2d Gauge/Bethe correspondence from String Theory

Susanne Reffert
CERN

based on work with D. Orlando and S. Hellerman
(arXiv:1106.2097, 1108.0644, 1111.4811)
The Gauge/Bethe Correspondence

Relates susy gauge theories in 2d/4d to (quantum) integrable systems.

The susy vacua of the gauge theory correspond to a sector of the Bethe spectrum of the spin chain.

Generators of chiral ring correspond to commuting Hamiltonians.

Integrable model: spectrum determined by Bethe equations.
Gauge theory: ground states determined by eff. twisted superpotential.

Correspondence works for all Bethe solvable integrable models.

Nekrasov, Shatashvili
The Gauge/Bethe Correspondence

What are the parameters of a spin chain?

- boundary conditions
- length of chain
- symmetry group
- inhomogeneities
- rank of symm. group
- representation
- Cartan
- number of particle species
- rapidities
- spectrum is given by solutions of
  \[ e^{2\pi i d Y(\lambda)} = 1 \]
  Yang counting fn (potential for Bethe equations)
The Gauge/Bethe Correspondence

General case: quiver gauge theories

What are the parameters?

- number of nodes
- bifundamental fields
- adjoint fields
- fundamental and antifundamental fields
- twisted masses
- gauge groups
- flavor groups

\[ \text{General case: quiver gauge theories} \]

\[ U(L_a) \]

\[ U(N_a) \]

\[ U(N_b) \]

\[ U(L_b) \]

\[ \frac{1}{2} \Lambda_k^a \pm \nu_k^{(a)} \]

\[ \frac{1}{2} \Lambda_k^b \pm \nu_k^{(b)} \]

\[ \frac{1}{2} C_{ab} \]

\[ \frac{1}{2} C_{bb} \]

\[ \Phi^a \]

\[ B_{ab} \]

\[ B_{ba} \]

\[ Q_k^a, \overline{Q}_k^a \]

\[ \hat{\theta}_a \]

\[ \text{The Gauge/Bethe Correspondence} \]
The Dictionary

<table>
<thead>
<tr>
<th>gauge theory</th>
<th>integrable model</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of nodes in the quiver</td>
<td>( r )</td>
</tr>
<tr>
<td>gauge group at ( a )-th node</td>
<td>( U(N_a) )</td>
</tr>
<tr>
<td>effective twisted superpotential</td>
<td>( \tilde{W}_{\text{eff}}(\sigma) )</td>
</tr>
<tr>
<td>equation for the vacua</td>
<td>( e^{2\pi i \tilde{W}_{\text{eff}}} = 1 )</td>
</tr>
<tr>
<td>flavor group at node ( a )</td>
<td>( U(L_a) )</td>
</tr>
<tr>
<td>lowest component of the twisted chiral superfield</td>
<td>( \sigma_i^{(a)} )</td>
</tr>
<tr>
<td>twisted mass of the fundamental field</td>
<td>( \tilde{m}_k^{(a)} )</td>
</tr>
<tr>
<td>twisted mass of the anti–fundamental field</td>
<td>( \tilde{m}_k^{(a)} )</td>
</tr>
<tr>
<td>twisted mass of the adjoint field</td>
<td>( \tilde{m}_{\text{adj}}^{(a)} )</td>
</tr>
<tr>
<td>twisted mass of the bifundamental field</td>
<td>( \tilde{m}_{\text{b}}^{(ab)} )</td>
</tr>
<tr>
<td>FI–term for ( U(1) )-factor of gauge group ( U(N_a) )</td>
<td>( \tau_a )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{i}{2} C^{aa} & \quad \Phi^a \\
\frac{i}{2} C^{ab} & \quad B^{ba} \\
\frac{i}{2} C^{bb} & \quad B^{ab}
\end{align*}
\]

\[
\begin{align*}
\frac{i}{2} \Lambda_k^a + \nu_k^{(a)} & \quad Q_k^a, Q_k^a \\
\frac{i}{2} \Lambda_k^b + \nu_k^{(b)} & \quad Q_k^b, Q_k^b
\end{align*}
\]

\[
\begin{align*}
U(N_a) & \quad U(N_b) \\
U(L_a) & \quad U(L_b)
\end{align*}
\]
For the integrable system, all $N$ magnon sectors must be considered together. This is usually not done for $U(N)$ gauge theories with different $N$.

Yet, there is evidence that it would make sense to do so. Study the simplest example of the spin $1/2$ XXX spin chain (Heisenberg model).

The low energy limit of the corresponding gauge theory is given by the NLSM with target space the cotangent bundle of $\text{Gr}(N,L)$.

$$\text{Gr}(N, L) = \{W \subset \mathbb{C}^L \mid \dim W = N\},$$

$$T^*\text{Gr}(N, L) = \{(X, W), W \in \text{Gr}(N, L), X \in \text{End}(\mathbb{C}^L) \mid X(\mathbb{C}^L) \subset W, X(W) = 0\}$$

Its ground states are given by the cohomology of $T^*\text{Gr}(N,L)$.
The Big Picture

Study the spectrum of $L=4$ spin 1/2 XXX spin chain:

The Gauge/Bethe correspondence relates gauge theories with different gauge groups!
A Brane Realization

Realize 2d gauge theories of the correspondence in string theory.

In the last configuration, the fundamental strings are charged under the enhanced symmetry $su(2)$ for coinciding D5 branes.

The symmetry of the integrable model becomes manifest in the D brane set up in the limit of coincident NS5 branes.
Twisted masses from bulk

How to turn on twisted masses in the gauge theory on the worldvolume of the D2 branes?

Answer: they are inherited from the background!

\[
\begin{align*}
4D & \quad \text{Wilson line b.c.} \\
\downarrow \text{reduction} & \quad \downarrow \text{effective theory} \\
3D & \quad \text{real mass} \\
\downarrow \text{effective theory} & \quad \downarrow \text{T-duality} \\
\text{D3–brane in fluxbrane} & = \Omega \text{ background} \\
\downarrow \text{effective theory} & \quad \downarrow \\
\text{D2–brane in fluxtrap} &
\end{align*}
\]
Gauge/Bethe correspondence relates susy gauge theories in 2d/4d to (quantum) integrable systems. Find/study string theory realization!

**Summary**

- **fluxbrane BG**
  - T duality
  - **fluxtrap BG**
    - + D2, NS5
    - 2d N=2 gauge theory
      - w. tw. masses
      - (2d Gauge/Bethe corr.)
    - + D4, NS5
    - 4d N=2 gauge theory
      - in Omega BG
      - (4d Gauge/Bethe corr.)
      - M Theory lift
Outline

* Introduction/Overview
  * Gauge/Bethe correspondence
* The Bulk
  * fluxbrane
  * fluxtrap
  * generalizations
* Branes
  * 2d gauge theory
  * D brane setup
  * SU(r) symmetry
  * SO(2r) symmetry
* Summary
The Bulk
The Fluxbrane Background

Use a flat bulk background with identifications:

$$\theta_1 \cong \theta_1 + 2\pi k_2,$$

$$\theta_2 \cong \theta_2 + 2\pi k_3,$$

$$x_8 = \tilde{R}\tilde{u},$$

$$\tilde{u} \cong \tilde{u} + 2\pi k_1,$$

$$\begin{cases}
\tilde{u} \cong \tilde{u} + 2\pi k_1, \\
\theta_1 \cong \theta_1 + 2\pi m\tilde{R}k_1, \\
\theta_2 \cong \theta_2 - 2\pi m\tilde{R}k_1,
\end{cases}$$

impose identifications

This corresponds to the well known Melvin or fluxbrane background.

To disentangle periodicities, introduce new coordinates:

$$\begin{cases}
\phi_1 = \theta_1 - m\tilde{R}\tilde{u}, \\
\phi_2 = \theta_2 + m\tilde{R}\tilde{u},
\end{cases}$$

fluxbrane parameter
The Fluxbrane Background

Write down metric:

\[
\tilde{g}_{\mu \nu} d\tilde{X}^\mu d\tilde{X}^\nu = dx_0^2 + \sum_{i=4}^{7} (dx_i + m V^i dx_8)^2 + dx_8^2 + dx_9^2
\]

\[
V^i \partial_i = -x^5 \partial_x + x^4 \partial_x + x^7 \partial_x - x^6 \partial_x = \partial_{\phi_1} - \partial_{\phi_2}
\]

This corresponds to the Omega deformation of flat space in the (4567) directions with \( \varepsilon_1 = -\varepsilon_2 = m \)

Locally, the metric is still flat, but some of the rotation symmetries are broken.

Eliminate degrees of freedom which are incompatible with the identifications via T duality.

Nekrasov, Okounkov
The Fluxtrap Background

To arrive at the fluxtrap background, we perform a T duality in the 8 direction.

The bulk fields after T duality are

\[ ds^2 = dx_0^2 + \ldots + 3 + \rho_1^2 + d\rho_1^2 + \rho_2^2 + \rho_2^2 d\phi_1^2 + \rho_2^2 d\phi_2^2 + \frac{-m^2 (\rho_1^2 d\phi_1 - \rho_2^2 d\phi_2)^2 + dx_8^2}{1 + m^2 (\rho_1^2 + \rho_2^2)} + dx_9^2, \]

\[ B = m \frac{\rho_1^2 d\phi_1 - \rho_2^2 d\phi_2}{1 + m^2 (\rho_1^2 + \rho_2^2)} \wedge dx_8, \]

\[ e^{-\Phi} = \sqrt{1 + m^2 (\rho_1^2 + \rho_2^2)} \frac{g_3^2 \sqrt{\alpha'}}{g_3^2 \sqrt{\alpha'}} \]

This background now only contains the physical degrees of freedom.
The Fluxtrap Background

Study supersymmetries preserved by fluxbrane/fluxtrap BG.

The Killing spinor for a flat BG in type IIB is

\[ K^{IIB} = \exp\left[ \frac{1}{2} \phi_1 \Gamma_{45} + \frac{1}{2} \phi_2 \Gamma_{67} \right] \exp\left[ \frac{m \tilde{R} u}{2} (\Gamma_{45} - \Gamma_{67}) \right] \epsilon_0 \]

In order for this to be preserved in the BG with identifications, we must additionally impose the projector

\[ \Pi_{\pm}^{\text{flux}} = \frac{1}{2} (\mathbb{1} \pm \Gamma_{4567}) \]

Half of the supersymmetries are left (16 real supercharges).

T dualize to type IIA:

\[ K^{IIA} = \epsilon_L + \epsilon_R \]

\[ \left\{ \begin{array}{l} 
\epsilon_L = e^{-\Phi/8} (1 + \Gamma_{11}) \Pi_{\text{flux}}^{\text{flux}} \exp\left[ \frac{1}{2} \phi_1 \Gamma_{45} + \frac{1}{2} \phi_2 \Gamma_{67} \right] \epsilon_0 , \\
\epsilon_R = e^{-\Phi/8} (1 - \Gamma_{11}) \Gamma_u \Pi_{\text{flux}}^{\text{flux}} \exp\left[ \frac{1}{2} \phi_1 \Gamma_{45} + \frac{1}{2} \phi_2 \Gamma_{67} \right] \epsilon_1 , 
\end{array} \right. \]

\[ \Gamma_u = \frac{m \rho_1}{\Delta} \Gamma_5 - \frac{m \rho_2}{\Delta} \Gamma_7 + \frac{1}{\Delta} \Gamma_8 \]  
Gamma matrix in u direction
It is also possible to construct more general fluxtrap backgrounds. So far, we used

\[ m_1 = -m_2 = m \in \mathbb{R} \]

It is however possible to construct backgrounds with

\[ m \in \mathbb{C} \text{ or } m_1 \neq -m_2 \]

1. Complex fluxtrap BG:

We need now two periodic variables giving rise to two shift parameters and perform two T dualities:

\[ \tilde{x}_8 = \tilde{R}_8 \tilde{u}, \quad \tilde{x}_9 = \tilde{R}_9 v \]

\[
\begin{align*}
\tilde{u} &\simeq \tilde{u} + 2\pi k_1, \\
\theta_1 &\simeq \theta_1 + 2\pi m_1 \tilde{R}_8 k_1, \\
\theta_2 &\simeq \theta_2 - 2\pi m_1 \tilde{R}_8 k_1,
\end{align*}
\]

\[
\begin{align*}
\tilde{v} &\simeq \tilde{v} + 2\pi k_2, \\
\theta_1 &\simeq \theta_1 + 2\pi m_2 \tilde{R}_9 k_2, \\
\theta_2 &\simeq \theta_2 - 2\pi m_2 \tilde{R}_9 k_2,
\end{align*}
\]

Preserves same amount of susy as real fluxtrap.
Corresponds to Omega BG with \( m = \frac{1}{2}(m_1 - im_2) \)
General Fluxtrap Backgrounds

2. Fluxtrap BG with \( m_1 + m_2 \neq 0 \):

For \( m_1, m_2 \) to be independent while still preserving some susy, we need to introduce identifications in another plane with a third identification parameter which fulfills

\[
m_1 + m_2 + m_3 = 0
\]

\[
\begin{cases}
\tilde{u} \simeq \tilde{u} + 2\pi k_1, \\
\theta_1 \simeq \theta_1 + 2\pi m_1 \tilde{R}_8 k_1, \\
\theta_2 \simeq \theta_2 + 2\pi m_2 \tilde{R}_8 k_1, \\
\theta_3 \simeq \theta_3 - 2\pi (m_1 + m_2) \tilde{R}_8 k_1.
\end{cases}
\]

Refined fluxtrap BG.

The refined fluxtrap preserves only half the susy of the fluxtrap (8 real supercharges)

It is possible in the same way to also construct a complex refined fluxtrap, however the possible brane configurations which can be realized in it are very limited.
Let's first discuss the properties of the $N=(2,2)$ gauge theories in 2d we want to realize on the branes:

Vector multiplet:

$$V = \theta^{-\bar{\theta}^-}(A_0 - A_1) + \theta^+\bar{\theta}^+(A_0 + A_1) - \theta^{-\bar{\theta}^+}\sigma - \theta^+\bar{\theta}^-\bar{\sigma}$$

$$+i\theta^-\theta^+(-\bar{\lambda}_- + \bar{\theta}^+\lambda_+) + i\bar{\theta}^+\bar{\theta}^-(-\theta^-\lambda_- + \theta^+\lambda_+) + \theta^-\bar{\theta}^+\bar{\theta}^-D$$

Chiral multiplet:

$$\Phi = \phi(y^\pm) + \theta^\alpha\psi_\alpha(y^\pm) + \theta^+\theta^-F(y^\pm)$$

$$y^\pm = x^\pm - i\theta^\pm\bar{\theta}^\pm$$

$$x^\pm = x_0 \pm x^1$$

Twisted chiral multiplet:

$$\Sigma = \sigma(\tilde{y}^\pm) + i\theta^+\bar{\lambda}_+(\tilde{y}^\pm) - i\bar{\theta}^-\lambda_-(\tilde{y}^\pm) + \theta^+\bar{\theta}^-[D(\tilde{y}^\pm) - iA_{01}(\tilde{y}^\pm)] + ...$$

$$A_{01} = \partial_0A_1 - \partial_1A_0 + [A_0, A_1]$$

$$\tilde{y}^\pm = x^\pm \mp i\theta^\pm\bar{\theta}^\pm$$
2d Gauge Theories

Action: D terms, F terms, twisted F terms

Twisted F term: $\int d^2 x \, d\bar{\theta} - d\theta^+ \, \tilde{W} \bigg|_{\theta^+ = \theta^- = 0} + \text{h.c.}$

Twisted superpotential

Kinetic term of action:

$$L_{\text{kin}} = \int d^4 \theta \left( \sum_k X_k^+ e^V X_k - \frac{1}{2e^2} \text{Tr}(\Sigma^\dagger \Sigma) \right),$$

Twisted masses:

$$L_{\text{tw}} = \int d^4 \theta \left( X^+ e^{\theta - \bar{\theta}^+} \tilde{m} X + \text{h.c.} \right)$$

Want to consider the Coulomb branch.

Calculate eff. action for slowly varying $\sigma$ fields.

Integrate out all massive matter fields.
2d Gauge Theories

Most general action (at most 4 fermions, 2 derivatives):

\[ S_{\text{eff}}(\Sigma) = - \int d^4\theta K_{\text{eff}}(\Sigma, \bar{\Sigma}) + \frac{1}{2} \int d^2\theta \tilde{W}_{\text{eff}}(\Sigma) + \text{h.c.} \]

Integrate out massive fields (S is quadratic in Q)

\[ e^{iS_{\text{eff}}(\Sigma)} = \int DQ e^{iS(\Sigma, Q)} \]

This calculation is exact (protected by supersymmetry).

\[ \tilde{W}_{\text{eff}}(\Sigma) = \frac{1}{2\pi} (\Sigma - \bar{m}_Q) (\log(\Sigma - \bar{m}_Q) - 1) - i\tau \Sigma \]

Vacuum equation:

\[ \exp \left[ 2\pi \frac{\partial \tilde{W}_{\text{eff}}(\sigma)}{\partial \sigma_i} \right] = 1 \]
2d Gauge Theories

We can construct $N=2$ gauge theories in 2d by studying the low energy theory on the worldvolume of D2 branes suspended between NS5 branes.

<table>
<thead>
<tr>
<th>direction</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS5</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluxtrap</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>m</td>
<td>−m</td>
<td>o</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D4</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Separation of NS5s in 3 direction: FI term
Separation of NS5s in 2 direction: $1/g^2$
2d Gauge Theories

Why is the fluxtrap called a fluxtrap?
In the static embedding, \( x^0 = \zeta^0, \ x^1 = \zeta^1, \ x^2 = \zeta^3 \) the e.o.m. are solved for the D2 branes sitting in \( x_3 = x_4 = x_5 = x_6 = x_7 = 0 \).
The D2s are trapped at the origin.

Adding only D2 branes to the fluxtrap preserves 8 supercharges (static embedding).
Adding also NS5 branes preserves 4 supercharges, N=(2,2)

\[
\begin{align*}
\epsilon_L &= e^{-\Phi/8} (1 + \Gamma_{11}) \Pi_+^{NS5} \Pi_-^{flux} \Gamma_{1208} \exp[\frac{1}{2} (\phi_1 + \phi_2) \Gamma_{67}] \epsilon, \\
\epsilon_R &= e^{-\Phi/8} (1 - \Gamma_{11}) \Gamma_u \Pi_+^{NS5} \Pi_-^{flux} \exp[\frac{1}{2} (\phi_1 + \phi_2) \Gamma_{67}] \epsilon.
\end{align*}
\]

\[\Pi_\pm^{NS5} = \frac{1}{2} (1 \pm \Gamma_{2345})\]
2d Gauge Theories

The fluxtrap deformation gives rise to the twisted masses! Start with (kappa fixed) DBI action (democratic formulation):

\[ S = -\mu_2 \int d^3 \zeta \, e^{-\Phi} \sqrt{-\det(g_{\alpha\beta} + B_{\alpha\beta})} \left[ 1 - \frac{1}{2} \bar{\psi} \left( (g + B)^{\alpha\beta} \Gamma_{\beta} D_{\alpha} + \Delta^{(1)} \right) \psi \right] \]

\[ D_{\alpha} = \partial_{\alpha} X^{\mu} \left( \nabla_{\mu} + \frac{1}{8} H_{\mu mn} \Gamma^{mn} \right), \]

\[ \Delta^{(1)} = \frac{1}{2} \Gamma^m \partial_m \Phi - \frac{1}{24} H_{mnp} \Gamma^{mnp} \]

After expanding to quadratic order in the fields, we get

\[ S = -\frac{1}{8\pi^2 g_3^2 (\alpha')^2} \int d^3 \zeta \left[ -\dot{X}^\sigma \dot{X}_\sigma + m^2 (\rho_1^2 + \rho_2^2) + \bar{\psi} \Gamma_{0} \psi + \frac{m}{2} \bar{\psi} (\Gamma_{45} - \Gamma_{67}) \Gamma_{8} \psi \right] + \ldots \]
An important ingredient of the Gauge/Bethe correspondence is the symmetry group of the integrable system, which also relates gauge theories with different gauge groups.

The example with two NS5 branes treated so far corresponds to the simplest case with symmetry group $\text{su}(2)$.

Spin chains can have any Lie group as symmetry, even supergroups. Can we realize all those via a brane construction?

So far, we are able to reproduce the A and D series.
**SU(r) Quiver Gauge Theories**

An SU(r) quiver gauge theory corresponds to a spin chain with SU(r) symmetry. Such a theory can be constructed by varying the brane set up.

![Diagram](attachment:diagram.png)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>fluxbrane</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>NS5</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mathcal{N} = (2,2))</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mathcal{N} = (1,1))</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**SU(r) Gauge Theories**

$r+1$ NS5 with stacks of D2s suspended in between.

- Adjoint fields
- Bifundamental fields
- Fundamentals and antifundamentals
- Bifundamental fields
**$SO(2r)$ Quiver Gauge Theories**

$SO(2r)$ quiver gauge theories can be constructed from $SU(r)$ theories with a further variation.

The quivers shown above represent the construction:

- **(a)** Represents a quiver with $r-1$ nodes, where nodes 1 and $r$ are joined by a diagonal line, indicating a fluxbrane action.
- **(b)** Represents a quiver with $N_r$ nodes, where nodes $N_1$ and $N_r$ are joined by a diagonal line, indicating a fluxbrane action.

### Table

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>fluxbrane</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>NS5</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO5</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{N} = (2,2)$</td>
<td>D4</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{N} = (1,1)$</td>
<td>D4'</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**SO(2r) Gauge Theories**

NO5: S dual of a D5 coincident with an O5, here

**Preserves same supersymmetries as NS5**

6d theory on NO5 has SO(2) symmetry: +,− charged D2s

\[
U(N_{r-1} + N_r) \rightarrow U(N_{r-1}) \times U(N_r)
\]

Seni Kapustini, Hanany, Zaffaroni

Fluxbrane identifications are compatible with orbifold action.
Summary
Summary

We choose a flat background with identifications (fluxbrane/Melvin BG). After T Duality, it turns into a fluxtrap background.

By placing D2 branes suspended between NS5 branes into the background, we arrive at an N=(2,2) gauge theory in 2d, with twisted masses, as studied in the Gauge/Bethe correspondence.

The symmetry group of the corresponding integrable system is encoded in the brane configuration.

By instead placing D4 branes into the fluxtrap, we get an N=2 gauge theory in 4d in the Omega BG (4d Gauge/Bethe). This configuration can be lifted to M Theory.
Summary

Gauge/Bethe correspondence relates susy gauge theories in 2d/4d to (quantum) integrable systems.

Find/study string theory realization!

- fluxbrane BG
  - T duality
  - fluxtrap BG
    - + D2, NS5
      - 2d N=2 gauge theory w. tw. masses
        - (2d Gauge/Bethe corr.)
    - + D4, NS5
      - 4d N=2 gauge theory in Omega BG
        - (4d Gauge/Bethe corr.)
  - M Theory lift
Thank you for your attention!