# High Energy Scattering in AdS/CFT

## Applications to N=4 SYM and to low-$x$ QCD

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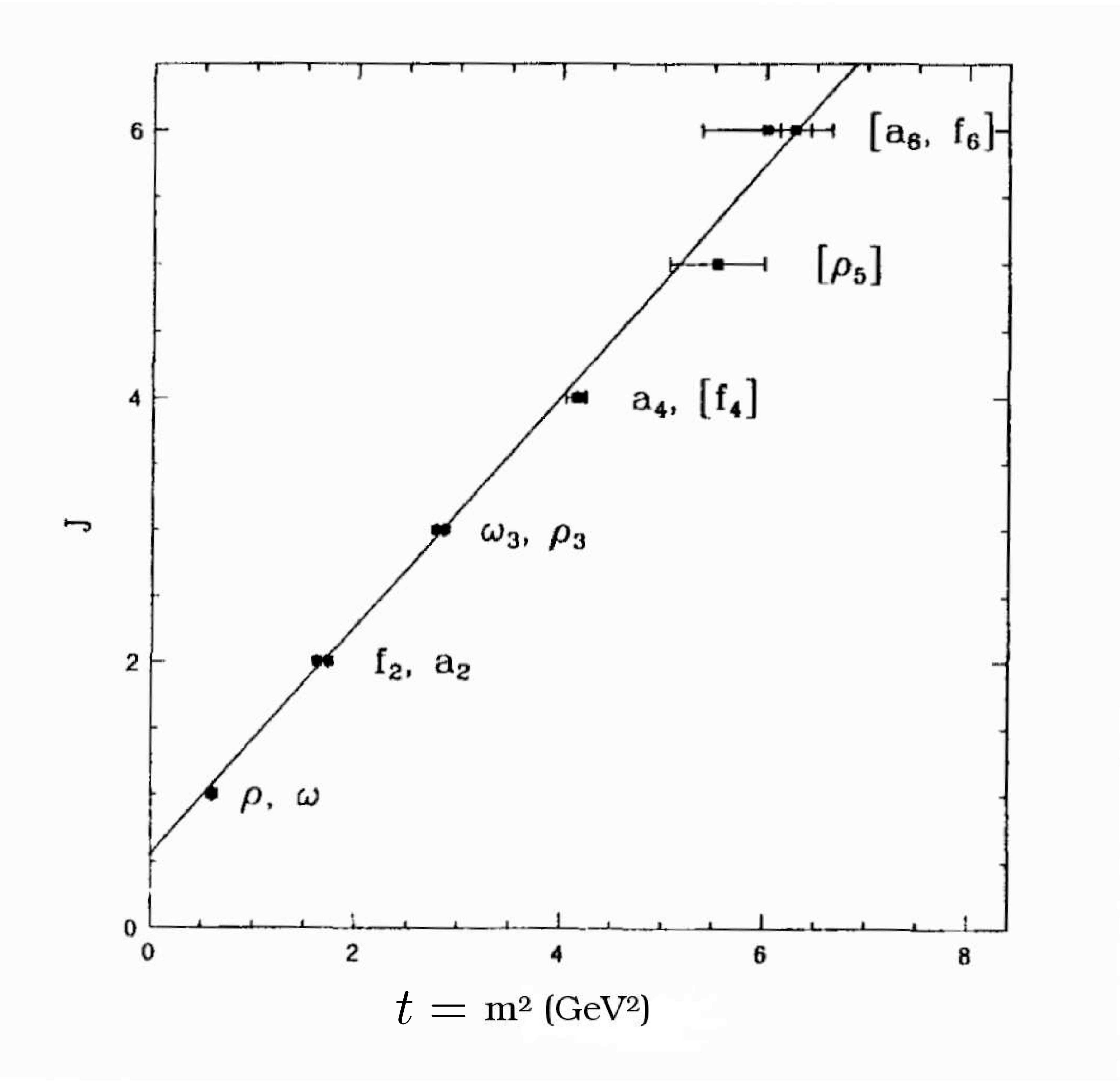
# Motivation

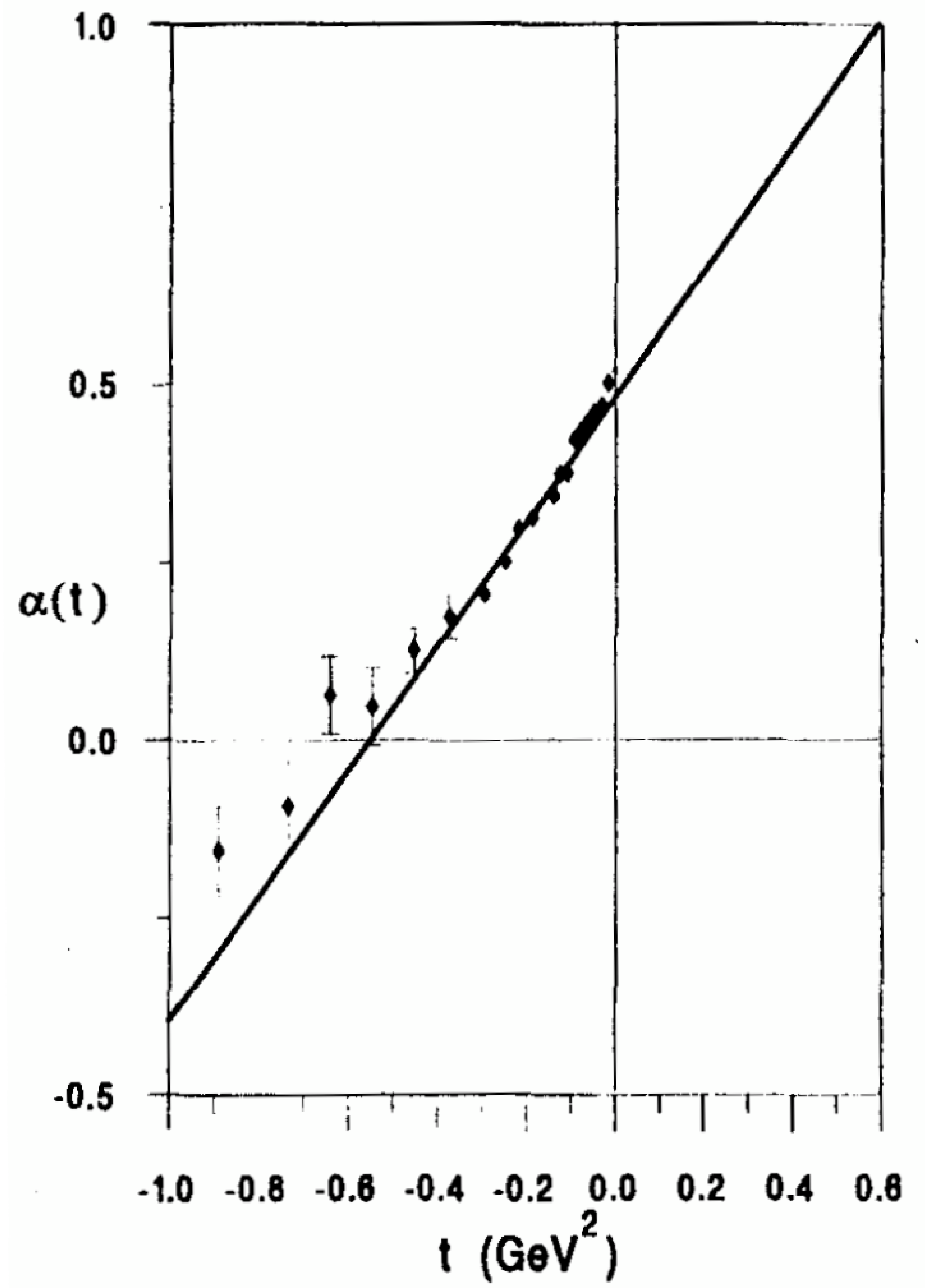
- Regge theory gives important physical information in QCD

Regge trajectory for isospin $I = 1$ even parity mesons.




These mesons dominate exchange in

$$\pi^- + p \to \pi^0 + n$$




$$s \gg -t$$

$$A(s,t) \sim \beta(t)\, s^{\alpha(t)}$$

$$\alpha(t) = \alpha(0) + \alpha' t$$

intercept $j_0$

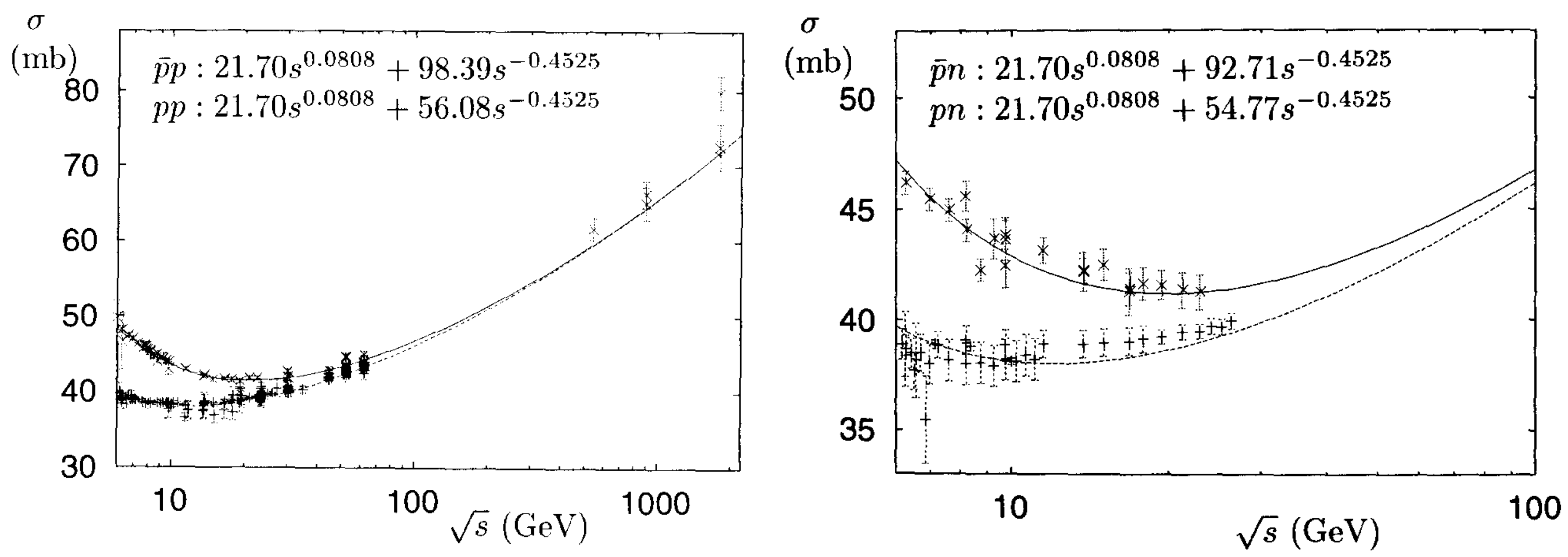
slope

- Trajectory that dominates a given process determined by exchanged quantum numbers. For elastic scattering these are the vacuum quantum numbers.

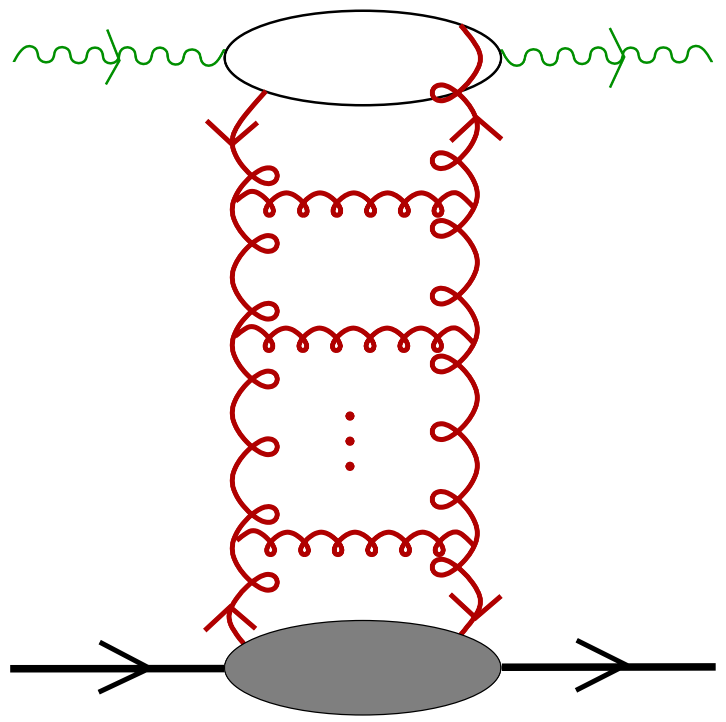
**Soft Pomeron** [Landshoff-Donnachie]

$$\alpha_P \approx 1.08 + 0.25\,t \quad (\text{GeV units})$$

(Evidence from lattice QCD that there are glueballs on this trajectory with $J \geq 2$)




- Pomeron enters also in diffractive processes. For example DIS.

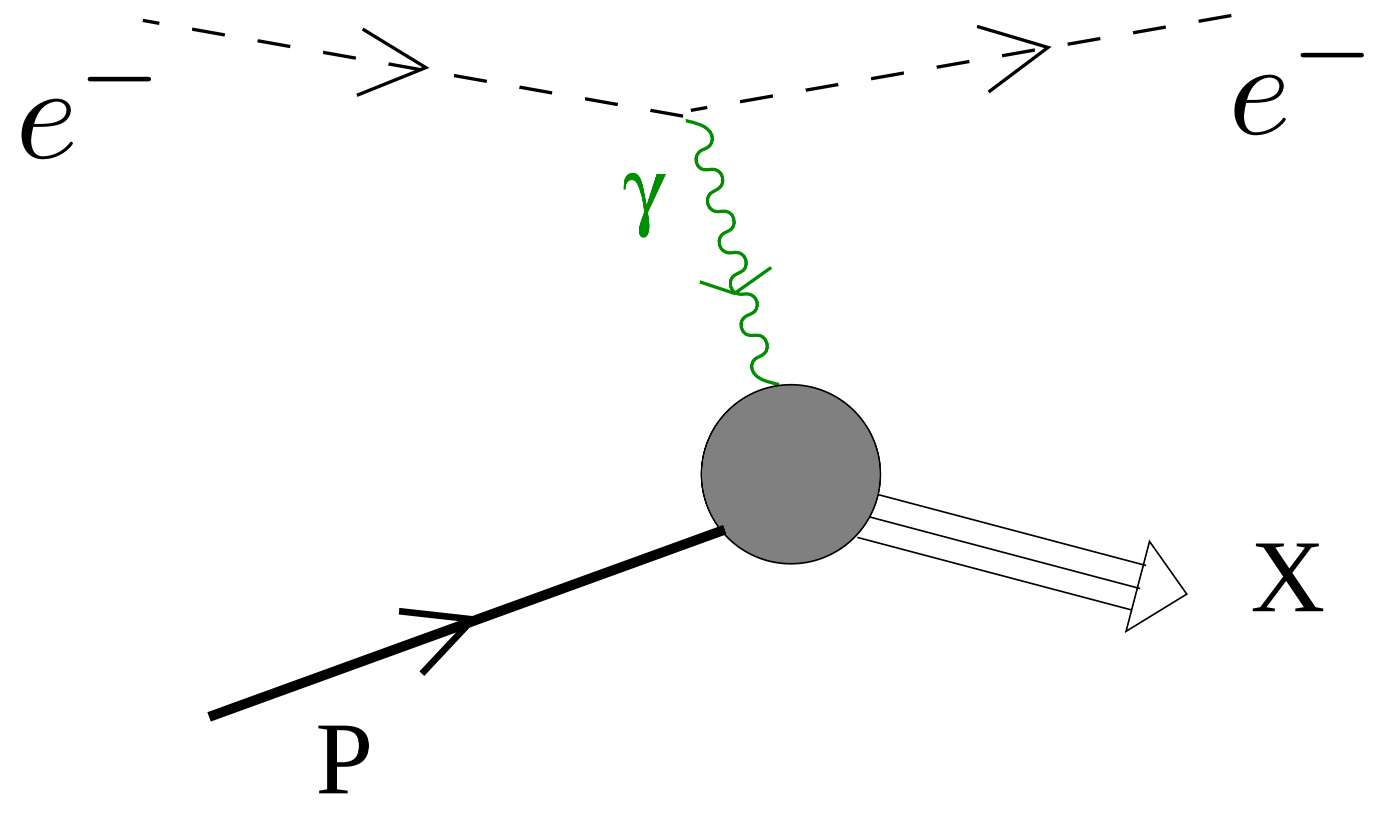


**Hard Pomeron**

[BFKL - Balitsky, Fadin, Kuraev & Lipatov]

**In DIS much larger intercept is observed**

$$j_0 = 1.2 - 1.4$$



## Basic idea & two goals

- Explore high energy scattering in the Regge limit in AdS/CFT context

Strings exhibit Regge behaviour

**Regge theory in CFT's**

[works with Cornalba, Penedones]

[Kotikov, Lipatov, Staudacher,Velizhanin 07]

- Obtain new information about anomalous dimensions and OPE coefficients in N=4 Super Yang-Mills, and also AdS graviton Regge trajectory

[MSC, Penedones, Gonçalves 12]

- Phenomenology of low x physics in QCD (Connection with pomeron physics by BPST 2006)

[works with Cornalba, Penedones,Djuric; MSC, Djuric, Evans to appear]

# Regge theory in String Theory

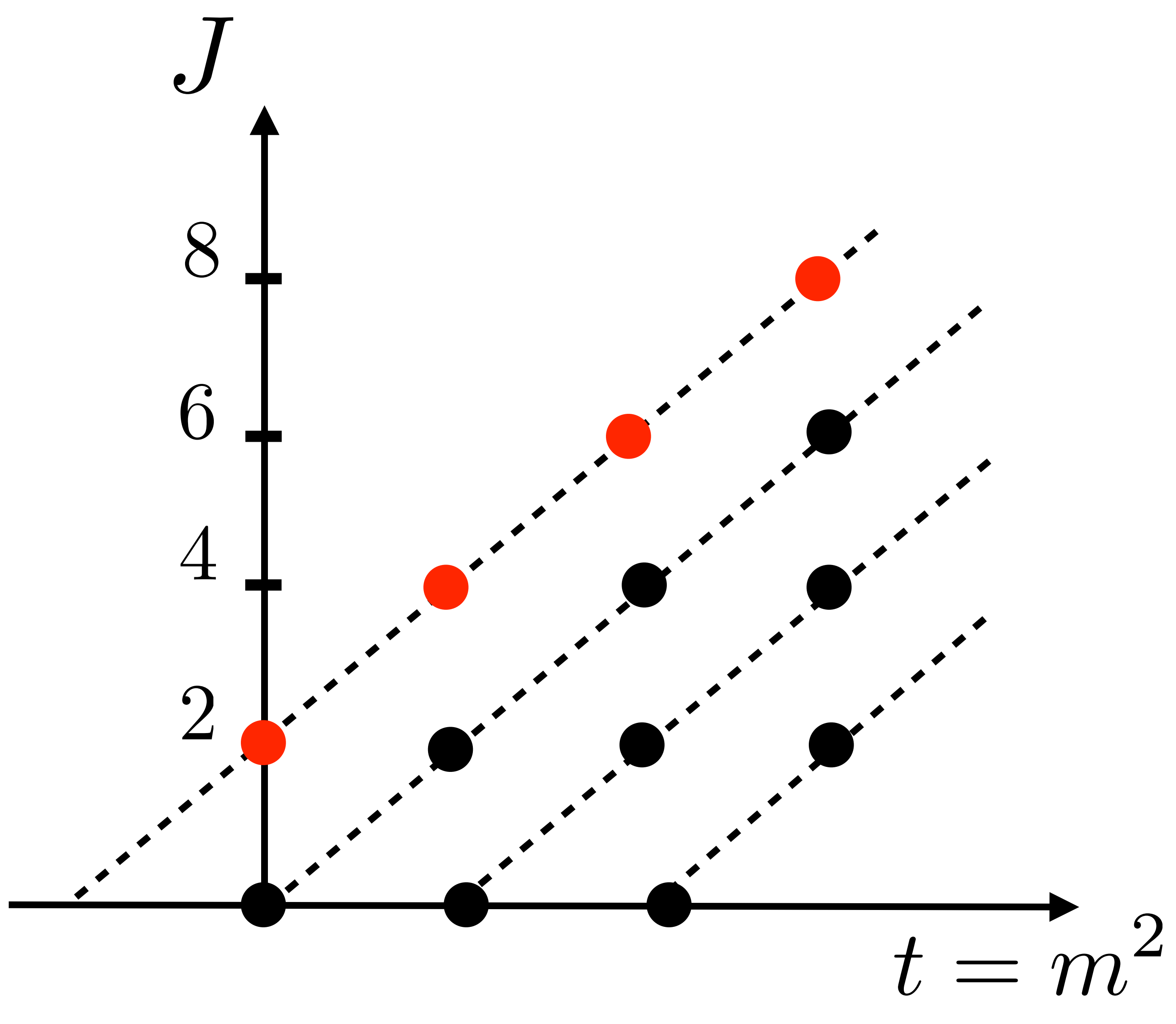
- Virasoro-Shapiro S-matrix element

$$\mathcal{T}(s,t) = 8\pi G_N \left(\frac{tu}{s} + \frac{su}{t} + \frac{st}{u}\right) \frac{\Gamma\left(1-\frac{\alpha' s}{4}\right)\Gamma\left(1-\frac{\alpha' u}{4}\right)\Gamma\left(1-\frac{\alpha' t}{4}\right)}{\Gamma\left(1+\frac{\alpha' s}{4}\right)\Gamma\left(1+\frac{\alpha' u}{4}\right)\Gamma\left(1+\frac{\alpha' t}{4}\right)}$$

Regge limit $s \gg -t$

$$\approx \frac{32\pi G_N}{\alpha'} e^{-\frac{i\pi\alpha' t}{4}} \frac{\Gamma\left(-\frac{\alpha' t}{4}\right)}{\Gamma\left(1+\frac{\alpha' t}{4}\right)} \left(\frac{\alpha' s}{4}\right)^{2+\frac{\alpha' t}{2}}$$

$\beta(t)$ $\quad j(t)$




- Amplitude contains poles for each physical exchange. The Regge behaviour can be obtained only from exchange of particles in leading Regge trajectory.

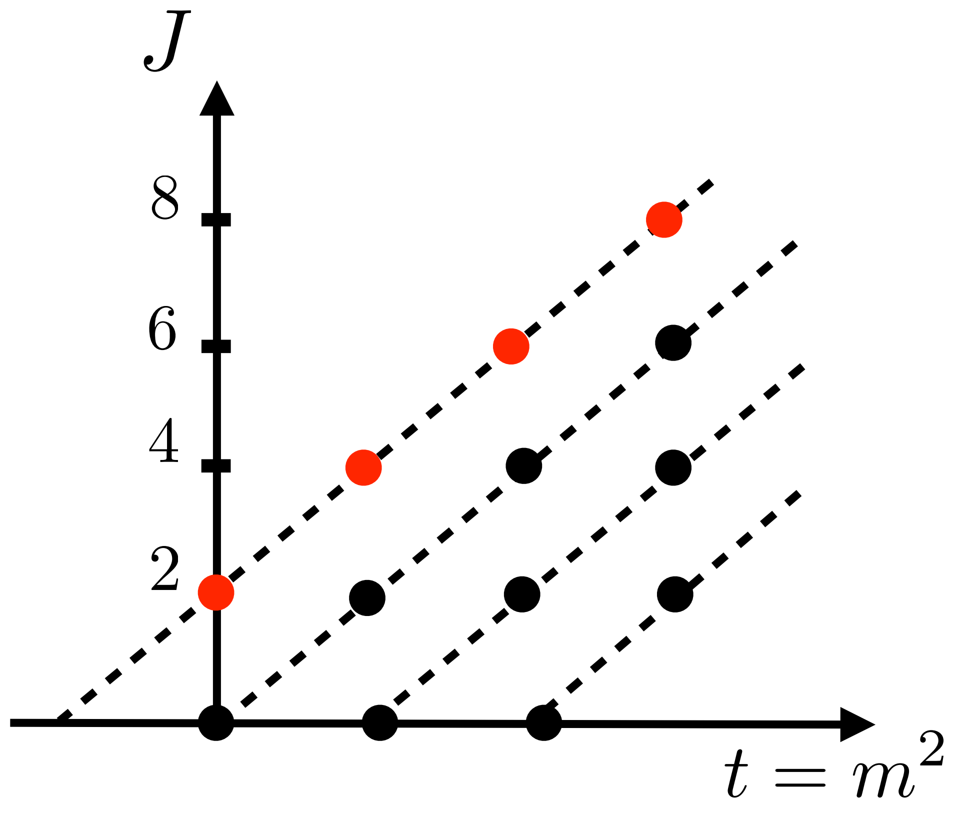
- t-channel partial wave expansion

$$\mathcal{T}(s,t) = \sum_{J=0}^{\infty} a_J(t)\, P_J\left(1 + 2\frac{s}{t}\right) \sim \left(\frac{s}{t}\right)^J$$

- Exchange of spin $J$ field has pole at $t = m^2(J)$

$$a_J(t) \approx \frac{r(J)}{t - m^2(J)}$$

- Sum exchanges in leading Regge trajectory and Sommerfeld-Watson transform

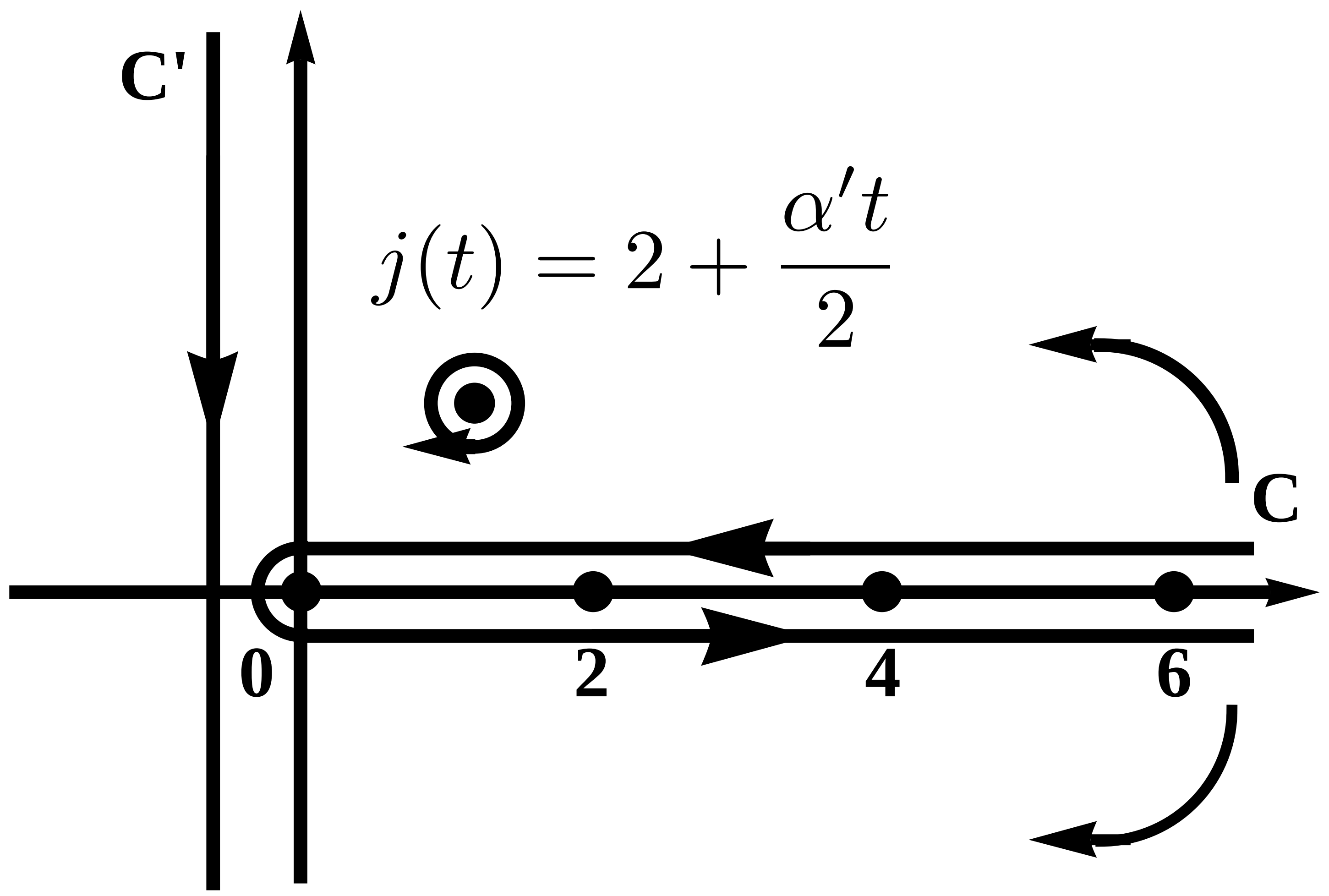



$$\sum_J \rightarrow \int_C \frac{dJ}{2\pi i} \frac{\pi}{\sin(\pi J)}$$

- Analytically continue in $J$ and pick leading pole from

$$\mathcal{T}(s,t) \approx \beta(t) s^{j(t)}$$

$$a_J(t) \approx -\frac{j'(t)\, r(j(t))}{J - j(t)}$$



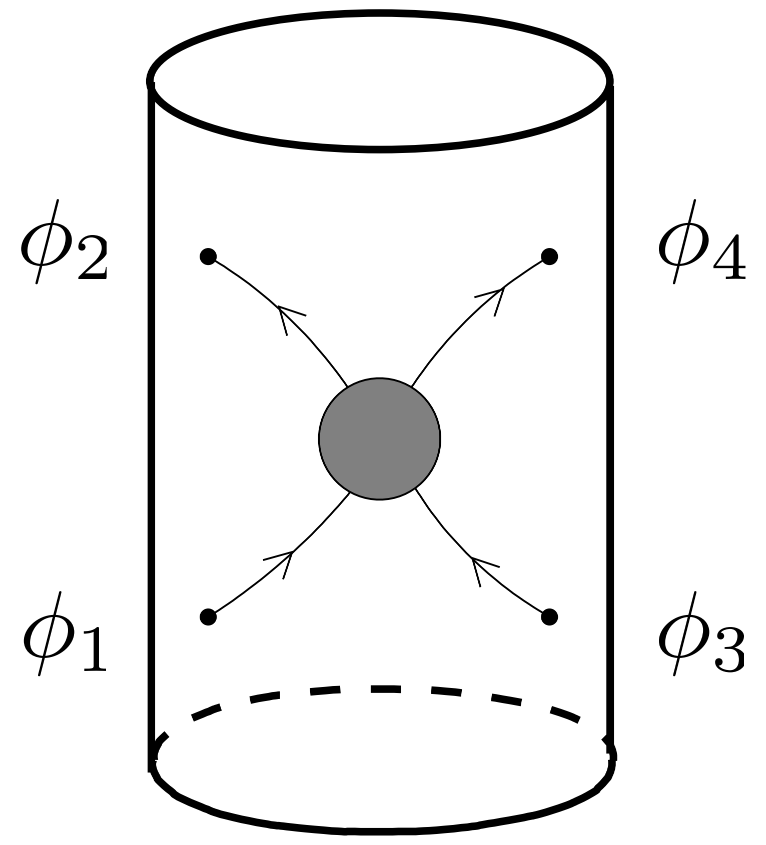
# AdS/CFT duality

Strings in AdS (d+1 dimensions) ⟷ Conformal Field Theory (d dimensions)

Tree level $g_s \to 0$ — Planar level $N \to \infty$

Finite string length $l_s = \sqrt{\alpha'}$ — Finite ‘t Hooft coupling $\lambda = g_{YM}^2 N = \frac{R^4}{\alpha'^2}$

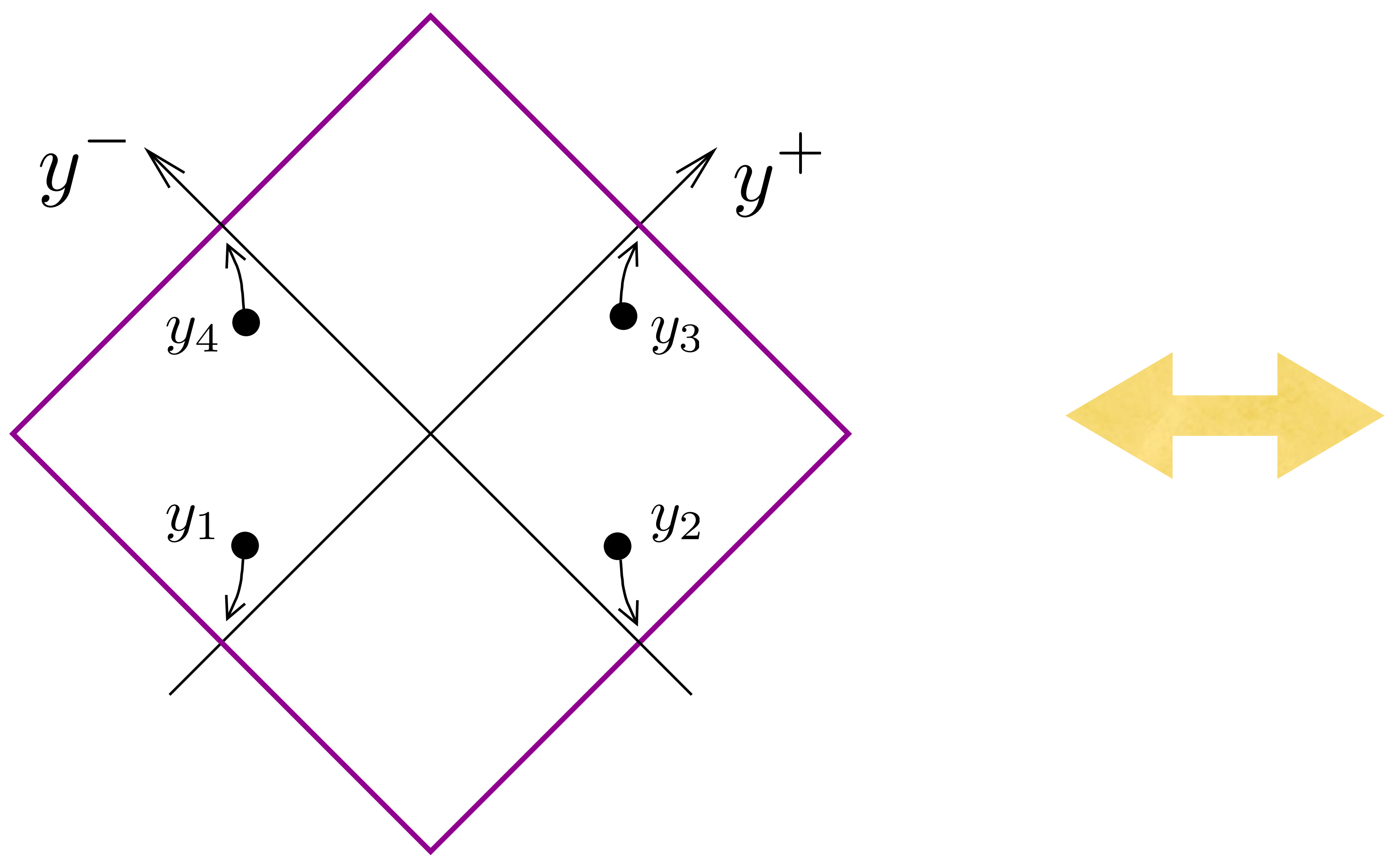
String fields $\phi$ — Single trace operators $\mathcal{O}$



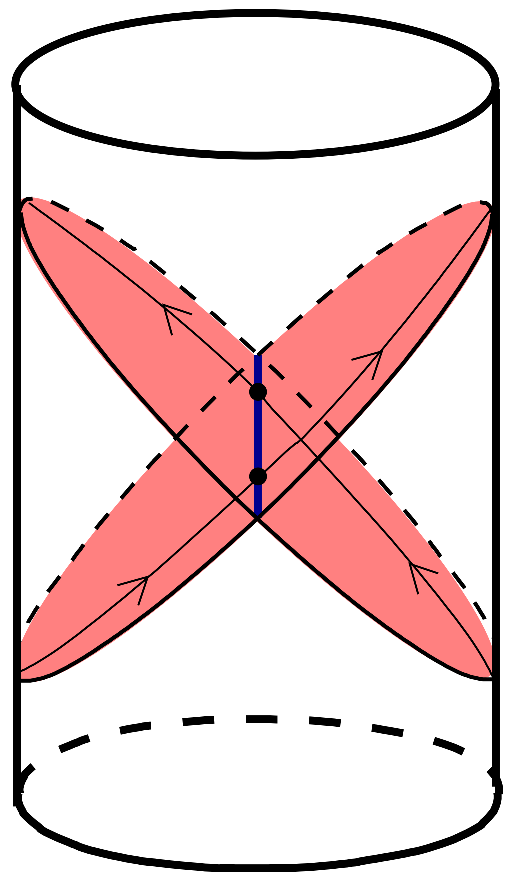

$$\langle \mathcal{O}_1(y_1)\, \mathcal{O}_2(y_2)\, \mathcal{O}_3(y_3)\, \mathcal{O}_4(y_4) \rangle$$

## Conformal Regge theory

- 4-pt correlator $A(y_i) = \langle \mathcal{O}_1(y_1)\mathcal{O}_2(y_2)\mathcal{O}_3(y_3)\mathcal{O}_4(y_4)\rangle$
- CFT Regge limit




AdS scattering process



- After Sommerfeld-Watson transform in Mellin space exchange of operators in leading Regge trajectory $\Delta = \Delta(J)$

$$M(s,t) \approx \int d\nu\, \beta(\nu)\, \omega_{\nu,j(\nu)}(t)\, s^{j(\nu)}$$

Reggeon spin $J = j(\nu)$ defined by inverse function

$$\nu^2 + (\Delta(J) - 2)^2 = 0$$

Residue related to OPE coeffs

$$\beta(\nu) \;\rightarrow\; C_{13j(\nu)}C_{24j(\nu)}$$

# N=4 Super Yang Mills

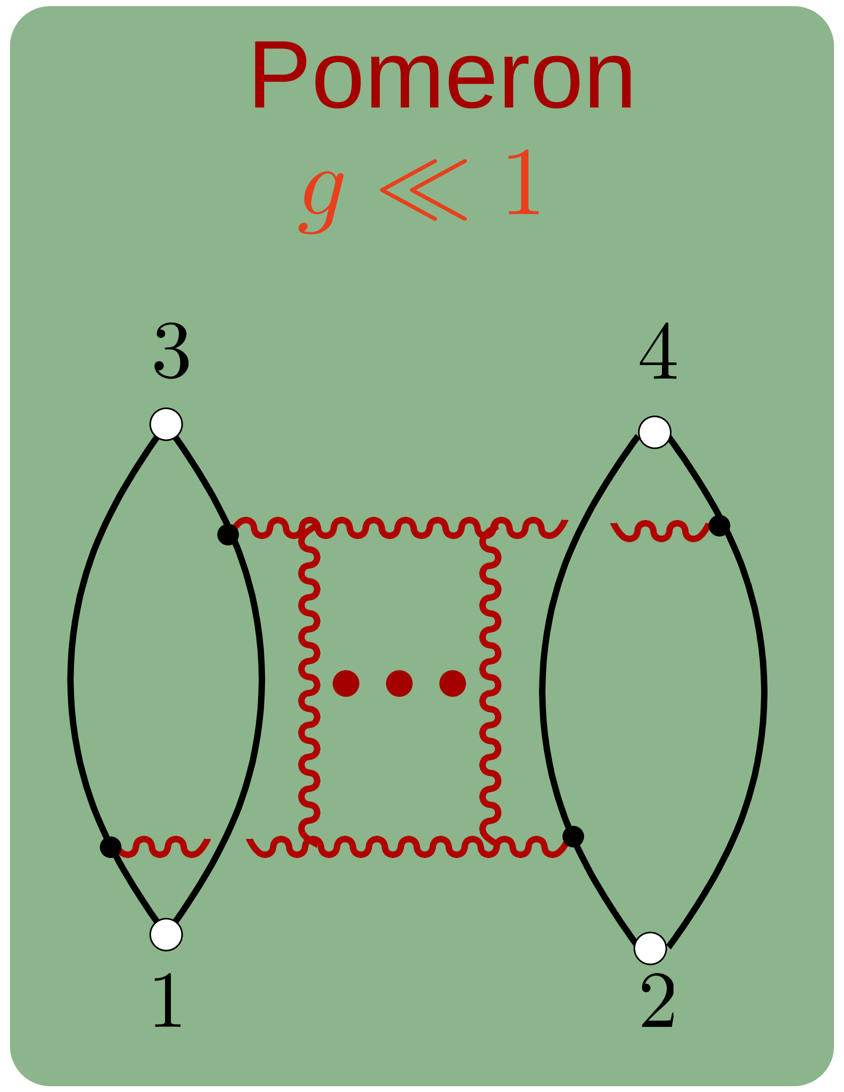
- Correlation functions that exchange vacuum quantum numbers are dominated in Regge limit by exchange of pomeron/graviton Regge trajectory (twist 2)

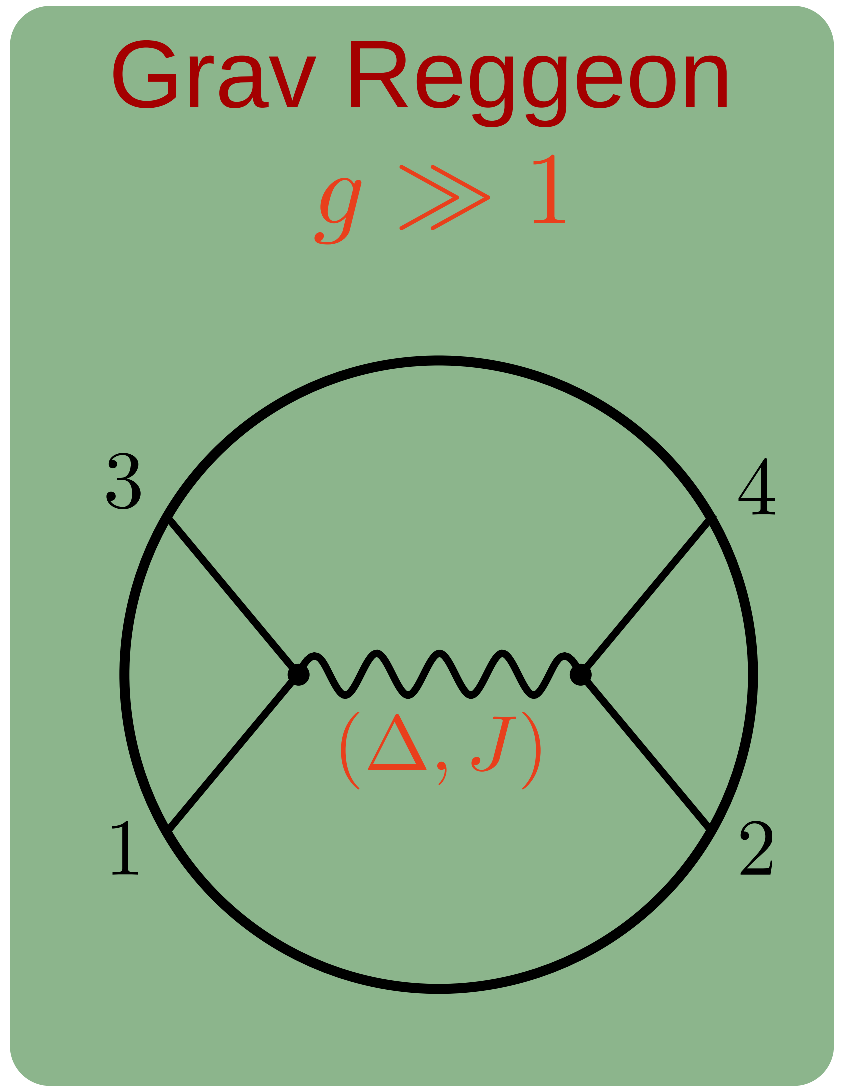
$$\mathcal{O}_1 = \mathcal{O}_3 = \mathrm{tr}\left(\phi_{12}\phi^{12}\right)$$

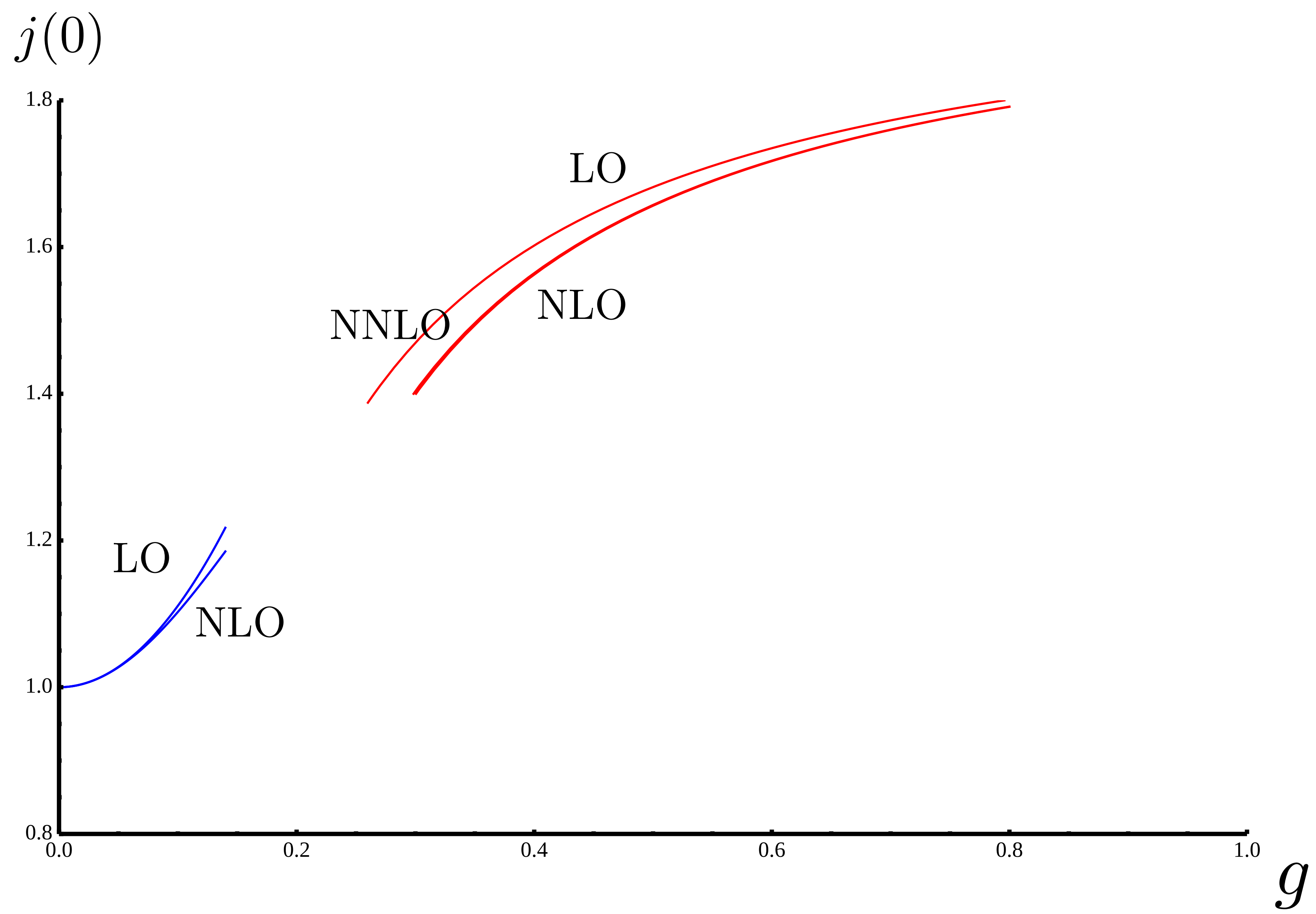
$$\mathcal{O}_2 = \mathcal{O}_4 = \mathrm{tr}\left(\phi_{34}\phi^{34}\right)$$

$$\mathcal{O}_J = \begin{cases} \mathrm{tr}\left(F_{\mu\nu_1} D_{\nu_2} \ldots D_{\nu_{J-1}} F_{\nu_J}{}^{\mu}\right) \\ \mathrm{tr}\left(\phi_{AB} D_{\nu_1} \ldots D_{\nu_J} \phi^{AB}\right) \\ \mathrm{tr}\left(\bar{\psi}_A D_{\nu_1} \ldots D_{\nu_{J-1}} \Gamma_{\mu_J} \psi^A\right) \end{cases}$$

- Weak coupling
- Strong coupling

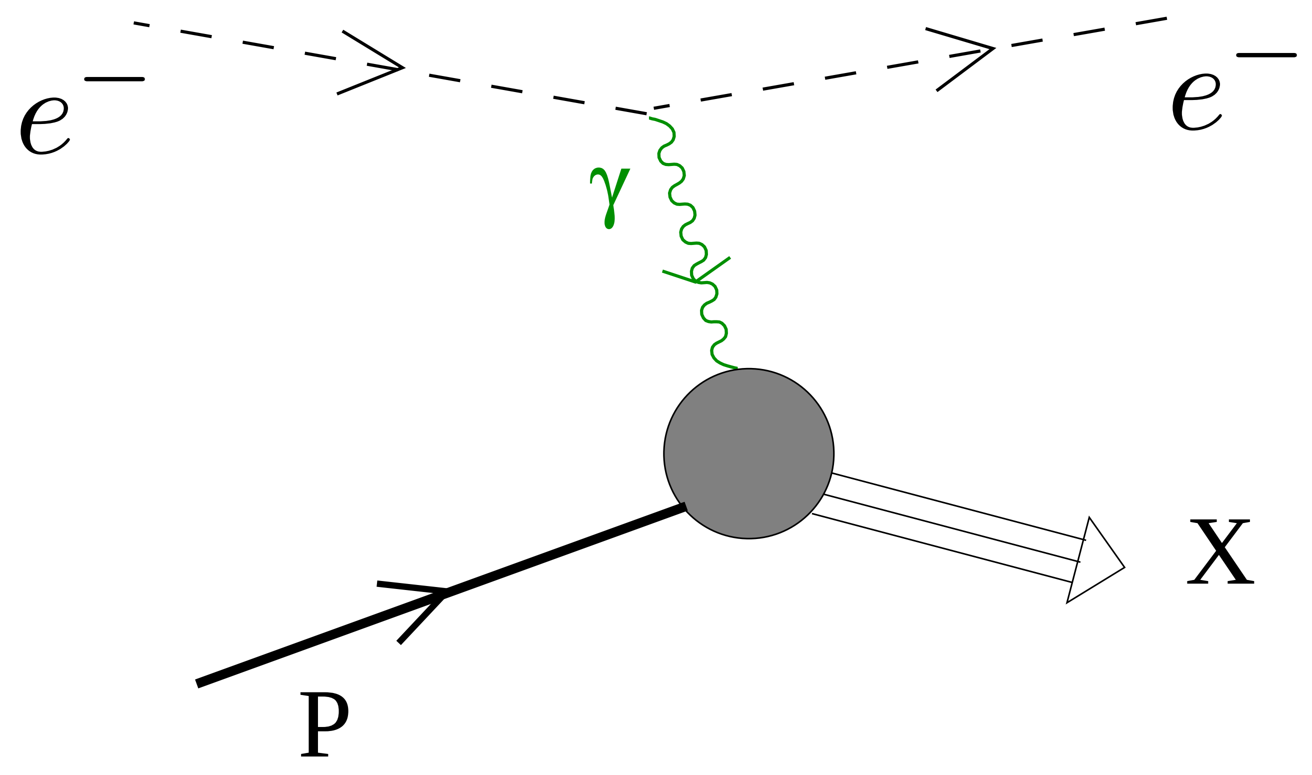


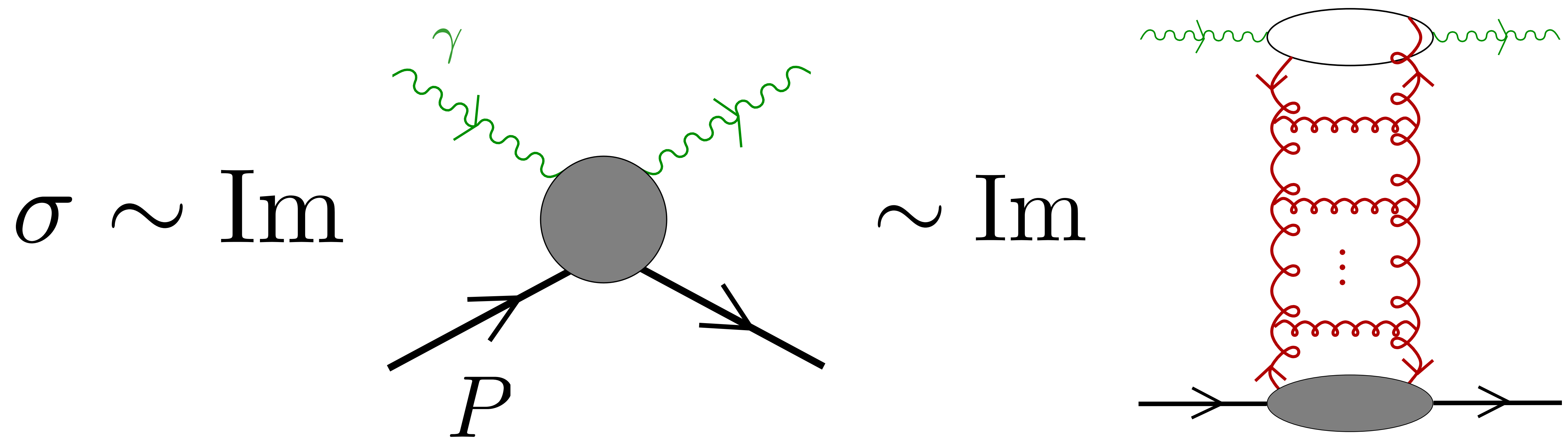



# Low-$x$ QCD (DIS, DVCS & VMP)

- Deep inelastic scattering

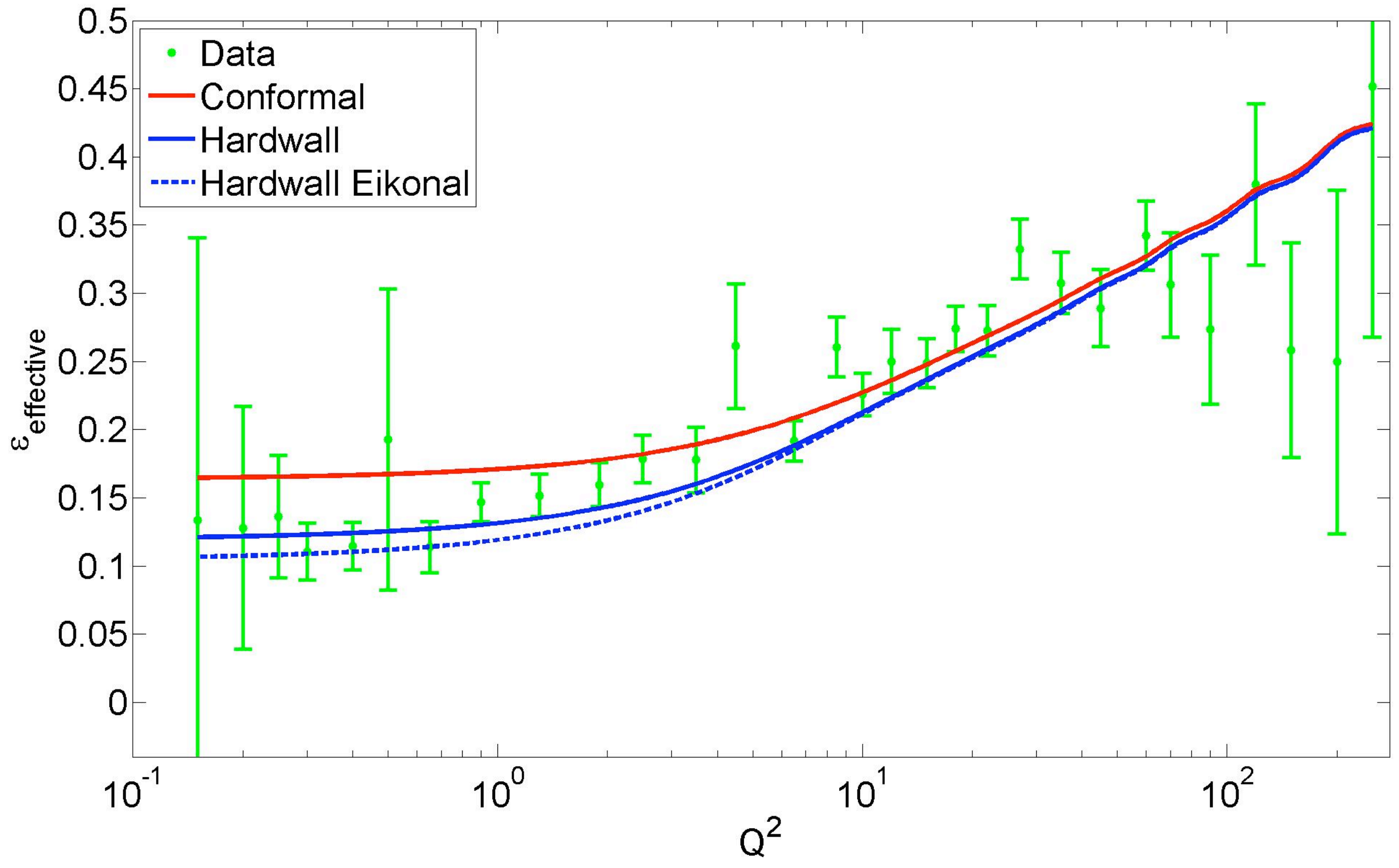



- At low-$x$




- BFKL pomeron is conformal, so it is particular case of conformal Regge theory. Use AdS model to fit data, therefore including strong coupling effects.

Effective Pomeron [Brower, Djuric, Sarcevic, Tan 10]



# 5min BREAK

## Regge Kinematics in CFTs [Cornalba 07; Cornalba, MSC, Penedones 08,09]

- Consider correlator with EMG current and scalar operators in position space

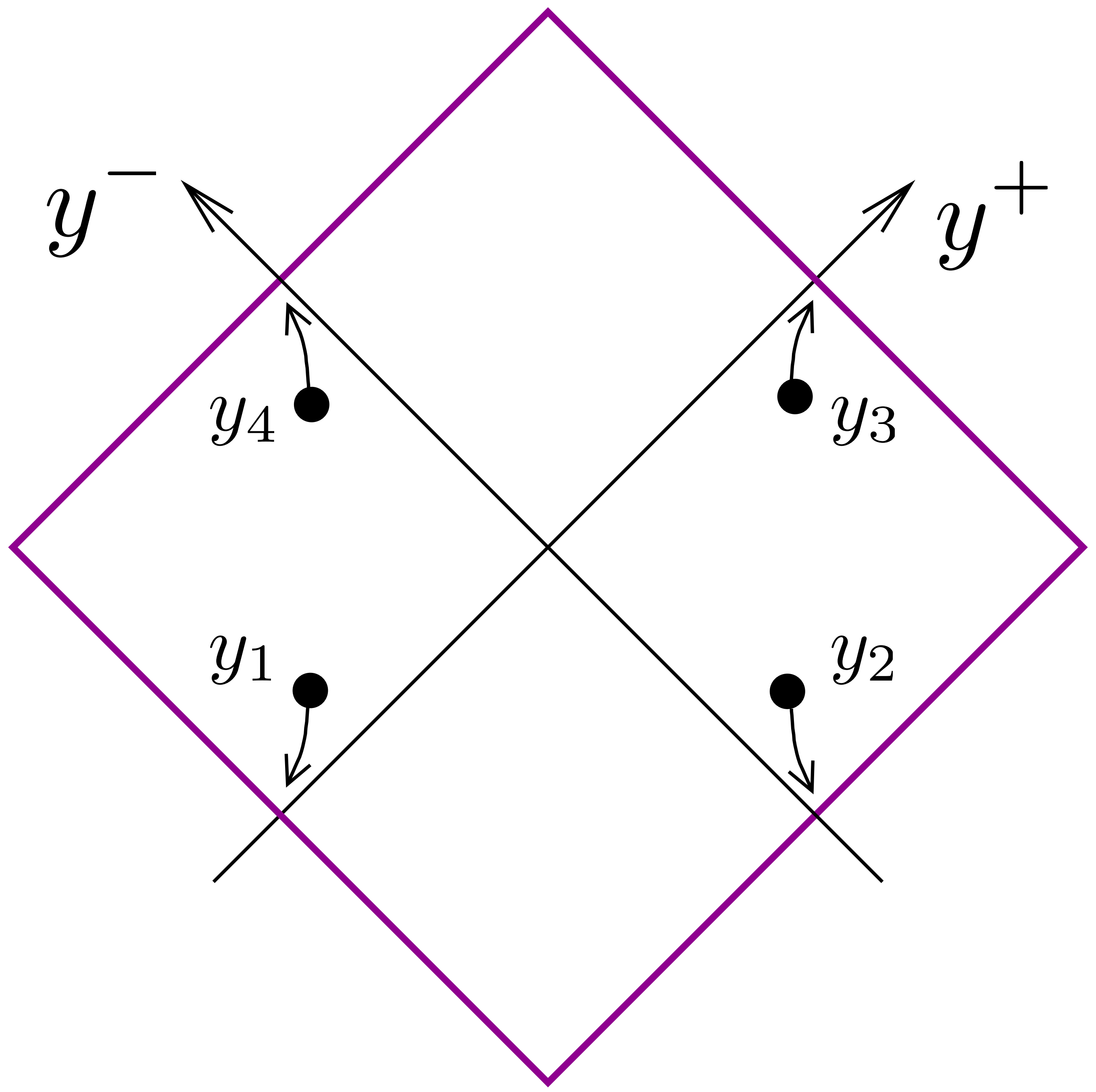
$$A(y_i) = \langle \mathcal{O}_1(y_1)\mathcal{O}_2(y_2)\mathcal{O}_3(y_3)\mathcal{O}_4(y_4)\rangle$$

$$\mathcal{O}_1 = \mathcal{O}_3 \equiv j^a$$
$$\mathcal{O}_2 = \mathcal{O}_4$$

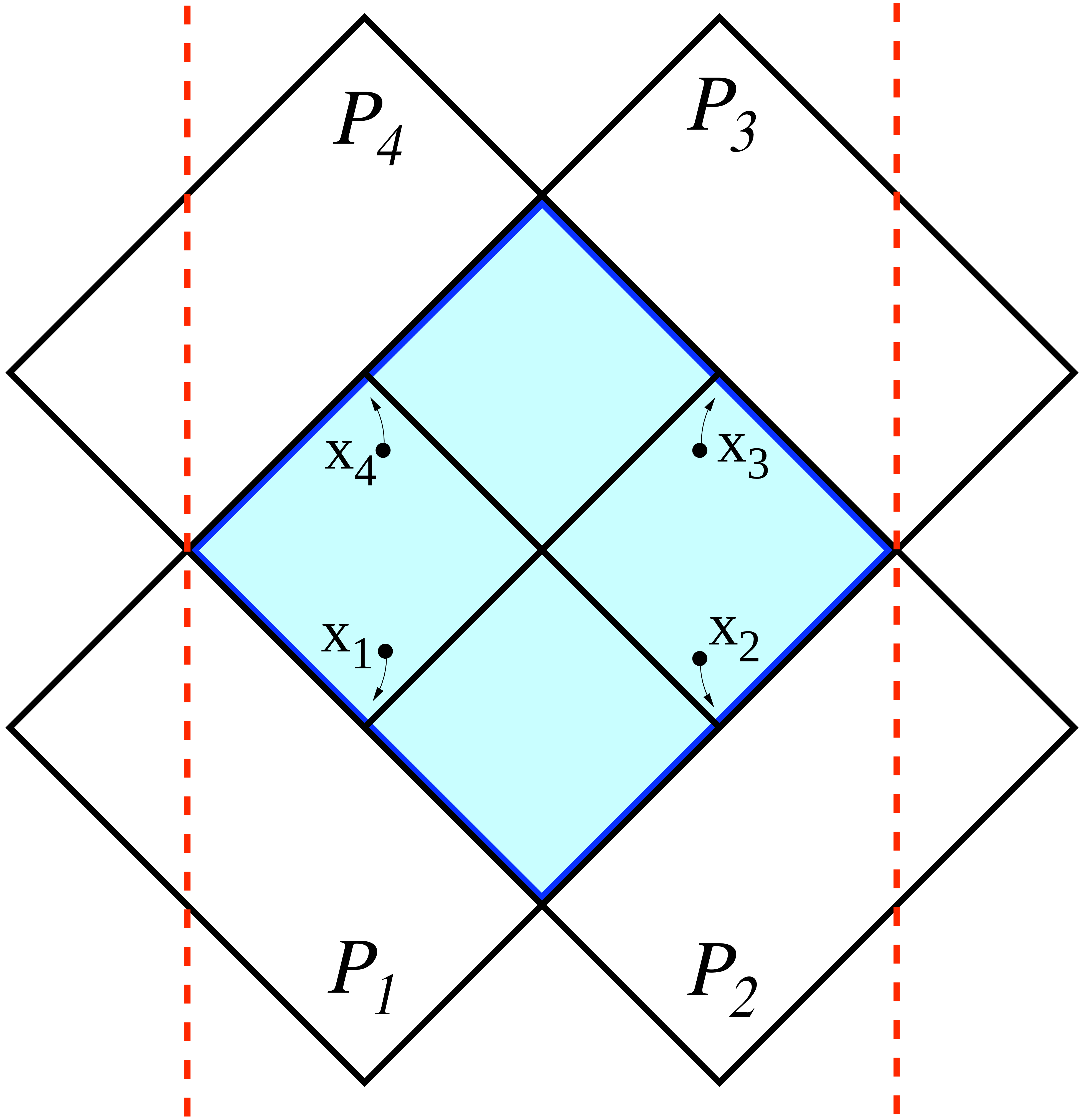
- Regge limit $\quad y = (y^+, y^-, y_\perp)$

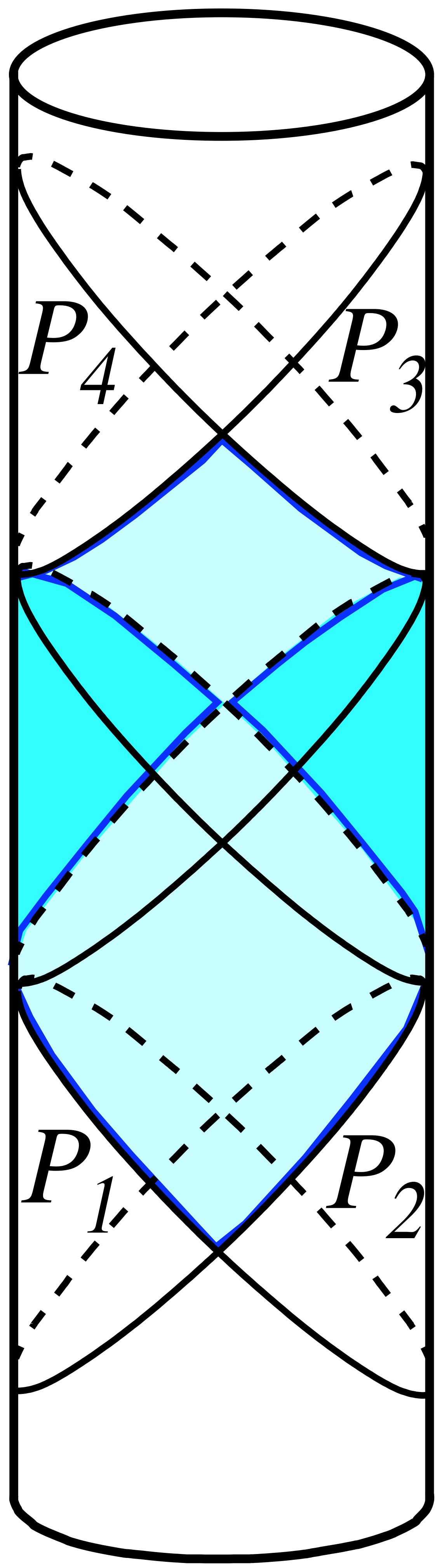
$$y_1^+ \to -\infty \qquad y_2^- \to -\infty$$
$$y_3^+ \to +\infty \qquad y_4^- \to +\infty$$

$y_i^2,\ y_{i\perp}^2$ fixed



- Use different Poincaré patches to cover each operator



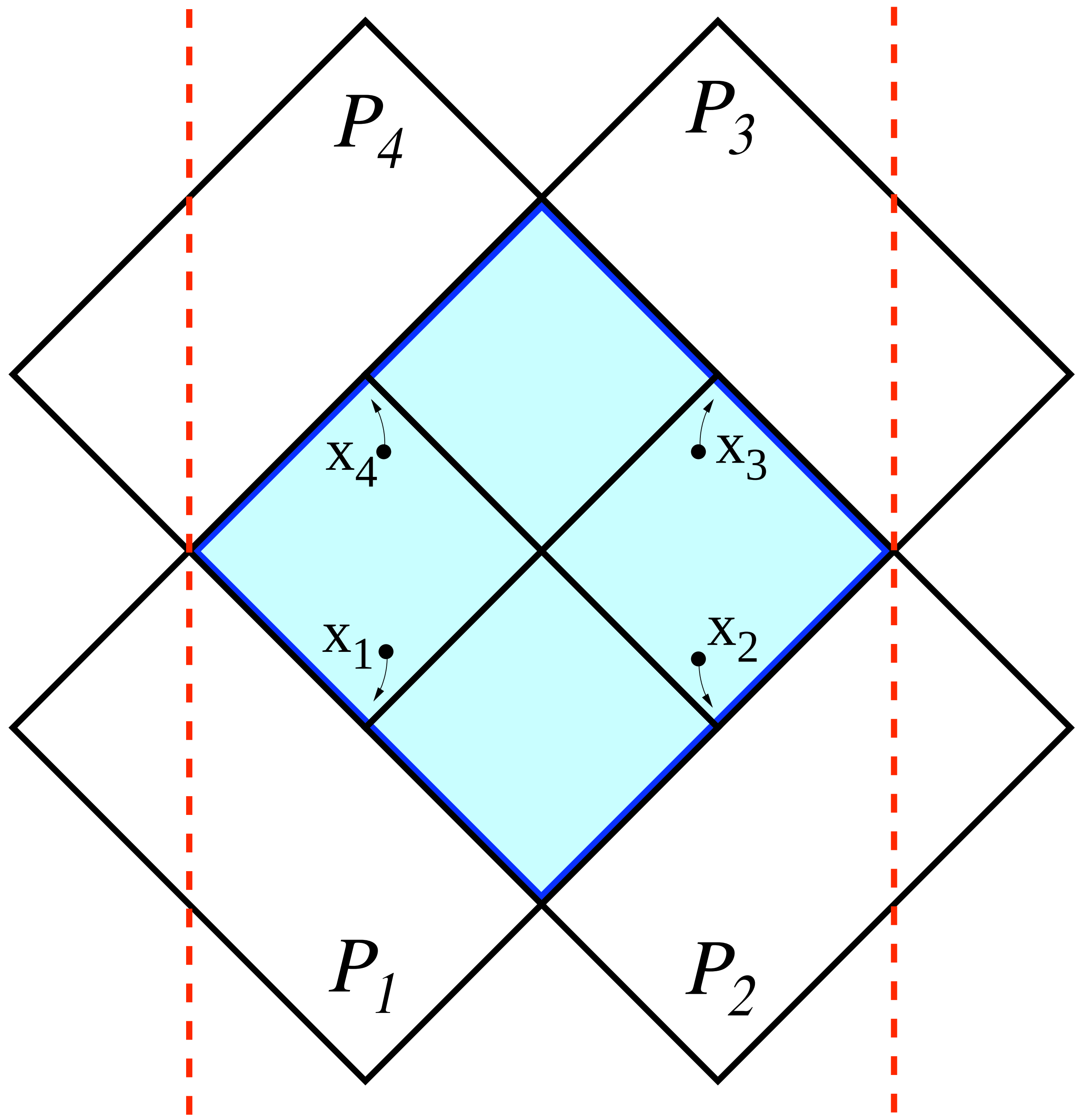



- Conformal transformation for each operator

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^+}(1, y_i^2, y_{i\perp}) \,, \qquad i = 1,3$$

$$x_i = (x_i^+, x_i^-, x_{i\perp}) = -\frac{1}{y_i^-}(1, y_i^2, y_{i\perp}) \,, \qquad i = 2,4$$

$$-dy^+dy^- + dy_\perp^2 = \frac{1}{(x^+)^2}\left(-dx^+dx^- + dx_\perp^2\right)$$




- In CFT Regge limit useful to consider correlator

$$A(x_i) = \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_1(x_3)\mathcal{O}_2(x_4)\rangle$$

Regge limit $x_i \to 0$

- Cross ratios

$$x \approx x_1 - x_3$$

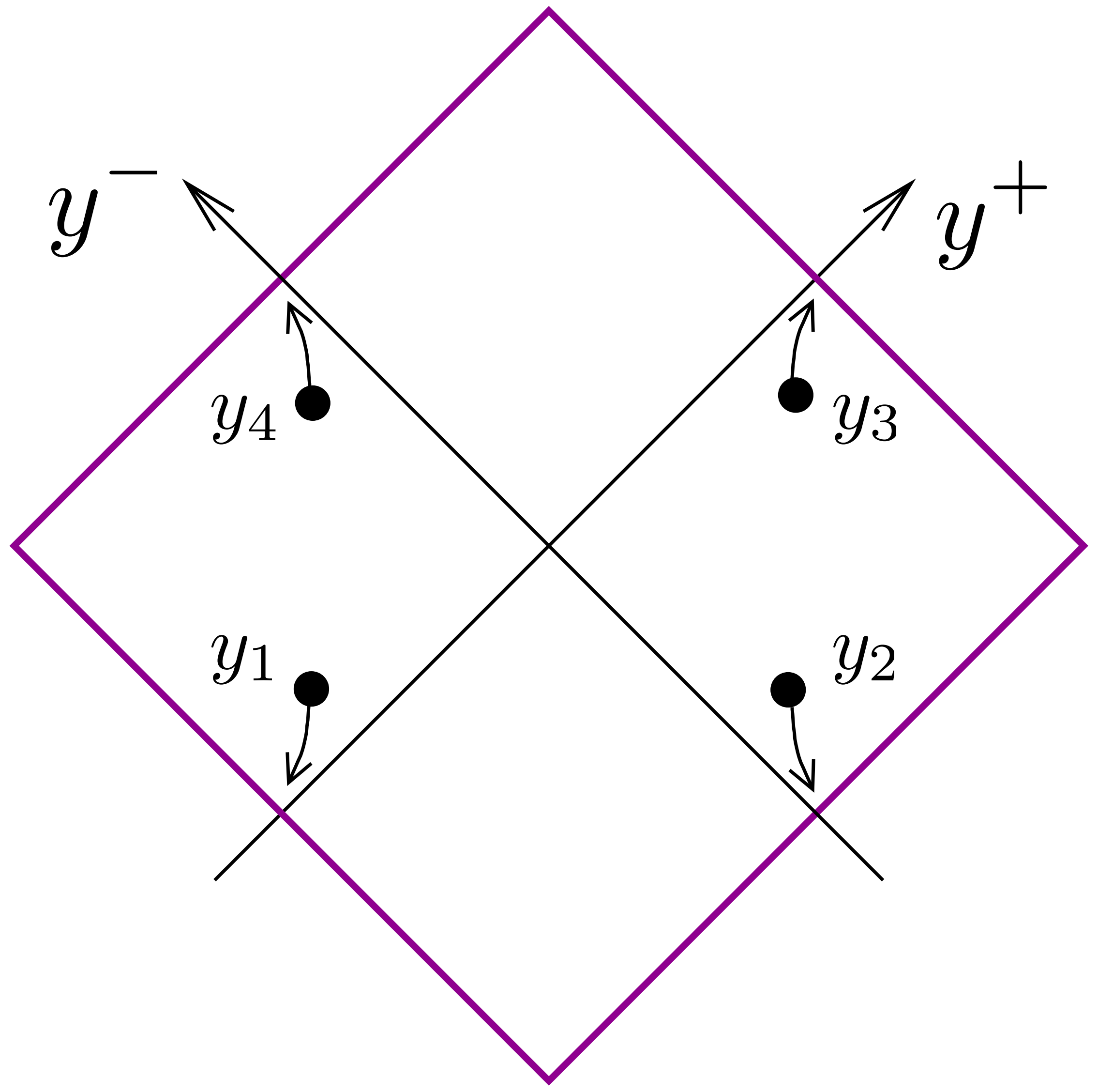
$$\bar{x} \approx x_2 - x_4$$

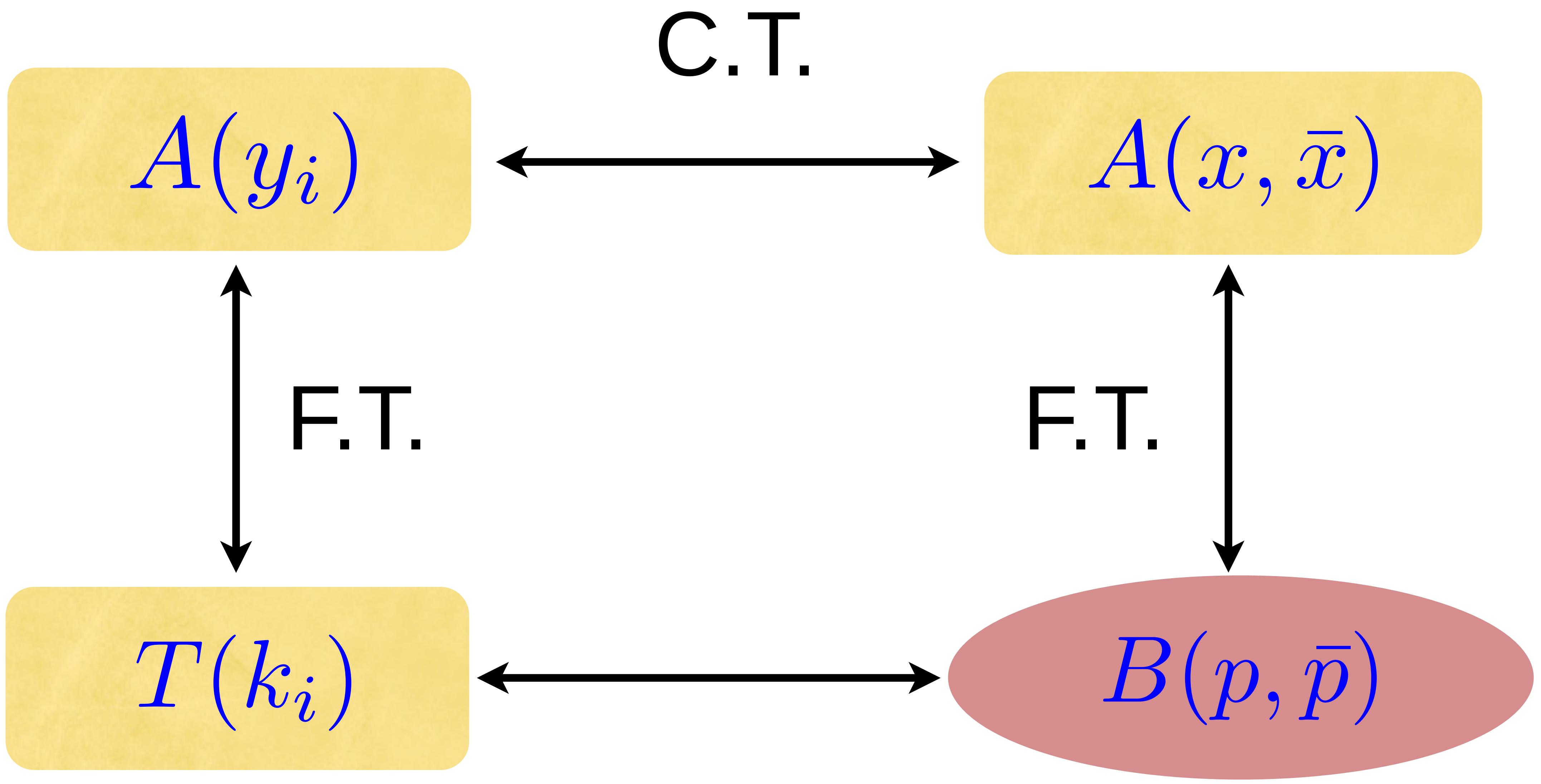
$$\sigma^2 = x^2\bar{x}^2 \,, \qquad \cosh\rho = -\frac{x\cdot\bar{x}}{|x||\bar{x}|}$$

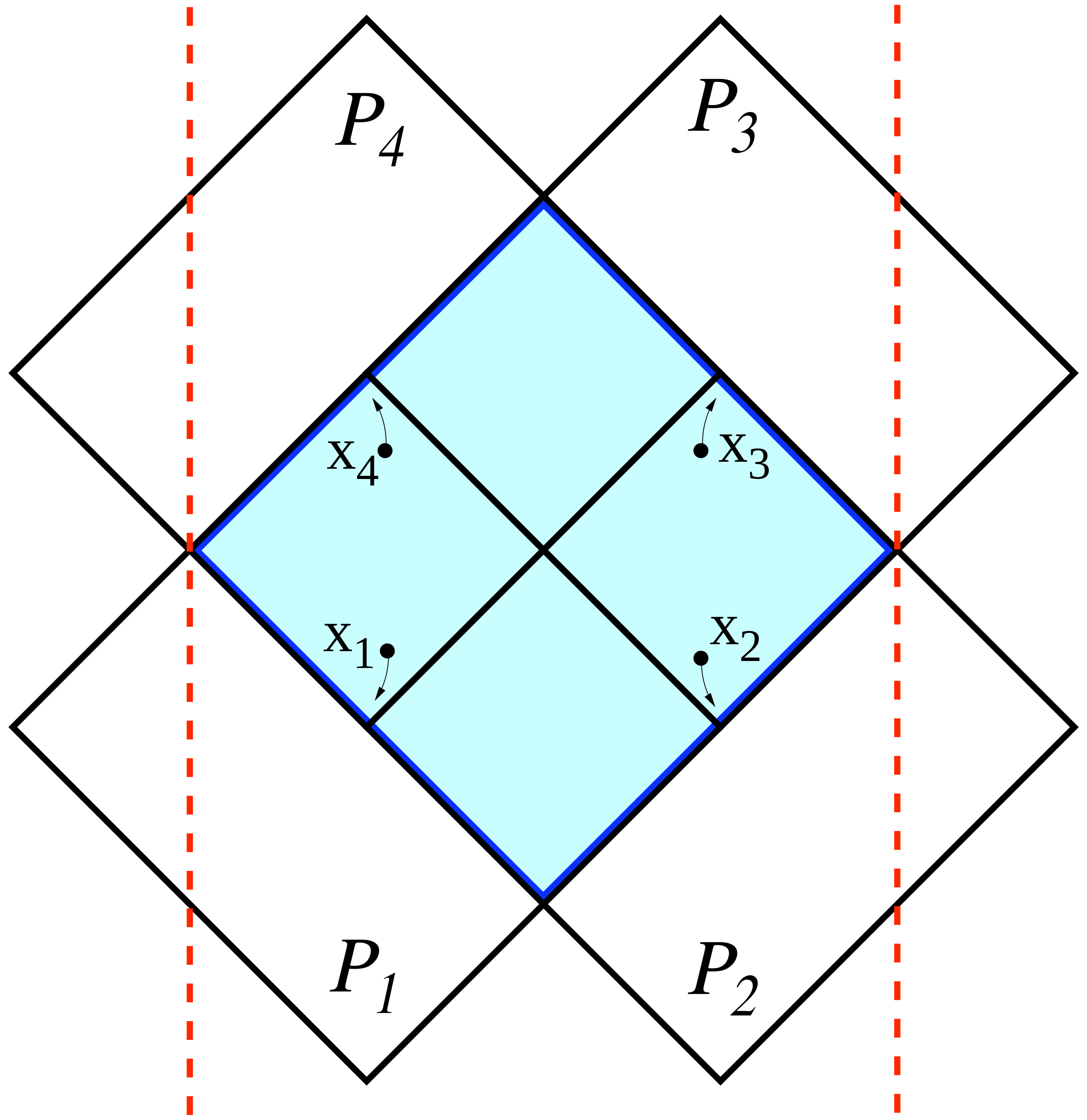
$$A(x,\bar{x}) = \frac{\mathcal{A}(\sigma,\rho)}{x^{2\Delta_1}\bar{x}^{2\Delta_2}}$$

**Regge limit** $\sigma \to 0 \,, \quad \rho$ fixed

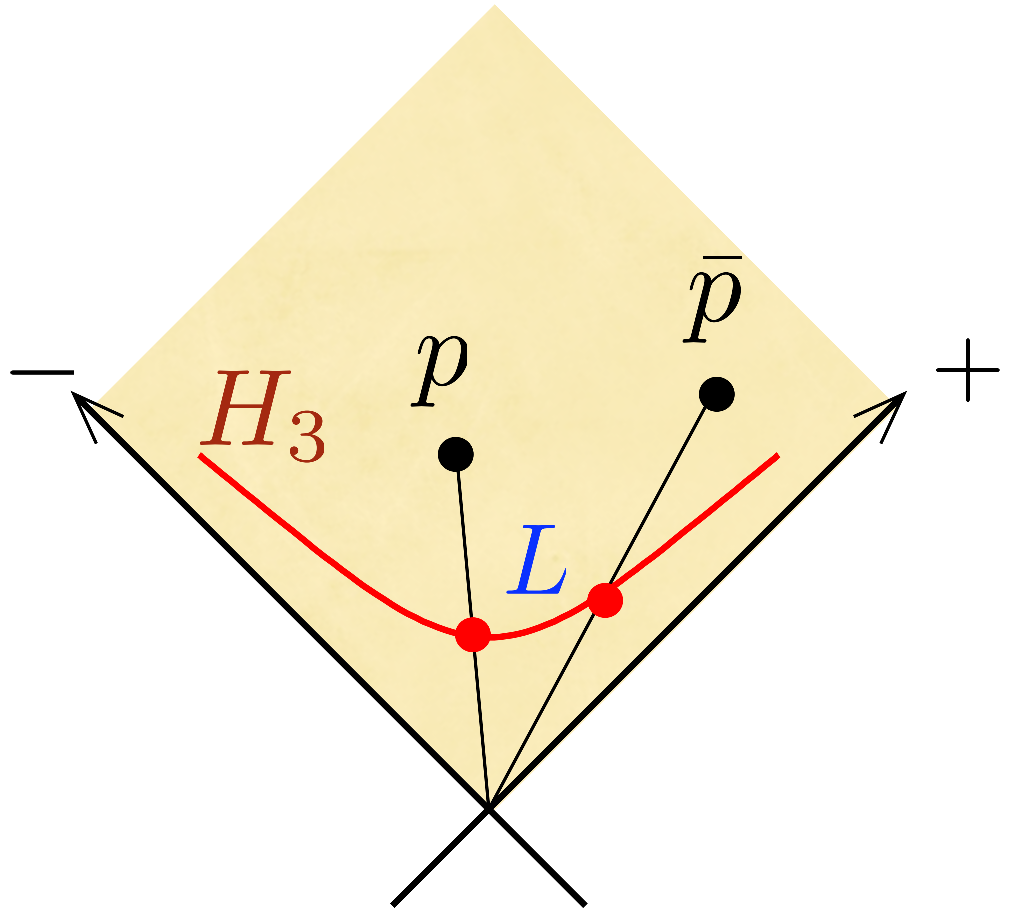
# Overview




- Where is AdS?




$$S = 4|p||\bar{p}|$$

$$\cosh L = -\frac{p \cdot \bar{p}}{|p||\bar{p}|}$$

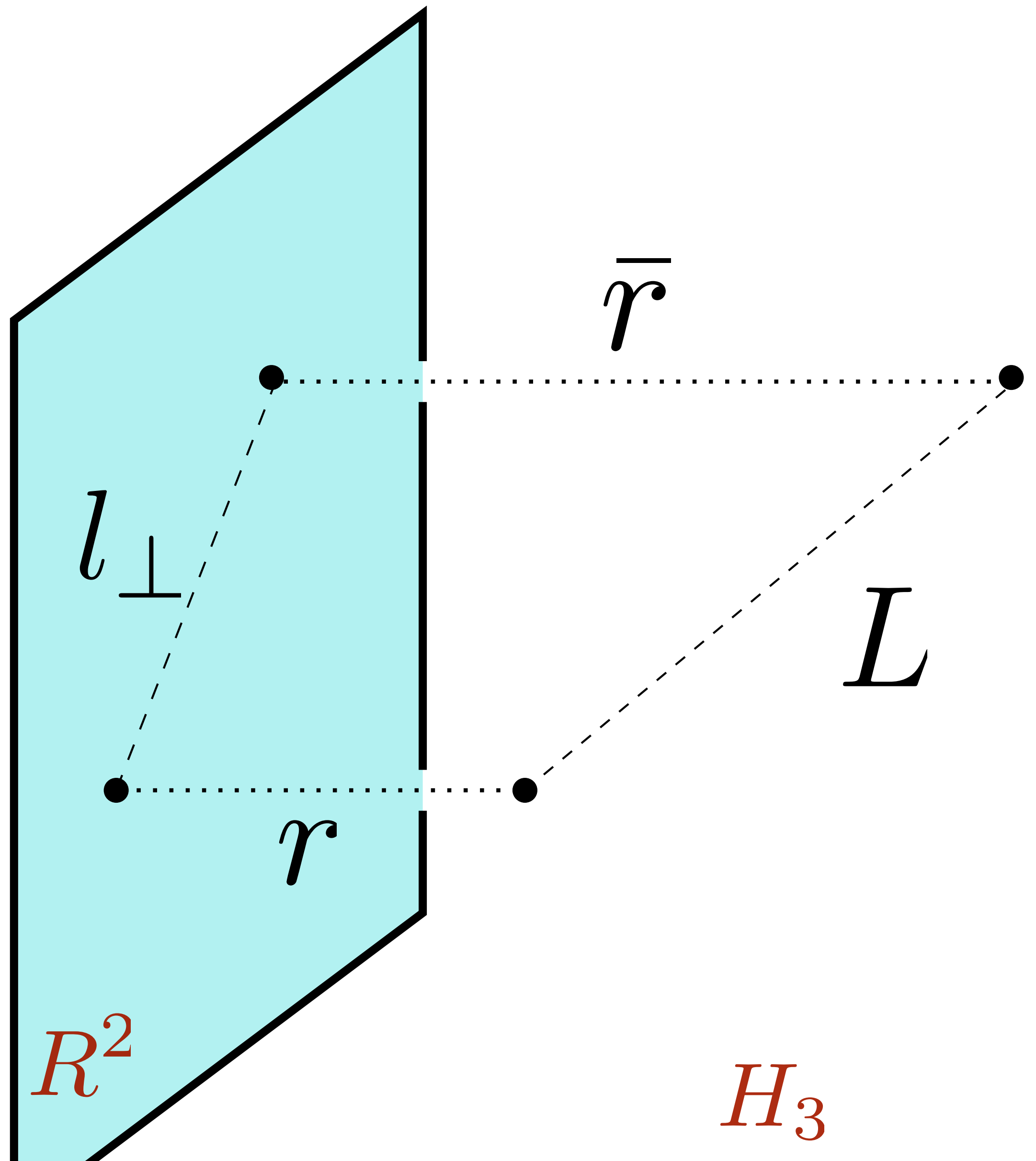
- Conformal (AdS) impact parameter representation [ Cornalba, MSC, Penedones, Schiappa 06]

Bulk-boundary propagators for scalar field coupled to Reggeon

$$T(k_j) \approx 2is \int dl_\perp \, e^{iq_\perp \cdot l_\perp} \int \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3} \, \Psi(r) \, \Phi(\bar{r}) \, \mathcal{B}(S,L)$$

idem for gauge field

$$\mathcal{B}(S,L) \rightarrow \left[1 - e^{i\delta(s,l_\perp)}\right]$$




$$ds^2(H_3) = \frac{dr^2 + ds^2(R^2)}{r^2}$$

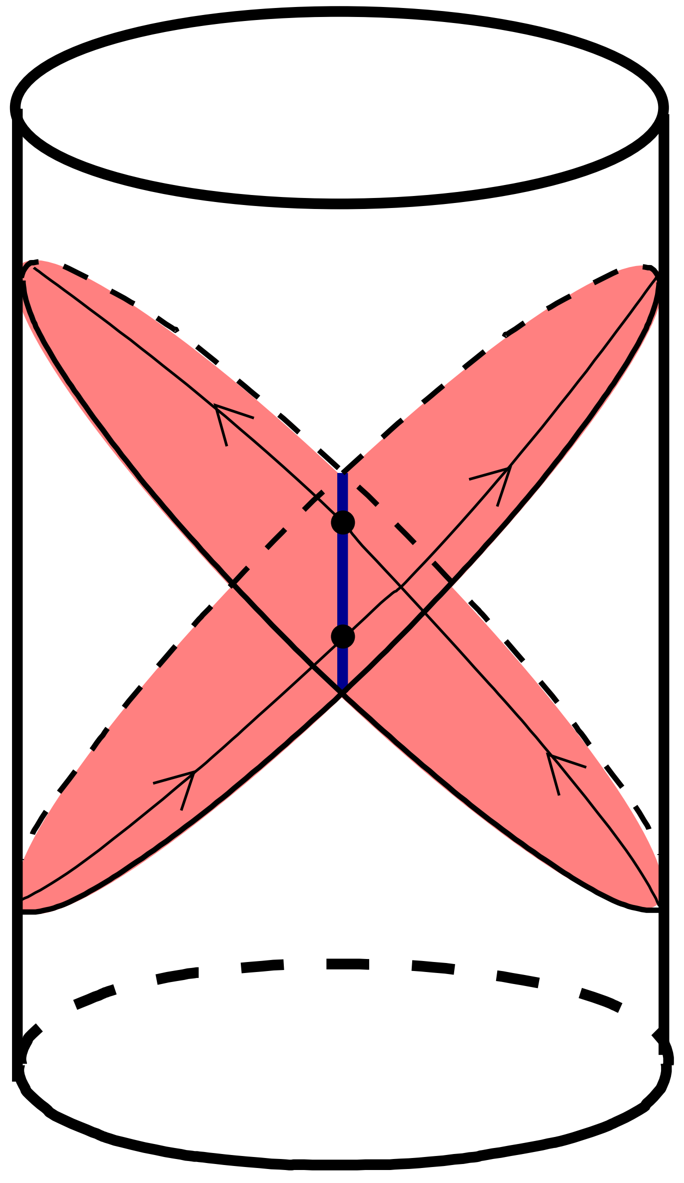
$S = r\bar{r}s$ , AdS energy squared

$$\cosh L = \frac{r^2 + \bar{r}^2 + l_\perp^2}{2r\bar{r}}$$ , impact parameter

- **Only used conformal symmetry**
- **Holography in Regge limit**

SO(3,1) is $H_3$ isometry group

AdS scattering process (Witten diagram)



# Conformal Regge theory

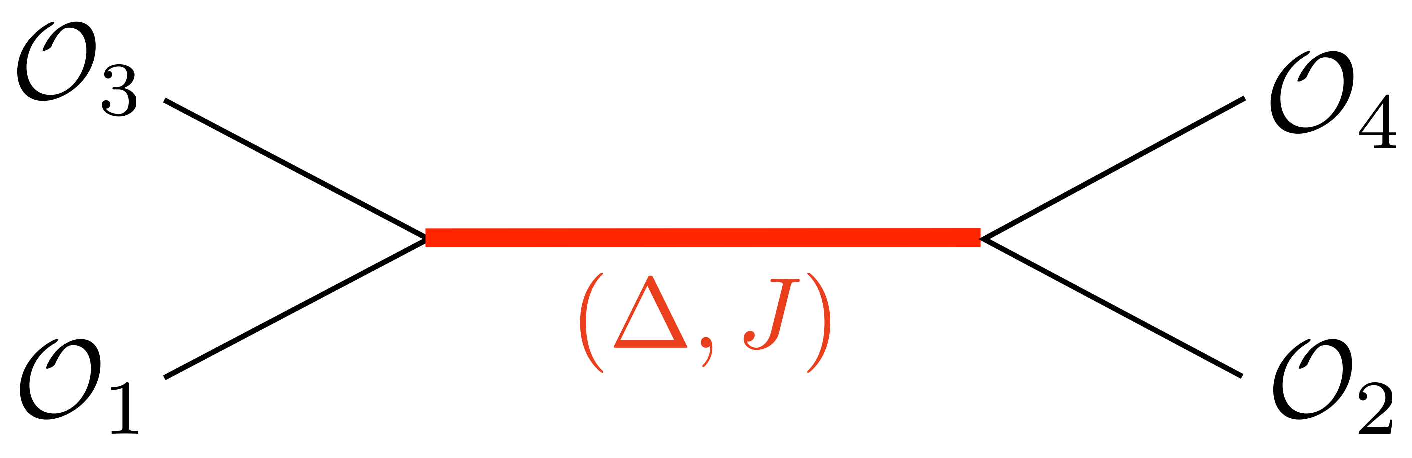
- Correlators can be thought as S-matrix elements for AdS scattering.
Mellin amplitudes make analogy explicit (Feynman rules) [Mack 09; Penedones 10]

$$\mathcal{A}(u,v) = \int_{-i\infty}^{i\infty} \frac{dtds}{(4\pi i)^2} M(s,t)\, u^{t/2} v^{-(s+t)/2} \quad \times \text{ product of } \Gamma \text{ functions}$$

- Can write partial wave expansion $\quad M(s,t) = \sum_{J=0}^{\infty} \int_{-\infty}^{\infty} d\nu\, b_J(\nu^2)\, M_{\nu,J}(s,t)$

- Exchange of operator of dimension $\Delta$ and spin $J$

$$b_J(\nu^2) \approx C_{13k} C_{24k} \frac{K_{\Delta,J}}{\nu^2 + (\Delta-2)^2}$$

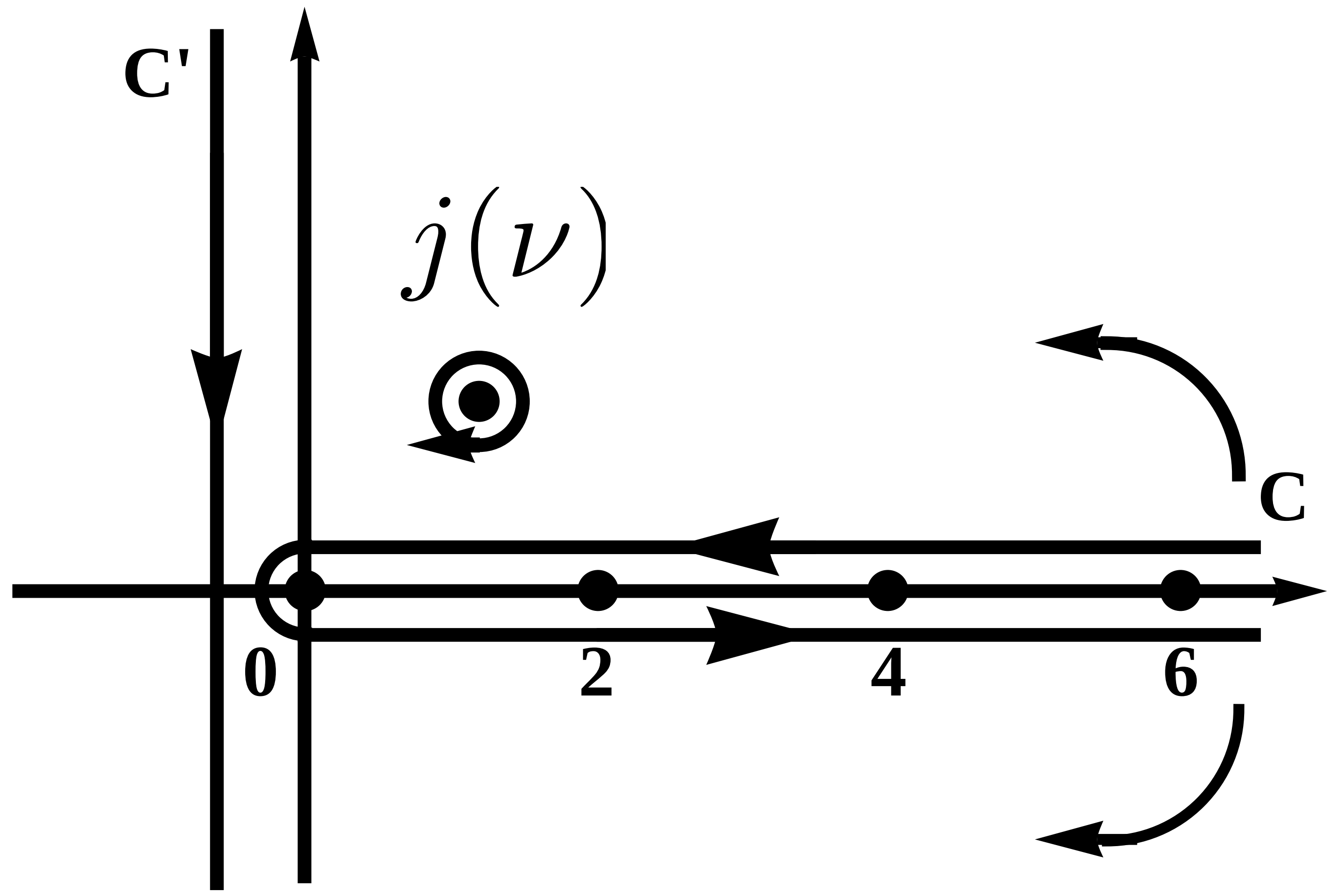



- Regge limit is again $\; s \gg t \qquad M_{\nu,J}(s,t) \approx \omega_{\nu,J}(t) s^J$

- Sommerfeld-Watson transform in CFT

$$M(s,t) = \sum_{J=0}^{\infty} \int_{-\infty}^{\infty} d\nu \, b_J(\nu^2) \, M_{\nu,J}(s,t)$$

$$\sum_J \to \int_C \frac{dJ}{2\pi i} \frac{\pi}{\sin(\pi J)}$$




$$b_J(\nu) \approx \frac{r(J)}{\nu^2 + (\Delta(J)-2)^2} \approx -\frac{j'(\nu)\, r(j(\nu))}{2\nu\,(J - j(\nu))}$$

Reggeon spin $J = j(\nu)$ defined by inverse function of $\Delta(J)$

$$\nu^2 + (\Delta(J)-2)^2 = 0$$

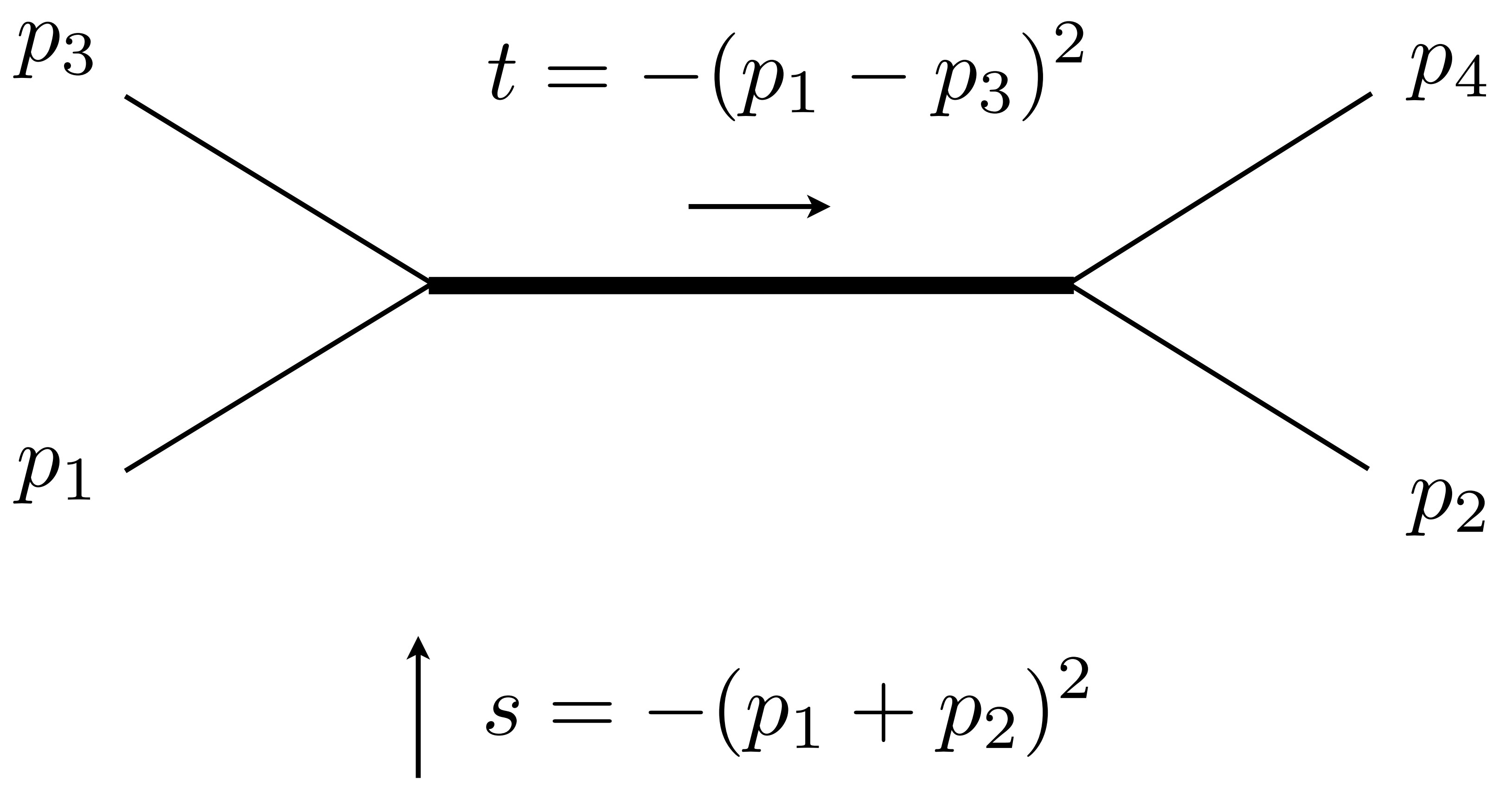
Residue related to OPE coeffs

$$r(J) = C_{13J} C_{24J} K_{\Delta(J),J}$$

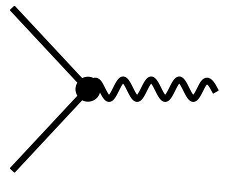
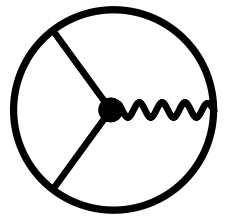
$$M(s,t) \approx \int d\nu \, \beta(\nu) \, \omega_{\nu,j(\nu)}(t) \, s^{j(\nu)}$$

$$\beta(\nu) \to C_{13j(\nu)} C_{24j(\nu)}$$

# Resume




$$\tau(s,t) = $$
$$= \sum_J \int d\mu\, a_J(\mu)\, \delta(\mu^2 - t)\, P_J(\cos\theta)$$

| **Strings in flat spacetime** | **$\mathrm{CFT}_d$ or Strings in $\mathrm{AdS}_{d+1}$** |
|---|---|
| Scattering amplitude<br>$\mathcal{T}(s,t)$ | Correlation function or Mellin amplitude<br>$M(s,t)$ |
| Partial wave expansion<br>$\mathcal{T}(s,t) = \sum_J a_J(t) \underbrace{P_J(\cos\theta)}_{\text{partial wave}}$ | Conformal partial wave expansion<br>$M(s,t) = \sum_J \int d\nu\, b_J(\nu^2) \underbrace{M_{\nu,J}(s,t)}_{\text{partial wave}}$ |
| On-shell poles<br>$a_J(t) \sim \frac{C^2(J)}{t - m^2(J)}$ | On-shell poles<br>$b_J(\nu^2) \sim \frac{C^2(J)}{\nu^2 + \left(\Delta(J) - \frac{d}{2}\right)^2}$ |
| Leading Regge trajectory<br>$m^2(J) = \frac{2}{\alpha'}(J-2)$ | Leading twist operators<br>$\Delta(J) = d - 2 + J + \underbrace{\gamma(J,g^2)}_{\text{anomalous dimension}}$ |
| Cubic couplings<br>$C(J) \sim$  | 3-pt functions or OPE coefficients<br>$C(J) \sim$  |
| Regge limit: $s \to \infty$ with fixed $t$<br>$P_J(\cos\theta) \approx \left(\frac{2s}{t}\right)^J$<br>$T(s,t) \approx \beta(t)\, s^{j(t)}$ | Regge limit: $s \to \infty$ with fixed $t$<br>$M_{\nu,J}(s,t) \approx \omega_{\nu,J}(t)\, s^J$<br>$M(s,t) \approx \int d\nu\, \omega_{\nu,j(\nu)}(t)\, \beta(\nu)\, s^{j(\nu)}$ |
| Regge pole and residue<br>$t - m^2(J) = 0 \;\Rightarrow\; J = j(t)$<br>$\beta(t) \sim C^2\big(j(t)\big)$ | Regge pole and residue<br>$\left(\Delta(J) - \frac{d}{2}\right)^2 + \nu^2 = 0 \;\Rightarrow\; J = j(\nu)$<br>$\beta(\nu) \sim C^2\big(j(\nu)\big)$ |

# N=4 Super Yang Mills

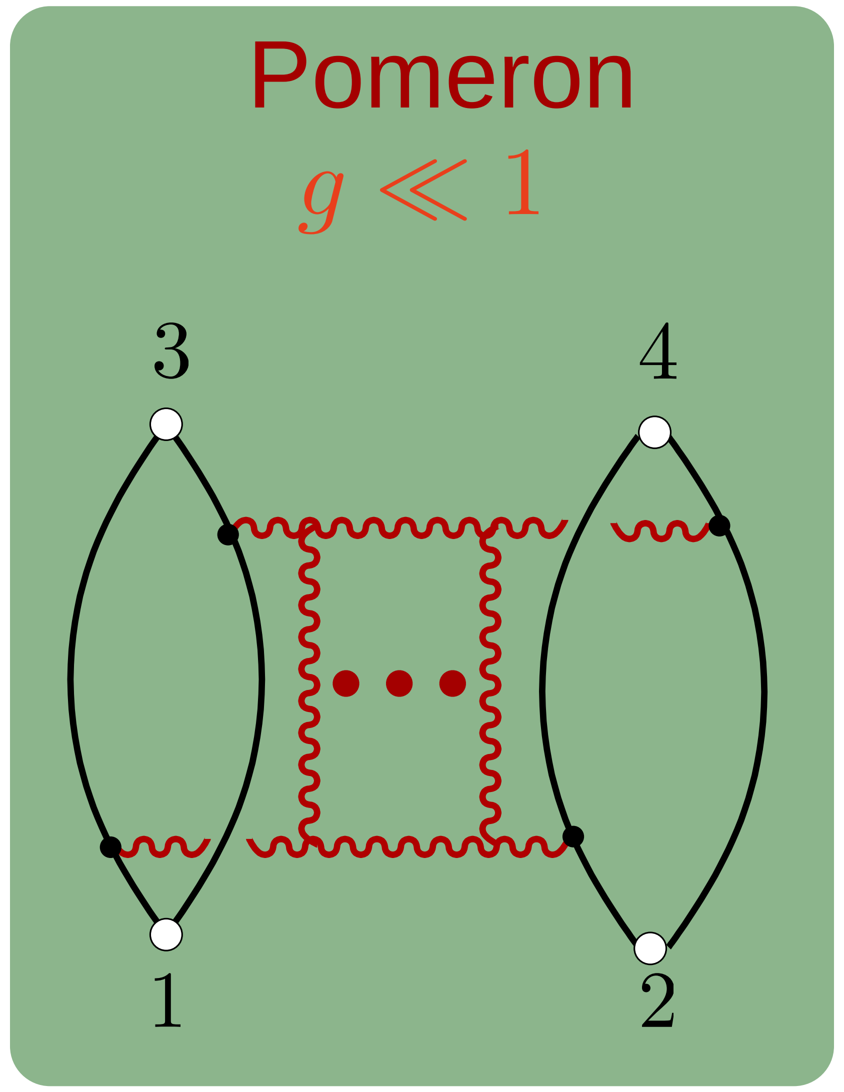
- Correlation functions that exchange vacuum quantum numbers are dominated in Regge limit by exchange of pomeron/graviton Regge trajectory (twist 2)

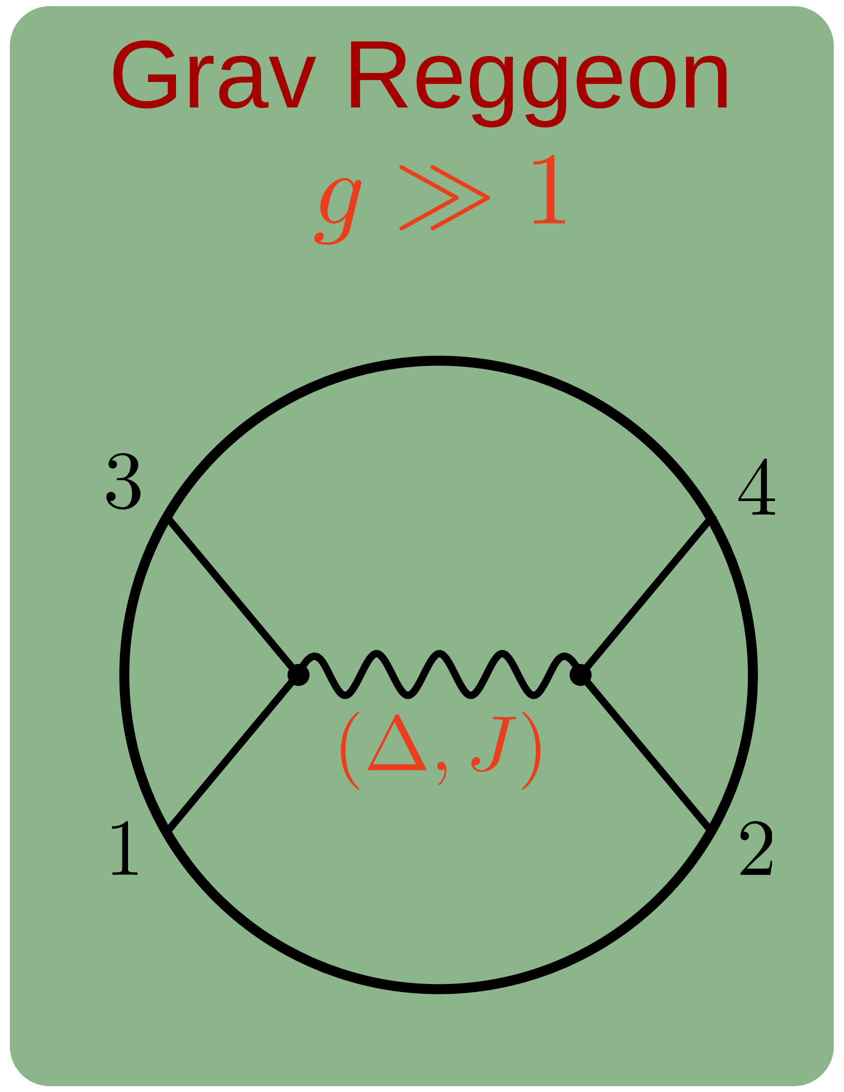
$$\mathcal{O}_1 = \mathcal{O}_3 = \mathrm{tr}\left(\phi_{12}\phi^{12}\right)$$

$$\mathcal{O}_2 = \mathcal{O}_4 = \mathrm{tr}\left(\phi_{34}\phi^{34}\right)$$

$$\mathcal{O}_J = \begin{cases} \mathrm{tr}\left(F_{\mu\nu_1} D_{\nu_2} \dots D_{\nu_{J-1}} F_{\nu_J}{}^{\mu}\right) \\ \mathrm{tr}\left(\phi_{AB} D_{\nu_1} \dots D_{\nu_J} \phi^{AB}\right) \\ \mathrm{tr}\left(\bar{\psi}_A D_{\nu_1} \dots D_{\nu_{J-1}} \Gamma_{\mu_J} \psi^A\right) \end{cases}$$

- Weak coupling
- Strong coupling



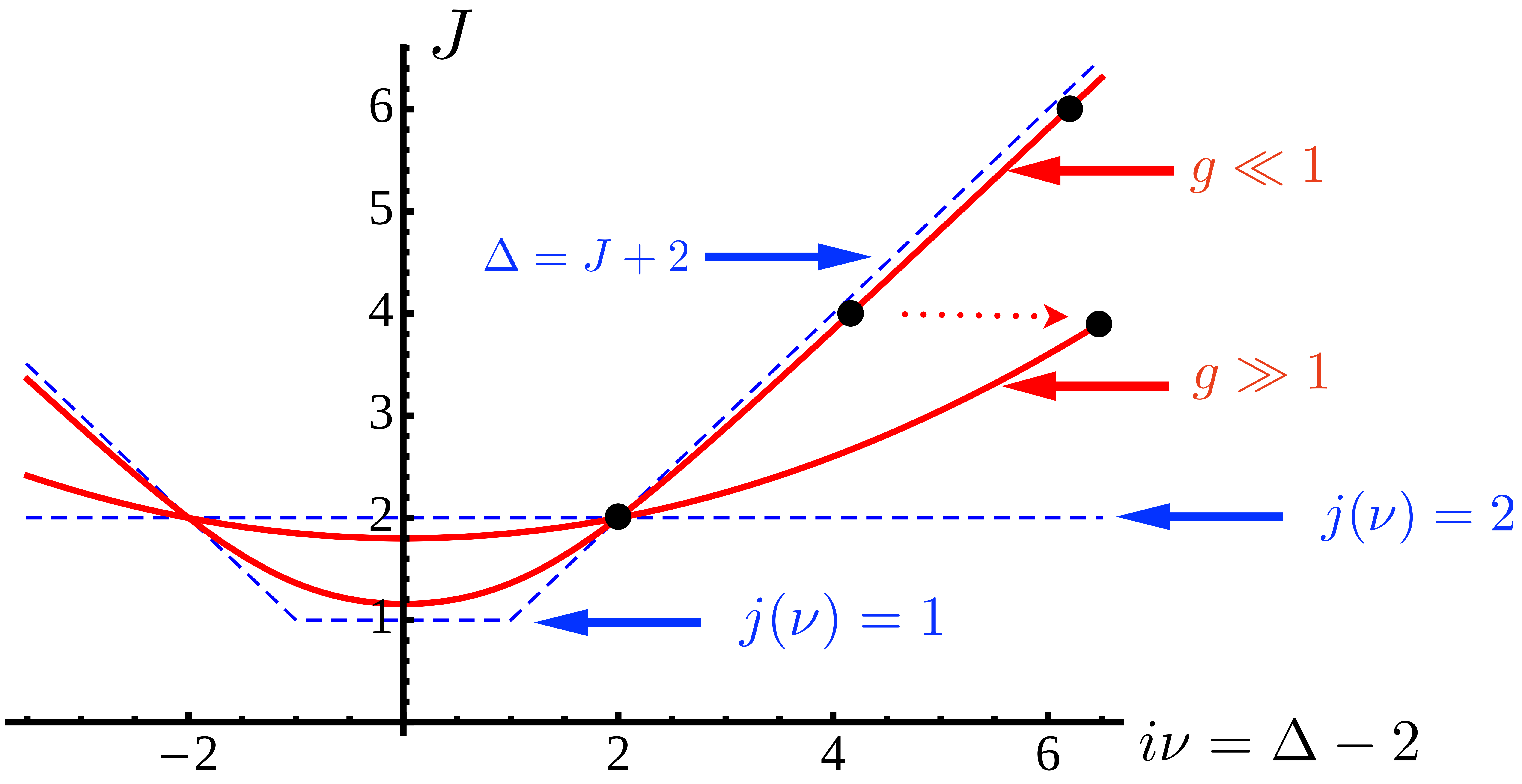

't Hooft coupling

$$\lambda = g_{YM}^2 N$$

$$= (4\pi g)^2$$

# Reggeon spin & dimension of twist 2 operators

$\Delta = \Delta(J)$ or $J = j(\nu)$ $\Delta\big(j(\nu)\big) = 2 + i\nu$



# N=4 Super Yang Mills - anomalous dimension at weak coupling [Kotikov et al 07]

- Anomalous dimension (integrability) $$\gamma(J) = \Delta(J) - J - 2 = \sum_{n=1}^{\infty} g^{2n}\gamma_n(J)$$
- Spin of BFKL pomeron $$j(\nu) = 1 + \sum_{n=1}^{\infty} g^{2n} j_n(\nu)$$ $$\Delta(j(\nu)) = 2 + i\nu$$
- Consider limit $j \to 1,\ g^2 \to 0$ of $$\frac{j(\nu)-1}{g^2} = \frac{-8}{i\nu - 1} + \sum_{k=0}^{\infty} a_k\,(i\nu-1)^k$$
- Inversion around $i\nu = 1$ gives prediction for behaviour of $\Delta(J)$ around $J=1$ to arbitrary high order in coupling (wrapping [Bajnok et al 08]). From leading BFKL spin

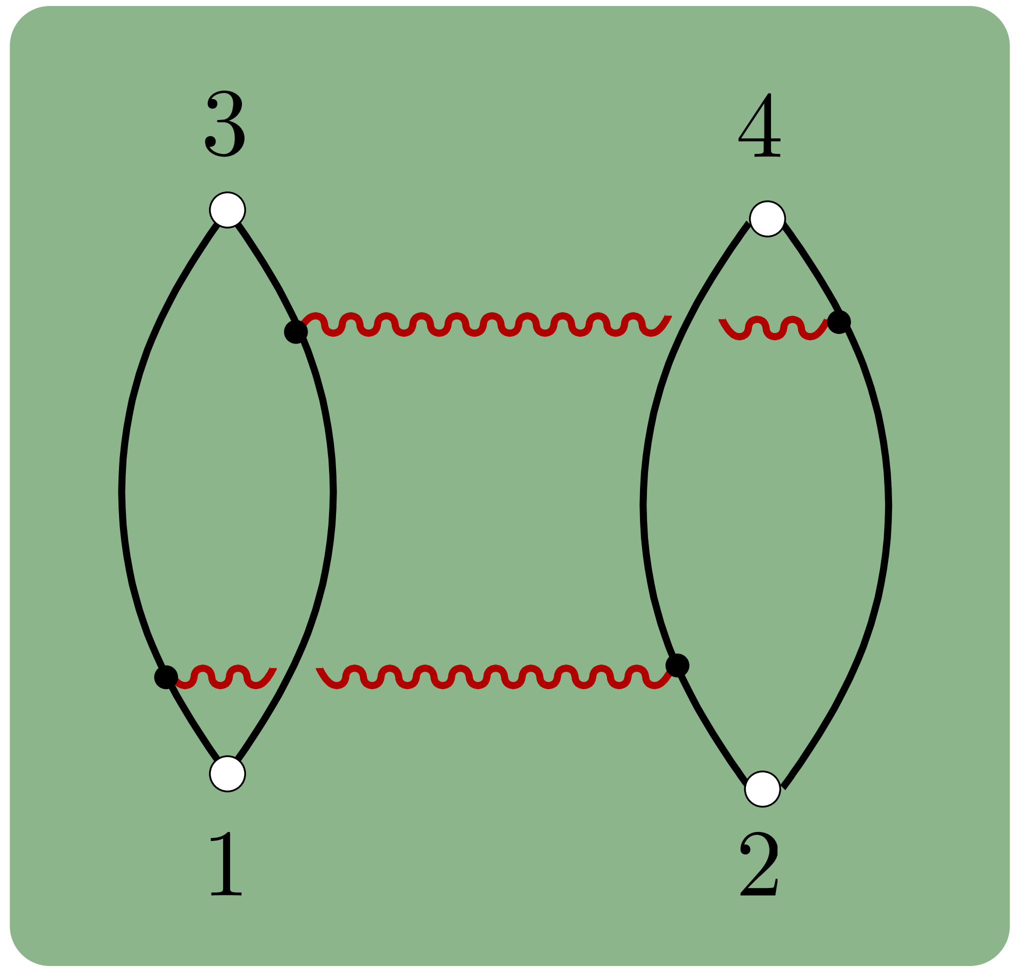
$$\Delta(J) - 3 = 2\left(\frac{-4g^2}{J-1}\right) + 0\left(\frac{-4g^2}{J-1}\right)^2 + 0\left(\frac{-4g^2}{J-1}\right)^3 - 4\zeta(3)\left(\frac{-4g^2}{J-1}\right)^4 + \cdots$$

# N=4 Super Yang Mills - OPE coefficients at weak coupling

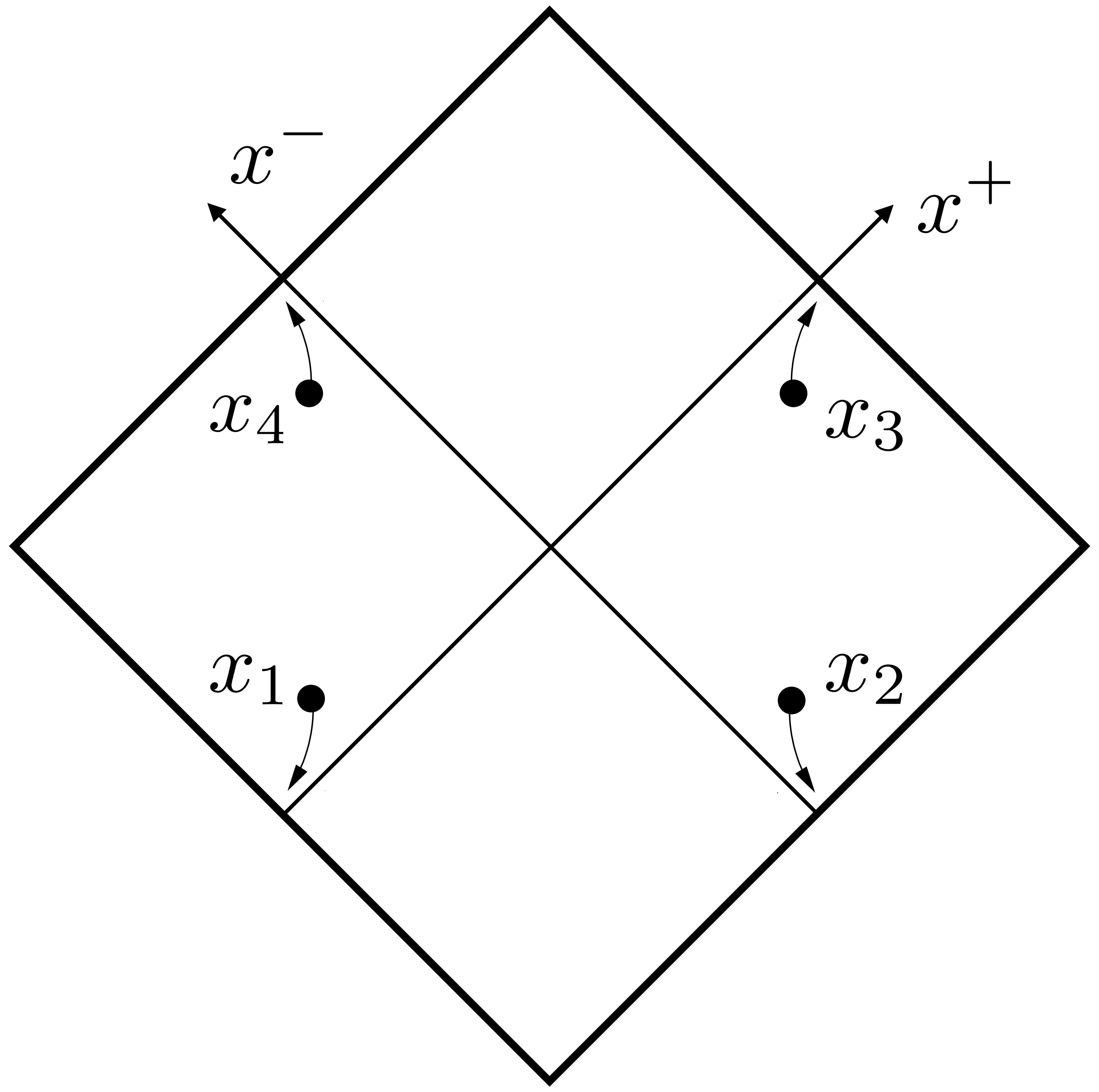
- From known form of 4pt correlation function at two loop obtain prediction for behaviour of OPE coefficients between external operators and operators in the leading Regge trajectory around $J=1$ to arbitrary high order in coupling

$$\mathcal{O}_1 = \mathcal{O}_3 = \mathrm{tr}\left(\phi_{12}\phi^{12}\right)$$

$$\mathcal{O}_2 = \mathcal{O}_4 = \mathrm{tr}\left(\phi_{34}\phi^{34}\right)$$




Regge limit in position space




$$\mathcal{A}(\sigma,\rho) \approx 2\pi i \int d\nu\, \alpha(\nu)\, \sigma^{1-j(\nu)}\, \Omega_{i\nu}(\rho)$$

$$M(s,t) \approx \int d\nu\, \beta(\nu)\, \omega_{\nu,j(\nu)}(t)\, s^{j(\nu)}$$

$$C_{11j(\nu)} C_{22j(\nu)}$$

# N=4 Super Yang Mills - OPE coefficients at weak coupling

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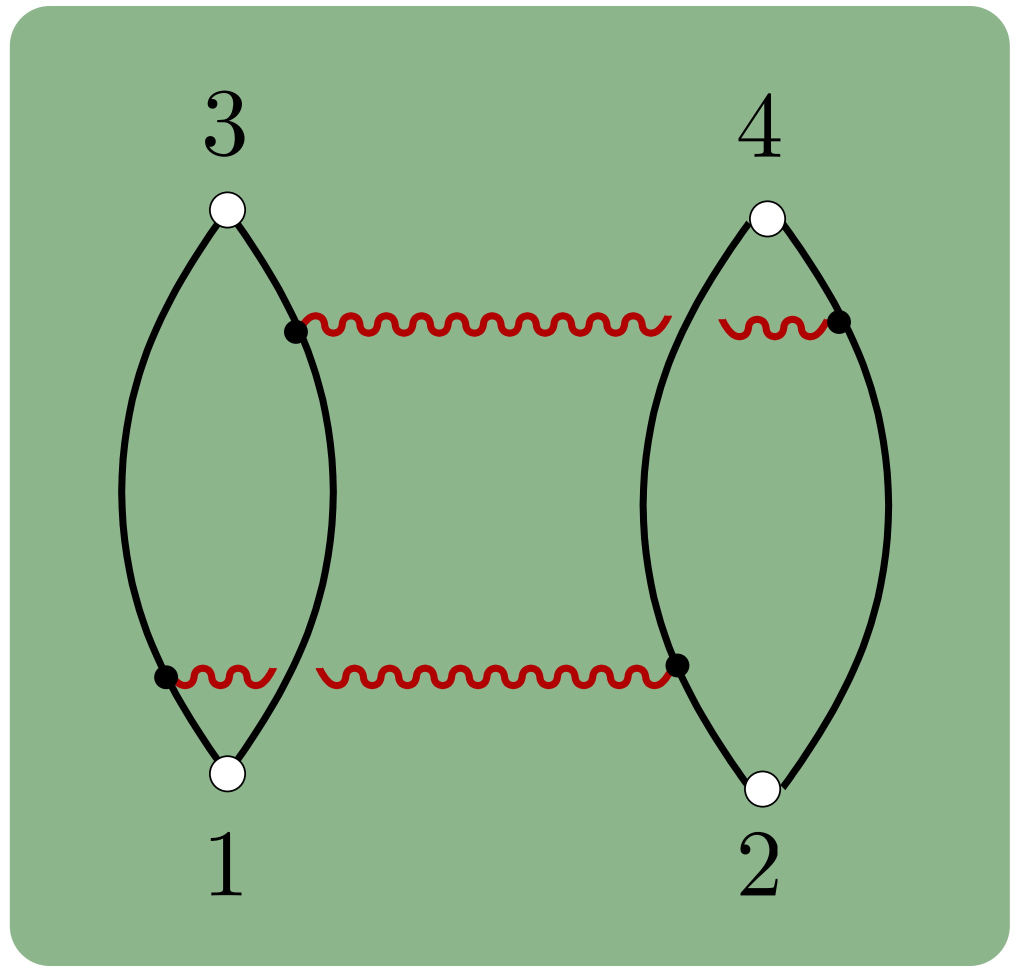



$$\begin{aligned} C_{11J}C_{22J} \;=\; & g^0\left[(J-1)\,\frac{2}{3} + O(J-1)^2\right] + \\ & g^2\left[\frac{64}{9} + O(J-1)\right] + \\ & g^4\left[\frac{1}{J-1}\,\frac{32}{27}\,\left(61-3\pi^2\right) + O(J-1)^0\right] + \\ & \ldots \end{aligned}$$

Free theory
(Wick contractions)

## N=4 Super Yang Mills - Reggeon spin at strong coupling

- Anomalous dimension of string states in leading Regge trajectory know up to next to next leading order [Basso 11; Gromov et al 11]

$$x = J - 2$$

$$\Delta(J)(\Delta(J)-4) = x\left[2\sqrt{\lambda} + \left(-1+\frac{3x}{2}\right) - \frac{3}{8}\left(-10 + x(8\zeta(3)-1) + x^2\right)\frac{1}{\sqrt{\lambda}} + \cdots\right]$$

- Can invert, $\Delta(j(\nu)) = 2 + i\nu$, to learn about behaviour of graviton Regge trajectory around $J=2$ to arbitrary high order in strong coupling expansion

$$j(\nu) = 2 - \frac{4+\nu^2}{2\sqrt{\lambda}}\left(1 + \sum_{n=2}^{\infty}\frac{\tilde{j}_n(\nu^2)}{\lambda^{(n-1)/2}}\right)$$

$\tilde{j}_n(\nu^2)$ is a polynomial of degree $n-2$

$$\tilde{j}_n(\nu^2) = \sum_{k=0}^{n-2} c_{n,k}\,\nu^{2k}$$

$$c_{2,0} = \frac{1}{2}\,, \quad c_{3,0} = -\frac{1}{8}\,, \quad c_{3,1} = \frac{3}{8}\,, \quad c_{4,1} = -\frac{3}{32}\left(8\zeta(3)-7\right), \quad c_{5,2} = \frac{21}{64}\,, \quad c_{n,k} = 0 \text{ for } \left[\frac{n}{2}\right] \le k \le n-2 \text{ with } n \ge 4$$

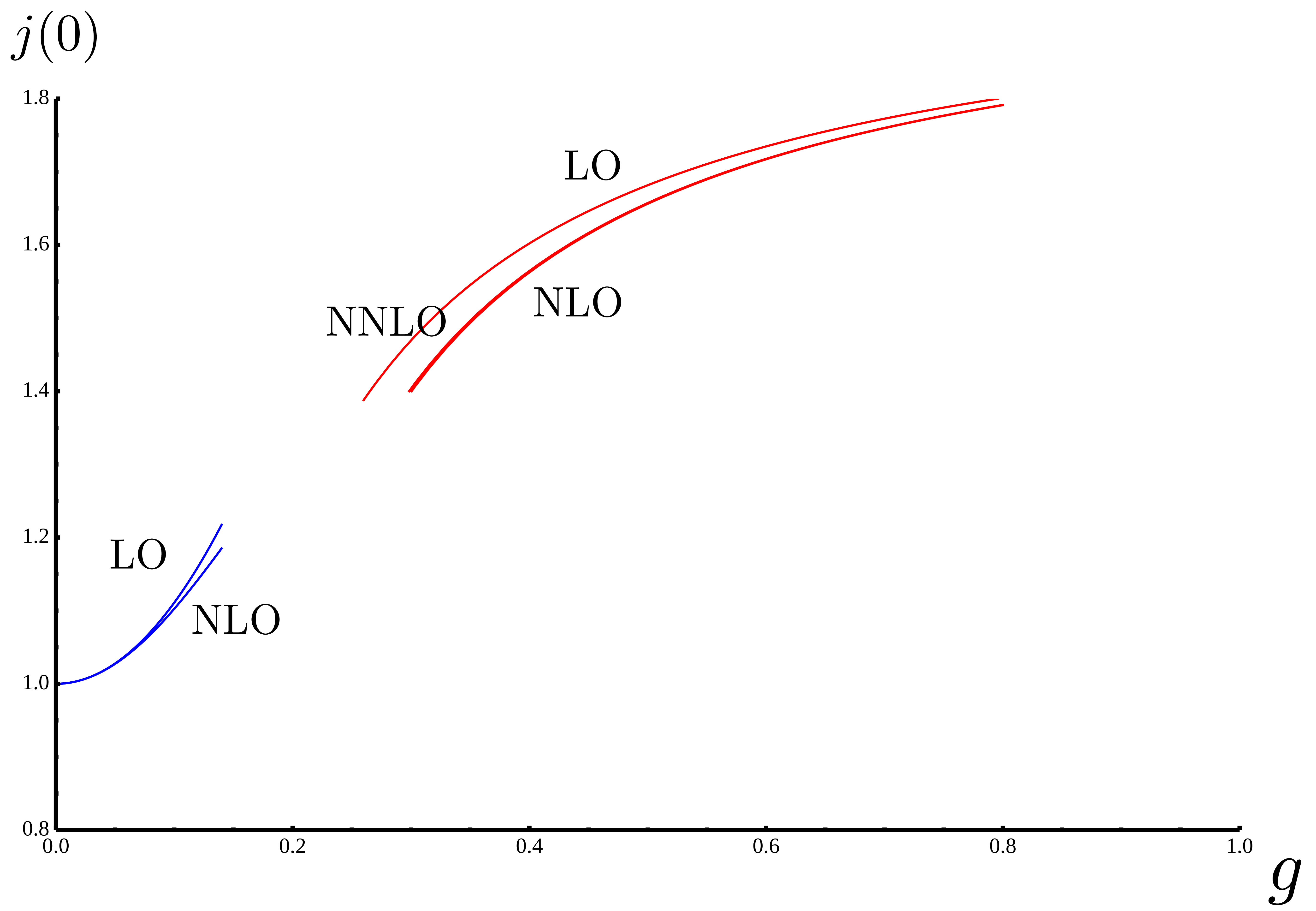
[Janik, work in progress]

- New prediction for the strong coupling expansion of intercept

$$j(0) = 2 - \frac{2}{\sqrt{\lambda}}\left(1 + \frac{1}{2\sqrt{\lambda}} - \frac{1}{8\lambda}\right)$$

$$+2(1-\zeta_3)\frac{1}{\lambda^2}$$

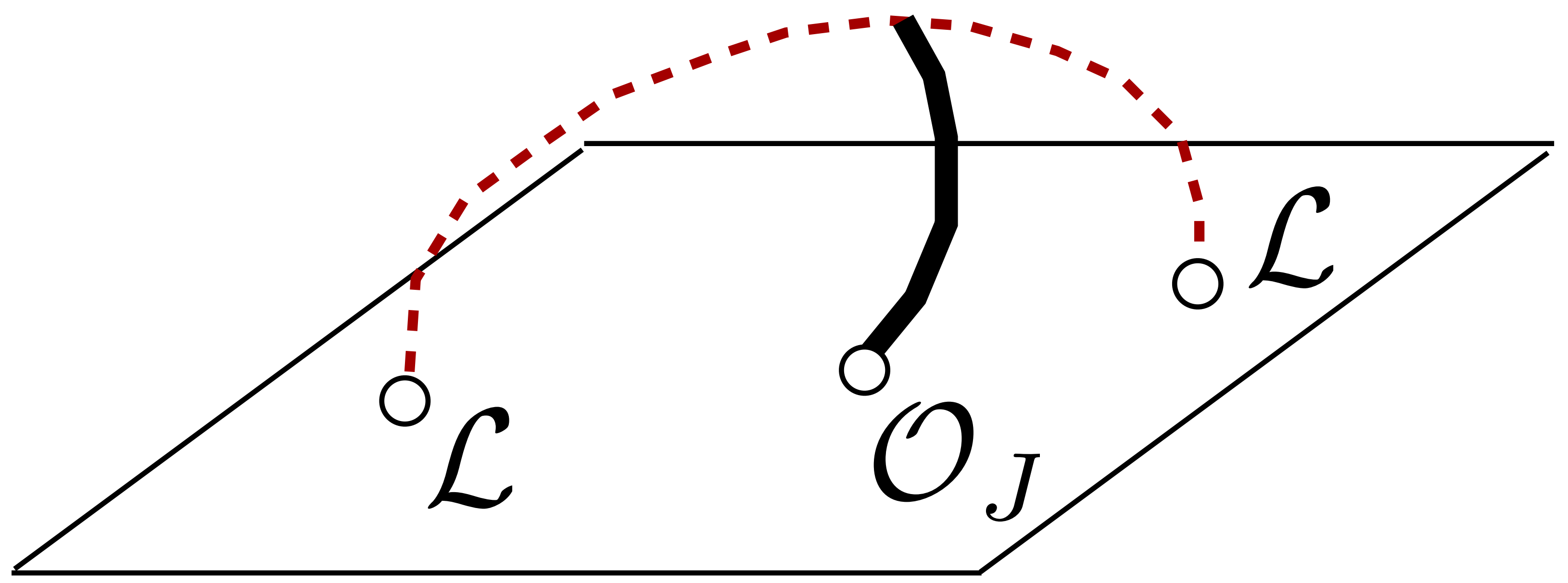
[Kotikov and Lipatov 13]



## N=4 Super Yang Mills - OPE coefficients at strong coupling

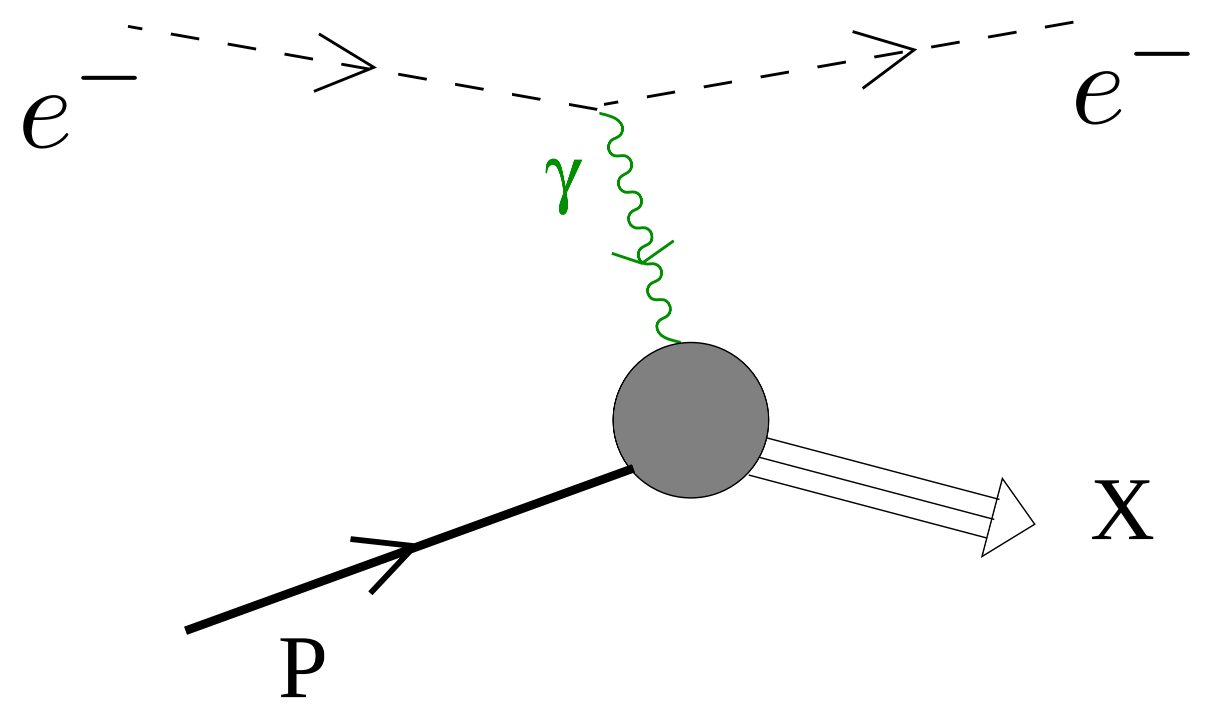
- Equating flat space limit of amplitude to Virasoro-Shapiro in Regge limit can make prediction for strong coupling OPE coefficients involving Lagrangian and operators in leading Regge trajectory

$$C_{\mathcal{L}\mathcal{L}J} = \frac{\pi^{\frac{3}{2}}}{3N} \frac{(J-2)^{\frac{5+J}{2}}}{2^{1+J}\Gamma\left(\frac{J}{2}\right)} \lambda^{\frac{7}{4}} 2^{-\lambda^{1/4}\sqrt{2(J-2)}}$$

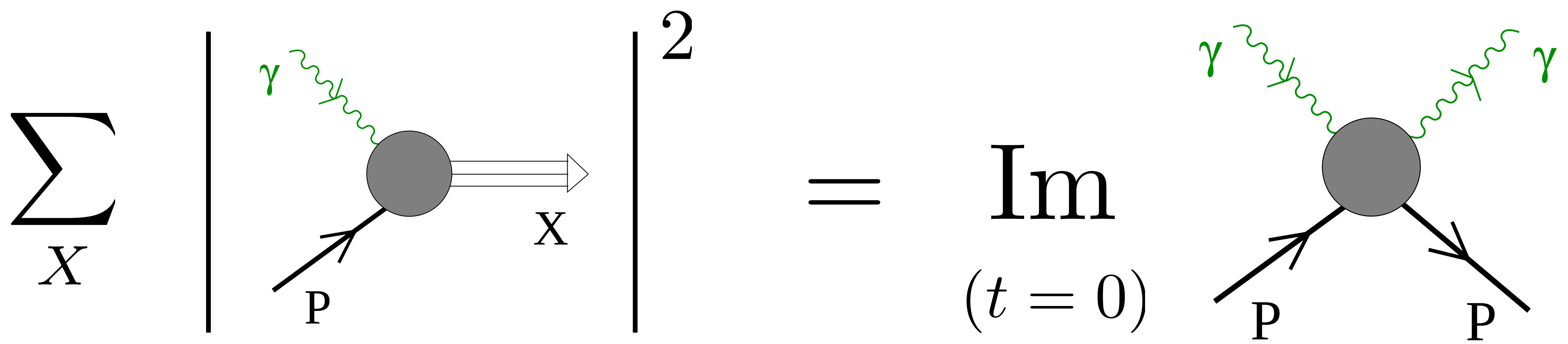


# Applications to low $x$ physics in QCD

- Deep inelastic scattering (DIS)

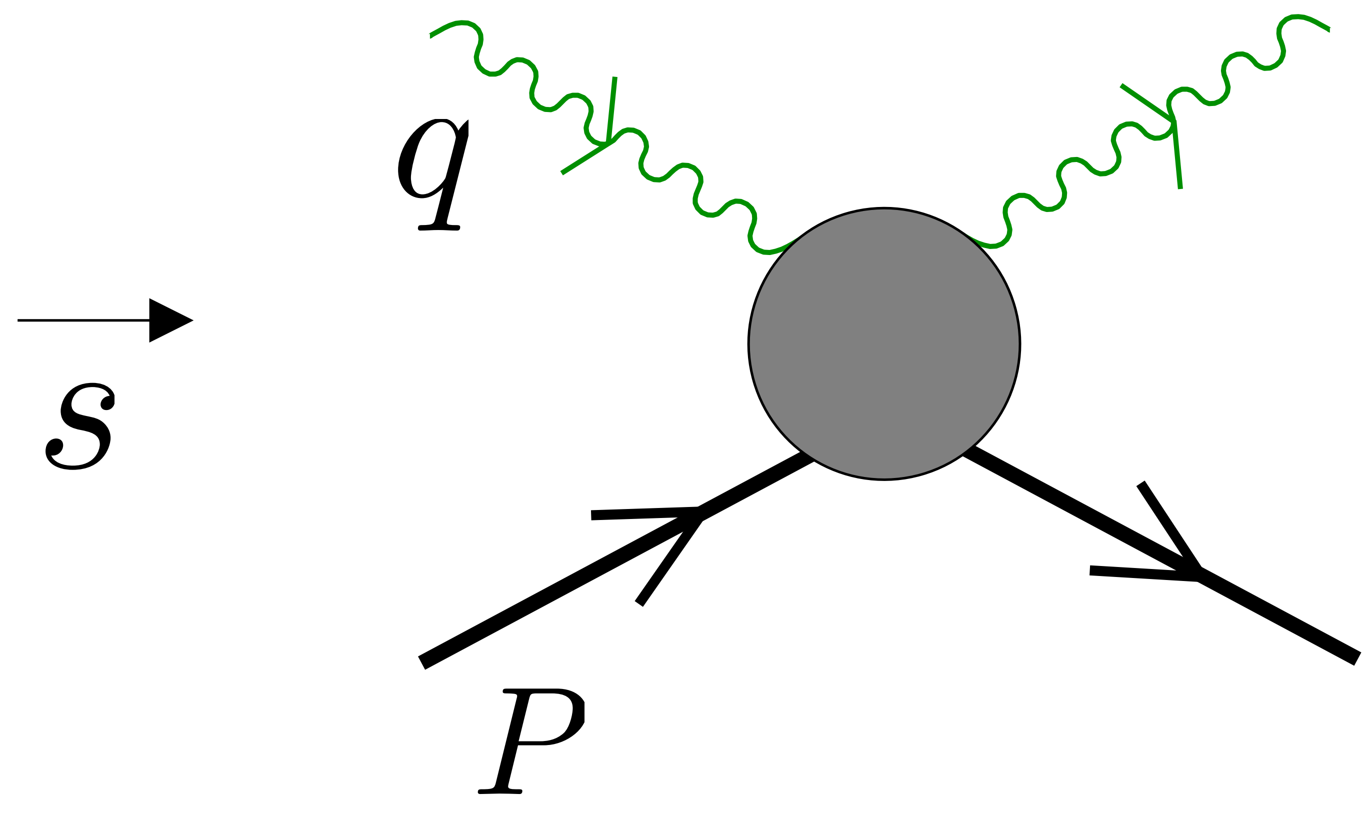



- Optical theorem




- Hadronic tensor

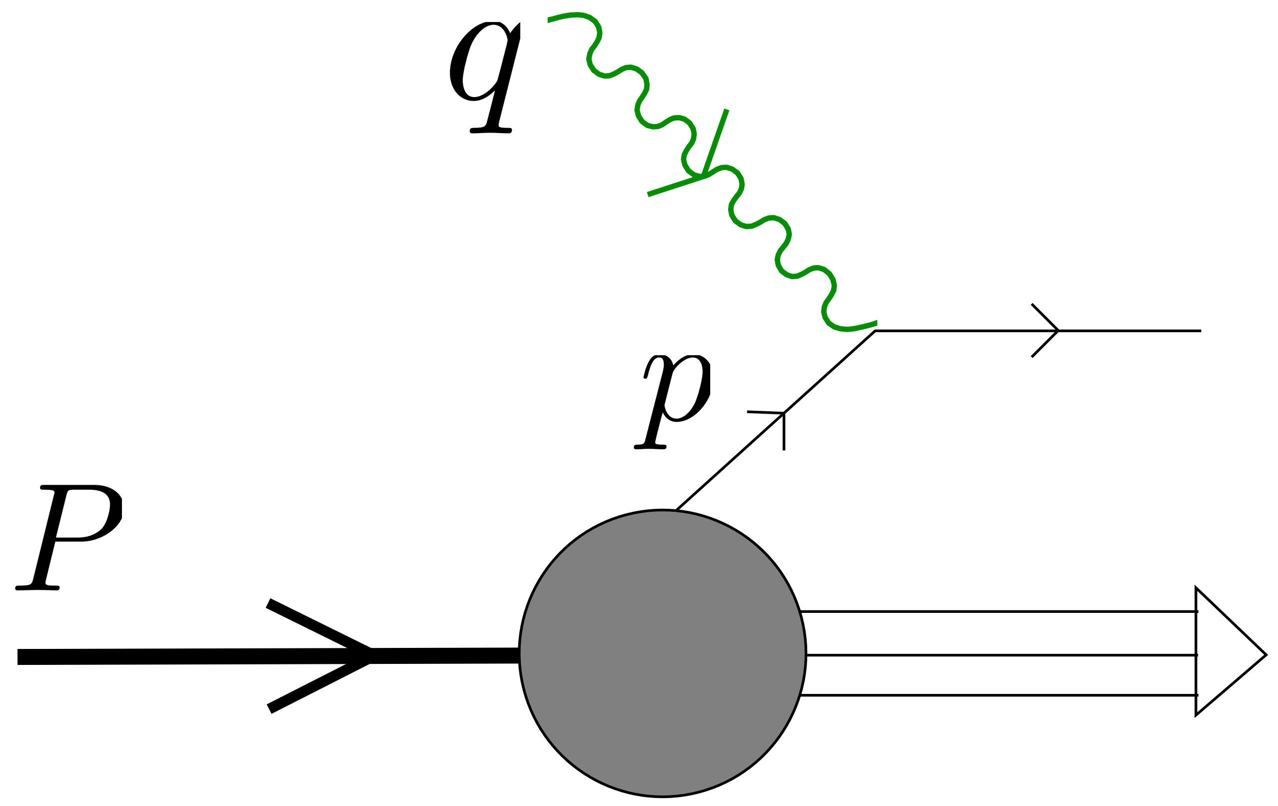
$$W^{ab}(x,Q,t) = i\int d^4y\, e^{iq\cdot y}\langle P|T\{j^a(y)\, j^b(0)\}|P'\rangle$$

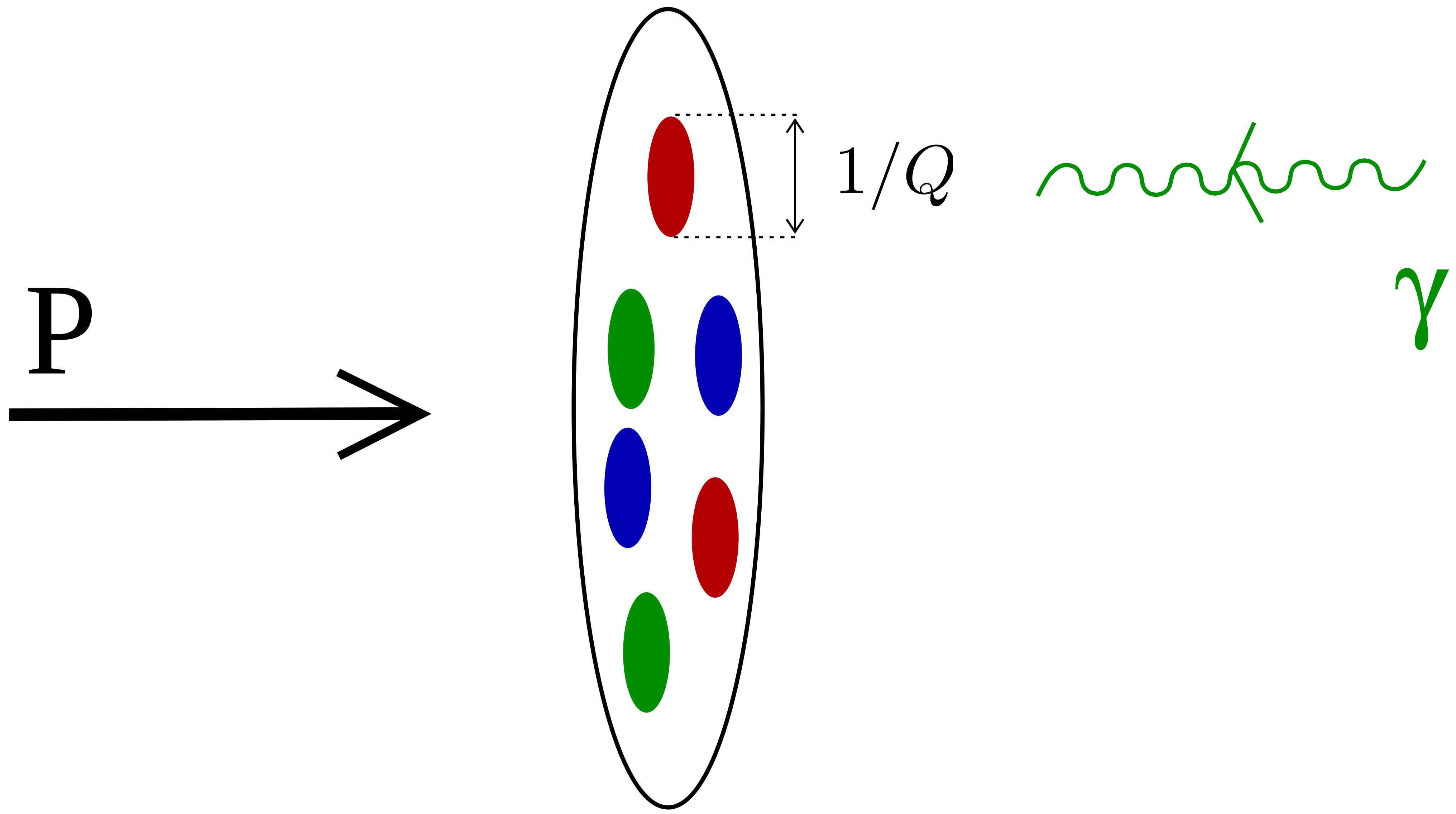



$$s = -(q+P)^2$$

$$Q^2 = q^2$$

- Bjorken $x$
- Transverse resolution $1/Q$



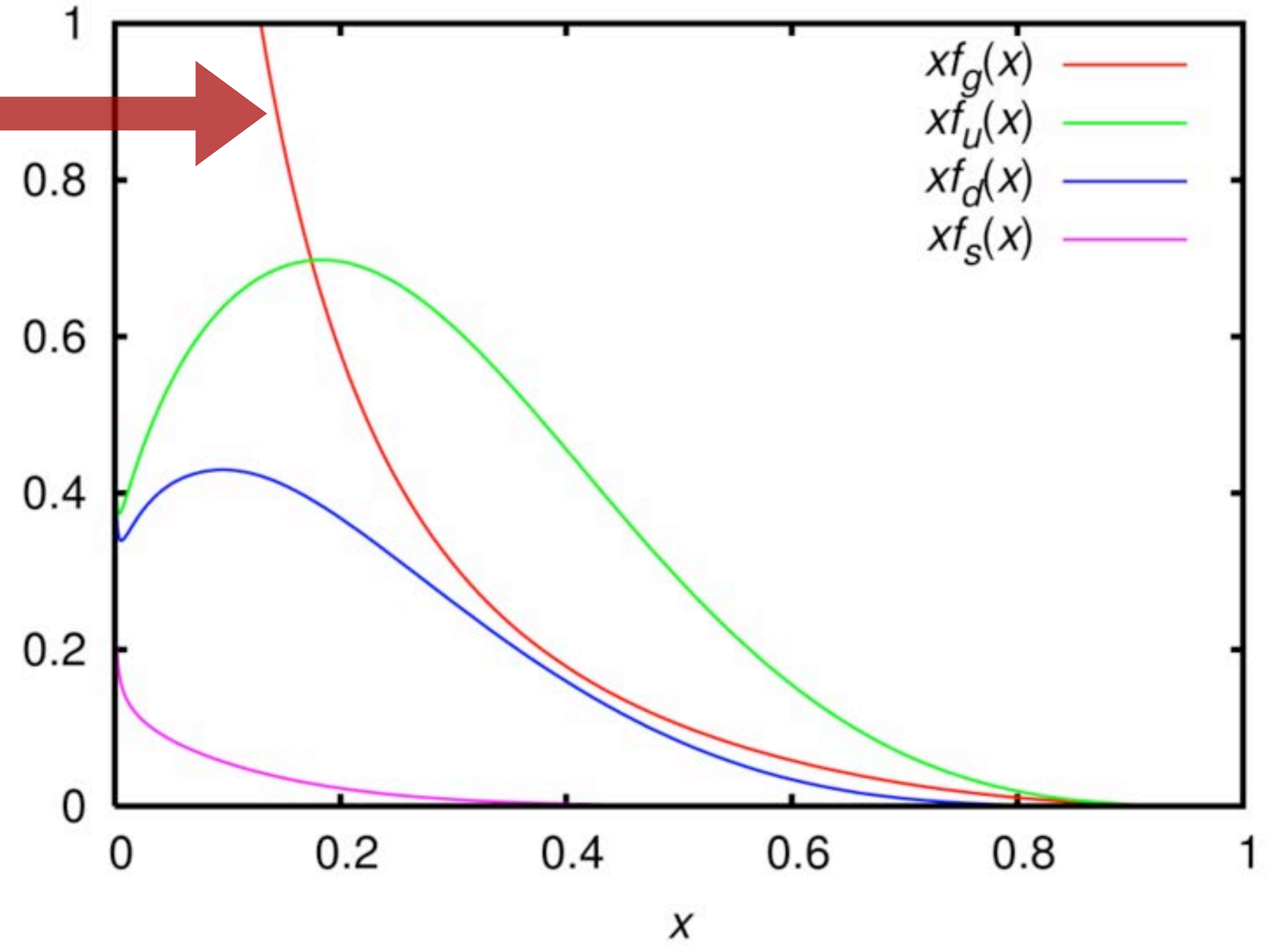

$$p = xP$$

$$s \approx \frac{Q^2}{x}$$

large $s$ $\Rightarrow$ small $x$

- Parton distribution functions $f_i\left(x, Q^2\right)$

Gluons dominate at small $x$




For $x \lesssim 10^{-2}$ much steeper $x$ - dependence

$$\sigma \sim x^{1-j_0}$$

with a intercept

$$\alpha(0) = j_0 \sim 1.2 - 1.3$$

**One or two pomerons (soft and hard)? Is it the same Regge trajectory?**

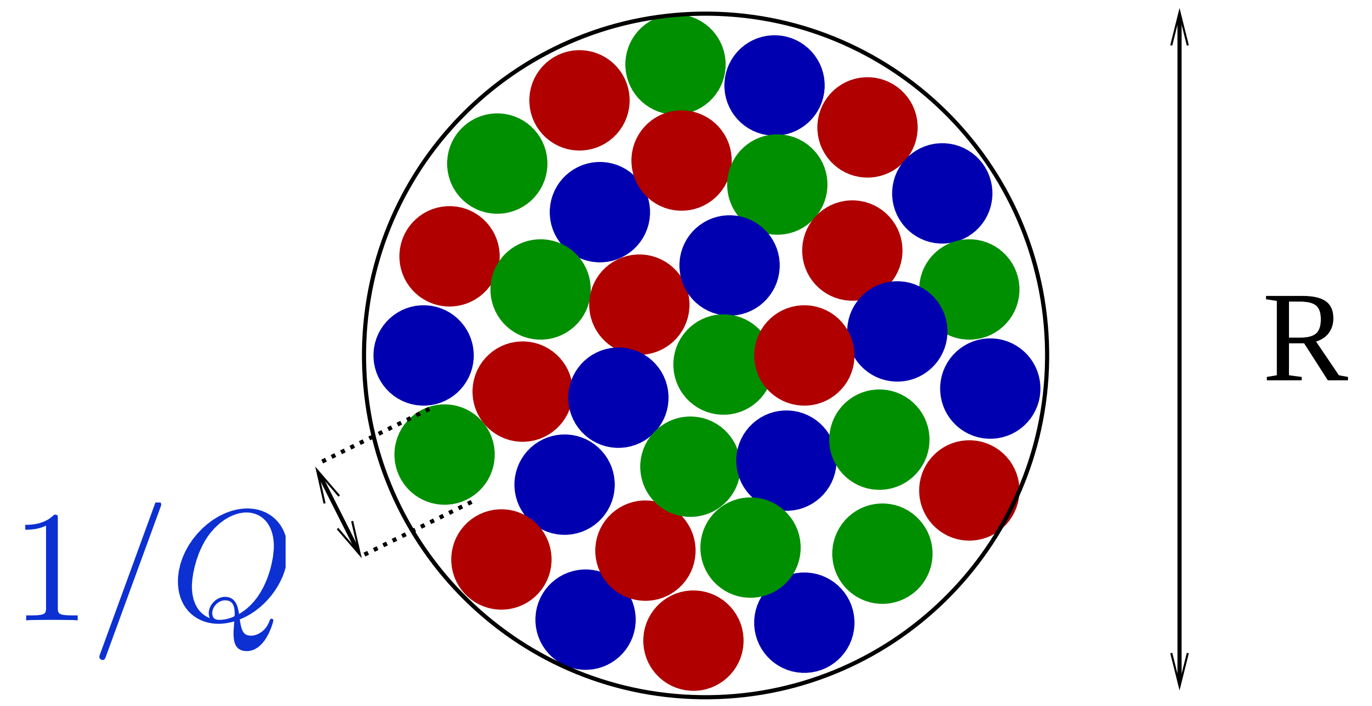
Hard Pomeron explains well data for DIS outside the confining region $Q \sim \Lambda_{QCD}$ [Kowalski, Lipatov, Ross, Watt 10]

Exponent is smaller in confining region (more like soft pomeron)

- Strong rise in $1/x$ , violating Froissart bound

$$\sigma \lesssim m_\pi \, (\ln s)^2$$

- Perturbation theory will break down, even for small coupling, because there will be gluon saturation at very low x.



# DIS, DVCS & VMP from AdS/CFT

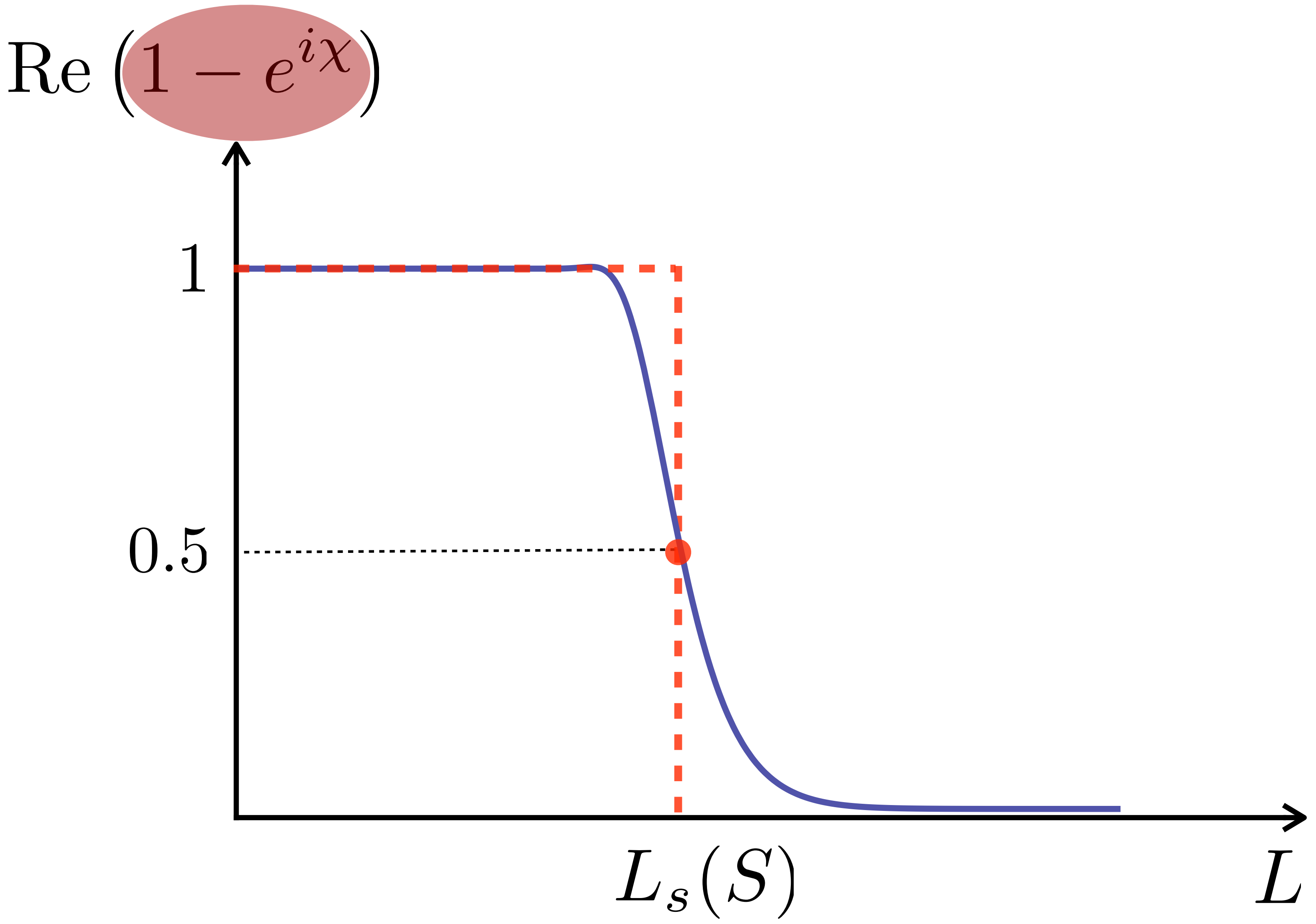
- Hadronic tensor

$$W = 2is \int d^2 l_\perp \, e^{iq_\perp \cdot l_\perp} \int \frac{dr}{r^3} \frac{d\bar{r}}{\bar{r}^3} \, \Psi(r) \, \Phi(\bar{r}) \, \mathcal{B}(S, L)$$

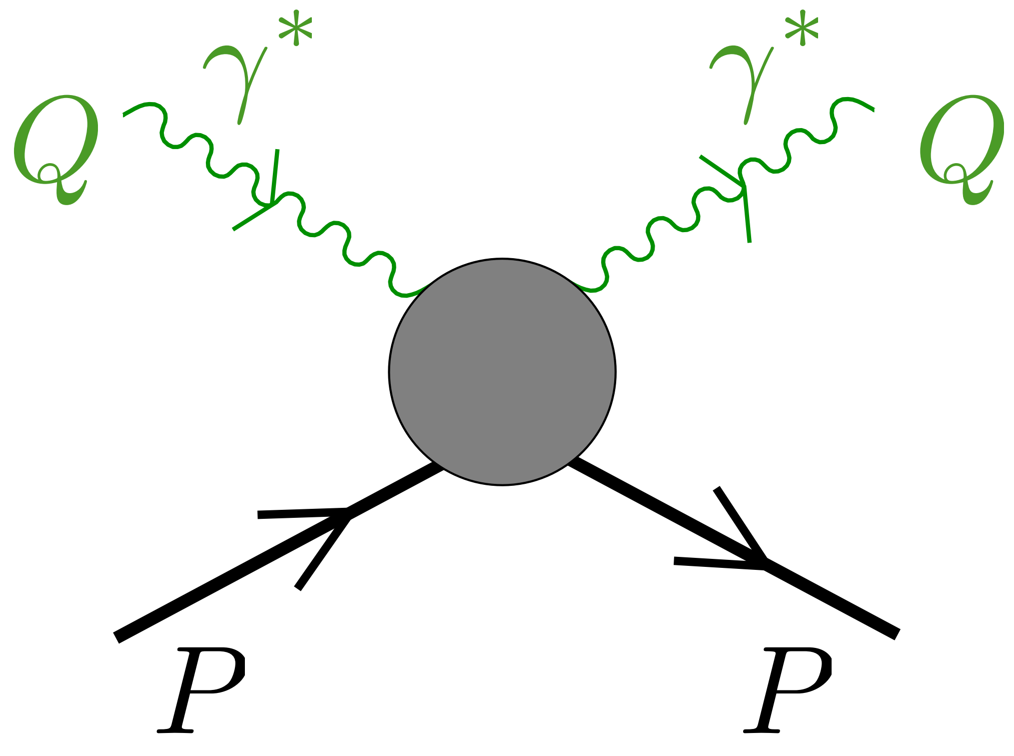
AdS black disk
and
AdS pomeron

non-normalizable ($0 < r < 1/Q$);
if out-going photon on-shell ($Q' \to 0$)

normalizable ($\bar{r} \sim 1/M$), use delta function




- DIS

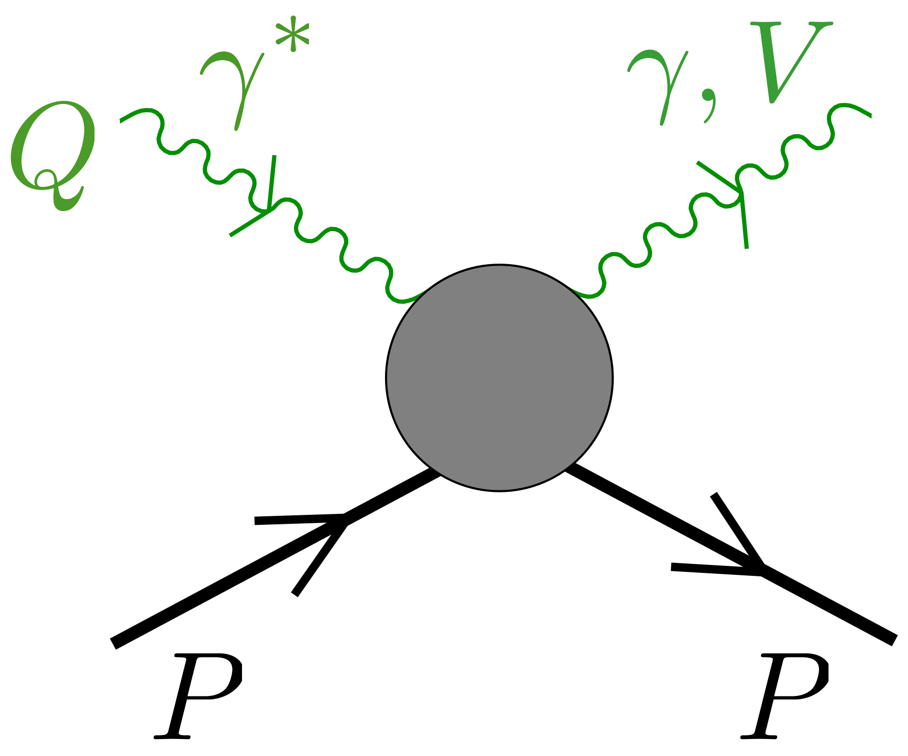



$\sigma(Q, x) \propto \operatorname{Im} W$

$(t = 0)$

(structure function $F_2$)

- DVCS & VMP




$$\frac{d\sigma}{dt}(Q, x, t) \propto |W|^2$$

$\sigma_{tot}(Q, x)$

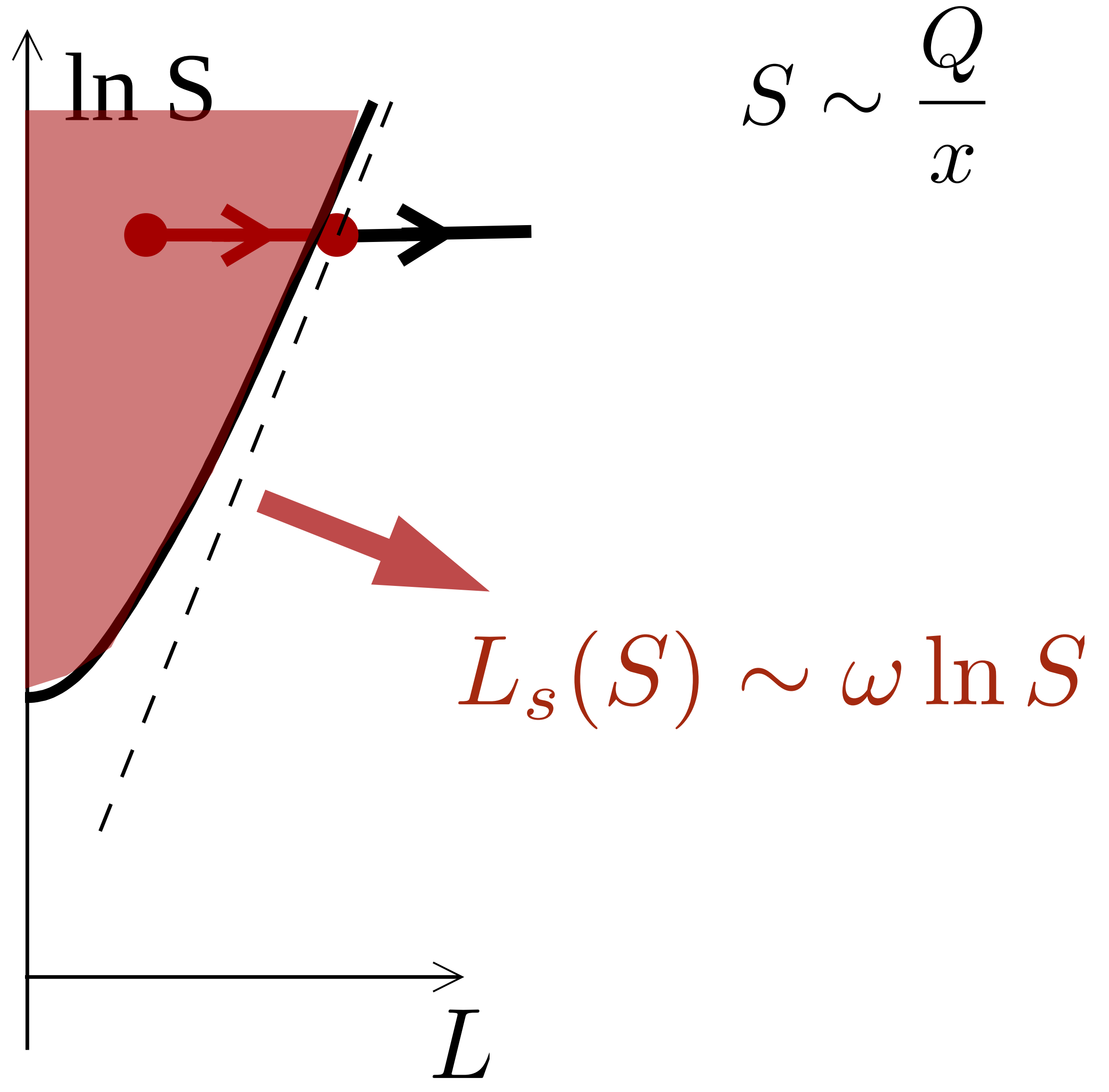
# AdS black disk model for saturation [Cornalba, MSC 08]

- Black disk in AdS (or in conformal QCD)

$$\mathcal{B}(S,L) = \left[1 - e^{i\chi(S,L)}\right] = \Theta\big(L_s(S) - L\big)$$

Non-linear effects become important for $\text{Im}\ \chi(S,L_s) \sim 1$. Both in weak coupling QCD and AdS gravi-Reggeon, this happens for

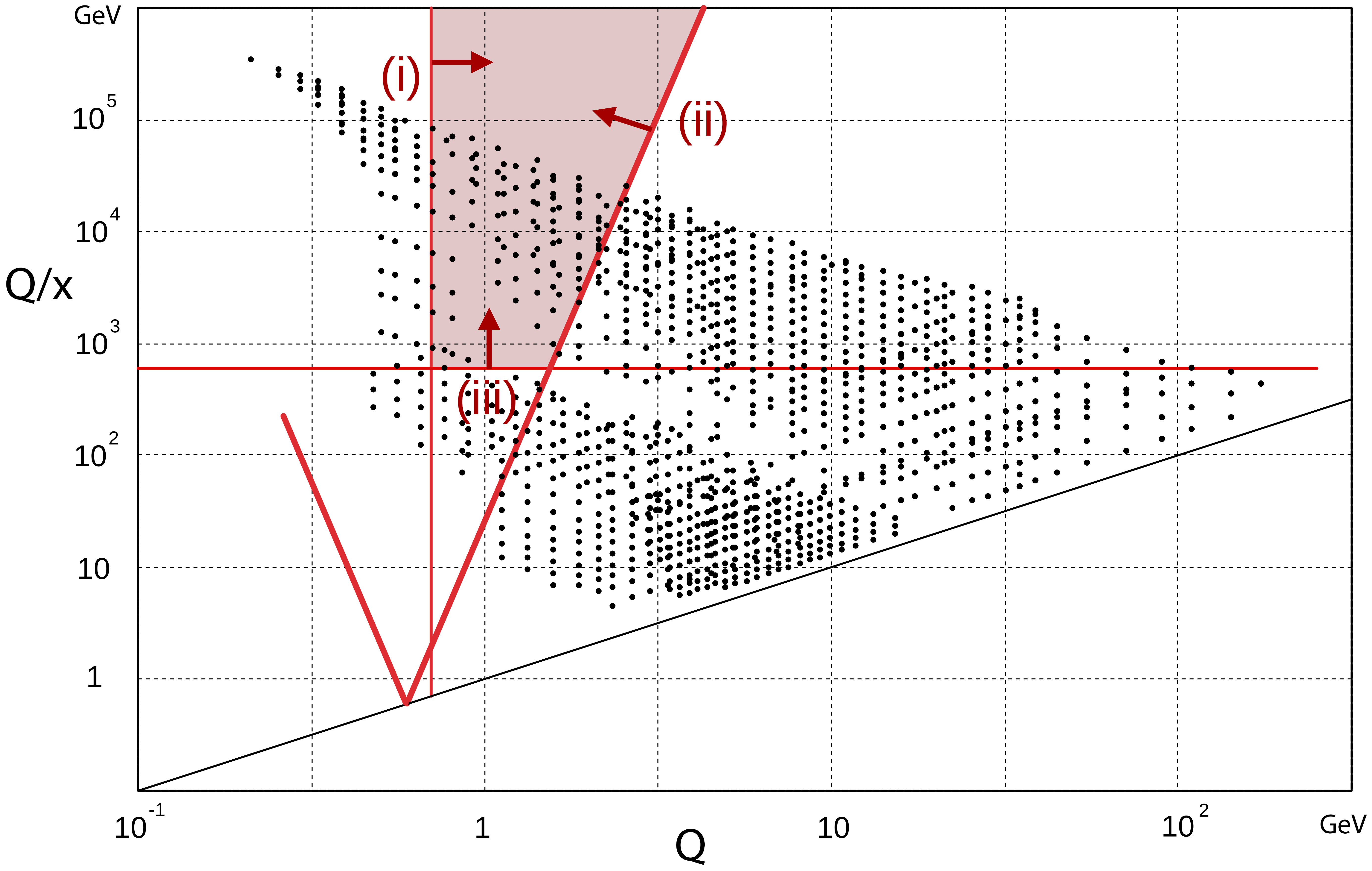
$$L_s(S) \sim \omega \ln S$$




- It is all AdS (or CFT) kinematics. Only dynamical information is the on-set of black disk region → $\omega$ ($j_0 \equiv 1 + \omega$ so $\sigma \sim x^{-\omega}$)

- Target wave function → $r_*$ ; Normalization of current operator → $C$

- Data selection (171 points)




[Debbio et al; mostly Zeus & Hera]

(i) Weak coupling

$$Q > Q_{min} \sim 1 \text{ GeV}$$

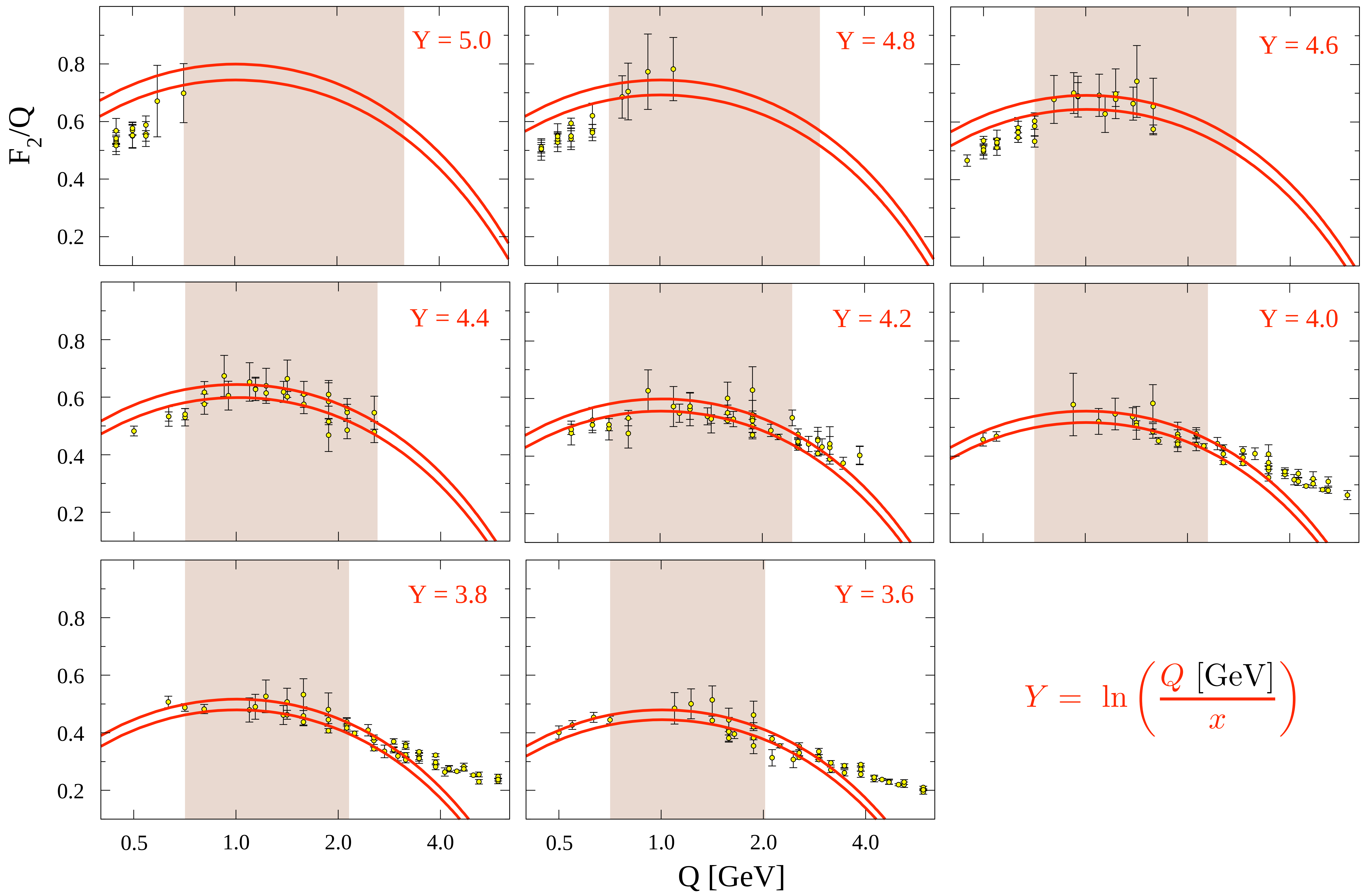
(ii) Inside saturation

$$\omega \ln \frac{Q}{xM} \gtrsim \ln \frac{Q}{M}$$

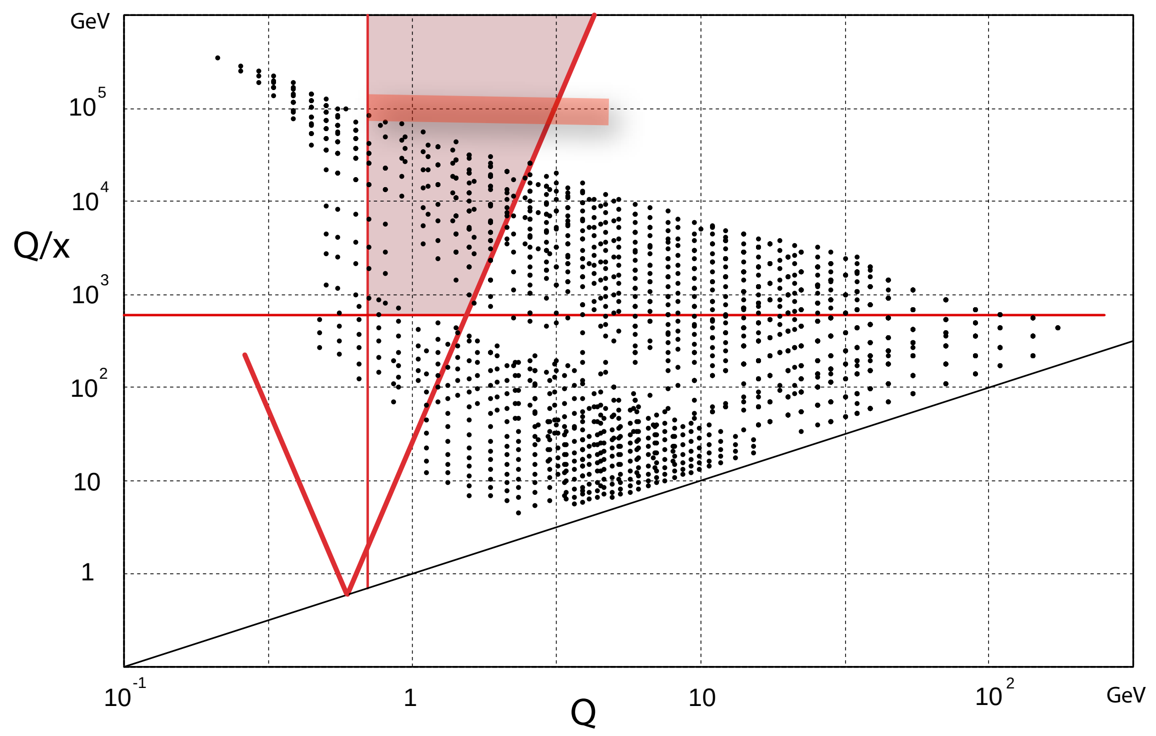
(iii) Regge limit of large S

$$\frac{Q}{xM} \gtrsim 10^3$$

- Fit to data




$$Y = \ln\left(\frac{Q\,[\mathrm{GeV}]}{x}\right)$$




- Matches data with 6% accuracy in kinematical range

$$0.5 < Q^2 < 10\ \mathrm{GeV}^2\,, \quad x < 10^{-2}$$

- Predict $\omega = 0.15$ . Compactible with geometric scaling ($\lambda = 0.32$ )

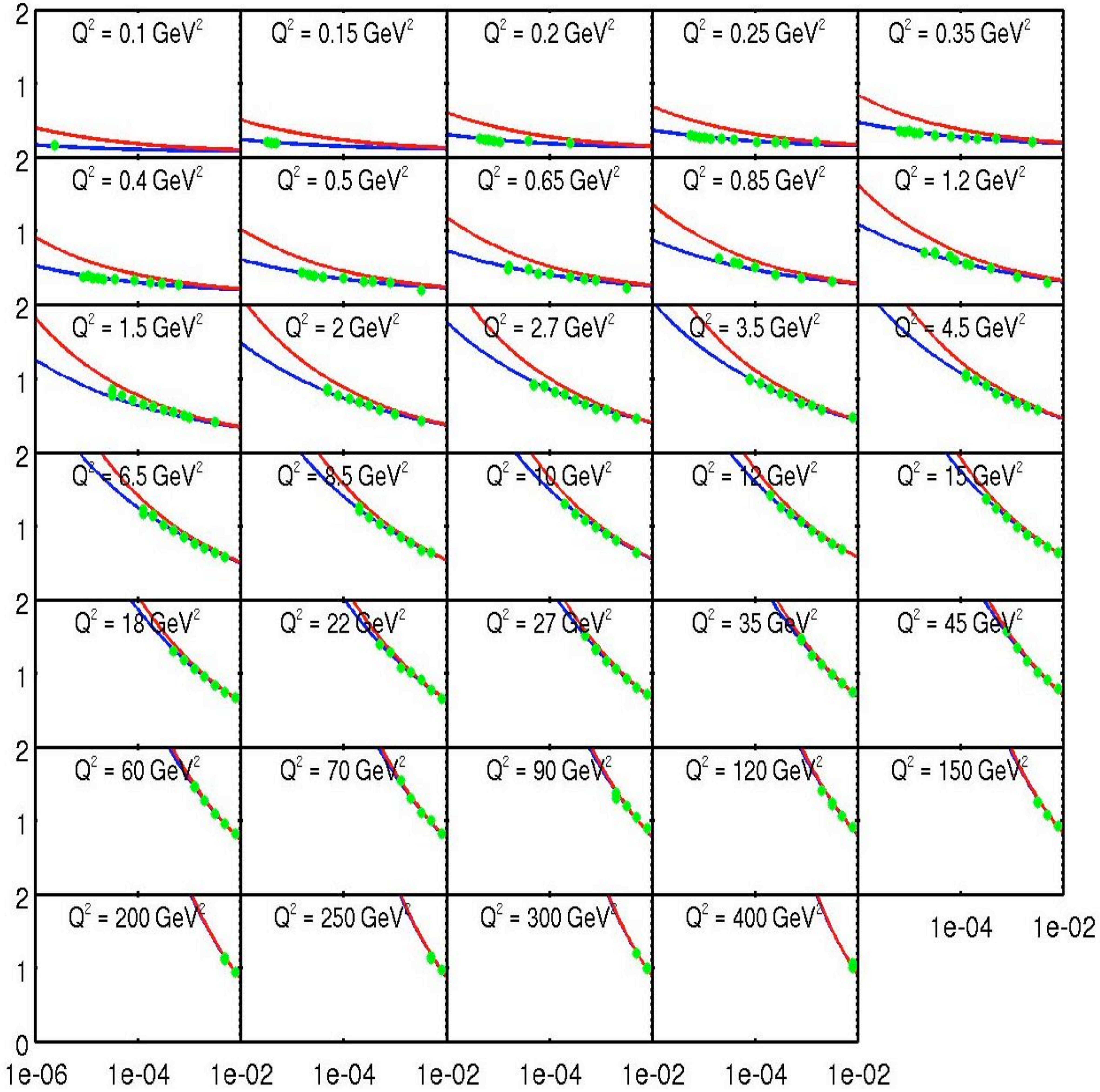
$$\sigma = \sigma\left(\frac{Q}{Q_s}\right)\,, \quad Q_s^2 = M^2 x^{-\lambda}$$

$$\lambda = \frac{2\omega}{1-\omega} \quad \longrightarrow \quad \omega = 0.14$$

- New prediction

$$\frac{F_L}{F_T} \approx \frac{F_2 - 2xF_1}{2xF_1} \approx \frac{1+\omega}{3+\omega}$$

# DIS - AdS Pomeron (with hard wall) [Brower, Djuric, Sarcevic, Tan 10]




Four parameters: $g_0^2\,,\quad j_0\,,\quad r_*\,,\quad r_0$

HERA combined data by H1 and ZEUS experiments [Aaron et al 10] with

$$0.10 < Q^2 < 400\ GeV^2,\ x < 10^{-2}$$

For hard wall model obtained excellent fit with (249 points)

$$\chi^2_{d.o.f.} = 1.07$$

$$j_0 = 1.22$$

$$r_* = 2.31\ \mathrm{GeV}^{-1}$$

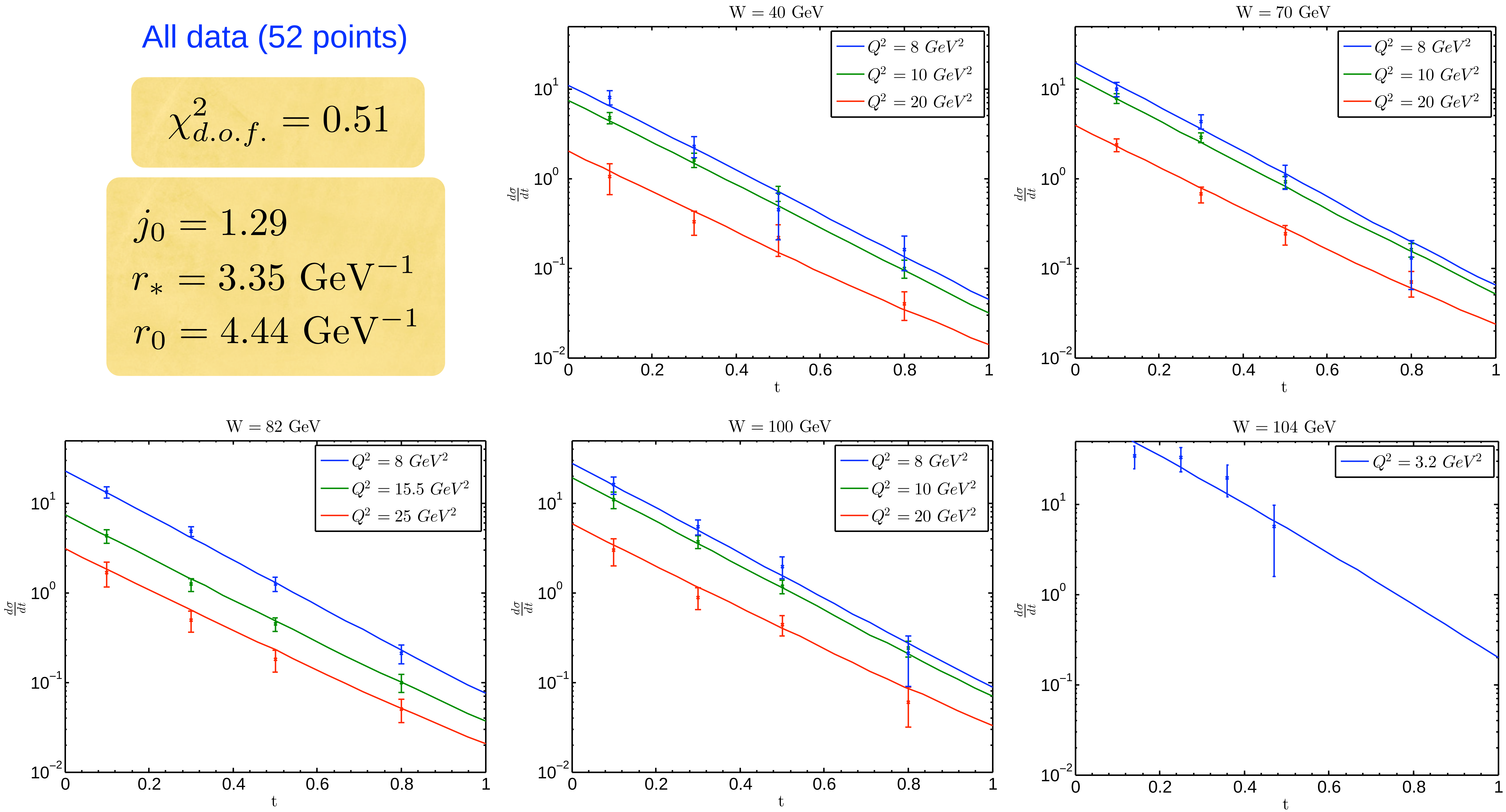
$$r_0 = 4.96\ \mathrm{GeV}^{-1}$$

# DVSC (differential cross section) [MSC, Djuric 12]

All data (52 points)

$$\chi^2_{d.o.f.} = 0.51$$

$$j_0 = 1.29$$
$$r_* = 3.35\ \mathrm{GeV}^{-1}$$
$$r_0 = 4.44\ \mathrm{GeV}^{-1}$$

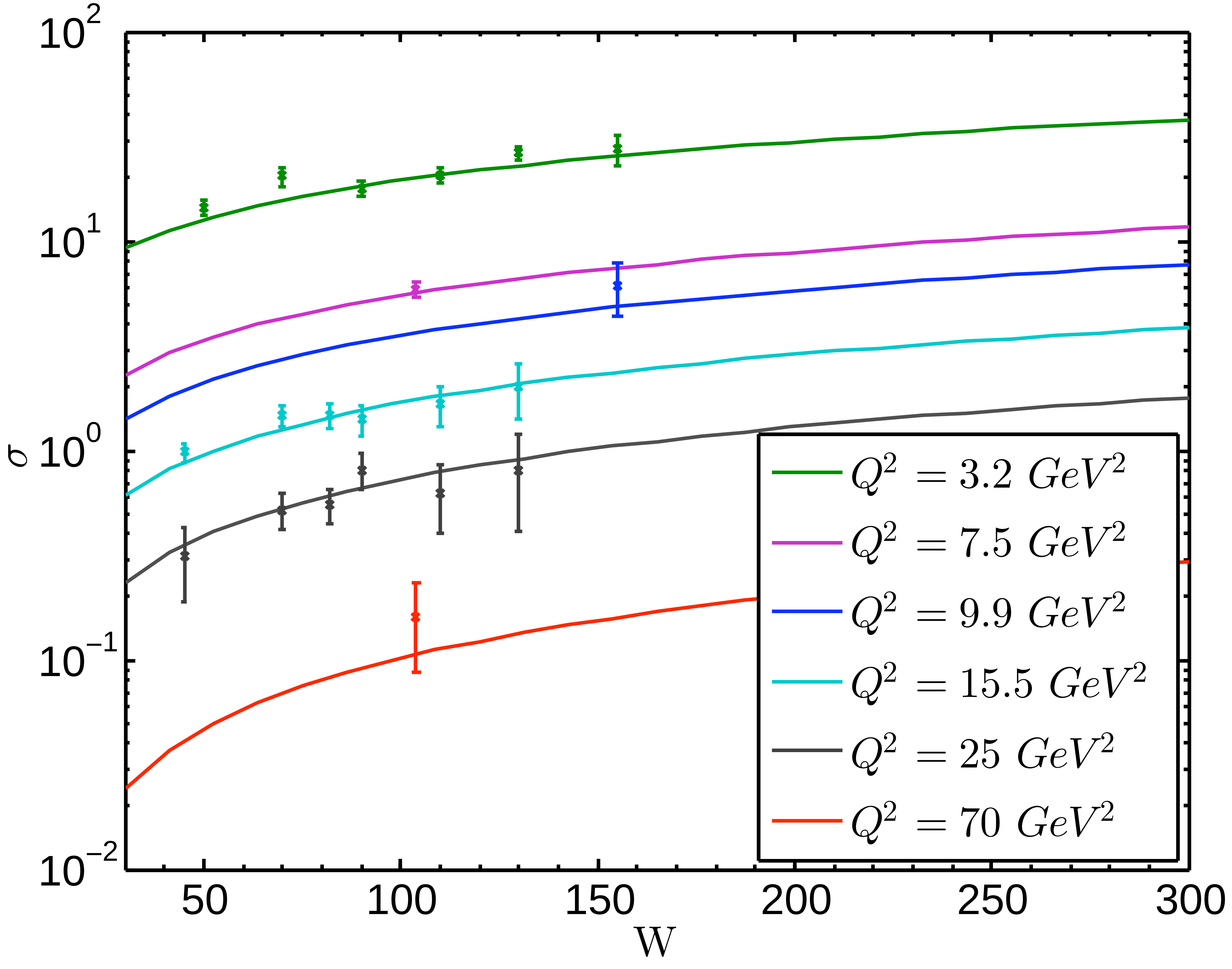


# DVSC (total cross section) [MSC, Djuric 12]

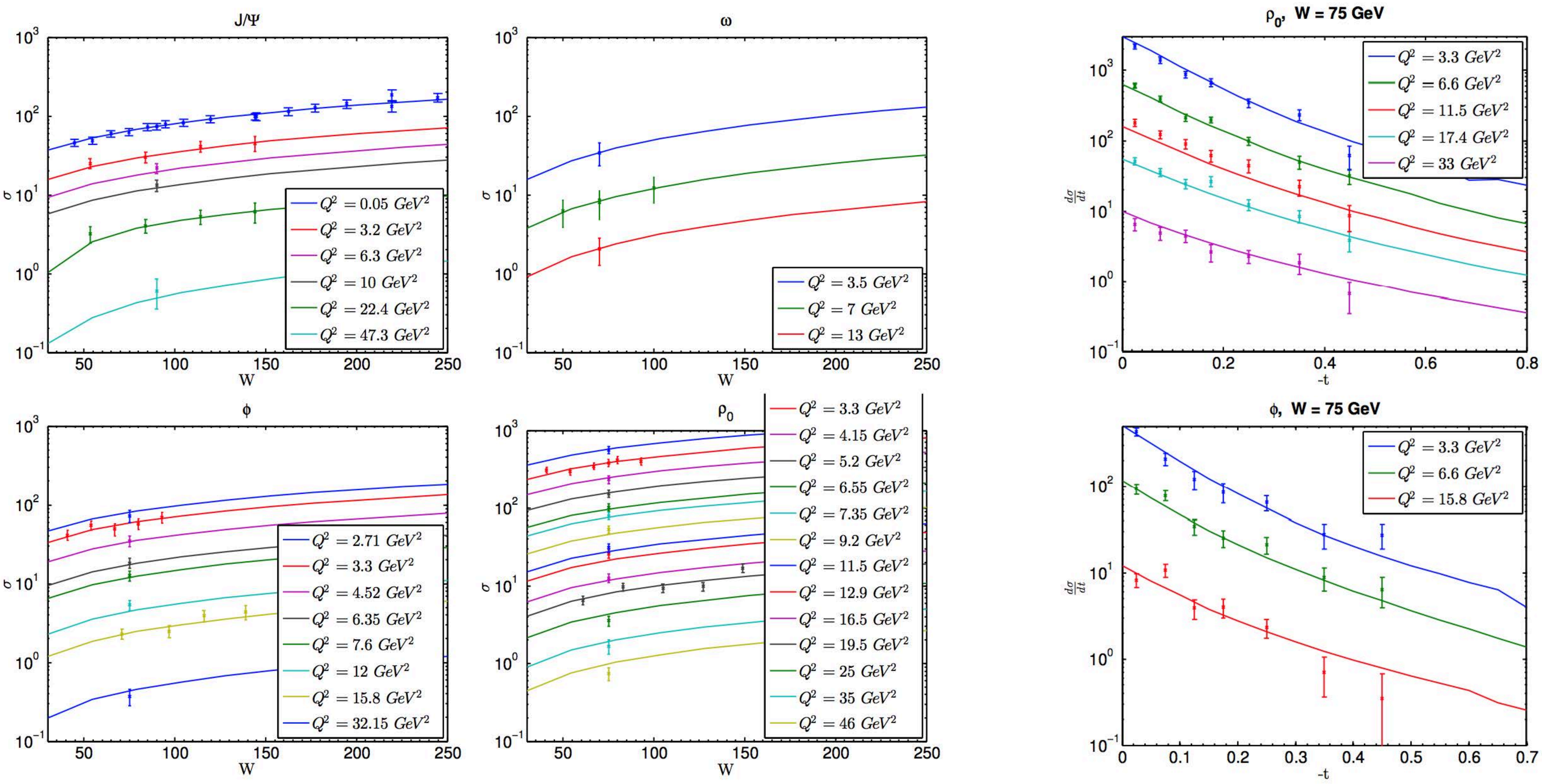
All data (44 points)

$$\chi^2_{d.o.f.} = 1.03$$

$$j_0 = 1.19$$
$$r_* = 4.86 \text{ GeV}^{-1}$$
$$r_0 = 8.14 \text{ GeV}^{-1}$$



# VMP ($J/\Psi,\ \omega,\ \phi,\ \rho_0$) [MSC, Djuric, Evans to appear]

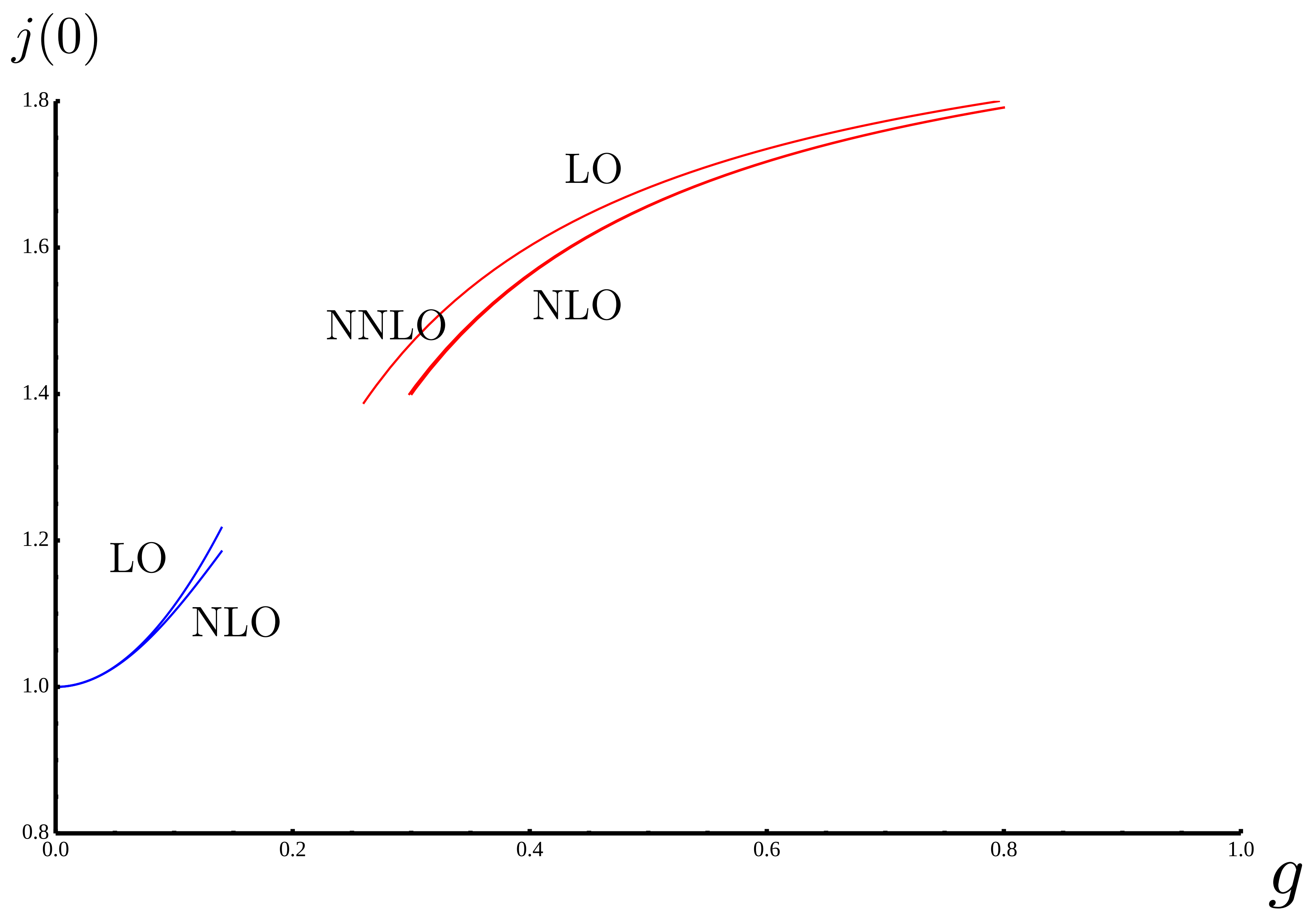


# Concluding remarks & future directions

• Constructed the formalism for Regge theories for CFT's or, equivalently, for scattering in AdS spaces.

• Explored consequences of Conformal Regge theory in N=4 SYM and gave many new predictions - useful data for program of solving theory exactly using integrability. Explore other trajectories, e.g. $\mathcal{O}_J = \text{Tr}\left(ZD^J Z\right)$ .

• In N=4 SYM can we derive spin of pomeron/graviton Regge trajectory using integrability for any value of the coupling (like Y-system for anomalous dimensions)?




• Pomeron exchange from strong coupling (AdS) computation matches data in very large kinematical range (for DIS, DVSC and VMP). Can we use AdS inspired IR cut-off to analise weak coupling BFKL?

• In a restricted kinematical window (inside saturation) DIS and DVSC show a black disk in AdS (or in conformal QCD).

• From DIS analysis, in confinement region of Q, effective intercept is decreasing (soft pomeron region). Is this evidence for a single pomeron?

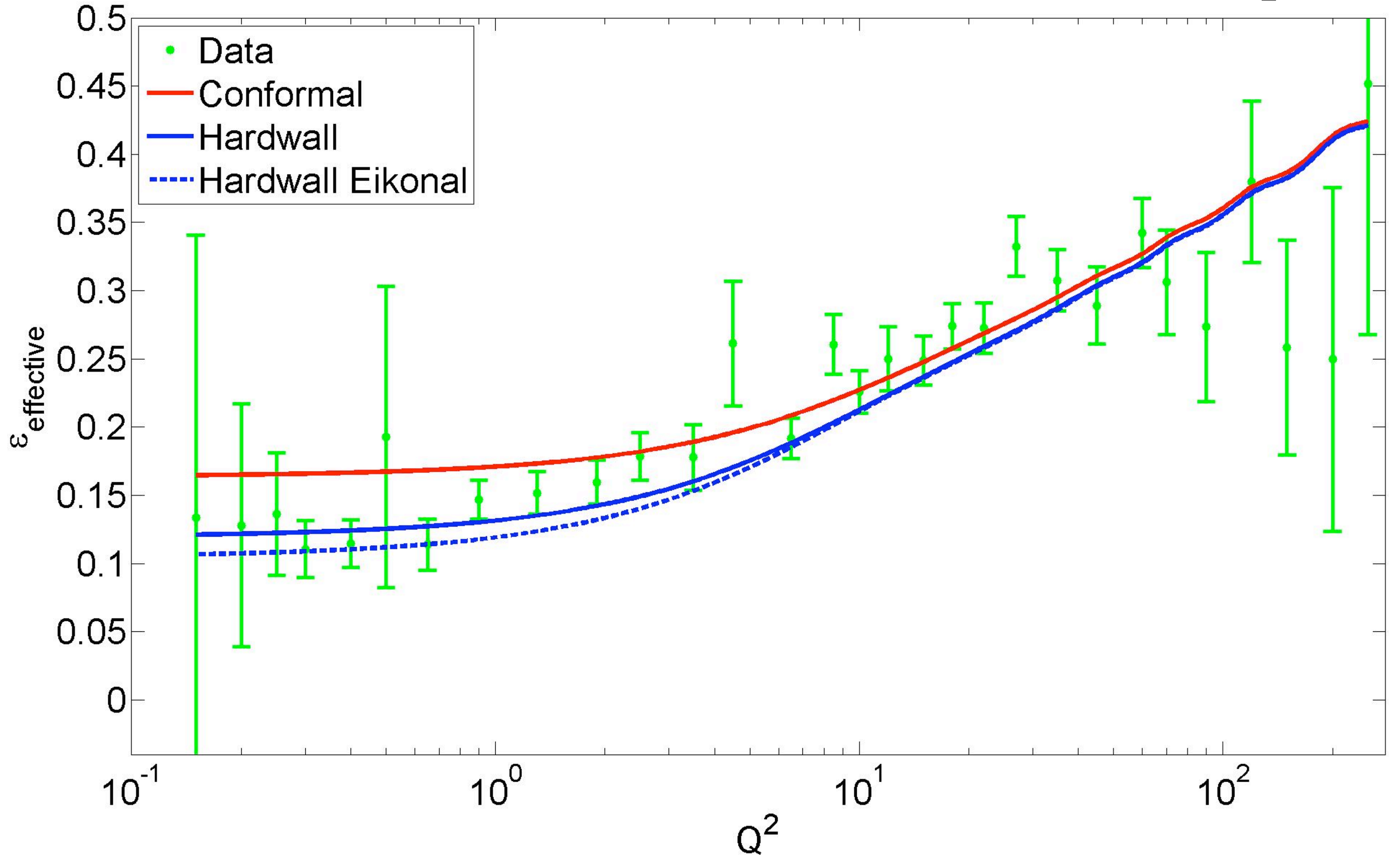
$$F_2(x,Q^2) \sim (1/x)^{\epsilon_{eff}}$$

- Intercept: 1.2 - 1.4 (hard pomeron)
  1.08 (soft pomeron)

- What about Regge slope?
  (0.25 for soft pomeron)

Effective Pomeron intercept

[Brower, Djuric, Sarcevic, Tan 10]




• One can interpolate between a CFT in UV and a confined gauge theory in IR where standard Regge theory applies. Can we understand better how conformal and standard Regge theories interpolate? (single Regge trajectory becomes infinite sequence of trajectories)

• Further model testing with other processes where pomeron plays a role (e. g. vector meson production [in progress], double diffractive Higgs production [Brower, Djuric, Tan 12], elastic hadron-hadron scattering). What happens at lower x values? Is it an AdS black disk? Can we turn this approach into a precise phenomenological model?

# THANK YOU