Conformal/supersymmetric interfaces for string theory

part I

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Based on

Y.S., JHEP 1203 (2011) 072 [arXiv: 1112.5935]

1. Introduction

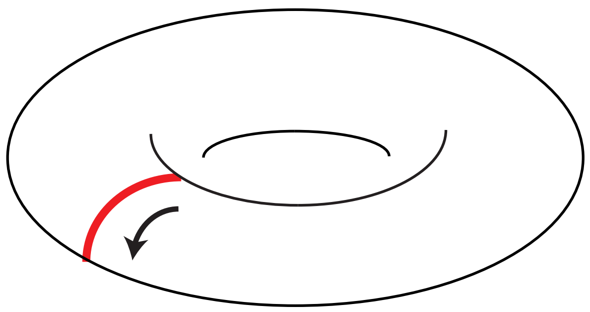
☆ Conformal (world-sheet) interface :
natural extension of conformal boundary

boundary | interface (defect)

cond. mat. w/ boundary | cond. mat. w/defect

D-brane | ??

- originated from
  - condensed matter w/ defect [Won-Affleck '94]
  - twisted partition fn. [Petkova-Zuber '00]



- may play an interesting role in
  - condensed matter phys.
  - conformal field theory (CFT)
  - string theory

In fact,

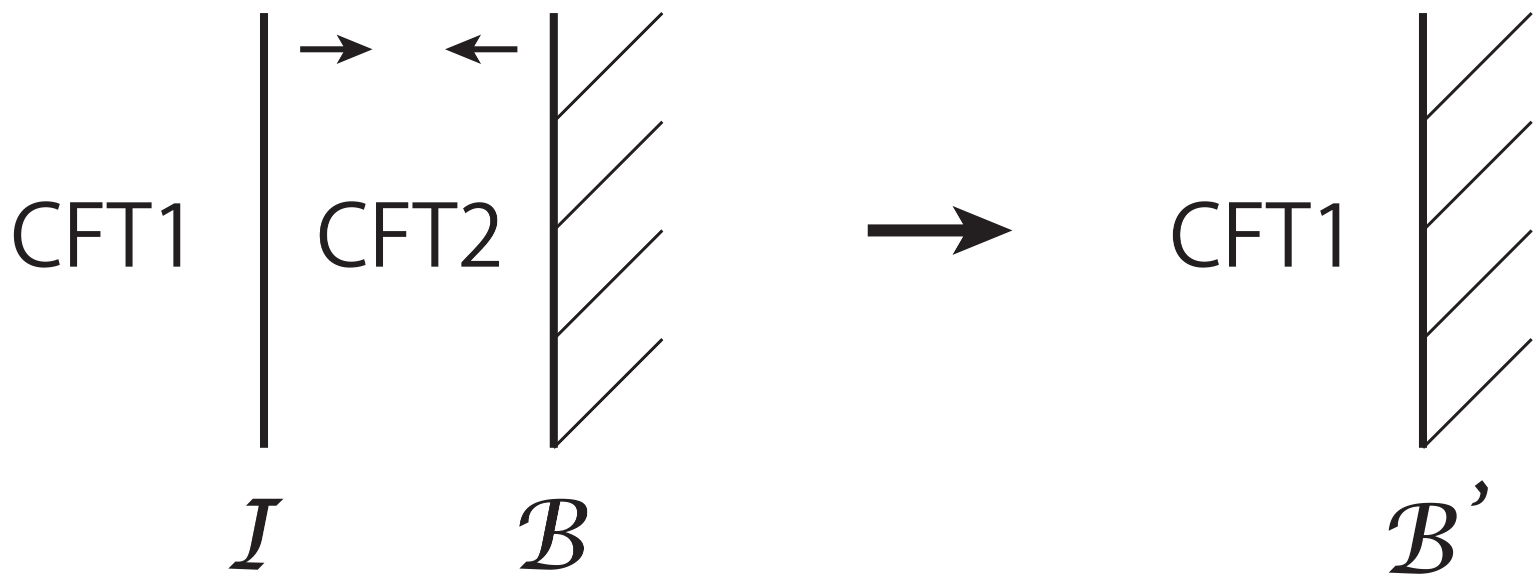
- interesting properties have been found

- permeable interfaces [Bachas–de Boer–Dijkgraaf–Ooguri '01]
- transform set of D–branes to another [Graham–Watts '03]
- "generator" of RG flow [Graham–Watts '03] [cf. Gaiotto '12]
- "generator" of symmetry (duality) of RCFT
  [Frohlich–Fuchs–Runkel–Scweigert '04, '07]
- target–space interpretation as "bi–brane"
  in G x G (WZNW model) [Fuchs–Schweigert–Waldorf '07]
- fusion of interfaces [Bachas–Brunner '07]
  ≈ spectrum generating "algebra" of string theory?
  [cf. Geroch group, U–duality group]

⋮

Transformation of D-branes [Graham-Watts '03, ...]

- interfaces transform
  a set of conformal boundaries/D-branes to another



non-perturbative trans. in string theory

cf. target space geometry of D-branes

- flat space-time : hyperplane $\mathbb{R}^{1,p} \subset \mathbb{R}^{1,9}$

- group manifold G : conjugacy class

$$\mathcal{B}_h = \left\{ g \;\middle|\; \exists x \in G : g = xhx^{-1} \right\}$$

- Calabi–Yau manifold :
  - holomorphic cycle
  - special Lagrangian submanifold

Target space interpretation

[Fuchs-Schweigert-Waldorf '07]

- world-sheet interface in WZNW model
  - not domain wall in target G (group mfd)
  - but "bi-brane" or "bi-conjugacy class" in G x G

$$\mathcal{I}_{h_1,h_2} = \left\{ (g_1, g_2) \,\middle|\, \exists x, y \in G : g_1 = x h_1 y^{-1}, g_2 = x h_2 y^{-1} \right\}$$

However,

- interfaces : not fully understood,
  especially, in string theory

- generally, only 1 Virasoro is preserved
  $\Longrightarrow$ ghost problem if embedded in string theory?
  [Bachas–de Boer–Dijkgraaf–Ooguri '01]

- conformal interface in full string
  world-sheet had not been discussed

Motivation of this work is simple :

- to construct interface in string theory [fixed genus]
- to study its basic properties
- to address issue of ghost, if possible

To avoid subtleties concerning ghosts, work w/ Green-Schwarz formulation in light-cone gauge

2. Results

- obtained 2 classes of susy (≈ conformal) interfaces for type II GS strings in flat spacetime
  - factorized D–branes
  - "topological" interfaces
- properties :
  - generate T–duality/Fourier–Mukai trans.
  - interpreted as a submanifold in doubled target space (bi–brane)
  - transform (rotate) D–branes
- correspond to those preserving 2 Virasoros

  $\Rightarrow$ evade ghost problem

3. Outlook

- NSR formulation ? $\Longrightarrow$ [Bachas–Brunner–Roggenkamp '12]

- more general interfaces ? $\Longrightarrow$ probably, No [Bachas et al.]

- rich algebra among interfaces when compactified

  $\Longrightarrow$ monoid (semi–group) extension of $O(d,d\,|\,\mathbb{Q})$

  [Bachas et al.]

- double field theory ?

- symmetry of string theory ?

  applications ?

$\vdots$

Conformal/supersymmetric interfaces for string theory

part II

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Plan

1. Introduction (part I)
2. Conformal interfaces
3. Supersymmetric boundary states for GS strings
4. Supersymmetric interfaces for string theory
5. Properties
6. Summary

2. Conformal interfaces

World-sheet conformal interface

- consider 1 dim. defect/interface in 2dim. world-sheet
- condition to keep conformal Ward id.

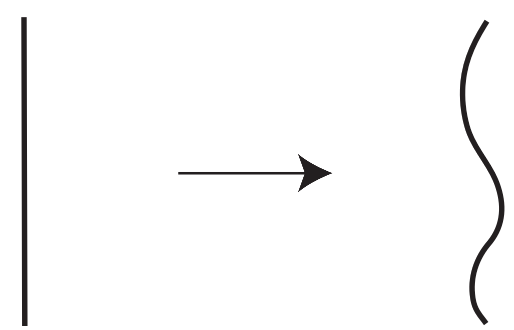
$$T_1(z) - \tilde{T}_1(\bar{z}) \approx T_2(z) - \tilde{T}_2(\bar{z})$$

[along interface]

- when $T_1(z) \approx T_2(z)\,, \quad \tilde{T}_1(\bar{z}) \approx \tilde{T}_2(\bar{z})$

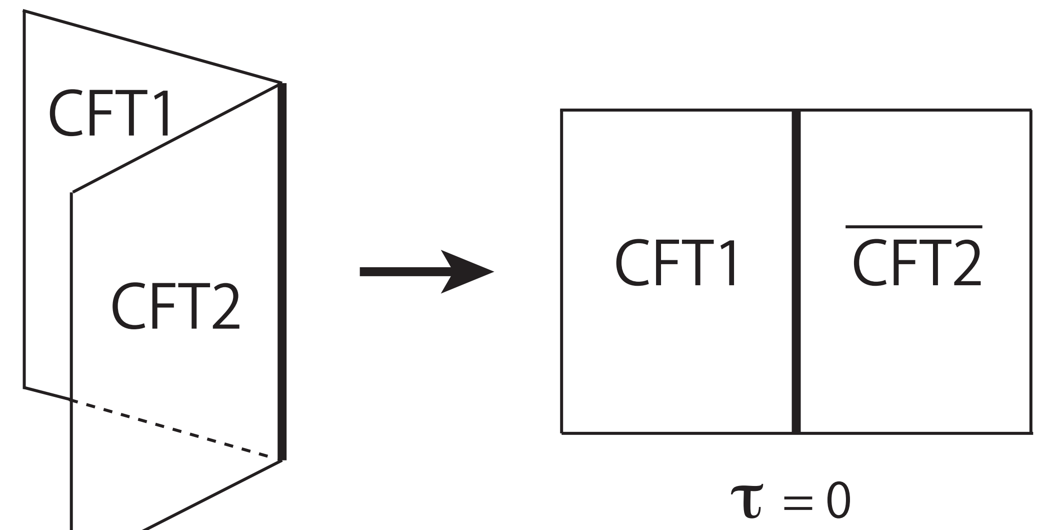
$\Rightarrow$ ▪ interface : freely deformed

▪ called topological interface



(Un)folding trick

- a way to construct interface : unfold conformal boundary



$[\ \bar{}\ : \tau \to -\tau\ ]$

$(\alpha^2_n, \tilde{\alpha}^2_n) \to (-\tilde{\alpha}^2_{-n}, -\alpha^2_{-n})$

- take b.d. state in CFT1 $\otimes$ CFT2

$$|\mathcal{B}\rangle = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \otimes |\mathcal{B}_j\rangle_2$$

s.t. $(L^1_n + L^2_n - \tilde{L}^1_{-n} - \tilde{L}^2_{-n})|\mathcal{B}\rangle = 0$

then, $\mathcal{I} = \sum_{i,j} c_{ij} |\mathcal{B}_i\rangle_1 \cdot {}_2\overline{\langle\mathcal{B}_j|}$

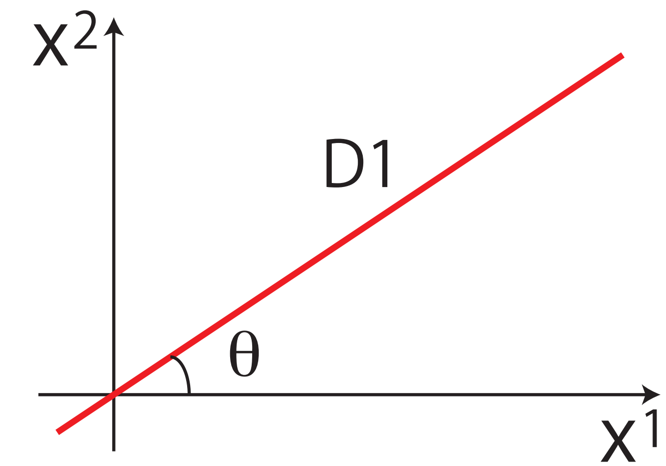
satisfies $(L^1_n - \tilde{L}^1_{-n})\mathcal{I} = \mathcal{I}(L^2_n - \tilde{L}^2_{-n})$ $\Leftarrow$ interface

Permeable inerface

[Bachas–de Boer–Dijkgraaf–Ooguri ’01]

- unfolding D–brane in c=2 theory $\Rightarrow$ “permeable” interface

$$|\mathcal{B}\rangle = \mathcal{C}\prod_{n=1}\exp\left[\frac{1}{n}M_{ij}\alpha^i_{-n}\tilde{\alpha}^j_{-n}\right]|\mathcal{B}\rangle_0$$



e.g. $$M_{ij} = \begin{pmatrix} -\cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

  - boundary cond. $(\alpha^i_n - M_{ij}\tilde{\alpha}^j_{-n})|\mathcal{B}\rangle = 0$

$$\Rightarrow \mathcal{I} = \mathcal{C}\prod_{n=1}\exp\left[\frac{1}{n}\left(M_{11}\alpha^1_{-n}\tilde{\alpha}^1_{-n} - M_{12}\alpha^1_{-n}\alpha^2_n - M_{21}\tilde{\alpha}^2_n\tilde{\alpha}^1_{-n} + M_{22}\tilde{\alpha}^2_n\alpha^2_n\right)\right]\mathcal{I}_0$$

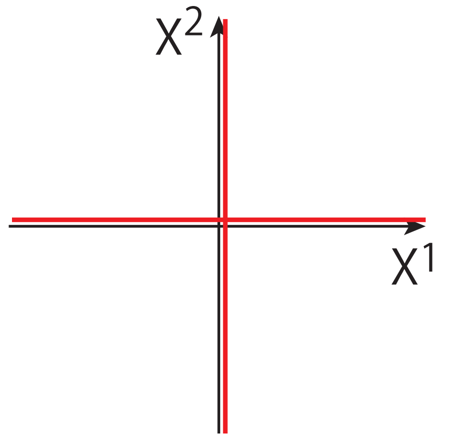
  - $(\alpha^1_{-n},\tilde{\alpha}^1_{-n}),(\alpha^2_n,\tilde{\alpha}^2_n)$ act from left and right, respectively

e.g. $$\exp\left[\alpha^1_{-n}\alpha^2_n\right]\cdot\mathcal{I}_0 = \sum_l \frac{1}{l!}(\alpha^1_{-n})^l\cdot\mathcal{I}_0\cdot(\alpha^2_n)^l$$

- when $\theta = \pi k/2 \quad (k \in \mathbb{Z})$

$$\alpha^i_n \pm \tilde{\alpha}^i_{-n} \approx 0\,, \quad L^i_n - \tilde{L}^i_{-n} \approx 0 \quad [\, i = 1, 2\,]$$

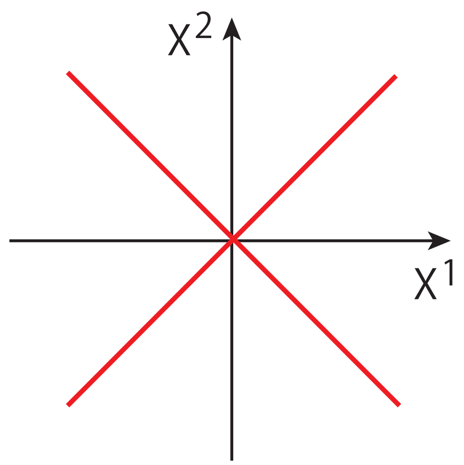
$\Rightarrow$ factorized D-branes $\quad \mathcal{I} \sim |\mathcal{B}_1\rangle \cdot \langle \mathcal{B}_2|$



- when $\theta = \pi(1 + 2k)/4 \quad (k \in \mathbb{Z})$

$$\alpha^1_n \approx \pm\alpha^2_n\,, \quad \tilde{\alpha}^1_n \approx \pm\tilde{\alpha}^2_n$$
$$L^1_n \approx L^2_n\,, \quad \tilde{L}^1_n \approx \tilde{L}^2_n$$

$\Rightarrow$ topological



- generic θ : “permeable”

$$\alpha^1_n + \cos 2\theta \cdot \tilde{\alpha}^1_{-n} - \sin 2\theta \cdot \alpha^2_n \approx 0$$
$$\tilde{\alpha}^2_{-n} - \sin 2\theta \cdot \tilde{\alpha}^1_{-n} + \cos 2\theta \cdot \alpha^2_n \approx 0$$

Generating renormalization group flow

[Graham-Watts '03]

- another way to view conformal interface :
  turn on relevant operator only on one side

$$\mathrm{CFT}_{\mathrm{UV}} \; \Big| \; \mathrm{CFT}_{\mathrm{UV}} \quad \longrightarrow \quad \mathrm{CFT}_{\mathrm{UV}} \; \Big| \; \mathrm{CFT}_{\mathrm{IR}}$$

$$\int_{\tau>0} d^2x \, \Phi_{\text{relevant}}$$

[different central charge]

- Gaiotto constructed RG interface [non-rational/Cardy]

[Gaiotto '12]

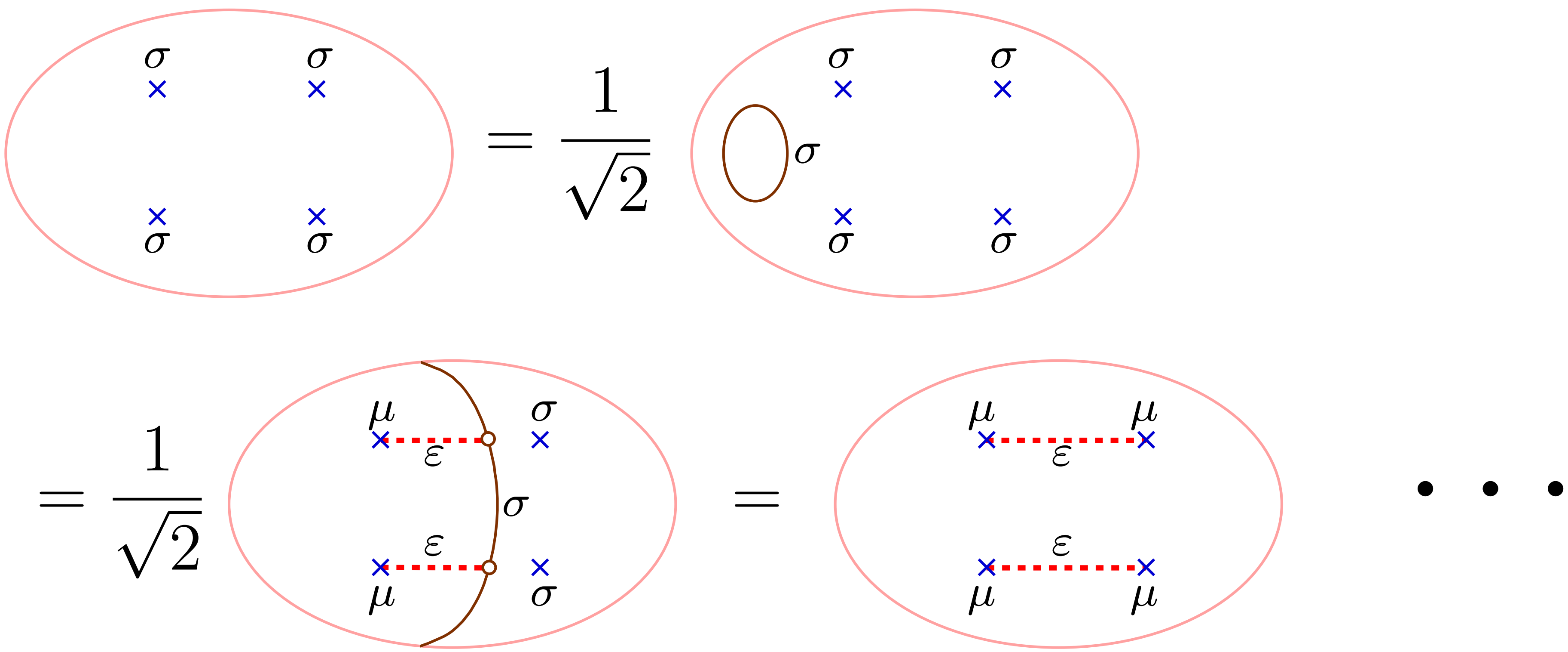
$$\mathcal{M}_{p+1,p} \;\to\; \mathcal{M}_{p,p-1} \,, \qquad \Phi_{\text{relevant}} = \phi_{1,3}$$

Generating symmetry of RCFT

[Frohlich−Fuchs−Runkel−Scweigert ’04, ’07]

- topological interfaces generate symm. of RCFT

e.g.) order/disorder duality in Ising CFT



[from Frohlich et al ’04]

Target space interpretation [Fuchs-Schweigert-Waldorf '07]

- world-sheet interface in WZNW model
  - not domain wall in target G
  - but "bi-brane" or "bi-conjugacy class" in G x G

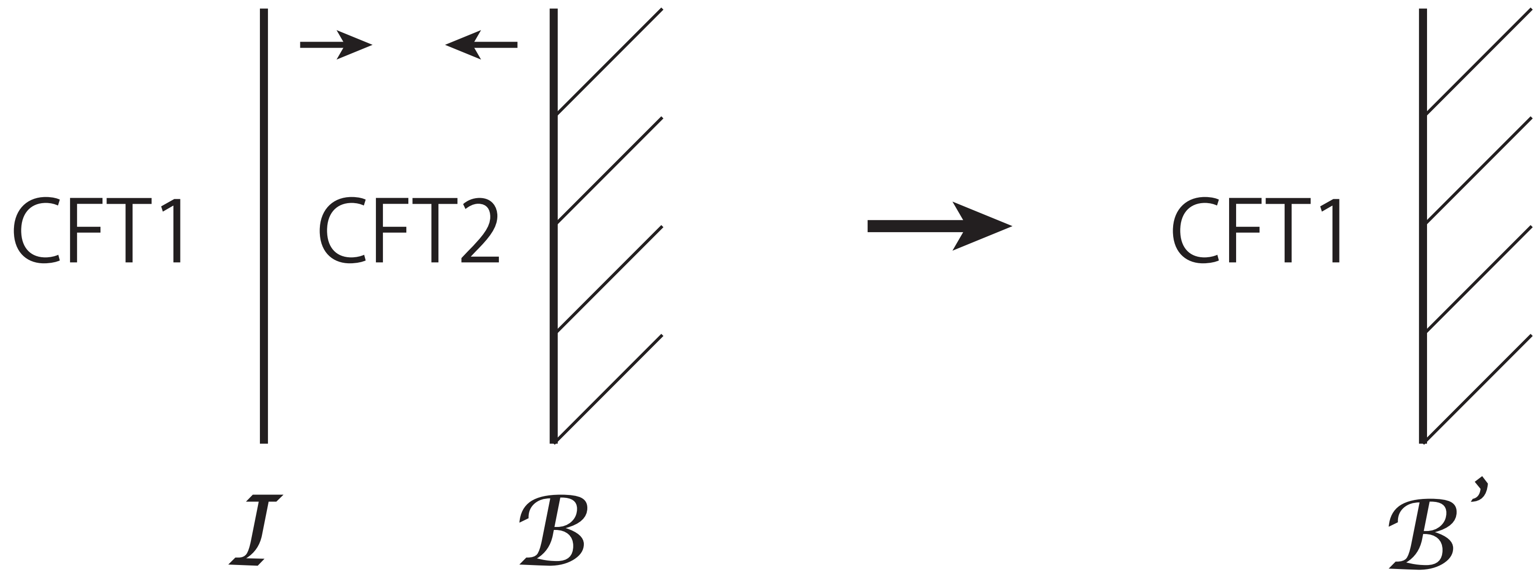
$$\mathcal{I}_{h_1,h_2} = \left\{ (g_1, g_2) \mid \exists x, y \in G : g_1 = x h_1 y^{-1}, g_2 = x h_2 y^{-1} \right\}$$

[cf. D-brane in G : conjugacy class]

$$\mathcal{B}_h = \left\{ g \mid \exists x \in G : g = x h x^{-1} \right\}$$

Transformation of D-branes [Graham-Watts '03, ...]

- interfaces transform a set of D-branes to another



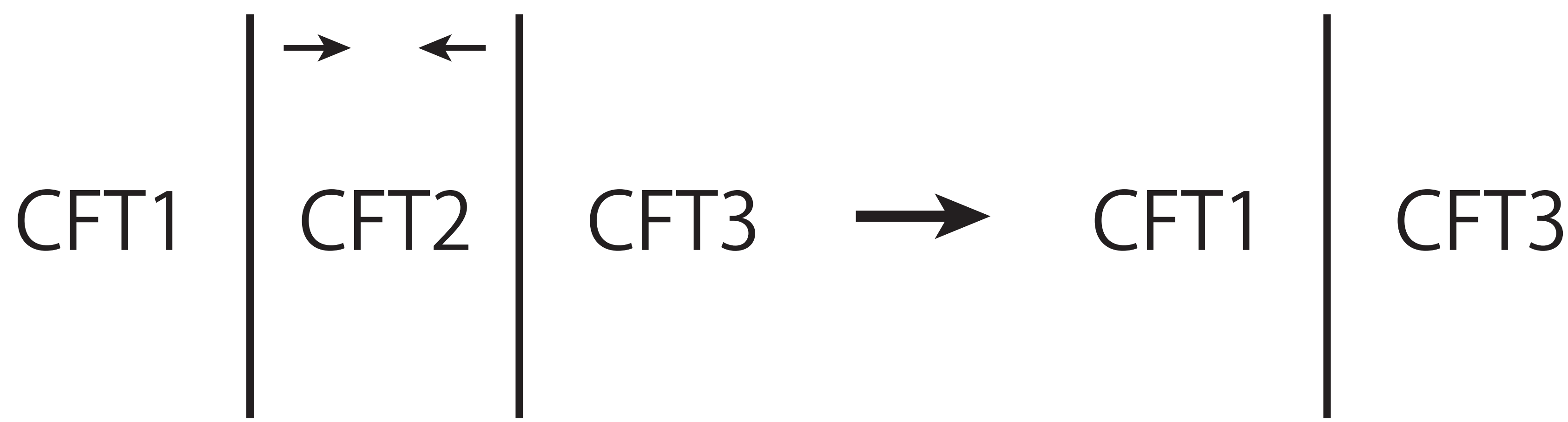
non-perturbative trans. in string theory

<u>Fusion of interfaces</u> [Bachas-Brunner '07]

- conformal boundary (D-brane)

  ≈ interface w/ one side being empty

- interfaces fuse to a new interface



- form "algebra"
- spectrum generating algebra of string theory?

  [cf. Geroch group, U-duality group]

3. Supersymmetric boundary states for GS string

Type II GS string

- in the following, we work w/ type II GS string
  in flat spacetime in light-cone gauge

- notation (IIB):

$\alpha_n^I,\ \tilde{\alpha}_n^I\ (I = 1, \ldots, 8)$: right, left bosonic modes

$S_n^a,\ \tilde{S}_n^a\ (a = 1, \ldots, 8)$: $so(8)$ spinor modes

$\gamma^I = \begin{pmatrix} 0 & \sigma^I \\ \bar{\sigma}^I & 0 \end{pmatrix}$: 8-dim. gamma matrices

- supercharges

  linear susy $\quad Q^a := \sqrt{2p^+} S_0^a$

  non-linear susy $\quad Q^{\dot{a}} := \frac{1}{\sqrt{p^+}} \sigma^I_{a\dot{a}} \sum_n S^a_{-n} \alpha^I_n$

$$\{Q^a, Q^b\} = 2p^+ \delta^{ab}\,,\ \{Q^a, Q^{\dot{a}}\} = \sqrt{2}\alpha^I_0 \sigma^I_{a\dot{a}}\,,\ \{Q^{\dot{a}}, Q^{\dot{b}}\} = P^- \delta^{\dot{a}\dot{b}}$$

  and $\quad \tilde{Q}^a,\ \tilde{Q}^{\dot{a}}$

- spinor zero-modes, e.g.,

$$S_0^a |\dot{a}\rangle_R = \frac{1}{\sqrt{2}} \bar{\sigma}^I_{\dot{a}a} |I\rangle_R\,, \quad S_0^a |I\rangle_R = \frac{1}{\sqrt{2}} \sigma^I_{a\dot{a}} |\dot{a}\rangle_R$$

D-branes [Green-Gutperle '96]

- GS string in l.c.-gauge: not a CFT
- Conformal boundary states

$\Rightarrow$ space-time susy boundary states s.t.

$$(Q^a + iM_{ab}\tilde{Q}^b)|\mathcal{B}\rangle = (Q^{\dot{a}} + iM_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}})|\mathcal{B}\rangle = 0$$

$$\Rightarrow |\mathcal{B}(M)\rangle = \mathcal{C}\prod_{n=1} \exp\left[\frac{1}{n}M_{IJ}\alpha^I_{-n}\tilde{\alpha}^J_{-n} - iM_{ab}S^a_{-n}\tilde{S}^b_{-n}\right]|\mathcal{B}\rangle_{\mathrm{b0}}|\mathcal{B}\rangle_{\mathrm{f0}}$$

$$\begin{aligned} |\mathcal{B}\rangle_{\mathrm{b0}} &= \sum |k^I, k^K M_{KJ}\rangle \\ |\mathcal{B}\rangle_{\mathrm{f0}} &= M_{IJ}|I\rangle_R|J\rangle_L - iM_{\dot{a}\dot{b}}|\dot{a}\rangle_R|\dot{b}\rangle_L \end{aligned}$$

$$M_{KL} = \left(e^{\omega_{IJ}\Sigma^{IJ}}\right)_{KL},\ M_{\alpha\beta} = \left(e^{\frac{1}{2}\omega_{IJ}\gamma^{IJ}}\right)_{\alpha\beta} = \begin{pmatrix} M_{ab} & 0 \\ 0 & M_{\dot{a}\dot{b}} \end{pmatrix}$$

- b.d. cond. $(\alpha^I_n - M_{IJ}\tilde{\alpha}^J_{-n})|\mathcal{B}\rangle = (S^a_n + iM_{ab}\tilde{S}^b_{-n})|\mathcal{B}\rangle = 0$

- simple Dp–brane [(p+1)–instanton]

$$M_{IJ} = \begin{pmatrix} -\mathbf{1}_{p+1} & 0 \\ 0 & \mathbf{1}_{7-p} \end{pmatrix}, \quad M_{\alpha\beta} = \gamma^1 \cdots \gamma^{p+1}$$

- IIA case : $\tilde{S}^a_n \rightarrow \tilde{S}^{\dot{a}}_n$ etc.

- In GS, "conformal" $\Rightarrow$ "space–time supersymmetric"

Conformal interface $\Longrightarrow$ susy interface

4. Susy interface for GS string

- we would like to find susy interface defined by

$$(Q_1^a + iR^1_{ab}\tilde{Q}_1^b)\mathcal{I} = \mathcal{I}(Q_2^a + iR^2_{ab}\tilde{Q}_2^b)$$
$$(Q_1^{\dot{a}} + iR^1_{\dot{a}\dot{b}}\tilde{Q}_1^{\dot{b}})\mathcal{I} = \mathcal{I}(Q_2^{\dot{a}} + iR^2_{\dot{a}\dot{b}}\tilde{Q}_2^{\dot{b}})$$

W. S. 1 | W. S. 2

for some $R^A_{ab}\,, R^A_{\dot{a}\dot{b}}\quad (A=1,2)$ [IIB case]

- for this,
  - prepare boundary states w/ “doubled” fields

$$(\alpha_n^{AI}, \tilde{\alpha}_n^{AI})\,,\ (S_n^{Aa}, \tilde{S}_n^{Aa})$$

[just as an intermediate step]

  - unfold this to interface

- we then obtain

$$\mathcal{I} = \mathcal{C}\,\mathcal{I}_{\mathsf{b}} \cdot \mathcal{I}_{\mathsf{f}}$$

$$\mathcal{I}_{\mathsf{b}} = \prod_{n=1} \exp\left[\frac{1}{n}\beta^{AI}_{-n} \cdot \mathcal{S}^{IJ}_{AB} \cdot \tilde{\beta}^{BJ}_{-n}\right] \cdot \mathcal{I}_{\mathsf{b0}}$$

$$\mathcal{I}_{\mathsf{f}} = \prod_{n=1} \exp\left[-iT^{Aa}_{-n} * \mathcal{S}^{ab}_{AB} * \tilde{T}^{Bb}_{-n}\right] \cdot \mathcal{I}_{\mathsf{f0}}$$

$$\beta^{AI}_{n} := (\alpha^{1I}_{n}, -\tilde{\alpha}^{2I}_{-n})\,, \quad T^{Aa}_{n} := (S^{1a}_{n}, -\tilde{S}^{2a}_{-n}) \quad \text{etc.}$$

$$\mathcal{I}_{\mathsf{b0}} = \sum |k^{1I}, k^{BI'}\mathcal{S}^{I'J}_{B1}\rangle_1 \cdot {}_2\langle -k^{2K}, -k^{BI'}\mathcal{S}^{I'L}_{B2}|$$

$$\mathcal{I}_{\mathsf{f0}} = M_{ijkl}|i\rangle_{1R}|j\rangle_{1L} \cdot {}_{2L}\langle k|_{2R}\langle l|\,, \qquad i = (I, \dot{a})$$

$$U_A * V_A := \eta_{AB} U_A V_B\,, \quad \eta_{AB} = \mathrm{diag}(+1, -1)$$

- need to determine $R^A,\ \mathcal{S}_{AB}\,,\ M_{ijkl}$ by susy cond.

Condition for linearly realized susy

- only zero-modes are relevant

$$0 = M_{\dot{a}JK\dot{d}}\bar{\sigma}^{I}_{\dot{a}a} + iR^{1}_{ab}M_{I\dot{b}K\dot{d}}\bar{\sigma}^{J}_{\dot{b}b} - M_{IJKL}\sigma^{L}_{a\dot{d}} + iR^{2}_{ab}M_{IJ\dot{c}\dot{d}}\bar{\sigma}^{K}_{\dot{c}b}$$

$$0 = M_{\dot{a}J\dot{c}L}\bar{\sigma}^{I}_{\dot{a}a} + iR^{1}_{ab}M_{I\dot{b}\dot{c}L}\bar{\sigma}^{J}_{\dot{b}b} - M_{IJ\dot{c}\dot{d}}\bar{\sigma}^{L}_{\dot{d}a} - iR^{2}_{ab}M_{IJKL}\sigma^{K}_{b\dot{c}}$$

$$0 = M_{\dot{a}\dot{b}KL}\bar{\sigma}^{I}_{\dot{a}a} + iR^{1}_{ab}M_{IJKL}\sigma^{J}_{b\dot{b}} - M_{I\dot{b}K\dot{d}}\bar{\sigma}^{L}_{\dot{d}a} - iR^{2}_{ab}M_{I\dot{b}\dot{c}L}\bar{\sigma}^{K}_{\dot{c}b}$$

$$0 = M_{IJKL}\sigma^{I}_{a\dot{a}} - iR^{1}_{ab}M_{\dot{a}\dot{b}KL}\bar{\sigma}^{J}_{\dot{b}b} - M_{\dot{a}JK\dot{d}}\bar{\sigma}^{L}_{\dot{d}a} - iR^{2}_{ab}M_{\dot{a}J\dot{c}L}\bar{\sigma}^{K}_{\dot{c}b}$$

$$0 = M_{I\dot{b}\dot{c}L}\sigma^{I}_{a\dot{a}} - iR^{1}_{ab}M_{\dot{a}J\dot{c}L}\sigma^{J}_{b\dot{b}} - M_{\dot{a}\dot{b}\dot{c}\dot{d}}\bar{\sigma}^{L}_{\dot{d}a} - iR^{2}_{ab}M_{\dot{a}\dot{b}KL}\sigma^{K}_{b\dot{c}}$$

$$0 = M_{I\dot{b}K\dot{d}}\sigma^{I}_{a\dot{a}} - iR^{1}_{ab}M_{\dot{a}JK\dot{d}}\sigma^{J}_{b\dot{b}} - M_{\dot{a}\dot{b}KL}\sigma^{L}_{a\dot{d}} + iR^{2}_{ab}M_{\dot{a}\dot{b}\dot{c}\dot{d}}\bar{\sigma}^{K}_{\dot{c}b}$$

$$0 = M_{IJ\dot{c}\dot{d}}\sigma^{I}_{a\dot{a}} - iR^{1}_{ab}M_{\dot{a}\dot{b}\dot{c}\dot{d}}\bar{\sigma}^{J}_{\dot{b}b} - M_{\dot{a}J\dot{c}L}\sigma^{L}_{a\dot{d}} + iR^{2}_{ab}M_{\dot{a}JK\dot{d}}\sigma^{K}_{b\dot{c}}$$

$$0 = M_{\dot{a}\dot{b}\dot{c}\dot{d}}\bar{\sigma}^{I}_{\dot{a}a} + iR^{1}_{ab}M_{IJ\dot{c}\dot{d}}\sigma^{J}_{b\dot{b}} - M_{I\dot{b}\dot{c}L}\sigma^{L}_{a\dot{d}} + iR^{2}_{ab}M_{I\dot{b}K\dot{d}}\sigma^{K}_{b\dot{c}}$$

- simple solutions :

(1) $$R^A_{ab} = M^A_{ab}$$

$$M^{\mathrm{FD}}_{ijkl} = N^1_{ij} N^2_{kl}$$

$$N^2_{KL} = M^2_{LK}\,,\ N^2_{\dot{c}\dot{d}} = -iM^2_{\dot{d}\dot{c}}$$

$$N^1_{IJ} = M^1_{IJ}\,,\ N^1_{\dot{a}\dot{b}} = -iM^1_{\dot{a}\dot{b}}$$

(2) $$R^A_{ab} = M^A_{ab}$$

$$M^{\mathrm{TP}}_{ijkl} = N^{\mathrm{id}}_{il} N^{\mathrm{rot}}_{jk}$$

$$N^{\mathrm{id}}_{IL} = \delta_{IL}\,,\ N^{\mathrm{id}}_{\dot{a}\dot{d}} = \delta_{\dot{a}\dot{d}}$$

$$N^{\mathrm{rot}}_{JK} = M^1_{IJ} M^2_{IK}\,,\ N^{\mathrm{rot}}_{\dot{b}\dot{c}} = M^1_{\dot{a}\dot{b}} M^2_{\dot{a}\dot{c}}$$

Sufficient condition for non-linearly realized susy

- after some algebras

$$0 = \sigma^I_{a\dot{a}} \mathcal{S}^{IJ}_{12} - i\sigma^J_{b\dot{a}} \mathcal{S}^{ab}_{12} \,, \qquad 0 = \sigma^I_{a\dot{a}} \mathcal{S}^{ab}_{12} + i\sigma^J_{b\dot{a}} \mathcal{S}^{IJ}_{12}$$

$$0 = \sigma^I_{a\dot{a}} \mathcal{S}^{IJ}_{11} - \sigma^J_{b\dot{b}} R^1_{\dot{a}\dot{b}} \mathcal{S}^{ab}_{11} \,, \qquad 0 = \sigma^I_{a\dot{a}} \mathcal{S}^{ab}_{11} - \sigma^J_{b\dot{b}} R^1_{\dot{a}\dot{b}} \mathcal{S}^{IJ}_{11}$$

$$0 = \sigma^I_{a\dot{a}} \mathcal{S}^{JI}_{22} - \sigma^J_{b\dot{b}} R^2_{\dot{a}\dot{b}} \mathcal{S}^{ba}_{22} \,, \qquad 0 = \sigma^I_{a\dot{a}} \mathcal{S}^{ba}_{22} - \sigma^J_{b\dot{b}} R^2_{\dot{a}\dot{b}} \mathcal{S}^{JI}_{22}$$

$$0 = \sigma^I_{a\dot{b}} R^1_{\dot{a}\dot{b}} \mathcal{S}^{JI}_{21} + i\sigma^J_{b\dot{b}} R^2_{\dot{a}\dot{b}} \mathcal{S}^{ba}_{21} \,, \qquad 0 = \sigma^I_{a\dot{b}} R^1_{\dot{a}\dot{b}} \mathcal{S}^{ba}_{21} - i\sigma^J_{b\dot{b}} R^2_{\dot{a}\dot{b}} \mathcal{S}^{JI}_{21}$$

- simple solutions :

$$R^A_{\dot{a}\dot{b}} \;=\; M^A_{\dot{a}\dot{b}}$$

$$\mathcal{S}^{IJ}_{AB} = \begin{pmatrix} a_{11} M^1_{IJ} & a_{12}\delta_{IJ} \\ a_{21} M^1_{KJ} M^2_{KI} & a_{22} M^2_{JI} \end{pmatrix}, \quad \mathcal{S}^{ab}_{AB} = \begin{pmatrix} a_{11} M^1_{ab} & -i a_{12}\delta_{ab} \\ i a_{21} M^1_{cb} M^2_{ca} & a_{22} M^2_{ba} \end{pmatrix}$$

a_{AB} : orthogonal

- consistency : same form of continuity cond. for zero, and non-zero modes

$$a_{AB} \ :\Rightarrow \quad a^{\mathrm{FD}}_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{or} \quad a^{\mathrm{TP}}_{AB} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

- find 2 classes of interfaces

factorized D-branes

$$\mathcal{I}(M^1, M^2) = \mathcal{C} \prod_{n=1} e^{\frac{1}{n} M^1_{IJ} \alpha^{1I}_{-n} \tilde{\alpha}^{1J}_{-n} - i M^1_{ab} S^{1a}_{-n} \tilde{S}^{1b}_{-n}} \cdot \mathcal{I}_{\mathrm{b0}} \mathcal{I}_{\mathrm{f0}}$$

$$\times \prod_{n=1} e^{\frac{1}{n} M^2_{IJ} \alpha^{2I}_{n} \tilde{\alpha}^{2J}_{n} + i M^2_{ab} S^{2a}_{n} \tilde{S}^{2b}_{n}}$$

$$\mathcal{I}_{\mathrm{b0}} = \sum |k^{1I}, \tilde{k}^{1J}\rangle\langle \tilde{k}^{2K}, k^{2L}|, \qquad k^{AI} = M^A_{IJ} \tilde{k}^{AJ}$$

$$\mathcal{I}_{\mathrm{f0}} = \left(M^1_{IJ} |I\rangle|J\rangle - i M^1_{\dot{a}\dot{b}} |\dot{a}\rangle|\dot{b}\rangle\right)\left(\langle K|\langle L| M^2_{LK} - i M^2_{\dot{d}\dot{c}} \langle \dot{c}|\langle \dot{d}|\right)$$

$$\Rightarrow \quad 0 \approx Q^a_A + i M^A_{ab} \tilde{Q}^b_A, \qquad 0 \approx Q^{\dot{a}}_A + i M^A_{\dot{a}\dot{b}} \tilde{Q}^{\dot{a}}_A$$

$$0 \approx S^{Aa} + i M^A_{ab} \tilde{S}^{Ab}, \qquad 0 \approx \partial_- X^{AI} - M^A_{IJ} \partial_+ \tilde{X}^{AJ}$$

$$0 \approx T_A - \tilde{T}_A$$

$$\Rightarrow \quad \mathcal{I} \sim |\mathcal{B}_1\rangle\langle\mathcal{B}_2|$$ "factorized"

"topological" interface

$$\mathcal{I}(M^1, M^2) \;=\; \mathcal{C} \prod_{n=1} e^{\frac{1}{n}(\alpha^{1I}_{-n}\alpha^{2I}_{n} + M^1_{KI}M^2_{KJ}\tilde{\alpha}^{1I}_{-n}\tilde{\alpha}^{2J}_{n})} \times e^{S^{1a}_{-n}S^{2a}_{n} + M^1_{ca}M^2_{cb}\tilde{S}^{1a}_{-n}\tilde{S}^{2b}_{n}} \cdot \mathcal{I}_{\mathsf{b0}}\mathcal{I}_{\mathsf{f0}}$$

$$\mathcal{I}_{\mathsf{b0}} = \sum |k^{1I}, \tilde{k}^{1J}\rangle\langle\tilde{k}^{2K}, k^{2L}|$$

$$k^{1I} = k^{2I}\,, \qquad M^1_{IJ}\tilde{k}^{1J} = M^2_{IJ}\tilde{k}^{2J}$$

$$\mathcal{I}_{\mathsf{f0}} = |I\rangle T_{\mathcal{I}}\langle I| + |\dot{a}\rangle T_{\mathcal{I}}\langle\dot{a}|$$

$$T_{\mathcal{I}} = M^1_{PJ}M^2_{PK}|J\rangle\langle K| + M^1_{\dot{p}\dot{b}}M^2_{\dot{p}\dot{c}}|\dot{b}\rangle\langle\dot{c}|$$

$$\Rightarrow \qquad
\begin{aligned}
Q^a_1 &\approx Q^a_2\,, & M^1_{ab}\tilde{Q}^b_1 &\approx M^2_{ab}\tilde{Q}^b_2 \\
Q^{\dot{a}}_1 &\approx Q^{\dot{a}}_2\,, & M^1_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}_1 &\approx M^2_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}_2 \\
S^{1a} &\approx S^{2a}\,, & M^1_{ab}\tilde{S}^{1b} &\approx M^2_{ab}\tilde{S}^{2b} \\
\partial_- X^{1I} &\approx \partial_- X^{2I}\,, & M^1_{IJ}\partial_+ X^{1J} &\approx M^2_{IJ}\partial_+ X^{2J}
\end{aligned}$$

$$T_1 \approx T_2 \qquad\qquad \tilde{T}_1 \approx \tilde{T}_2$$

"topological"

5. Properties

- in the following, we
  - concentrate on topological, un–compactified case
  - set for simplicity

$$M^A_{IJ} = \begin{pmatrix} -\mathbf{1}_{p_A+1} & 0 \\ 0 & \mathbf{1}_{7-p_A} \end{pmatrix} \qquad (p_1 < p_2)$$

$$\Rightarrow \qquad \mathcal{I}_{\mathsf{b0}}(Y) = \int \frac{d^8 k}{(2\pi)^8}\, e^{-ik\cdot Y} |k\rangle\langle k| \prod_{J=1}^{p_2+1} 2\pi\delta(k^J)$$

Target space geometry

- target space geometry is probed by localized states

$$|x\rangle = \int d^8 k \, e^{-ik\cdot x} |k\rangle$$

- then,

$$\langle x| \mathcal{I}_{\mathsf{b0}}(Y)\, |x'\rangle \;\sim\; \prod_{I=p_2+2}^{8} \delta(x_I - x'_I - Y^I)$$

$\Rightarrow$ interface : localized in a submanifold (bi-brane)

$$x = x' + Y$$

in common Dirichlet directions

in doubled (transverse) target space $\mathbb{R}^8 \times \mathbb{R}^8 \ni (x, x')$

Coupling through interfaces

- consider massless NS–NS fields $|\zeta\rangle\!\rangle := \zeta_{IJ}|I\rangle|J\rangle$
- coupling is read off from

$$\langle\!\langle\zeta|\,\mathcal{I}_{\mathrm{f0}}\,|\zeta'\rangle\!\rangle = \zeta^*_{IJ} M_{IJKL} \zeta'_{LK}$$

- when $p_1 + 1 = p_2 =: p$ (IIB–IIA)

$$-\zeta^*_{p+1\,p+1}\zeta'_{p+1\,p+1} + \zeta^*_{(p+1\,I)}\zeta'_{[p+1\,I]} + \zeta^*_{[p+1\,I]}\zeta'_{(p+1\,I)} + \sum_{I,J\neq p+1}\zeta^*_{IJ}\zeta'_{IJ}$$

$\Longrightarrow$ nothing but Buscher rules

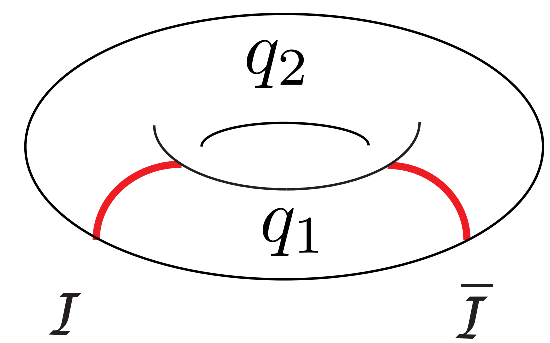
$$g'_{p+1\,p+1} = \frac{1}{g_{p+1\,p+1}}\,, \quad g'_{p+1\,I} = \frac{b_{p+1\,I}}{g_{p+1\,p+1}}\,, \quad b'_{p+1\,I} = \frac{g_{p+1\,I}}{g_{p+1\,p+1}}$$

$$\phi' = \phi - \frac{1}{2}\log g_{p+1\,p+1}$$

Partition fn. w/ interfaces inserted

[cf. Bachas–Brunner '07, Sakai–YS '08]

- one can compute partition fn. w/ interfaces inserted



- e.g., insert $\mathcal{I}$ and its conjugate $\overline{\mathcal{I}}$

$$\Rightarrow \quad Z = \mathrm{tr}\Big(\mathcal{I} q_2^{L_0^2+\tilde{L}_0^2} \overline{\mathcal{I}} q_1^{L_0^1+\tilde{L}_0^1}\Big) = \mathcal{C}^2 Z_{\mathsf{b0}} Z_{\mathsf{f0}} Z_{\mathsf{b}}^{\mathsf{osc}} Z_{\mathsf{f}}^{\mathsf{osc}}$$

$$\big(Z_{\mathsf{b}}^{\mathsf{osc}}\big)^{-1} = Z_{\mathsf{f}}^{\mathsf{osc}} = \prod_{n=1} (1 - q_1^n q_2^n)^{16}$$

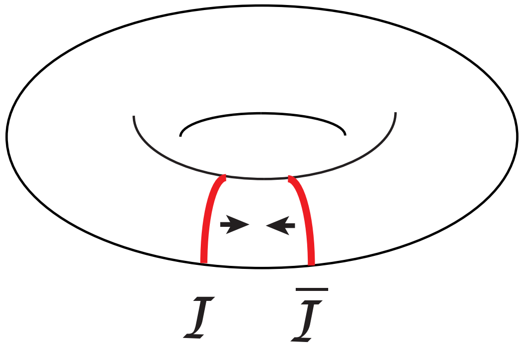
$$Z_{\mathsf{b0}} = V^{p_2+9}(\pi\sqrt{2t})^{p_2-7}\,,\ Z_{\mathsf{f0}} = (\delta_{II} - \delta_{\dot{a}\dot{a}})^2 = 0$$

$$[\, q_1 q_2 = e^{-2\pi t}\,]$$

- when q_1 (or q_2) $\to 1$, $\mathcal{I}$ and $\overline{\mathcal{I}}$ are fused

  $\Rightarrow$ modes coupled to $\mathcal{I}$ are read off

  $\Rightarrow$ same as D-brane



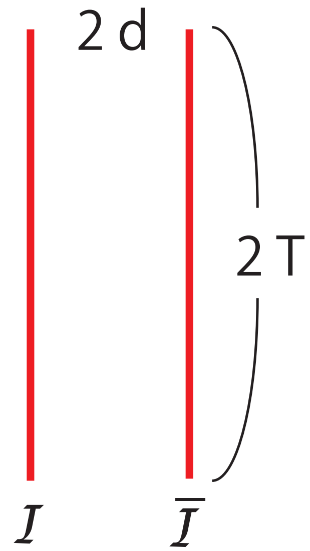
- $q_2 \to 0$ and $q_1 = e^{-2\pi d/T} \to 1$

[Bachas-de Boer-Dijkgraaf-Ooguri '01]

  $\Rightarrow$ Casimir energy between $\mathcal{I}$ and $\overline{\mathcal{I}}$

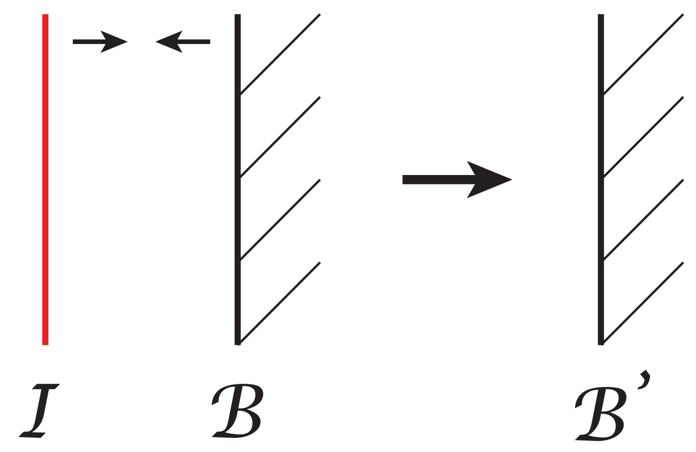
$$\mathcal{E} = \lim_{T\to\infty} \frac{-1}{2T} \log Z = \mathcal{E}_{\mathsf{b}} + \mathcal{E}_{\mathsf{f}}$$

$$\mathcal{E}_{\mathsf{b}} = -\mathcal{E}_{\mathsf{f}} = 0$$



- other pairs
  - susy is broken
  - $\mathcal{E} \neq 0$

Transformation of D-branes



- interface transform D-branes as

$$|\mathcal{B}'(M')\rangle = \lim_{q\to 1} \mathcal{I}(M^1, M^2)\, q^{L_0^2+\tilde{L}_0^2} |\mathcal{B}(M)\rangle$$

- in the present case, results in SO(8) trans.

$$M' = M(M^2)^{\mathsf{T}} M^1$$

6. Summary

- conformal interfaces : (expected to)
  play an interesting role in CFT, string theory, ...

- we have constructed [fixed genus]
  susy (≈ conformal) interfaces for type II GS string

- they
  - generate T−duality (Buscher rules)
  - are interpreted as a submanifold
    in doubled target space (bi−brane)
  - transform (rotate) D−branes

- partition fn. w/ interfaces inserted
    - coupling of massive modes
    - Casimir energy

- our interfaces correspond to
  those preserving 2 Virasoros

  $\Rightarrow$ evade ghost problem

Future directions

- NSR formulation ? $\Rightarrow$ [Bachas–Brunner–Roggenkamp '12]
- more general interfaces ?

  $\Rightarrow$ probably, No [Bachas et al.]
- richer algebras among interfaces when compactified

  $\Rightarrow$ monoid (semi–group) extension of $O(d, d \,|\, \mathbb{Q})$

  [Bachas et al.]
- double field theory ?
- symmetry of string theory ?

  applications ?

$\vdots$