Higher spin AdS_3 supergravity and its dual CFT

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Overview

- Backgrounds
- Our proposal

AdS/CFT correspondence and higher spin gauge theory

BACKGROUNDS

AdS/CFT correspondence

- Duality based on Dp-branes in superstring theory
  - Superstring theory on AdS_{p+2}
    ⇔ Low energy effective theory on Dp-brane
- Examples
  - AdS_5/CFT_4 [Maldacena '97]
    - Superstring on $AdS_5 \times S^5$ ⇔ N=4 Super Yang-Mills theory
  - AdS_3/CFT_2
    - Superstring on $AdS_3 \times S^3 \times M_4$ ⇔ N=(4,4) SCFT on Sym(M_4)
  - AdS_4/CFT_3 [Aharony, Bergman, Jafferis, Maldacena '08]
    - Superstring on $AdS_4 \times CP^3$ ⇔ Superconformal Chern-Simons theory
- Difficulties
  - Strong/weak duality
    - Difficult to derive it
    - Essential to study strongly coupled physics
  - Superstring theory is difficult to analyze

Higher spin gauge theory

- Gauge theory with higher spin *s* [Fronsdal '78]

$$\phi_{\mu_1\cdots\mu_s} \sim \phi_{\mu_1\cdots\mu_s} + \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}, \quad \phi^{\lambda\sigma}{}_{\lambda\sigma\mu_5\cdots\mu_s} = 0$$

  - Yang-Mills (*s*=1), Gravity (*s*=2), …
- Vasiliev and collaborators theory
  - Defined on AdS space
  - Includes interactions
  - All spin *s* should be included, less degrees of freedom
  - Only equations of motion are known, no action
- Motivations to study
  - A toy model of string theory
  - Black holes, singularity resolution
  - Simple versions of AdS/CFT

Simple AdS_4/CFT_3

- Klebanov-Polyakov conjecture '02
  - 4d Vasiliev theory ⇔ 3d O(*N*) vector model
  - A weak/weak duality
- State counting
  - Free action of O(*N*) vector model

$$S = \frac{1}{2}\int d^3x \sum_{a=1}^{N}(\partial_\mu h^a)^2$$

  - O(*N*) singlet conserved currents $\left(\text{c.f. } \mathrm{tr}[\Phi\nabla^{l_1}\Phi\nabla^{l_2}\cdots\Phi]\right)$

$$J_{\mu_1\cdots\mu_s} = h^a\partial_{(\mu_1}\cdots\partial_{\mu_s)}h^a + \cdots \Longleftrightarrow \phi_{\mu_1\cdots\mu_s}$$

- RG flow [Klebanov-Witten '99]
  - Two types of boundary condition can be assigned for a scalar field and RG flow interchanges the two.
  - The IR fixed point of O(*N*) vector model

$$S = \int d^3x \left[\frac{1}{2}\sum_{a=1}^{N}(\partial_\mu h^a)^2 + \frac{\lambda}{2N}(h^a h^a)^2\right]$$

- Correlation functions [Giombi, Yin '09, '10]

Simple AdS$_3$/CFT$_2$

- Gaberdiel-Gopakumar conjecture '10
  - 3d Vasiliev theory ⇔ W_N minimal model

A bosonic subsector of Prokushkin-Vasiliev '98

$$\frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$$

$$M^2 = -1 + \lambda^2 \qquad k, N \to \infty, \; 0 < \lambda = \frac{N}{k+N} < 1$$

- Checks of the proposal
  - Symmetry (a large *N* limit of W_N symmetry)
  - RG flow
  - Partition function [Gaberdiel, Gopakumar, Hartman, Raju '11]
  - Correlation functions [Chang-Yin '11, …]
- Generalizations
  - SU => SO [Ahn '11, Gaberdiel-Vollenweider '11]
  - Supersymmetric extension [Creutzig-YH- Rønne '11]

Supersymmetric extension

- Our conjecture '11
  - 3d full Vasiliev theory ⇔ *N*=2 CP^N Kazama-Suzuki model

    Full sector of Prokushkin-Vasiliev

    $$\frac{SU(N+1)_k \otimes SO(2N)_1}{SU(N)_{k+1} \otimes U(1)_{N(N+1)(k+N+1)}}$$
- Checks of the proposal
  - Symmetry
    - Asymptotic symmetry is *N*=(2,2) *W* algebra
    - The Kazama-Suzuki model has the same symmetry
  - Partition function
    - Gravity partition function is computed
    - Some parts can be reproduced by CFT partition function
    - We now try to establish the complete match
  - Need more
    - RG flow, correlation functions,…

Higher spin AdS_3 supergravity and its dual CFT

OUR PROPOSAL

Chern-Simons formulation

- 3d Vasiliev theory
  - Massive sector
    - Complex scalar fields, Dirac fermions
  - Massless sector
    - Gauge fields with integer or half-integer spin *s*
    - Chern-Simons formulation if we neglect coupling to matters (special for 3d case)
- Chern-Simons theory

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}], \;\; S_{\rm CS}[A] = \frac{k_{\rm CS}}{4\pi} \int {\rm tr}(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$$

  - Examples of GxG Chern-Simons formulation
    - G=SL(2): Einstein gravity on AdS_3
    - G=OSP(*p*|2): AdS supergravity [Achucarro, Townsend '86]
    - G=SL(*N*): Higher spin AdS gravity theory, $N=\infty$ for GG [Blencowe '89]
    - G=SL(*N*+1|*N*): Higher spin AdS supergravity, $N=\infty$ for ours

Asymptotic symmetry

- Chern-Simons theory with boundary
  - Chern-Simons theory is a topological theory on bulk
  - Degrees of freedom exists at boundary
  - Boundary theory is G WZNW model
- Asymptotic symmetry
  - Assign the condition of asymptotically AdS space
  - The condition is equivalent to Drinfeld-Sokolov Hamiltonian reduction [Campoleoni, Fredenhagen, Pfenninger, Theisen '10, Campoleoni, Fredenhagen, Pfenninger '11]
  - Examples
    - SL(2) => Virasoro [Brown, Henneaux '86]
    - SL(N) => W_N [Henneaux, Rey '10, Campoleni, Fredenhagen, Pfenninger, Theisen '10]
    - SL(N+1|N) => N=2 W_{N+1} [Creutzig-YH- Rønne '11]

Gravity partition function of GG

- Massless sector
  - Partition function for higher spins [Gaberdiel, Gopakumar, Saha '10]

$$Z_B^{(s)} = \prod_{n=s}^{\infty} |1-q^n|^{-2}, \quad Z_B^{\mathrm{HS}} = \prod_{s=2}^{\infty} Z_B^{(s)}$$

- Massive sector
  - Two complex scalars with mass $M^2 = -1 + \lambda^2$
    - Boundary conditions are chosen differently for the two scalars
  - Partition function for massive scalar with *h* [Giombi, Maloney, Yin '08]

$$Z_{\mathrm{scalar}}^{h} = \prod_{l,l'=0}^{\infty} (1-q^{h+l}\bar{q}^{h+l'})^{-2}, \quad Z_B^{\mathrm{matter}} = Z_{\mathrm{scalar}}^{\frac{1+\lambda}{2}} Z_{\mathrm{scalar}}^{\frac{1-\lambda}{2}}$$

- Comparison to CFT partition function [Gaberdiel, Gopakumar, Hartman, Raju '11]
  - Agreed at large *k*,*N* but fixed $\lambda = \frac{N}{k+N}$

Supergravity partition function

- Massless sector
  - Partition function for spin *s*+1/2 [Creutzig-YH- Rønne '11]

$$Z_F^{(s)} = \prod_{n=s}^{\infty} |1+q^{n+\frac{1}{2}}|^2 , \quad Z^{\text{HS}} = \prod_{s=2}^{\infty} Z_B^{(s)} (Z_F^{(s-1)})^2 Z_B^{(s-1)}$$

- Massive sector
  - 4 complex scalars + 4 Dirac fermions with mass [Prokushkin, Vasiliev '98]

$$(M_B^{+})^2 = -1+\lambda^2 , \quad (M_B^{-})^2 = -1+(1-\lambda)^2 , \quad (M_F^{\pm})^2 = (\tfrac{1}{2}-\lambda)^2$$

    - Divide into two groups, which have different boundary conditions
  - Partition function for massive fermions

$$Z_{\text{spinor}}^{h} = \prod_{l,l'=0}^{\infty} (1+q^{h+l}\bar{q}^{h-\frac{1}{2}+l'})(1+q^{h-\frac{1}{2}+l}\bar{q}^{h+l'})$$

$$Z^{\text{matter}} = Z_{\text{susy}}^{\frac{\lambda}{2}} Z_{\text{susy}}^{\frac{1-\lambda}{2}} , \quad Z_{\text{susy}}^{h} = Z_{\text{scalar}}^{h} (Z_{\text{spinor}}^{h+\frac{1}{2}})^2 Z_{\text{scalar}}^{h+\frac{1}{2}}$$

- Comparison with CFT partition function
  - Vacuum character and first few terms agree

Proposed dual CFT

- $\mathcal{N}$=(2,2) CP^N Kazama-Suzuki model

$$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)_{N(N+1)(k+N+1)}}$$

  - Take the 't Hooft limit

$$k, N \to \infty, \ 0 < \lambda = \frac{N}{k+N} < 1$$

  - The model has $\mathcal{N}$=(2,2) W_{N+1} symmetry
  - The central charge

$$c = \frac{3Nk}{k+N+1} \to 3N(1-\lambda)$$

  - First non-trivial states are dual to massive fields
- Further checks of the proposal
  - Complete the comparison of partition function
  - Study the RG flow of $\mathcal{N}$=2 CP^N model
  - Extensions to other cosets

Details

- Chern-Simons formulation
- Gravity partition function
- Dual $\mathcal{N}$=2 CP^N model

Higher spin gravity theory and asymptotic symmetry

CHERN-SIMONS FORMULATION

Chern-Simoms gravity

- SL(2) Chern-Simons theory
  - Action

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$$

$$S_{\rm CS}[A] = \frac{k_{\rm CS}}{4\pi} \int {\rm tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right), \quad k_{\rm CS} = \frac{\ell}{4G}$$

  - Gauge transformation

$$A = A_\mu^a J_a dx^\mu, \;\; J_a (a = 1, 2, 3) : {\rm sl}(2) \text{ generator}$$

$$\delta A = d\lambda + [A, \lambda], \;\; \delta \tilde{A} = d\tilde{\lambda} + [\tilde{A}, \tilde{\lambda}]$$

- Relation to Einstein Gravity
  - Einstein-Hilbert action with a negative cosmological constant in the first order formulation
    - Dreibein: $e_\mu^a = \frac{\ell}{2}(A_\mu^a - \tilde{A}_\mu^a)$
    - Spin connection: $\omega_{\mu,a,b} = \frac{1}{2}\epsilon_{abc}\omega_\mu^c, \;\; \omega_\mu^c = \frac{1}{2}(A_\mu^c + \tilde{A}_\mu^c)$

Chern-Simons supergravity

- SL(N+1|N) Chern-Simons theory
  - We decompose sl(N+1|N) in terms of sl(2) as

$$\mathrm{sl}(N+1|N) = \mathrm{sl}(2) \oplus \left(\bigoplus_{s=3}^{N+1} \mathrm{g}^{(s)} \right) \oplus \left(\bigoplus_{s=1}^{N} \mathrm{g}^{(s)} \right) \oplus 2 \left(\bigoplus_{s=1}^{N+1} \mathrm{g}^{(s+\frac{1}{2})} \right)$$

Gravitational sl(2) Grassmann even Grassmann odd

  - Generators

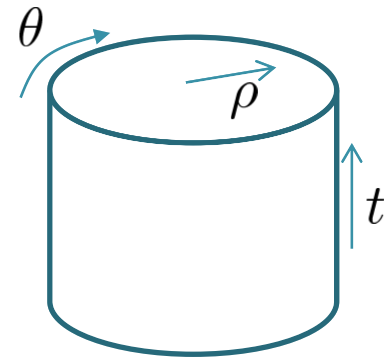
$$V_n^{(s)+} \; (s = 2, 3, \cdots), \; V_n^{(s)-} \; (s = 1, 2, \cdots), \; F_r^{(s)\pm} \; (s = 1, 2, \cdots)$$
$$(|n| \leq s - 1, \; |r| \leq s - 1/2)$$

  - Fields

$$e_{\mu,\pm}^{(s)n}, \; \omega_{\mu,\pm}^{(s)n}, \; \psi_{\mu.\pm}^{(s)r}, \; \tilde{\psi}_{\mu.\pm}^{(s)r} \; \left(e_{\mu,+}^{(2)n} \Leftrightarrow \mathrm{Dreibein} \right)$$

- Comments
  - Spin-statistic holds for this embedding
  - Different embedding leads to different asymptotic symmetry

Gauge fixings & conditions



- Coordinate system
  - t: time coordinate, (ρ,θ): coordinates of disk
  - Boundary at $\rho \to \infty$
- Restrictions
  - Gauge fixing $(A_\pm = A_\theta \pm A_t)$

$$A_+ = e^{-\rho V_0^{(2)+}} a(t+\theta) e^{\rho V_0^{(2)+}}, \ A_- = 0, \ A_\rho = e^{-\rho V_0^{(2)+}} \partial_\rho e^{\rho V_0^{(2)+}}$$

  - The condition of asymptotically AdS space

$$\begin{aligned} a(t+\theta) = V_1^{(2)+} &+ \sum_{s\geq 2} L_s^+(t+\theta) V_{-s+1}^{(s)+} + \sum_{s\geq 1} L_s^-(t+\theta) V_{-s+1}^{(s)-} \\ &+ \sum_{s\geq 1} G_s^+(t+\theta) F_{-s+\frac{1}{2}}^{(s)+} + \sum_{s\geq 1} G_s^-(t+\theta) F_{-s+\frac{1}{2}}^{(s)-} \end{aligned}$$

  - Same as the constraints for the Hamiltonian reduction

Asymptotic symmetry

- Residual gauge transformation

$$\Lambda(\theta) = e^{-\rho V_0^{(2)+}} \lambda(\theta) e^{\rho V_0^{(2)+}}, \quad \delta_\lambda a(\theta) = \partial_\theta \lambda(\theta) + [a(\theta), \lambda(\theta)]$$

  - $\lambda(\theta)$ not vanishing at the boundary generates physical symmetry
- Asymptotic symmetry
  - Generator

$$Q(\lambda) = -\frac{k}{2\pi} \int d\theta \, \mathrm{str}\left(\lambda(\theta) a(\theta)\right)$$

  - Poisson brackets

$$\{Q(\lambda), Q(\eta)\} = -\frac{k}{2\pi} \int d\theta \, \mathrm{str}\left(\eta(\theta) \delta_\lambda a(\theta)\right)$$

- Symmetry algebra
  - Same as the one from the Hamiltonian reduction
    - SL(2): Virasoro symmetry, $c=3l/2G$
    - SL(N+1|N): N=2 W_N with Virasoro sub-algebra, $c=3l/2G$

One loop determinant from heat kernel method

GRAVITY PARTITION FUNCTION

Partition function at 1-loop level

- Total contribution
  - Higher spin sector + Matter sector

$$Z^{\mathrm{Bulk}} = Z^{\mathrm{HS}} Z^{\mathrm{matter}}$$

- Higher spin sector
  - Two series of bosons and fermions

$$Z^{\mathrm{HS}} = \prod_{s=2}^{\infty} Z_B^{(s)} (Z_F^{(s-1)})^2 Z_B^{(s-1)}$$

$$Z_B^{(s)} = \prod_{n=s}^{\infty} |1-q^n|^{-2}, \quad Z_F^{(s)} = \prod_{n=s}^{\infty} |1+q^{n+\frac{1}{2}}|^2$$

- Matter part sector
  - 4 massive complex scalars and 4 massive Dirac fermions

$$Z^{\mathrm{matter}} = Z_{\mathrm{susy}}^{\frac{\lambda}{2}} Z_{\mathrm{susy}}^{\frac{1-\lambda}{2}}, \quad Z_{\mathrm{susy}}^{h} = Z_{\mathrm{scalar}}^{h} (Z_{\mathrm{spinor}}^{h+\frac{1}{2}})^2 Z_{\mathrm{scalar}}^{h+\frac{1}{2}}$$

$$Z_{\mathrm{scalar}}^{h} = \prod_{l,l'=0}^{\infty} (1 - q^{h+l} \bar{q}^{h+l'})^{-2}$$

$$Z_{\mathrm{spinor}}^{h} = \prod_{l,l'=0}^{\infty} (1 + q^{h+l} \bar{q}^{h-\frac{1}{2}+l'})(1 + q^{h-\frac{1}{2}+l} \bar{q}^{h+l'})$$

Heat kernel method

- General method
  - Definition of the heat kernel on EAdS_3

$$K^{(s)}_{ab}(x,y;t) = \langle y, b| e^{t\Delta_{(s)}} |x, a\rangle = \sum_n \psi^{(s)}_{n,a}(x)\psi^{(s)}_{n,b}(y)^* e^{t\lambda^{(s)}_n}$$

Laplacian on EAdS_3 Eigen-function of $\Delta_{(s)}$ Eigen-value

  - One-loop determinant

$$\ln\det(-\Delta_{(s)}) = \mathrm{tr}\ln(-\Delta_{(s)}) = -\int_0^\infty \frac{dt}{t}\int \sqrt{g}\, d^3x K^{(s)}_{aa}(x,x;t)$$

- Explicit formula [David, Gaberdiel, Gopakumar '09]
  - Consider thermal EAdS_3, whose boundary is torus with modulus $q = e^{i\tau}$
  - For transverse and traceless components

$$K^{(s)}(\tau,\bar{\tau};t) = (2-\delta_{s,0})\sum_{m=1}^{\infty}\frac{(-1)^{2sm}\tau_2}{4\sqrt{\pi t}|\sin\frac{m\tau}{2}|^2}\cos(sm\tau_1)e^{-\frac{m^2\tau_2^2}{4t}}e^{-(s+1)t}$$

  - Explicit computation leads to results for *s*=0 and *s*=1/2

Integer spin bosonic field

- Integer spin *s* gauge field [Fronsdal '79]
  - Symmetric traceless tensor of rank *s*

$$\phi^{\pm}_{\mu_1\cdots\mu_s} = \frac{1}{s}\bar{e}_{(\mu_1}{}^{a_1}\cdots\bar{e}_{\mu_{s-1}}{}^{a_{s-1}}e^{\pm}_{\mu_s)a_1\cdots a_{s-1}}, \quad \bar{e}^a_\mu : \text{background dreibein}$$

  - Constraint & gauge transformation

$$\phi^{\lambda\sigma}{}_{\lambda\sigma\mu_5\cdots\mu_s} = 0, \ \ \delta\phi_{\mu_1\cdots\mu_s} = \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

  - Decomposition

$$\phi_{\mu_1\cdots\mu_s} = \phi^{\mathrm{TT}}_{\mu_1\cdots\mu_s} + g_{(\mu_1\mu_2}\tilde{\phi}_{\mu_3\cdots\mu_s)} + \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

$$\xi_{\mu_1\cdots\mu_{s-1}} = \xi^{\mathrm{TT}}_{\mu_1\cdots\mu_{s-1}} + \nabla_{(\mu_1}\sigma_{\mu_2\cdots\mu_{s-1})} + \cdots$$

- One loop determinant [Gaberdiel, Gopakumar, Saha '10]

$$Z^{(s)}_B = \det{}^{-\frac{1}{2}}\left(-\Delta + \frac{s(s-3)}{\ell^2}\right)^{\mathrm{TT}}_{(s)} \det{}^{\frac{1}{2}}\left(-\Delta + \frac{s(s-1)}{\ell^2}\right)^{\mathrm{TT}}_{(s-1)}$$

  - Only transverse and traceless components contribute
  - The heat kernel leads to the result for integer s > 0

Half-integer spin fermionic field

- Half integer spin s+1/2 gauge field [Fang, Fronsdal '80]
  - Symmetric traceless rank s tensor-spinor with two components

$$\psi^{\pm,\alpha}_{\mu_1\cdots\mu_s} = \frac{1}{s}\bar{e}_{(\mu_1}{}^{a_1}\cdots\bar{e}_{\mu_{s-1}}{}^{a_{s-1}}\psi^{\pm,\alpha}_{\mu_s)a_1\cdots a_{s-1}}$$

  - Constraint & gauge transformation

$$\psi^{\lambda}_{\lambda\mu_4\cdots\mu_s} = 0, \;\; \delta\psi^{\alpha}_{\mu_1\cdots\mu_s} = \nabla_{(\mu_1}\epsilon^{\alpha}_{\mu_2\cdots\mu_s)} + \frac{1}{2\ell}\Gamma_{(\mu_1}\epsilon^{\alpha}_{\mu_2\cdots\mu_s)}$$

  - Decomposition

$$\psi^{\alpha}_{\mu_1\cdots\mu_s} = \psi^{\mathrm{TT}\alpha}_{\mu_1\cdots\mu_s} + \Gamma_{(\mu_1}\hat{\psi}^{\alpha}_{\mu_2\cdots\mu_s)} + \nabla_{(\mu_1}\eta^{\alpha}_{\mu_2\cdots\mu_s)} + \cdots$$

$$\xrightarrow{\text{Gauge fixing}} \quad \hat{\psi}^{\alpha}_{\mu_1\ldots\mu_{s-1}} = \frac{1}{2}\Gamma_{(\mu_1}\tilde{\psi}^{\alpha}_{\mu_2\cdots\mu_{s-1})}$$

- One loop determinant [Creutzig-YH- Rønne '11]

$$Z^{(s)}_B = \det{}^{\frac{1}{2}}\left(-\Delta + \frac{(s+\frac{1}{2})(s-\frac{5}{2})}{\ell^2}\right)^{\mathrm{TT}}_{(s+\frac{1}{2})} \det{}^{-\frac{1}{2}}\left(-\Delta + \frac{(s-\frac{1}{2})(s+\frac{1}{2})}{\ell^2}\right)^{\mathrm{TT}}_{(s-\frac{1}{2})}$$

  - The heat kernel leads to the result for half-integer s+1/2
  - For gravitino with spin 3/2 [David, Gaberdiel, Gopakumar '09]

CFT partition function and relation to dual gravity theory

DUAL $\mathcal{N}$=2 CP^N MODEL

Dual CFTs

- Dual for a bosonic sub-sector of Vasiliev theory
  [Gaberdiel, Gopakumar '10]
  - Minimal model with W_N symmetry

$$\frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}} \qquad k, N \to \infty,\ 0 < \lambda = \frac{N}{k+N} < 1$$

  - Evidences
    - Partition function, RG flow, correlation functions
- Dual for the full Vasiliev theory [Creutzig-YH- Rønne '11]
  - Minimal model with N=2 W_N symmetry

$$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)} \qquad k, N \to \infty,\ 0 < \lambda = \frac{N}{k+N} < 1$$

  - Level-rank duality [Gepner '88]

$$\frac{\mathrm{SU}(N+1)_k}{\mathrm{SU}(N)_k \otimes \mathrm{U}(1)_{N(N+1)k}} \simeq \frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}}$$

States of the coset

- $\mathcal{N}$=2 CP^N Kazama-Suzuki model

$$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)_{N(N+1)(k+N+1)}} \qquad c = \frac{3Nk}{k+N+1}$$

  - States are labeled by $(\rho, s; \nu, m)$
    - ρ, ν: highest weights for $\mathrm{SU}(N+1)_k$, $\mathrm{SU}(N)_{k+1}$
    - $s=0,2$, m takes values in $Z_{N(N+1)(k+N+1)}$
  - Conformal weights

$$h(\rho, s; \nu, m) = n + \tfrac{s}{4} + \tfrac{1}{(k+N+1)}\left(C_{N+1}(\rho) - C_N(\nu) - \tfrac{m^2}{2N(N+1)}\right)$$

$$\left(\tfrac{|\rho|}{N+1} + \tfrac{s}{2} - \tfrac{|\nu|}{N} - \tfrac{m}{N(N+1)} = 0 \bmod 1\right)$$

  - First non-trivial states

$$h(\mathrm{f}, s : 0, N) \sim \tfrac{s}{4} + \tfrac{\lambda}{2}, \;\; h(0, s : \mathrm{f}, -N-1) \sim \tfrac{4-s}{4} - \tfrac{\lambda}{2}$$

    - States are products of chiral and anti-chiral parts
    - Dual to massive scalars and fermions

Partition function

- Vacuum character of $\mathcal{N}$=2 W_N algebra
  - Definition of the vacuum

$$L^{(s)\pm}_{n-s+1}|0\rangle = G^{(s)\pm}_{n-s+\frac{1}{2}}|0\rangle = 0, \text{ for } n \geq 0$$

  - Vacuum character

$$Z_B^{(s)} = \prod_{n=s}^{\infty} |1-q^n|^{-2}, \; Z_F^{(s)} = \prod_{n=s}^{\infty} |1+q^{n+\frac{1}{2}}|^2$$

$$\lim_{N\to\infty} \chi_0 = \prod_{s=2}^{\infty} Z_B^{(s)} (Z_F^{(s-1)})^2 Z_B^{(s-1)} = Z^{\mathrm{HS}}$$

- Bosonic subsectors
  - Factorization & level-rank duality

$$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)} \simeq \frac{\mathrm{SU}(k)_N \otimes \mathrm{SU}(k)_1}{\mathrm{SU}(k)_{N+1}} \times \frac{\mathrm{SU}(N)_k \otimes \mathrm{SU}(N)_1}{\mathrm{SU}(N)_{k+1}} \times \mathrm{U}(1)$$

  - The partition function is $Z_{GG}(1-\lambda)$ x $Z_{GG}(\lambda)$ x (fermions)

$$\left(Z_{\mathrm{GG}}(\lambda) = Z_{\mathrm{scalar}}^{\frac{1+\lambda}{2}} Z_{\mathrm{scalar}}^{\frac{1-\lambda}{2}} \prod_{s=2}^{\infty} Z_B^{(s)} \right)$$

Checks of the duality

- CFT partition function
  - Sum over characters of all states

$$Z_{\rm CFT}(N,k) = \sum_{\Lambda_+,\Lambda_-} |\chi_{(\Lambda_+,0;\Lambda_-)}(q) + \chi_{(\Lambda_+,2;\Lambda_-)}(q)|^2$$

  - For bosonic subsector, it matches to gravity partition function at large *N*,*k* [Gaberdiel, Gopakumar, Hartman, Raju, '11]
  - For our case, only first terms are checked
- Further checks of the proposal
  - Comparison of partition function
    - Explicit expression of CFT partition function cannot be found
    - Supersymmetry, elliptic genus, expansion for q~0, free field realization
  - RG flow (c.f. $(k+N)^{\rm th}$ minimal => $(k+N-1)^{\rm th}$ minimal)
  - Other cosets

$$\frac{{\rm G}_k \otimes {\rm SO(dim.)}_1}{{\rm SU}(N)_{k+1}\otimes {\rm U}(1)}\,, \quad {\rm G} = {\rm SO(2N)}, {\rm Sp}(2N)$$

Our conjecture of duality and related works

CONCLUSION

Summary

- Our proposal
  - 3d full Vasiliev theory ⇔ N=2 CP^N Kazama-Suzuki model

    Full sector of Prokushkin-Vasiliev
    $$\frac{\mathrm{SU}(N+1)_k \otimes \mathrm{SO}(2N)_1}{\mathrm{SU}(N)_{k+1} \otimes \mathrm{U}(1)_{N(N+1)(k+N+1)}}$$
  - Checks of the proposal
    - N=2 W_N Symmetry
    - Comparison of partition function
    - Need more (RG flow, correlation functions,…)
    - Possible to derive the duality?
- Related works
  - Resolution of black hole singularity [Ammon, Gutperle, Kraus, Perlmutter '11,…]
  - 1/N corrections [Castro, Lepage-Jutier, Maloney '10,…]
  - Gravity dual of Ising model [Castro, Gaberdiel, Hartman, Maloney, Volpato '11]
  - 3d CS theory with vector matters [Aharony, Gur-Ari, Ycoby '11, Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin'11]
  - Realization in superstring theory?