Astronomical Imaging and Photometry

Methods and Techniques

How do you measure the brightness of an object?

We showed that the signal-tp-noise ratio for a star observed in some aperture of radius r was the signal, just the flux within r, divided by the square root of the signal plus the sky within r plus any additional source of noise such as read noise, if we express all the signals in electrons.

$S_r/N = Sr/sqrt(S_r + B_r + RN^2)$

If we make r very large, S goes to the total flux in the object, but $B_r \sim r^2$*, so the S/N ~ 1/r – we are just swamping the signal with sky noise. If we make r very small, we do not get very much of the flux; S and B are proportional to* r^2*, and S/N ~ r*

So there is some OPTIMUM aperture size for which you get most of the flux and not too much sky which maximizes the S/N. For a gaussian PSF, this is ~1.5 times the FWHM, but it depends on the size/shape of the object, and of course, the seeing.

Is this what you should do???   NO!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Why not??

1. *The fraction of the light you get, even for an unresolved source, depends on seeing, aberrations, and the exact form of the PSF. which changes with time—the time average of a varying Kolmogorov PSF is not a Kolmogorov PSF.*

2. *The fraction of the light you get depends sensitively on the radial form factor of the object you are observing; even for galaxies of a given effective radius, it depends on the shape of the radial brightness distribution.*

3. *Even though there is an aperture with optimal S/N for any brightness distribution, that optimal S/N is NOT as high as can be obtained by properly weighting the image—which also solves the problems of (1) and (2).*

So suppose you have an image with pixel values p_i in some units. Suppose we have subtracted the sky, so that the mean pixel values are zero away from our object (this is not trivial, but one thing at a time)

Then doing aperture photometry is just adding the pixel values up within some radius r—that is

$S = \Sigma p_i w_i$ where the weights wi are 0 outside the aperture and 1 inside.

We clearly want a LINEAR expression for the signal, so we want an expression like this, but any set of weights will give us something proportional to the flux as long as they are nonzero where the object is.

*There is an optimal choice, which gives the highest possible signal-to-noise, called the North-Wiener-Kolmogorov matched filter. It is actually only applicable for the faint limit, where photon noise in the object is dominant, but this is clearly where we are interested in maximizing the S/N. In this limit, and in the case that the pixel noise is the same in all pixels and is independent from pixel to pixel, the optimal filter is $w_i \sim p_i$, (hence the name *matched* filter).*

Makes sense – weights are high where there is lots of flux, low where there is not much, so the sky noise is deweighted toward the outside of the object where there is lots of sky but not much flux.

In detail, we do photmetry by measuring the shape of the object or class of objects, and we then just do a least-squares fit of the profile normalized somehow, say $\Sigma w_i = 1$, so the filter has unit flux.

This is a simple linear problem. We let $p_i = \alpha\, w_i$. Since the w_i have unit flux and has the same shape as the object, the estimate for α IS the estimate for the flux:

$$\alpha = \Sigma p_i w_i / \Sigma w_i^2$$

The variance in this estimate is

$$\text{var}\, \alpha = \sigma_p^2 / \Sigma w_i^2 = N_{eff}\, \sigma_p^2$$

$N_{eff} = 1/\Sigma w_i^2$ is the "noise-effective number of pixels" in this filter. these expressions for α and the variance hold for any set of weights w_i --- but α is not necessarily an estimate for the total flux unless the $w_i \sim p_i$

The noise-effective number of pixels

$N_{eff} = 1/\Sigma w_i^2$

For an aperture, N_{eff} is just the number of pixels in the aperture (remember $\Sigma w_i = 1$) but α is an estimate of the the flux in the aperture, not the total flux. For a gaussian PSF, $N_{eff} = 4\pi\sigma^2$. Here σ is the gaussian width parameter, about $2.9\pi(HWHM)^2$. For a much more reasonable approximation to the Kolmogorov PSF, the standard double gaussian,

$$p(r) = 0.9 \exp(-r^2/2\sigma_1^2) + 0.1 \exp(-r^2/2\sigma_2^2), \quad \sigma_2 = 2\sigma_1.$$

is about $3.8\pi(HWHM)^2$, reflecting the more extended wings.

For galaxies, the situation is MUCH more complex and the use of matched filters very much more important.

Galaxy Photometry

Galaxies are extended objects, and so N_{eff} will be larger than for stars; one must use a filter such that there is more sky contamination and sky noise, so the S/N for a galaxy of a given brightness will be smaller than for a star of the same brightness, or, equivalently, the brightness at a given S/N will be larger; in magnitudes, $\delta m = -1.25 \log (Neff/Neff_{,PSF})$ at fixed S/N.

Galaxies can be described roughly by the SERSIC surface brightness profile

$$B(r) = B_e \exp(1 - C_n(r/r_e)^{1/n}),$$

*where n is the *SERSIC INDEX*, r_e is the effective radius (half-light radius), B_e is the surface brightness at r_e, and C_n is a fudge factor which makes the radial scale correct (i.e. so that r_e contains half the integrated light.)*
An approximate expession is

$$C_n = 1.678 + 2.01(n-1) - .016*(n-1)^2$$

Clearly n=1 is an exponential profile; r_e is about 1.7 scale lengths.
n=4 is a deVaucouleurs profile ($r^{1/4}$). Most galaxies are intermediate $1<n<4$.

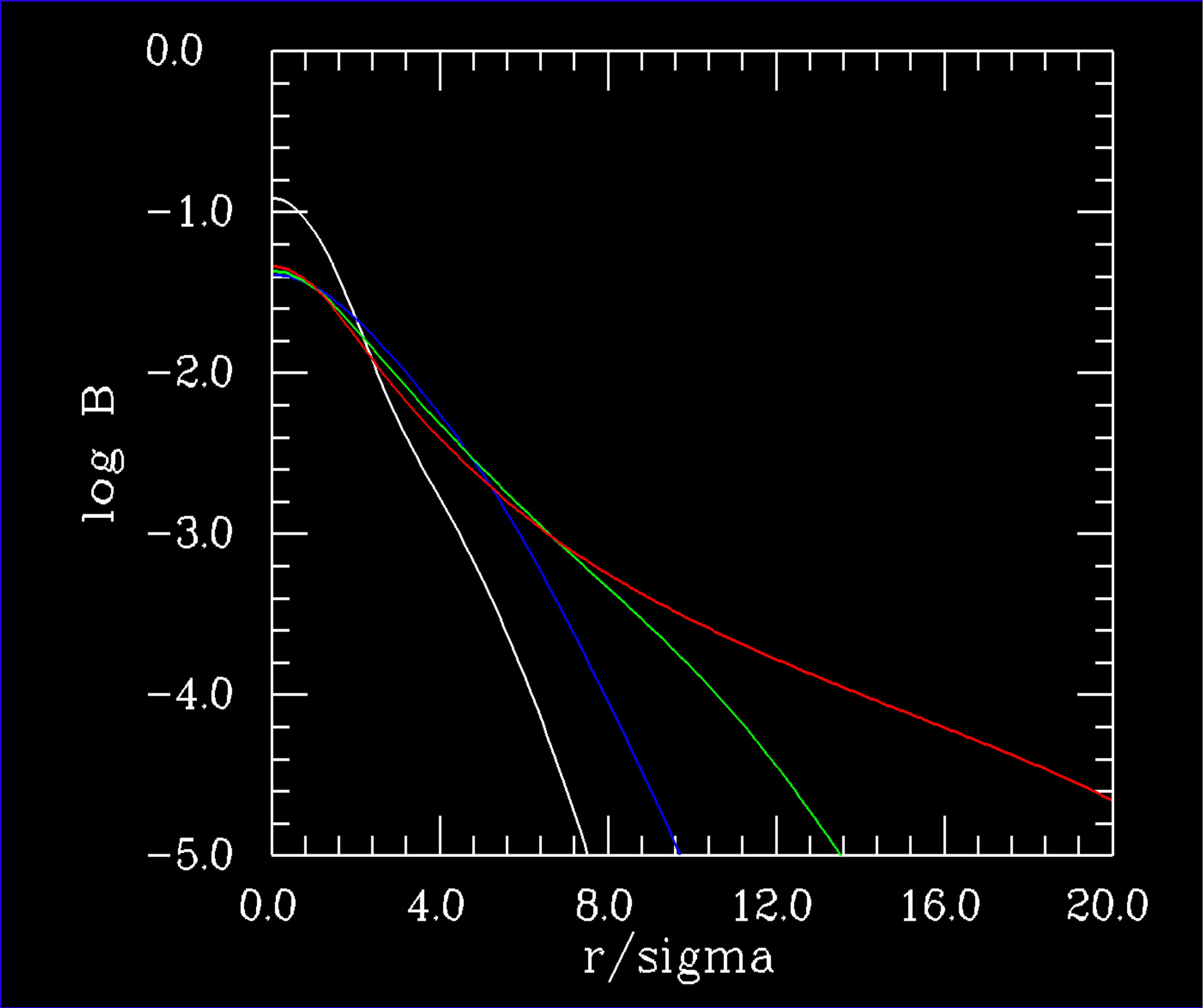
Galaxy Photometry, Cont.

The Sersic profiles must be convolved with the PSF for fitting, clearly; that is what we see on the CCD. If the galaxy is small, the seeing makes all of these profiles very similar, and the fitting is difficult.

What the profiles look like:

The profiles for various Sersic indices are very similar in the presence of seeing
(here Re = 0.7" FWHM, r_e = 2 kpc, z~1)

white: star
blue: n=1
green: n=2.5
red: n=4



Galaxy Photometry, Cont.

Let us think about the statistics a little. For each Sersic profile, one can calculate the variance in the flux, effective radius, and Sersic index if one fits to an image which has noise in it, The radius and Sersic index are nonlinear parameters, but one can use standard first-order expansions to get at the errors. One can calculate the constrained errors, if you ONLY allow that parameter to be free, and the case in which you fit for all of them and want the error for one when you marginalize over all the others. Both are interesting, as we shall see.

Galaxy Photometry, Cont.

Let us look at those quantities for an n=2.5 (average??) galaxy as observed by Subaru.

type	Re (kpc)	A /PSF	δmag	FWHM (")	σlnrc	σlnnc	σlnfm	σlnrm	σlnnm
psf	0.00	1.00	-0.00	0.70					
S2.5	1.13	1.30	-0.14	0.78	3.76	9.18	1.77	15.26	38.21
S2.5	2.26	1.92	-0.36	0.86	2.30	7.64	2.42	4.53	14.83
S2.5	3.19	2.56	-0.51	0.92	1.91	7.38	2.78	4.27	11.56
S2.5	4.52	3.61	-0.70	0.98	1.64	6.63	3.12	4.63	9.54
S2.5	6.39	5.40	-0.92	1.05	1.44	5.27	3.40	5.01	8.08
S2.5	9.04	8.54	-1.16	1.14	1.30	3.98	3.61	5.28	6.95

*Using this table is a little complicated. Recall that δmag is the magnitude difference between a star at a given S/N and the galaxy at the same S/N, presuming you know the profile. We call this the standard photometric S/N. Say that a star at i=22 has an S/N = 200 (S/N = 5 at i=26) and one observes a galaxy with n=2.5 and re = 3.2 kpc. That galaxy has an S/N of about 125 (200*10(-0.4*δmag)). The sigmas in the table are referred to this standard error as unity, so to get the S/N for any of the quantities in the table, you DIVIDE that standard S/N by the sigma value. So if you want the marginalized S/N for the sersic index, the value is 125/11.6 ~ 11. The marginalized S/N for the radius is 125/4.3 ~ 30, for the total flux 125/2.8~45.*

Galaxy Photometry, Cont.

So not knowing the profile of your galaxy costs you quite a lot. You have a S/N of 45 on an image on which the stars of the same brightness have S/N=200, 1.5 magnitudes at a given S/N.

But if you do aperture photometry, you have no idea what you are measuring, because the answer depends on the Sersic index of the galaxy and on the details of the seeing profile. The errors in total photometry are much larger than your photon errors, and you do not have a handle on them at all.

Galaxy Photometry: Measuring colors

Are we ever interested in the CONSTRAINED sigmas? Yes, when we are measuring colors.

The exact meaning of the total magnitude of a galaxy is not well defined, because galaxies have no edges and may just merge smoothly into their neighbors, in all their components.

For the same reason, colors are poorly defined.

But if one uses a set of average weights, same in each band, one can measure a color which is well defined, and reflects the bright part of the galaxy in a well-defined way. These are the so-called MODEL colors in Sloan, and we will do this in the Subaru survey with the best-fitting Sersic model, either in I or some composite band. For this, the S/N in each band is just the standard one of the stellar S/N decreased by $10^{-.4\delta mag}$, and the S/N of the color is just $((S/N)_1^{-2} + (S/N)_2^{-2})^{-1/2}$

Photometry: Internal Calibration (Flat Fields)

Flat fielding is a complex subject, but most workers do not realize that it is.

We will adopt a somewhat simplified model of the response of the telescope/ detector to investigate this issue. We will neglect distortion, which complicates the equations a great deal because it preserves flux but does not preserve surface brightness; the effective solid angle of a pixel is not constant over the field if the distortion is nonzero.

For most optical systems, the distortion is radial, and maps an angle in the sky to a radius in the focal plane with a function which is not linear. For HSC, the function is

$$r' = r(1 + .0463r^2 + .0177r^4)$$

when normalized to unity at the origin (r is in degrees). At the edge of the field, $r' \sim 1.03r$, which does not sound so bad. But what one really wants is the Jacobian of the map, $(r'/r)(dr'/dr)$, which is approximately

$$J = (1 + .0463r^2 + .0177r^4)(1 + .139r^2 + .088r^4) \sim (1 + .185r^2 + .106r^4)$$

This is 14% at the edge, and the distortion of SHAPES is 7% (10.6,3.2)

Flat fields, continued

Distortion must be dealt with by remapping pixels or by constructing nonrectangular pixel grids, both of which are mathematically and statistically complex. We will not discuss it further, and assume that there is no distortion in our pictures.

There CAN be vignetting. We did not discuss vignetting when we were talking about optics, but the concept is simple. If there is an obstruction which is not at the pupil, some rays will run into it and others will not, and the fraction of the pupil area which is covered will depend on the field angle. For HSC, the vignetting is caused by the entrance lens for the big corrector, which could not be made big enough without destroying the telescope. It begins 7.5' off axis, and reaches 26 percent at the edge of the 75' radius field, roughly linear in the angle. We will not need to worry about it explicitly.

So let R(x,y) be the pixel values in some image.

let Rf(x,y) be the pixel values in a well-controlled dome flat field, taken aways under exactly the same conditions with a lamp which reasonably matches the spectrum of the sources we are interested in (!) Normalize Rf(x,y) so that its median or mean or something is unity.

Flat fields, continued

Dome flats are best because you control them and the opportunity for scattered light is much less than twilight or other sky flats. But scattered light is the bugaboo of flat fields no matter what.

In general,

$R(x,y) = G(x,y)(P(x,y) + S(x,y))$

Where $P(x,y)$ is the flux which would fall on the pixel at x,y from the sky if the system were perfect, $G(x,y)$ is the GAIN function, which is the quantum efficiency of the system at x,y—ie some ratio of the response of the system to the incident flux. This includes the individual detector QEs, the vignetting, everything.

$S(x,y)$ is the bad guy. It is the flux which arrives on the detector NOT directly from the sky through the optics in the `right' place, but light scattered off the telescope structure, off dust in the optics, from direct skylight from incorrect baffling of the telescope, etc....

This is usually larger than you would like, at least several percent of the real signal, and need have NO relation to $P(x,y)$.

Flat fields, continued

Likewise, the flat field has some (hopefully smaller) scattered light contribution.

$Rf(x,y) = G(x,y)(Pf(x,y) + Sf(x,y))$

Pf should be almost constant, but if it is not, we can deal with it.

So when we `flatten' the frame R(x,y) by dividing by the flat, we produce

$R'(x,y) = (P(x,y) + S(x,y)) / (Pf(x,y) + Sf(x.y)) = P(x,y)/(Pf(x,y) + Sf(x,Y)) + S'(x,y)$

Where S' is a new ADDITIVE scattering term. G(x,y) has disappeared from this equation. If Pf is a constant and Sf is ZERO, we have successfully `flattened' the frame, which means that the measured fluxes of objects all over the frame are proportional to their real fluxes. The frame will not LOOK flat because of the additive S'(x,y) term.

BUT Pf is probably not exactly constant, and Sf is not zero. What do we do?

We fix it with stellar photometry is what we do.

Flat fields, continued

$R'(x,y) = P(x,y)/[Pf(x,y) + Sf(x,Y)] \quad + \quad S'(x,y)$

We make a further assumption, that Pf and Sf are smooth, so we do not need a lot of information to characterize their deviation from constant. If Pf were constant and Sf were zero, the photometry of stars would be perfect over the field. We simply DO photometry over the field, by connecting to a known catalog (SDSS, for example) and solve for a function Q(x,y) which is the ratio of the real stellar flux to that measured in the flattened frame. Q will be smooth and not deviate very much from unity. A 2D Chebyshev polynomial or a Zernike polynomial expansion will serve fine.

Then $R''(x,y) = P(x,y)*Q(x,y)[Pf(x,y) + Sf(x,Y)] \quad + \quad S''(x,y)$

This frame is photometrically correct, by construction, and a little thought will convince you that we have in fact found the gain function.

But the frame is still not flat, because of the ADDITIVE scattering term, though it is photometrically correct. What do we do? We can ADD or SUBTRACT ANYTHING, so we subtract something which makes the sky in our frame flat—namely the sky in another frame, or collection of frames.

Flat fields, continued

The construction of this sky frame is not trivial, because the scattering term S''(x,y) is almost certainly dependent on almost everything—zenith angle, moon brightness and location in the sky, cloudiness, etc, etc.

Objects must be removed, carefully, the frames normalized properly, probably high-pass filtered and medianed or sigma-clipped. The less done to the sky data the better, but we do not know yet how much has to be done.

There are still unsolved problems, but they must be solved if the deep and, especially, ultradeep, layers of the HSC survey are to succeed. Any residual small-scale structure In the sky will prevent reaching the photon-noise limit at some level; we need that level to be very faint.

In the ultra-deep field, the goal is 28^{th} magnitude; the sky is 22^{nd}, per square arcsecond; the objects have central surface brightnesses of about 29^{th}, 7 magnitudes, 600 times fainter than the sky, which we need to measure to a few percent, say an error of 1/20,000 of the sky. We therefore need to subtract the sky to an accuracy of .005 percent.