Exotic Branes, Double Bubbles, & Superstrata

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1004.2521, 1107.2650, 1110.2781

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for the Origin of Particles and the Universe

String
Introduction
Exotic branes
Exotic branes

“Forgotten” branes in string theory

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9712047 Blau+O’Loughlin]
[9809039 Obers+Pioline]

<table>
<thead>
<tr>
<th>Type IIA</th>
<th>P (7), F1 (7), D0 (1), D2 (21), D4 (35), D6 (7), NS5 (21), KKM (42), 5^2 (21), 0^7 (1), 2^5 (21), 4^2 (35), 6^1 (7), 0^{(1,6)} (7), 1^6 (7)</th>
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<tbody>
<tr>
<td>Type IIB</td>
<td>P (7), F1 (7), D1 (7), D3 (35), D5 (21), D7 (1), NS5 (21), KKM (42), 5^2 (21), 1^6 (7), 3^4 (35), 5^2 (21), 7_3 (1), 0^{(1,6)} (7), 1^6 (7)</td>
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<tr>
<td>M-theory</td>
<td>P (8), M2 (28), M5 (56), KKM (56), 5^3 (56), 2^6 (28), 0^{(1,7)} (8)</td>
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Exotic branes

- “Forgotten” branes in string theory
- Co-dimension 2

[Diagram showing a codim-2 hypersurface]
Exotic branes

- “Forgotten” branes in string theory
- Co-dimension 2
- Charge = U-duality monodromy

Jump by a U-duality as one goes around it

Generalization of F-theory 7-branes
Exotic branes

- “Forgotten” branes in string theory
- Co-dimension 2
- Charge = U-duality monodromy
- Non-geometric

Even metric can jump!

“U-fold”
Supertube effect – “bubbling”

- Codim-2 object problematic?
  - Log divergences
    \[ V \sim \frac{1}{r^{d-2}} \quad \text{d=2} \quad V \sim \log \left( \frac{\mu}{r} \right) \]

- Supertube effect = spontaneous polarization
  [Mateos+Townsend]

[Diagram showing the puffs up of D2(1\psi): dipole charge]
Exotic bubbling

- Ordinary branes can puff up to produce exotic dipole charges

- No log divergence

- Exotic branes relevant to non-exotic physics; More common than previously thought!
Double bubbling

- Bubbling can occur at multiple stages

→ “Superstratum” – arbitrary surface
→ Bubbling may occur repeatedly, producing all kinds of exotic charges
Black hole microstates

- Black holes: bound states of branes
  - Generic microstates involve exotic superstrata
  - Microstate (non-)geometries?
Claim:

Generic microstates of black holes involve exotic non-geometric superstrata.
Outline

- Introduction ✓
- Exotic states & their higher-D origin
- Supertube effect
- Black hole microstates
- Conclusion
Exotic states and their higher-D origin
Compactification to 3D

- **M on $T^8$ or Type II on $T^7$**
  - $3D \, \mathcal{N} = 16$ sugra
  - $U$-duality group $E_{8(8)}(\mathbb{Z})$ : generated by T- and S-dualities
  - 128 moduli scalars (in 3D, scalar = vector) $\in SO(16) \backslash E_{8(8)}(\mathbb{R}) / E_{8(8)}(\mathbb{Z})$

- **Particle multiplet:**
  - Start from a point-like object
    - e.g. D7(3456789) wrapped on $T^7$
  - Take T- and S-dualities to get other states
Exotic states in 3D

- **Particle multiplet:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle States</th>
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<td>Type IIA</td>
<td>P ( (7) ), F1 ( (7) ), D0 ( (1) ), D2 ( (21) ), D4 ( (35) ), D6 ( (7) ), NS5 ( (21) ), KKM ( (42) ), ( \frac{5^2}{2} ) ( (21) ), ( \frac{0^7}{3} ) ( (1) ), ( \frac{2^5}{3} ) ( (21) ), ( \frac{4^2}{3} ) ( (35) ), ( 6^1 ) ( (7) ), ( 0^1 ) ( (7) ), ( 1^6 ) ( (7) )</td>
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<td>P ( (7) ), F1 ( (7) ), D1 ( (7) ), D3 ( (35) ), D5 ( (21) ), D7 ( (1) ), NS5 ( (21) ), KKM ( (42) ), ( \frac{5^2}{2} ) ( (21) ), ( 1^6 ) ( (7) ), ( 3^4 ) ( (35) ), ( 5^2 ) ( (21) ), ( 7^3 ) ( (1) ), ( 0^1 ) ( (7) ), ( 1^6 ) ( (7) )</td>
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<td>P ( (8) ), M2 ( (28) ), M5 ( (56) ), KKM ( (56) ), ( 5^3 ) ( (56) ), ( 2^6 ) ( (28) ), ( 0^1 ) ( (7) )</td>
</tr>
</tbody>
</table>

- **Notation for exotic states**

\[
b_n^c : M = \frac{R^b (R^c)^2}{g_s^n}
\]

\[
b_n^{(d,c)} : M = \frac{R^b (R^c)^2 (R^d)^3}{g_s^n}
\]

**Example:** \( 5_2^2 (34567,89) : \)

\[
M = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^8}
\]
Duality rules

- Duality rules can be read off from:

\[ T_y: \quad R_y \rightarrow \frac{l_y^2}{R_y}, \quad g_s \rightarrow \frac{l_s}{R_y} g_s \quad S: \quad g_s \rightarrow \frac{1}{g_s}, \quad l_s \rightarrow g_s^{1/2} l_s \]

- Example:

\[
\text{NS5}(34567) \xrightarrow{T_8} \text{KKM}(34567,8) \xrightarrow{T_9} 5_2^2(34567,89)
\]

\[
M = \frac{R_3 \cdots R_7}{g_s^2 l_s^6} \xrightarrow{T_8} \frac{R_3 \cdots R_7}{(g_s l_s/R_8)^2 l_s^6} = \frac{R_3 \cdots R_7 R_8^2}{g_s^2 l_s^8} : 5_2^1 = \text{KKM}
\]

\[
\xrightarrow{T_9} \frac{R_3 \cdots R_7 R_8^2}{(g_s l_s/R_9)^2 l_s^8} = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^{10}} : 5_2^2
\]
Higher D origin = U-folds (1)

- Claim: higher D origin is
  
  U-fold = non-geometric background

E.g. D7 on T^7

- (Magnetically) coupled to RR 0-form C_0
- 3D scalar \( \phi = C_0 \)
- Monodromy: \( \phi \rightarrow \phi + 1 \) shift (part of \( SL(2, \mathbb{Z}) \) duality of IIB)
Higher D origin = U-folds (2)

- $\phi$ gets combined with other scalars to form moduli matrix

$$M \in \mathcal{M} = SO(16)\backslash E_{8(8)}(\mathbb{R})/E_{8(8)}(\mathbb{Z})$$

- Shifting symmetry + S, T-dualities

- Can consider a particle with general U-duality monodromy

$$q \in E_{8(8)}(\mathbb{Z}) \equiv G(\mathbb{Z})$$

“Charge” of a 3D particle is U-duality monodromy around it!
Higher D origin = U-folds (3)

- In 10D/11D, we have a non-geometric U-fold

3D

\[ \begin{align*}
3D \text{ particle} & \quad M \\
Mq & \quad \text{codim-2 hypersurface}
\end{align*} \]

10D/11D

\[ \begin{align*}
\text{Start from a configuration...} & \\
\text{...and come back to a U-dual version}
\end{align*} \]

Exotic branes = U-folds

Cf. F-theory, U-branes & non-geom bg

- [Greene+Shapere+Vafa+Yau] [Vafa] F-theory
- [Kumar+Vafa] [Liu+Minasian] [Hellerman+McGreevy+Williams] contractible U-branes
- [Dabholkar+Hull] [Flournoy+Wecht+Williams] ... non-contractible U-branes & moduli stabl’n
Sugra solution for $5_2^2$

$5_2^2(56789,34)$ metric:

$$ds^2 = -dt^2 + H(dr^2 + r^2 d\theta^2) + HK^{-1}dx_{34}^2 + dx_{56789}^2$$

$$B_{34} = -K^{-1} \theta \sigma, \quad e^{2\Phi} = HK^{-1}, \quad K \equiv H^2 + \sigma^2 \theta^2$$

$$H(r) = h + \sigma \log \left( \frac{\mu}{r} \right) \quad \sigma = \frac{R_3 R_4}{2\pi \alpha'}$$

Cf. [Blau+O'Loughlin]: 63

T-fold structure:

- $\theta = 0 : \quad G_{33} = G_{44} = H^{-1},$
- $\theta = 2\pi : \quad G_{33} = G_{44} = \frac{H}{H^2 + (2\pi \sigma)^2}$

$\Rightarrow x_3$-$x_4$ torus size doesn’t come back to itself!
T-fold structure of $5^2_2$

- Package 3-4 part of G, B:

$$M = \begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

T-duality acts as

$$M \rightarrow M' = \Omega^t M \Omega, \quad \Omega \in SO(2,2, \mathbb{R})$$

T-duality monodromy around $5^2_2$:

$$\Omega = \begin{pmatrix} 1 & 0 \\ 2\pi \sigma & 1 \end{pmatrix}: \quad M(\theta = 0) \rightarrow M(\theta = 2\pi)$$
Comments

- Not well-defined as stand-alone objects
  - Log divergence
    → Superpositions (cf. F-theory 7-branes)
    → Configs with higher codims. (we turn to this next!)

- Easy to get sugra metric for other exotic branes
  - Questionable for states with $M \sim g_s^{-3}, g_s^{-4}$
Lesson 1:

Exotic branes

= Non-geometric U-folds
Supertube effect and exotic branes
Supertube effect

D0 + F1(1) \rightarrow \text{“puffs up”} \rightarrow \text{D2(1} \psi \text{)}

- Spontaneous polarization effect (cf. Myers effect)
- Produces new dipole charge
- Cross section = arbitrary curve
- Fluctuations of curve $\rightarrow$ large degeneracy $\sim e^{\#\sqrt{N_{F1}N_{D0}}}$
Dualizing supertubes

Original supertube effect:

$$D0 + F1(1) \rightarrow D2(1\psi)$$

dualize!

Various other known puff-ups:

$$F1(1) + P(1) \rightarrow F1(\psi)$$  \hspace{2cm} FP sys

$$D1(1) + D5(12345) \rightarrow KKM(2345\psi, 1)$$  \hspace{2cm} LM geom

$$M2(12) + M2(34) \rightarrow M5(1234\psi)$$  \hspace{2cm} black ring
“Exotic” supertubes

\[ \text{D0} + \text{F1}(1) \rightarrow \text{D2}(1\psi) \]
dualize

**Exotic puff-up:**

\[ \text{D4}(6789) + \text{D4}(4589) \rightarrow 5_2^2(4567\psi, 89) \]

**More exotic:**

\[ \text{D3}(589) + \text{NS5}(46789) \rightarrow 5_3^2(4567\psi, 89) \]

**Still more exotic:**

\[ \text{NS5}(46789) + \text{KKM}(46789, 5) \rightarrow 1_4^6(\psi, 456789) \]

- Ordinary branes can polarize into exotic branes
- Only dipoles \( \rightarrow \) no log divergence
Example: $D4+D4 \rightarrow 5^2_2$

- Basic sugra supertube

- Exotic 2-charge solution
Metric for $D4+D4 \rightarrow 5^2_2$

$D4(6789)+D4(4589) \rightarrow 5^2_2 (4567\psi,89)$

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}} (dt - A)^2 + \sqrt{f_1} f_5 \, dx_{123}^2 + \sqrt{f_2} f_4 \, dx_{45}^2 + \sqrt{f_2} f_6 \, dx_{67}^2 + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2} \, dx_{89}^2,$$

$f_i, A$: sourced along curve

$$f_1 = 1 + \frac{Q_1}{L} \int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L} \int_0^L \frac{|\dot{\vec{F}}(v)|^2}{|\vec{x} - \vec{F}(v)|} \, dv, \quad A_i = -\frac{Q_1}{L} \int_0^L \frac{\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|} \, dv$$

$$d\gamma = *_3 dA, \quad d\beta_i = *_3 d f_i$$

- $\gamma, \beta_i$ have monodromy around curve
  $$\gamma \rightarrow \gamma - 2q, \quad \beta_i \rightarrow \beta_i - 2 Q_i \rightarrow T$-fold structure just as before$

- Asymptotically flat 4D
Metric for D4+D4→5^2/2

Other fields:

\[ e^{2\Phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}, \quad B_{89}^{(2)} = \frac{\gamma}{f_1 f_2 + \gamma^2}, \quad C^{(3)} = -\gamma \rho + \sigma \]

\[
\rho = (f_2^{-1} + dt - A) \wedge dx^4 \wedge dx^5 + (f_1^{-1} + dt - A) \wedge dx^6 \wedge dx^7
\]

\[
\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7
\]
Lesson 2:

Exotic branes are important for generic physics of string theory

Exotic branes

Non-geometric U-folds

Supertube effect
Exotic branes and black hole microstates
Black hole microstates

- BH has thermodynamic entropy

- Is a BH an ensemble? If so, where are microstates?
  - Not visible in gravity?
  - No-hair theorem in 4D Einstein gravity

\[ S_{BH} = \frac{A}{4G_N} \]
Mathur’s fuzzball proposal

- A BH is filled with stringy “fuzz”

- BH microstates = different configurations of the fuzz

- BH entropy = stat. mech. entropy of the fuzz

\[ S_{BH} = \frac{A}{4G_N} \equiv S_{fuzz} \]
Sugra microstates

- **Supersym. BHs**
- **Microstates describable within sugra as smooth, horizonless solutions?**

### 2-charge system: **Successful**
- All microstates have been constructed within sugra
- Counting sugra sol’ns reproduces micro entropy: $S_{sugra} = S_{micro}$
- But horizon vanishes classically

### 3-charge system: **Not so successful**
- Many sugra microstates have been constructed
- Evidence that they are *not* enough

---

[Boer+El-Showk+Messamah+Van de Bleeken]
[Bena+Bobev+Ruef+Warner]
[Lunin+Mathur]
[Lunin+Maldacena+Maoz]
[Rychkov]
[Bena+Warner]
[Berglund+Gimon+Levi]
They looked for *geometric* solutions in sugra and didn’t find enough microstates.

But this seems just in accord with what we saw:

**Generic BH microstates involve exotic charges and are non-geometric!**
“Single bubbling”

- 2-charge system (“small” BH)

\[ \text{D}1(5) \quad \text{D}5(56789) \quad \text{puff up} \]

\[ \text{KKM}(6789\psi;5) \]

1-dim curve \( \in \mathbb{R}^4 \)

geometric microstates
(Lunin-Mathur)

\[ S_{\text{micro}} = S_{\text{geom}} \]
“Double bubbling” (1)

3-charge system: 5D BH : Well studied for microstate geometry

M2(56)  
M2(78)  
M2(9A)  

M5(789Aψ)  
M5(569Aψ)  
M5(5678ψ)  

5³(789Aφ, 56ψ)  
5³(569Aφ, 78ψ)  
5³(5678φ, 9Aψ)  

arbitrary curve “supertube”

arbitrary surface “superstratum”

non-geometric microstates

S_{micro} \neq S_{nongeom}
“Double bubbling” (2)

- 4-charge system: 4D BH

: Well studied for microstate counting

[Maladacena+Strominger+Witten]

\[
\begin{align*}
D_0 & \quad \text{NS5}(6789\psi) \ 5^2_2(6789,45\psi) \\
D_4(6789) & \quad \text{NS5}(4589\psi) \ 5^2_2(4589,67\psi) \\
D_4(4589) & \quad \text{NS5}(4567\psi) \ 5^2_2(4567,89\psi) \\
D_4(4567) & \\
\end{align*}
\]

More exotic branes

\[
S_{\text{micro}} \equiv S_{\text{nongeom}}
\]

exotic supertube

exotic superstratum
Endless bubbling??

Presumably, a black hole is made of an extremely complicated structure (fuzzball) of exotic branes.
Susy in supertube

Preserves
\[ \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ susy} \]

Locally \( \frac{1}{2} \) BPS
Which \( \frac{1}{2} \) is preserved depends on local orientation
Common susy preserved = original \( \frac{1}{4} \) susy

This is why supertube can be along an arbitrary curve.
Susy in superstratum

3-charge sys.

Preserves
\[
\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}
\]

Locally \(\frac{1}{2}\) BPS

Which \(\frac{1}{2}\) is preserved depends on local orientation

Common susy preserved = original \(\frac{1}{8}\) susy

If true, superstratum can be along an arbitrary surface in principle!
Example: F1-P $\rightarrow$ D2

Susy preserved by original config:

$$
\Pi_{F1(z)} Q = \Pi_{P(z)} Q = 0,
Q = \begin{pmatrix} Q \\ \tilde{Q} \end{pmatrix}
$$

$$
\Pi_{F1(z)} = \frac{1}{2} \left( 1 + \Gamma^{0z} \sigma_3 \right)
$$

$$
\Pi_{P(z)} = \frac{1}{2} \left( 1 + \Gamma^{0z} \right)
$$

Tilted and boosted F1-P

Projector after puffing up:

$$
\Pi = \frac{1}{2} \left[ 1 - s \left( c \Gamma^{0z} - s \Gamma^{01} \right) + c \left( c \Gamma^{01} + s \Gamma^{0z} \right) \sigma_3 \right]
$$

$$
= c \left( c + s \Gamma^{z \psi} \right) \Pi_{F1(z)} + s \left( s - c \Gamma^{z \psi} \sigma_3 \right) \Pi_{P(z)},
$$

$$
c = \cos \alpha, \quad s = \sin \alpha.
$$

Common susy preserved

= same as original $\frac{1}{4}$ susy
General formula for $1 \rightarrow 2$ puff-up

Projectors before puffing up:

$$
\Pi_1 = \frac{1}{2} (1 + P_1), \quad \Pi_2 = \frac{1}{2} (1 + P_2)
$$

Projector after puffing up:

$$
\Pi = \frac{1}{2} \left[ 1 + c^2 P_1 + s^2 P_2 - sc\Gamma^{0\psi} + sc\Gamma^{0\psi} P_1 P_2 \right]
$$

$$
= c(c - s\Gamma^{0\psi})\Pi_1 + s(s - c\Gamma^{0\psi})\Pi_2 + 2sc\Gamma^{0\psi}\Pi_1\Pi_2
$$
D1-D5-P (1)

Original config.  Infinite straight supertube = tilted and boosted D1-D5

puffs up just like D1-D5 → KKM (LM geom.)

Infinite straight superstratum

Special case; superstratum is purely geometric
D1-D5-P (2)

\[ \Pi_1 = \frac{1}{2} (1 + \Gamma^{0z} \sigma_1) \]
\[ \Pi_2 = \frac{1}{2} (1 + \Gamma^{01234z} \sigma_1) \]
\[ \Pi_3 = \frac{1}{2} (1 + \Gamma^{0z}) \]

\[ \hat{\Pi}_i = \frac{1}{2} (1 + \hat{\rho}_i) \]
\[ \hat{\rho}_1 = c_1 c_2 \Gamma^{0\hat{z}} \sigma_1 + s_1 s_2 \Gamma^{01234\hat{z}} \sigma_1 + c_1 s_2 \Gamma^{0\hat{\theta}} - s_1 c_2 \Gamma^{01234\hat{\theta}} \]
\[ \hat{\rho}_2 = s_1 s_2 \Gamma^{0\hat{z}} \sigma_1 + c_1 c_2 \Gamma^{01234\hat{z}} \sigma_1 - s_1 c_2 \Gamma^{0\hat{\theta}} + c_1 s_2 \Gamma^{01234\hat{\theta}} \]
\[ \Gamma^{\hat{z}} = c_3 \Gamma^z + s_3 \Gamma^\theta, \quad \Gamma^{\hat{\theta}} = c_3 \Gamma^\theta - s_3 \Gamma^z \]

\[ \hat{\Pi} = \frac{1}{2} (1 + s_4^2 \hat{\rho}_1 + c_4^2 \hat{\rho}_2 - s_4 c_4 \Gamma^{0\psi} + s_4 c_4 \Gamma^{0\psi} \hat{\rho}_1 \hat{\rho}_2) \]

Same 1/8 susy preserved
Backreacted strata

- Dynamics of superstrata
  - Arbitrary surface possible?

- 6D sugra (D1-D5-P) [Gutowski+Martelli+Reall] [Cariglia+Mac Conamhna] [Bena+Giusto+Warner+MS 1110.2781]
  - Linear problem, if solved in the right order
    - Linearity was crucial for previous microstate solutions.
    - $x^5$-dep 4D almost-HK base. Functions & forms on it.

- Given superstratum data, should be possible to find solutions
Double bubbling of D1-D5-P system

D1-D5-KK tube (Lunin-Mathur geometry)

D1-P supertube spiral

D5-P spiral

Geometric superstrum
So far we could construct:

\[ x^5 \]

\[ \mathbf{x} = \mathbf{F}^{(i)}(t - x^5) \]

\( N \) strands of D1-P and D5-P supertube spirals along arbitrary curves \( \mathbf{F}^{(i)}, i = 1, \ldots, N \)

Next step: KKM dipole (work in progress)
Lesson 3:
Exotic branes are ingredients of BHs
Conclusions
Conclusions

- Exotic branes = non-geometries (U-folds)
- Exotic charges = U-duality monodromies
- Relevant even for non-exotic physics by supertube effect

Exotic branes are not at all exotic; They are everywhere!

- Essential ingredients of BHs
Future directions

- Geometric superstrata
- Non-geometric superstrata
  - More generic
  - Locally geometric
  - Generalize susy sol’n ansatz
  - Generalized geometry, DFT

[Berman, Hohm, Hull, Perry, Zwiebach, …]
Conjecture:

Generic microstates of black holes involve non-geometric superstrata.
Thanks!