Supersymmetric Boundary Conditions in Three Dimensional N=2 Theories

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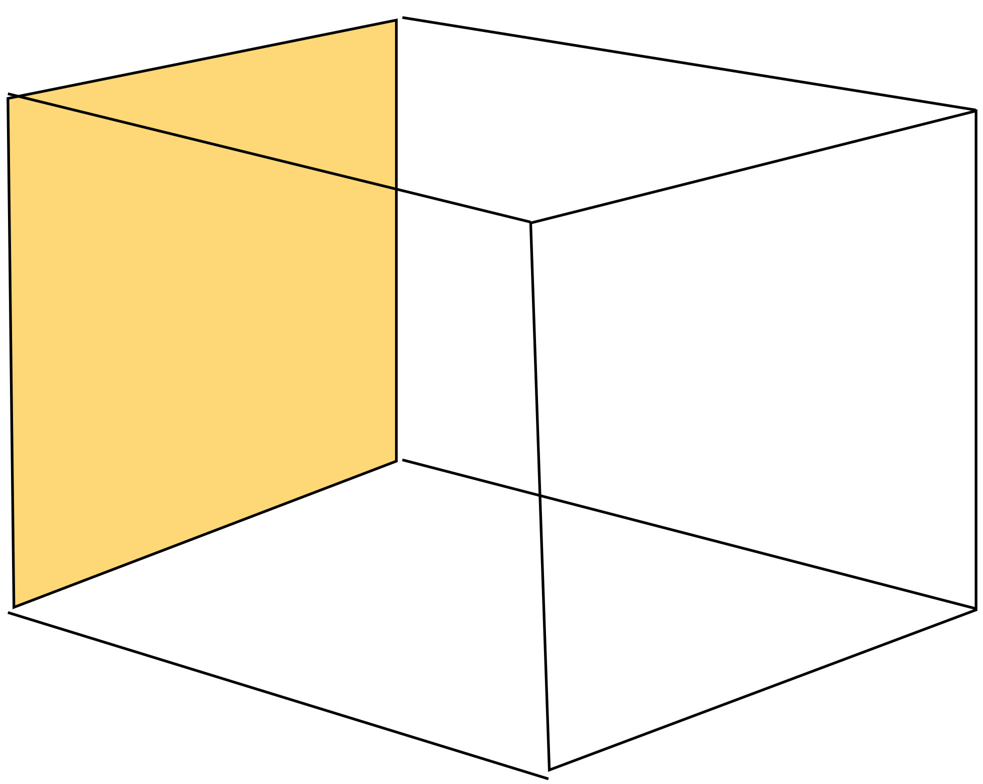
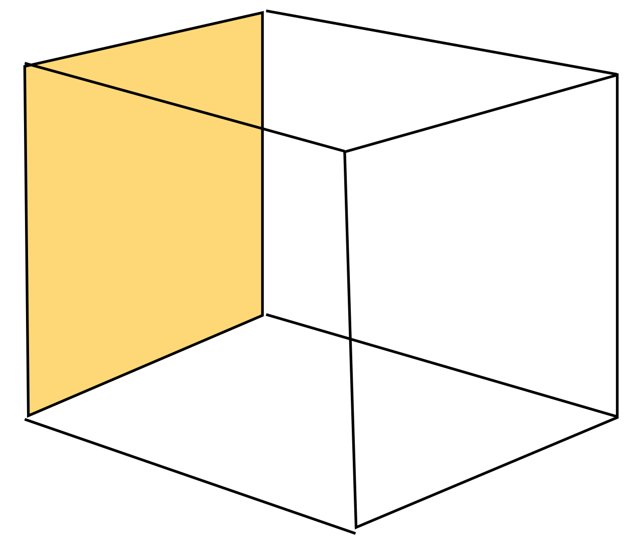
Based on T. Okazaki, SY, arXiv:1302.6593

Quantum field theory on the spacetime with

Boundary



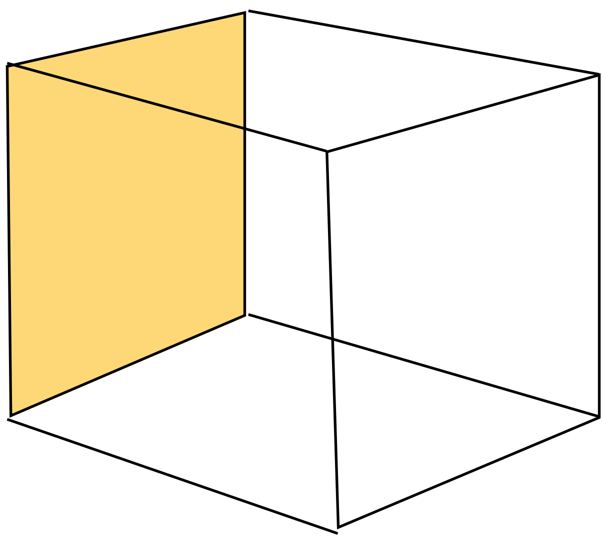
QFT with boundary



Not often appear in particle physics

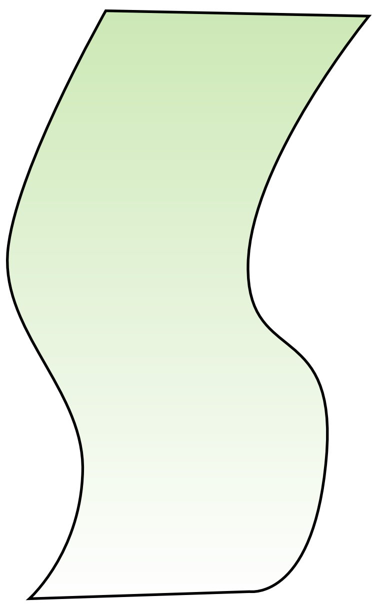
Ex. phenomenology of extra dimensions
(boundary of internal space)

QFT with boundary



Worldsheet theory of open string

2D boundary CFT



D-brane

Other dimensional spacetime?

Today's talk

3-dimensional QFT with boundary

"D-branes of membrane theory"

Boundary and symmetry

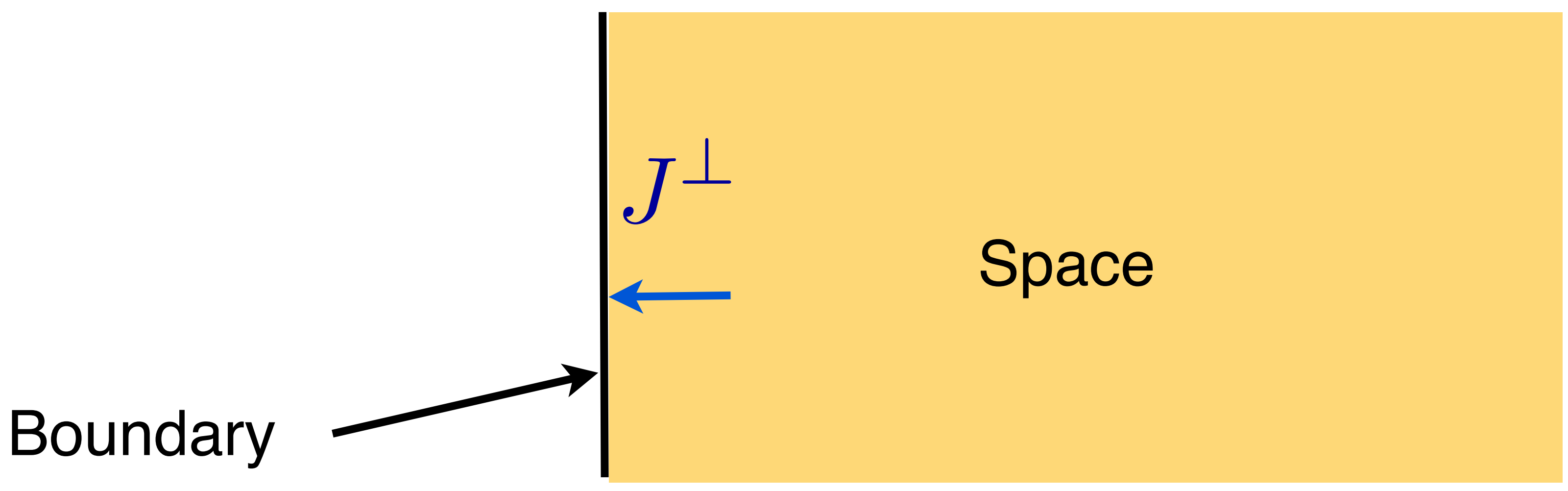
Symmetry $\Longleftrightarrow$ Current J^{μ}

$$\partial_\mu J^\mu = 0$$

$Q = \int_{\text{space}} J^0$ conserved

Generator of the transformation

When boundary exists...



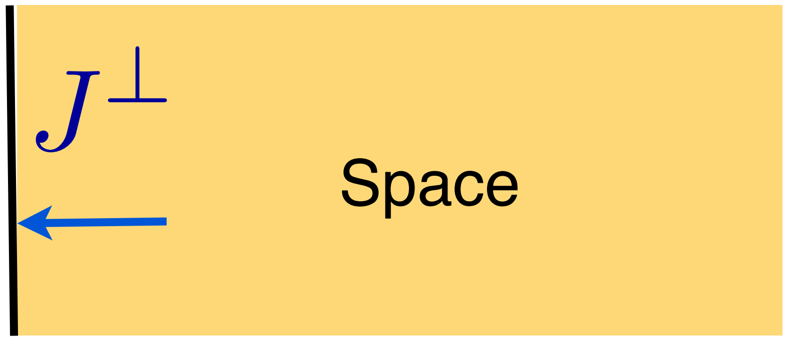
If

$$J^{\perp}|_{\text{boundary}} \neq 0$$

⇒ Charge is NOT conserved

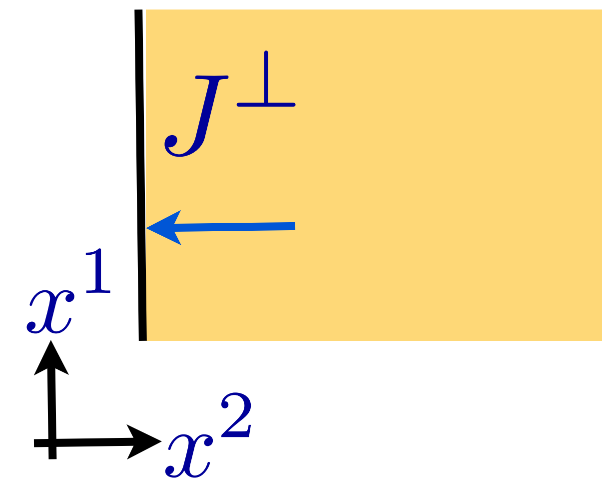
⇒ Symmetry is broken

Symmetry is preserved



$$J^{\perp}|_{\text{boundary}} = 0$$

Example: Complex scalar



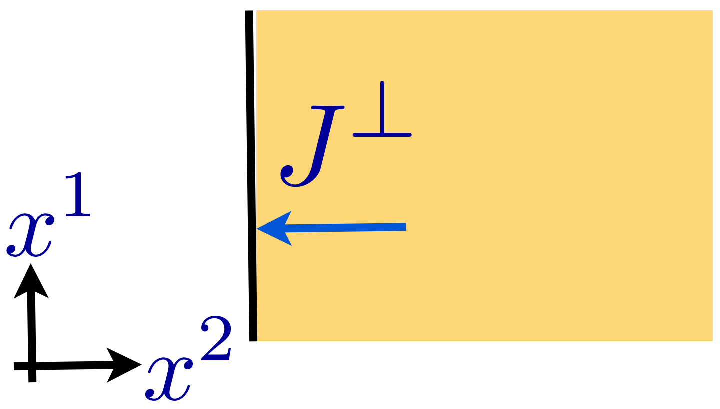
$$S = \int d^3x(-\partial_\mu\phi\partial^\mu\bar{\phi} - V(|\phi|))$$

U(1) symmetry $\phi \to e^{i\alpha}\phi$

Current $J^\mu = i\left(\bar{\phi}\partial^\mu\phi - \phi\partial^\mu\bar{\phi}\right)$

Example: Complex scalar

$$S = \int d^3x(-\partial_\mu \phi \partial^\mu \bar{\phi} - V(|\phi|))$$



At the boundary

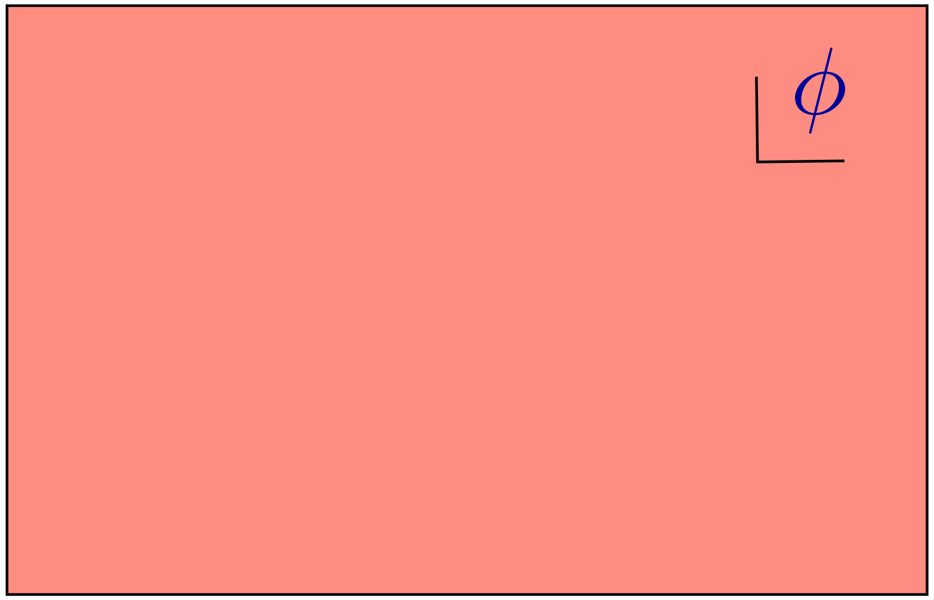
$$J^\perp = J^2 = i\left(\bar{\phi}\partial_2\phi - \phi\partial_2\bar{\phi}\right) = 0$$

Ex. 1 $\quad \partial_2\phi = 0$

Ex. 2 $\quad \phi = 0$

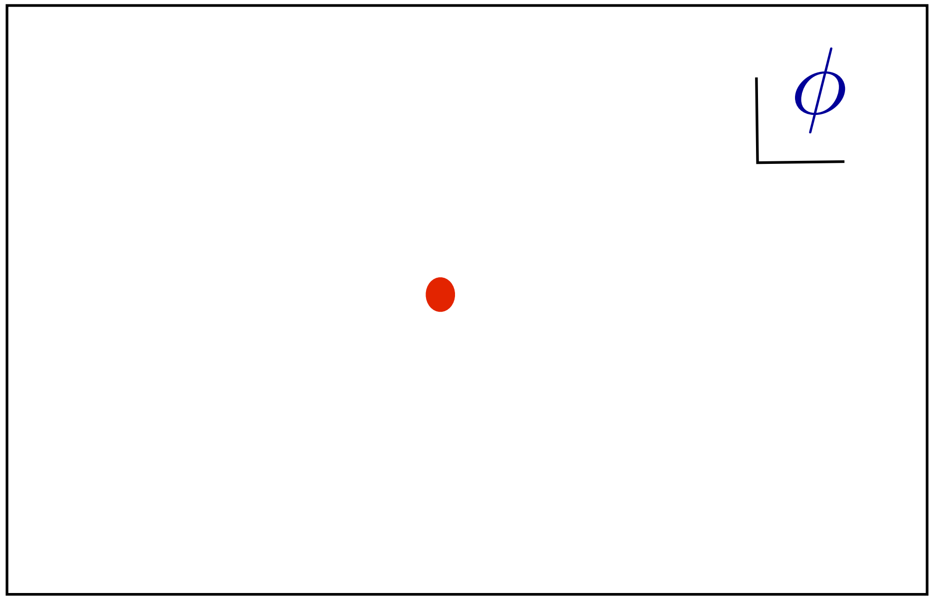
Ex. 1 $\partial_2\phi = 0$

(Neumann)



Ex. 2 $\phi = 0$

(Dirichlet)



Today’s talk

Supersymmetry and bounday

What is 1/2 SUSY preserving boundary condition?

Motivation

“D-brane of membrane”

Duality with boundary

c.f. 2dim [Ooguri,Oz,Yin], [Hori,Iqbal,Vafa]

(2d SUSY)-(4d non-SUSY) relation

[Alday,Gaiotto,Tachikawa],
[Terashima, Yamazaki], [Gaiotto,Gukov,Dimofte]

Our result 1

N=2 SUSY Landau-Ginzburg model

A-type: Lagrangian submanifold

$$\mathrm{Im}W = (\mathrm{constant})$$

B-type: Complex submanifold

$$W = (\mathrm{constant})$$

Our result 2

N=2 Maxwell theory

Mapping of the “brane” under the abelian duality

Our result 3

N=2 SUSY QED

Mapping of the “brane” under the mirror symmetry

(Conjecture)

Let us take a break

1/2 BPS boundary condition

3D N=2 SUSY similar to 4D N=1 SUSY

$$Q_\alpha, \quad \alpha = \pm$$ Complex

Real central charge

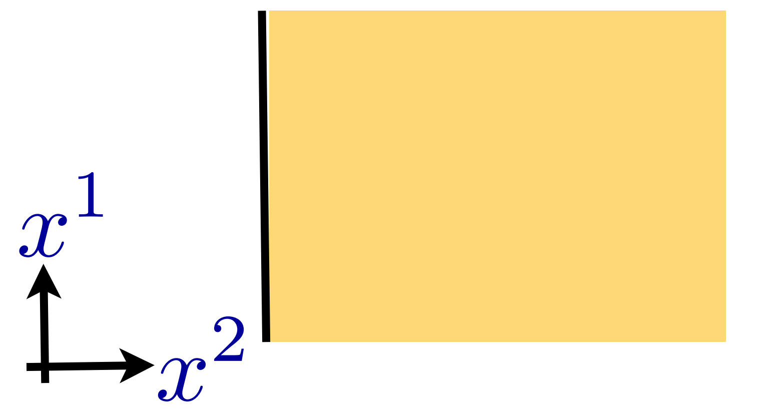
$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu + i C_{\alpha\beta} Z$$

No dotted spinor

Parameter ϵ^α

$$\delta_\epsilon = \epsilon Q - \bar{\epsilon}\bar{Q}$$

When boundary exists



It is impossible to preserve all the SUSY

because

P_2 is not preserved

P_2 should not appear in $[\delta_\epsilon, \delta_{\epsilon'}]$

Two possibilities for 1/2 BPS boundary condition

[Berman, Thompson]

N=(1,1) type (A-type) $\gamma^2\epsilon = \bar{\epsilon}$

N=(2,0) type (B-type) $\gamma^2\epsilon = \epsilon$

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$\gamma^2 = \gamma^0\gamma^1$ Chirality in 2D sense

$$\epsilon = \epsilon_1 + i\epsilon_2$$

Majorana

N=(1,1) type (A-type) $\gamma^2\epsilon = \bar{\epsilon}$

$$\gamma^2\epsilon_1 = \epsilon_1 \qquad \gamma^2\epsilon_2 = -\epsilon_2$$

N=(2,0) type (B-type) $\gamma^2\epsilon = \epsilon$

$$\gamma^2\epsilon_1 = \epsilon_1 \qquad \gamma^2\epsilon_2 = \epsilon_2$$

Remark

A-type and B-type are completely distinct.

They cannot be equivalent.

Landau-Ginzburg model

Chiral super field

$$\Phi^i(x,\theta,\bar{\theta}) = \phi^i(x) + \sqrt{2}\theta\psi^i(x) + \theta\theta F^i(x) + \cdots$$

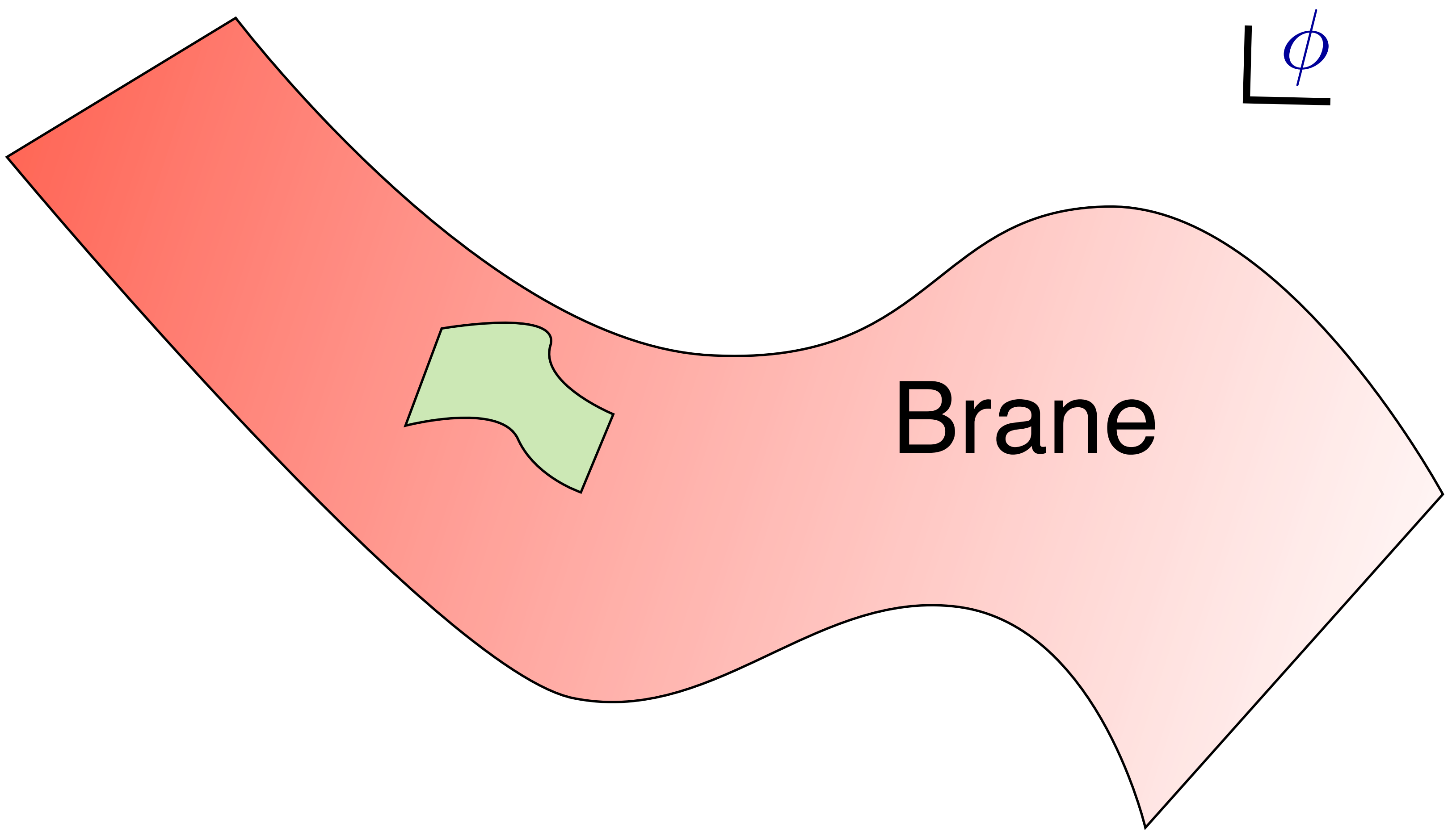
Lagrangian

$$L = K(\Phi,\bar{\Phi})|_D + W(\Phi)|_F + (c.c.)$$

SUSY current

$$J^\mu = -\sqrt{2}K_{i\bar{j}}(\partial^\mu\bar{\phi}^{\bar{j}})\psi^i + \sqrt{2}K_{i\bar{j}}(\partial_\nu\bar{\phi}^{\bar{j}})\gamma^{\mu\nu}\psi^i - \sqrt{2}i\gamma^\mu\bar{\psi}^{\bar{i}}\bar{W}_{\bar{i}},$$
$$\bar{J}^\mu = -\sqrt{2}K_{i\bar{j}}(\partial^\mu\phi^i)\bar{\psi}^{\bar{j}} + \sqrt{2}K_{i\bar{j}}(\partial_\nu\phi^i)\gamma^{\mu\nu}\bar{\psi}^{\bar{j}} + \sqrt{2}i\gamma^\mu\psi^i W_i.$$

Boundary condition $\longrightarrow$ Brane



Results

A-type: Lagrangian submanifold

$$\mathrm{Im}W = (\mathrm{constant})$$

B-type: Complex submanifold

$$W = (\mathrm{constant})$$

A-type $\gamma^2\epsilon = \bar\epsilon$

$$\epsilon J^2 - \bar\epsilon\bar J^2 = 0 \qquad\qquad \gamma^2\psi^i = S^i{}_{\bar j}\bar\psi^{\bar j}$$

$$v^I := \begin{cases} \epsilon\psi^i & (I = i) \\ -\bar\epsilon\bar\psi^{\bar i} & (I = \bar i) \end{cases} \qquad w^{Ia} := \begin{cases} \bar\epsilon\sigma^a\psi^i & (I = i) \\ -\epsilon\sigma^a\bar\psi^{\bar i} & (I = \bar i) \end{cases}$$

$$g_{IJ}\partial_2\phi^I v^J = 0,$$

$$g_{IJ}\partial_a\phi^I w^{Ja} = 0,$$

$$iW_i v^i - i\bar W_{\bar i}\bar v^{\bar i} = 0, \qquad \text{where} \quad a = 0, 1$$

A-type $\partial_2 \phi^I$ Normal to the brane

$\partial_a \phi^I$ Tangent to the brane

$$g_{IJ}\partial_2\phi^I v^J = 0, \longrightarrow v^I \text{ tangent}$$

$$g_{IJ}\partial_a\phi^I w^{Ja} = 0, \longrightarrow w^{Ia} \text{ normal}$$

$$\underline{iW_i v^i - i\bar{W}_{\bar{i}}\bar{v}^{\bar{i}} = 0,} \qquad \text{where} \quad a = 0, 1$$

$$v^I \partial_I \mathrm{Im} W = 0 \longrightarrow \mathrm{Im} W = (\mathrm{constant})$$

To show Lagrangian

$$S^I{}_J = \begin{pmatrix} 0 & S^{*\bar{i}}{}_j \\ S^i{}_{\bar{j}} & 0 \end{pmatrix}$$

From the definition

$$Sv = +v \qquad Sw = -w$$

(Tangent space)=(+1 eigen space of S)

$\Rightarrow$ The brane is middle dimensional

$S^2 = 1, \ \mathrm{Tr} S = 0$

ω_{IJ} Kähler form

We can show

$\omega_{IJ} v^I v'^J$ for arbitrary tangent vectors v^I v'^I

The A-type brane is a Lagrangian submanifold

B-type $\gamma^2\epsilon = \epsilon$

$$\epsilon J^2 - \bar{\epsilon}\bar{J}^2 = 0 \qquad\qquad \gamma^2\psi^i = R^i{}_j\psi^j$$

$$\Longrightarrow \quad v^I := \begin{cases} \epsilon\psi^i & (I = i) \\ -\bar{\epsilon}\bar{\psi}^{\bar{i}} & (I = \bar{i}) \end{cases} \qquad z^{Ia} := \begin{cases} \epsilon\sigma^a\psi^i & (I = i) \\ -\bar{\epsilon}\sigma^a\bar{\psi}^{\bar{i}} & (I = \bar{i}) \end{cases}$$

$$\begin{aligned} g_{IJ}\partial_2\phi^I v^J &= 0, \\ g_{IJ}\partial_a\phi^I z^{Ja} &= 0, \\ u^i W_i + \bar{u}^{\bar{i}}\bar{W}_{\bar{i}} &= 0. \end{aligned}$$

B-type

$$g_{IJ}\partial_2\phi^I v^J = 0, \longrightarrow v^I \text{ tangent}$$

$$g_{IJ}\partial_a\phi^I z^{Ja} = 0, \longrightarrow z^{Ia} \text{ normal}$$

$$u^i W_i + \bar{u}^{\bar{i}} \bar{W}_{\bar{i}} = 0.$$

B-type

$$S^I{}_J = \begin{pmatrix} R^i{}_j & 0 \\ 0 & R^{*\bar{i}}{}_{\bar{j}} \end{pmatrix}$$

$$Sv = -v \qquad\qquad Sz = z$$

tangent normal

We can show

$$\omega_{IJ} v^I z^J = 0$$

Complex submanifold

$$u^I := \begin{cases} i\bar{\epsilon}\psi^i & (I = i) \\ i\epsilon\bar{\psi}^{\bar{i}} & (I = \bar{i}) \end{cases}$$

u is tangent

$$u^i W_i + \bar{u}^{\bar{i}} \bar{W}_{\bar{i}} = 0$$

W is constant on the brane

Results

A-type: Lagrangian submanifold

$$\mathrm{Im}W = (\mathrm{constant})$$

B-type: Complex submanifold

$$W = (\mathrm{constant})$$

Maxwell theory

Vector super field

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta\bar{\theta}\sigma - i\theta\theta\bar{\theta}\bar{\lambda} + i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x).$$

Field strength

$$\Sigma := -\frac{i}{2}\bar{D}DV.$$

Maxwell action

$$L = -\frac{1}{e^2}\Sigma^2|_D$$

$$= \frac{1}{e^2}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\bar{\lambda}\sigma^\mu\partial_\mu\lambda - \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}D^2\right)$$

SUSY current

$$J^{\mu} = -iF^{\mu\nu}\gamma_{\nu}\bar{\lambda} + \frac{i}{2}\epsilon^{\mu\rho\sigma}\bar{\lambda}F_{\rho\sigma} + \gamma^{\mu\nu}\bar{\lambda}\partial_{\nu}\sigma - \bar{\lambda}\partial^{\mu}\sigma,$$

$$\bar{J}^{\mu} = +iF^{\mu\nu}\gamma_{\nu}\lambda - \frac{i}{2}\epsilon^{\mu\rho\sigma}\lambda F_{\rho\sigma} + \gamma^{\mu\nu}\lambda\partial_{\nu}\sigma - \lambda\partial^{\mu}\sigma.$$

Duality

Vector superfield

$$V$$

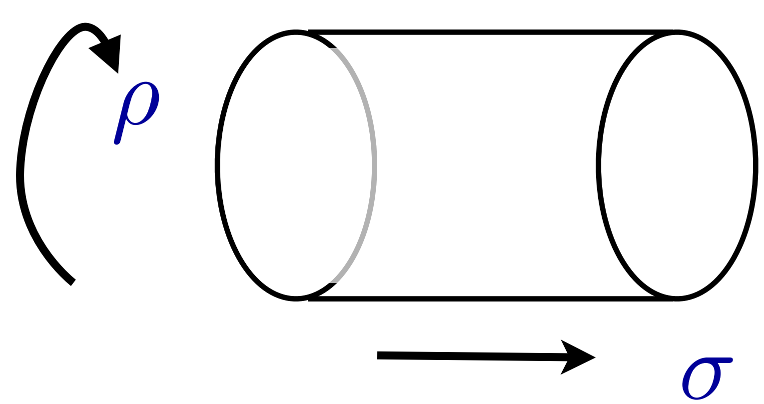
$$V = -\theta\sigma^{\mu}\bar{\theta}A_{\mu} + i\theta\theta\bar{\theta}\sigma - i\theta\theta\bar{\theta}\bar{\lambda} + i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x).$$

Chiral superfield

$$\Phi$$

$$\phi = \sigma + i\rho$$

dual photon

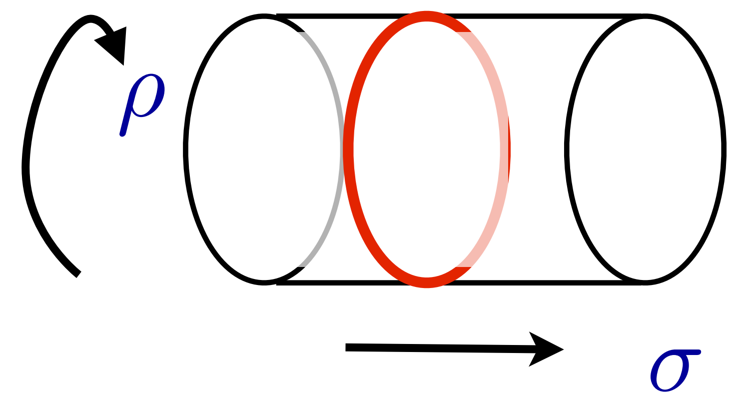


A-type

Ex.1

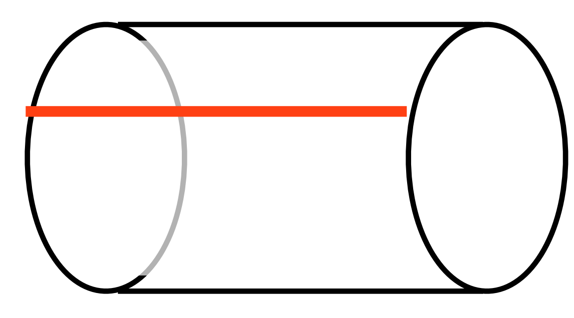
$$\sigma = (\text{const.}), \quad F_{01} = 0$$

Dirichlet



Ex.2

$$\partial_2 \sigma = 0, \quad F_{2a} = 0$$



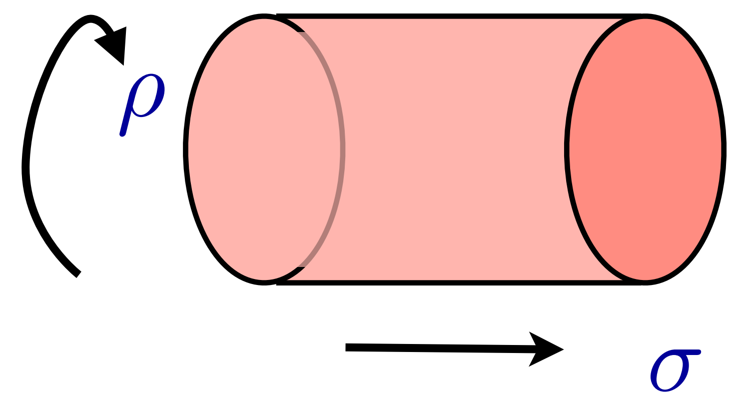
These branes are actually Lagrangian submanifolds

B-type

Ex.3

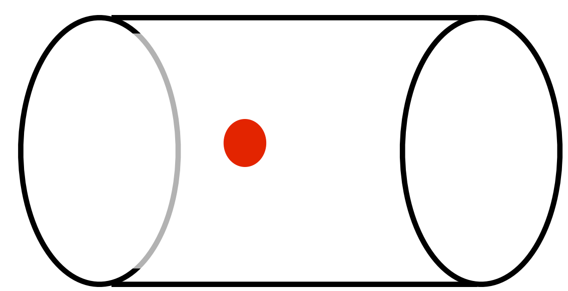
$$\partial_2\sigma = 0,\ \ F_{01} = 0$$

Dirichlet



Ex.4

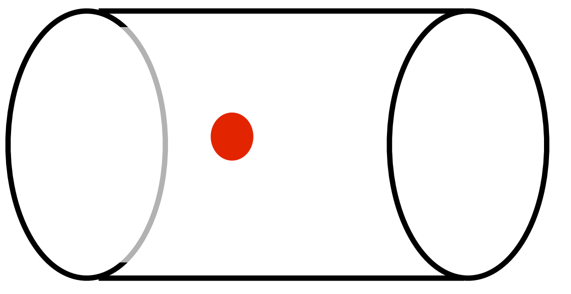
$$\sigma = (\text{const.}),\ \ F_{2a} = 0$$



They are actually complex submanifolds

Ex.4

$\sigma = (\text{const.}),\ \ F_{2a} = 0$



Position in ρ direction

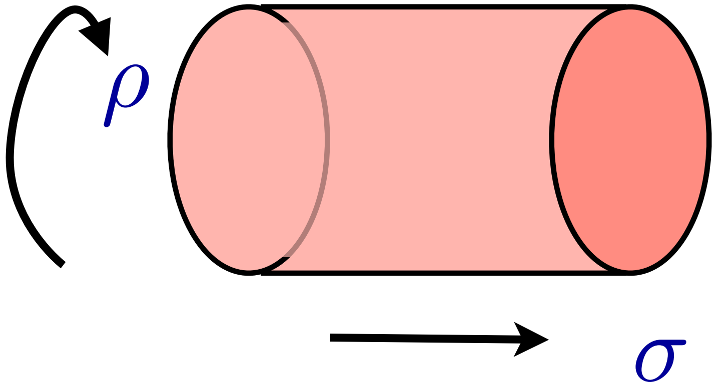
$\updownarrow$

Boundary theta term

$$S_\vartheta = \frac{\vartheta}{2\pi}\int_{x^2=0} dx^0 dx^1 F_{01}$$

Ex.3

$\partial_2\sigma = 0,\ \ F_{01} = 0$



No boundary theta term

QED

Field contents

Field strength

Vector super field V $\longrightarrow$ Σ

Charged chiral super fields $\Phi_+, \ \Phi_-$

Lagrangian

$$L = \left[-\frac{1}{e^2}\Sigma^2 + \bar{\Phi}_+ e^{-2V} \Phi_+ + \bar{\Phi}_- e^{+2V} \Phi_- \right] |_D$$

Example of B-type boundary condition

$$\gamma^2\psi_\pm = \psi_\pm, \quad \gamma^2\lambda = -\lambda, \quad \phi_\pm = 0, \quad \partial_2\sigma = F_{01} = 0$$

Mirror symmetry

[Aharony, Hanany, Intriligator, Seiberg, Strassler],
[de Boer, Hori, Oz]

Ex.

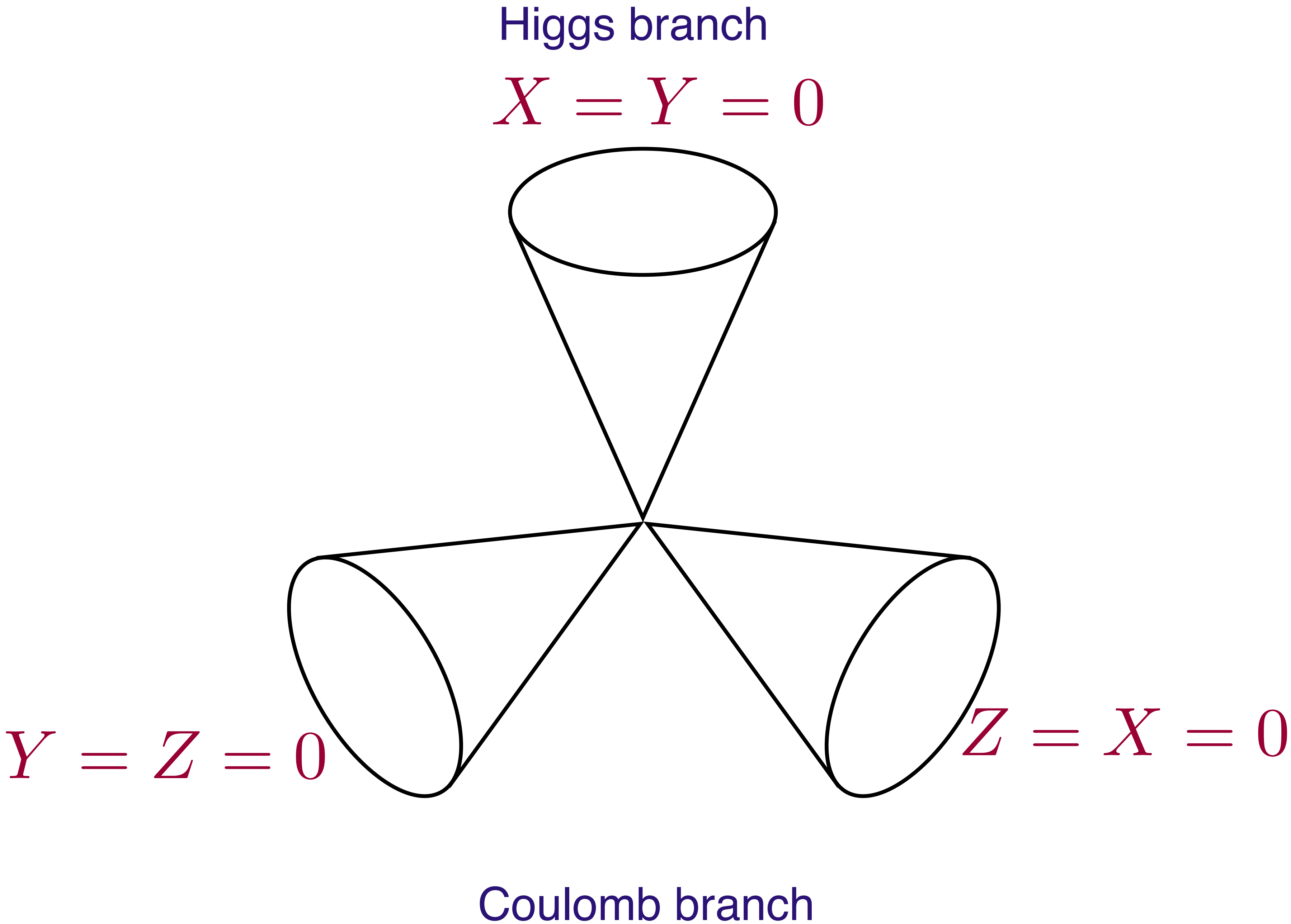
QED = XYZ model

LG model with 3 chiral super fields

$$X, Y, Z$$

$$W = XYZ$$

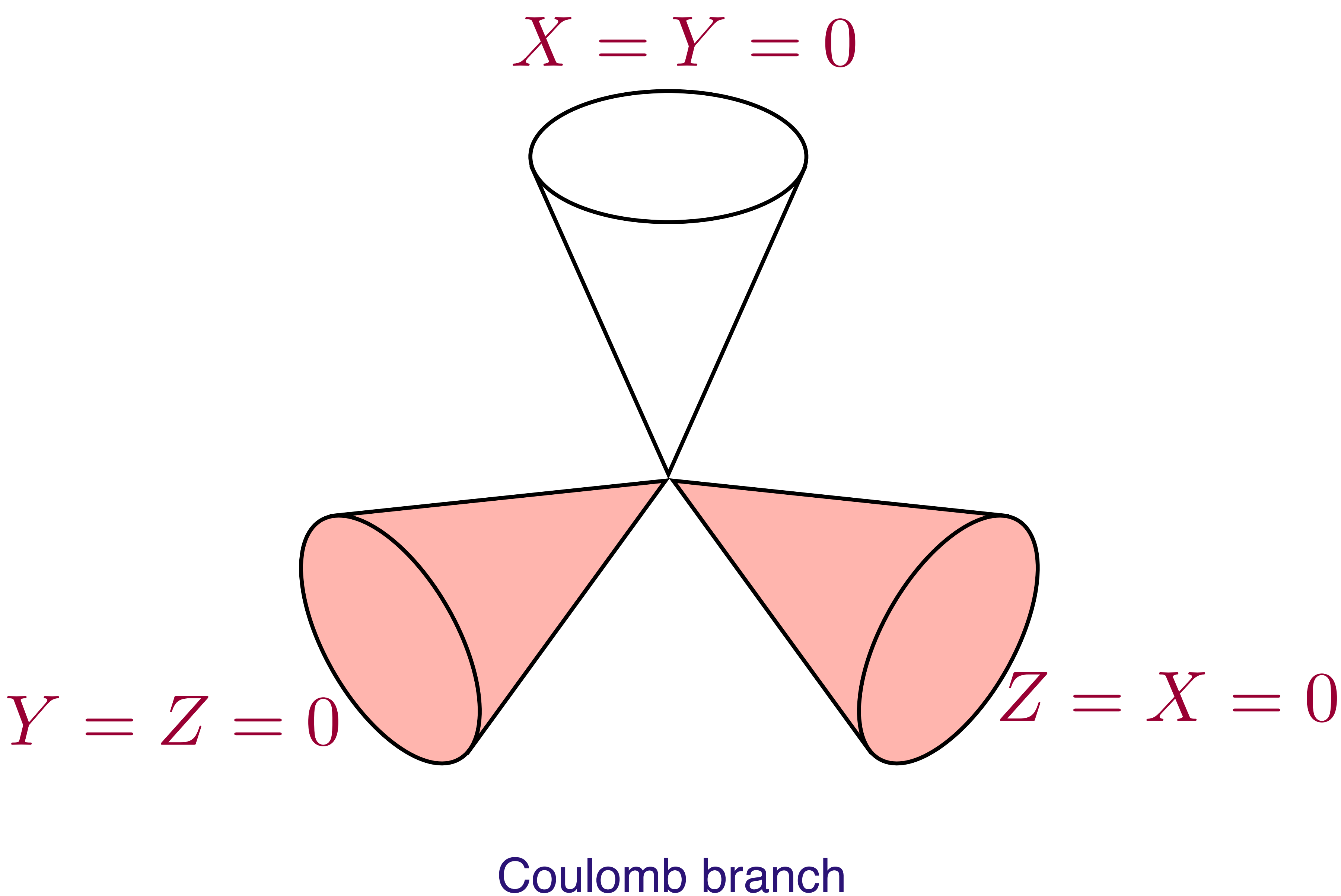
Moduli space



Example of map of B-type boundary condition

QED $\phi_\pm = 0, \quad \partial_2 \sigma = F_{01} = 0.$ $\Leftrightarrow$ XYZ $Z = 0$

Complex submanifold
superpotential is constant



Summary

3D N=2 theories

1/2 BPS boundary condition are explored

Landau-Ginzburg

Maxwell

Duality

QED

Future problem

Calculating superconformal index

(For B-type boundary,
superconformal index will exists)