Imperial College London

Brane Tiling Mutations and beyond

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Kavli IPMU, 5th September 2012

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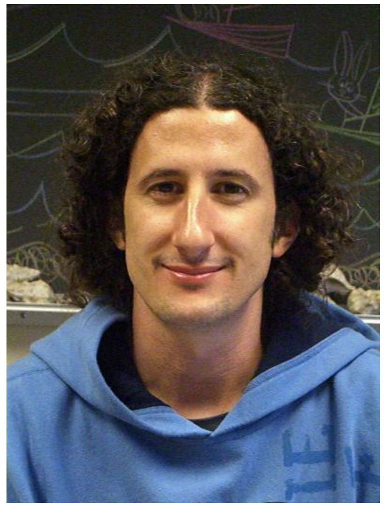
London

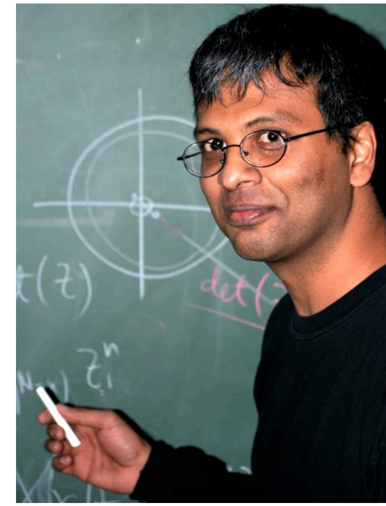
based on work with

Amihay Hanany



Sebastian Franco

Sanjaye Ramgoolam



Vishnu Jejjala

Brane Tilings and Specular Duality [hep-th/1206.2386]

Brane Tilings and Reflexive Polygons [hep-th/1201.2614]

Calabi-Yau Orbifolds and Torus Coverings [hep-th/1105.3471]

What are we interested in?

- study the moduli space of supersymmetric gauge theories
- moduli spaces are parameterized by VEV of gauge invariant quantities
- moduli spaces can have intricate characteristics: Calabi-Yau spaces, orbifolds, complete intersections …
- moduli spaces are everywhere:
  - *N=1 theories:* brane tilings, SQCD, …
  - *N=2 theories:* M2 brane theories, Instantons, …

What are we interested in?

Physics

- *gauge invariants* = polynomials; *relations* among gauge invariants
- can assign *global charges* to gauge invariants, e.g. R-charges
- *partition function* of spectrum of gauge invariant quantities

Mathematics

- moduli space is an algebraic variety
- graded coordinate ring of the algebraic variety (graded by charge weights)
- spectrum of graded coordinate ring has a Hilbert series

Motivations

- *N=1* theories with product gauge groups play a central role

SUSY Standard Model (e.g. MSSM)

AdS/CFT correspondence (orbifolds, generalized conifold…)

- naturally arise as worldvolume theories of D-brane probes of Calabi-Yau singularities

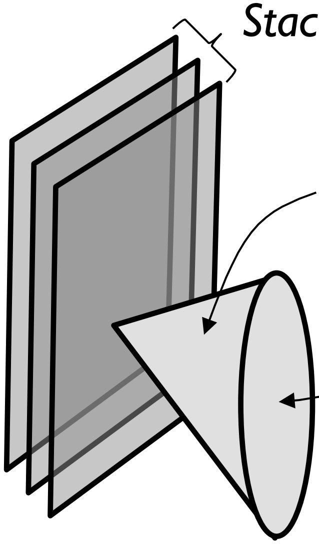
open strings between branes give bifundamentals
with product gauge groups

↓

Quivers & Superpotentials

Branes and SCFTs

4d supersymmetric gauge theories with *N=1* SUSY



worldvolume theory of a stack of *N* D3 branes probing singular CY

9+1d Type IIB on $\mathrm{AdS}_5 \times \mathrm{SE}^5$

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○						
CY3					○	○	○	○	○	○

Marriage

Geometry

Calabi-Yau cone C(H)
over
Sasaki-Einstein base H

Gauge Theory

4d N=1
Lagrangian fixed by
quiver (field content) &
superpotential

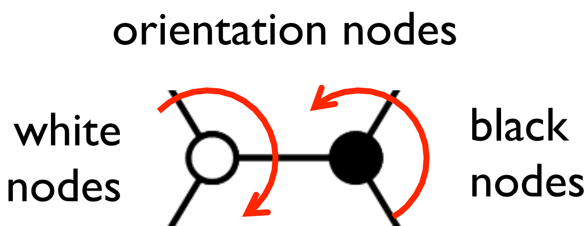
Brane Tiling
(Dimer)

Recent Developments

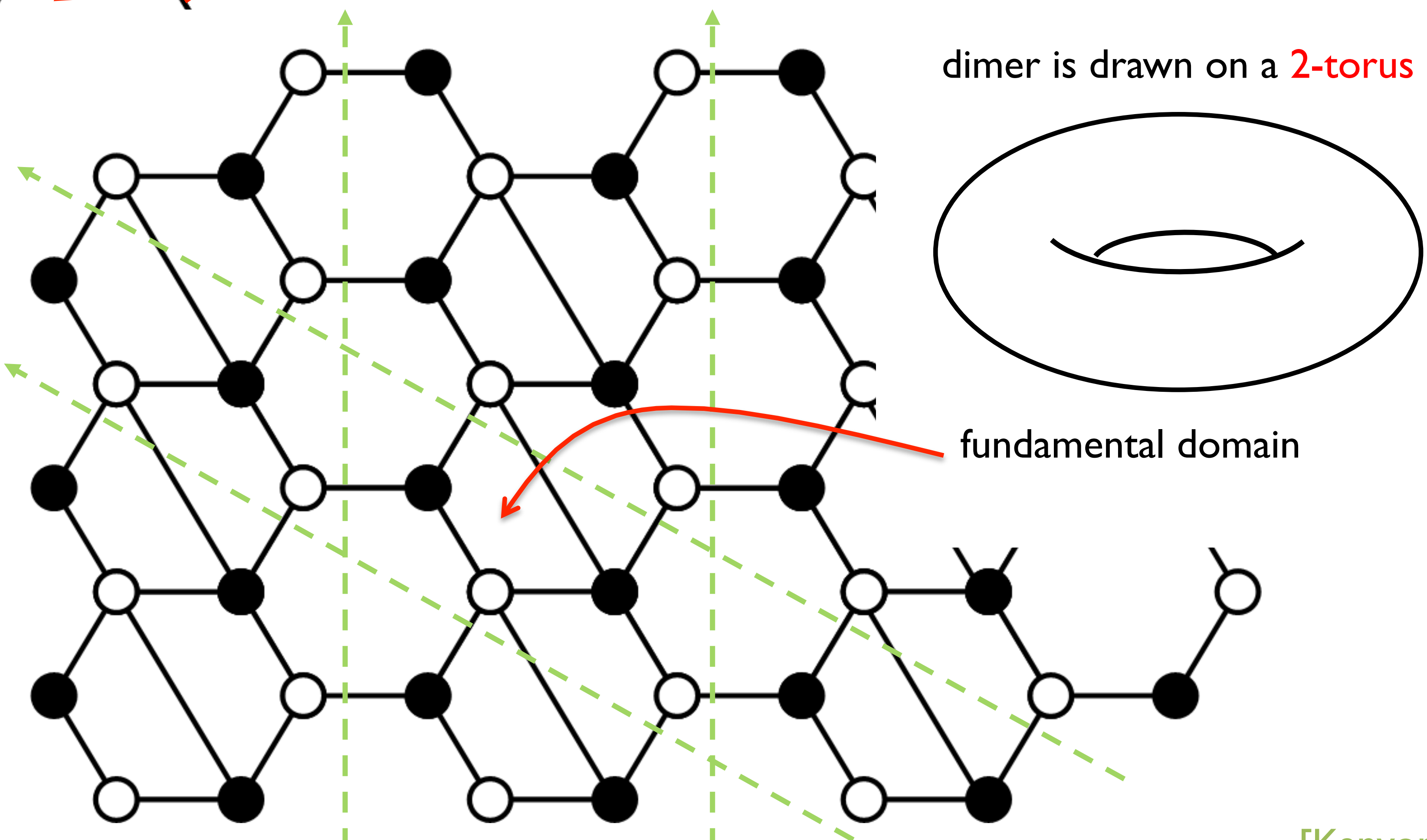
Recent developments on Brane Tilings

- Quantum Integrable Systems — [Franco 2012] [Goncharov, Kenyon 2011]
- 4d Superconformal Index — [Yamazaki, Xie 2012] [Eager, Schmude, Tachikawa 2012]
- BPS Quivers, Spectral Networks — [Vafa, Cecotti 2011, 2012] [Gaiotto, Moore, Neitzke 2012]
- Scattering Amplitudes — [Arkani-Hamed, Cachazo]

Dimers



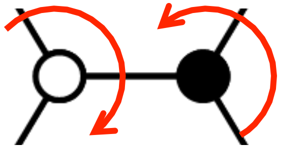
- an edge connects always a white and a black node
- the number of white and black nodes in the fundamental domain is the same



dimer is drawn on a 2-torus

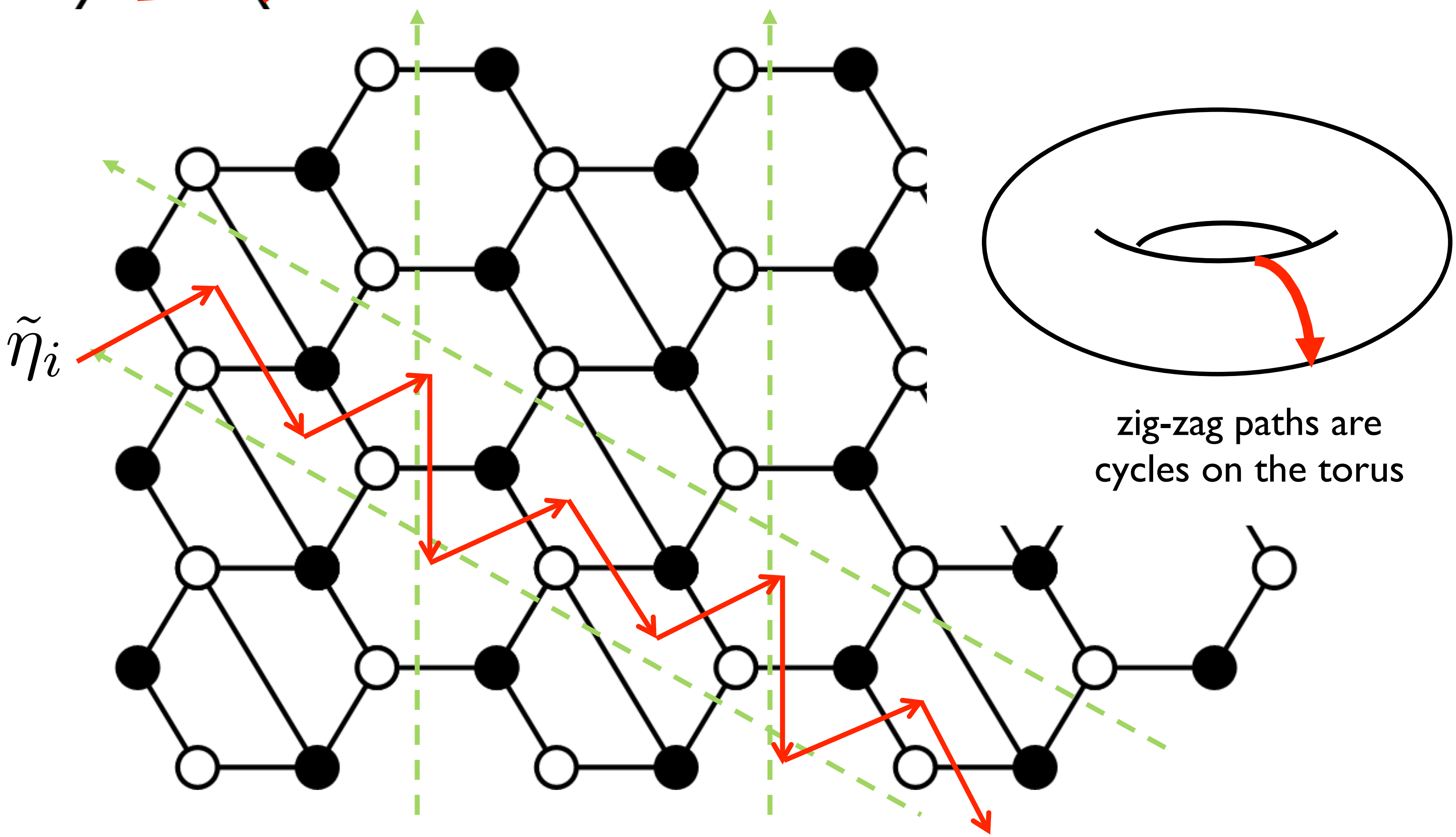
[Kenyon]

Special Paths on a Dimer



zig-zag paths

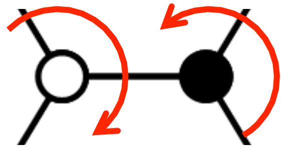
follow node orientation alternating between white and black nodes

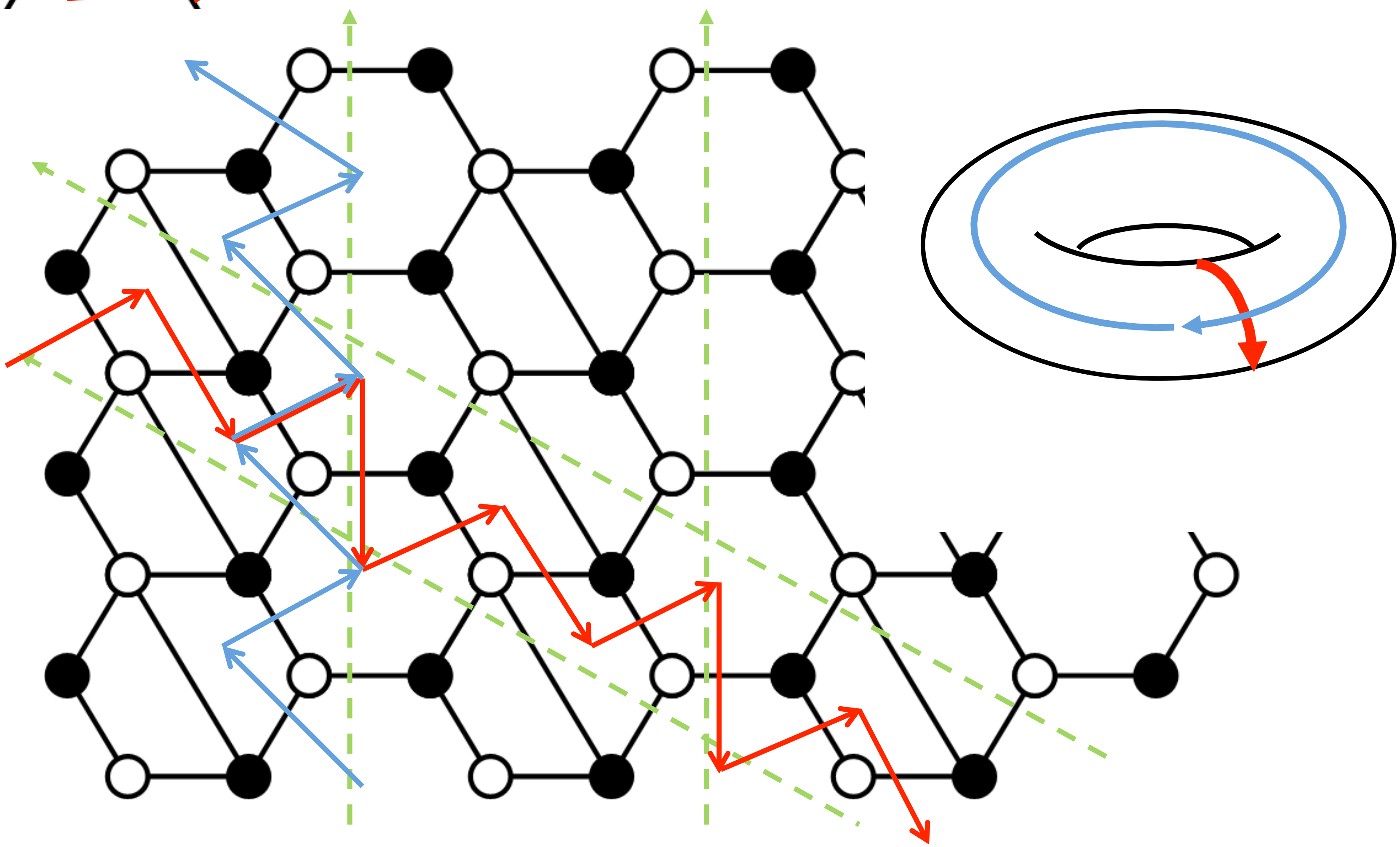


zig-zag paths are cycles on the torus

Special Paths on a Dimer

here are two zig-zag paths representing the a- and b-cycles of the torus

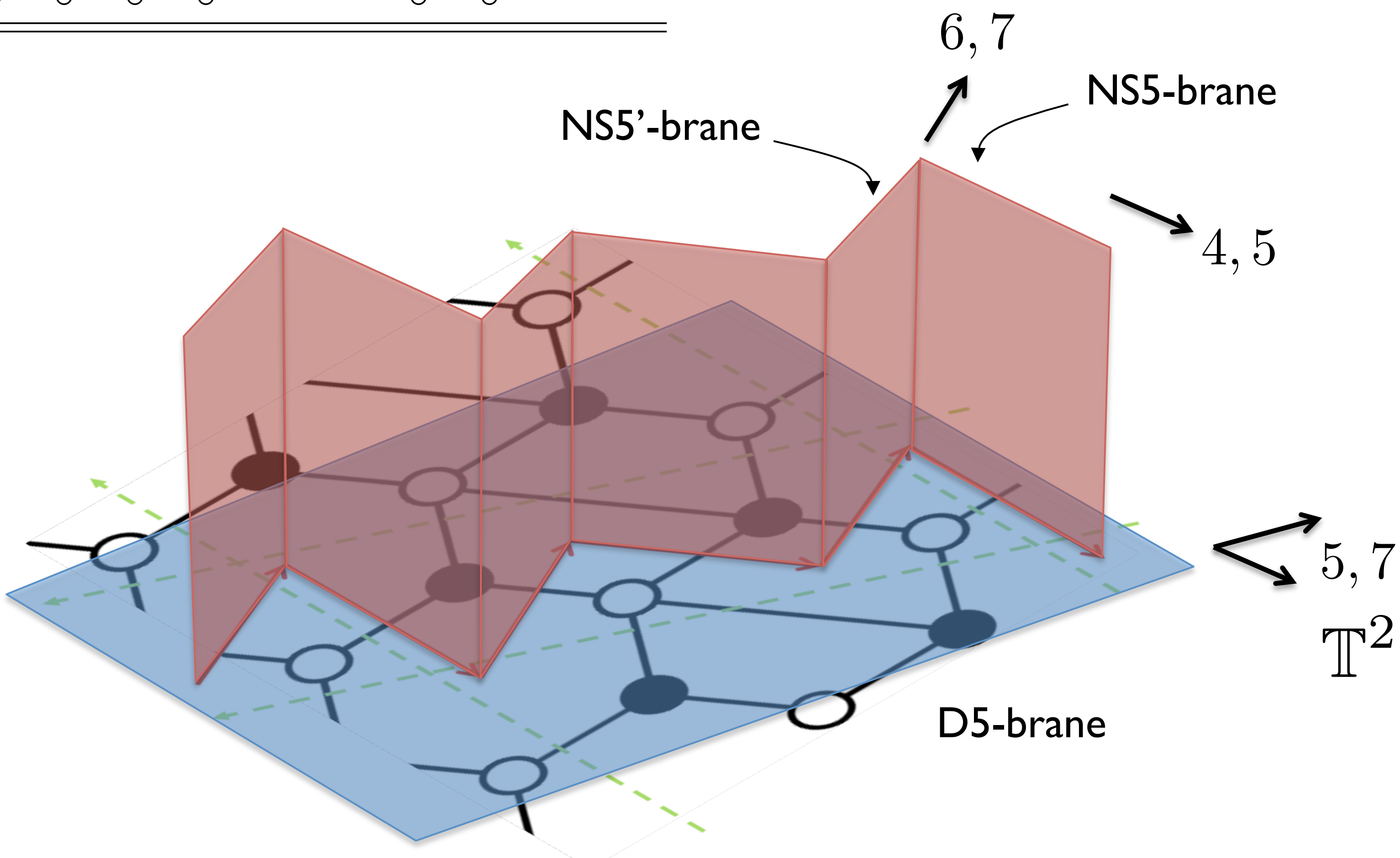




Brane Boxes

	0	1	2	3	4	5	6	7	8	9
D5	○	○	○	○		○		○		
NS5	○	○	○	○	○	○				
NS5'	○	○	○	○			○	○		

the dimer mimics a brane construction

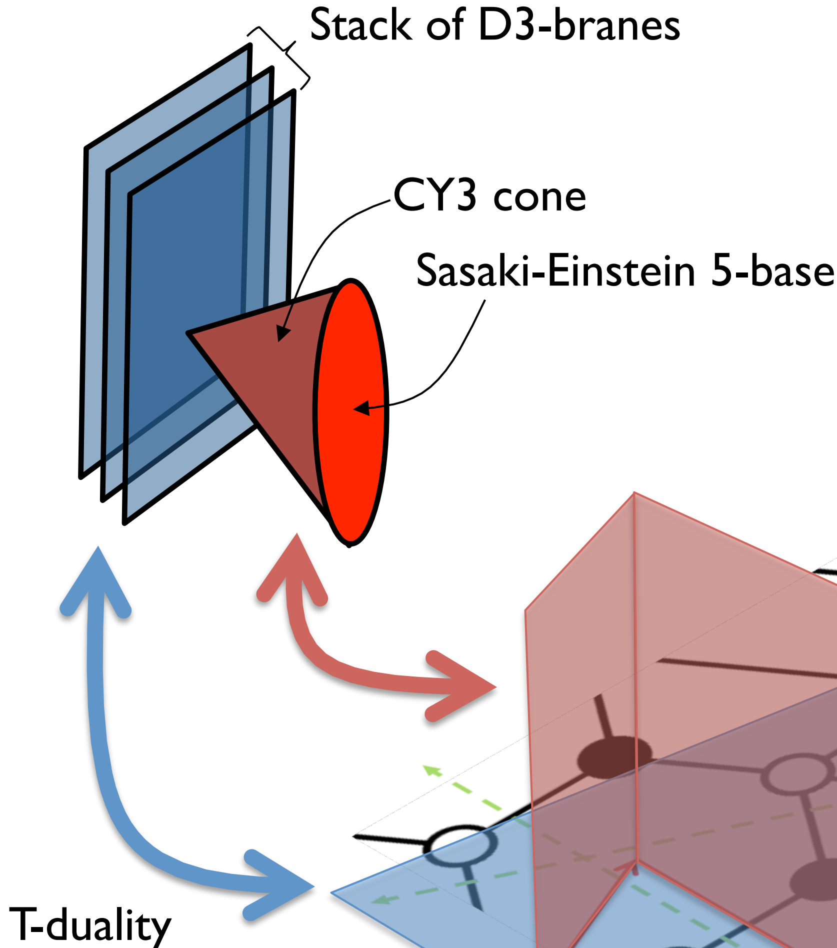


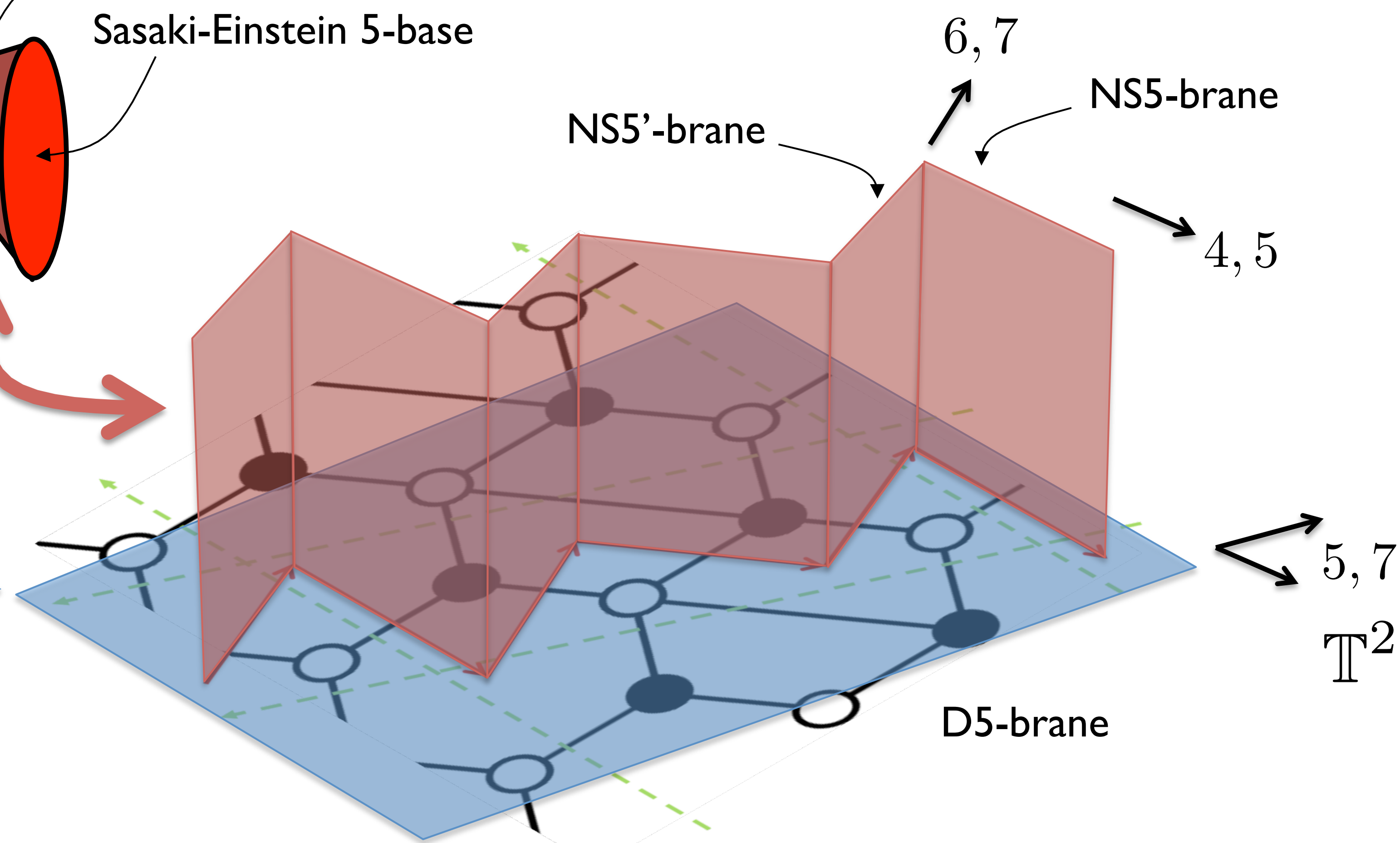
[Hanany, Kennaway 2005] [Imamura, Yamazaki 2007]

Brane Boxes

taking the T-dual of the NS5-D5 brane construction leads to a familiar picture

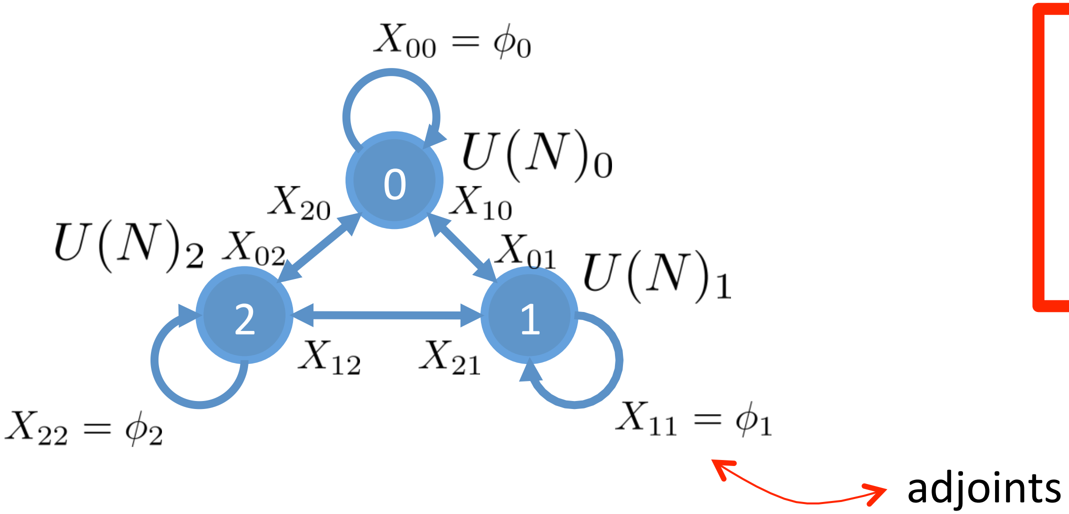
dimers are also known as **Brane Tilings**





Quivers and Superpotentials

Quiver



At a Quiver vertex:

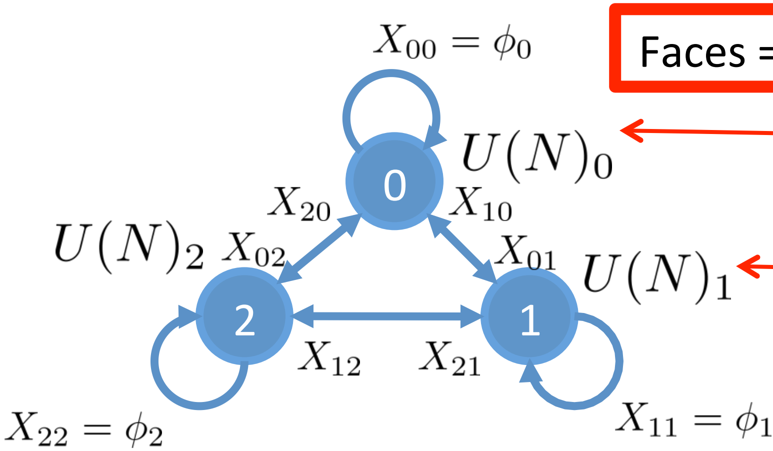
 # Incoming Arrows =
 # Outgoing Arrows

Superpotential

$$
\begin{aligned}
W \quad = \quad & \phi_1 \boxed{X_{10}} X_{01} + \phi_2 X_{21} X_{12} \\
& + \phi_0 X_{02} X_{20} - \boxed{X_{10}} \phi_0 X_{01} \\
& - X_{21} \phi_1 X_{12} - X_{02} \phi_2 X_{20}
\end{aligned}
$$

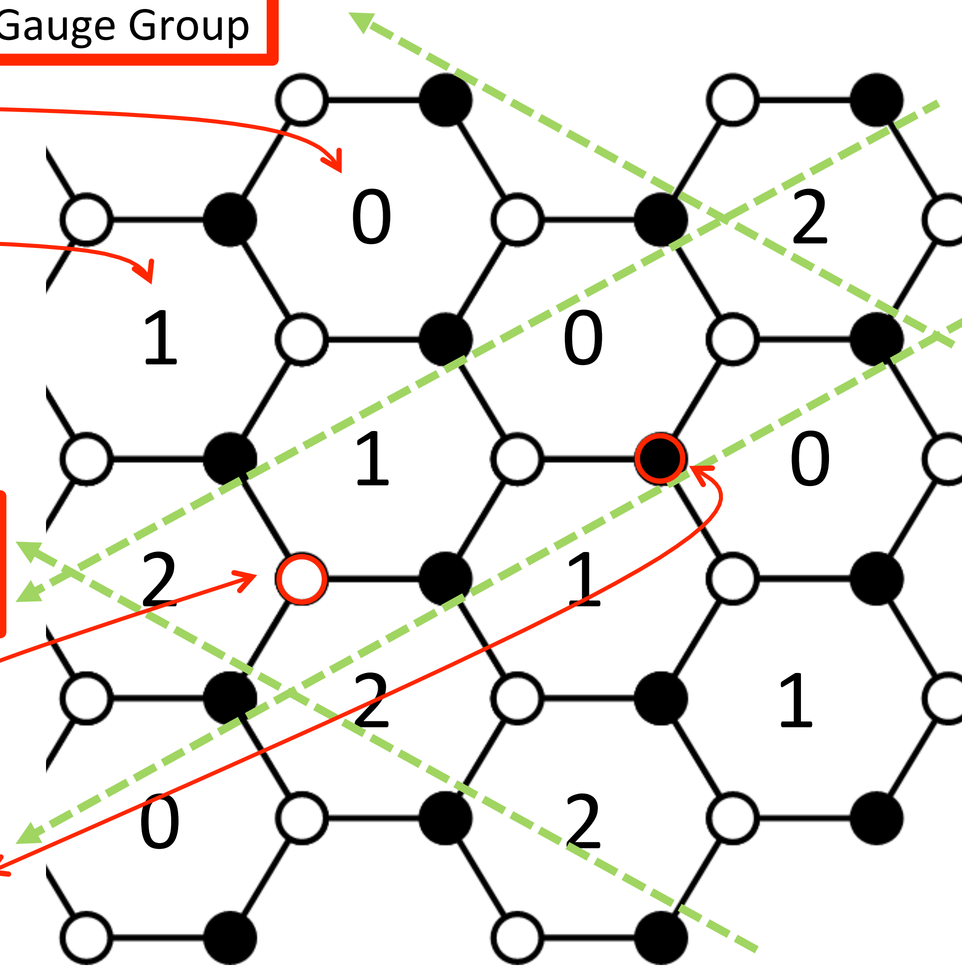
A chiral field appears exactly **twice** in the superpotential (positive & negative term)

Brane Tilings



Faces = U(N) Gauge Group

$\mathbb{T}^2$

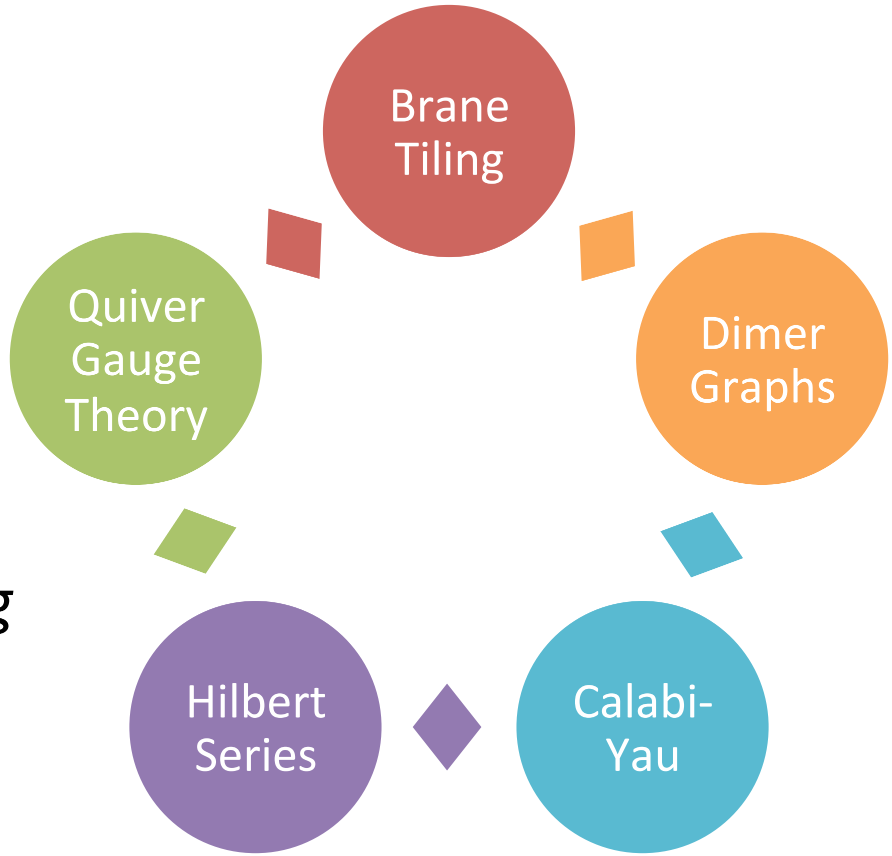


$$
\begin{aligned}
W \quad = \quad & \phi_1 X_{10} X_{01} + \phi_2 X_{21} X_{12} \\
& + \phi_0 X_{02} X_{20} - X_{10} \phi_0 X_{01} \\
& - X_{21} \phi_1 X_{12} - X_{02} \phi_2 X_{20}
\end{aligned}
$$

[Hanany, Kennaway’05]

After the break

Brane Tilings represent one of the largest known classes of supersymmetric quiver gauge theories

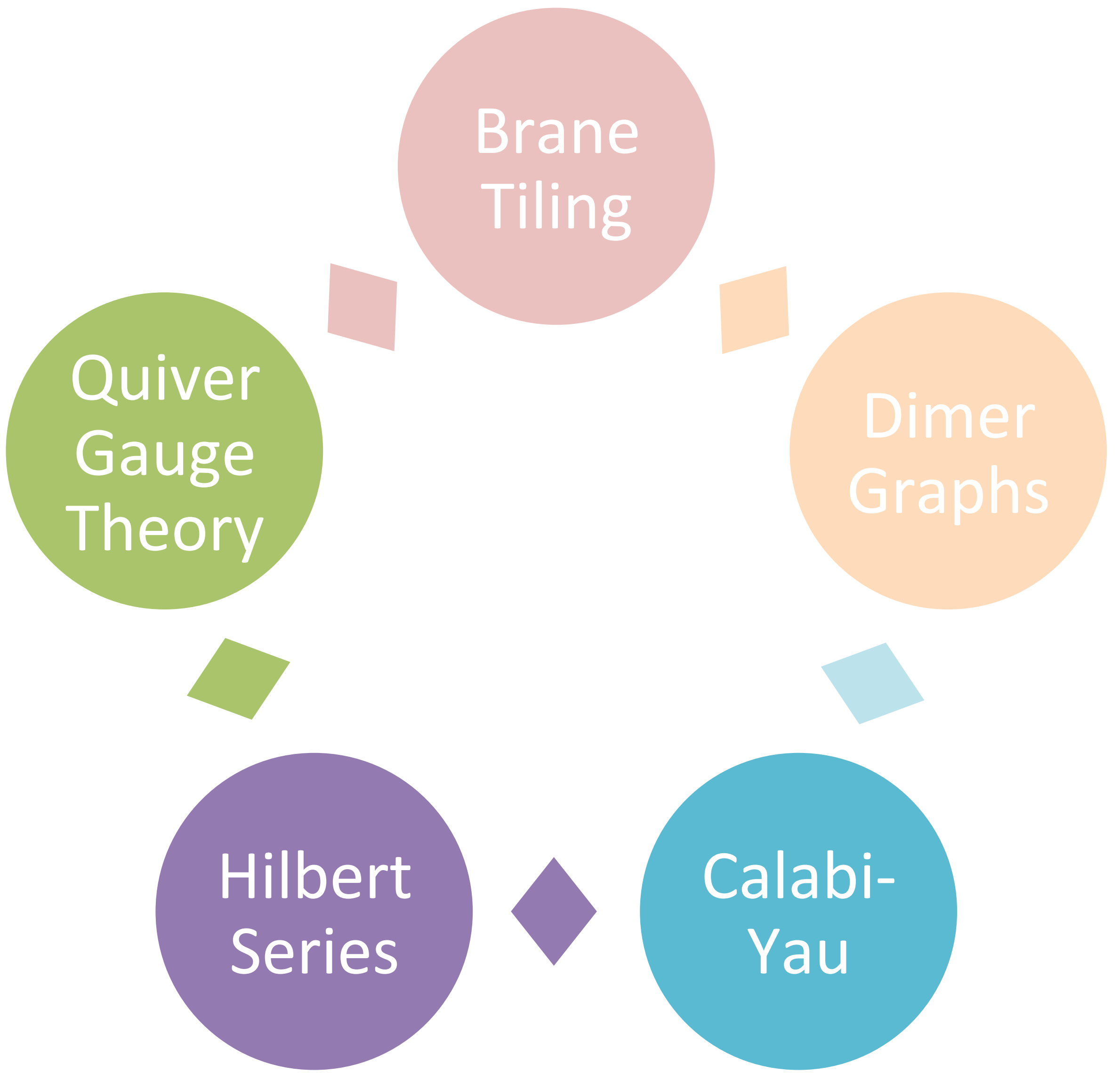


new hidden physics by studying the behavior of "ensembles of theories"

tools from mathematics help us to identify even faster the moduli space

Mutations and Dualities

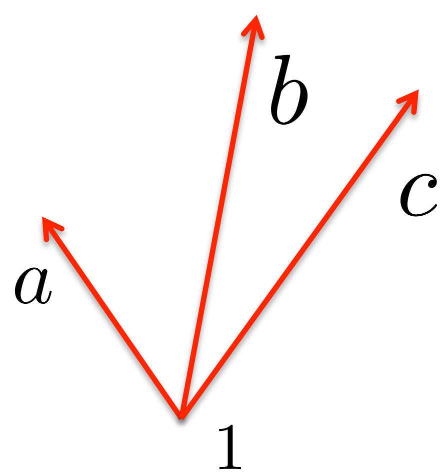
Break



Brane Tiling
Dimer Graphs
Calabi-Yau
Hilbert Series
Quiver Gauge Theory

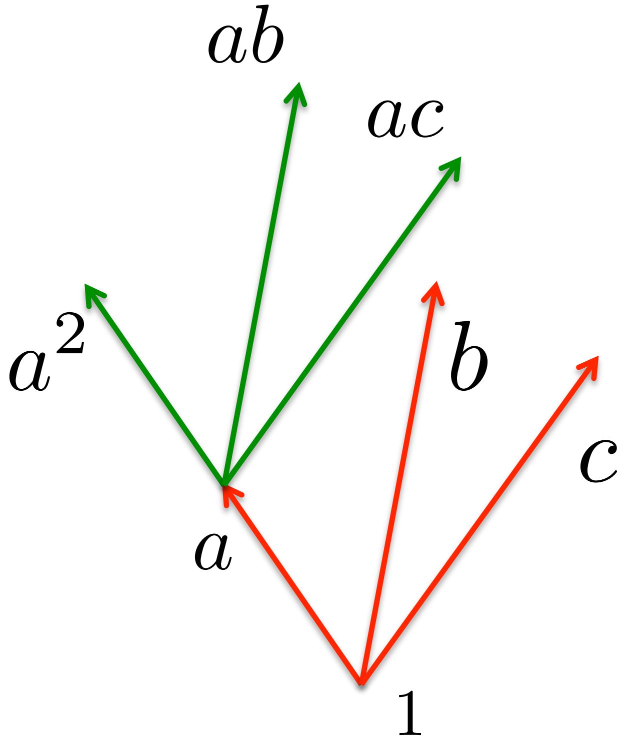
Counting Points

$$\{1, a, b, c\}$$



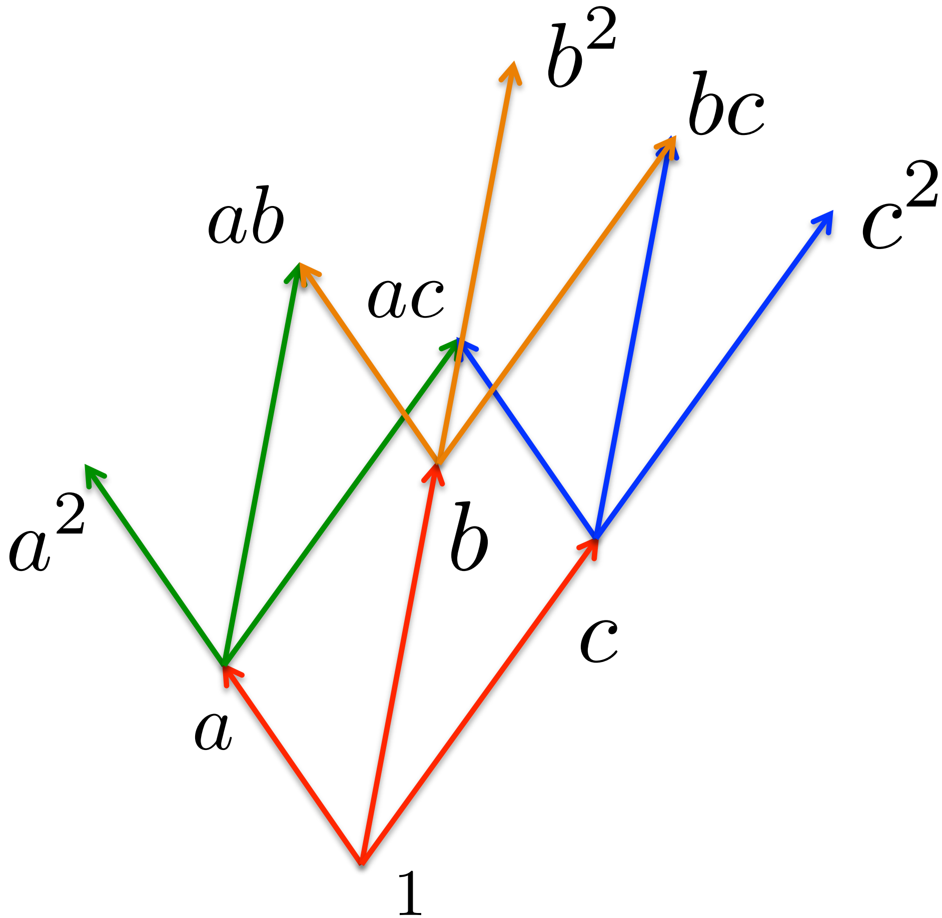
Counting Points

$$\{1, a, b, c, a^2, ab, ac\}$$



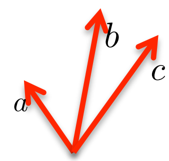
Counting Points

$$\{1, a, b, c, a^2, ab, ac, b^2, bc, c^2, \dots\}$$



lattice cone

Counting Points

complex ring generated by 

$$\{1, a, b, c, a^2, ab, ac, b^2, bc, c^2, \dots\}$$

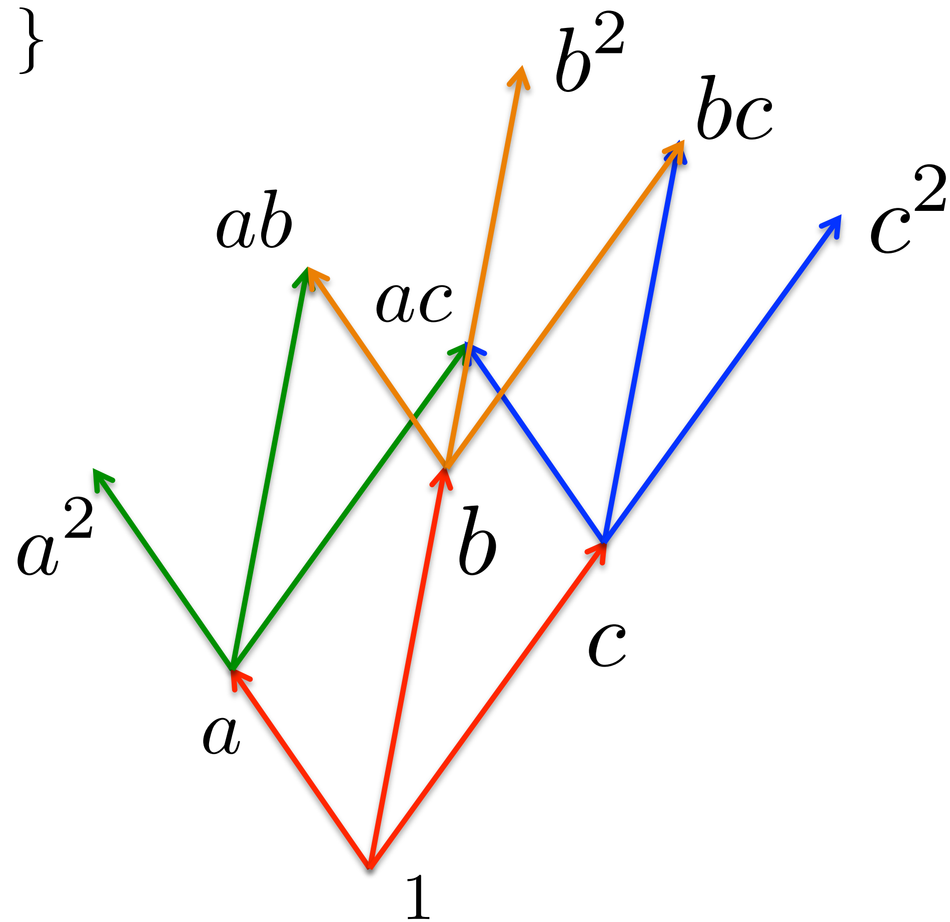
t_1 t_2 t_3

fugacities

power = degree

coefficient = count

$$1 + t_1 + t_2 + t_3 + t_1^2 + t_1 t_2$$
$$+ t_1 t_3 + t_2^2 + t_2 t_3 + t_3^2 + \dots$$
$$= \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$



Plethystic Exponential

Hilbert Series of freely generated spaces

$\mathbb{C}^3$ $$HS(\mathbb{C}^3; t_i) = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$\mathbb{C}^d$ $$HS(\mathbb{C}^d; t_i) = \frac{1}{\prod_{i=1}^{d}(1-t_i)}$$

Plethystic Exponential

$$\text{PE}[f(t_1, \ldots, t_m)] = exp\left[\sum_{k=1}^{\infty} \frac{f(t_1^k, \ldots, t_m^k)}{k}\right]$$

$\longrightarrow$ $$\text{PE}\left[\sum_{i=1}^{d} t_i\right] = \frac{1}{\prod_{i=1}^{d}(1-t_i)}$$

Symmetries and refinement

reassignment of fugacities and a new grading of the ring

$\mathbb{C}^3$

$$HS(\mathbb{C}^3; t_i) = \frac{1}{(1-t_1)(1-t_2)(1-t_3)}$$

$$t_1 = x_1 t \ , \ t_2 = \frac{x_2}{x_1} t \ , \ t_3 = \frac{1}{x_2} t$$

$$= 1 + \left(x_1 + \frac{1}{x_2} + \frac{x_2}{x_1} \right) t + \left(x_1^2 + \frac{x_1}{x_2} + x_2 + \frac{1}{x_2^2} + \frac{1}{x_1} + \frac{x_2^2}{x_1^2} \right) t^2 + \ldots$$

characters of SU(3) irreps

$[1,0]_{SU(3)}$ $\quad$ $[2,0]_{SU(3)}$

no integer coefficients if refined with full symmetry

U(1) SU(3) "fully refined"

$$HS(\mathbb{C}^3; t, x_1, x_2) = \sum_{n=0}^{\infty} [n,0]_{SU(3)} t^n$$

Symmetries and refinement

reassignment of fugacities and a new grading of the ring

$\mathbb{C}^d$

	$SU(d)$					$U(1)$	
a_1	1	0	$\ldots$	0	0	1	$x_1 t$
a_2	-1	1	$\ldots$	0	0	1	$\frac{x_2}{x_1} t$
$\vdots$							$\vdots$
a_{d-1}	0	0	$\ldots$	-1	1	1	$\frac{x_d}{x_{d-1}} t$
a_d	0	0	$\ldots$	0	-1	1	$\frac{1}{x_d} t$

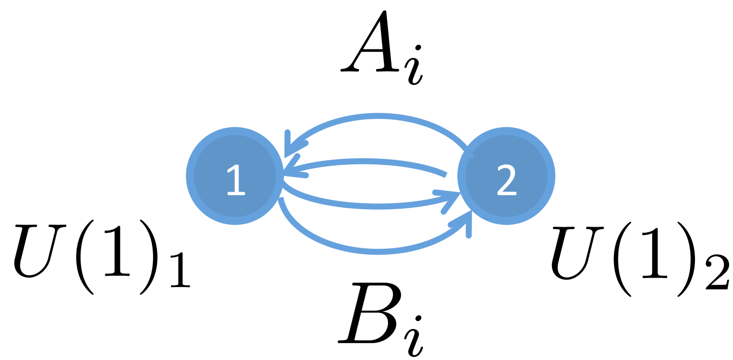
$$HS(\mathbb{C}^3; t, x_i) = \sum_{n=0}^{\infty} [n, 0, \ldots, 0]_{SU(d)} t^n$$

$$\mathrm{PE}\left[[1, 0, \ldots, 0]_{SU(d)}\right] = \sum_{n=0}^{\infty} [n, 0, \ldots, 0]_{SU(d)} t^n$$

The U(1) Conifold

The Conifold theory is a 4d quiver theory with *N=1* SUSY

Quiver



Superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

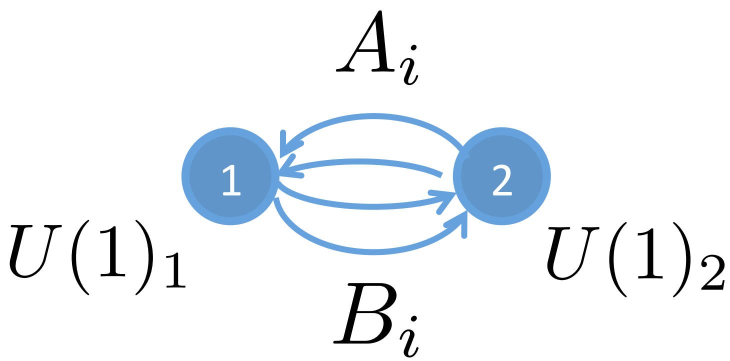
implicit trace

note: for U(1) we have Abelian superpotential (W=0)

The U(1) Conifold

The Conifold theory is a 4d quiver theory with *N=1* SUSY

Quiver



Gauge & R-charges

	gauge charges		R-charge	
	$U(1)_1$	$U(1)_2$	$U(1)_R$	fugacity
A_1	$+1$	-1	r_1	$\frac{b_1}{b_2}t_1$
A_2	$+1$	-1	r_2	$\frac{b_1}{b_2}t_2$
B_1	-1	$+1$	r_3	$\frac{b_2}{b_1}t_3$
B_2	-1	$+1$	r_4	$\frac{b_2}{b_1}t_4$

$$\mathrm{PE}\left[\frac{b_1}{b_2}t_1+\frac{b_1}{b_2}t_2+\frac{b_2}{b_1}t_3+\frac{b_2}{b_1}t_4\right]=\frac{1}{\left(1-\frac{b_1}{b_2}t_1\right)\left(1-\frac{b_1}{b_2}t_2\right)\left(1-\frac{b_2}{b_1}t_3\right)\left(1-\frac{b_2}{b_1}t_4\right)}$$

The U(1) Conifold

freely generated space

$$\mathrm{PE}\left[\frac{b_1}{b_2}t_1 + \frac{b_1}{b_2}t_2 + \frac{b_2}{b_1}t_3 + \frac{b_2}{b_1}t_4\right] = \frac{1}{\left(1-\frac{b_1}{b_2}t_1\right)\left(1-\frac{b_1}{b_2}t_2\right)\left(1-\frac{b_2}{b_1}t_3\right)\left(1-\frac{b_2}{b_1}t_4\right)}$$

Molien Integral

U(1) Haar measures

$$g_1(\mathcal{C}; b_i, t_i) = \frac{1}{(2\pi i)^2}\oint_{|b_1|=1}\frac{\mathrm{d}b_1}{b_1}\oint_{|b_2|=1}\frac{\mathrm{d}b_2}{b_2}\frac{1}{\left(1-\frac{b_1}{b_2}t_1\right)\left(1-\frac{b_1}{b_2}t_2\right)\left(1-\frac{b_2}{b_1}t_3\right)\left(1-\frac{b_2}{b_1}t_4\right)}$$

$$= \frac{1-t_1t_2t_3t_4}{(1-t_1t_3)(1-t_2t_3)(1-t_1t_4)(1-t_2t_4)}$$

Hilbert Series of

the gauge invariant space (mesonic moduli space)

The U(1) Conifold

a different grading

$$g_1(\mathcal{C}; b_i, t_i) = \frac{1 - t_1 t_2 t_3 t_4}{(1 - t_1 t_3)(1 - t_2 t_3)(1 - t_1 t_4)(1 - t_2 t_4)}$$

global symmetry charges

	flavor charges: $\mathrm{SU}(2)_1$	flavor charges: $\mathrm{SU}(2)_2$	R-charge: $\mathrm{U}(1)_R$	fugacity
A_1	+1	0	$\frac{1}{2}$	$t_1 = x_1 t$
A_2	-1	0	$\frac{1}{2}$	$t_2 = \frac{1}{x_1} t$
B_1	0	+1	$\frac{1}{2}$	$t_3 = x_2 t$
B_2	0	-1	$\frac{1}{2}$	$t_4 = \frac{1}{x_2} t$

$$g_1(\mathcal{C}; t_i, x_i) = \frac{1 - t^4}{(1 - x_1 x_2 t^2)(1 - x_1^{-1} x_2 t^2)(1 - x_1 x_2^{-1} t^2)(1 - x_1^{-1} x_2^{-1} t^2)}$$

$$= \sum_{n=0}^{\infty} [n]_{SU(2)_1} [n]_{SU(2)_2} t^{2n} = \sum_{n=0}^{\infty} [n; n] t^{2n}$$

The U(1) Conifold

encoded moduli space

$$g_1(\mathcal{C}; b_i, t_i) = \frac{\boxed{1 - t_1 t_2 t_3 t_4}}{(1-t_1t_3)(1-t_2t_3)(1-t_1t_4)(1-t_2t_4)}$$

generators:
(mesons, GIOs)

$$A_1B_1 \quad A_2B_1 \quad A_1B_2 \quad A_2B_2$$

$$\mathbb{C}^4 \quad a \quad b \quad c \quad d$$

relation: $ad = bc$

relation of degree $t_1t_2t_3t_4$

$$\mathcal{C} \sim \mathbb{C}^4 / \langle ad = bc \rangle$$

complete intersection

Complete Intersections

complete intersection moduli spaces are generated by a finite number of operators which satisfy a finite number of relations

the Hilbert series of any complete intersection moduli space takes the form:

$$g_1(\mathcal{M};t_i) = \frac{\prod_{m=1}^{N_r}(1-t_m^{n_m})}{\prod_{k=1}^{N_o}(1-t_k^{n_k})}$$

degree of relation

degree of generator

for non-complete intersection moduli spaces, the numerator is a non-factorizeable polynomial

there are relations among relations called syzygies

$$g_1(\mathcal{M};t_i) = \frac{P(t_i)}{\prod_{k=1}^{N_o}(1-t_k^{n_k})}$$

if the polynomial is palindromic, the moduli space is Calabi-Yau

[Stanley's Theorem 1978]

Plethystic Logarithm

Plethystic Exponential

Möbius function

$$\mathrm{PL}\left[f(t_1,\ldots,t_d)\right] = \sum_{k=1}^{\infty} \frac{\mu(k)}{k} \log\left[f(t_1^k,\ldots,t_d^k)\right]$$

inverse of the plethystic exponential

complete intersection: conifold

generators relation

$$\mathrm{PL}\left[\frac{1-t_1t_2t_3t_4}{(1-t_1t_3)(1-t_2t_3)(1-t_1t_4)(1-t_2t_4)}\right] = t_1t_3+t_2t_3+t_1t_4+t_2t_4-t_1t_2t_3t_4$$

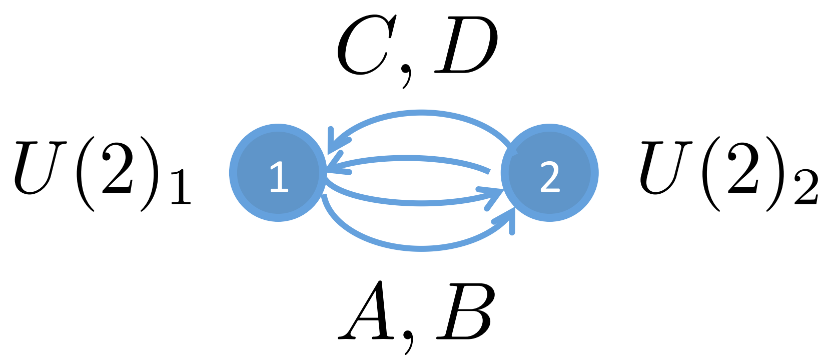
non-complete intersection: dP_0

infinite expansion relating to relations among relations (syzygies)

$$\mathrm{PL}\left[\frac{1+t_1^2t_2+t_1t_2^2+t_1^2t_3+t_1t_2t_3+t_2^2t_3+t_1t_3^2+t_2t_3^2+t_1^2t_2^2t_3^2}{(1-t_1^3)(1-t_2^3)(1-t_3^2)}\right] =$$

$$t_3^2+t_1^3+t_1^2t_2+t_1t_2^2+t_2^3+t_1^2t_3+t_1t_2t_3+t_2^2t_3+t_1t_3^2+t_2t_3^2-t_1^4t_2^2-t_1^3t_2^3+\ldots$$

The U(2) Conifold



non-Abelian superpotential

$$W = ACBD - ADBC$$

F-terms:

$$\frac{\partial W}{\partial X} = 0$$

we have to consider the quotient ring of the form

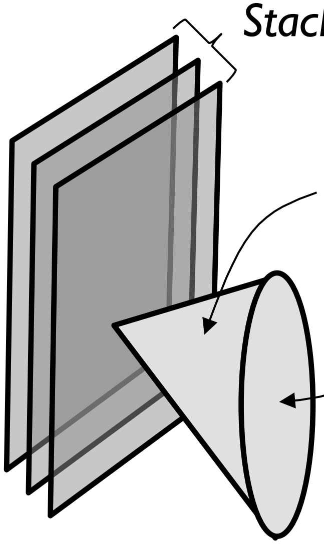
$$\mathbb{C}^d / \langle \text{F-terms} \rangle$$

↓ Molien Integral

gauge invariant space (mesonic moduli space)

Branes and SCFTs

4d supersymmetric gauge theories with *N=1* SUSY



worldvolume theory of a stack of *N* D3 branes probing singular CY

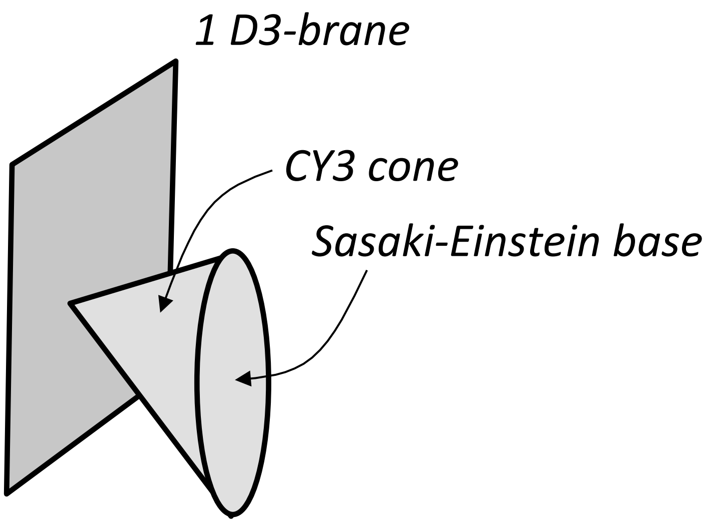
9+1d Type IIB on $\mathrm{AdS}_5 \times \mathrm{SE}^5$

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○						
CY3					○	○	○	○	○	○

Branes and SCFTs

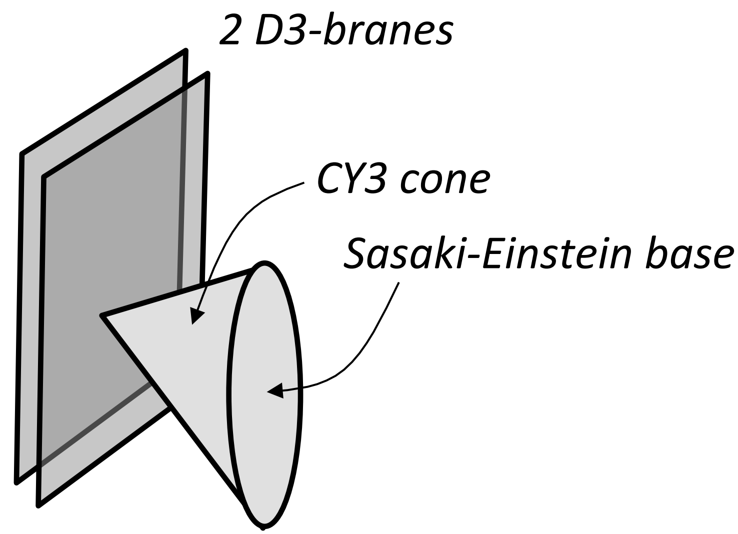
U(1) Conifold Theory

= worldvolume theory



$$\mathcal{M}_1 = \mathcal{C}$$

U(2) Conifold Theory



$$\mathcal{M}_2 = \frac{(\mathcal{M}_1)^2}{S_2}$$

Symmetric Product $= \mathrm{Sym}^2(M_1)$

Symmetric Products

the symmetric product has a Hilbert series implementation:

$$PE_v[g_1(t_1,\ldots,t_d)] = exp\left[\sum_{k=1}^{\infty} \frac{g_1(t_1^k,\ldots,t_d^k)}{k}\ v^k\right]$$

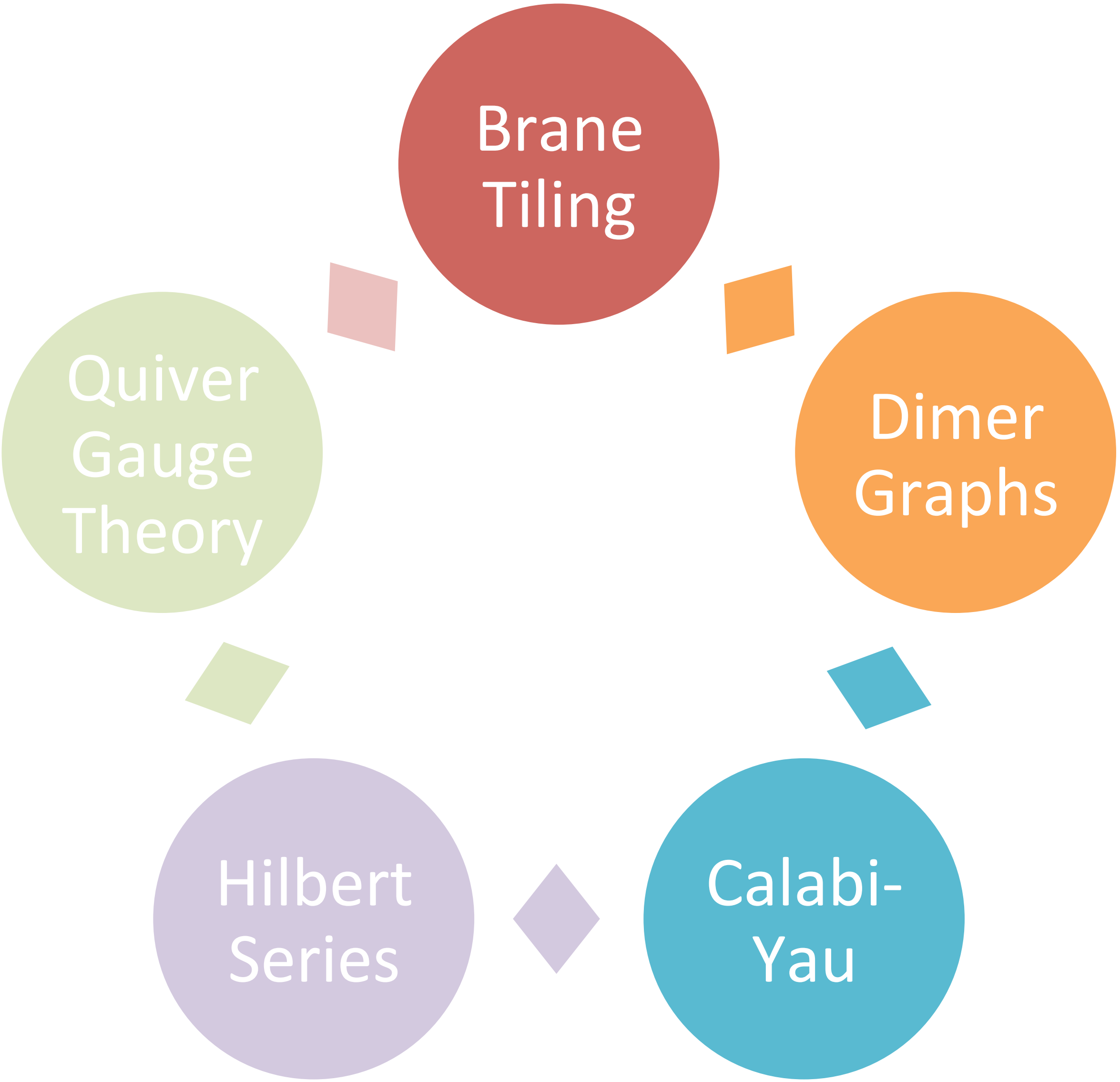
$$= 1 + g_1(t_i)v + g_2(t_i)v^2 + g_3(t_i)v^3 + \ldots$$

$$\mathcal{M}_1 \qquad \mathrm{Sym}^2(\mathcal{M}_1) \qquad \mathrm{Sym}^3(\mathcal{M}_1)$$

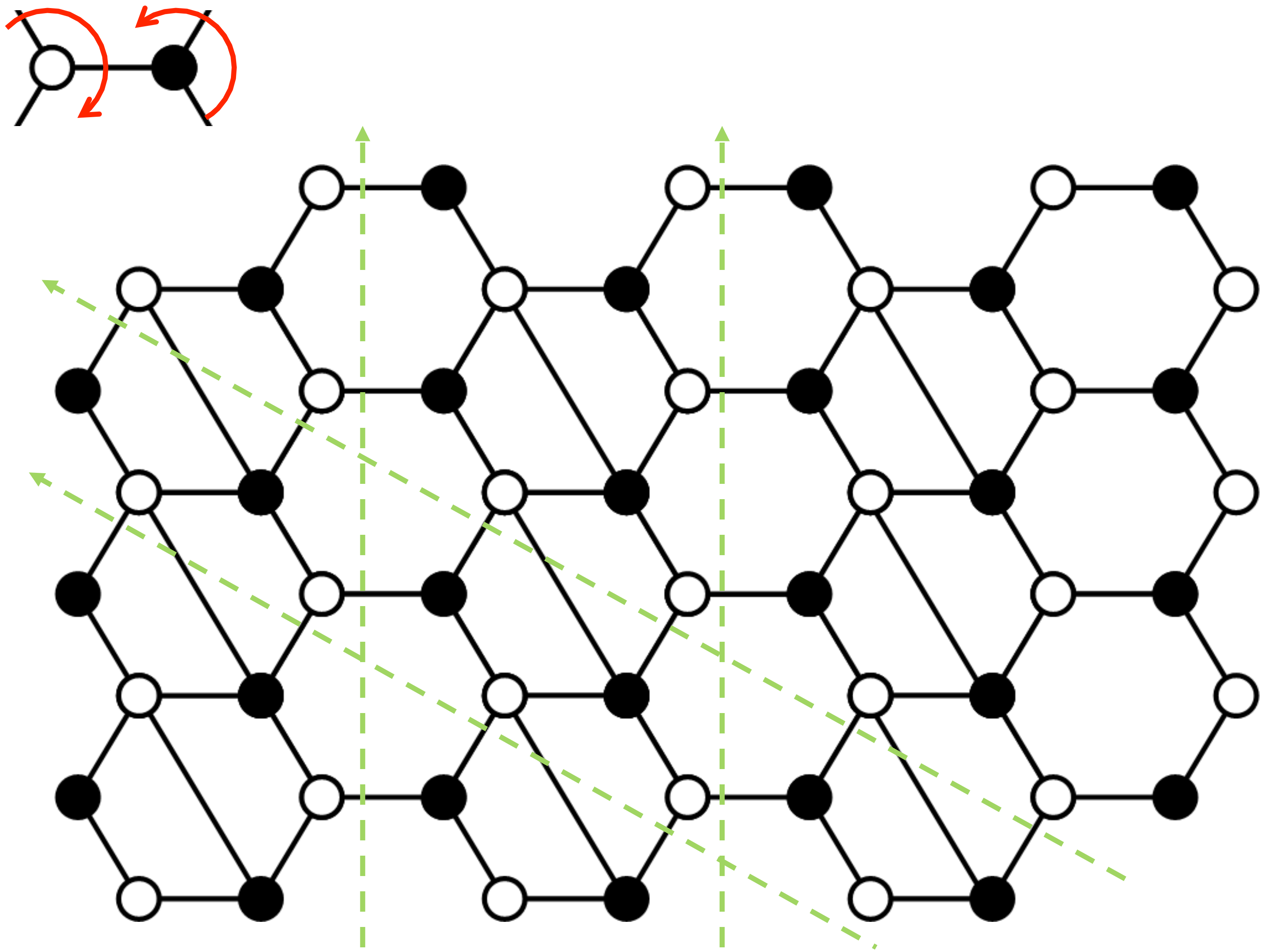
for the U(N) conifold theories, we have:

$$PE_v[g_1(\mathcal{C};t)] = 1 + \frac{1-t^4}{(1-t^2)^4}v + \frac{1+t^2+7t^4+3t^6+4t^8}{(1-t^2)^3(1-t^4)^3}$$

$$+\frac{1+7t^4+13t^6+18t^8+31t^{10}+34t^{12}+18t^{14}+16t^{16}+6t^{18}}{(1-t^2)^4(1-t^4)^2(1-t^6)^3}v^3+\ldots$$

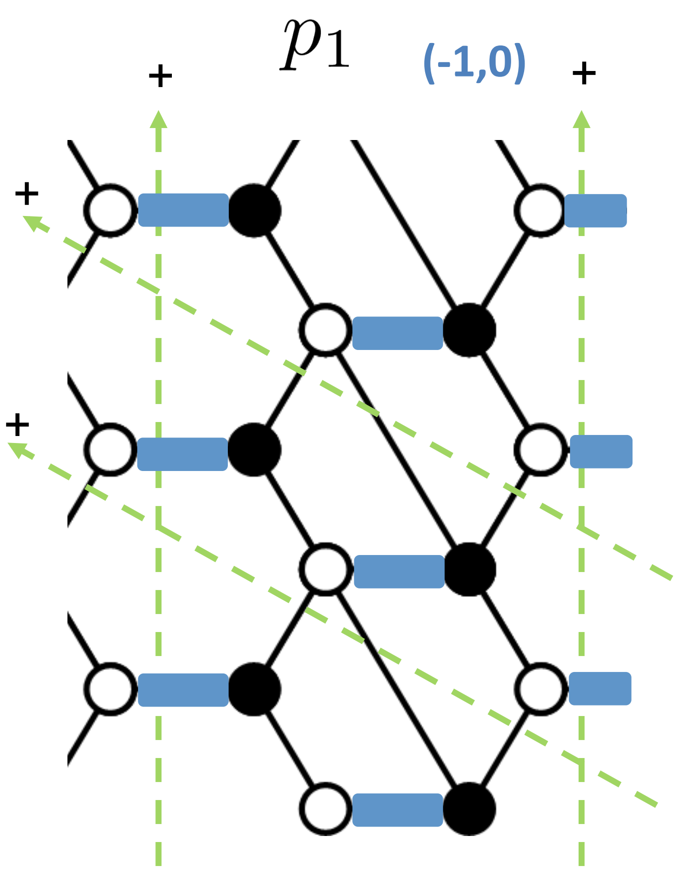

Brane Tiling
Dimer Graphs
Calabi-Yau
Hilbert Series
Quiver Gauge Theory

Example: Suspended Pinch Point

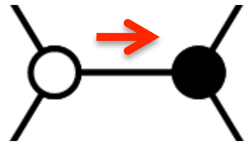


Perfect Matchings

perfect matchings are a set of edges that connect to every white and black node uniquely once



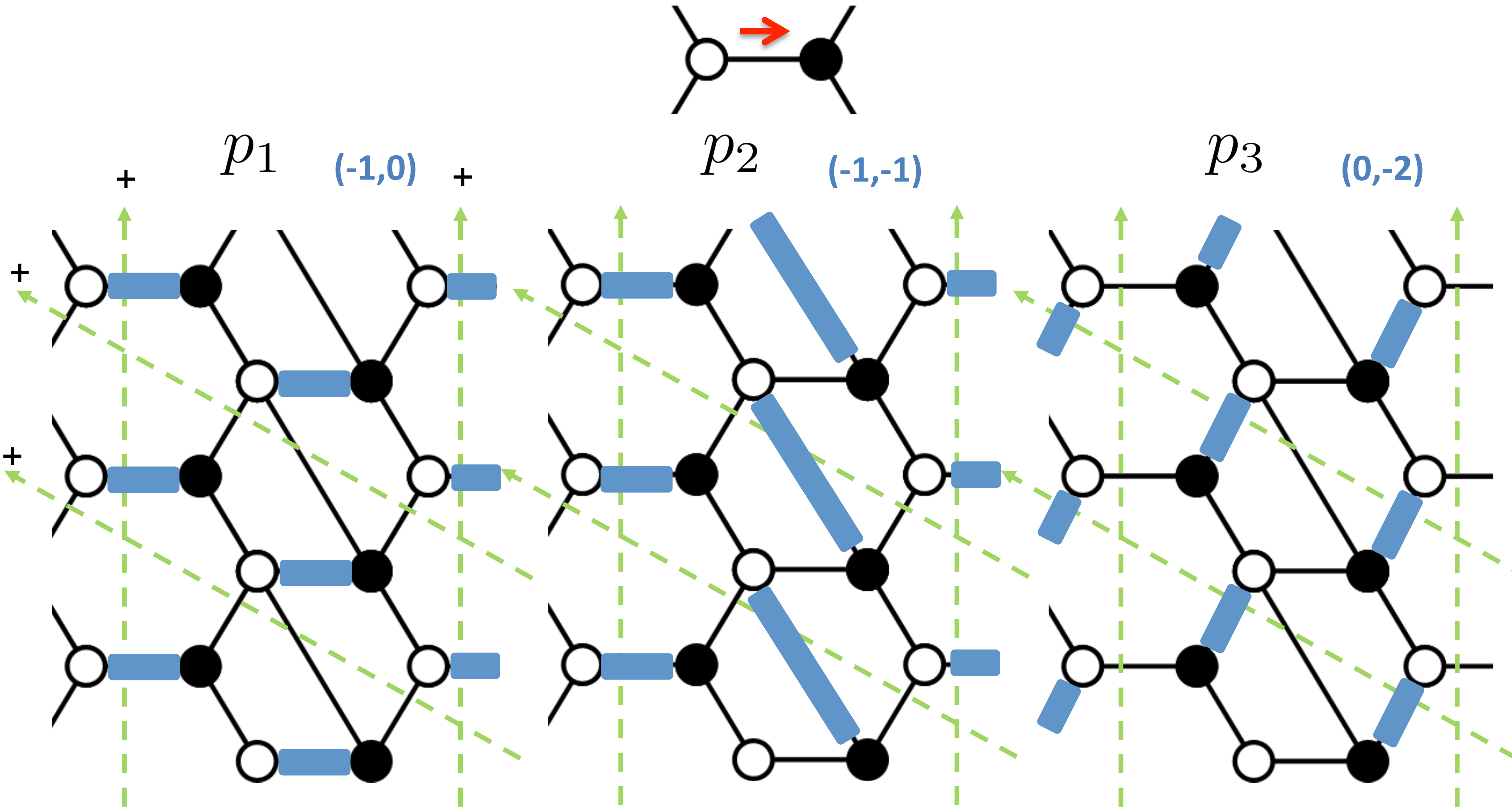
by setting a standard orientation from white to black nodes



every perfect matchings can be assigned a winding number on T^2

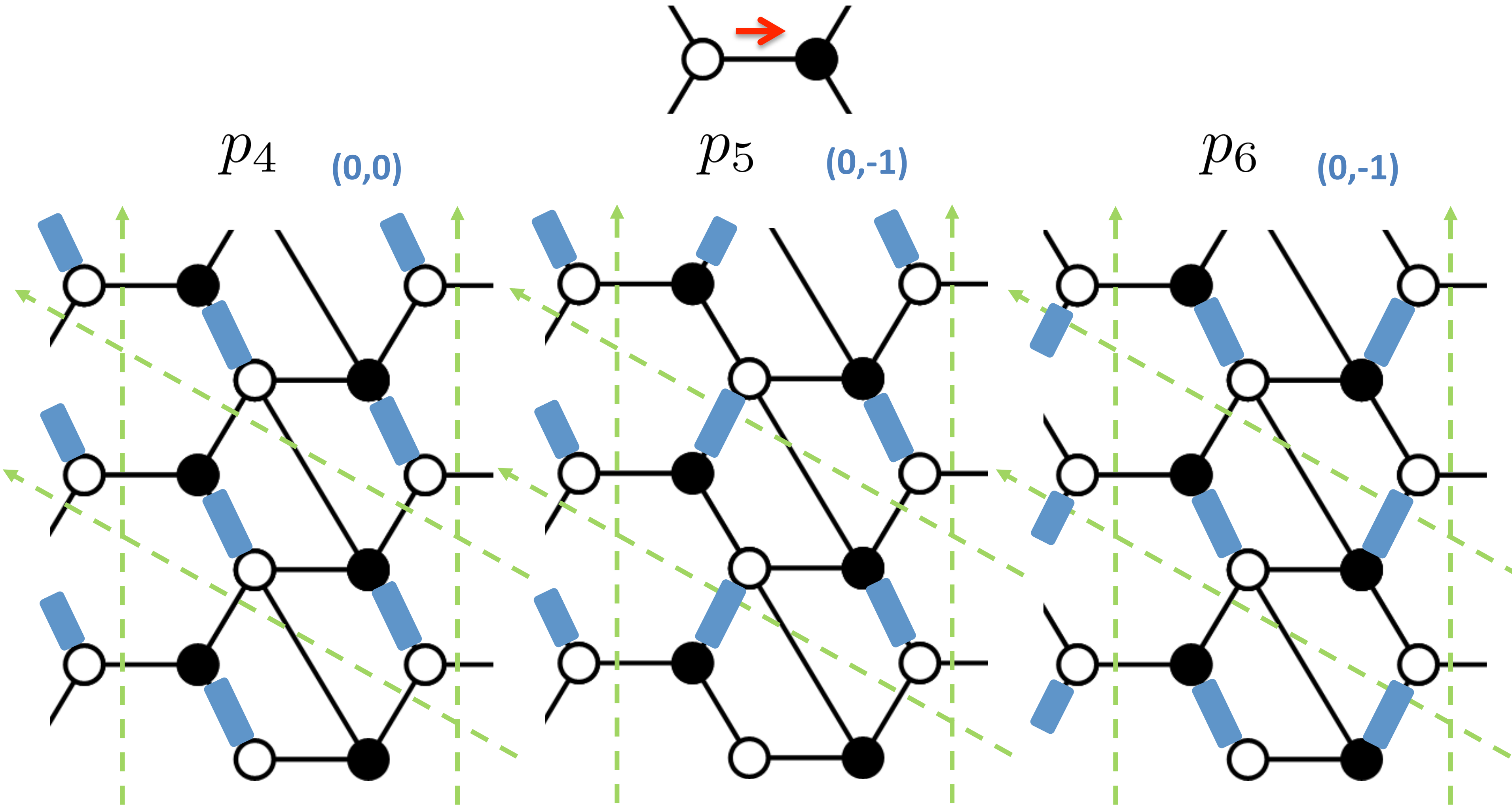
Perfect Matchings

there are in total 6 distinct perfect matchings for SPP

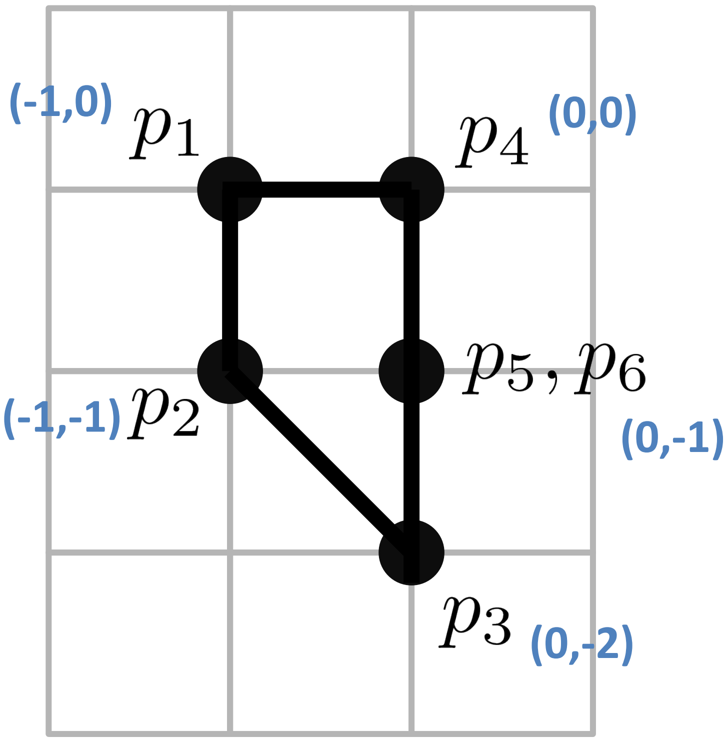


Perfect Matchings

there are in total 6 distinct perfect matchings for SPP

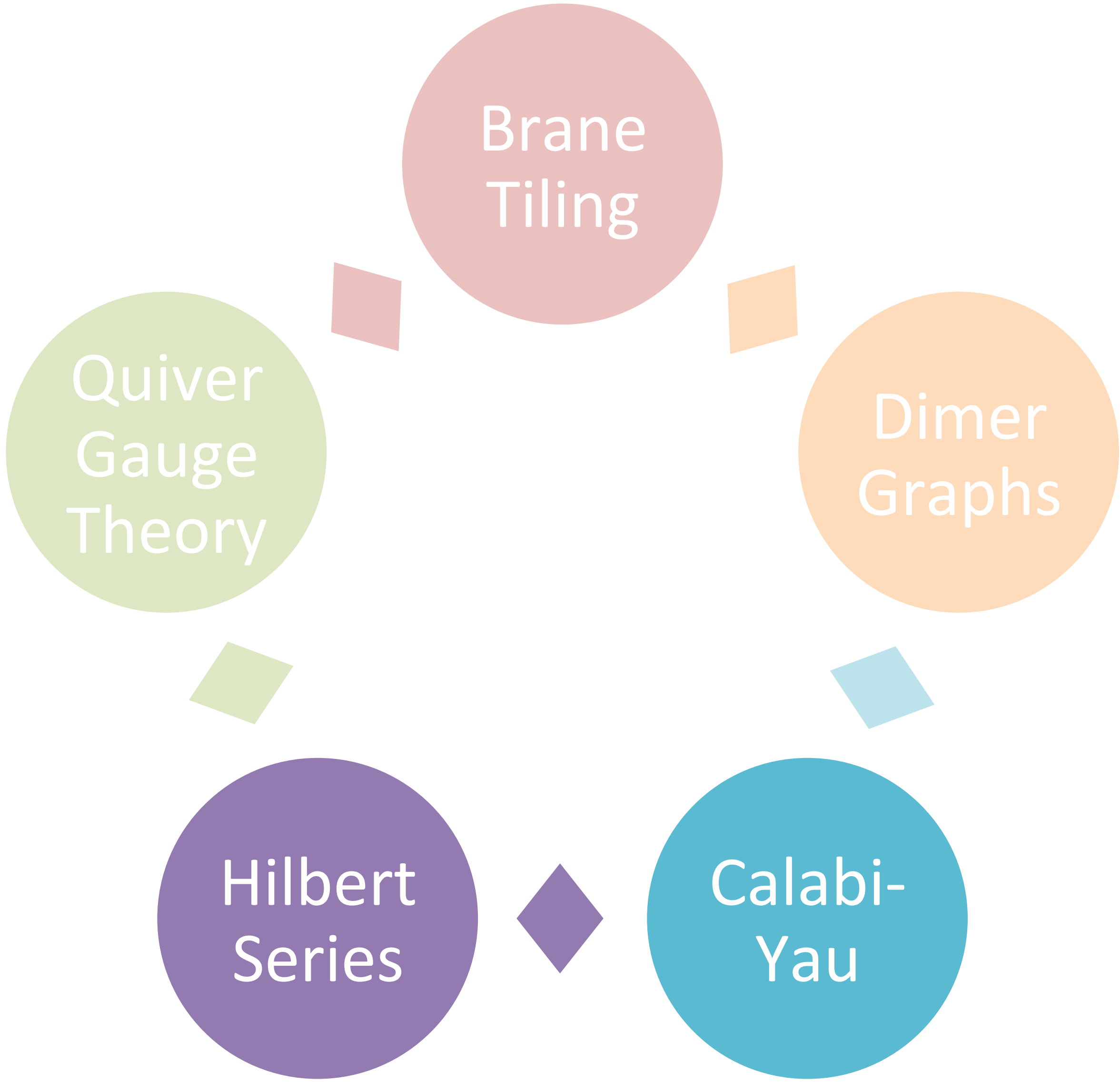


Toric Diagram

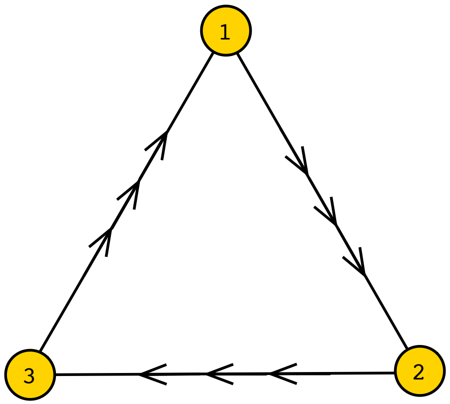


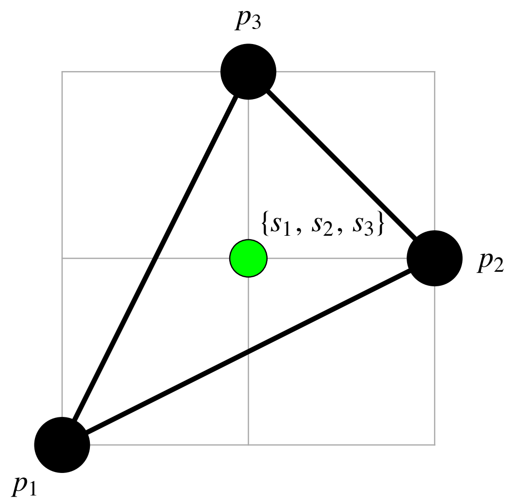
- every perfect matching corresponds to a toric point
- the coordinates correspond to perfect matching winding numbers
- internal points correspond to multiple perfect matchings, corner points do not

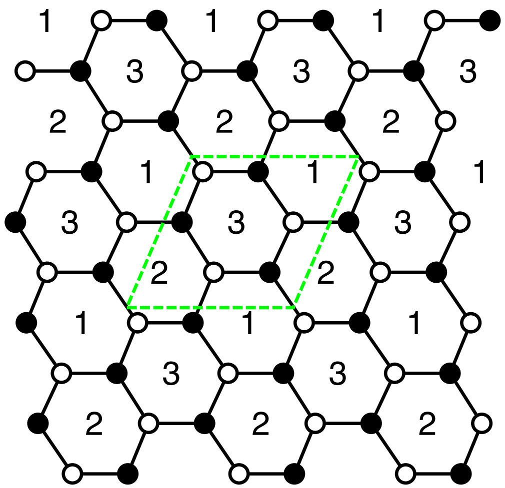
1. the toric diagram characterizes the Calabi-Yau moduli space
2. perfect matchings correspond to Gauge Linear Sigma Model fields


Brane Tiling
Dimer Graphs
Calabi-Yau
Hilbert Series
Quiver Gauge Theory

Example: dP_0







Hilbert Series of the mesonic moduli space

$$g_1(t, \tilde{x}_1, \tilde{x}_2; \mathcal{M}^{mes}_{16}) = \sum_{n=0}^{\infty} [3n, 0]_{(\tilde{x}_1, \tilde{x}_2)} \; t^{3n}$$

Lattice of Generators

$$PL[g_1(t,\tilde{x}_1,\tilde{x}_2;\mathcal{M}^{mes}_{16})] = [3,0]_{(\tilde{x}_1,\tilde{x}_2)}t^3 - [2,2]_{(\tilde{x}_1,\tilde{x}_2)}t^6 + ([1,1]_{(\tilde{x}_1,\tilde{x}_2)} + [1,4]_{(\tilde{x}_1,\tilde{x}_2)} + [2,2]_{(\tilde{x}_1,\tilde{x}_2)} + [4,1]_{(\tilde{x}_1,\tilde{x}_2)})t^9 - (2[0,3]_{(\tilde{x}_1,\tilde{x}_2)} + 2[1,1]_{(\tilde{x}_1,\tilde{x}_2)} + 2[1,4]_{(\tilde{x}_1,\tilde{x}_2)} + 2[2,2]_{(\tilde{x}_1,\tilde{x}_2)} + [2,5]_{(\tilde{x}_1,\tilde{x}_2)} + 2[3,0]_{(\tilde{x}_1,\tilde{x}_2)} + 2[3,3]_{(\tilde{x}_1,\tilde{x}_2)} + 2[4,1]_{(\tilde{x}_1,\tilde{x}_2)} + [5,2]_{(\tilde{x}_1,\tilde{x}_2)})t^{12} + \ldots \ .$$

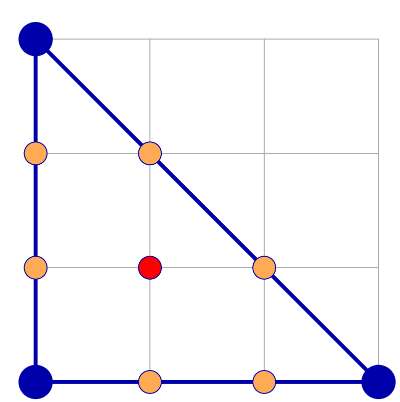
moduli space generators

$$[3,0]_{(\tilde{x}_1,\tilde{x}_2)}t^3 = \left(\tilde{x}_1^3 + \tilde{x}_1\tilde{x}_2 + \frac{\tilde{x}_1^2}{\tilde{x}_2} + \frac{\tilde{x}_2^2}{\tilde{x}_1} + 1 + \frac{\tilde{x}_2^3}{\tilde{x}_1^3} + \frac{\tilde{x}_1}{\tilde{x}_2^2} + \frac{\tilde{x}_2}{\tilde{x}_1^2} + \frac{1}{\tilde{x}_1\tilde{x}_2} + \frac{1}{\tilde{x}_2^3}\right)t^3$$

lattice of generators

$$\tilde{x}_1 = \frac{1}{x_1^{1/3}x_2^{1/3}} \ , \ \tilde{x}_2 = \frac{x_1^{1/3}}{x_2^{2/3}}$$

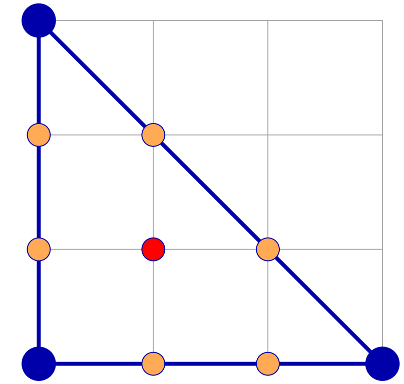
Generator	$SU(3)_{(x_1,x_2)}$
$X_{12}^3X_{23}^1X_{31}^2$	(-1, -1)
$X_{12}^1X_{23}^1X_{31}^2 = X_{12}^3X_{23}^1X_{31}^3 = X_{12}^3X_{23}^2X_{31}^2$	(0, -1)
$X_{12}^1X_{23}^1X_{31}^3 = X_{12}^1X_{23}^2X_{31}^2 = X_{12}^3X_{23}^2X_{31}^3$	(1, -1)
$X_{12}^1X_{23}^2X_{31}^3$	(2, -1)
$X_{12}^2X_{23}^1X_{31}^2 = X_{12}^3X_{23}^1X_{31}^1 = X_{12}^3X_{23}^3X_{31}^2$	(-1, 0)
$X_{12}^1X_{23}^1X_{31}^1 = X_{12}^1X_{23}^3X_{31}^2 = X_{12}^2X_{23}^1X_{31}^3 = X_{12}^2X_{23}^2X_{31}^2 = X_{12}^3X_{23}^2X_{31}^1 = X_{12}^3X_{23}^3X_{31}^3$	(0, 0)
$X_{12}^1X_{23}^2X_{31}^1 = X_{12}^1X_{23}^3X_{31}^3 = X_{12}^2X_{23}^2X_{31}^3$	(1, 0)
$X_{12}^2X_{23}^1X_{31}^1 = X_{12}^2X_{23}^3X_{31}^2 = X_{12}^3X_{23}^3X_{31}^1$	(-1, 1)
$X_{12}^1X_{23}^3X_{31}^1 = X_{12}^2X_{23}^2X_{31}^1 = X_{12}^2X_{23}^3X_{31}^3$	(0, 1)
$X_{12}^2X_{23}^3X_{31}^1$	(-1, 2)



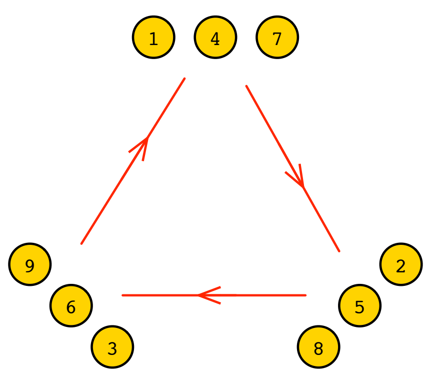
dP_0 and C^3/Z_3xZ_3

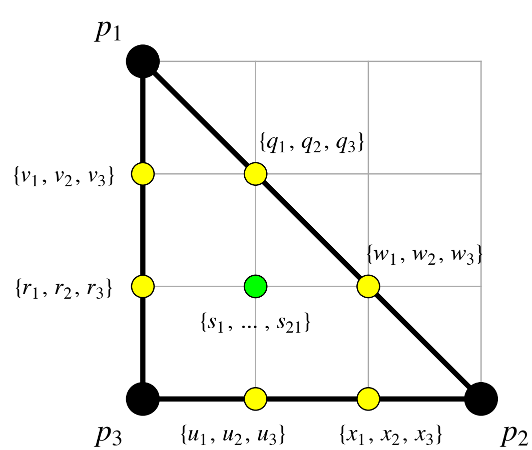
lattice of generators looks like a toric diagram

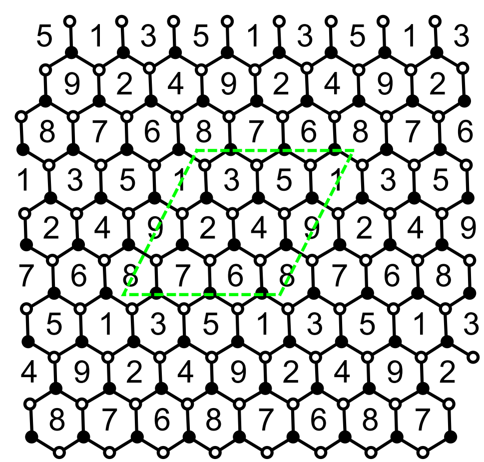
Generator	$SU(3)_{(x_1,x_2)}$
$X^3_{12}X^1_{23}X^2_{31}$	(-1, -1)
$X^1_{12}X^1_{23}X^2_{31} = X^3_{12}X^1_{23}X^3_{31} = X^3_{12}X^2_{23}X^2_{31}$	(0, -1)
$X^1_{12}X^1_{23}X^3_{31} = X^1_{12}X^2_{23}X^2_{31} = X^3_{12}X^2_{23}X^3_{31}$	(1, -1)
$X^1_{12}X^2_{23}X^3_{31}$	(2, -1)
$X^2_{12}X^1_{23}X^2_{31} = X^3_{12}X^1_{23}X^1_{31} = X^3_{12}X^3_{23}X^2_{31}$	(-1, 0)
$X^1_{12}X^1_{23}X^1_{31} = X^1_{12}X^3_{23}X^2_{31} = X^2_{12}X^1_{23}X^3_{31} = X^2_{12}X^2_{23}X^2_{31} = X^3_{12}X^2_{23}X^1_{31} = X^3_{12}X^3_{23}X^3_{31}$	(0, 0)
$X^1_{12}X^2_{23}X^1_{31} = X^1_{12}X^3_{23}X^3_{31} = X^2_{12}X^2_{23}X^3_{31}$	(1, 0)
$X^2_{12}X^1_{23}X^1_{31} = X^2_{12}X^3_{23}X^2_{31} = X^3_{12}X^3_{23}X^1_{31}$	(-1, 1)
$X^1_{12}X^3_{23}X^1_{31} = X^2_{12}X^2_{23}X^1_{31} = X^2_{12}X^3_{23}X^3_{31}$	(0, 1)
$X^2_{12}X^3_{23}X^1_{31}$	(-1, 2)



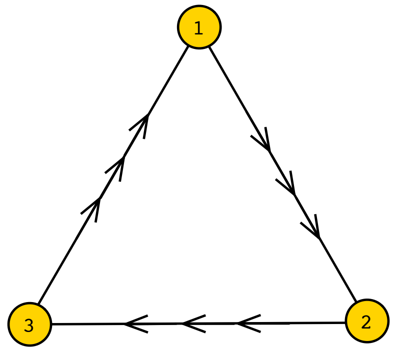
Model for $\mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$

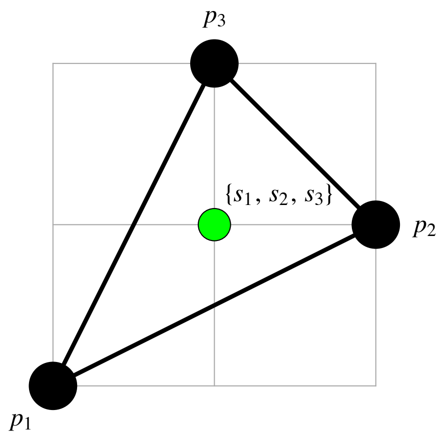


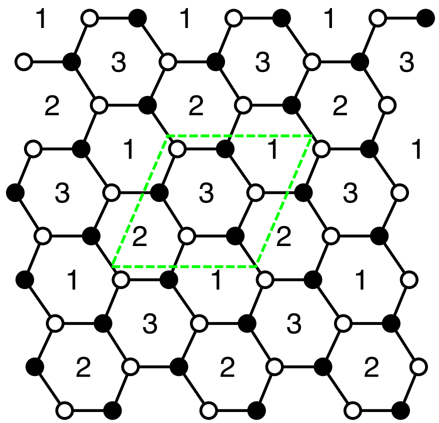


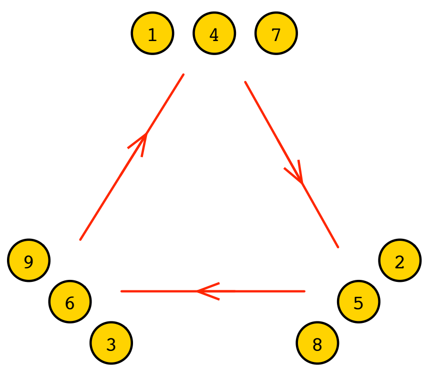


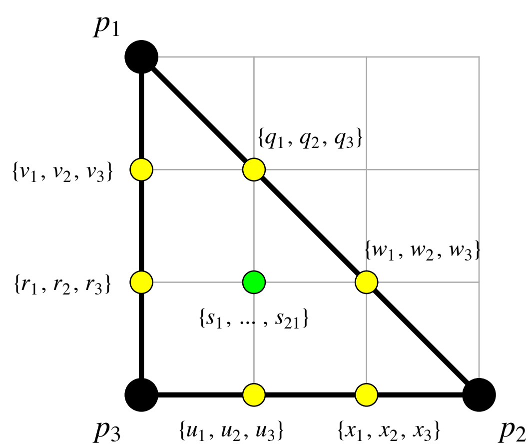
dP_0 and C^3/Z_3xZ_3

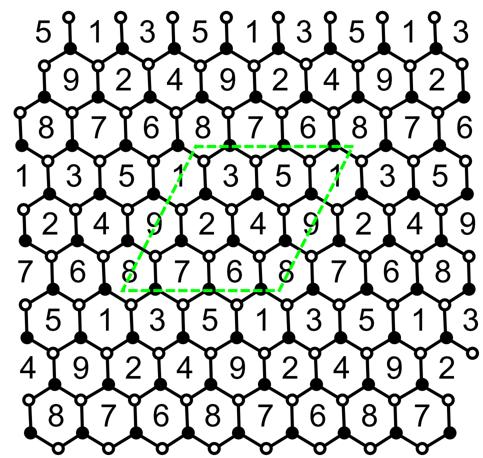




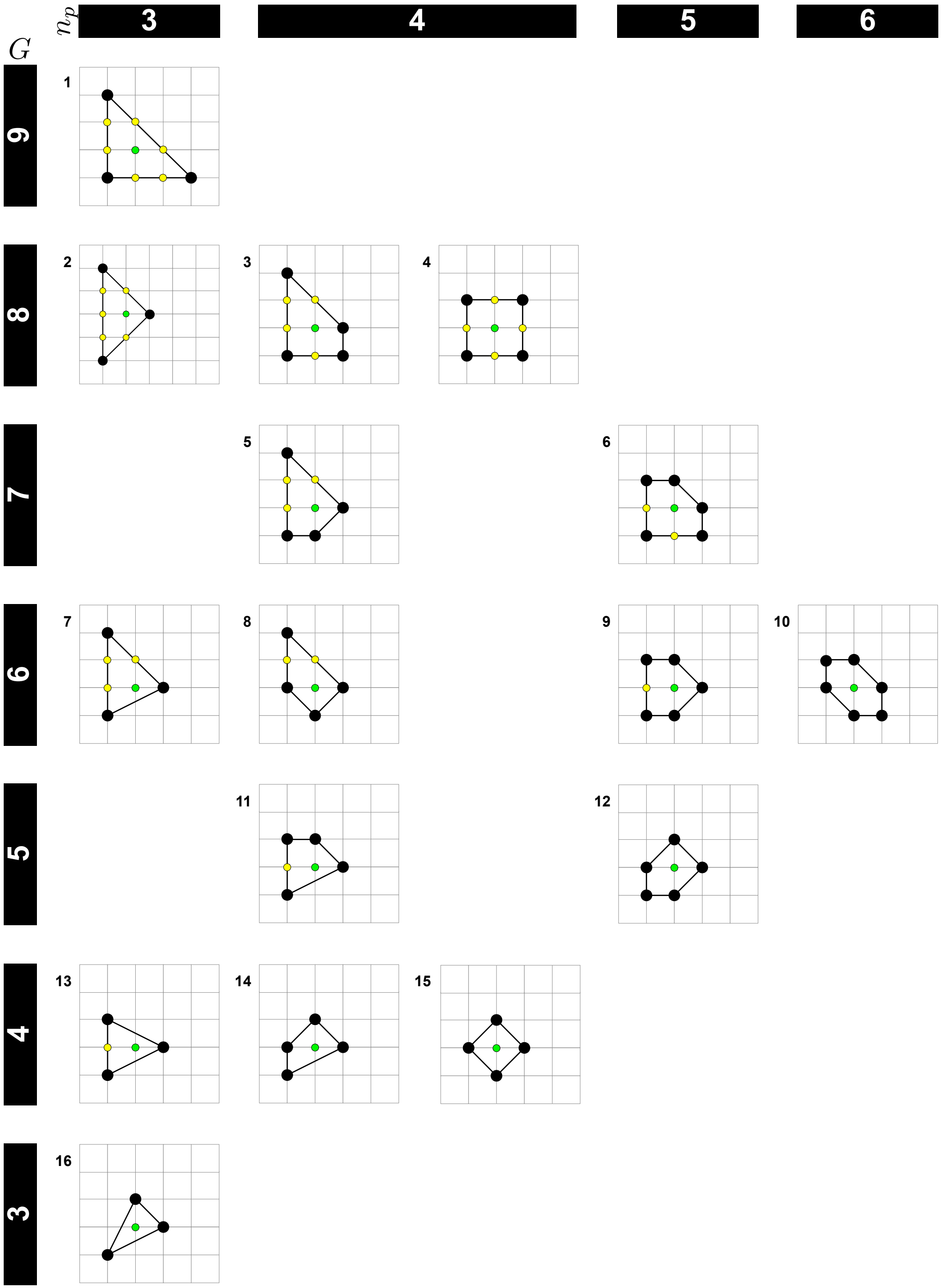








Reflexive Polygons

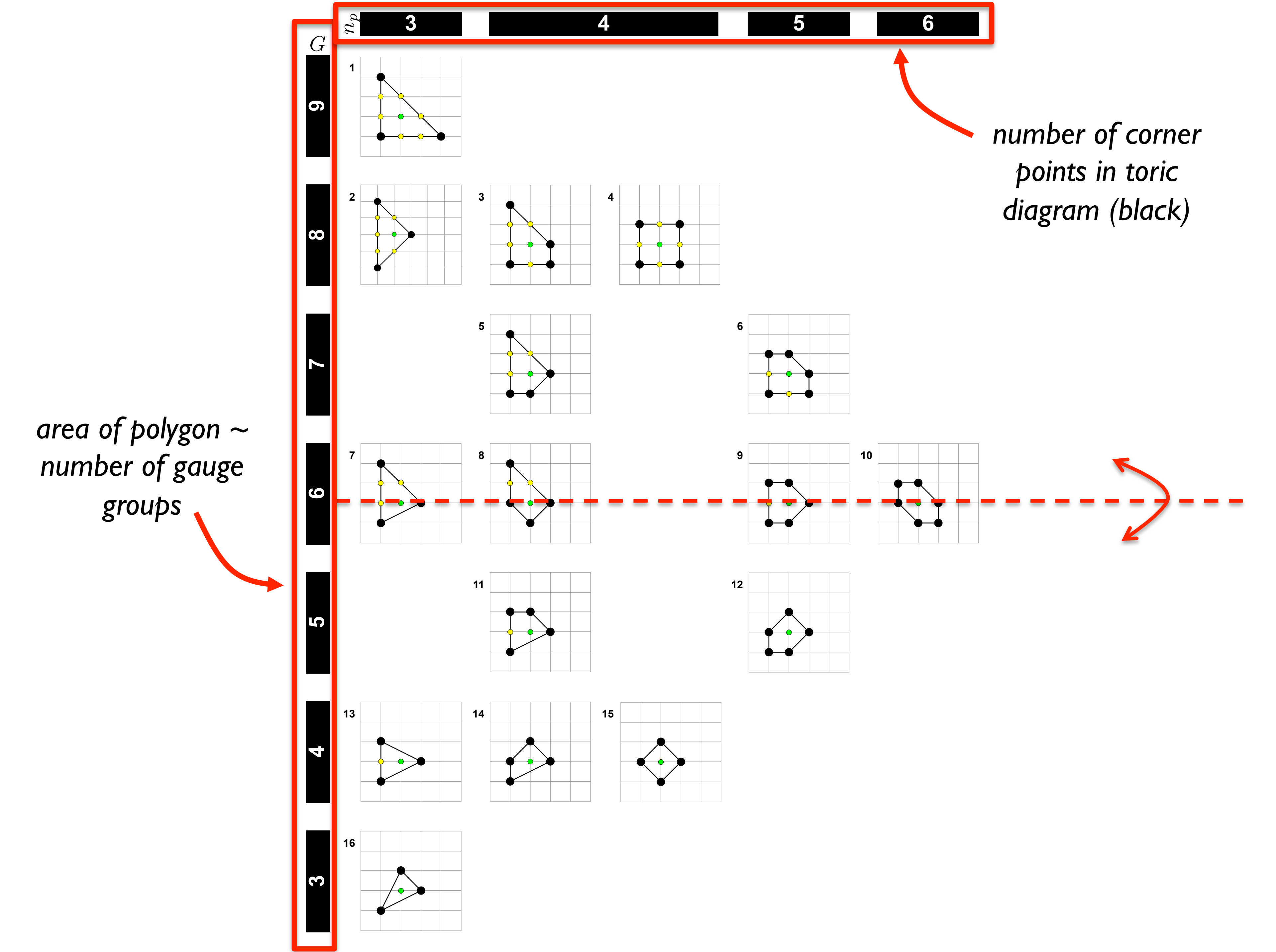


there is a class of polygons known as Reflexive Polygons

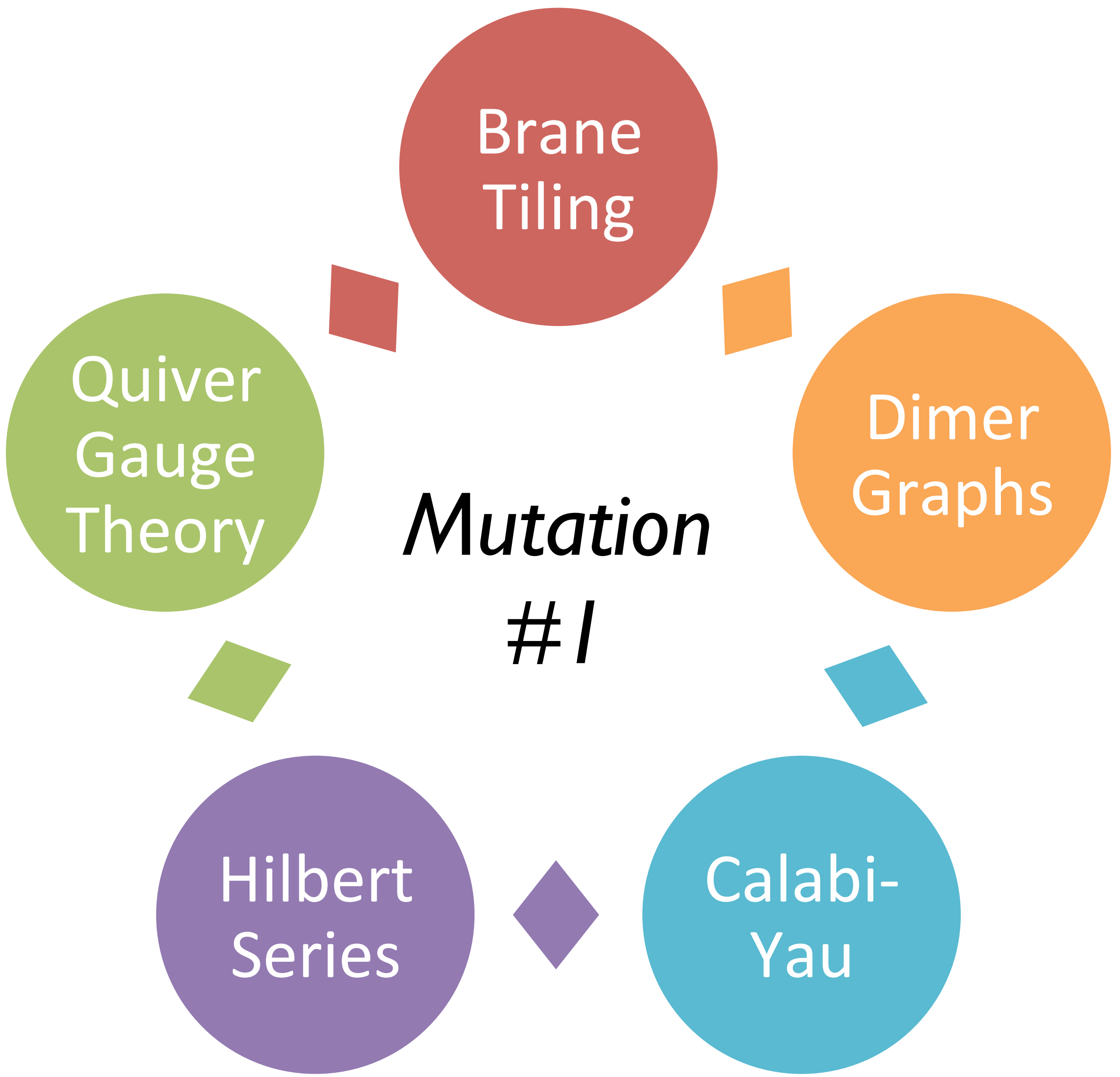
they are convex lattice polygons with a single internal point

there are precisely 16 distinct reflexive polygons (2D)

d	Number of Polytopes
1	1
2	16
3	4319
4	473800776

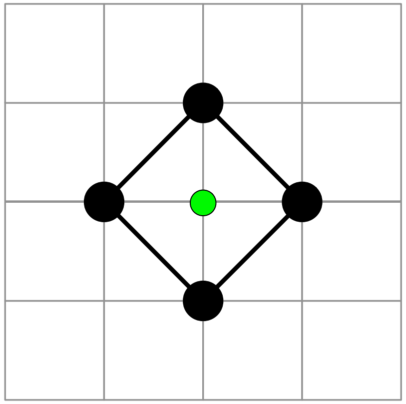


n_p
3
4
5
6
G
9
8
7
6
5
4
3
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of corner points in toric diagram (black)
area of polygon ~ number of gauge groups

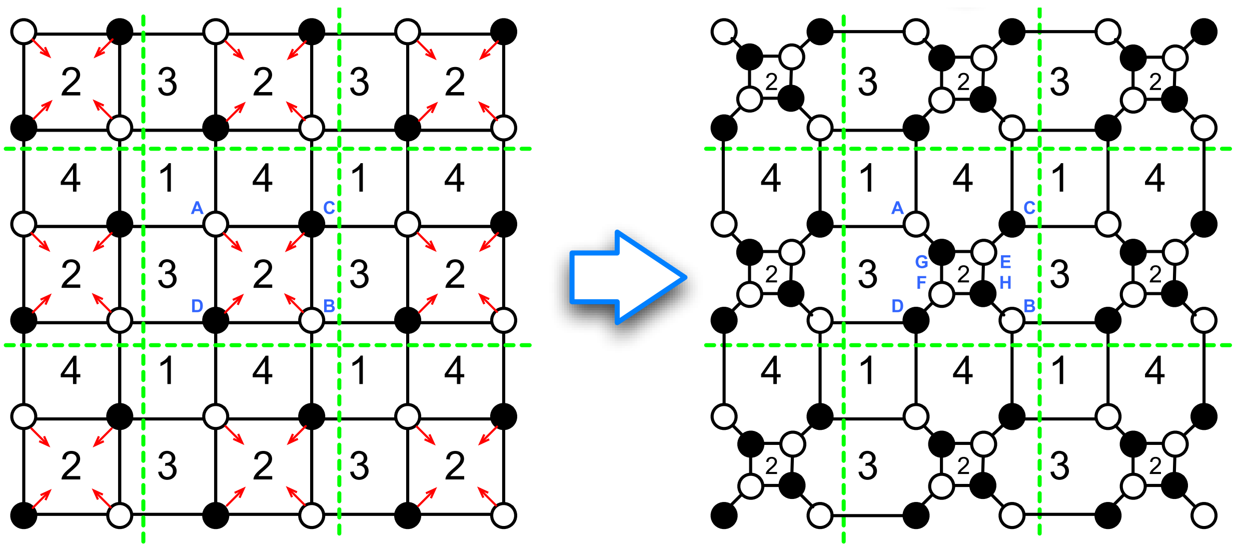

Brane Tiling
Dimer Graphs
Calabi-Yau
Hilbert Series
Quiver Gauge Theory
Mutation #1

Toric Duality

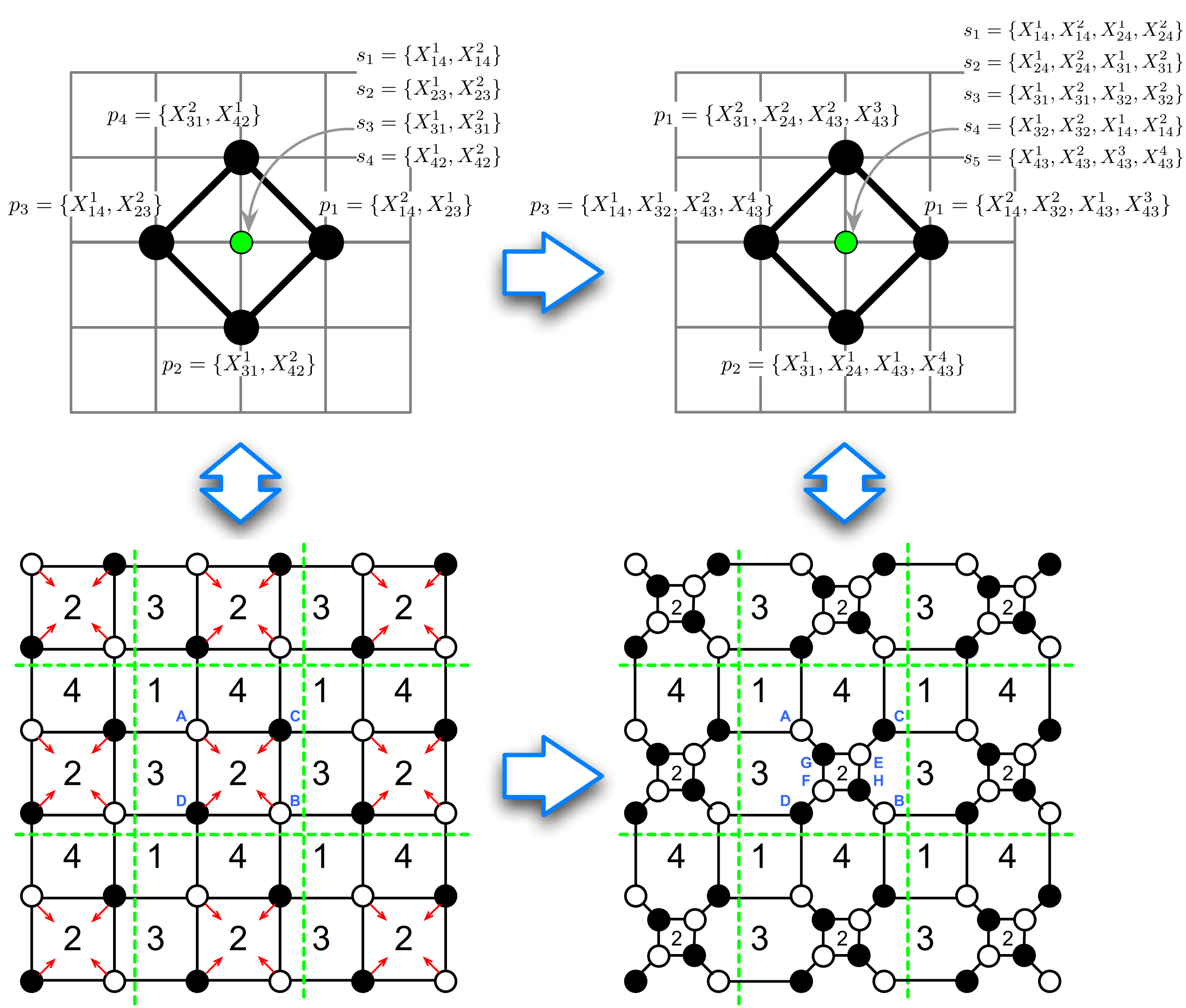
Multiple brane tilings can correspond to the same toric diagram



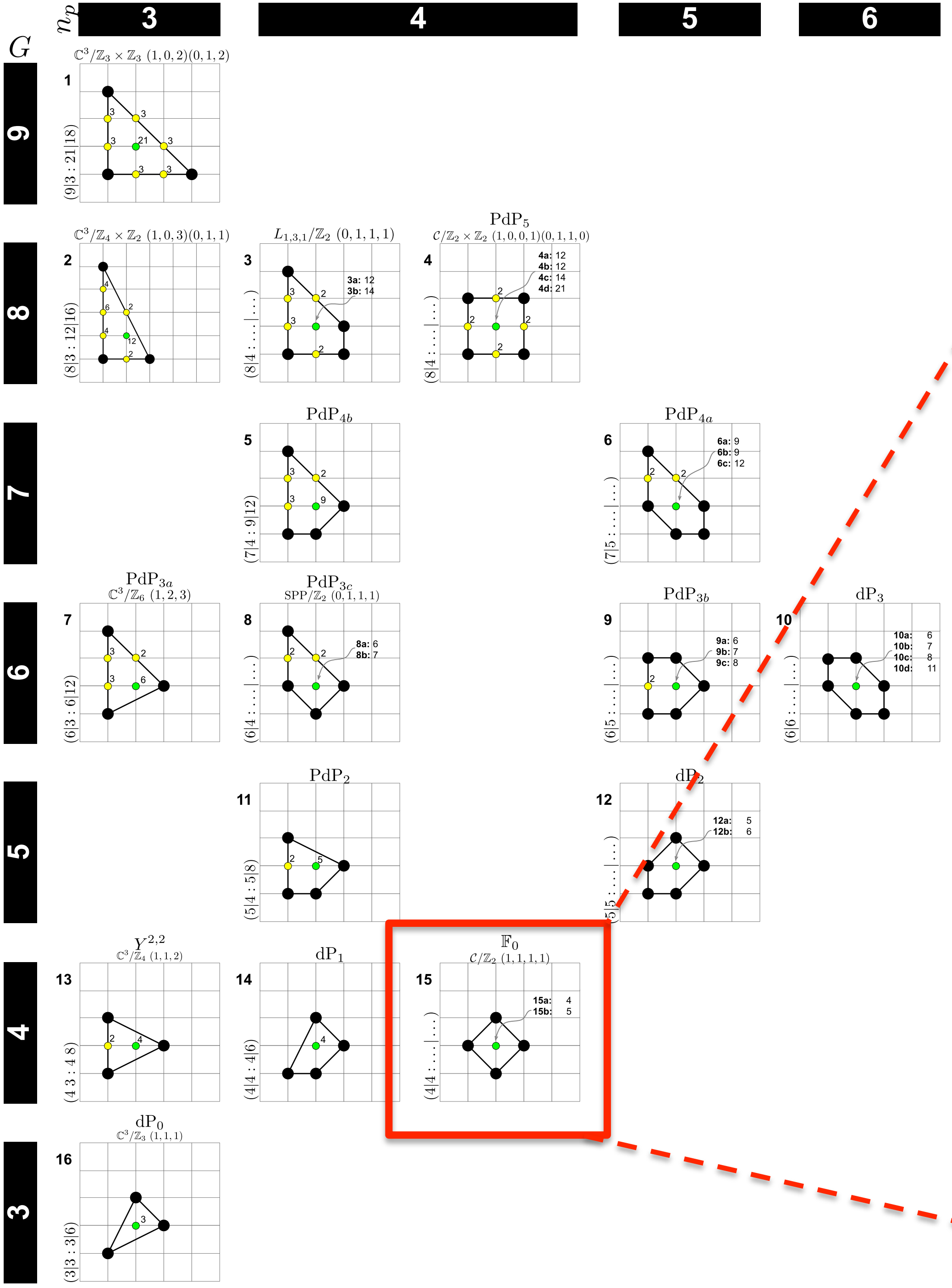
The dual brane tilings are related by a mutation known as urban renewal

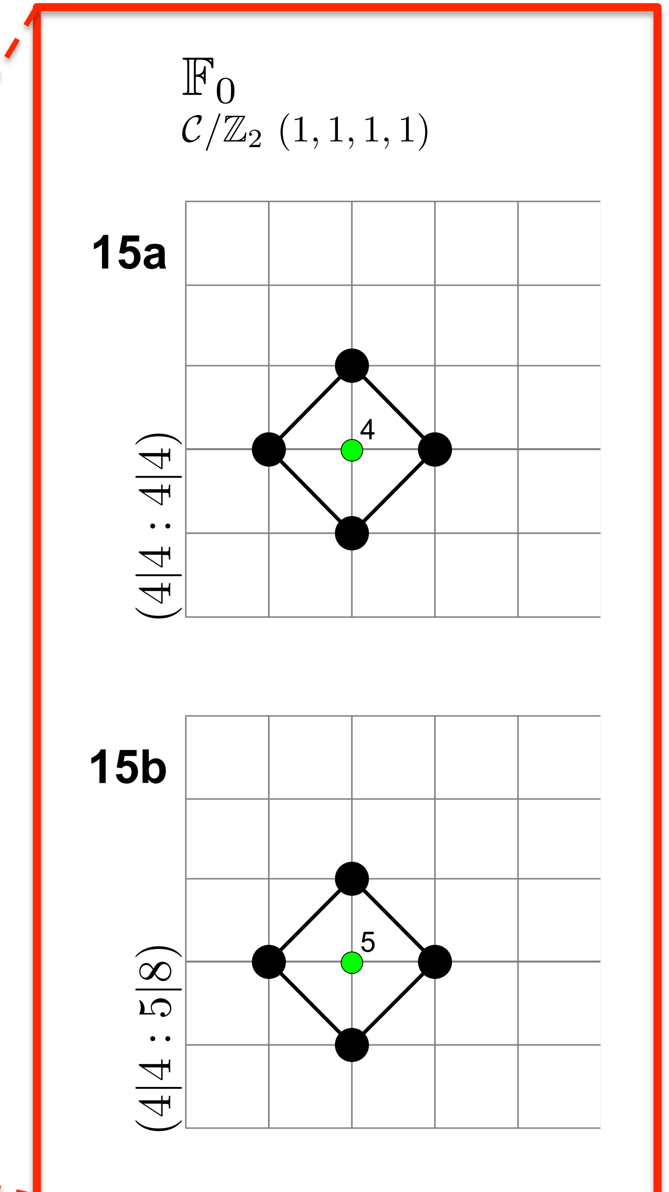


Toric Duality

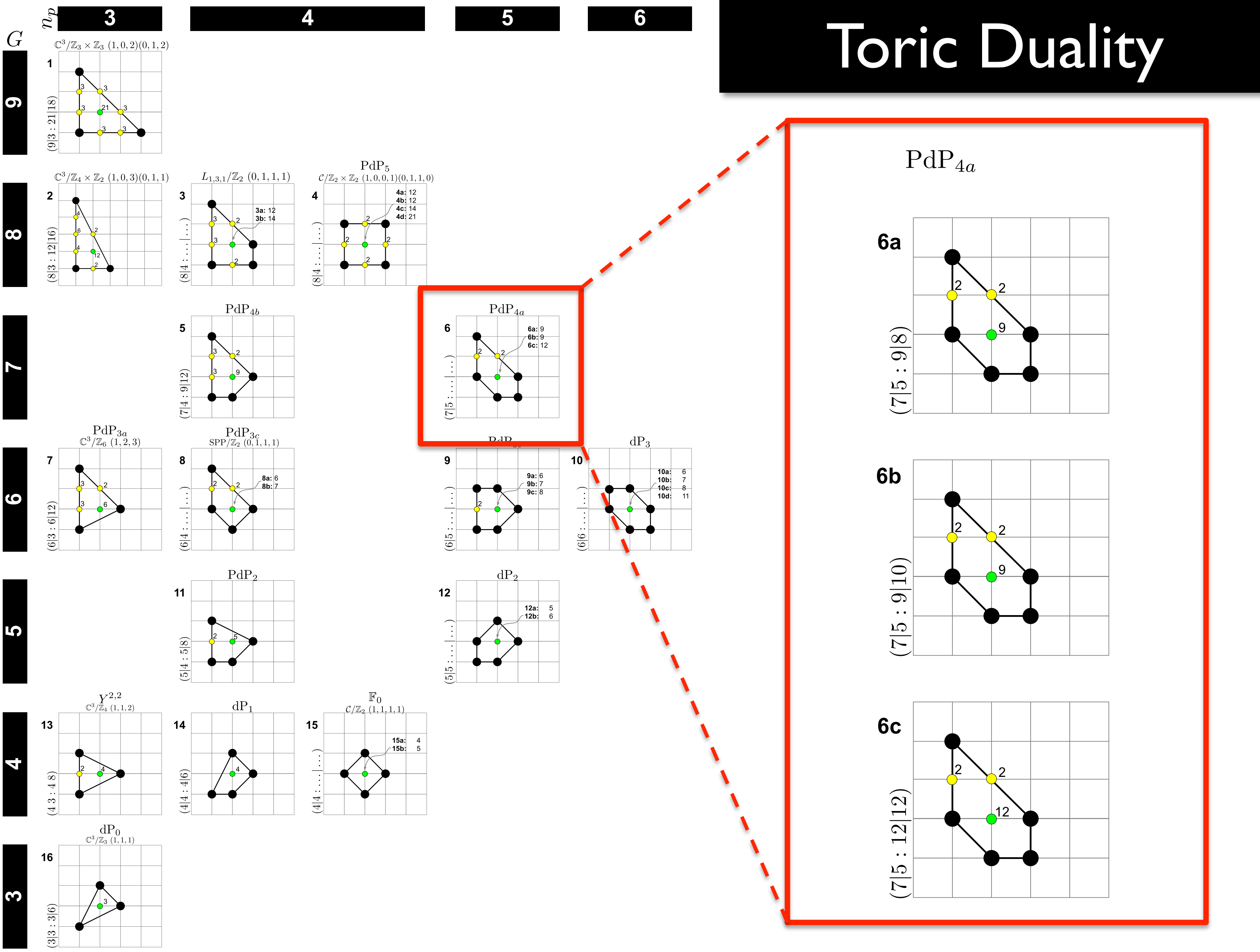


Toric Duality



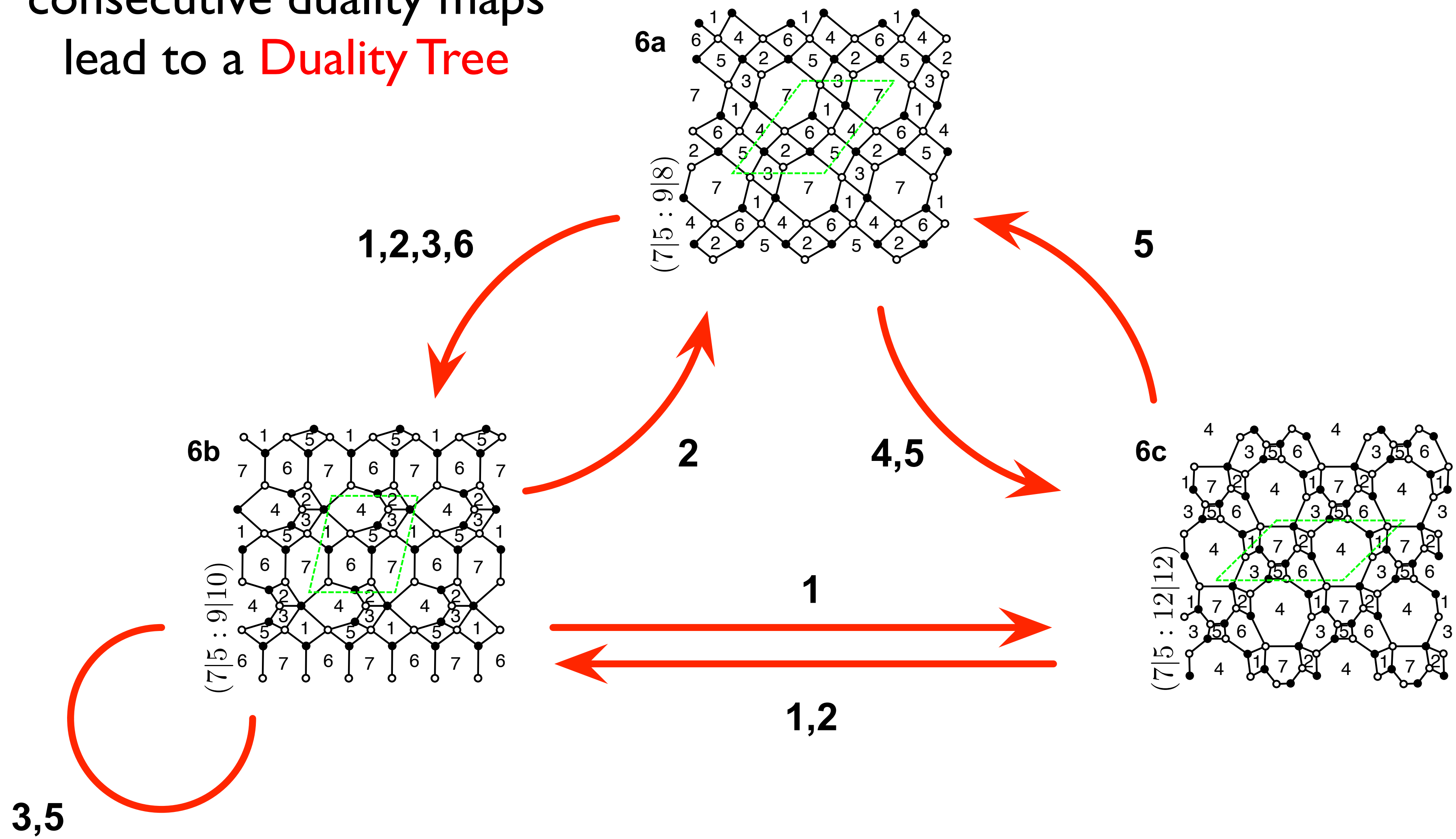


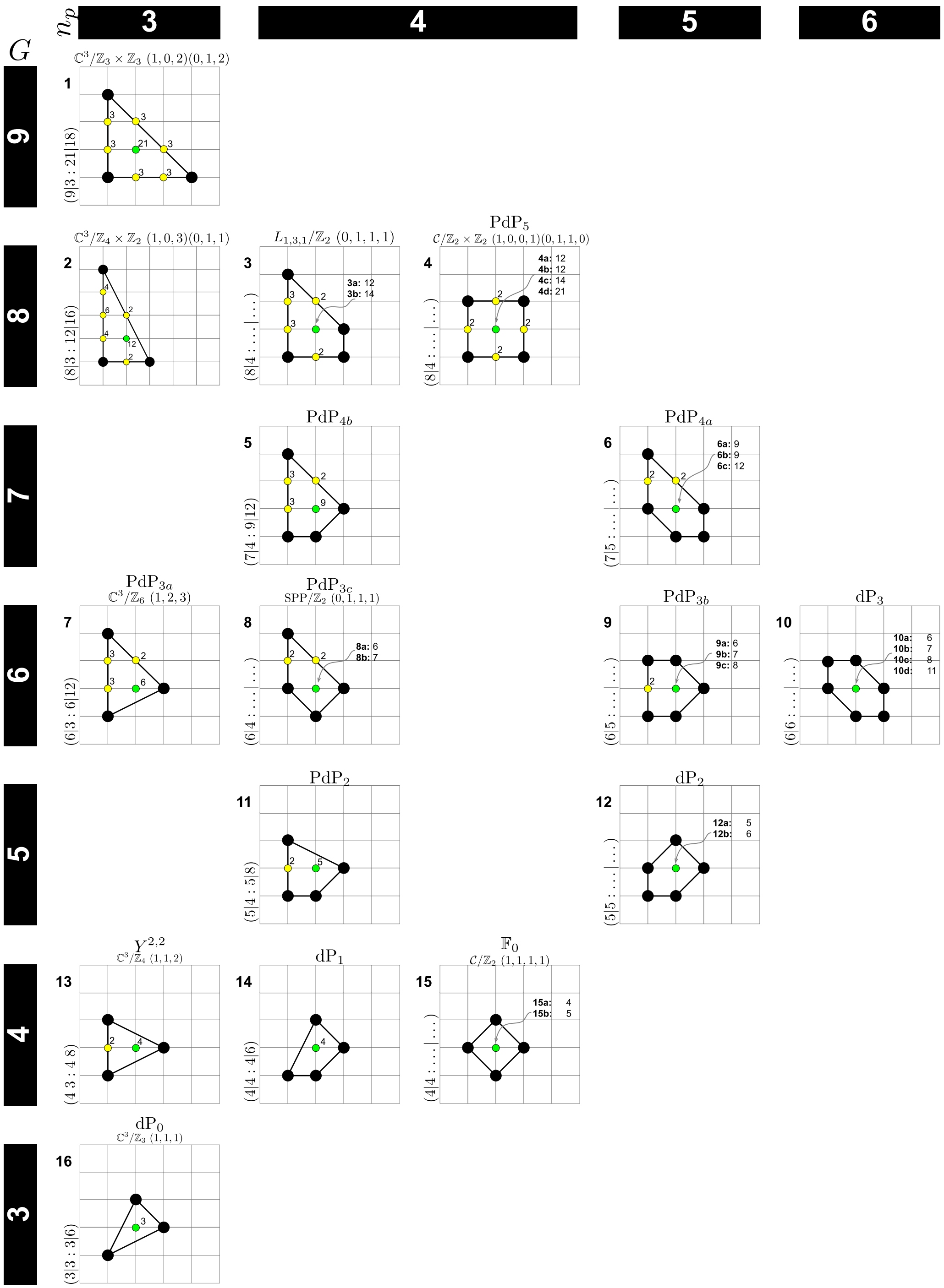
Toric Duality



Toric Duality Tree

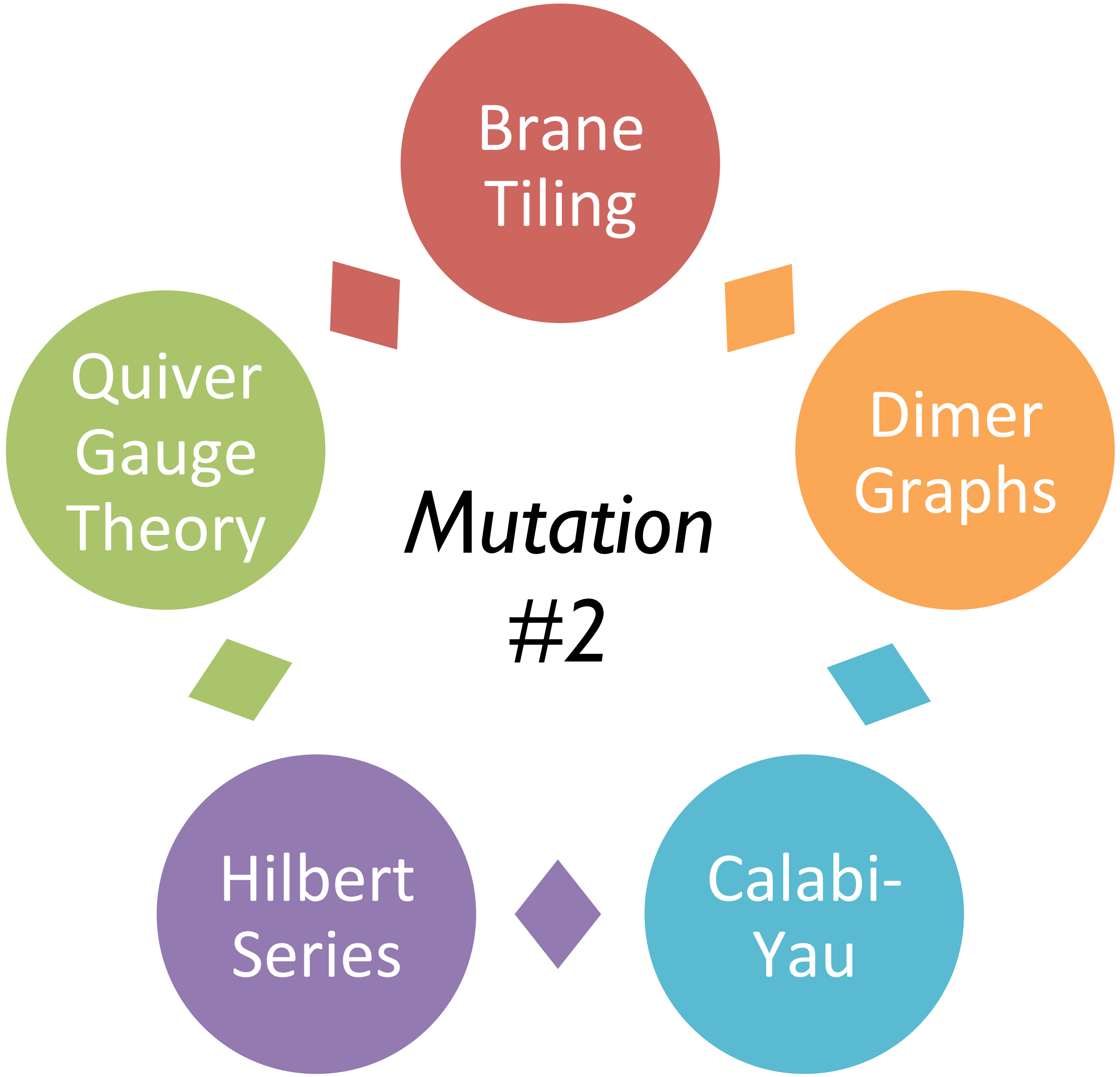
consecutive duality maps lead to a Duality Tree

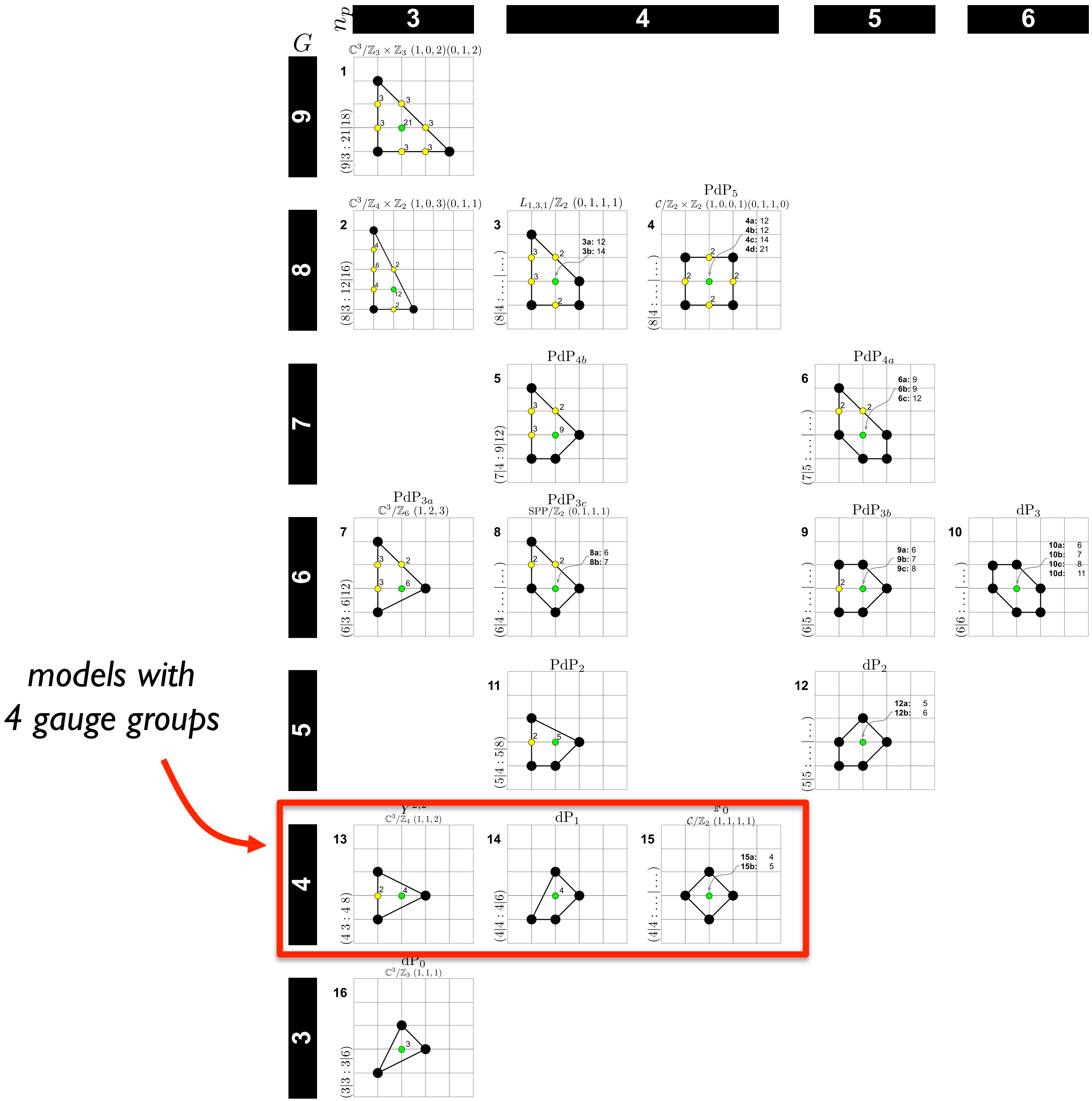




Toric Duality

- there are 30 brane tilings for the 16 reflexive polygons
- 8 reflexive polygons have multiple brane tilings associated to them


Brane Tiling
Dimer Graphs
Calabi-Yau
Hilbert Series
Quiver Gauge Theory
Mutation #2


models with
4 gauge groups
dP$_0$
dP$_1$
dP$_2$
dP$_3$
PdP$_2$
PdP$_{3a}$
PdP$_{3b}$
PdP$_{3c}$
PdP$_{4a}$
PdP$_{4b}$
PdP$_5$

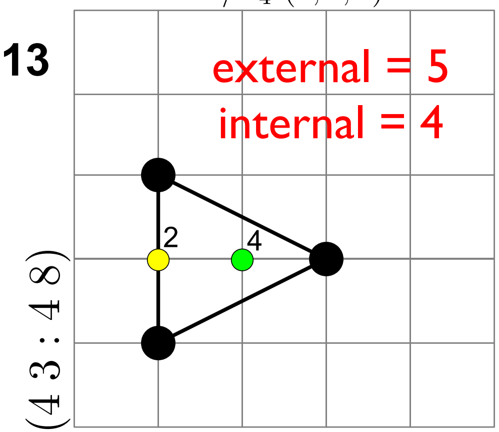
A new correspondence?

4

$Y^{2,2}$

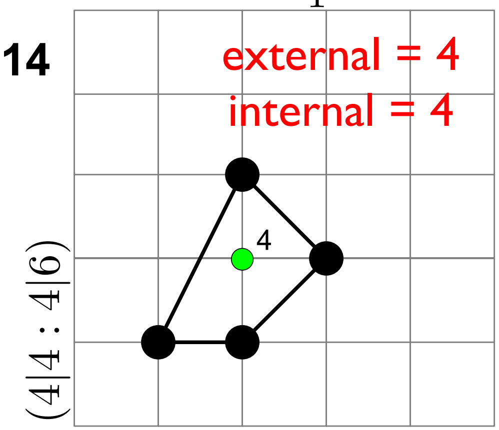
$\mathbb{C}^3/\mathbb{Z}_4\ (1,1,2)$

13



dP_1

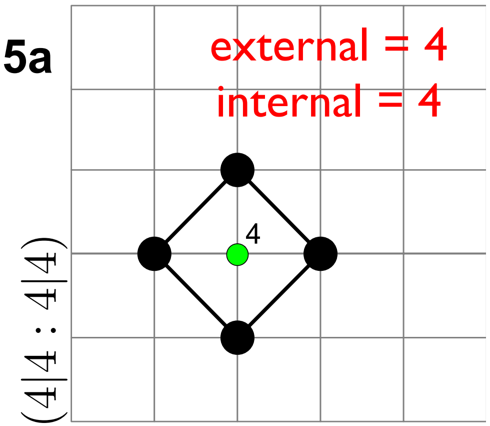
14



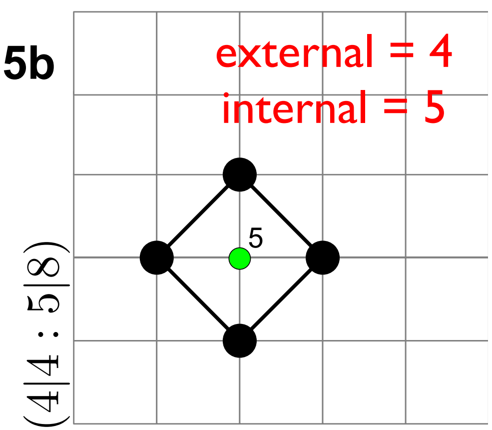
$\mathbb{F}_0$

$\mathcal{C}/\mathbb{Z}_2\ (1,1,1,1)$

15a

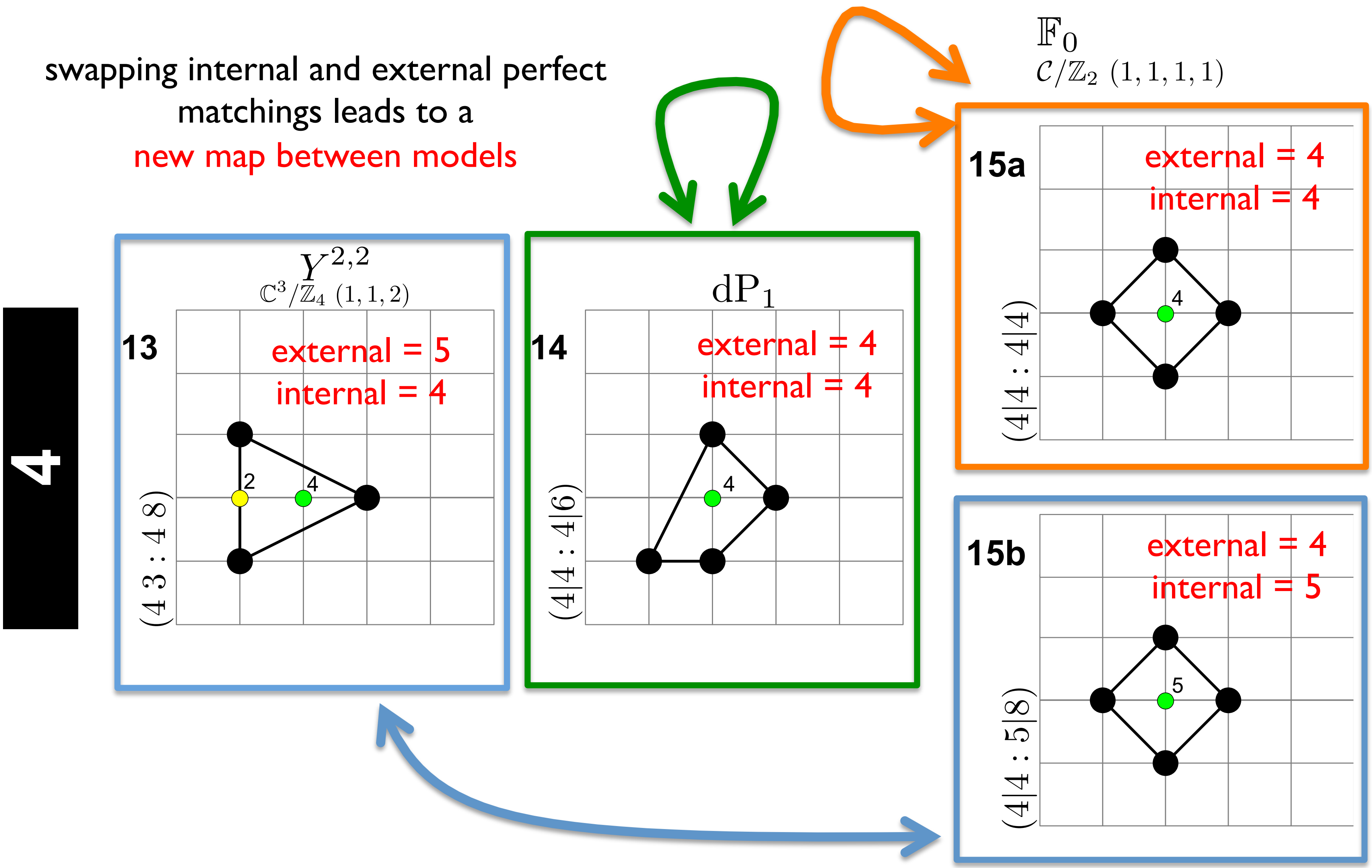


15b



A new correspondence?

swapping internal and external perfect matchings leads to a new map between models



F-term (Master) Space

$$g_1(\tilde{t}, x, f, h_i, b; {}^{\text{Irr}}\mathcal{F}^\flat_{13}) =$$
$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} \tilde{f}^{n_1+n_2-2n_3} b^{-n_1+n_2} [n_1+n_2; n_2+n_3; n_1+n_3] \tilde{t}^{n_1+n_2+2n_3}$$

Model 13

$$SU(2)_x \times U(1)_f \times U(1)_R \times SU(2)_{h_1} \times SU(2)_{h_2} \times U(1)_b$$

mesonic flavor symmetry | R-symmetry | hidden baryonic symmetry | remaining baryonic symmetry

mesonic flavor symmetry | R-symmetry | hidden baryonic symmetry | remaining baryonic symmetry

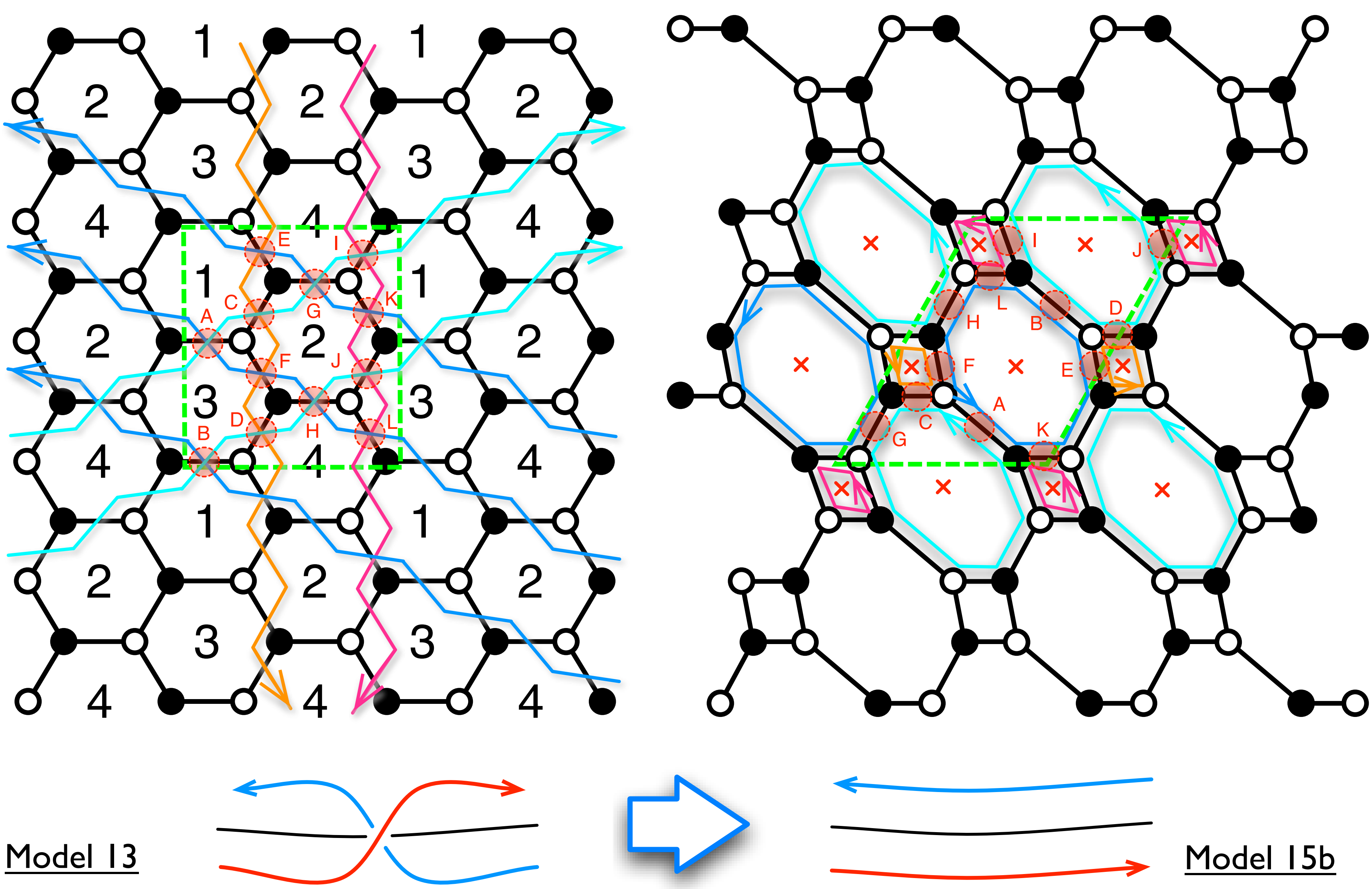
Model 15b

$$SU(2)_x \times SU(2)_y \times U(1)_R \times SU(2)_{h_1} \times U(1)_{h_2} \times U(1)_b$$

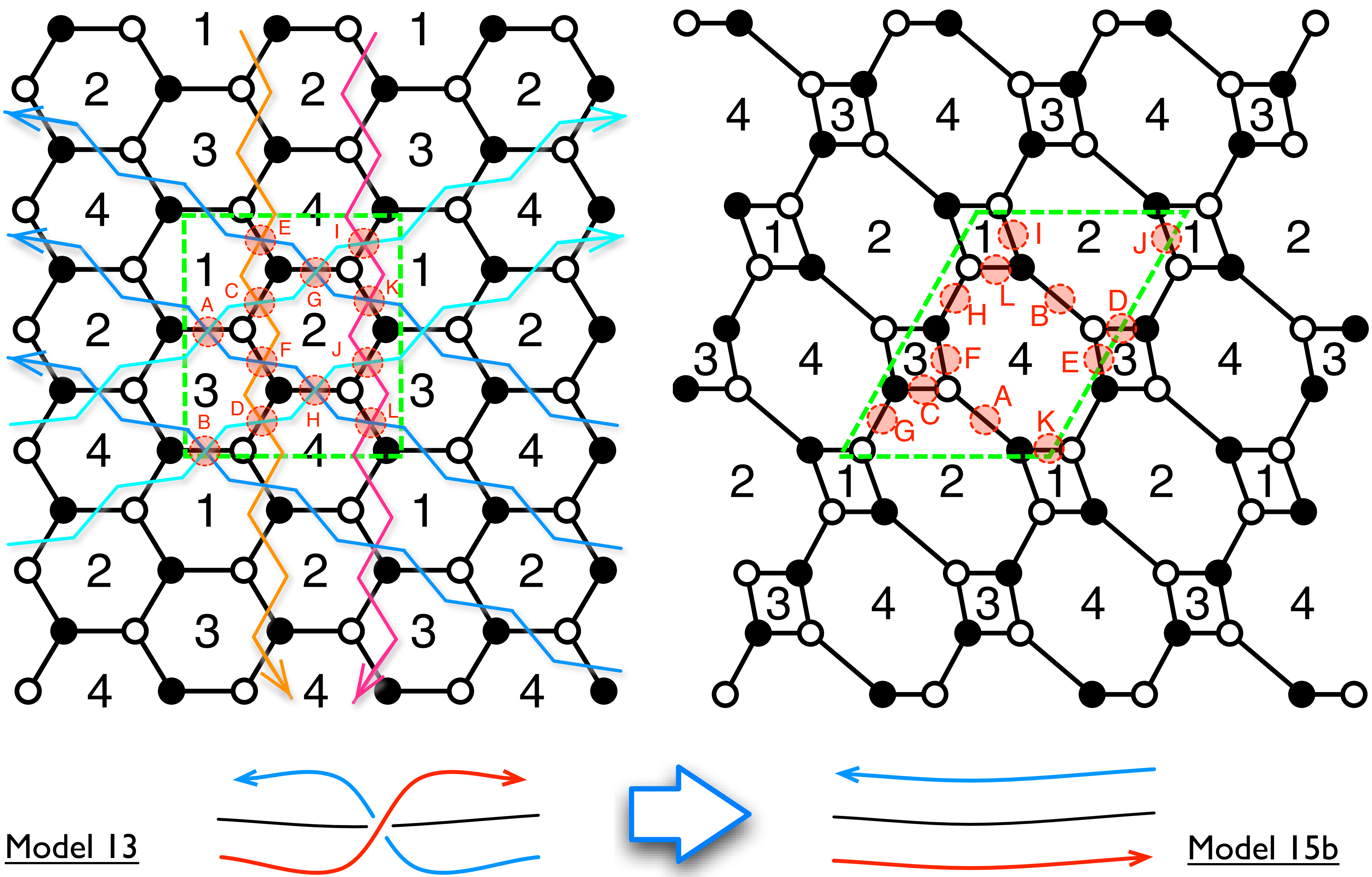
$$g_1(t, x, y, h_i, b; {}^{\text{Irr}}\mathcal{F}^\flat_{15b}) =$$
$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} h_2^{n_1+n_2-2n_3} b^{-n_1+n_2} [n_2+n_3; n_1+n_3; n_1+n_2] t^{n_1+n_2+2n_3}$$

full spectrum of mesonic and baryonic BPS operators matches -
Hilbert series matches up to swap of charge fugacities

Brane Tiling Mutation

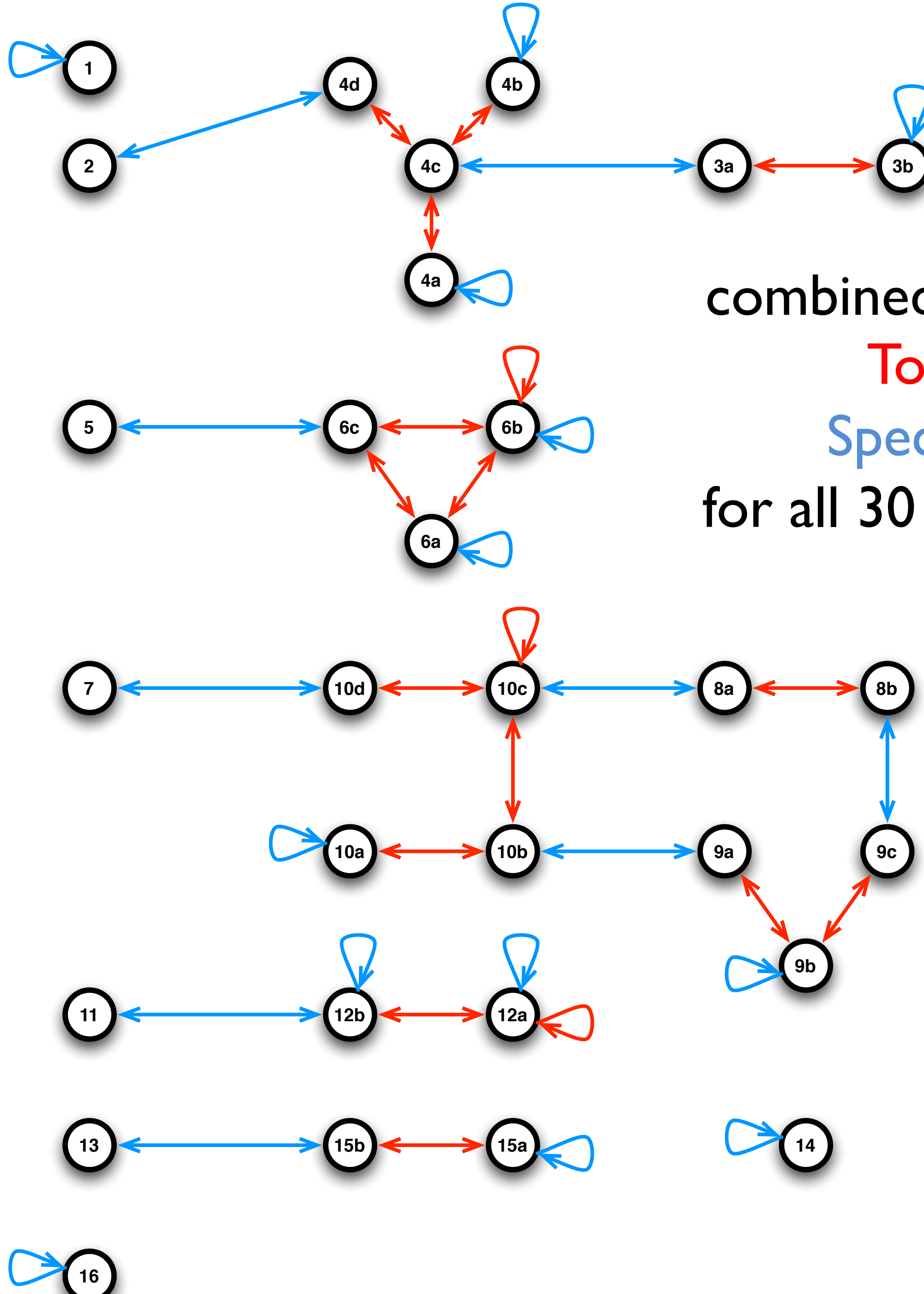


Brane Tiling Mutation



A complete mapping

all 30 brane tilings are mapped into each other by “Specular Duality”



combined duality trees for
Toric Duality
Specular Duality
for all 30 reflexive polygon models

Beyond T^2

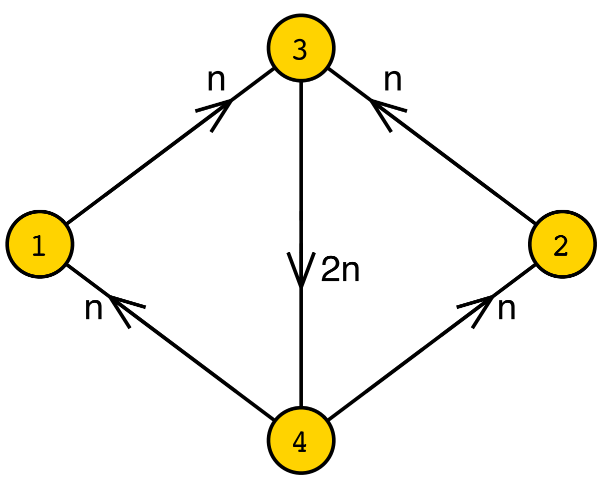
the twisting mutation leads to brane tilings/dimers on Riemann surfaces with arbitrary genus

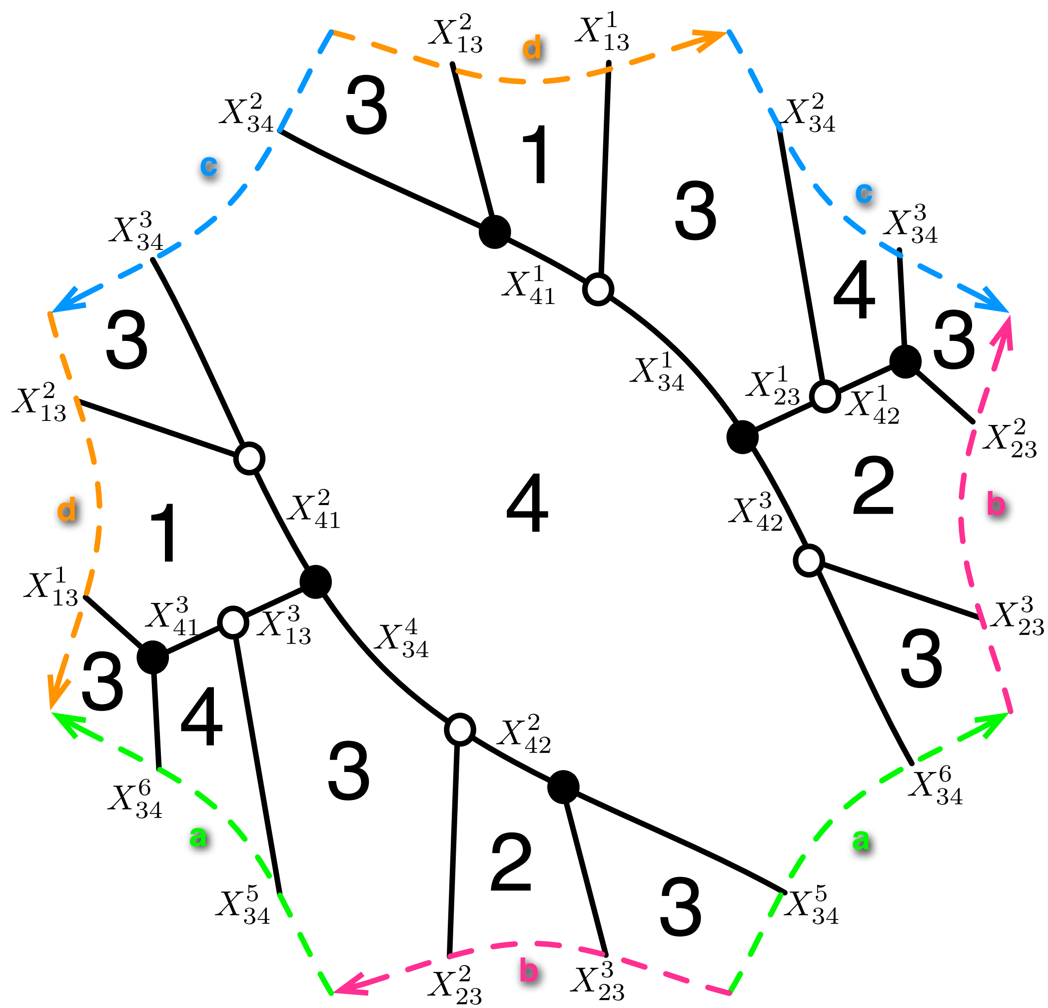
g=2 tiling corresponding to T^2 tiling for

$\mathbb{C}^3/\mathbb{Z}_6\ (1,1,4)$

the quiver for dual of

$\mathbb{C}^3/\mathbb{Z}_{2n}\ (1,1,-2)$

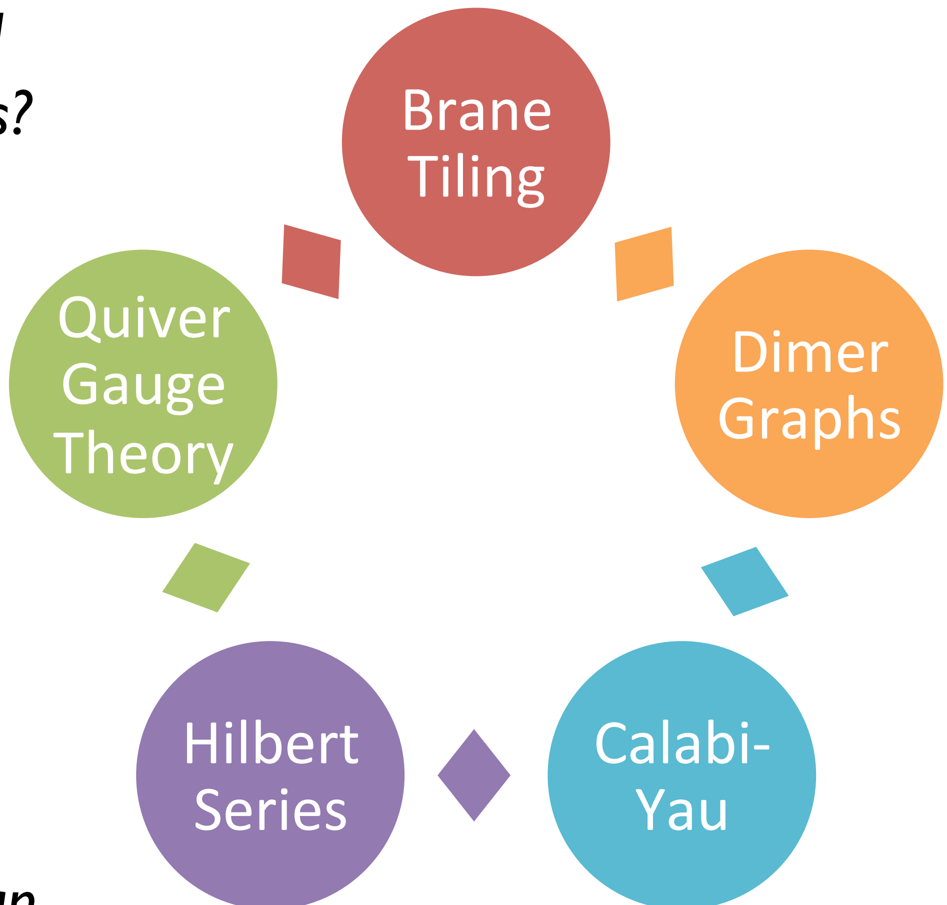




Future Directions

Does this diagram hold beyond T^2 tilings?

Are the new field theories conformal?



What new phsyics and mathematics can we learn?

New applications of tilings/dimers in string theory?