\pi R)^2+n^2\pi^2)}}$$

for n even and zero otherwise.



Boundary localized terms

Fermions: [Csaki,Hubisz,Meade(2001);Aguila, Perez-Victoria, Santiago(2003); TF, Gerstenlauer (in preparation)];
The fermion Lagrangian including BLKTs ("non-minimal UED") has the form:

$$\mathcal{S} = \int_{\mathbb{M}} \int_{S^1/\mathbb{Z}_2} \mathrm{d}^5 x \left[\frac{i}{2} \left(\overline{\Psi}\, \Gamma^M D_M \Psi - D_M \overline{\Psi}\, \Gamma^M \Psi \right) + \mathcal{L}_{BLKT} \right]$$

with

$$\mathcal{L}_{BLKT} = a_h \left[\delta\left(y - \frac{\pi R}{2} \right) + \delta\left(y + \frac{\pi R}{2} \right) \right] i \overline{\Psi}_h \not{D} \Psi_h \; ,$$

where $h = R, L$ represents the chirality and Γ^M is defined as $(\gamma^\mu, i\gamma^5)$.

For left-handed BLKTs, the KK decomposition leads to

KK zero modes	even numbered KK-modes	odd numbered KK-modes
$f_L^{(0)}(y) = \frac{1}{\sqrt{2a_L+\pi R}}$ $f_R^{(0)}(y) = 0$ $m_0 = 0$	$f_L^{(n)}(y) = -N\cos(m_n y)$ $f_R^{(n)}(y) = N\sin(m_n y)$ $\tan(\frac{\pi R}{2} m_n) = -a_L m_n$	$f_L^{(n)}(y) = N\sin(m_n y)$ $f_R^{(n)}(y) = N\cos(m_n y)$ $\cot(\frac{\pi R}{2} m_n) = a_L m_n.$

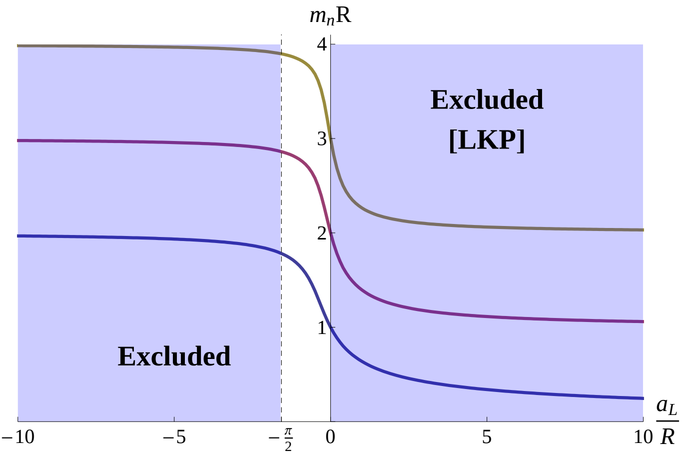
The fermion wave functions satisfy modified orthogonality relations

$$\delta_{mn} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\; f_L^{(n)}(y) f_L^{(m)}(y) \left(1 + a_L[\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2})]\right)$$

$$\delta_{mn} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\; f_R^{(n)}(y) f_R^{(m)}(y).$$

Results for right-handed BLKTs are given by $L \leftrightarrow R$.

nUED Fermion Mass Spectrum



Masses of the first three fermion KK modes in the presence of BLKTs. [D. Gerstenlauer, Diploma Thesis, Würzburg (2011)]

nUED: Electroweak sector and coupling constants

The BLKTs in the electroweak sector are:

$$\mathcal{L}_{BLKT,EW} = \left(-\frac{a_B}{4\hat{g}_1^2} B_{\mu\nu} B^{\mu\nu} - \frac{a_W}{4\hat{g}_2^2} W^a_{\mu\nu} W^{a,\mu\nu} - a_H (D_\mu H)^\dagger D^\mu H\right) \times \left[\delta\left(y - \frac{\pi R}{2}\right) + \delta\left(y + \frac{\pi R}{2}\right)\right]$$

For simplicity, we consider a common EW boundary parameter $a_B = a_W = a_H \equiv a_{ew}$.

For the generic case, *c.f.* [TF,Menon,Phalen(2009)].

KK decomposition yields the mass spectrum and wave functions of the Higgs and gauge bosons which resemble the fermionic wave functions.

Integrating over the extra dimension yields couplings of zero mode fermions to even KK mode gauge bosons:

$$g_{eff}^{00n} = g_0 \mathcal{F}_{00n} = g_0 \frac{(-1)^{n/2}\sqrt{2}(a_f - a_{ew})}{\pi R/2 + a_f} \sqrt{\frac{1 + a_{ew} 2/\pi R}{\sec^2(k_n R/2) + a_{ew} 2/\pi R}},$$

where a_f and a_{ew} are the fermion- and electroweak BLKT parameters, and k_n is determined from $k_n a_{ew} = -\tan[k_n R\pi/2]$.

Phenomenology: Electroweak precision constraints on sUED and nUED

If corrections to the SM only influence the gauge boson propagators, they can be parameterized by the Peskin-Takeuchi Parameters

$$\alpha S = 4e^2 \left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right) \quad ,$$
$$\alpha T = \frac{e^2}{\hat{s}_Z^2 \hat{c}_Z^2 M_Z^2} \left(\Pi_{11}(0) - \Pi_{33}(0)\right) \quad ,$$
$$\alpha U = 4e^2 \left(\Pi'_{11}(0) - \Pi'_{33}(0)\right)$$

where $\Pi(0)$ is the respective two-point function evaluated at a reference scale $p^2 = 0$,
and $\Pi'(0) = \frac{d\Pi}{dp^2}\Big|_{p^2=0}$.

Experimental values: [Gfitter(2011)]

$$S_{BSM} = 0.04 \pm 0.10$$
$$T_{BSM} = 0.05 \pm 0.11$$
$$U_{BSM} = 0.08 \pm 0.11$$

reference point: $m_h = 120\,\mathrm{GeV}$, $m_t = 173\,\mathrm{GeV}$,

with correlations of $+0.89$ $(S - T)$, -0.45 $(S - U)$, and -0.69 $(T - U)$.

In MUED, vertex corrections are small, and couplings of zero mode fermions to KK mode gauge bosons are only induced at loop level.
$\Rightarrow$ EW corrections in MUED can be parameterized via S, T and U.

Problem in nUED/sUED:
Fermion-to-KK-gauge-boson couplings are not small. This in particular leads to modifications to muon-decay $\Leftrightarrow$ determination of the Fermi-constant G_f

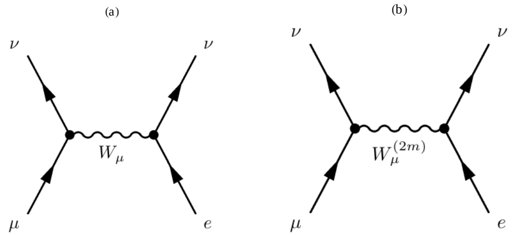


Figure: Muon decay. (a) The only diagram in the Standard Model. (b) additional diagrams for sUED/nUED where the KK modes of the W boson couple to the muon.

Solution: [Carena, Ponton, Tait, Wagner (2002)]

If the corrections are universal (which we for now assume), one can consider

$$G_{XY} \equiv \sum_{n=0}^{\infty} G_{XY}^{(n)}$$

as a generalized gauge boson propagator.

For LEP measurements (at $p^2 \sim m_Z^2$), the zero mode propagator is resonant, but for the G_f measurement (at $p^2 \sim m_\mu^2$), all propagators are off-resonance and contribute.

The measured value of G_f enters the S, T, U parameters, because the underlying SM parameters (g, g', v) are fixed from the observables (G_f, α, m_Z)

This effect can be compensated for by introducing the effective parameters

$$S_{eff} = S_{UED}$$

$$T_{eff} = T_{UED} + \Delta T_{UED} = T_{UED} - \frac{1}{\alpha} \frac{\delta G_f}{G_f^{obl}}$$

$$U_{eff} = U_{UED} = \Delta U_{UED} = U_{UED} + \frac{4 \sin^2 \theta_W}{\alpha} \frac{\delta G_f}{G_f^{obl}}$$

At tree level In nUED/sUED, the only contributions to the effective parameters arise from W KK excitations, so that

$$\frac{\delta G_f}{G_f^{obl}} = m_W^2 \sum_{n=1}^{\infty} \frac{(\mathcal{F}_{002n})^2}{m_W^2 + \left(\frac{2n}{R}\right)^2},$$

where again, $\mathcal{F}_{002n}$ are the overlap integrals which depend on μ (sUED) or respectively a_f, a_{ew} (nUED).
The leading one-loop contributions are

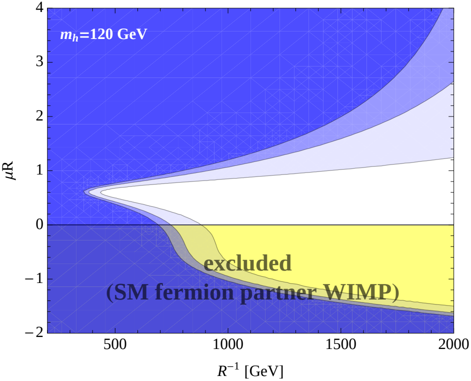
$$S_{UED} \approx \frac{4\sin^2\theta_W}{\alpha}\left[\frac{3g^2}{4(4\pi)^2}\left(\frac{2}{9}\sum_n \frac{m_t^2}{m_{t(n)}^2}\right) + \frac{g^2}{4(4\pi)^2}\left(\frac{1}{6}\sum_n \frac{m_h^2}{m_{h(n)}^2}\right)\right],$$

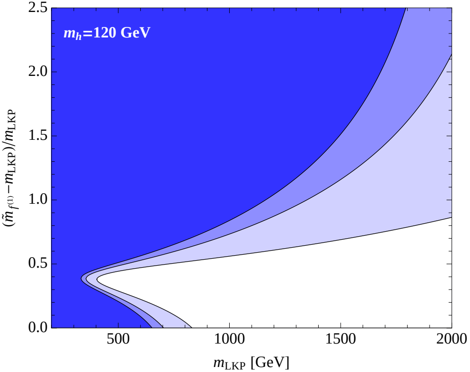
$$T_{UED} \approx \frac{1}{\alpha}\left[\frac{3g^2}{2(4\pi)^2}\frac{m_t^2}{m_W^2}\left(\frac{2}{3}\sum_n \frac{m_t^2}{m_{t(n)}^2}\right) + \frac{g^2\sin^2\theta_W}{(4\pi)^2\cos^2\theta_W}\left(-\frac{5}{12}\sum_n \frac{m_h^2}{m_{h(n)}^2}\right)\right],$$

$$U_{UED} \approx -\frac{4g^2\sin^4\theta_W}{(4\pi)^2\alpha}\left[\frac{1}{6}\sum_n \frac{m_W^2}{m_{W(n)}^2} - \frac{1}{15}\sum_n \frac{m_h^2 m_W^2}{m_{W(n)}^4}\right].$$

Compare to experimental values (χ^2-test) $\Rightarrow$ Constraints on parameter space.

Constraints on the sUED parameter space and 1st KK-mode masses



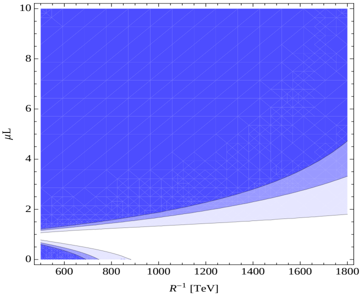


Left: 99%, 95%, and 68% exclusion contours in the μR vs. R^{-1} parameter space.

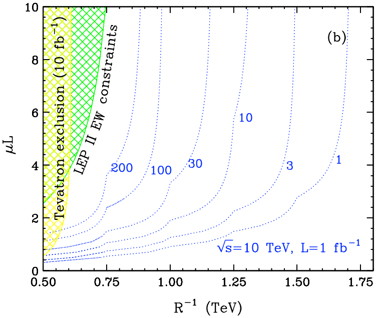
Right: Bounds on the rel. mass splitting at the first KK level vs. the mass of the LKP.

Comparison to LHC predictions

Comparison to potential LHC signals

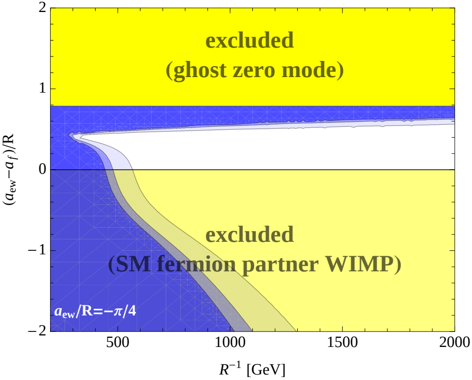


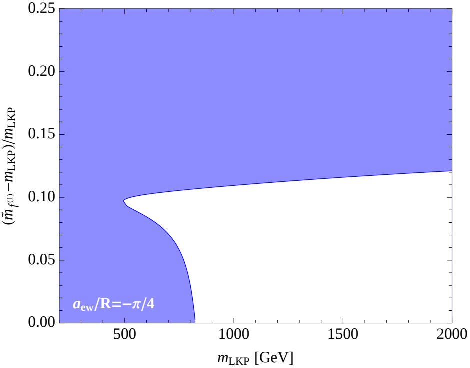
EW constraints on sUED



Predicted number of events in the di-lepton channel at LHC from [Kong, Park, Rizzo, JHEP 1004 (2010) 081]

Constraints on the nUED parameter space and 1^{st} KK-mode masses

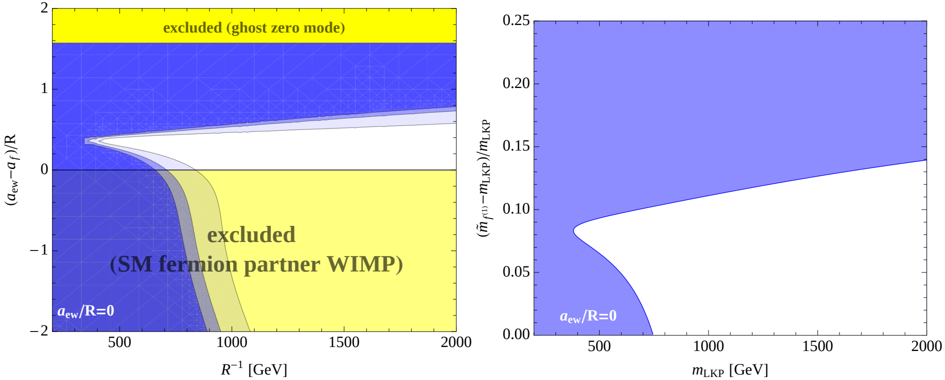




Left: 99%, 95%, and 68% exclusion contours in the $(a_{ew} - a_f)/R$ vs. R^{-1} parameter space for $a_{ew}/R = -\pi/4$

Right: 95% exclusion on the relative mass splitting at the first KK level vs. the mass of the LKP for $a_{ew}/R = -\pi/4$.

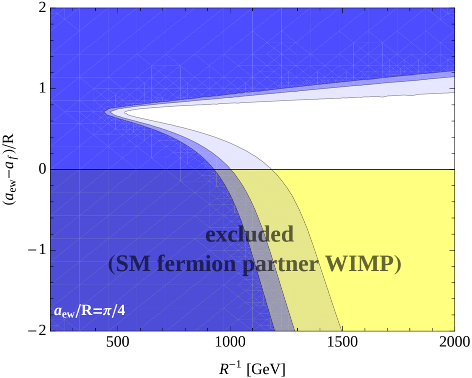
Constraints on the nUED parameter space and 1^{st} KK-mode masses

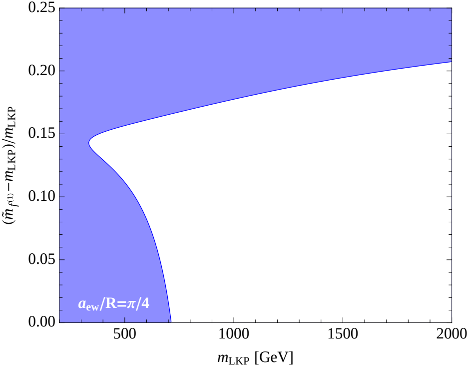


Left: 99%, 95%, and 68% exclusion contours in the $(a_{ew} - a_f)/R$ vs. R^{-1} parameter space for $a_{ew}/R = 0$

Right: 95% exclusion on the relative mass splitting at the first KK level vs. the mass of the LKP for $a_{ew}/R = 0$.

Constraints on the nUED parameter space and 1st KK-mode masses



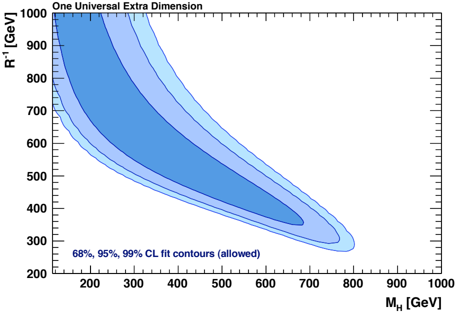


Left: 99%, 95%, and 68% exclusion contours in the $(a_{ew} - a_f)/R$ vs. R^{-1} parameter space for $a_{ew}/R = \pi/4$

Right: 95% exclusion on the relative mass splitting at the first KK level vs. the mass of the LKP for $a_{ew}/R = \pi/4$.

Interlude: News from the Higgs

On Tuesday, the bound on a heavy Higgs was lifted to $m_h > 600\,\mathrm{GeV}$ (CMS).



← left: Gfitter (2011)

Flavor constraint FCNC:
$R^{-1} \gtrsim 600\,\mathrm{GeV}$ at 95% c.l. Weiler, Haisch (2007)

$\Rightarrow$ for MUED, the heavy Higgs solution is in tension with combined EW, Higgs and flavor bounds.

This cannot be resolved by flavor-blind sUED or nUED with $B^{(1)}$ dark matter because both corrections reduce the allowed R^{-1}.

Flavor violation nUED and sUED

Including Flavor, the most general sUED / nUED action for fermions reads

$$S = \int d^5x \mathcal{L}_f + \mathcal{L}_{Yuk}$$

with

$$\begin{aligned} \mathcal{L}_f &= \sum_{ij} \left\{ \frac{i}{2}\delta_{ij} \left(D_M \overline{\Psi}_i \Gamma^M \Psi_j - \overline{\Psi}_i \Gamma^M D_M \Psi_j \right) - \overline{\Psi}_i M^{\Psi}_{ij}(y) \Psi_j \right\}, \\ \mathcal{L}_{Yuk} &= \sum_{ij} \left\{ \lambda^U_{ij} \overline{Q}_i \tilde{H} U_j + \lambda^D_{ij} \overline{Q}_i H D_j + \lambda^E_{ij} \overline{L}_i H E_j \right\} + \text{h.c.}. \end{aligned}$$

$$\text{where } M^{\psi}_{ij}(y) = \begin{cases} M^{\psi}\left[\delta(y+\pi R/2) + \delta(y-\pi R/2)\right] \not{D} & \text{for nUED} \\ M^{\psi}\theta(y) & \text{for sUED} \end{cases},$$

$M^{Q,u,d,L,e}$ are 3×3 hermitian matrices in flavor space,
$\lambda^{U,D,E}$ are 3×3 matrices in flavor space.

Calculating the 4D effective action

Via field redefinitions, the matrices M^ψ can be diagonalized, and the fermion zero mode Lagrangian in the zero mode approximation reads

$$
\begin{aligned}
\mathcal{L}_{kin} &= \overline{\psi}^{(0)} i\gamma^\mu \partial_\mu \psi^{(0)} \\
\mathcal{L}_{f,g} &= \sum_{n=0} \left[\overline{\psi}^{(0)} i\gamma^\mu (D_\mu - \partial_\mu)^{(2n)} \psi^{(0)} \mathcal{F}^{\psi,\psi}_{002n} \right] \\
\mathcal{L}_{Yuk} &= \overline{u^q}^{(0)}_{L,i} \frac{\lambda'^U_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L,u^j_R}_{000} u^{(0)}_{R,j} + \overline{d^q}^{(0)}_{L,i} \frac{\lambda'^D_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{q^i_L,d^j_R}_{000} d^{(0)}_{R,j} + \overline{e^l}^{(0)}_{L,i} \frac{\lambda'^E_{ij}}{\sqrt{2}} v_5 \mathcal{F}^{l^i_L,e^j_R}_{000} e^{(0)}_{R,j} \\
&\quad + \text{h.c.}
\end{aligned}
$$

The basis in which the M^ψ are diagonal thus signifies the gauge eigenbasis.

Transformation to the quark mass eigenbasis by bi-unitary transformations:

$$u_L = S_u^\dagger (u^q)_L^{(0)}, \quad d_L = S_d^\dagger (d^q)_L^{(0)}, \quad e_L = S_e^\dagger (e^l)_L^{(0)}$$
$$u_R = T_u^\dagger u_R^{(0)}, \quad d_R = T_d^\dagger d_R^{(0)}, \quad e_R = T_e^\dagger e_R^{(0)}.$$

In the fermion mass eigenbasis, the couplings to the gauge bosons read

$$\mathcal{L}_{q,eff} \subset \sum_{n=0} \eta^{\mu\nu} \left[g_3 G_\mu^{A(2n)} J_{q\nu}^{A(2n)} + \left(\frac{g_2}{\sqrt{2}} W_\mu^{+(2n)} J_{q\nu}^{+(2n)} + \text{h.c.} \right) \right.$$
$$\left. + e A_\mu^{(2n)} J_{q\nu}^{em,(2n)} + \frac{g_2}{\cos(\theta_W)} Z_\mu^{(2n)} J_{q\nu}^{0(2n)} \right],$$

with *e.g.*

$$J_{q\nu}^{A(2n)} = \left(V_{L,ij}^{u(2n)} \overline{u}_{L,i} T^A \gamma_\nu u_{L,j} + V_{L,ij}^{d(2n)} \overline{d}_{L,i} T^A \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$
$$J_{q\nu}^{em(2n)} = \left(\frac{2}{3} V_{L,ij}^{u(2n)} \overline{u}_{L,i} \gamma_\nu u_{L,j} - \frac{1}{3} V_{L,ij}^{d(2n)} \overline{d}_{L,i} \gamma_\nu d_{L,j} \right) + (L \leftrightarrow R)$$

with

$$V_{L,ij}^{u(2n)} = (S_u^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_u)_{ij} \quad , \quad V_{R,ij}^{u(2n)} = (T_u^\dagger \mathcal{F}_{002n}^{u_R,u_R} T_u)_{ij}$$
$$V_{L,ij}^{d(2n)} = (S_d^\dagger \mathcal{F}_{002n}^{q_L,q_L} S_d)_{ij} \quad , \quad V_{R,ij}^{d(2n)} = (T_d^\dagger \mathcal{F}_{002n}^{d_R,d_R} T_d)_{ij}$$

Integrating out all but the zero modes leads the low-energy effective action

$$S_{eff,4q} = -\sum_{n=1}\eta^{\mu\nu}\left[\frac{g_3^2}{2m^2_{G^{(2n)}}}J^{A(2n)}_{q\mu}J^{A(2n)}_{q\nu}+\frac{g_2^2}{2m^2_{W^{(2n)}}}J^{+(2n)}_{q\nu}J^{-(2n)}_{q\nu}\right.$$
$$\left.+\frac{e^2}{2m^2_{A^{(2n)}}}J^{em(2n)}_{q\mu}J^{em(2n)}_{q\nu}+\frac{g_2^2}{2\cos^2\theta_W^{(2n)}m^2_{Z^{(2n)}}}J^{0(2n)}_{q\mu}J^{0(2n)}_{q\nu}\right],$$

which in particular contain the $\Delta F = 2$ effective operators of sUED, which can be parameterized according to

$$\mathcal{H}^{\Delta F=2}_{int} = \sum_{i=1}^{5} C^i_{q_iq_j}Q_i^{q_iq_j} + \sum_{i=1}^{3}\tilde{C}^i_{q_iq_j}\tilde{Q}_i^{q_iq_j}$$

with

$$Q_1^{q_i,q_j} = \left(\overline{q}^a_{L,j}\gamma_\mu q^a_{L,i}\right)\left(\overline{q}^b_{L,j}\gamma^\mu q^b_{L,i}\right)$$
$$Q_2^{q_i,q_j} = \left(\overline{q}^a_{R,j}q^a_{L,i}\right)\left(\overline{q}^b_{R,j}q^b_{L,i}\right), \quad Q_3^{q_i,q_j} = \left(\overline{q}^a_{R,j}q^b_{L,i}\right)\left(\overline{q}^b_{R,j}q^a_{L,i}\right)$$
$$Q_4^{q_i,q_j} = \left(\overline{q}^a_{R,j}q^a_{L,i}\right)\left(\overline{q}^b_{L,j}q^b_{R,i}\right), \quad Q_5^{q_i,q_j} = \left(\overline{q}^a_{R,j}q^b_{L,i}\right)\left(\overline{q}^b_{L,j}q^a_{R,i}\right),$$

and $\tilde{Q}_{1,2,3} = Q_{1,2,3}(L \leftrightarrow R)$.

For the Wilson coefficients in sUED, we find

$$C_K^1 = \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{L,ds} V^{d(2n)}_{L,ds}$$

$$\tilde{C}_K^1 = \sum_n \left(\frac{g_3^2}{3m^2_{G^{(2n)}}} + \frac{g_2^2}{2\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{2m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{R,ds}$$

$$C_K^4 = \sum_n - \left(\frac{g_3^2}{m^2_{G^{(2n)}}} + \frac{g_2^2}{\cos^2\theta_W^{(2n)} m^2_{Z^{(2n)}}} + \frac{e^2}{m^2_{A^{(2n)}}} \right) V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds}$$

$$C_K^5 = \sum_n \frac{g_3^2}{3m^2_{G^{(2n)}}} V^{d(2n)}_{R,ds} V^{d(2n)}_{L,ds},$$

where $V^{u(2n)}_{L,ij} = (S_u^\dagger \mathcal{F}^{q_L,q_L}_{002n} S_u)_{ij}$, $V^{u(2n)}_{R,ij} = (T_u^\dagger \mathcal{F}^{u_R,u_R}_{002n} T_u)_{ij}$

$V^{d(2n)}_{L,ij} = (S_d^\dagger \mathcal{F}^{q_L,q_L}_{002n} S_d)_{ij}$, $V^{d(2n)}_{R,ij} = (T_d^\dagger \mathcal{F}^{d_R,d_R}_{002n} T_d)_{ij}$

The analogous expressions for C^i_{D,B_d,B_s} follow from these by the replacements $(ds) \to (uc), (db), (sb)$.

Experimental constraints: [Bona *et al.* (UTfit Collaboration, 2007)]

Parameter	95% allowed $[TeV^{-2}]$	Parameter	95% allowed $[TeV^{-2}]$
$\mathrm{Re}C_K^1$	$[-9.6, 9.6] \cdot 10^{-7}$	$\mathrm{Im}C_K^1$	$[-4.4, 2.8] \cdot 10^{-9}$
$\mathrm{Re}C_K^2$	$[-1.8, 1.9] \cdot 10^{-8}$	$\mathrm{Im}C_K^2$	$[-5.1, 9.3] \cdot 10^{-11}$
$\mathrm{Re}C_K^3$	$[-6.0, 5.6] \cdot 10^{-8}$	$\mathrm{Im}C_K^3$	$[-3.1, 1.7] \cdot 10^{-10}$
$\mathrm{Re}C_K^4$	$[-3.6, 3.6] \cdot 10^{-9}$	$\mathrm{Im}C_K^4$	$[-1.8, 0.9] \cdot 10^{-11}$
$\mathrm{Re}C_K^5$	$[-1.0, 1.0] \cdot 10^{-8}$	$\mathrm{Im}C_K^5$	$[-5.2, 2.8] \cdot 10^{-11}$
$\lvert C_{B_d}^1 \rvert$	$< 2.3 \cdot 10^{-5}$	$\lvert C_{B_s}^1 \rvert$	$< 1.1 \cdot 10^{-3}$
$\lvert C_{B_d}^2 \rvert$	$< 7.2 \cdot 10^{-7}$	$\lvert C_{B_s}^2 \rvert$	$< 5.6 \cdot 10^{-5}$
$\lvert C_{B_d}^3 \rvert$	$< 2.8 \cdot 10^{-6}$	$\lvert C_{B_s}^3 \rvert$	$< 2.1 \cdot 10^{-4}$
$\lvert C_{B_d}^4 \rvert$	$< 2.1 \cdot 10^{-7}$	$\lvert C_{B_s}^4 \rvert$	$< 1.6 \cdot 10^{-5}$
$\lvert C_{B_d}^5 \rvert$	$< 6.0 \cdot 10^{-7}$	$\lvert C_{B_s}^5 \rvert$	$< 4.5 \cdot 10^{-5}$
$\lvert C_D^1 \rvert$	$< 7.2 \cdot 10^{-7}$		
$\lvert C_D^2 \rvert$	$< 1.6 \cdot 10^{-7}$		
$\lvert C_D^3 \rvert$	$< 3.9 \cdot 10^{-6}$		
$\lvert C_D^4 \rvert$	$< 4.8 \cdot 10^{-8}$		
$\lvert C_D^5 \rvert$	$< 4.8 \cdot 10^{-7}$		

How can FCNCs be avoided in the quark sector?

The constraints on the C^i's imply, that the products $\frac{1}{m^2_{G(2n)}} V^{u/d(2n)}_{L/R,ij} V^{u/d(2n)}_{L/R,ij}$ have to be very small. This can be achieved in three ways

1. High compactification scale:
   $R^{-1} \gtrsim 10^5\,\mathrm{TeV}$ satisfies all constraints for $S_{ij}, T_{ij}, \mathcal{F}_{ij}$ of $\mathcal{O}(1)$.
2. Degenerate mass matrices:
   All constraints are avoided if the matrices M^Q, M^U, and M^D are proportional to the unit-matrix up to corrections of $\mathcal{O}(10^{-6})$.
   Note: the eigenvalues of M^Q, M^U, and M^D can still differ
3. Alignment of the 5D mass matrices with the Yukawa couplings.

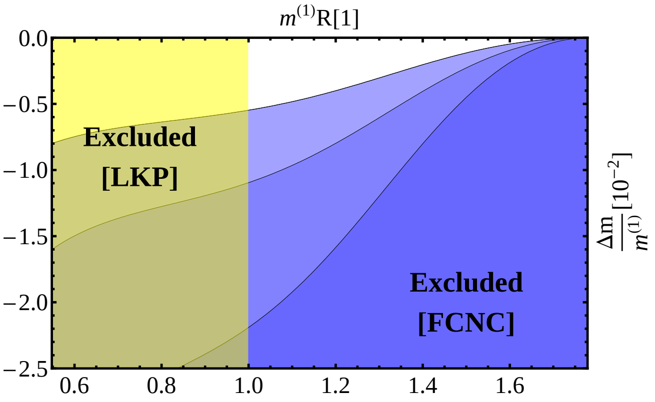
Alignment

- All Wilson coefficients would vanish if we could choose $S_d = T_d = T_u = S_u = \mathbb{1}$, *but* to obtain the SM at the fermion zero mode level, $S_u^\dagger S_d = U_{CKM}$ must be imposed.
- Choosing at least $S_d = T_d = \mathbb{1}$ avoids all bounds from the down-type sector.
- Furthermore choosing $T_u = \mathbb{1}$ avoids all constraints from the up-type sector apart from the C_D^1 constraint.
- The C_D^1 constraint with $T_u = \mathbb{1}$, $S_u = U_{CKM}^\dagger$ implies

$$|C_{1,gl}| \approx |\frac{g_s^2 \pi^2}{144} V_{12}^Q V_{12}^Q| < 7.2 \cdot 10^{-7} \mathrm{TeV}^{-2} \ ,$$

with no other constraints on $M^{Q,U,D}$.

Aligned ansatz in nUED: mass $m^{(1)}$ times R vs. mass degeneracy $\frac{\Delta m}{m^{(1)}}$]



First KK mode mass $m^{(1)}$ times R vs. the mass degeneracy $\frac{\Delta m}{m^{(1)}}$ plotted for different values of the compactification radius $R = \{0.5,1,2\}\mathrm{TeV}^{-1}$ from bottom to top in the aligned nUED scenario.

Implications for the sUED mass spectrum

The masses of the first KK mode fermions are $m^{(1)} = \sqrt{(\mu_5)^2 + k_1^2} + \delta_{HM}$ where k_1^2 is determined from $(\mu_5)^2 = k_1^2 \cot^2(k_1 \pi R/2)$.
δ_{HM} is a small correction $(m_\psi / m_\psi^{(1)})^2 m^{(1)}$ from mixing via the Yukawas.
The implications of the three solutions to the FCNC problem are therefore:

1. High complactification scale $\Rightarrow$ No new physics at the TeV scale.
2. Degenerate mass matrices $\Rightarrow$ First KK mode quarks come in three mass degenerate sets $\left(u_1^{(1)}, c_1^{(1)}, t_1^{(1)}\right)$, $\left(d_1^{(1)}, s_1^{(1)}, b_1^{(1)}\right)$, $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}, b_2^{(1)}, t_2^{(1)}\right)$.
3. Alignment $\Rightarrow$ The mass degenerate first KK mode sets are $\left(u_2^{(1)}, d_2^{(1)} c_2^{(1)}, s_2^{(1)}\right)$ and $\left(b_2^{(1)}, t_2^{(1)}\right)$, but the remaining first KK quark masses are not constrained by flavor physics at tree level.

Conclusions

- Modifications of the KK fermion mass spectrum can be reached with boundary localized kinetic terms or with fermion bulk mass terms.
- In both cases, this also alters the KK wave functions and induces interactions of Standard Model fermions with all even KK modes of the gauge bosons.
- If present in the lepton sector, these interactions modify muon-decay $\Rightarrow$ the electroweak constraints turn out to be stronger than naively expected.
- Interactions induce FCNCs which can only be avoided when
  - choosing the masses/BLKTs flavor-blind, implying 3- (6-)fold degeneracies in the KK mass spectrum.
  - delicately aligning the masses/BLKTs; even this only lifts part of the degeneracies.

Outlook

- Concerning Flavor Physics:
  - We only considered bounds on quark, but not on lepton flavor violation.
  - The requirements of mass degeneracy and/or alignment with the sUED setup calls for a flavor symmetry embedding.
- Concerning Electroweak Precision:
  We assumed universal couplings for fermions and quarks. Dropping this assumption requires an analysis beyond S, T, U.
  (work in progress)
- Concerning LHC Phenomenology:
  Signatures of second KK modes in UED-extensions are easy to search for, as they lead to s-channel resonances
  $\Rightarrow$ W'-like, Z'-like searches and searches for colored resonances
  $\Rightarrow$ very different from standard SUSY or minimal UED signatures.
  Note: LHC only now begins to probe parameter space not excluded by EWP.