Future foam: Non-trivial topology from bubble collisions in eternal inflation

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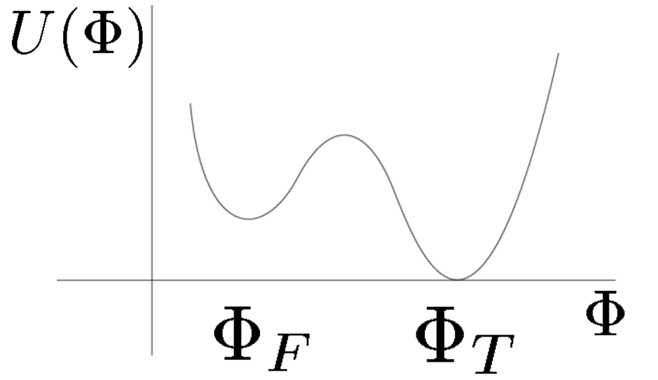
Based on
R. Bousso, B. Freivogel, Y. Sekino, S. Shenker, L. Susskind,
I.-S. Yang, and C.-P. Yeh, PRD78, 063538 (2008), arXiv: 0807.1947[hep-th]

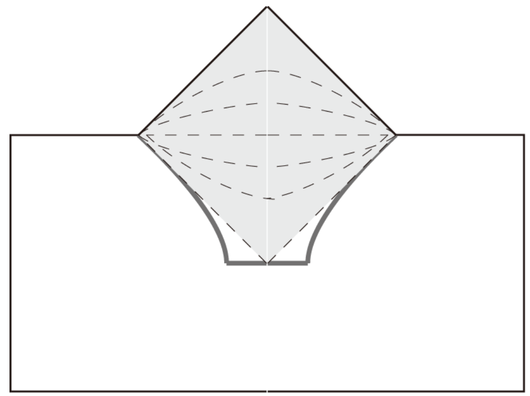
Bubble nucleation

- (As a model for landscape,)
  Gravity coupled to a scalar field whose potential has a metastable vacuum:

  Assume U(Φ_F)>0, U(Φ_T)=0.



- False vacuum: de Sitter space
- Creation of Universe:
  Tunneling (bubble nucleation); described by Coleman-De Luccia (CDL) instanton
- Inside the bubble, open FRW universe
- Nucleation rate : $\Gamma \sim \exp(-(S_{\mathrm{inst}} - S_{\mathrm{F}}))$

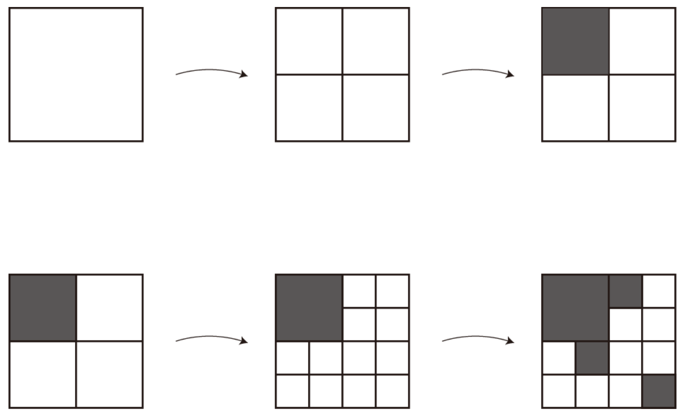


Eternal inflation

- If nucleation rate is small compared to expansion rate (H),

$$\Gamma < cH^4$$

true vacuum does not "percolate".

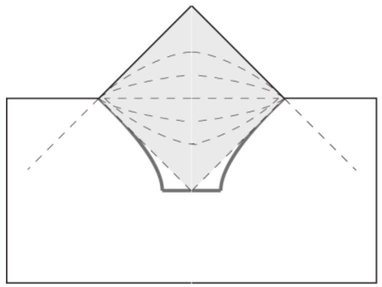


- Intuitive picture (Winitzki) :
Fractal percolation model
    white: false vacuum
    black: true vacuum
 Finite fractal dimension: $D_F = 2 - |\log(1-\Gamma)|/\log 2$
- Eternal inflation:
Fraction of false vacuum goes to zero: $f(t) \sim e^{-c(\Gamma H^{-3})t}$
But its physical volume grows indefinitely,

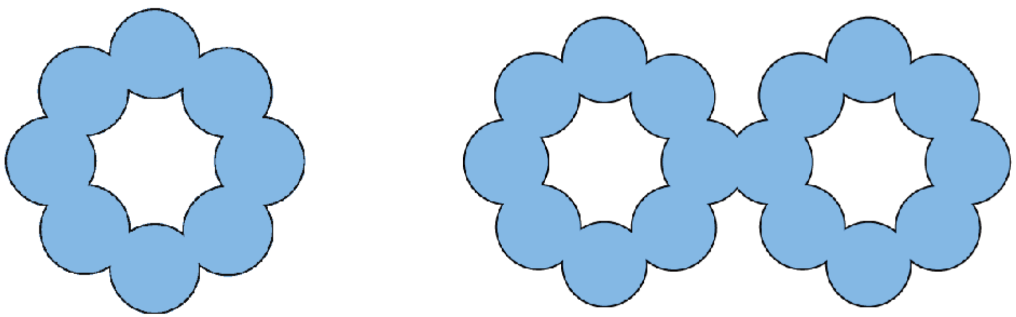
$$e^{3Ht} f(t) \sim e^{(3-c\Gamma H^{-4})Ht} \to \infty$$

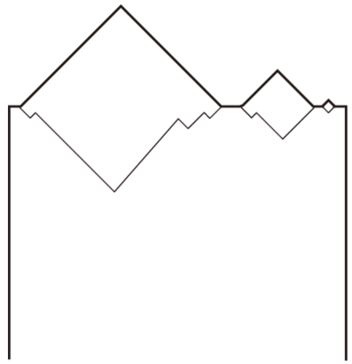
Bubble collisions

- Inevitable in eternal inflation
(Infinite 4-volume inside past light-cone)



- Due to bubble collisions, true vacuum region
(“pocket universe”) which has non-trivial
 boundary topology may occur:





- We perform detailed study especially on interior geometry
and causal structure; we take the thin-wall limit (in 3+1 dim);

Motivation for studying boundary topology

- <u>Observational consequence</u>:
  Identical objects on the sky
  (However, this is rare.)

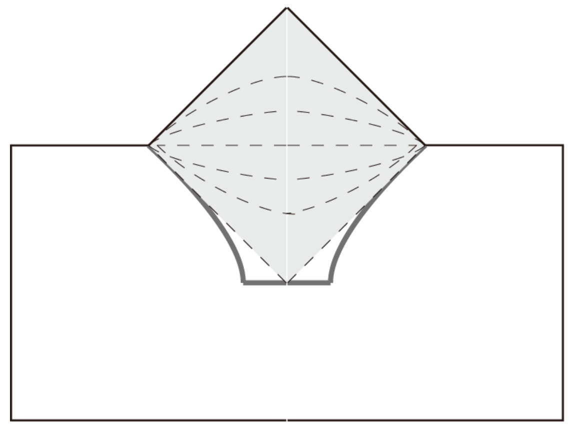
- <u>Holographic description of eternal inflation</u>
  FRW/CFT correspondence (Freivogel, Sekino, Susskind, Yeh, '06)
  - Dual theory: defined at the boundary (spatial infinity).
    (S^2 at the boundary of H^3 for the one bubble case)

  Importance of finding non-perturbative formulation:
  - "Definition" of de Sitter vacua
  - Mathematical framework for eternal inflation

FRW/CFT correspondence

(Freivogel, Sekino, Susskind, Yeh, '06)

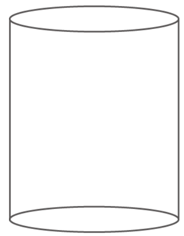
- Dual theory:
  - Conformal field theory on S^2
  - Contains 2D gravity (Liouville)
  - (matter c) ~ (de Sitter entropy)
  - The dual has 2 less dim than the bulk (Liouville plays the role of time)

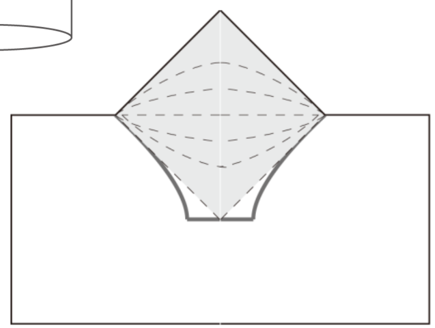


- Evidence:
  - SO(3,1) symmetry
  - Bulk correlators can be interpreted as CFT correlators.
  - Energy momentum tensor has dimension 2.

Boundary geometry

- <u>Difference with AdS/CFT:</u>
  - In AdS space, boundary condition should be fixed ("cold" boundary).
  - In our FRW, boundary condition of graviton should be integrated ("warm" boundary). Reason: Universe is embedded in de Sitter. (c.f. super-horizon correlations in de Sitter)





- If there can be universes w/ non-trivial boundary topology (and if the boundary is accessible to a single observer), we should include them in the dual theory:
  - Sum over topologies of base space of CFT.

Plan of the talk

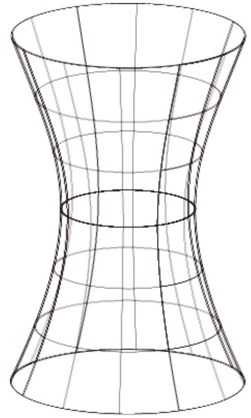
- Basic facts about bubble collisions
  - Two bubble collision: symmetries, assumptions on domain wall, causal structure, etc.
- Existence of non-trivial boundary topology
  - Heuristic argument: "Dust" wall approximation
  - Torus solution: sequence of collisions of radiation
  - "Coarse grained" smooth torus
- Multiple boundaries
- Implication for holographic duality

A bubble in de Sitter (CDL instanton)

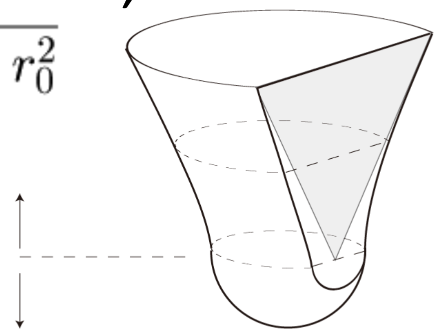
- De Sitter space: hyperboloid in $R^{4,1}$

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2$$

$$ds^2 = -dt^2 + \ell^2 \cosh^2(t/\ell) d\Omega_3^2$$



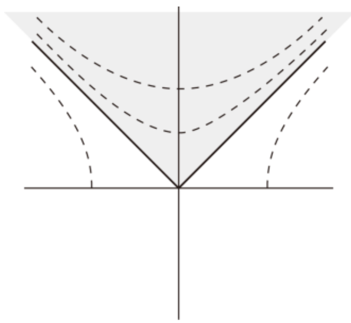
- A bubble (w/ zero vacuum energy, thin-wall limit; nucleated at t=0): plane at $X_4 = \text{const.} = \sqrt{\ell^2 - r_0^2}$
Preserves SO(3,1).

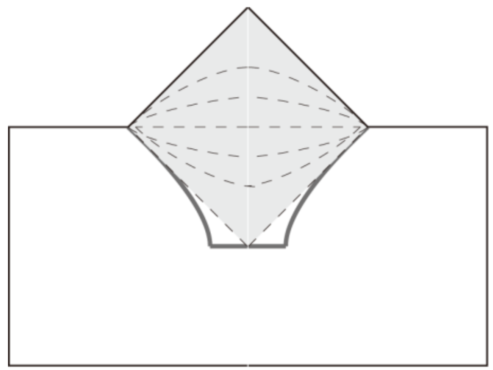


- Open FRW universe in the bubble:

$$ds^2 \;=\; -dt^2 + t^2 dH_3^2$$

Part of Minkowski space
Future asymptotics: "hat"

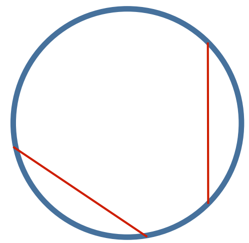




Collision of two bubbles

(Bousso, Freivogel, Yang, '07)

- Residual symmetry: SO(2,1)
  Two bubbles nucleated on the great circle
  (in the (X_3, X_4) plane)

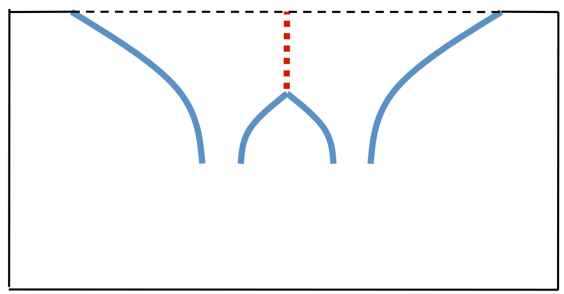


- Parametrization of de Sitter w/ manifest H^2 :

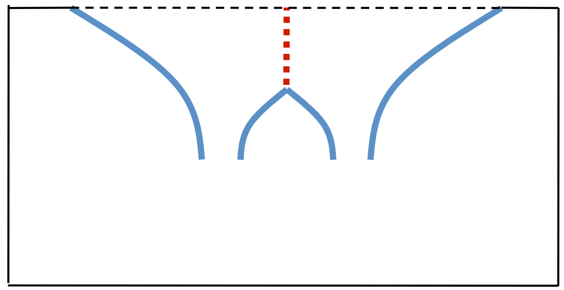
$$ds^2 = -f^{-1}(t)dt^2 + f(t)dz^2 + t^2 dH_2^2$$
$$f(t) = 1 + t^2/\ell^2, \quad (0 \leq z \leq 2\pi\ell)$$
$$\left(X_a = tH_a \ (a = 0, 1, 2), \quad X_3 = \sqrt{t^2 + \ell^2}\cos(z/\ell), \quad X_4 = \sqrt{t^2 + \ell^2}\sin(z/\ell)\right)$$

- Profile in the (t, z) space (H^2 is attached to each point) :

We want to find the interior geometry.



Finding interior geometry



- Flat space:

$$ds^2 = -dt^2 + dz^2 + t^2 dH_2^2$$

- Assume a domain wall forms after collision (effectively).
  Trajectory: $(t(\tau), z(\tau))$
  Intrinsic geometry: $ds_{\mathrm{DW}}^2 = -d\tau^2 + R^2(\tau) dH_2^2 \qquad (R(\tau) = t(\tau))$

- We patch another flat space across this domain wall.
  Israel junction condition:

$$K_{ab} - h_{ab} K = 8\pi G T_{ab}$$

h_{ab} : induced metric on hypersurface

K_{ab} : extrinsic curvature

Domain wall equation of state

- Domain wall: perfect fluid $T^a_b = (-\rho, p, p)$
  Conservation: $\dot{\rho} = -2(\rho + p)(\dot{R}/R)$

- Dust wall (p=0): $(\rho R^2) = \text{const.}$
  - Energy density is diluted as R gets large.
    (This prevents gravitational collapse.)

- "Vacuum domain wall" $(\rho = -p):$ $\rho = \text{const.}$
  - Realized by a scalar kink. Produced in the collision of two bubbles of different vacua.
  - (We don't consider this in the later discussion of topologies, assuming there is only one kind of true vacuum.)

Solving the junction condition

- Junction condition: $$\Delta K^\tau_\tau = -4\pi G(\rho + p)$$ $$\Delta K^1_1 = \Delta K^2_2 = -4\pi G\rho$$
- Extrinsic curvature: $$\Delta K^1_1 = \frac{1}{2} g^{11}\partial_n g_{11} = \frac{\sqrt{\dot{t}^2 - 1}}{t}$$
- Energy density: $\rho = \rho_0/t^2 \quad \text{(for dust wall)}$
- <u>"Effective potential" for t</u>: $\dot{t}^2 + V_{\text{eff}}(t) = 0$
  - For dust wall,

$$V_{\text{eff}}(t) = -1 - \frac{4\pi^2 G^2 \rho_0^2}{t^2}$$

$$\text{As } t \to \infty, \quad \dot{t} \to 1, \ \dot{z} \to 0$$

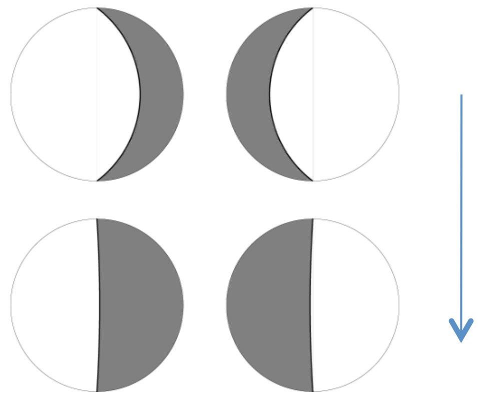
  - For vacuum domain wall,

$$V_{\text{eff}}(t) = -1 - 4\pi^2 G^2 \rho t^2$$

$$\text{As } t \to \infty, \quad \dot{t} \sim \dot{z} \to \infty$$

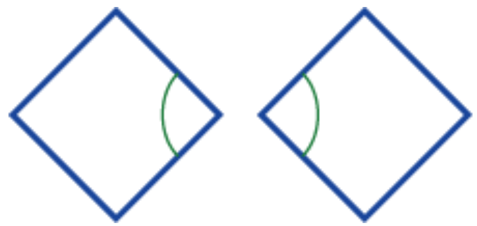
Causal structure

- Dust wall case: the geometry approaches empty open FRW (spatial geometry: H^3). (Right figure: DW seen in the H^3 slicing)

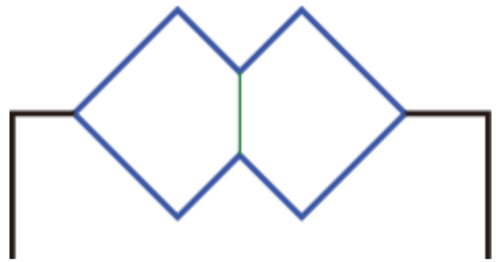


  - DW approaches minimal surface in H^3
  - A time-like observer can see the whole true vac. region.

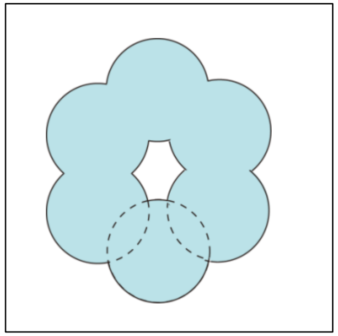
- For the vacuum wall case,



  - Domain wall is "repelled from either side" (Vilenkin, Ipser, Sikivie, '84).
  - There are two time-like infinities. (Right figure: causal structure)



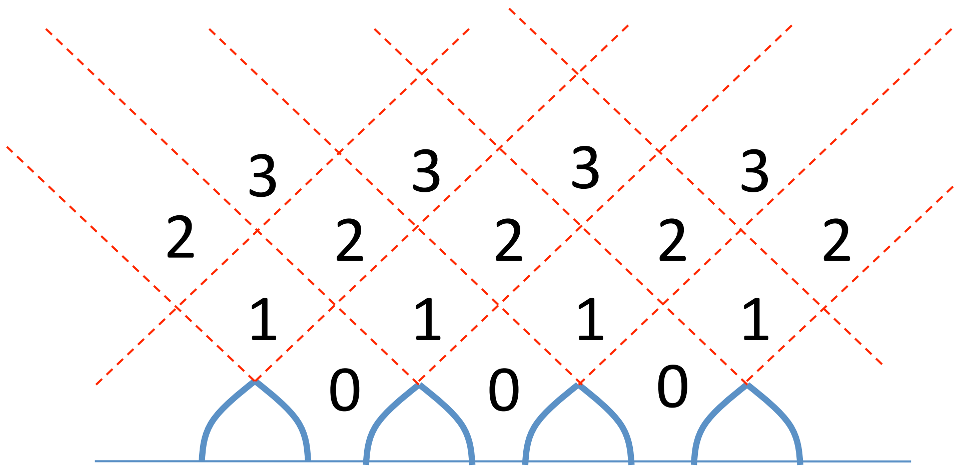
Existence of non-trivial topology (qualitative argument; dust wall assumption)



- Future (conformal) infinity of de Sitter:
  A bubble cuts out a ball
- <u>Interior geometry</u>:
  - If dust walls don't intersect, local geometry around the wall will be the same as in the 2-bubble case.
  - Dust walls don't intersect if the boundary S^1 's don't.
- We can produce arbitrary genus without letting boundary S^1 's intersect with each other.
  - Smooth geometry with arbitrary boundary genus should exist.

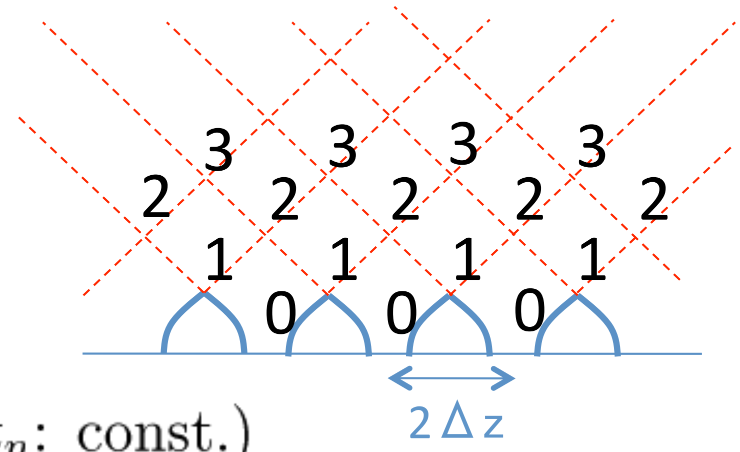
Torus solution: sequence of collisions

- Special configuration preserving SO(2,1):
  Bubbles nucleated at t=0, along the great circle of S^3 with equal spacing.
- Assumption: At the collision, bubbles walls instantaneously annihilates, emitting a shell of radiation.



  - Geometry behind the radiation is modified.
  - Solve the geometry recursively.

Iteration of junction conditions



- Metric:

$$ds^2 = -f(t)dt^2 + f^{-1}(t)dz^2 + t^2 dH_2^2$$
$$f(t) = 1 + t^2/\ell^2 \quad \text{(de Sitter)}$$
$$f(t) = 1 - t_n/t \equiv f_n(t) \quad \text{(in region } n\text{; } t_n\text{: const.)}$$

- Junction condition (consistency condition):

$$f_{n+2}(t_{*,n+2}) f_n(t_{*,n+2}) = \left(f_{n+1}(t_{*,n+2})\right)^2$$
$$t_{*,n} : \text{ time of the } n\text{-th collision}$$

- In the weakly curved limit (f ~1),

$$t_{n+2} - t_{n+1} = t_{n+1} - t_n \qquad \Rightarrow \qquad t_n = n t_1 \sim n(\Delta z)^3/\ell^2$$
$$t_{*,n} \sim n\Delta z, \quad (2\Delta z : \text{initial separation})$$

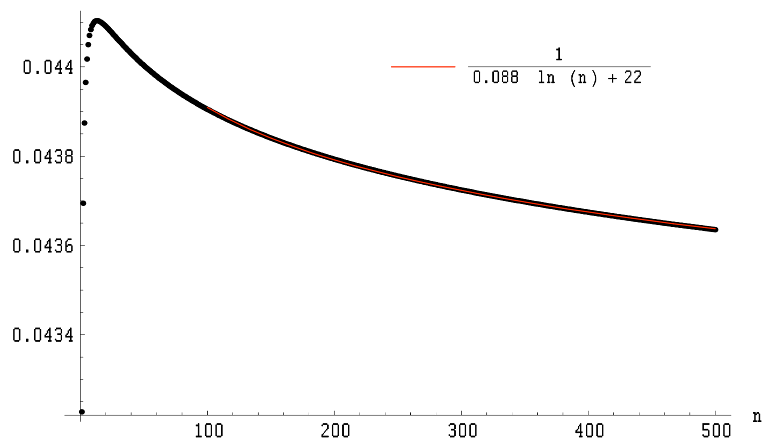
Properties of the solution

- In the weakly curved limit, deviation from flat space is

$$t_n/t_{n,*} \sim (\Delta z/\ell)^2$$

  - In the case of many bubbles with fine spacing, interior geometry is close to flat. (This is because bubbles collide quickly and do not have much energy when they collide.)

- At late time, geometry approaches flat space logarithmically. (From 2nd order, or numerical analysis.)



Torus solution: “coarse grained” version

- We patch flat space with de Sitter across a (smooth) toroidal domain wall (symmetry: U(1) x U(1)).
- Interior (flat) metric (“bulk” of torus):

$$ds^2 = -\gamma dt^2 + t^2 d\theta_1^2 + dr_2^2 + r_2^2 d\theta_2^2$$

$$(0 \leq \theta_1, \theta_2 \leq 2\pi; \;\; \gamma : \;\; \text{const.})$$

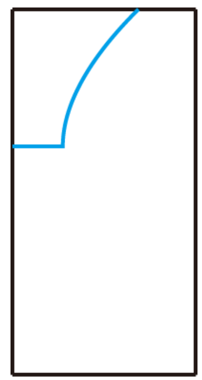
- Intrinsic geometry of DW:

$$ds_{\text{DW}}^2 = -d\tau^2 + r_1^2(\tau) d\theta_1^2 + r_2^2(\tau) d\theta_2^2$$

- To parametrize de Sitter with U(1) x U(1) sym, recall:

$$d\Omega_3^2 = d\alpha^2 + \sin^2 \alpha d\theta_1^2 + \cos^2 \alpha d\theta_2^2 \qquad (0 \leq \alpha \leq \pi/2)$$

Torus solution



- Size of the two boundary circles:

$$r_1(\tau) = \frac{1}{\gamma}\left[\epsilon\sinh(\tau/\epsilon) + \sqrt{1+\gamma^2}\sqrt{\ell^2-\epsilon^2}\right]$$
$$r_2(\tau) = \epsilon\cosh(\tau/\epsilon)$$

- Meaning of parameters:

At $\tau = 0$ ("nucleation time"), $\dot{r}_2 = 0, r_2 = \epsilon$
(Many bubbles with size ϵ are nucleated along a circle with radius $r_1(0)$.)

$\gamma \to 0$: Late nucleation, $\gamma \to \infty$: Nucleation at the minimal S^3

- Both circles grows to infinite size (except when $\gamma \to \infty$).
Asymptotic aspect ratio: $r_2/r_1 = \gamma$

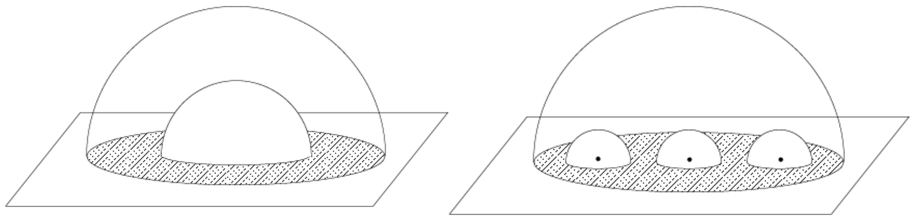
Summary of our analysis so far

- For the torus case, we have constructed explicit geometry.

- Boundary with any genus will occur.
  Interior geometry will be $ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma}$

  H/Γ : H^3 modded out by discrete element of isometry

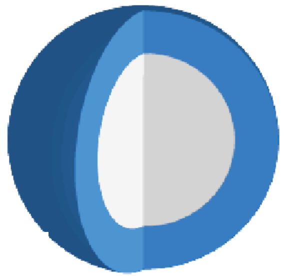
  Boundary of H/Γ can have arbitrary genus.
  (Krasnov, '00)



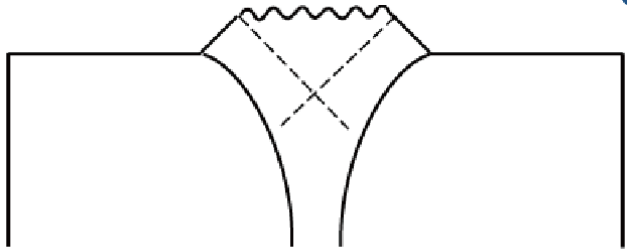
- A time-like observer can see the whole boundary (since orbifolding makes causal contact easier).

Multiple boundaries

- Two spherical boundaries? (Kodama, Maeda, Sasaki, Sato, '82)
  Initial condition: "Shell" of bubbles
  (for simplicity, assume spherical sym.)



- From Birkhoff's theorem,
  interior metric = Schwarzschild



- Singularity develops between two boundaries .
  (A time-like observer can see only one boundary.)

- Higher genus case: open question
  - Could there be multiple boundaries accessible to one observer? If there is, confusing in terms of dual theory.
    (Maldacena-Maoz, '04)

Implications for holographic duality

- Proposal for the dual theory (FSSY '06):
  - 2D gravity (Liouville field) coupled to a large number of matter ("super-critical")
  - (central charge) ~ (de Sitter entropy)
- We have to sum over genera of the base space, and integrate over the moduli (as in string perturbation).
  - We find peculiar behaviors compared to string theory.

Peculiarities in summing over topologies

- Moduli dependence for the torus case:
  - In the bulk, long thin torus is suppressed (we need many bubbles to produce it): $\sim \Gamma^{\tau_2} \quad (\tau_2 \equiv r_2/r_1)$
  - In super-critical string, there are "pseudo-tachyons". They seem to cause divergence at $\tau_2 \to \infty$
    (Aharony-Silverstein, Hellerman-Swanson, '06)

- Nature of the genus expansion:
  - "String coupling": We need at least two or three bubbles to increase genus by one: $g_s \sim \Gamma^k \quad (k = 2 \text{ or } 3)$
  - The series may converge. Consider sum over bubbles (of the same size): $\sum_n \Gamma^n e^{bn} \quad (b: \text{ order } 1)$

Conclusions

- Summary
  - There can be universe w/ arbitrary boundary genus.
  - There could be multiple boundaries. (Spherical case: separated by singularity.)
  - The dual theory should involve sum over genera of the base space.
- Open questions
  - Multiple boundaries (w/o singularity in bulk) exist?
  - Interpretation of the moduli dependence
  - Euclidean solution with non-trivial boundary topology?