Quantum quench in matrix models: Dynamical phase transitions, equilibration and the Generalized Gibbs Ensemble

10 May Seminar at IPMU

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(→ Kentucky (from Oct.))

Reference

1302.0859 with G. Mandal (TIFR, India)

Introduction

Introduction

◆ Understanding of time evolutions in string theory is almost unexplored topics.

Even gravity, which is just a low energy effective theory of string, exhibits highly interesting natures/questions.

- Cosmic censorship hypothesis: Do naked singularities appear?
- How the BH entropy production happens?
- Hawking radiation and Information paradox.
- Inflation, etc....

→ Why don’t we study time evolutions of large-N gauge theories, which is another aspect of string theories?

Introduction

But solving the time evolution of large-N gauge theories is generally very difficult... Even numerical computations will not work properly.

⇒ Today, I will introduce a simple matrix model and show interesting time evolutions, which may be related to string theory and gravity.

Introduction

◆ The model: unitary matrix model

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr} \left(|D_t U|^2 \right) - \frac{a}{2N} \left(\mathrm{Tr} U + \mathrm{Tr} U^\dagger \right) \right\}$$

U(t): N × N unitary matrix

- Integrable
- related to c=1 non-critical string theory through the double scaling limit (But we have not consider the double scaling limit yet.)

We will see time evolutions which are potentially related to gravity

- quantum quench & dynamical phase transition → appearance of naked singularities
- equilibration & entropy production → black hole formation and information loss

BUT do not expect too much. The connection to BH physics is unclear at all.

Introduction

◆ Integrability vs. thermodynamics

Does thermalization happen in the integrable system in which infinite conserved charges exist?

integrable system

$$Q_m \quad (m = 1, \cdots, \infty)$$

standard thermodynamics

$$E, Q_i \quad (i : \text{finite number})$$

⇨ Recently new thermal ensemble in integrable system called "Generalized Gibbs Ensemble (GGE) " is proposed in condensed matter physics and is confirmed in several models.
We will see our matrix model indeed obeys GGE.

Introduction

◆ Integrability vs. thermodynamics

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integrable system	standard thermodynamics
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⇨ Recently new thermal ensemble in integrable system called "Generalized Gibbs Ensemble (GGE) " is proposed in condensed matter physics and is confirmed in several models. We will see our matrix model indeed obeys GGE.

⇨
- Integrablity plays crucial roles in the recent developments in string theory, e.g. spin chain, BPS objects, fuzz ball conjecture.
  → GGE may be important in these studies.
- GGE may be related to the HS/CFT correspondence too, since infinite HS charges exist in this relation.

Introduction

(Ultimate) Goals of our study

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|D_t U|^2\right) - \frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right) \right\}$$

- Understand the time evolutions of the matrix model to reveal the time evolution of string/gravity.
- Study the GGE and consider the application to string and HS theories.
- Connect string theory to condensed matter physics through the quantum quench and GGE.
- How integrable systems break to ergodic systems. (→ related to "From BPS to our world.")

Plan of today's talk

1. Introduction
2. Review of the single trace matrix model
3. Time evolution of the single trace matrix model
4. Equilibration and Generalized Gibbs Ensemble
5. Role of the critical point in the quantum quench
6. D2 brane system and chaotic dynamics
7. Summary

Review of the single trace matrix models

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|D_t U|^2\right) - \frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right) \right\}$$

To analyze this model, Fermion description is convenient.

Separate the diagonal component as

$$U = V \begin{pmatrix} e^{i\theta_1} & & & \\ & e^{i\theta_2} & & \\ & & \ddots & \\ & & & e^{i\theta_N} \end{pmatrix} V^\dagger \qquad \theta_i = \theta_i + 2\pi$$

(θ_i can be regarded as the position of the i-th fermion on S^1.)

It is known that V can be gauged away and $\{\theta_i\}$ behave as N fermions on S^1.

∵) We can rewrite the kinetic term of the Hamiltonian as

$$H_{kin} \sim -\mathrm{Tr}\left(\frac{\partial}{\partial U}\right)^2 \sim -\frac{1}{\Delta(\theta)}\left(\frac{\partial}{\partial \theta_i}\right)^2 \Delta(\theta) + \cdots \qquad \Delta(\theta) \equiv \prod_{i<j} \sin\left(\left(\theta_i - \theta_j\right)/2\right)$$

original bosonic wave function

$$H_{kin}\chi(\theta) \rightarrow -\left(\frac{\partial}{\partial \theta_i}\right)^2 \Delta(\theta)\chi(\theta)$$

$$\psi(\theta) \equiv \Delta(\theta)\chi(\theta)$$

This new wave function ψ is fermionic, since it is anti-symmetric under $\theta_i \leftrightarrow \theta_j$

Review of the single trace matrix models

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|D_t U|^2\right) - \frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right)\right\}$$

In terms of the Fermions, the action can be rewritten as

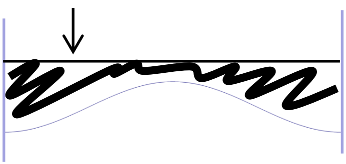
$$\frac{S}{N^2} = \int dt\ d\theta\ \psi^\dagger(\theta,t)[-i\partial_t - h(\theta,\partial_\theta)]\psi(\theta,t),$$

$$h(\theta,\partial_\theta) = -\frac{1}{N^2}\partial_\theta^2 - a\cos\theta$$: hamiltonian for a single fermion.

$\psi(\theta,t)$: second quantized fermion field.

$$\hbar = 1/N$$

fermi surface



N free fermions are in the cos potential

∵)

$$\mathrm{Tr}U + \mathrm{Tr}U^\dagger = \sum_{k=1}^{N}\left(e^{i\theta_k} + e^{-i\theta_k}\right) = 2\sum_{k=1}^{N}\cos\theta_k$$

Review of the single trace matrix models

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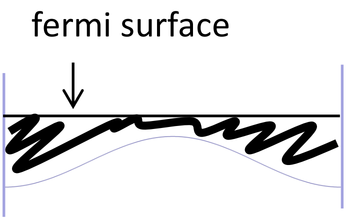

Important point

The eigen function is given by the Mathieu function.

$$h(\theta,\partial_\theta)\varphi_m(\theta) = \epsilon_m \varphi_m(\theta)$$

Mathieu function is available in Mathematica & Maple!

→ They tell us the answer! (But some critical BUGS exist in these softwares.)

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Time evolution of the single trace matrix model

◆ Quantum quench dynamics

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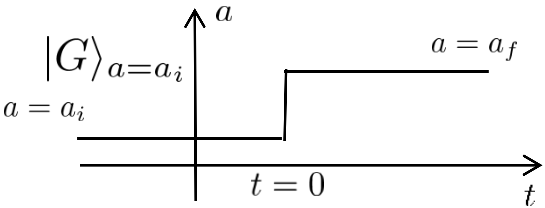
$$h(\theta,\partial_\theta) = -\frac{1}{N^2}\partial_\theta^2 - \underline{a}\cos\theta$$: The potential depth a controls the phases.

What will happen if we consider the ground state at t<0 and change the potential from a_i to a_f suddenly at t=0?



$|G\rangle_{a=a_i}$: the ground state at $a = a_i$

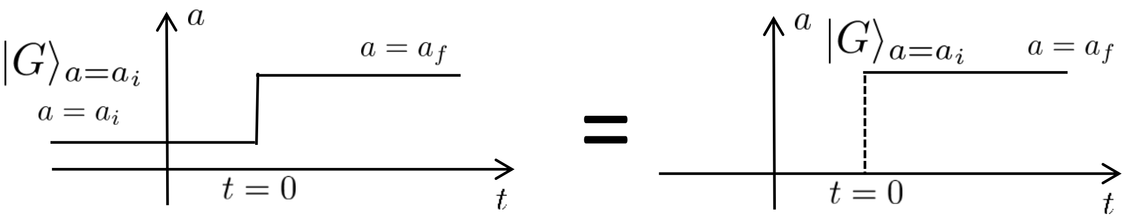
Time evolution of the single trace matrix model

◆ Quantum quench dynamics

Comment: Advantage of quantum quench dynamics

Generally solving the Schrödinger equation in a time dependent potential is difficult. However, in the quench case, what we need is just solving the equation with the Hamiltonian at $a = a_f$ with the initial configuration at t=0, which is the ground state at $a = a_i$.

→ We can avoid the time dependent potential!

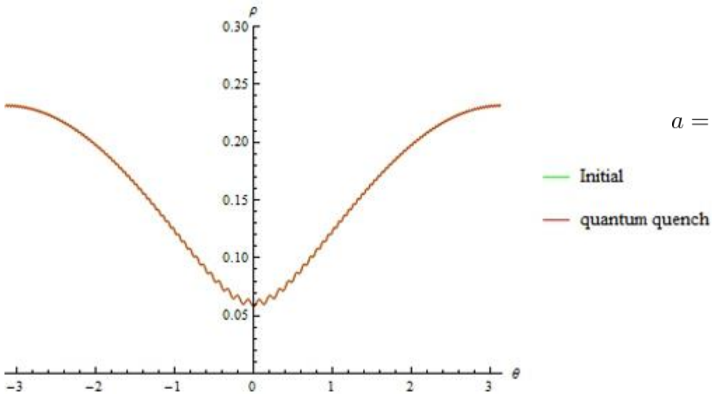


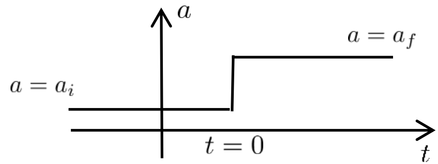
$|G\rangle_{a=a_i}$: the ground state at $a = a_i$

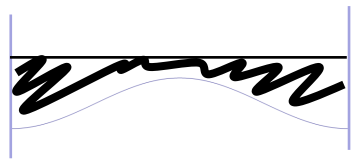
Time evolution of the single trace matrix model

◆ Quantum quench dynamics (Result at N=120)

Time evolution of the fermion density $\rho(\theta,t)=\frac{1}{N}\sum_{i=1}^{N}\delta(\theta-\theta_i)=\psi^\dagger(\theta,t)\psi(\theta,t)$







Initial density

The initial large oscillations subtle down to the small ripples.
→ Equilibration would happen even in free system.

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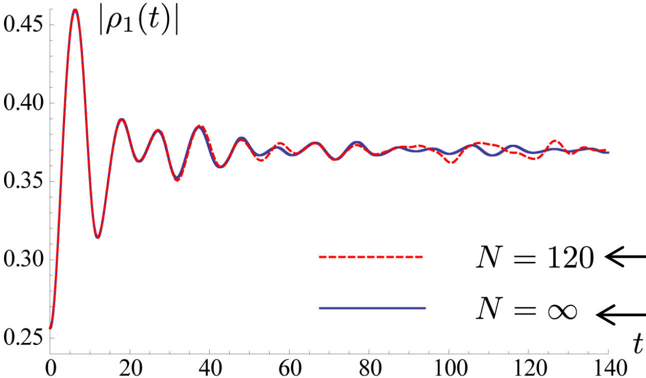
Equilibration and Generalized Gibbs Ensemble

To see the equilibration qualitatively, we evaluate the Fourier mode of ρ.

$$\rho_1(t) \equiv \int_{-\pi}^{\pi} d\theta \cos\theta \rho(\theta, t)$$: Characterize the shape of the density.

$$\rho(\theta, t) = \frac{1}{N}\sum_{i=1}^{N} \delta(\theta - \theta_i)$$



$N = 120$ ← exact analysis

$N = \infty$ ← semi-classical analysis

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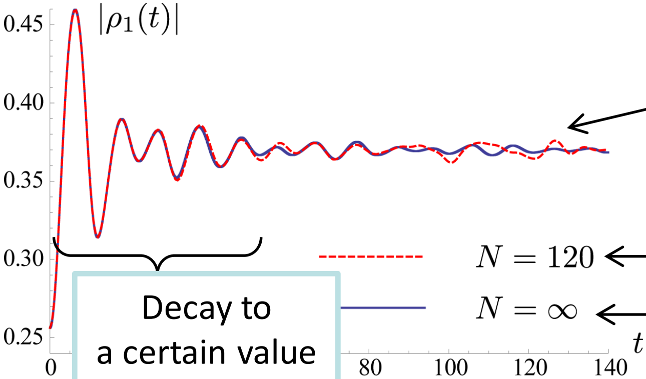

The small oscillations of N=120 are slightly larger than N=∞.
→ Poincaré recurrence

$N = 120$ ← exact analysis

$N = \infty$ ← semi-classical analysis

Equilibration and Generalized Gibbs Ensemble

◆ Poincaré recurrence

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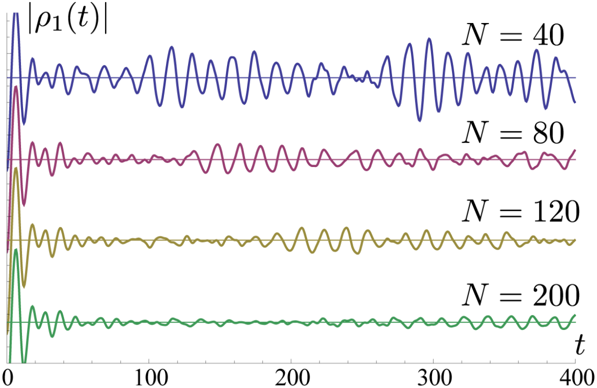
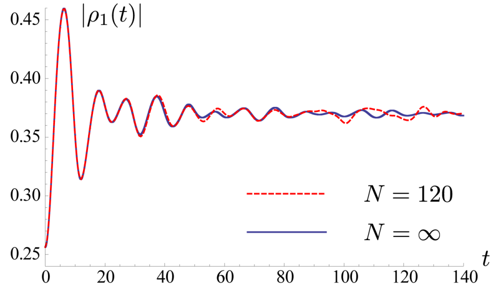




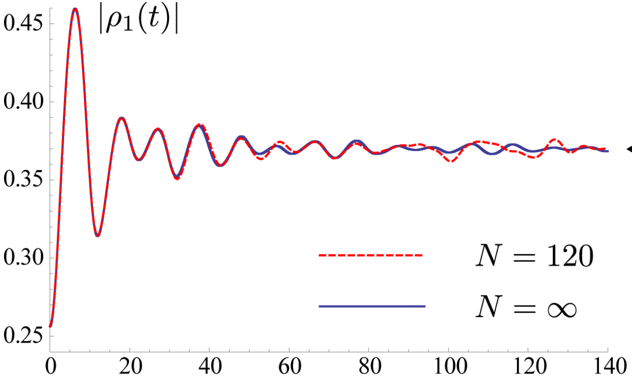
The late time oscillation decreases as N increase. → Poincaré recurrence

→ We expect the recurrence does not occur only at N=∞ and
it really equilibrates to an asymptotic state.

Equilibration and Generalized Gibbs Ensemble

◆ Equilibration and Generalized Gibbs Ensemble (GGE)

Q. Can we predict the equilibrated observables through any ensemble?



?? ←

$$\langle \rho_1 \rangle = \frac{1}{Z} \sum_n \langle n | \rho_1 e^{-\beta E_n} | n \rangle$$

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Equilibration and Generalized Gibbs Ensemble

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◆ Integrability of the free fermion system

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$$\psi(\theta,t) = \sum_m c_m \varphi_m(\theta) e^{-i\epsilon_m t}$$

eigen function

$$h(\theta,\partial_\theta)\varphi_m(\theta) = \epsilon_m \varphi_m(\theta)$$

We can define the following infinite number of conserved charges.

$$\hat{N}_m \equiv c_m^\dagger c_m \quad (m = 1, \cdots, \infty)$$

Since the the fermions are free, the fermion number $\langle \hat{N}_m \rangle$
at each level is conserved. → Infinite conserved charges → Integrable

Equilibration and Generalized Gibbs Ensemble

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◆ Integrability vs. thermodynamics

our system

$$N_m \quad (m = 1, \cdots, \infty)$$

standard thermodynamics

$$E, Q_i \quad (i : \text{finite number})$$

Number of the conserved quantities is quite different!
→ Standard thermodynamics will not work.

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⇨ Generalized Gibbs Ensemble, which was recently proposed,

may work for such integrable systems.

Equilibration and Generalized Gibbs Ensemble

◆ Generalized Gibbs Ensemble (GGE)

cf.) $\hat{\rho} = \frac{1}{Z} e^{-\beta(\hat{H} - \mu \hat{N})}$

$$\hat{\rho}_{GGE} \equiv \frac{1}{Z} \exp\left(- \sum_{m=1}^{\infty} \mu_m \hat{Q}_m \right)$$

: GGE density matrix

$Q_m, \ \hat{Q}_m$: the conserved charges in a integrable system and its operator

In our case $\hat{Q}_m \to \hat{N}_m = c_m^\dagger c_m$

μ_m : the chemical potential for each conserved charge, which is fixed at the initial state.

> A conjecture: GGE describes the asymptotic state of a generic quanum integrable model.

See a review by Polkovnikov, Sengupta, Silva, Vengalattore 2010

Equilibration and Generalized Gibbs Ensemble

◆ Generalized Gibbs Ensemble (GGE): example

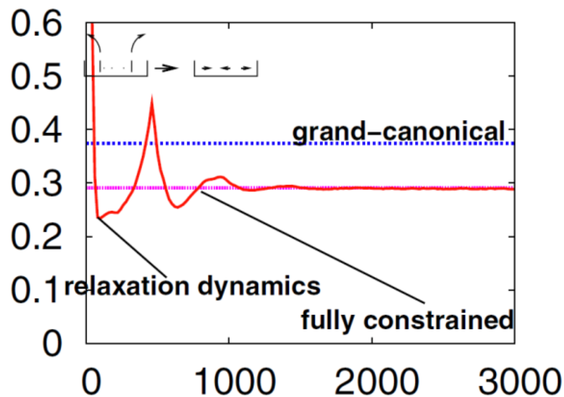
hard-core bosons on a one-dimensional lattice

$$\hat{H} = -J \sum_{i=1}^{L} (\hat{b}_i^\dagger \hat{b}_{i+1} + \mathrm{H.c.}),$$

where

$$[\hat{b}_i, \hat{b}_j^\dagger] = 0, \qquad [\hat{b}_i, \hat{b}_j] = [\hat{b}_i^\dagger, \hat{b}_j^\dagger] = 0$$
$$\text{for all } i \quad \text{and} \quad j \neq i;$$
$$\{\hat{b}_i, \hat{b}_i^\dagger\} = 1, \qquad (\hat{b}_i)^2 = (\hat{b}_i^\dagger)^2 = 0 \quad \text{for all } i.$$

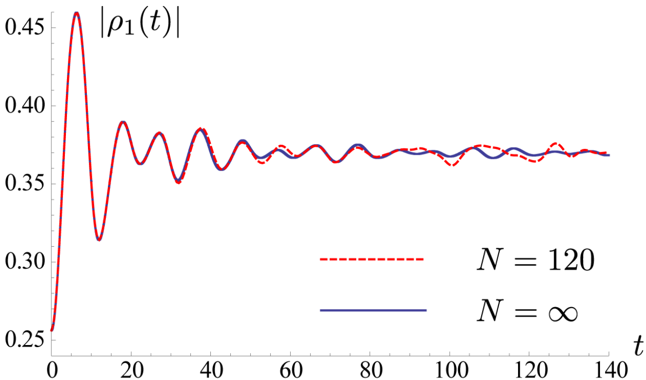
Rigol, Dunjko, Yurovsky and Olshanii 2007



``fully constrained"=GGE

Equilibration and Generalized Gibbs Ensemble

◆ Generalized Gibbs Ensemble (GGE)



μ: Fixed by the state at t=0.

cf.) total energy E → T

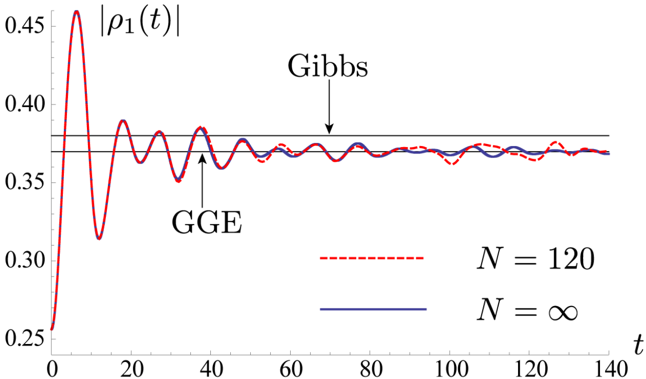
$$\hat{\rho}_{GGE} \equiv \frac{1}{Z}\exp\left(-\sum_m \mu_m \hat{N}_m\right)$$

$$\langle\rho_1\rangle_{GGE} = \mathrm{Tr}\hat{\rho}_{GGE}\rho_1$$

$$\psi(\theta,t) = \sum_m c_m \varphi_m(\theta) e^{-i\epsilon_m t} \qquad \hat{N}_m \equiv c_m^\dagger c_m$$

Equilibration and Generalized Gibbs Ensemble

◆ Generalized Gibbs Ensemble (GGE)



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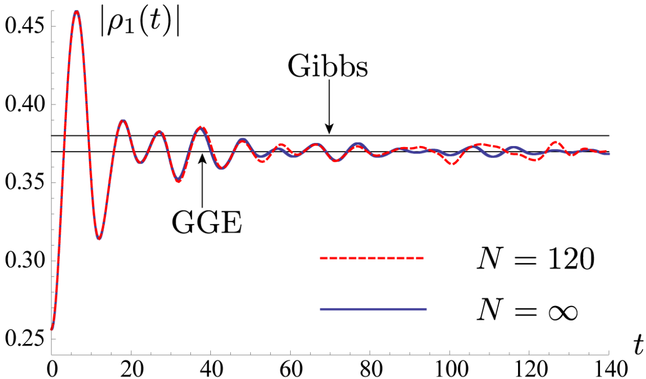
⇨ GGE works quite well in our model!

Equilibration and Generalized Gibbs Ensemble

◆ Entropy production in GGE



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$$\langle \rho_1 \rangle_{GGE} = \mathrm{Tr} \hat{\rho}_{GGE} \rho_1$$

This agreement implies that we can approximate the asymptotic states of this system by using $\hat{\rho}_{GGE}$. (coarse graining)

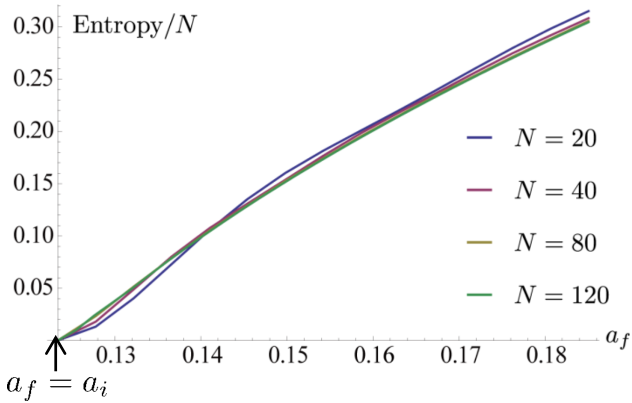
Since $\hat{\rho}_{GGE}$ is not a pure state, the von Neumann entropy is non-zero.

$$S = - \langle \log \hat{\rho}_{GGE} \rangle_{GGE} \neq 0$$

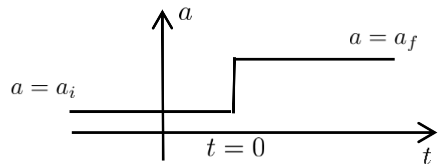
→ The equilibration causes an entropy production.
(Although the original state is pure state and the entropy=0.)

Equilibration and Generalized Gibbs Ensemble

◆ Entropy production in GGE



Our result shows that
Entropy is proportional to N.
(N fermion system)



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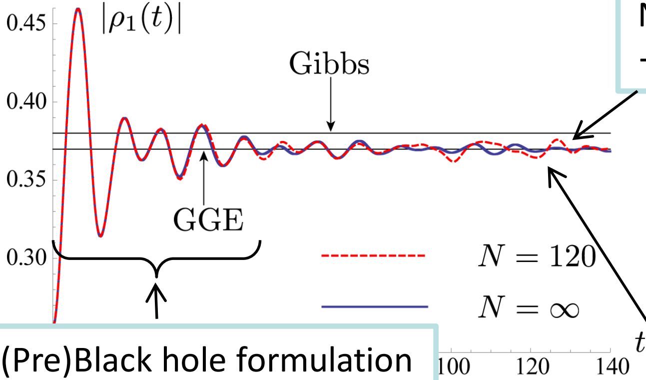
Equilibration and Generalized Gibbs Ensemble

◆ Interpretation as a black hole formulation in 2d string

NOTE: It is unclear at all that any dual black hole exists in our mod

Especially entropy is O(N) in our model and O(N^2) in 2d string.



N=∞ evolves to the GGE state.
→ A black hole is formulated.

Approximation by GGE is not so good in the finite N case, since it keeps on oscillating.

(Pre)Black hole formulation

Finite N starts the recurrence.

→ Hawking radiation + back reaction to BH (?)

$$1/N = \hbar \text{ in the gravity}$$

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Role of the critical point in the quantum quench

◆ Quantum phase transition

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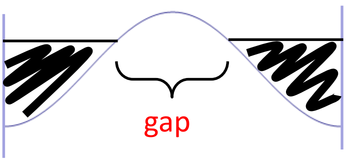
$$h(\theta,\partial_\theta) = -\frac{1}{N^2}\partial_\theta^2 - \underline{a}\cos\theta$$: The potential depth a controls the phases.

The Gross-Witten-Wadia type 3rd order transition happens at large-N.



The ground state for a large a.

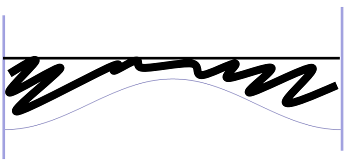
→ a gap exists.

(If N is finite, the gap is smeared through a quantum effect.)

$a > a_c$

↕ $a_c = \pi^2/64$

$a < a_c$



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Role of the critical point in the quantum quench

◆ Quantum quench dynamics

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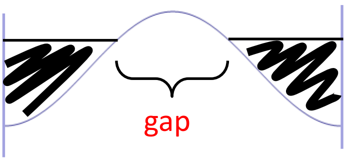
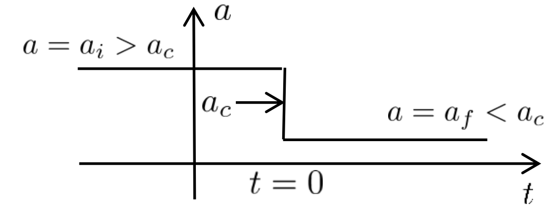
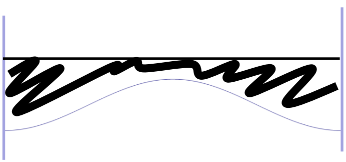


$a > a_c$





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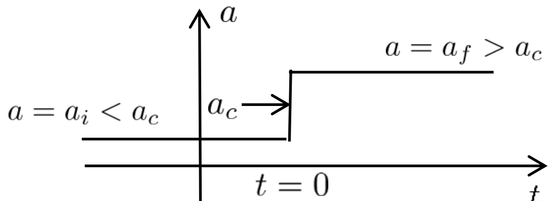
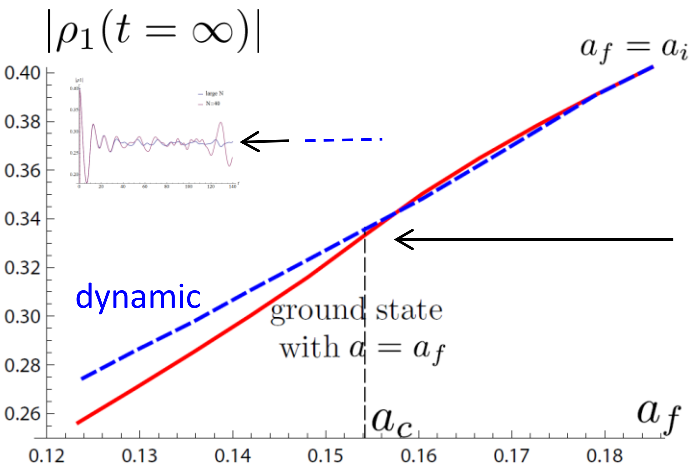


Role of the critical point in the quantum quench

Evaluate ρ1 by changing af, we found an importance of af=ac.



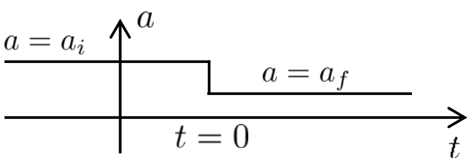
$$\rho_1(t) \equiv \int_{-\pi}^{\pi} d\theta \cos\theta \rho(\theta, t)$$

significant deviation

starts from af=ac.

⇩

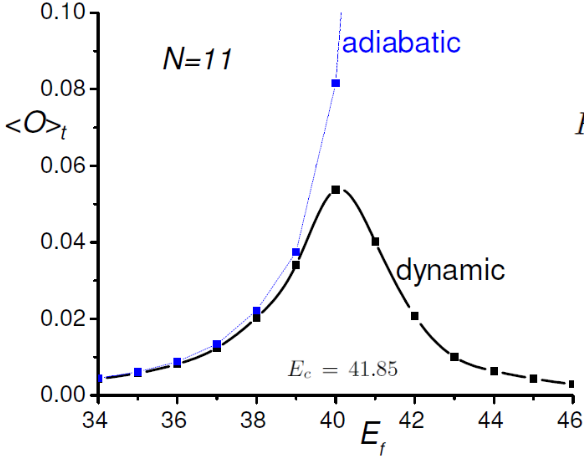
The criticality plays some role!



Role of the critical point in the quantum quench

Similar behaviour near of the critical point in the quantum quench has been observed in a different model too.

◆ One-dimensional dipole model of the Mott insulator



$$H_{1\mathrm{D}}[E] = -w\sqrt{n_0(n_0+1)}\sum_\ell (d_\ell^\dagger + d_\ell) + (U-E)\sum_\ell d_\ell^\dagger d_\ell .$$

⇨ Presumably a characteristic nature of the quantum quench near the critical point.

K. Sengupta, Stephen Powell, and Subir Sachdev (2003)

Plan of today's talk

1. Introduction
2. Review of the single trace matrix model
3. Time evolution of the single trace matrix model
4. Equilibration and Generalized Gibbs Ensemble
5. Role of the critical point in the quantum quench
6. D2 brane system and chaotic dynamics
7. Summary

D2 brane system and chaotic dynamics

◆ Matrix model from N D2 brane (cf. Witten's holographic QCD)

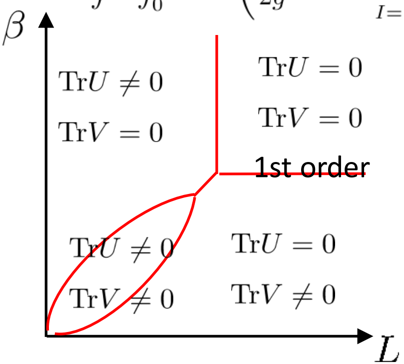
N D2 brane theory winding on $S^1_L \times S^1_{L_{KK}}$ $S^1_{L_{KK}}$ is a Scherk–Schwarz circle.
(breaks SUSY.)

$$S = \int dt \int_0^L dx \int_0^{L_{KK}} dy L_{3dSYM}$$ ⟹ The dual gravity exist at strong coupling.

↓ Take L_{KK} small.
Fermions are decoupled.

2d bosonic U(N) gauge theory on S^1_L

$$S = \int dt \int_0^L dx \, \mathrm{Tr}\left(\frac{1}{2g^2} F_{tx}^2 + \sum_{I=1}^{8} \frac{1}{2} \left(D_\mu Y^I\right)^2 + \frac{m^2}{2} (Y^I)^2 + \sum_{I,J} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right).$$



$$\begin{cases} \frac{1}{N}\mathrm{Tr}U = \frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^L A_x dx\right]\right) \\ \frac{1}{N}\mathrm{Tr}V = \frac{1}{N}\mathrm{Tr}P\left(\exp\left[i\int_0^\beta A_t dt\right]\right) \end{cases}$$

Four phases exist at finite temperature.

D2 brane system and chaotic dynamics

◆ Matrix model from N D2 brane (cf. Witten's holographic QCD)

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|\partial_t U|^2\right) - \frac{\xi}{N^2} (\mathrm{Tr}U)(\mathrm{Tr}U^\dagger) \right\}$$

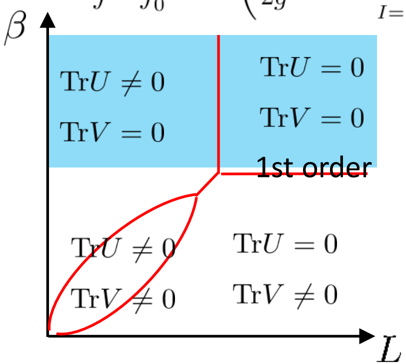
cf. the previous model

$$V = -\frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right)$$

↑ Integrate out Y (1/D expansion) in the confinement phase,
we obtain a one-dimensional unitary matrix model.

2d bosonic U(N) gauge theory on S^1_L

$$S = \int dt \int_0^L dx\, \mathrm{Tr}\left(\frac{1}{2g^2} F_{tx}^2 + \sum_{I=1}^{8} \frac{1}{2} \left(D_\mu Y^I\right)^2 + \frac{m^2}{2} (Y^I)^2 + \sum_{I,J} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right).$$



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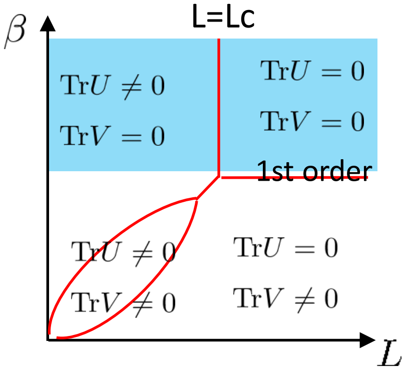
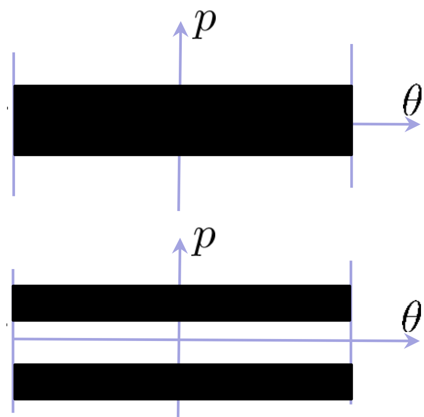
This model is approximately integrable if the kinetic term dominates.

→ Infinite number of excited states exist stably, like the GGE states.

(States in the one matrix model is characterized by droplets in the phase space.)





ground state (L>>Lc)

$\xi \propto e^{-Lm}$

an excited state

D2 brane system and chaotic dynamics

◆ Matrix model from N D2 brane (cf. Witten's holographic QCD)

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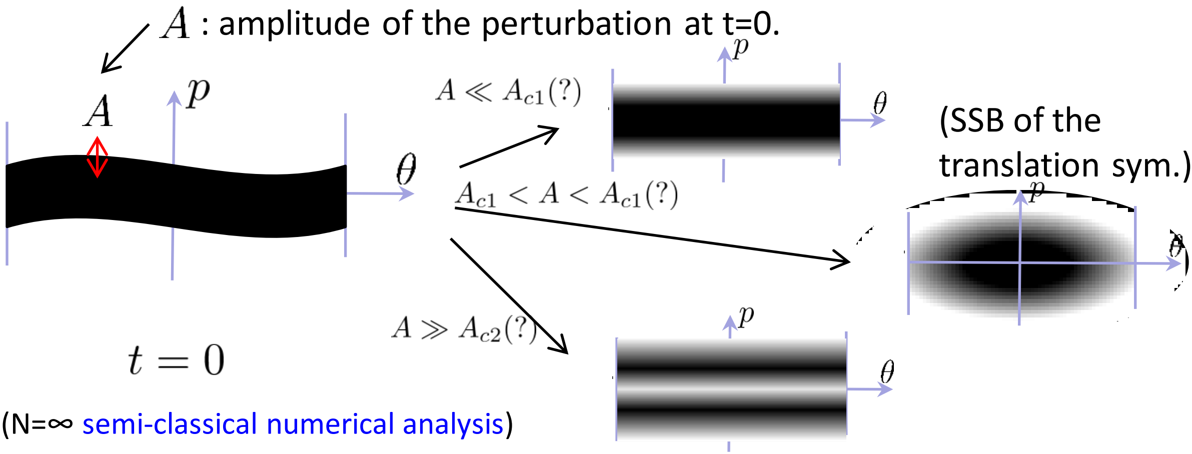
cf. the previous model

$$V = -\frac{a}{2N}\left(\mathrm{Tr} U + \mathrm{Tr} U^\dagger\right)$$

◆ Chaotic dynamics in the matrix model

The asymptotic states are completely different depending on the initial perturbations and ξ. (attractor structures??)



(N=∞ semi-classical numerical analysis)

D2 brane system and chaotic dynamics

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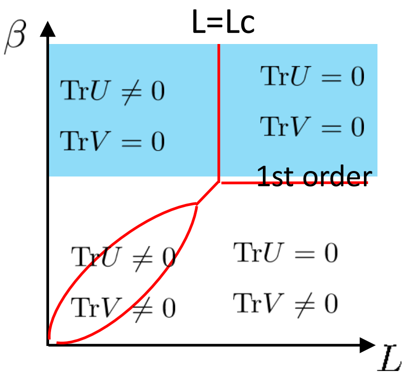
$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|\partial_t U|^2\right) - \frac{\xi}{N^2} (\mathrm{Tr}U)(\mathrm{Tr}U^\dagger) \right\}$$

cf. the previous model

$$V = -\frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right)$$

◆ The gravity duals are given by gravitational solutions in the confinement geometry background. (cf. AdS soliton.)

→ Surprisingly many stable solutions have been found in confinement geometries. Many hairs!!



ex)

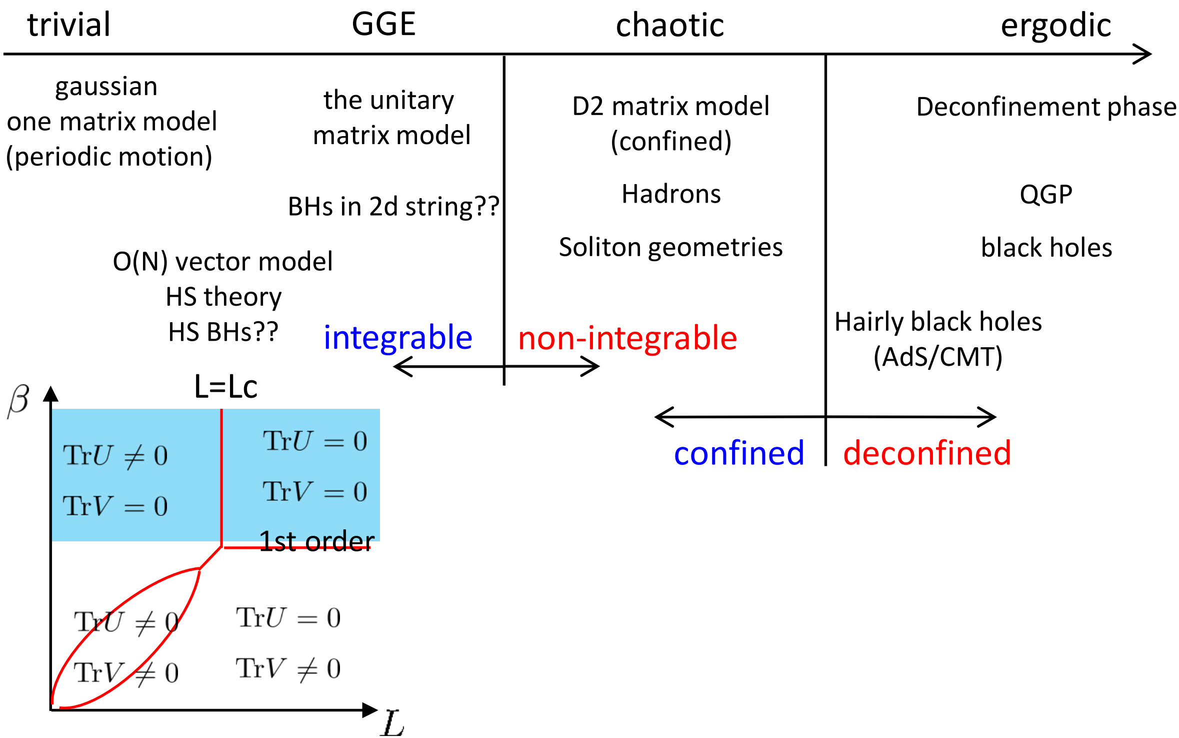
Boson stars, Geons (Dias-Horowitz-Marolf-Santos 2012)

Chaotic solutions (Basu-Ghosh 2013)

→ Candidates of the gravity duals of the solutions in the D2 matrix model.

D2 brane system and chaotic dynamics

◆ List of the typical time evolutions in string theories



Summary

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|D_t U|^2\right) - \frac{a}{2N}\left(\mathrm{Tr}U + \mathrm{Tr}U^\dagger\right) \right\}$$

Through the quantum quench dynamics, we observe several natures of the time evolution of the unitary matrix model at $N=\infty$ and $N < \infty$.

$N=\infty$ is qualitatively different from the finite N case.

- $N=\infty$: Equilibration to the GGE, and the entropy production.
- finite N: Tends to equilibrate but the recurrence starts later.

In the dual gravity (if exist), these qualitative differences are related to the differences between the classical and quantum gravity.

Summary

D2 brane model in the confinement phase exhibits the chaotic properties

$$S/N^2 = \int dt \left\{ \frac{1}{2N} \mathrm{Tr}\left(|D_t U|^2 \right) + \frac{\xi}{N^2} (\mathrm{Tr} U)(\mathrm{Tr} U^\dagger) \right\}$$

The related chaotic properties have been found
in the solitonic geometries in gravity.

→ New direction of the gauge/gravity correspondence toward chaotic systems.

Summary

Future directions

- Calculation of the entanglement entropy in the thermalization process.
- Application to the non-critical string theory by taking the double scaling limit.
- Application of GGE to other integrable systems in string theories.
- Application to the HS theory.
  Especially our entropy is O(N) and the HS BH may have O(N) entropy too.
- Understanging the role of the critical point in the quenched dynamics
- Systematic analysis of the D2 brane matrix model.

Thanks