Twisted space-time reduction in large N QCD with adjoint Wilson fermions

M. Okawa with A. Gonzalez-Arroyo

We study the large N QCD with adjoint fermions using the twisted space-time reduced model.

For two flavor theory ($N_f = 2$) with N=289, string tension is calculated which seems to vanish at $m_q = 0$, in a way consistent with the theory governed by an infrared fixed point.

For one flavor theory ($N_f = 1$), string tension remains finite at $m_q = 0$, indicating a confining theory .

Plan of the talk (first part)

- Twisted Eguchi-Kawai model
  for pure SU(N) gauge theory

- large N QCD with two adjoint fermions

- large N QCD with one adjoint fermion

● Eguchi-Kawai model

Eguchi-Kawai model is obtained from the usual SU(N) lattice gauge theory

$$Z_W = \int \prod_{x,\mu} dU_{x,\mu} \exp\left\{ bN \sum_x \sum_{\mu \neq \nu = 1}^{d} Tr\left(U_{x,\mu} U_{x+\mu,\nu} U^{\dagger}_{x+\nu,\mu} U^{\dagger}_{x,\nu} \right) \right\}$$

by neglecting the space-time dependence of the link variables

$$\boxed{U_{x,\mu} \rightarrow U_{\mu}}$$

$$Z_{EK} = \int \prod_{\mu} dU_{\mu} \exp\left\{ bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(U_{\mu} U_{\nu} U^{\dagger}_{\mu} U^{\dagger}_{\nu} \right) \right\}, \qquad b = \frac{1}{g^2 N}$$

In the same way, Wilson loop is defined by

$$W_W(C) = \left\langle Tr\left(U_{x,\mu} U_{x+\mu,\nu} \cdots U_{x-\rho,\rho} \right) \right\rangle$$

$$W_{EK}(C) = \left\langle Tr\left(U_{\mu} U_{\nu} \cdots U_{\rho} \right) \right\rangle$$

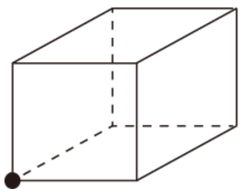
Eguchi and Kawai show that in the large N limit,
the Schwinger-Dyson eqs. satisfied by Wilson loops are identical in both theories provided that the $Z(N)^d$ symmetry

$$\boxed{U_\mu \rightarrow e^{i\theta_\mu} U_\mu,} \qquad e^{i\theta_\mu} \in Z(N)$$

of the EK model is not spontaneously broken.

Bhanot, Heller and Neuberger found, however, that this symmetry is broken spontaneously in the weak coupling region.

● Twisted Eguchi-Kawai model



EK model can be viewed as the usual Wilson gauge theory having only one site with the periodic boundary conditions.

We introduce twisted boundary conditions in $SU(N),\ N = L^2$ theory

(why it works. second part!)

$$S_{TEK} = bN \sum_{\mu\neq\nu=1}^{d} Tr\left(Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right)$$

$$Z_{\mu\nu} = \exp\left(k\frac{2\pi i}{L}\right), \quad Z_{\nu\mu} = -Z_{\mu\nu}, \qquad \mu > \nu$$

k , L : co-prime, k/L fixed as we go $N = L^2 \to \infty$

Our model is related to ordinary SU(N) lattice theory on $V = L^4$ space-time volume up to $O(1/N^2)$ corrections.

The number of degree of freedom of SU(N) matrix is $N^2 = L^4$

If the TEK model is correct nonperturbatively,
we should be able to calculate the string tension.

In our reduced model, the Wilson loop $W(R,T)$ is defined by

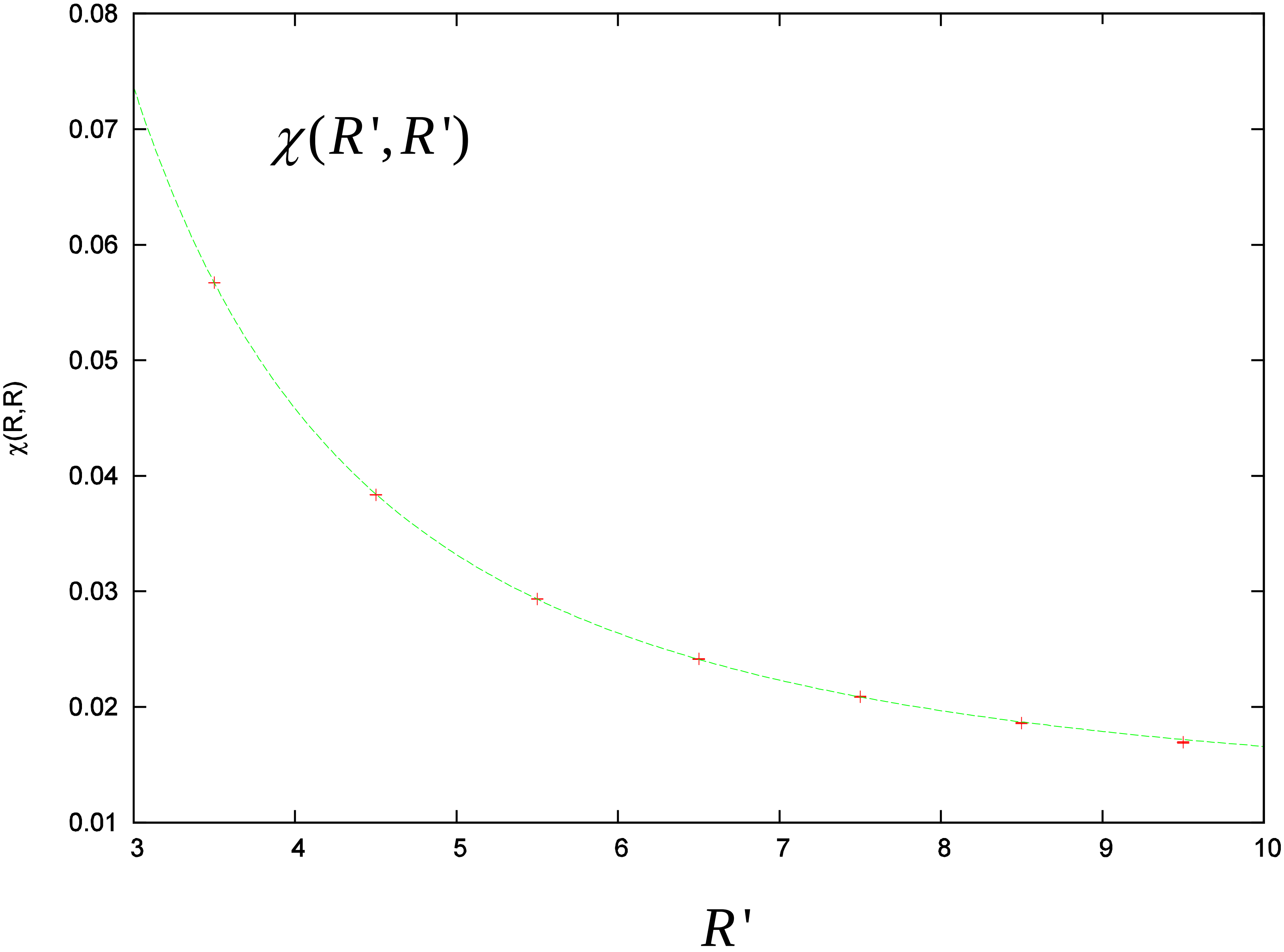
$$W(R,T) = Z_{\mu\nu}^{RT} \left\langle Tr\left(U_\mu^R U_\nu^T U_\mu^{\dagger R} U_\nu^{\dagger T}\right)\right\rangle \sim \sigma RT + \cdots$$

Then the string tension σ is obtained from Creutz ratio as

$$\chi(R',T') = -\log\frac{W(R'+0.5,T'+0.5)W(R'-0.5,T'-0.5)}{W(R'+0.5,T'-0.5)W(R'-0.5,T'+0.5)}$$

$$\chi(R',R') = \sigma + \frac{2\gamma}{R'^2} + \frac{\eta}{R'^4}$$

with half-integer R', T' .



$\chi(R',R')$
χ(R,R)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
3
4
5
6
7
8
9
10
R'

We calculate the continuum string tension by extrapolating the TEK data with $N = 841 = 29^2, \; k = 9$ at 6 values of b

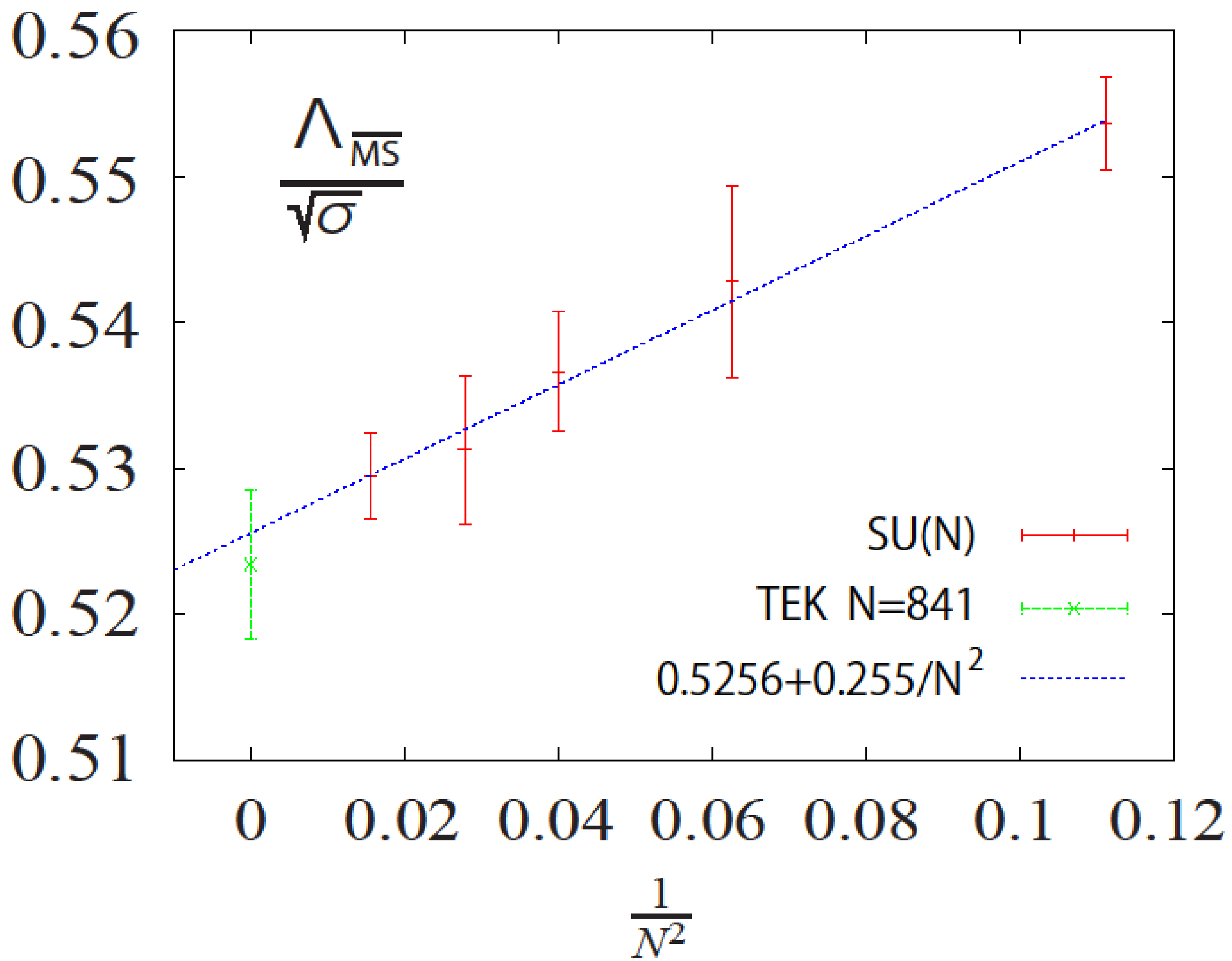
$$b = 0.36, \; 0.365, \; 0.37, \; 0.375, \; 0.38, \; 0.385$$

Our system should be related to the lattice theory with $V = 29^4$

For comparison, we also calculate the continuum string tension using ordinary SU(N) lattice gauge theory with $N = 3, 4, 5, 6, 8$ on a $V = 32^4$ lattice

Comparison of the continuum string tension $\Lambda_{\overline{MS}} / \sqrt{\sigma}$

TEK model with $N = 841 = 29^2$ and LGT with $N = 3, 4, 5, 6, 8$



Plan of the talk

- Twisted Eguchi-Kawai model
  for pure SU(N) gauge theory

- large N QCD with two adjoint fermions

- large N QCD with one adjoint fermion

Motivation for $N_f = 2$ adjoint fermions

SU(N) LGT with two adjoint fermions is thought to be conformal or nearly conformal for any value of N since the first two coefficient of beta functions expressed in term of ‘t Hooft coupling is independent of N.

$$b_0 = \frac{4N_f - 11}{24\pi^2}, \quad b_1 = \frac{16N_f - 17}{192\pi^4}$$

$$b_0 < 0 \rightarrow N_f < \frac{11}{4} = 2.75$$ asymptotic free

$$b_1 > 0 \rightarrow N_f > \frac{17}{16} = 1.08$$ infrared fixed point

● Twisted reduced model of large N QCD

with two adjoint Wilson fermions

We consider gauge group $SU(N),\ N = L^2$

$$S = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) + \sum_{j=1}^{N_f} \bar{\psi}_j D_W \psi_j$$

$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L}\right), \quad Z_{\nu\mu} = Z_{\mu\nu}^*, \qquad \mu > \nu$$

$$D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right], \quad U_\mu^{adj} \psi_j = U_\mu \psi_j U_\mu^\dagger$$

k , L : co-prime, $\quad m_q = (1/\kappa - 1/\kappa_c)/2$

$k = 0$ corresponds to periodic boundary condition

$k \neq 0$ corresponds to twisted boundary condition

We calculate the string tension with $N = 289 = 17^2, \ k = 5$
at 2 values of $b = 0.35, \ 0.36$ for various values of κ

Our system should be related to the lattice theory with $V = 17^4$

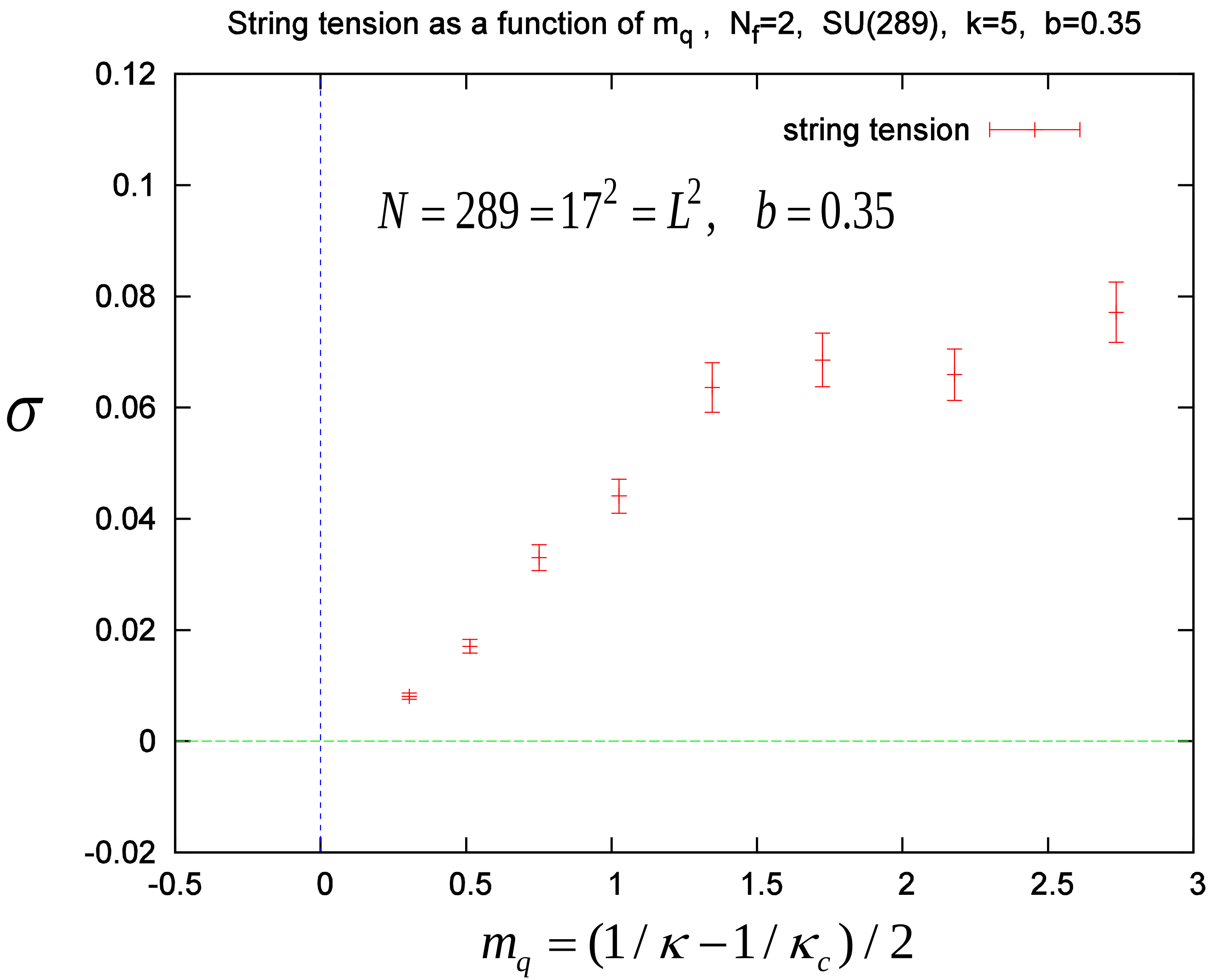
● For $N_f = 2$, we use the Hybrid Monte Carlo method.

Simulations have been done on Hitachi SR16000 at KEK

One node: 32 cores power 7,
peak speed 980 GFlops
256 GB shared memory

Sustained speed of our code in one node is
600 Gflops at N=289

We thank to Hitachi system engineers !



How to calculate κ_c , second part

If the theory is governed by an infrared fixed point with the relevant mass term $m_q \bar{\psi}\psi$, all physical quantity having mass dimension should vanish as $m_q \to 0.$

In particular, the string tension having mass square dimension should behave as

$$\sigma \sim m_q^{2/(1+\gamma_*)}$$

with γ_* the mass anomalous dimension at infrared fixed point. From our data, we have

$$\frac{2}{1+\gamma_*} = 1.34(11) \qquad \therefore \ \gamma_* = 0.49(4)$$

Simple derivation

$$\mu \frac{dm(\mu)}{d\mu} = -\gamma_* m(\mu)$$

$$\therefore \; m(\mu) \propto \mu^{-\gamma_*} \mu_0^{\gamma_*} m(\mu_0)$$

Define RG invariant mass M as $m(M) = M$.

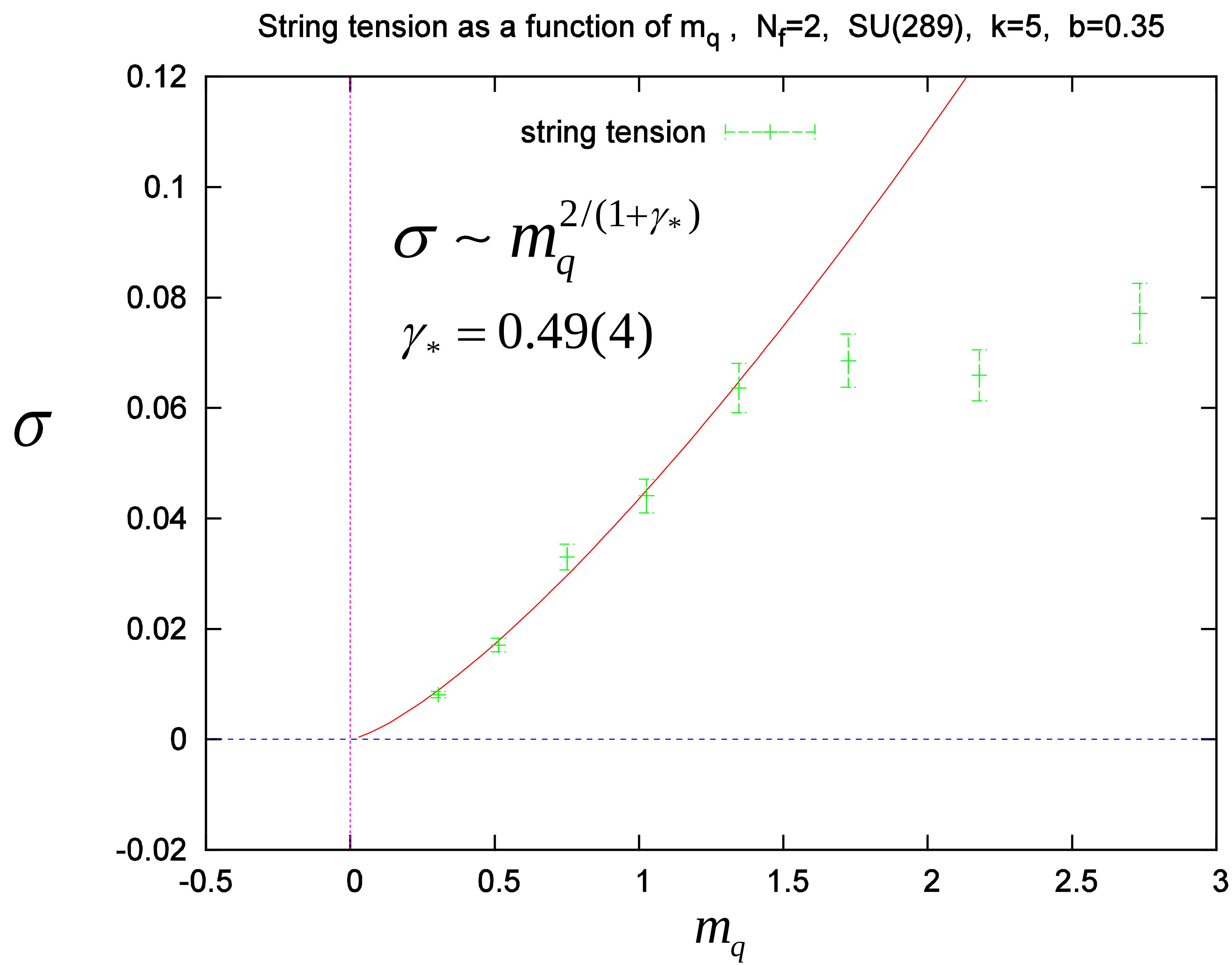
Then setting $\mu_0 = M$ and $\mu = a^{-1}$, we have

$$m(a^{-1}) = m_q \propto a^{-\gamma_*} M^{1+\gamma_*}$$

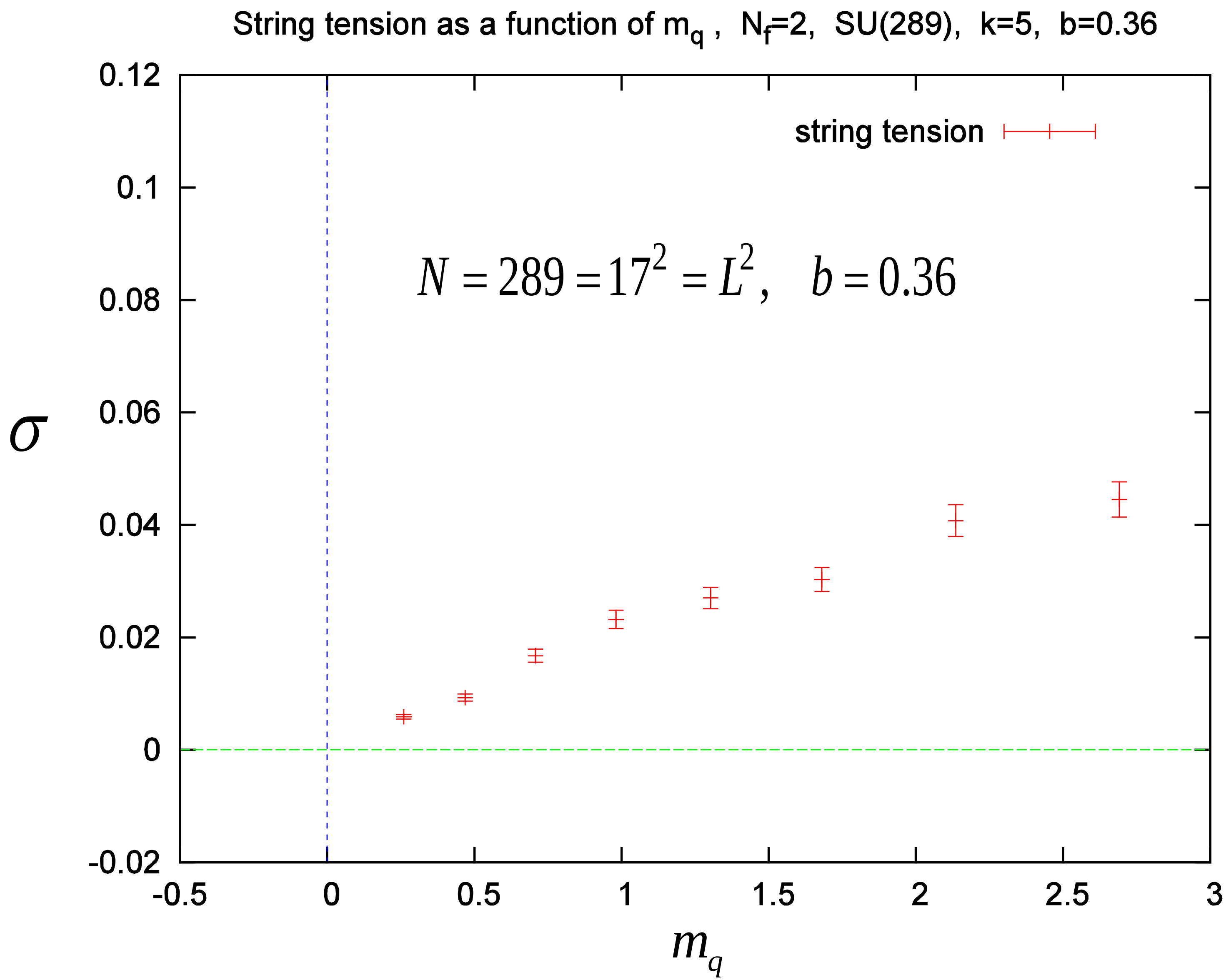
$$\therefore \; aM \propto (am_q)^{1/(1+\gamma_*)}$$

We can show that all physical quantity M_X having mass dimension is proportional to M , then

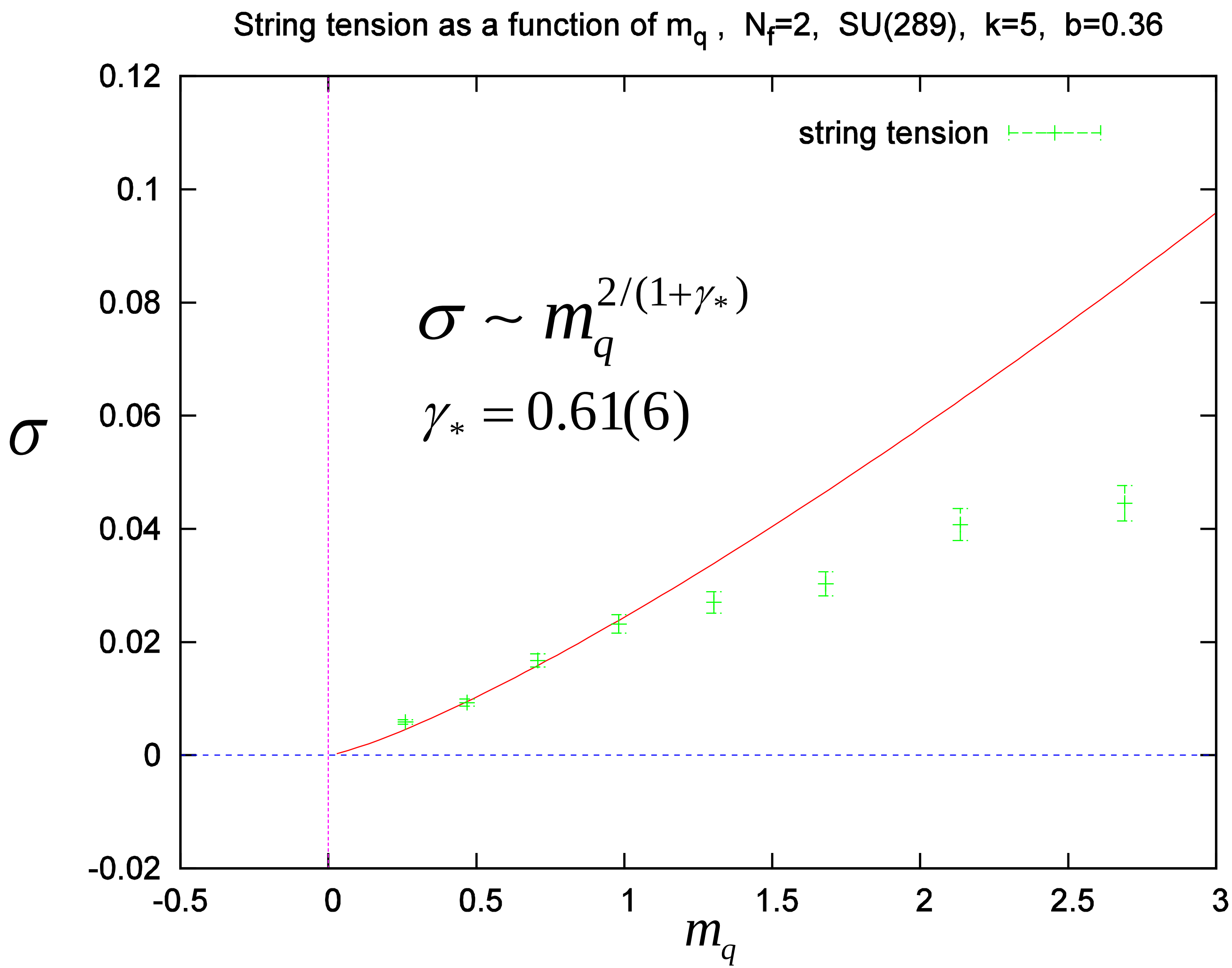
$$aM_X \propto (am_q)^{1/(1+\gamma_*)}$$



String tension as a function of m_q , N_f=2, SU(289), k=5, b=0.35
string tension
$\sigma \sim m_q^{2/(1+\gamma_*)}$
$\gamma_* = 0.49(4)$
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3



String tension as a function of m_q , N_f=2, SU(289), k=5, b=0.36
string tension
$N = 289 = 17^2 = L^2, \quad b = 0.36$
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3


String tension as a function of m_q , N_f=2, SU(289), k=5, b=0.36
string tension
$\sigma \sim m_q^{2/(1+\gamma_*)}$
$\gamma_* = 0.61(6)$
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3

Plan of the talk

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  for pure SU(N) gauge theory

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- large N QCD with one adjoint fermion

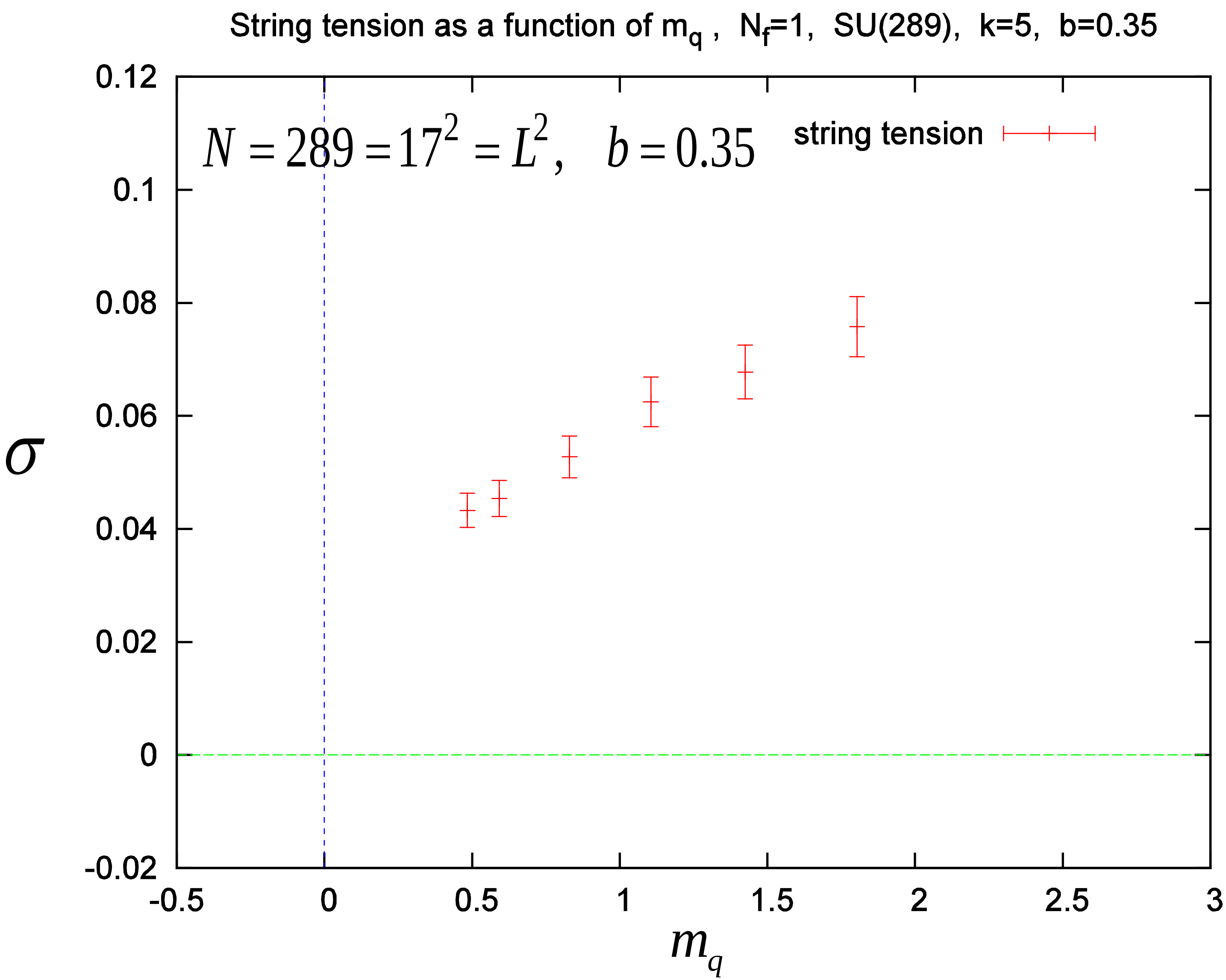
Motivation for $N_f = 1$ adjoint fermion

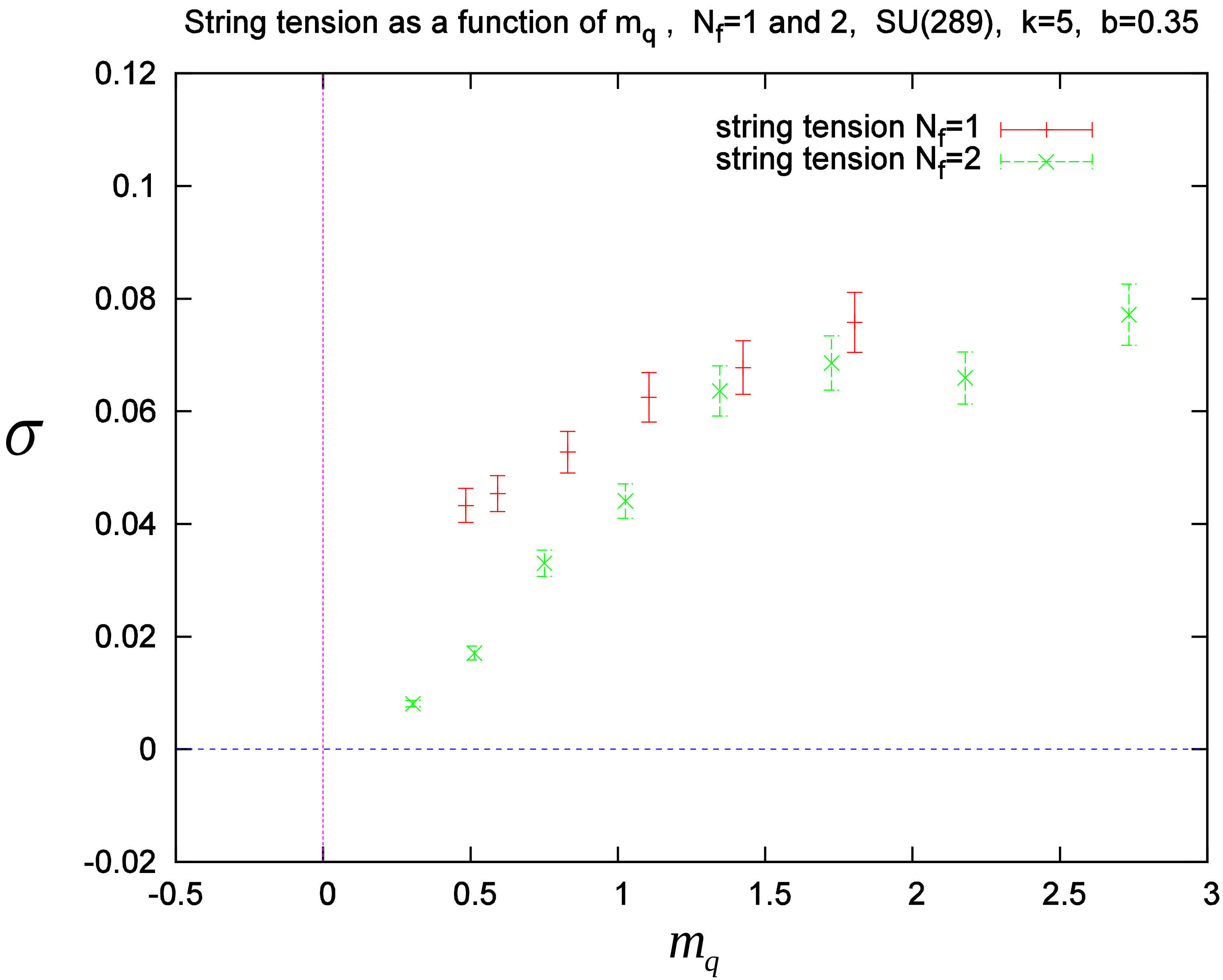
In the large N limit, $N_f = 1$ adjoint fermion is equivalent to $N_f = 2$ fundamental fermion in rank two anti-symmetric rep.
(Armoni, Shifman, Veneziano, Kovtun, Unsal, Yaffe)

For N=3, the latter theory is just two flavor QCD and our model corresponds to Corrigan-Ramond large-N limit.

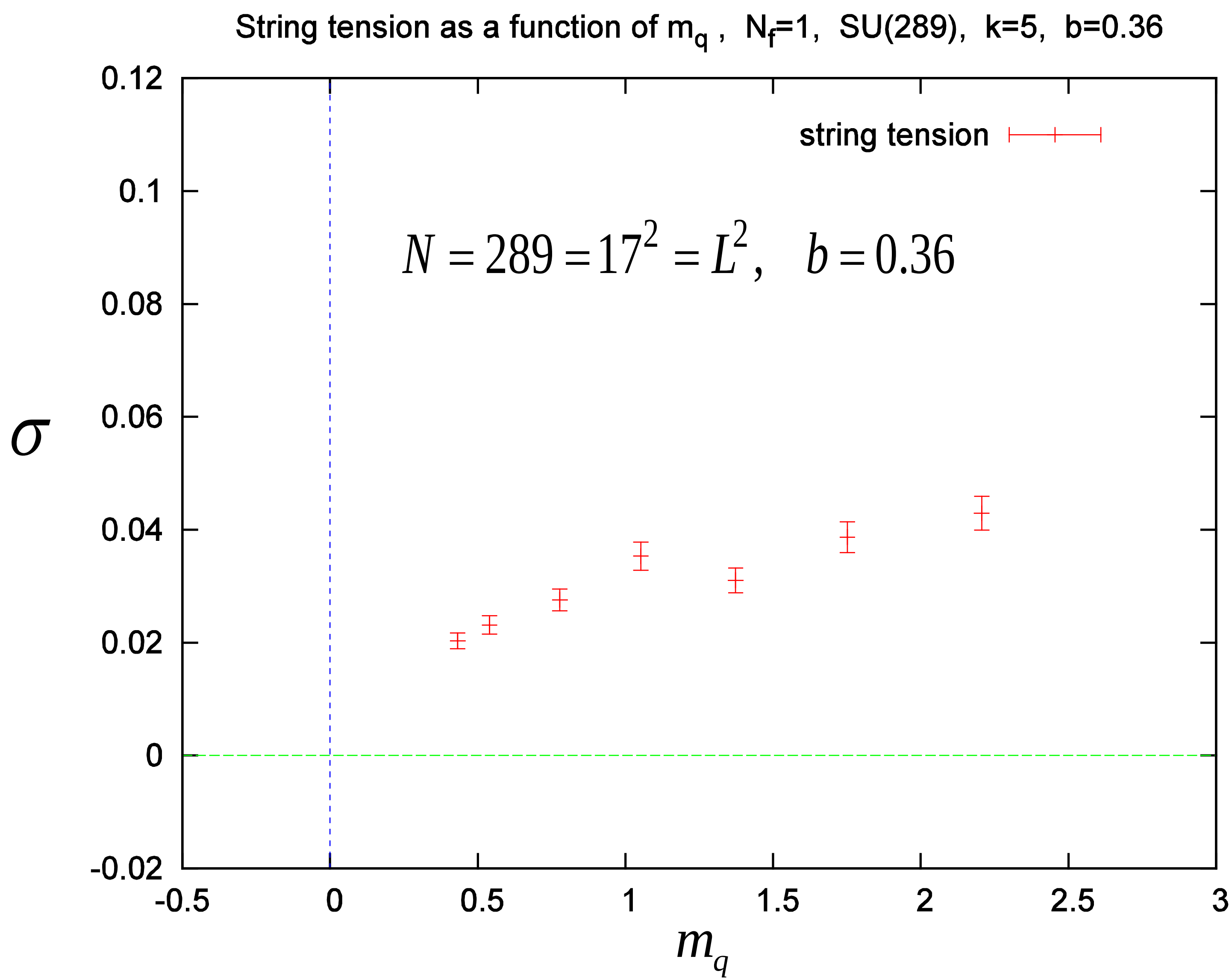
We then expect the reduced model of $N_f = 1$ adjoint fermion as

confining theory

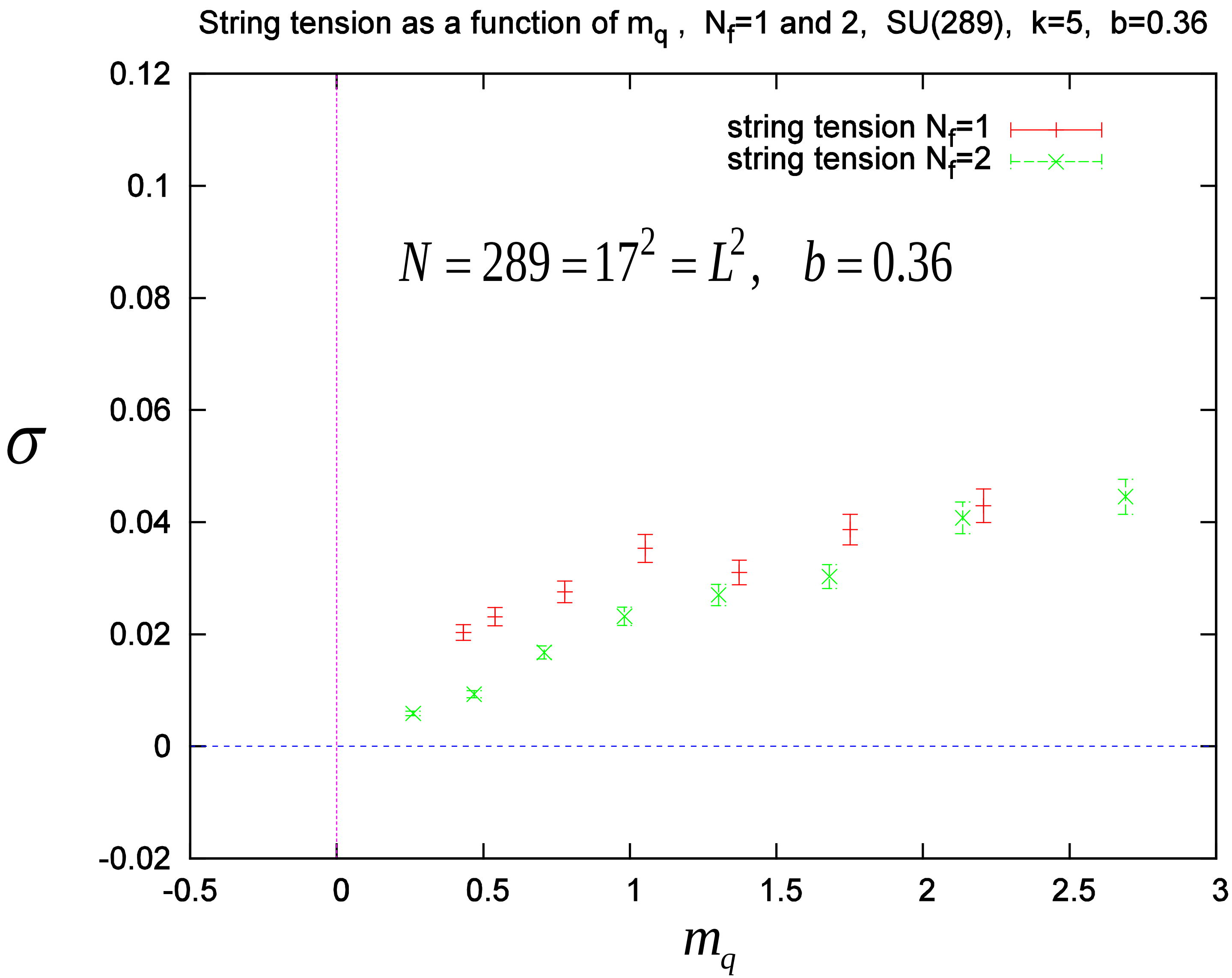

String tension as a function of m_q , N_f=1, SU(289), k=5, b=0.35
$N = 289 = 17^2 = L^2, \quad b = 0.35$
string tension
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3



String tension as a function of m_q, N_f=1 and 2, SU(289), k=5, b=0.35
string tension N_f=1
string tension N_f=2
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3
σ
m_q



String tension as a function of m_q , N_f=1, SU(289), k=5, b=0.36
string tension
$N = 289 = 17^2 = L^2, \quad b = 0.36$
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3



String tension as a function of m_q, N_f=1 and 2, SU(289), k=5, b=0.36
string tension N_f=1
string tension N_f=2
$N = 289 = 17^2 = L^2, \quad b = 0.36$
σ
m_q
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
0
0.5
1
1.5
2
2.5
3

Conclusion

We have demonstrated that the twisted reduced model of large N QCD with adjoint Wilson fermions works quite well.

$N_f = 2$

String tension is calculated at N=289, which clearly decreases as we increase kappa and seems to vanish around kappa ∼ 0.175 in a way consistent with the theory governed by an infrared fixed point .

$N_f = 1$

String tension is calculated at N=289, which clearly remains finite around kappa ∼ 0.16 strongly suggesting this is the confining theory. We also find that cg iteration does not converge for kappa > 0.16.

Remaining important problems

- We need to understand the finite N (finite volume) effects

  make simulation with larger $N = 529 = 23^2$

- calculate hadronic correlators

On going project

Wilson fermions with $N_f = 1/2$

overlap fermions with $N_f = 2$

with Garcia-Perez, Gonzalez-Arroyo, Ishikawa, Keegan

Twisted space-time reduction in large N QCD with adjoint Wilson fermions

Plan of the talk (second part)

- Why twisted reduction works ?
- Finite N corrections with two adjoint fermions
- How to calculate κ_c

Twisted reduction in theories having SU(N) internal symmetry

Let consider SU(N) with $N = L^2$ and
introduce the twisted space-time reduction as

$$U(n) \to U$$
$$U(n+\mu) \to \Gamma_\mu U \Gamma_\mu^\dagger$$

Γ_μ are four twist matrices satisfying

$$\Gamma_\nu \Gamma_\mu = z_{\mu\nu} \Gamma_\mu \Gamma_\nu$$

with $$z_{\mu\nu} = \exp(k\frac{2\pi i}{L}), \quad z_{\nu\mu} = z^*_{\mu\nu}, \quad \mu > \nu$$

k and L should be co-prime

We can construct Γ_μ from 'tHooft matrices P_L and Q_L

$$P_L = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \cdot & & \\ & & & \cdot & 1 \\ 1 & & & & 0 \end{pmatrix} \quad , \quad Q_L = \begin{pmatrix} 1 & & & & \\ & z & & & \\ & & z^2 & & \\ & & & \cdot & \\ & & & & z^{L-1} \end{pmatrix} \quad , \quad z = \exp\left(\frac{2\pi i}{L}\right)$$

$$P_L Q_L = z Q_L P_L \qquad\qquad P_L, Q_L : L \times L \quad \text{matrices}$$

$$\Gamma_1 = P_L \otimes I_L$$

$$\Gamma_2 = Q_L^k \otimes P_L$$

$$\Gamma_3 = Q_L^k \otimes P_L Q_L^k$$

$$\Gamma_4 = Q_L^k \otimes Q_L^k$$

$\Gamma_\mu : N \times N$ matrices with $N = L^2$

$$\Gamma_\nu \Gamma_\mu = z^k \Gamma_\mu \Gamma_\nu \ , \quad \mu > \nu$$

Let consider the SU(N) lattice gauge theory

$$S = bN \sum_{n} \sum_{\mu \neq \nu = 1}^{4} Tr\left[V_\mu(n) V_\nu(n+\mu) V_\mu^\dagger(n+\nu) V_\nu^\dagger(n) \right]$$

where b is the inverse 't Hooft coupling constant and $V_\mu(n) \in SU(N)$

Twisted reduction amounts to the following replacements

$$V_\mu(n) \to V_\mu$$

$$V_\mu(n+\nu) \to \Gamma_\nu V_\mu \Gamma_\nu^\dagger$$

Then we get $S = bN \sum_{\mu \neq \nu = 1}^{4} Tr\left[V_\mu \Gamma_\mu V_\nu \Gamma_\mu^\dagger \Gamma_\nu V_\mu^\dagger \Gamma_\nu^\dagger V_\nu^\dagger \right]$

Writing $U_\mu = V_\mu \Gamma_\mu$ and using $\Gamma_\nu \Gamma_\mu = z_{\mu\nu} \Gamma_\mu \Gamma_\nu$ we have

$$S = bN \sum_{\mu \neq \nu = 1}^{4} Tr\left[z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right]$$

This is the twisted Eguchi-Kawai (TEK) model

(A. Gonzalez-Arroyo and M. O. 1983, 2010)

Classical vacuum configuration $U_\mu^{(0)}$ satisfies

$$z_{\mu\nu} U_\mu^{(0)} U_\nu^{(0)} U_\mu^{(0)\dagger} U_\nu^{(0)\dagger} = 1$$

$$\therefore U_\nu^{(0)} U_\mu^{(0)} = z_{\mu\nu} U_\mu^{(0)} U_\nu^{(0)}$$

Thus, $U_\mu^{(0)} = \Gamma_\mu$. Perturbation theory can be obtained by expanding U_μ around classical vacuum $U_\mu = \exp(iA_\mu)\Gamma_\mu$

Usually, we expand A_μ in terms the generator of Lie algebra λ^a.
However, it is more convenient to use another bases.

We can show $\Gamma(n) = \Gamma_1^{n_1}\Gamma_2^{n_2}\Gamma_3^{n_3}\Gamma_4^{n_4}, \quad 1 \le n_\mu \le L$

are $L^4 = N^2$ independent matrices,
which span the bases of *NxN* matrix $A_\mu = \sum_n a_\mu(n)\Gamma(n),$

Since $\Gamma_\nu \Gamma_\mu = \exp\left(\pm k \frac{2\pi i}{L} \right) \Gamma_\mu \Gamma_\nu$, for $\Gamma(n) = \Gamma_1^{n_1} \Gamma_2^{n_2} \Gamma_3^{n_3} \Gamma_4^{n_4}$

$$\Gamma_\mu \Gamma(n) \Gamma_\mu^\dagger = \exp\left[k \frac{2\pi i}{L} \left(-\sum_{\nu=1}^{\mu-1} n_\nu + \sum_{\nu=\mu+1}^{4} n_\nu \right) \right] \Gamma(n)$$

$$\equiv \exp\left[\frac{2\pi i}{L} m_\mu \right] \Gamma(n), \quad m_\mu \quad \text{defined modulo} \quad L$$

$$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix} = k \begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix}, \qquad \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix} = \frac{1}{k} \begin{pmatrix} 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & 1 \\ -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{pmatrix}$$

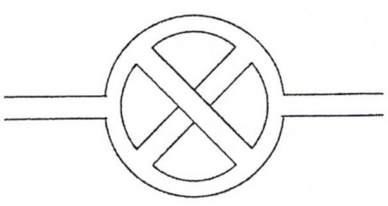
Then $\Gamma(n(m))$ is the momentum eigenstate on a L^4 lattice

Propagator is identical to that of the lattice theory on $V = L^4 = N^2$

If we introduce interactions, there appears phase factor in each vertex. However, they cancel completely in planar diagram.

For non-planar diagrams, phase factor survives, which oscillates very rapidly in the large N limit, and suppressing the contribution of non-planar diagram.

no phase factor remains



$$\int dp \exp\left[i\bar{k} L f(p) \right] \xrightarrow{L\to\infty} 0$$

$$k\bar{k} = 1 \quad \mathrm{mod}(L)$$

The order parameters of $Z(L)^4$ symmetry $U_\mu \to zU_\mu$ of the TEK action are

$$\left\langle Tr(U_\mu^\ell) \right\rangle, \quad \ell = 1 \sim (L-1)$$

For the classical vacuum $U_\mu^{(0)} = \Gamma_\mu$, it is straightforward to show

$$Tr(U_\mu^{(0)\ell}) = Tr(\Gamma_\mu^\ell) = 0, \quad \ell = 1 \sim (L-1)$$

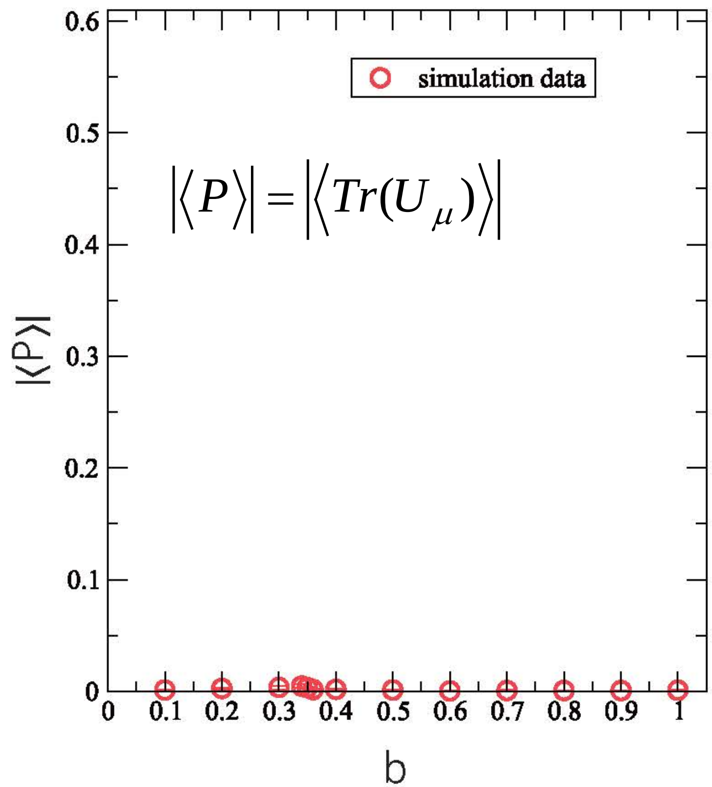
Then the Schwinger-Dyson eqs. satisfied by the TEK model and the corresponding lattice theory are identical.

We naturally expect that both theories are equivalent even nonperturbatively.

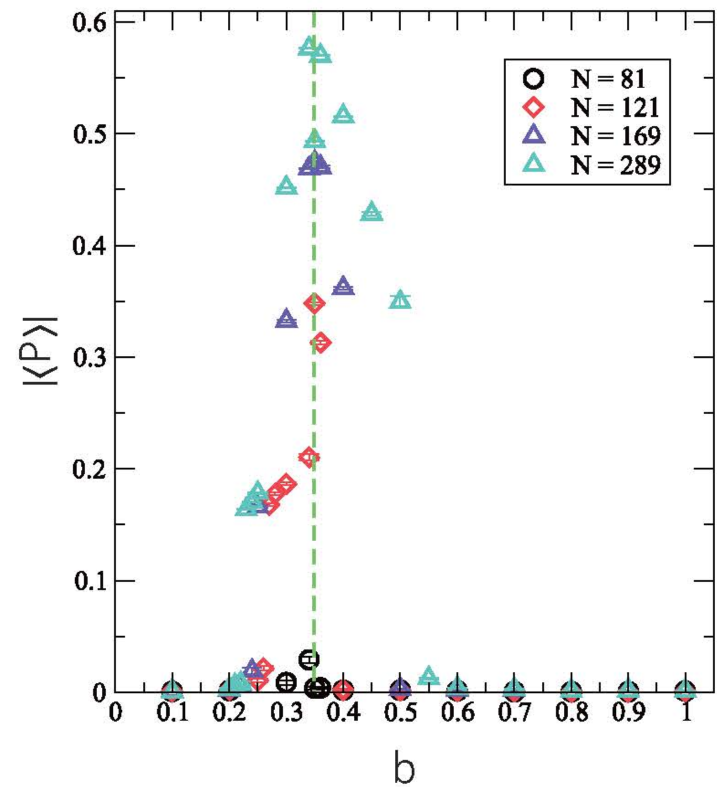
A. Gonzalez-Arroyo and M. O. 1983 $k = 1$

In 2003, Tomomi Ishikawa and M. O. found that in the intermediate coupling region, $\langle Tr(U_\mu)\rangle \neq 0$ for N>100 with *k*=1.

N=64, k=1



k=1



Z_L^4 symmery is not broken all over the coupling region.

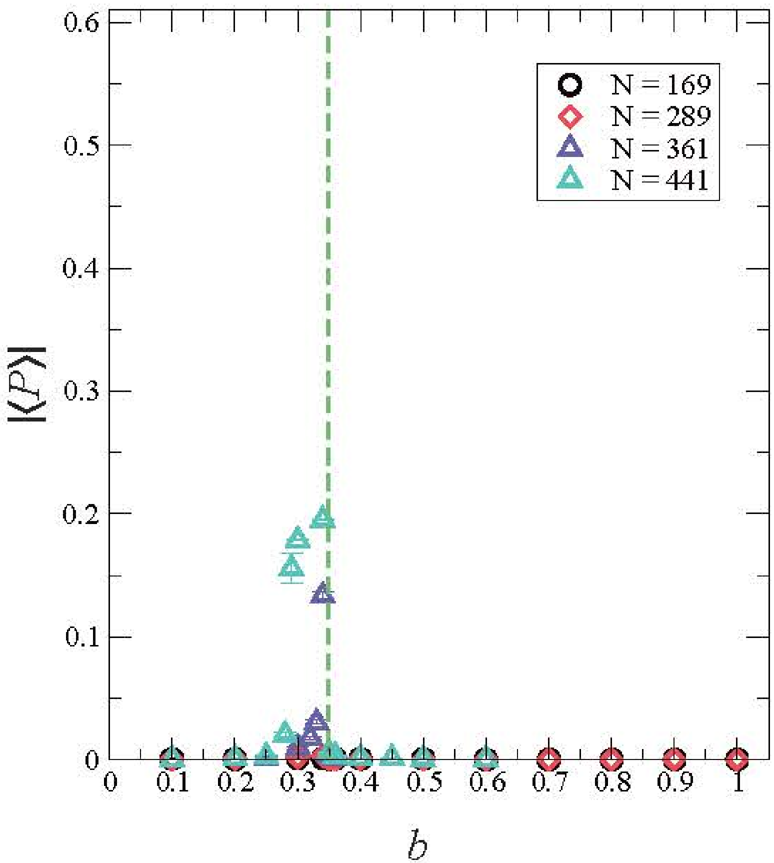
Z_L^4 symmetry is broken in the intermediate region for N> 100

We also found that

$Z(L)^4$ symmetry is broken

for $N \geq 360$ with k=2.

k = 2



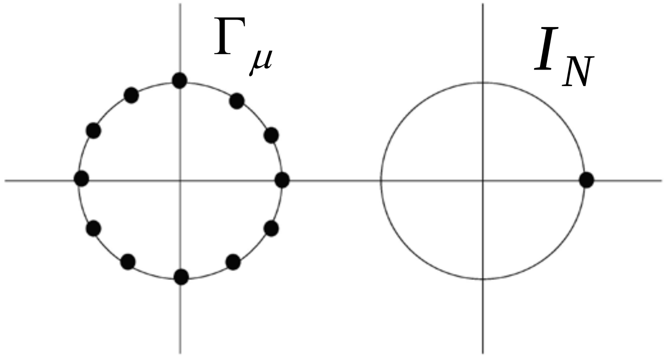
Z_L^4 **symmetry is broken in the intermediate region for** $N \gtrsim 360$.

Why $Z(L)^4$ symmetry is broken ?

M. Teper, H. Vairinhops (2007) *k*=1

A. Gonzalez-Arroyo, M. O. (2010) general *k*

In the $Z(L)^4$ symmetry broken phase,
the eigenvalues of U_μ attract each other
In the complex plane, thus $U_\mu^{(0)} \sim I_N$



It is then the competition of energy gap and entropy between two configurations

$$F(U_\mu^{(0)} = \Gamma_\mu) = \frac{3}{2}\log(bN)N^2$$

$$F(U_\mu^{(0)} = I_N) = 12bN^2\left(1 - \cos\left(\frac{2\pi k}{L}\right)\right) + \log(bN)N^2$$

$$\Delta F \equiv F(U_\mu^{(0)} = I_N) - F(U_\mu^{(0)} = \Gamma_\mu)$$

$$= 12bN^2\left(1 - \cos\left(\frac{2\pi k}{L}\right)\right) - \frac{1}{2}\log(bN)N^2$$

● For $b \to \infty$ with fixed L, N, k, $\Delta F > 0$ (weak coupling limit)

● For $N = L^2 \to \infty$ with fixed b, k, $\Delta F \sim -N^2/2 < 0$

The wrong vacuum $U_\mu^{(0)} = I_N$ wins

Our proposal

As we take $N = L^2 \to \infty$, we fix k/L. L and k co-prime.

In the same time, we scale $b \sim (11/48\pi^2)\log(N)$

then the physical lattice size $La(b)$ is fixed

So far, the arguments are perturbative, which we should not trust so much. We need non-perturbative study.

$k=0:\quad Z(L)$ symmetry is broken for $L>0$

$k=1:\quad Z(L)$ symmetry is broken for $L>10$

$k=2:\quad Z(L)$ symmetry is broken for $L>18$

$k=3:\quad Z(L)$ symmetry is broken for $L>28$

$k=4:\quad Z(L)$ symmetry is broken for $L>37$

The above numerical results strongly suggest that

$Z(L)^4$ symmetry is not broken for $\dfrac{k}{L} > \dfrac{1}{9}$

We also found that *k* should not be chosen too large .

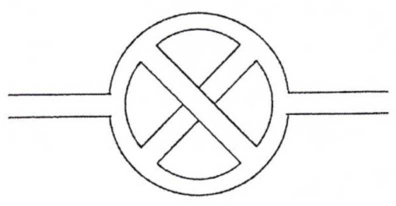
In fact, for $k = \frac{L-1}{2}$, we observe at $L = 17, 19, 21, 23$

$$\langle Tr(U_\mu) \rangle = 0 \quad \text{but} \quad \langle Tr(U_\mu^2) \rangle \neq 0$$

We notice $2k = L - 1 = 1 \,(\text{mod } L)$, then $\bar{k} = 2$ $\left[k\bar{k} = 1 \text{ mod } L \right]$

Might be related to the tachyonic instability of the non-commutative field theory ?

In any case, large value of $\bar{k}$ is desirable to suppress non planer diagrams



$$\int dp \exp\left[i\bar{k} L f(p) \right] \xrightarrow{L\to\infty} 0$$

Take large L keeping $k/L > 1/9$ with large $\bar{k}$. L and k co-prime.

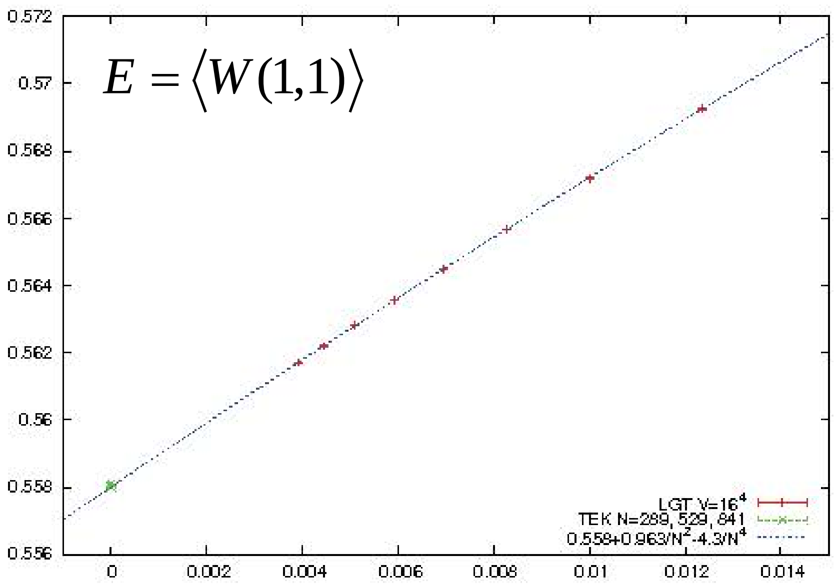
We mainly make numerical simulations
for the following four parameter sets

N	L	k	$\bar{k}$
289	17	5	7
529	23	7	10
841	29	9	13
1369	37	11	10

We note $k / L \sim 0.3$ for all cases !

N dependence of W(1,1)

Detailed comparison at b=0.36



$1/N^2$

For V $= 16^4$ N = 9 -16

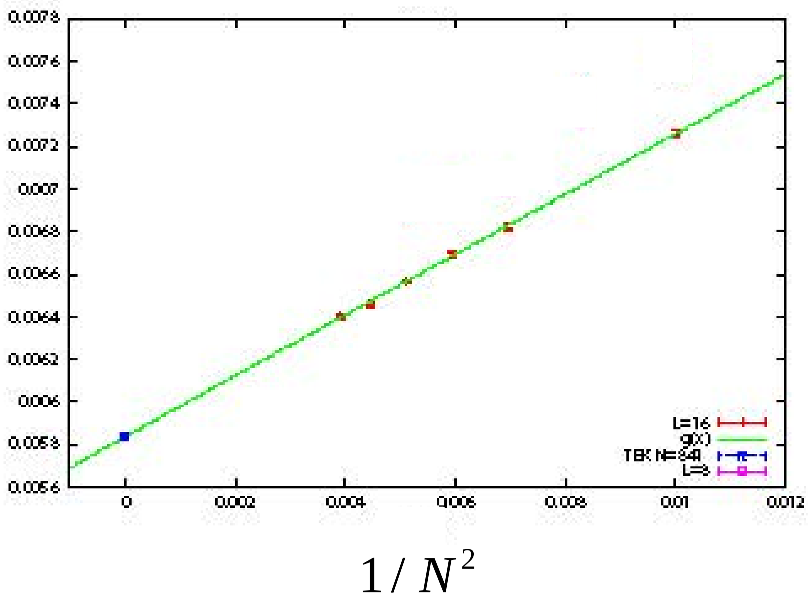
$$E=0.55800(2) + \frac{0.963(6)}{N^2} - \frac{4.3(4)}{N^4}$$

E=0.557998(5) (N=841, k=9)

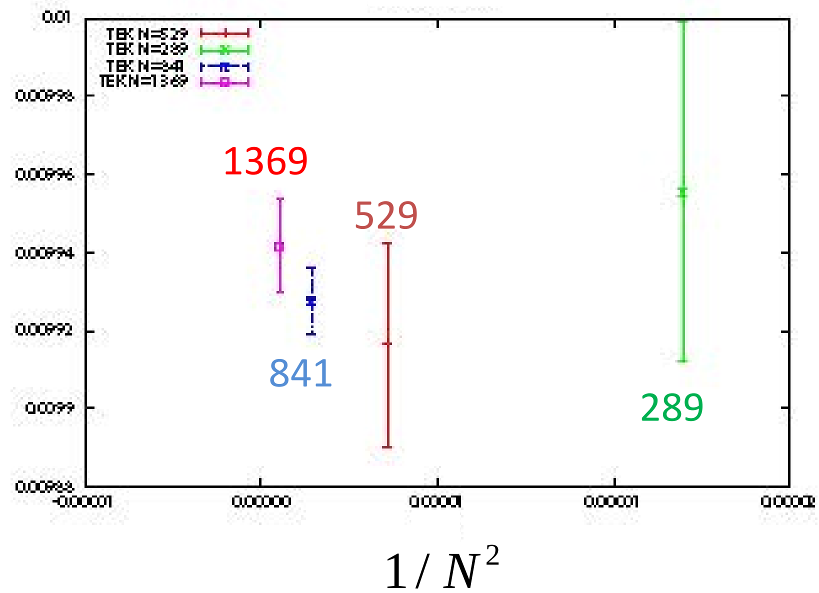
E=0.557999(19) (N=289, k=5)

E=0.557991(13) (N=529, k=7)

$\langle W(4,4) \rangle \quad b = 0.36$

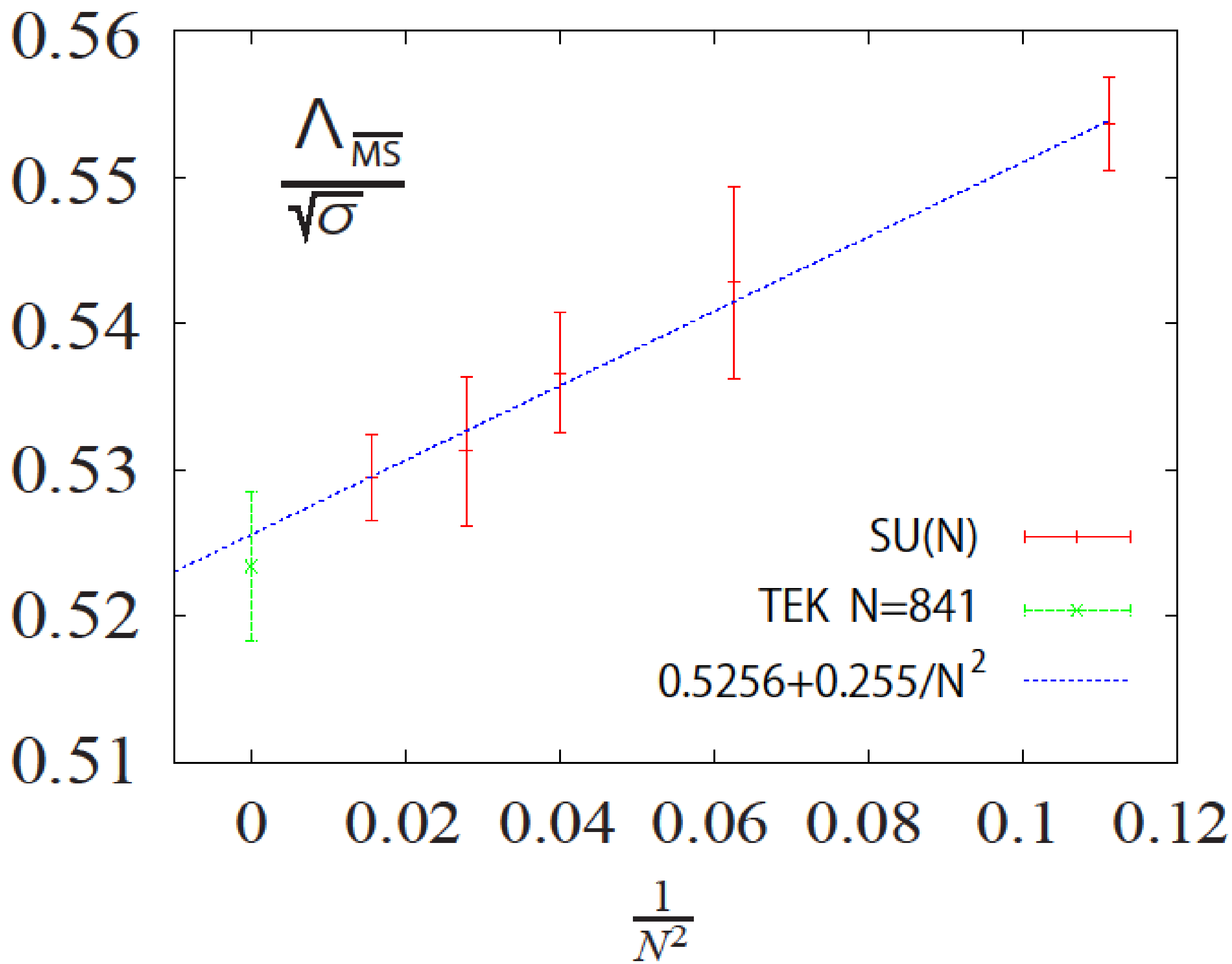


$\langle W(4,4) \rangle \quad TEK \quad b = 0.37$



Comparison of the continuum string tension $\Lambda_{\overline{MS}} / \sqrt{\sigma}$

TEK model with $N = 841 = 29^2$ and LGT with $N = 3, 4, 5, 6, 8$



Twisted space-time reduction in large N QCD with adjoint Wilson fermions

Plan of the talk (second part)

- Why twisted reduction works ?
- Finite N corrections with two adjoint fermions
- How to calculate κ_c

The action of the adjoint fermions in lattice theory is

$$S_f = \sum_{j=1}^{N_f}\sum_{n} Tr\Big[\bar{\psi}_j(n)\psi_j(n) - \kappa\sum_{\mu=1}^{4}\Big\{\bar{\psi}_j(n)(1-\gamma_\mu)V_\mu(n)\psi_j(n+\mu)V_\mu^\dagger(n)$$

$$+\bar{\psi}_j(n)(1+\gamma_\mu)V_\mu^\dagger(n-\mu)\psi_j(n-\mu)V_\mu(n-\mu)\Big\}\Big]$$

where ψ_j is the adjoint fermions in color $(N,\bar{N})$ representation

Reducing the lattice theory by

$$U_\mu(n) \to U_\mu \qquad \psi_j(n) \to \psi_j$$

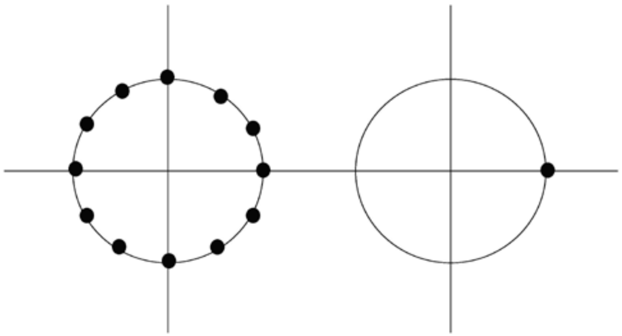
$$U_\mu(n+\nu) \to \Gamma_\nu U_\mu \Gamma_\nu^\dagger \qquad \psi_j(n+\nu) \to \Gamma_\nu \psi_j \Gamma_\nu^\dagger$$

$$U_\mu(n-\nu) \to \Gamma_\nu^\dagger U_\mu \Gamma_\nu \qquad \psi_j(n-\nu) \to \Gamma_\nu^\dagger \psi_j \Gamma_\nu$$

and writing $U_\mu = V_\mu \Gamma_\mu$ with $\Gamma_\nu\Gamma_\mu = z_{\mu\nu}\Gamma_\mu\Gamma_\nu$ we have

$$S_f = \sum_{j=1}^{N_f} Tr\left[\bar{\psi}_j\psi_j - \kappa\sum_{\mu=1}^{4}\left\{\bar{\psi}_j(1-\gamma_\mu)U_\mu\psi_j U_\mu^\dagger + \bar{\psi}_j(1+\gamma_\mu)U_\mu^\dagger\psi_j U_\mu\right\}\right]$$

Z(N) symmetry breaking of the Eguchi-Kawai model (k=0) without fermion occures since the eigenvalues of U_μ attract each other, then $U_\mu^{(0)} \sim I_N$.



It has been shown by several authors that

Dynamical quark effects of adjoint fermion induce repulsive force between the eigenvalues of U_μ , thus the symmetry breaking does not occurs even for k=0 .   Kovtun, Unsal, Yaffe, Bringoltz, Koren, Sharpe

It is true, however, it is not clear

how large is the finite N corrections especially for k=0.

Simulations have been done with $SU(N),\ N = L^2$

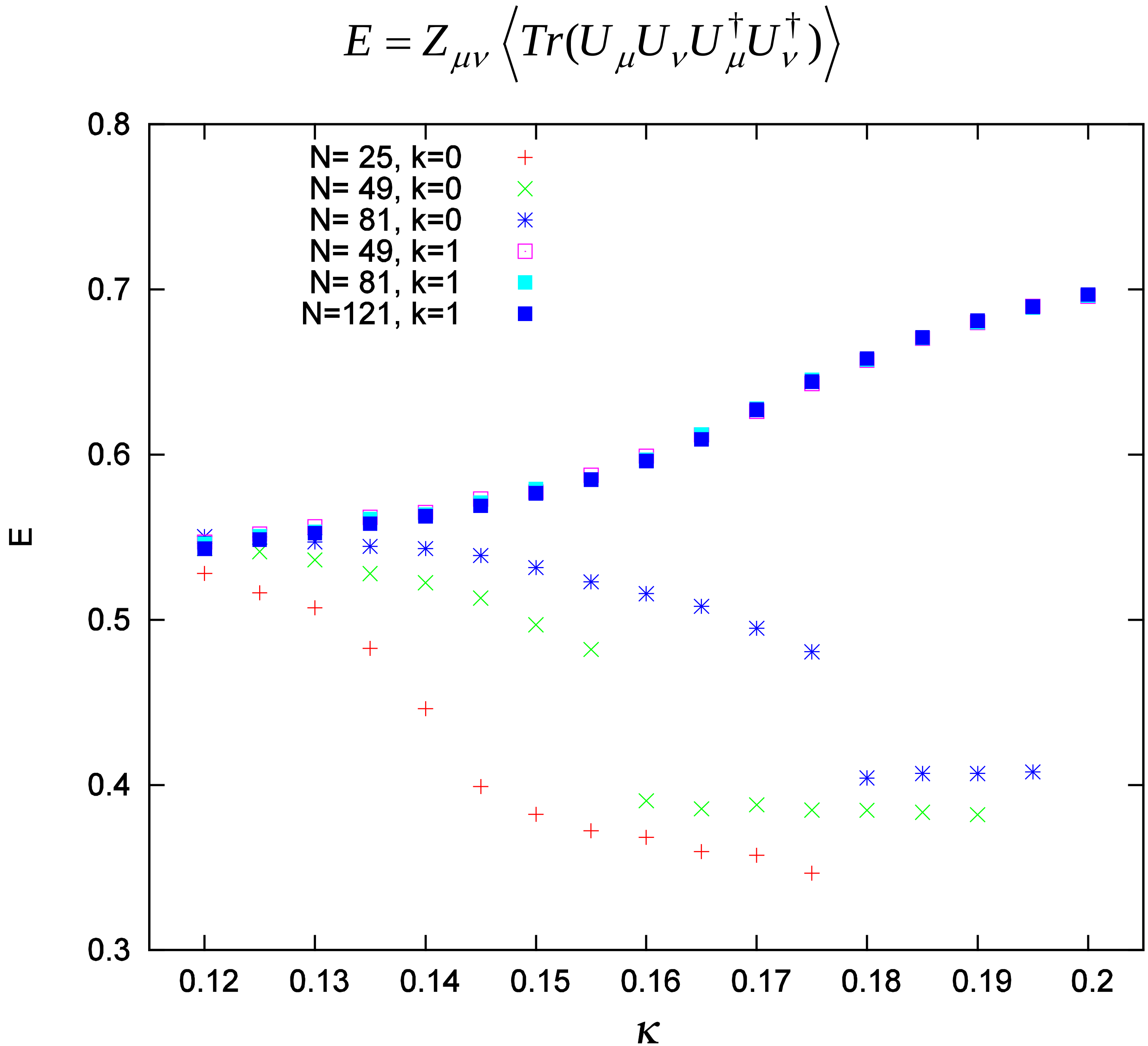
$$N = 25, 49, 81, 121, 169, 225, 289$$
$$(\ L = 5,\ 7,\ 9,\ 11,\ 13,\ 15,\ 17\ )$$

Twisted model is related to ordinary SU(N) lattice theory on $V = L^4$ space-time volume up to $O(1/N^2)$ corrections

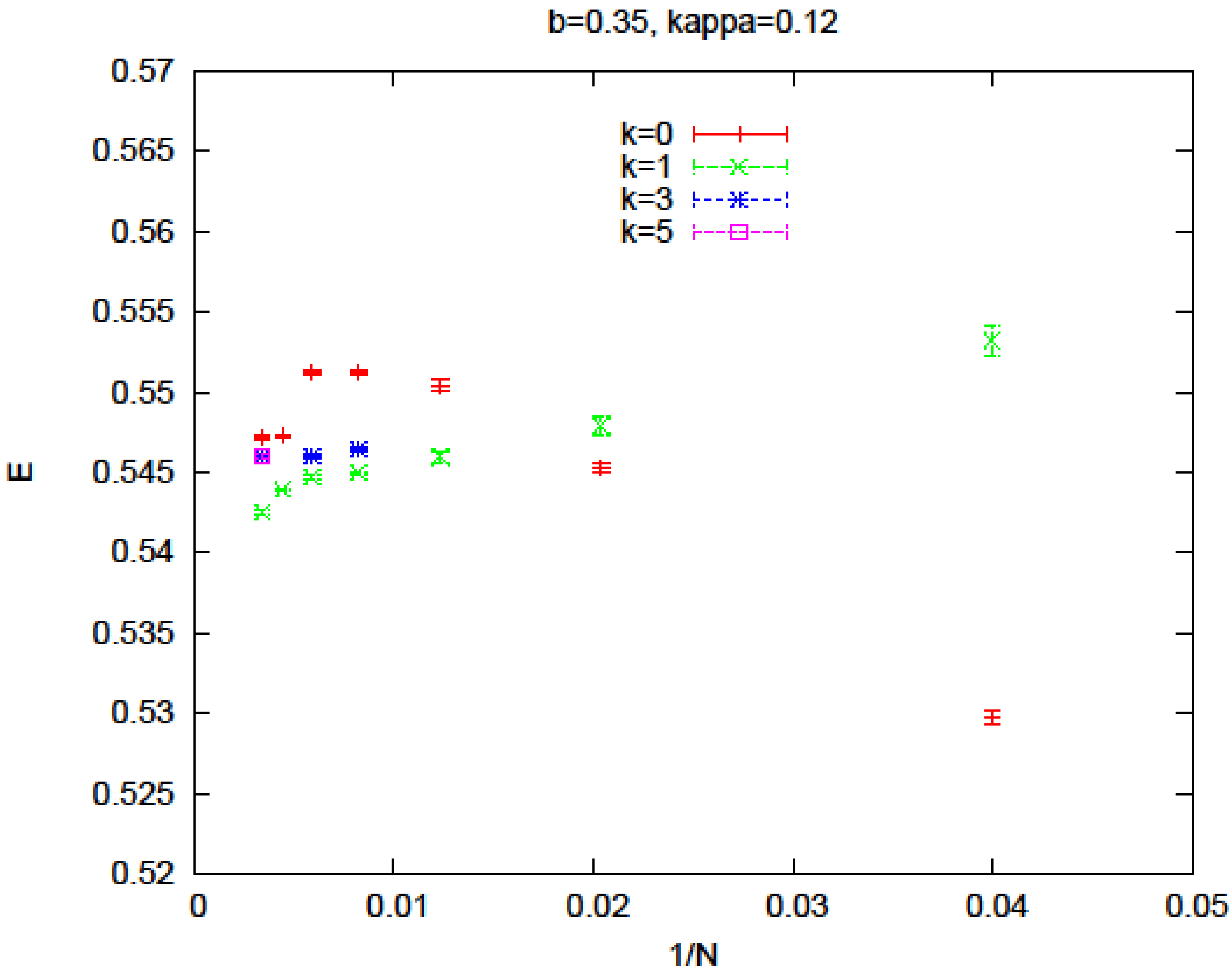
$$N = 25,\ 49,\ 81,\ 121,\ 169,\ 225,\ 289$$
$$V = 5^4,\ 7^4,\ 9^4,\ 11^4,\ 13^4,\ 15^4,\ 17^4$$

We can, then, calculate Wilson loop W(R,R) up to

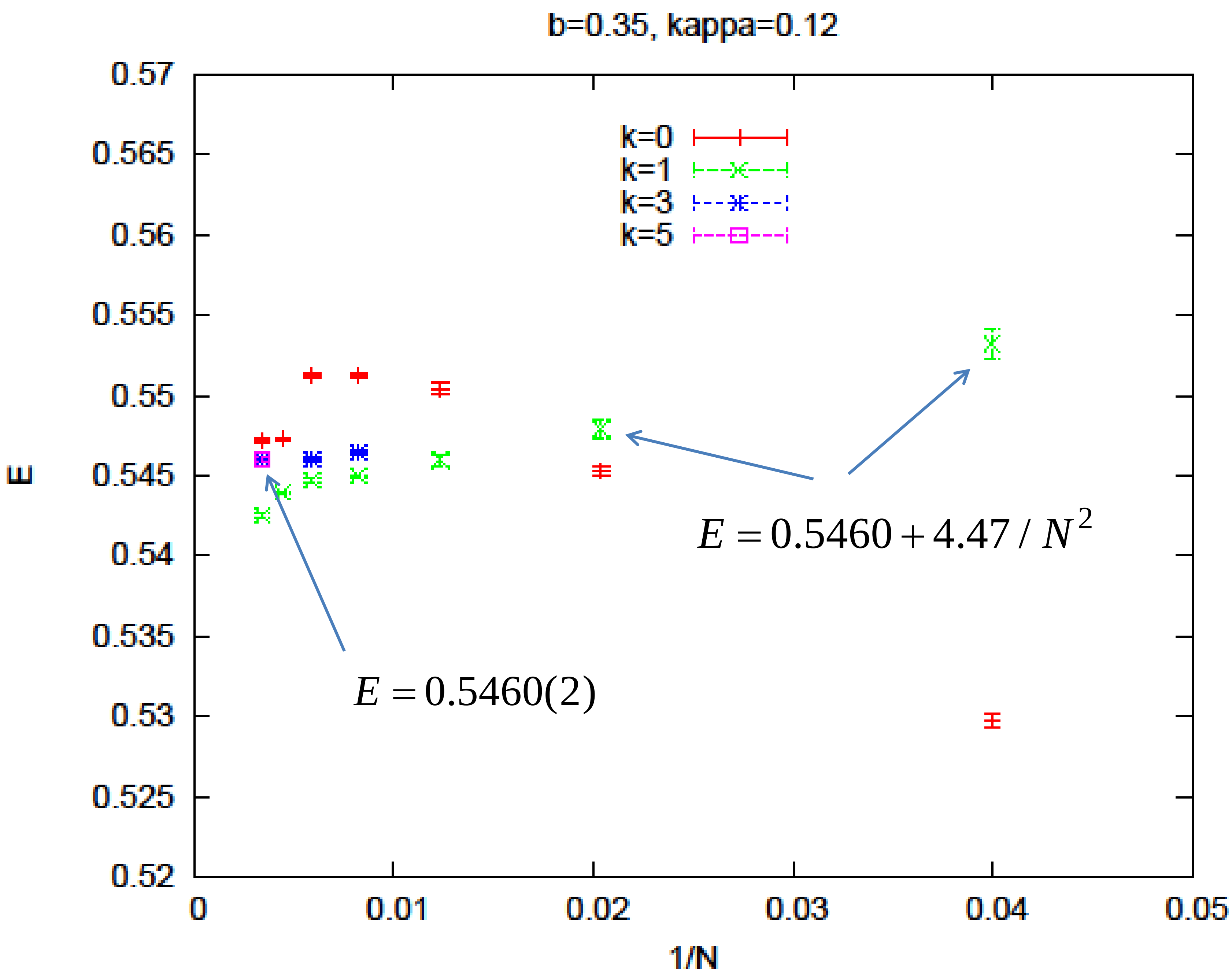
$$R = 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8$$



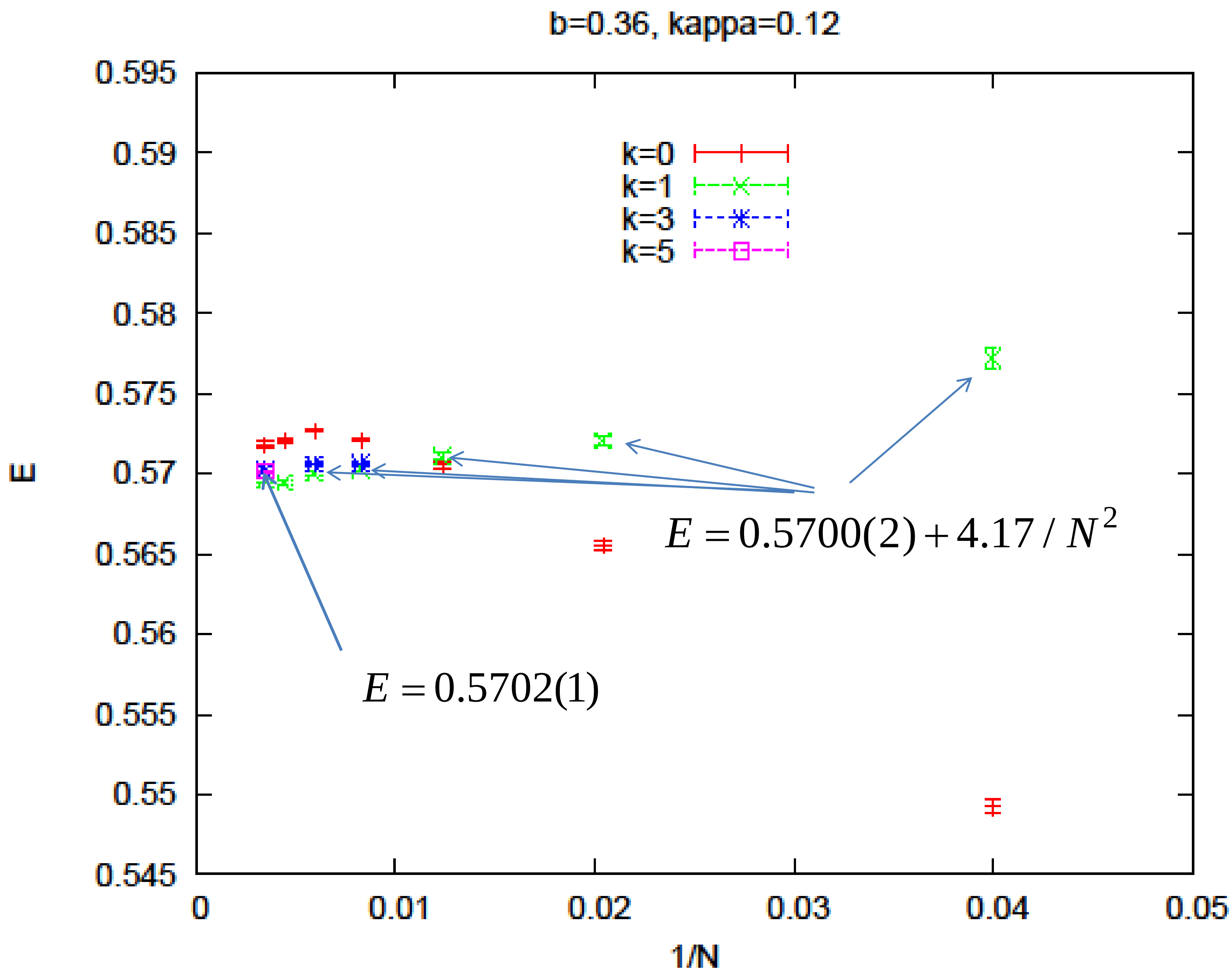
$E = Z_{\mu\nu}\left\langle Tr(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)\right\rangle$
N= 25, k=0
N= 49, k=0
N= 81, k=0
N= 49, k=1
N= 81, k=1
N=121, k=1
0.8
0.7
0.6
0.5
0.4
0.3
E
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
κ



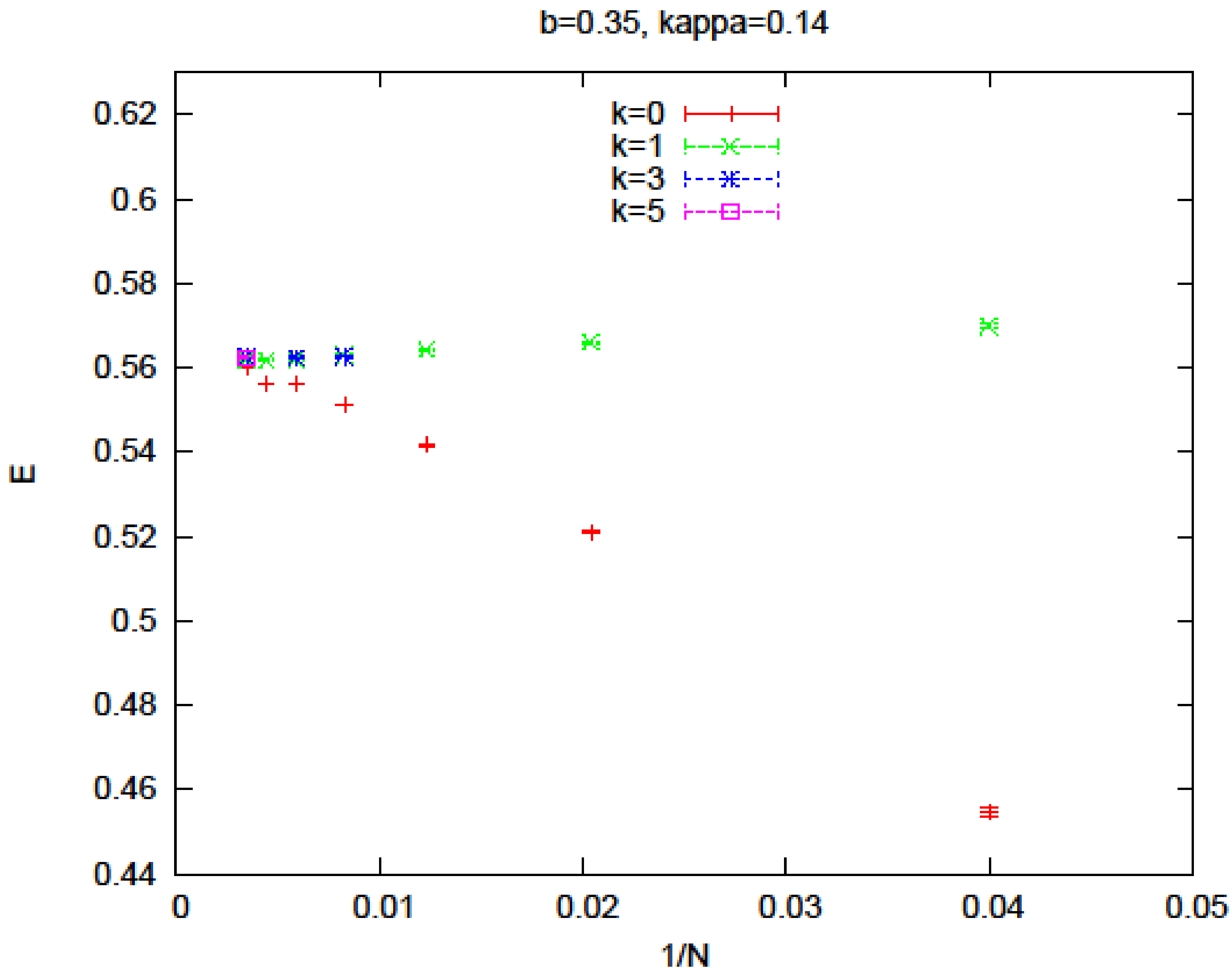
b=0.35, kappa=0.12
k=0
k=1
k=3
k=5
E
1/N
0.57
0.565
0.56
0.555
0.55
0.545
0.54
0.535
0.53
0.525
0.52
0
0.01
0.02
0.03
0.04
0.05



b=0.35, kappa=0.12
k=0
k=1
k=3
k=5
E
1/N
0.57
0.565
0.56
0.555
0.55
0.545
0.54
0.535
0.53
0.525
0.52
0
0.01
0.02
0.03
0.04
0.05
$E = 0.5460 + 4.47 / N^2$
$E = 0.5460(2)$



b=0.36, kappa=0.12
k=0
k=1
k=3
k=5
E
1/N
0.595
0.59
0.585
0.58
0.575
0.57
0.565
0.56
0.555
0.55
0.545
0
0.01
0.02
0.03
0.04
0.05
$E = 0.5700(2) + 4.17 / N^2$
$E = 0.5702(1)$


b=0.35, kappa=0.14
k=0
k=1
k=3
k=5
E
1/N
0.62
0.6
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0
0.01
0.02
0.03
0.04
0.05

The finite N correction of the model with twisted boundary condition ($k \neq 0$)

- significantly smaller than those of the model with periodic boundary condition ($k = 0$).

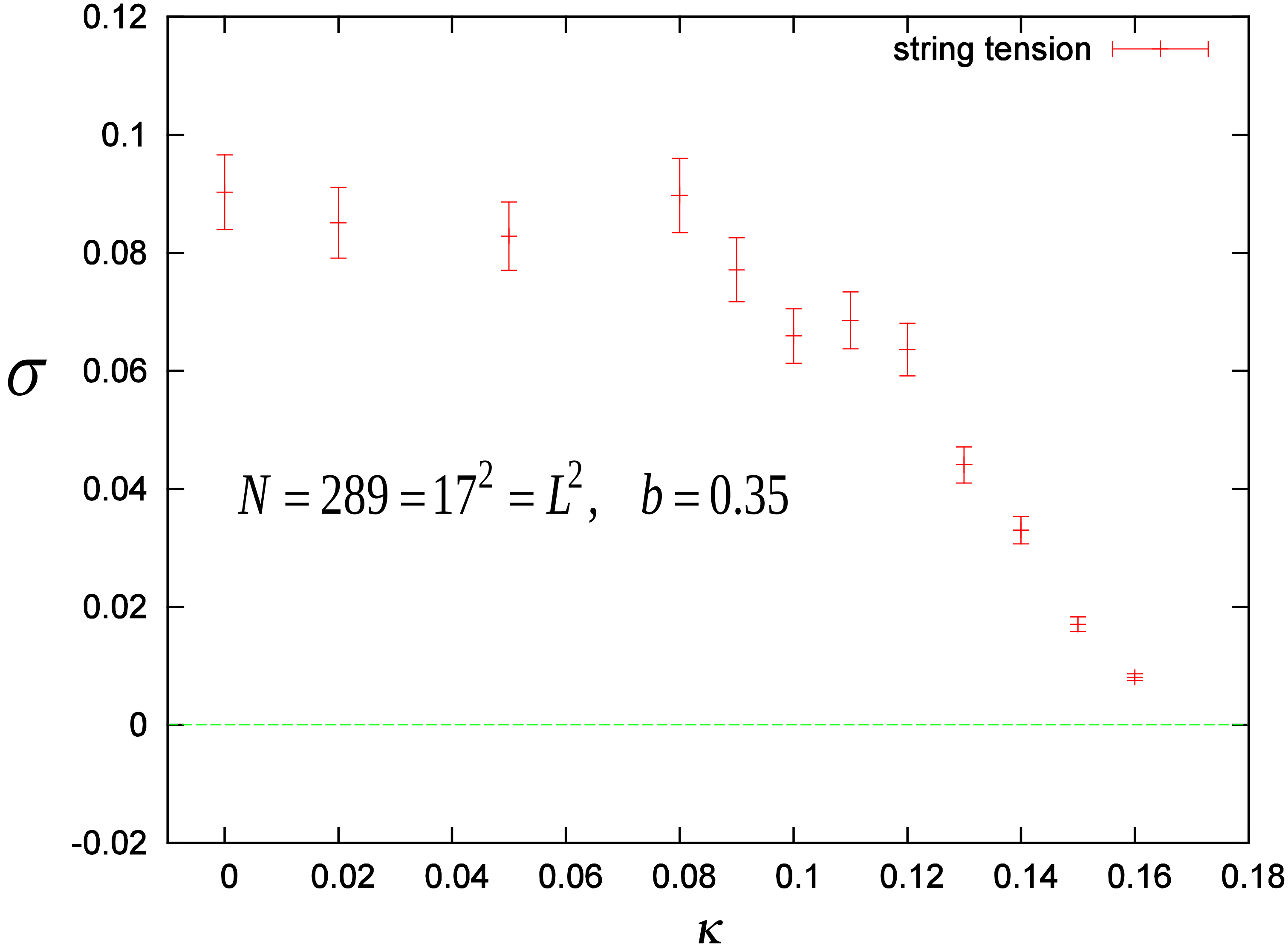
- can be fitted with the form $a + b / N^2 + \cdots$ for appropriately chosen range of $N < N_c$, with N_c roughly determined by the symmetry breaking pattern of the pure gauge theory.

Twisted space-time reduction in large N QCD with adjoint Wilson fermions

Plan of the talk (second part)

- Why twisted reduction works ?
- Finite N corrections with two adjoint fermions
- How to calculate κ_c

How to calculate κ_c



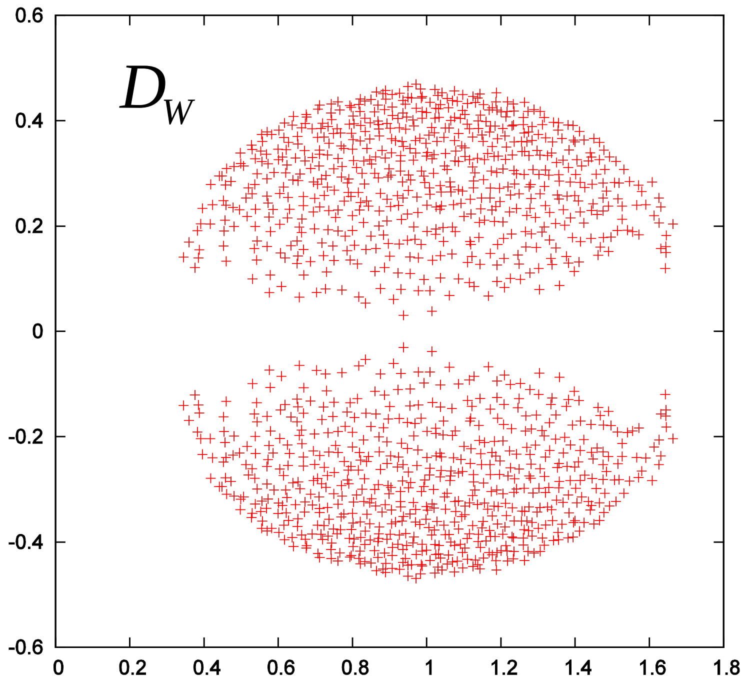
$$m_q = 1/\kappa - 1/\kappa_c$$

So far, we have not calculated any hadronic spectrum.

However, it is straightforward to calculate the lowest eigenvalue of positive hermitian Wilson Dirac operator $Q^2 = (D_W \gamma_5)^2$, which should be related to the physical quark mass square.

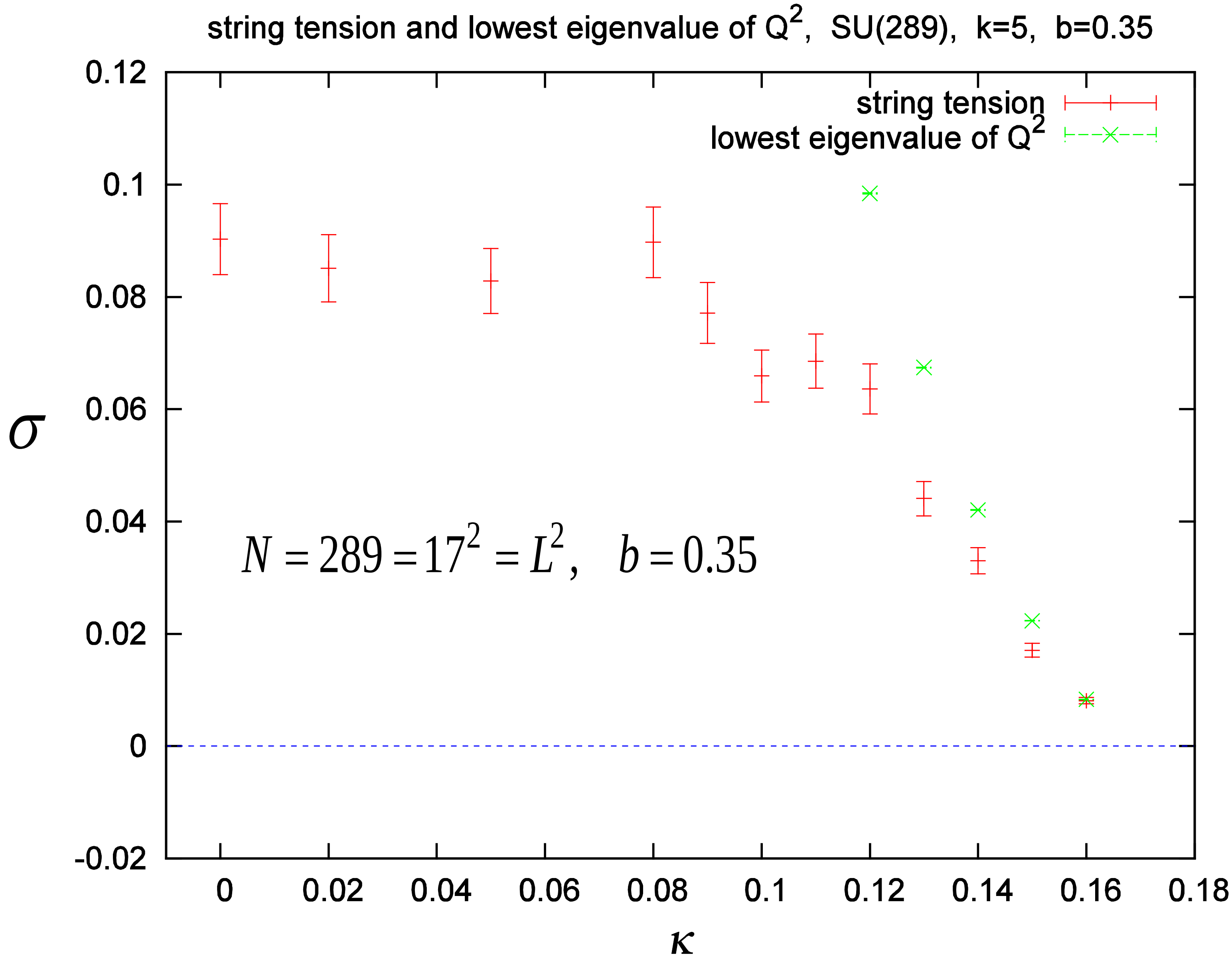
$$D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right] = \kappa \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right) + 2\kappa \partial_\mu \gamma^\mu + \cdots$$

$$= 2\kappa m_q + 2\kappa \partial_\mu \gamma^\mu + \cdots$$



lowest eigenvalue of positive Hermitian Wilson Dirac operator

$$\kappa^2 \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right)^2 = 4\kappa^2 m_q^2$$



$$m_q = 1/\kappa - 1/\kappa_c$$

We can fit the lowest eigenvalue of Q^2 with the following fitting form

$$\kappa^2 \left(1/\kappa - 1/\kappa_c\right)^2$$

We then have $\kappa_c = 0.1773(2)$

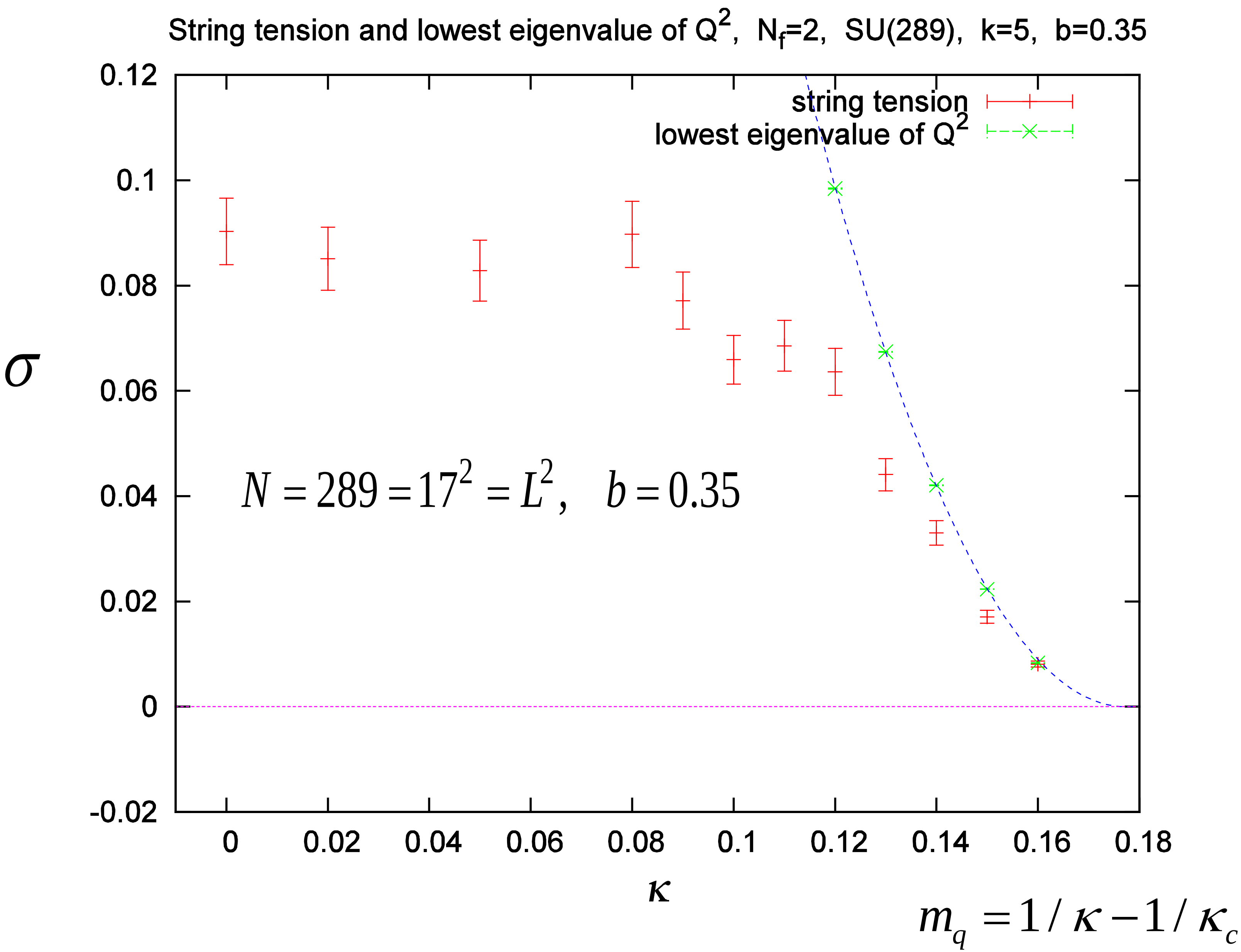
Now the string tension should behave as

$$\sigma \sim m_q^{2/(1+\gamma_*)} \sim (1/\kappa - 1/\kappa_c)^{2/(1+\gamma_*)}$$

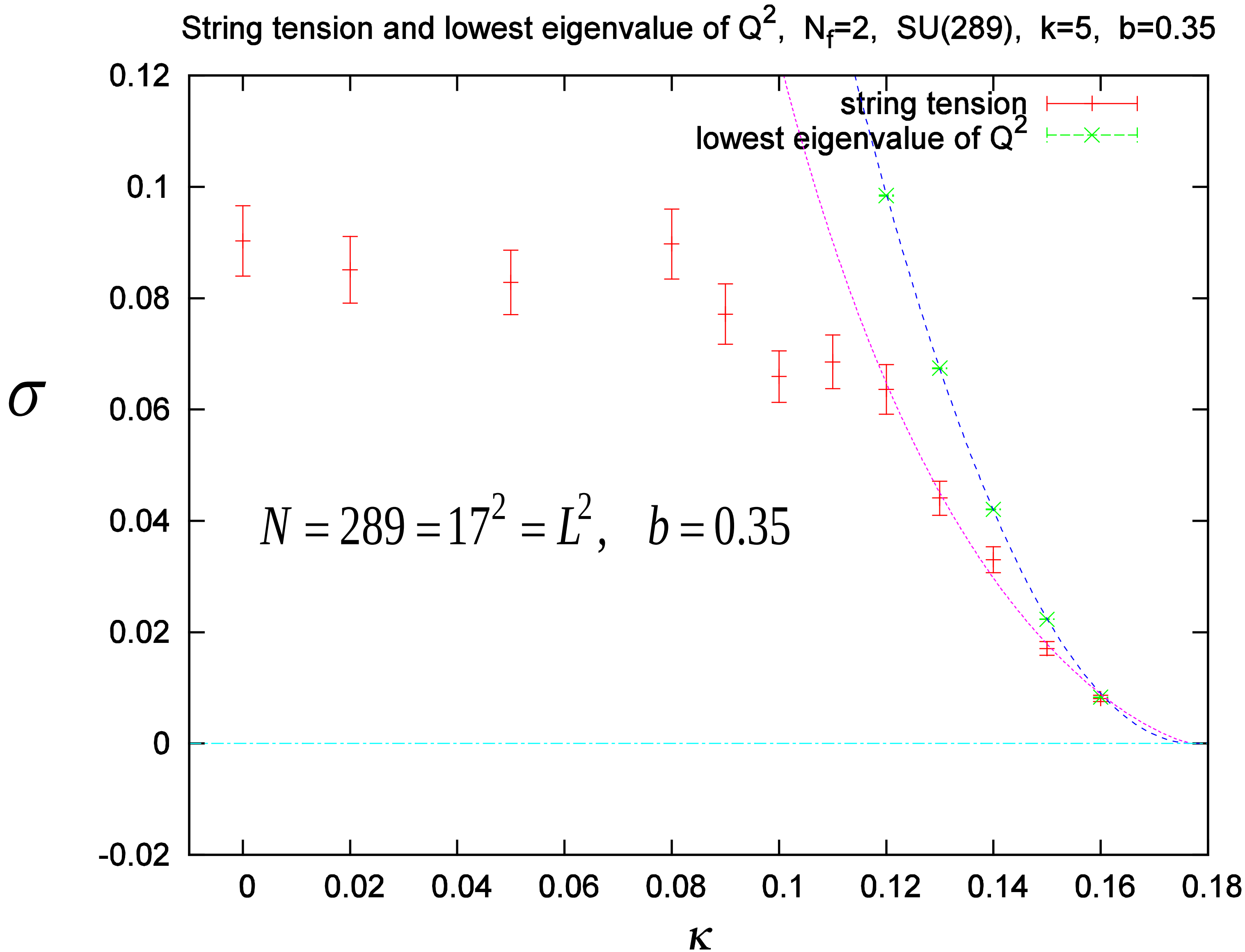
with γ_* the mass anomalous dimension at infrared fixed point.

From our data, we have

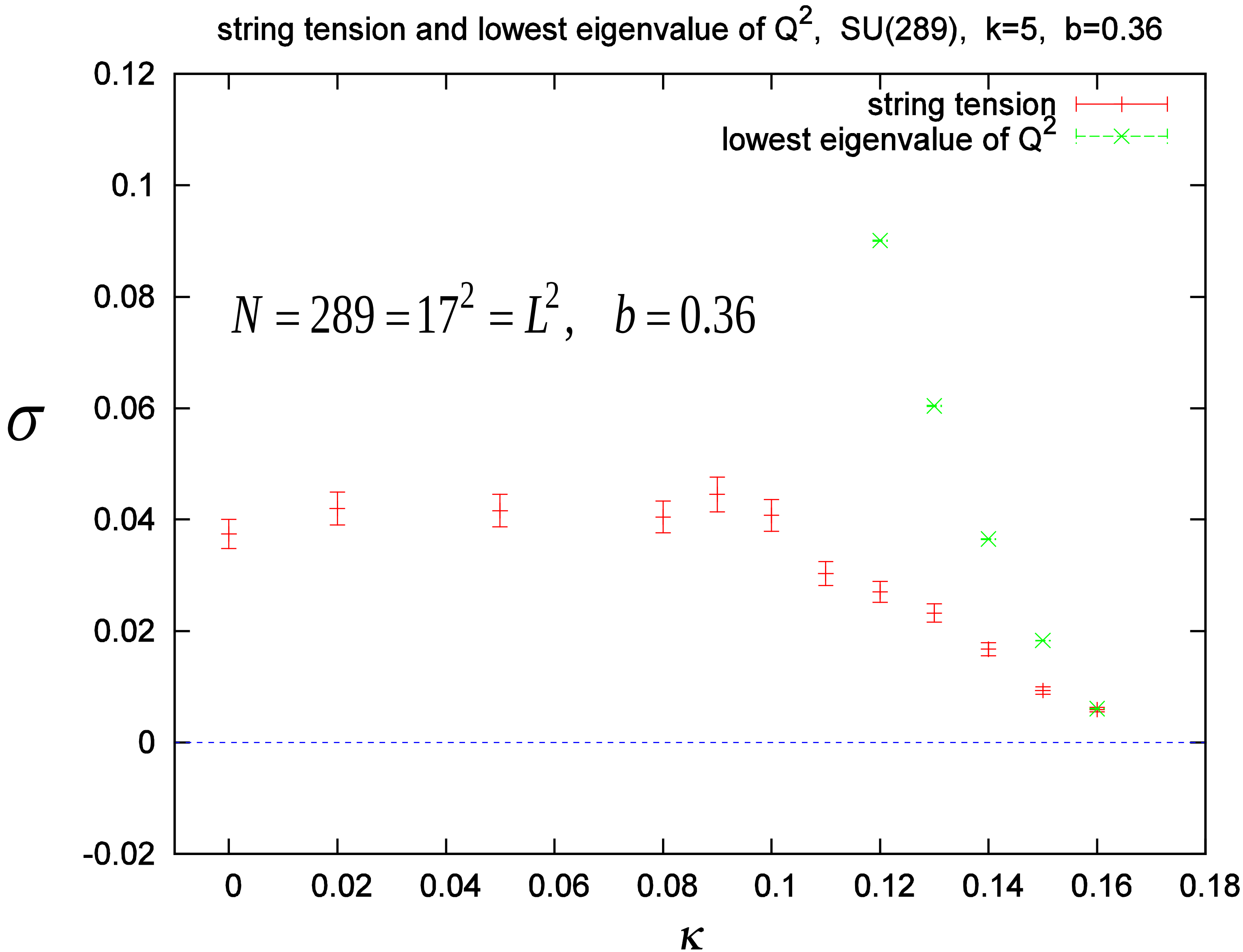
$$\frac{2}{1+\gamma_*} = 1.34(11) \qquad \therefore \ \gamma_* = 0.49(4)$$



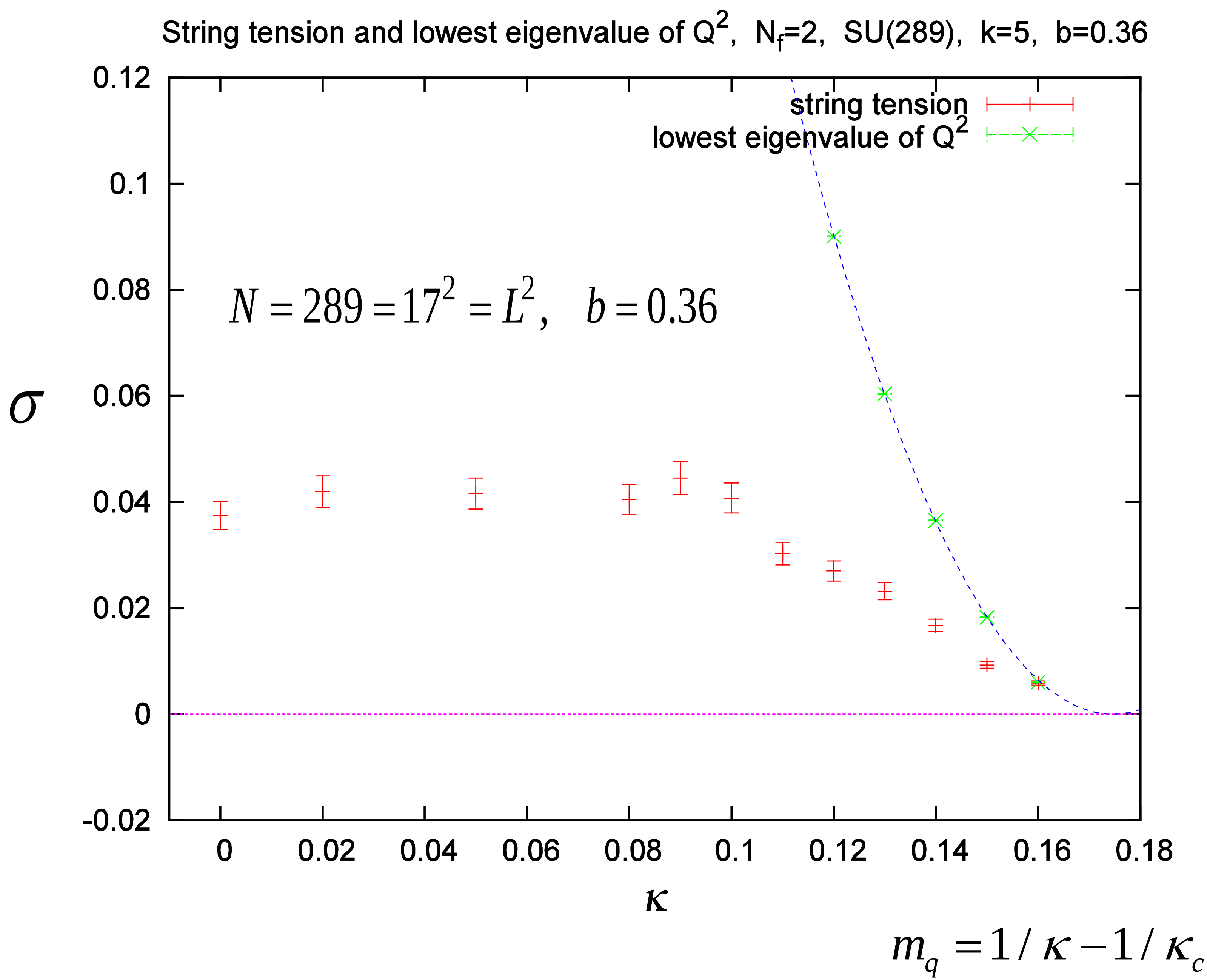
String tension and lowest eigenvalue of Q^2, N_f=2, SU(289), k=5, b=0.35
string tension
lowest eigenvalue of Q^2
σ
$N = 289 = 17^2 = L^2, \quad b = 0.35$
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
κ
$m_q = 1/\kappa - 1/\kappa_c$



$$m_q = 1/\kappa - 1/\kappa_c$$



$$m_q = 1/\kappa - 1/\kappa_c$$



String tension and lowest eigenvalue of Q^2, N_f=2, SU(289), k=5, b=0.36
string tension
lowest eigenvalue of Q^2
$N = 289 = 17^2 = L^2, \quad b = 0.36$
σ
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
κ
$m_q = 1/\kappa - 1/\kappa_c$

Conclusion and outlook

- Twisted space-time reduction works for $SU(N),\ N = L^2$ with

$$\frac{1}{9} < \frac{k}{L} < \frac{1}{2}\ .$$ L and k co-prime. large $\bar{k}$.

- For suitably chosen values of *N,* the finite *N* corrections of the model with fixed b, k, κ are of order $1/N^2$ as expected.

- Study of the eigenvalue distribution $\rho(\omega)$ of the Wilson Dirac fermion matrix is promising.

  Lowest eigenvalue determine the value of critical κ_c .
  We can determine γ_* from $\rho(\omega) \sim \omega^{(3-\gamma_*)/(1+\gamma_*)}$ near κ_c .

- We need to develop the method to calculate hadronic quantities!