INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Aspects of Horava-Lifshitz cosmology

arXiv:0904.2190 [hep-th]
arXiv:0905.3563 [hep-th]

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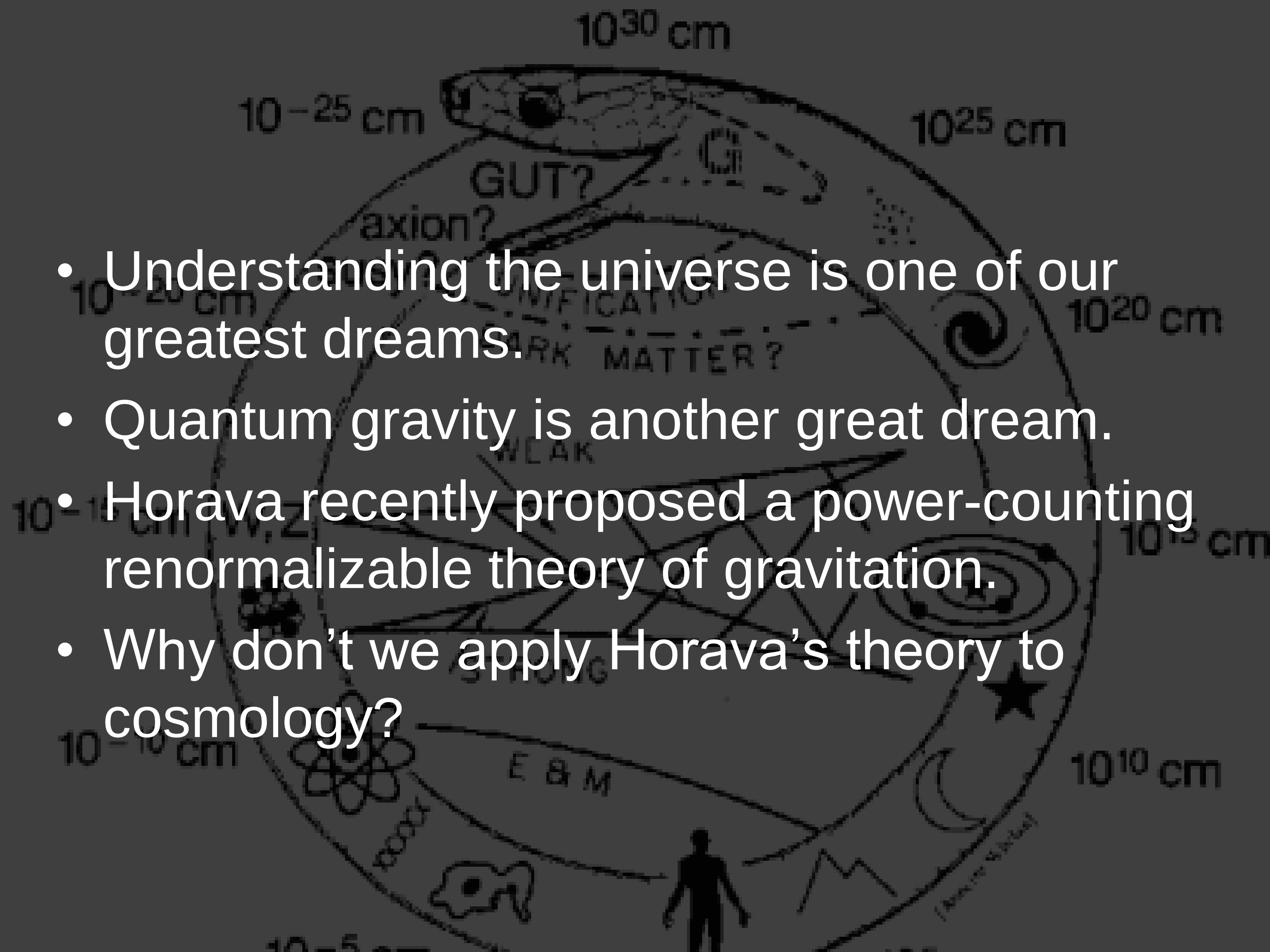


- Understanding the universe is one of our greatest dreams.
- Quantum gravity is another great dream.
- Horava recently proposed a power-counting renormalizable theory of gravitation.
- Why don’t we apply Horava’s theory to cosmology?

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**

  t → b t (E → b^{-1}E)

  x → b x

  ϕ → b^s ϕ

  1+3-2+2s = 0

  s = -1

- Renormalizability

  $$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  t → b^z t (E→b^{-z}E)

  x → b x

  ϕ → b^s ϕ

  z+3-2z+2s = 0

  s = -(3-z)/2

- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Scalar with z=3

free part $$I_\phi = \frac{1}{2}\int dt d^3x\left(\dot{\phi}^2 + \phi O \phi\right)$$

$$O = \underbrace{\frac{\Delta^3}{M^4}}_{\text{UV: z=3}} - \frac{\lambda\Delta^2}{M^2} + \underbrace{c_\phi^2\Delta - m_\phi^2}_{\text{IR: z=1}}$$

- **UV: z=3** , renormalizable nonlinear theory

  ⇓ RG flow

- **IR: z=1** , familiar Lorentz invariant theory

Note: we need a mechanism to make “limits of speed” of different species to be the same.

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse N, shift N^i, 3d spatial metric g_{ij}
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \rightarrow t'(t), \quad x^i \rightarrow x'^i(t,x^j)$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \Big[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \Big]$$

c.f. $D_i R_{ji} D^j R^{ki}$ is written in terms of other terms

Relevant deformations

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

IR action with z=1

- **UV: z=3** , renormalizable quantum gravity

  ⬇ RG flow

- **IR: z=1** , recovers familiar GR iff λ → 1

kinetic term

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(\overbrace{K_{ij}K^{ij}-\lambda K^2}^{\text{kinetic term}}+\underbrace{R-2\Lambda}_{\text{IR potential}}\right)$$

IR potential

note: RG flow has not yet been investigated.

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1$, $N^i = 0$, $g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $$\partial_t \partial_i H_{ij} = 0 \qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H\delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and may have strong self-coupling.

- This is not a problem if there is no vDVZ discontinuity or if there is Vainshtein effect, since HL gravity is supposed to be UV complete. More on related issue later.

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is the gauge field associated with the time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequencely, Hamiltonian constraint is an equation integrated over a whole space.

Note

- Imposing local Hamiltonian constraint would result in theoretical inconsistencies and phenomenological obstacles.
- “Strong coupling in Horava gravity”
  by C.Charmousis, et.al., arXiv:0905.2579
  “A trouble with Horava-Lifshitz gravity”
  by M.Li and Y.Pang, arXiv:0905.2751
- Those problems disappear once we notice that there is no local Hamiltonian constraint.
  (c.f. section 5 of arXiv:0905.3563)

Horava-Lifshitz cosmology

- It is interesting to investigate cosmological implications, in parallel with fundamental issues such as renormalizability and RG flow.
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- The z=3 scaling leads to scale-invariant cosmological perturbations in non-inflationary epoch (Mukohyama 2009).
- Absence of local Hamiltonian constraint leads to CDM as integration "constant" (Mukohyama 2009).

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \to b\,t$ ($E \to b^{-1}E$)

  $x \to b\,x$

  $\phi \to b^{-1}\,\phi$

  ⇒ $$\delta\phi \propto E \sim H$$

  Scale-invariance requires almost const. H, i.e. inflation.

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

- Scaling law

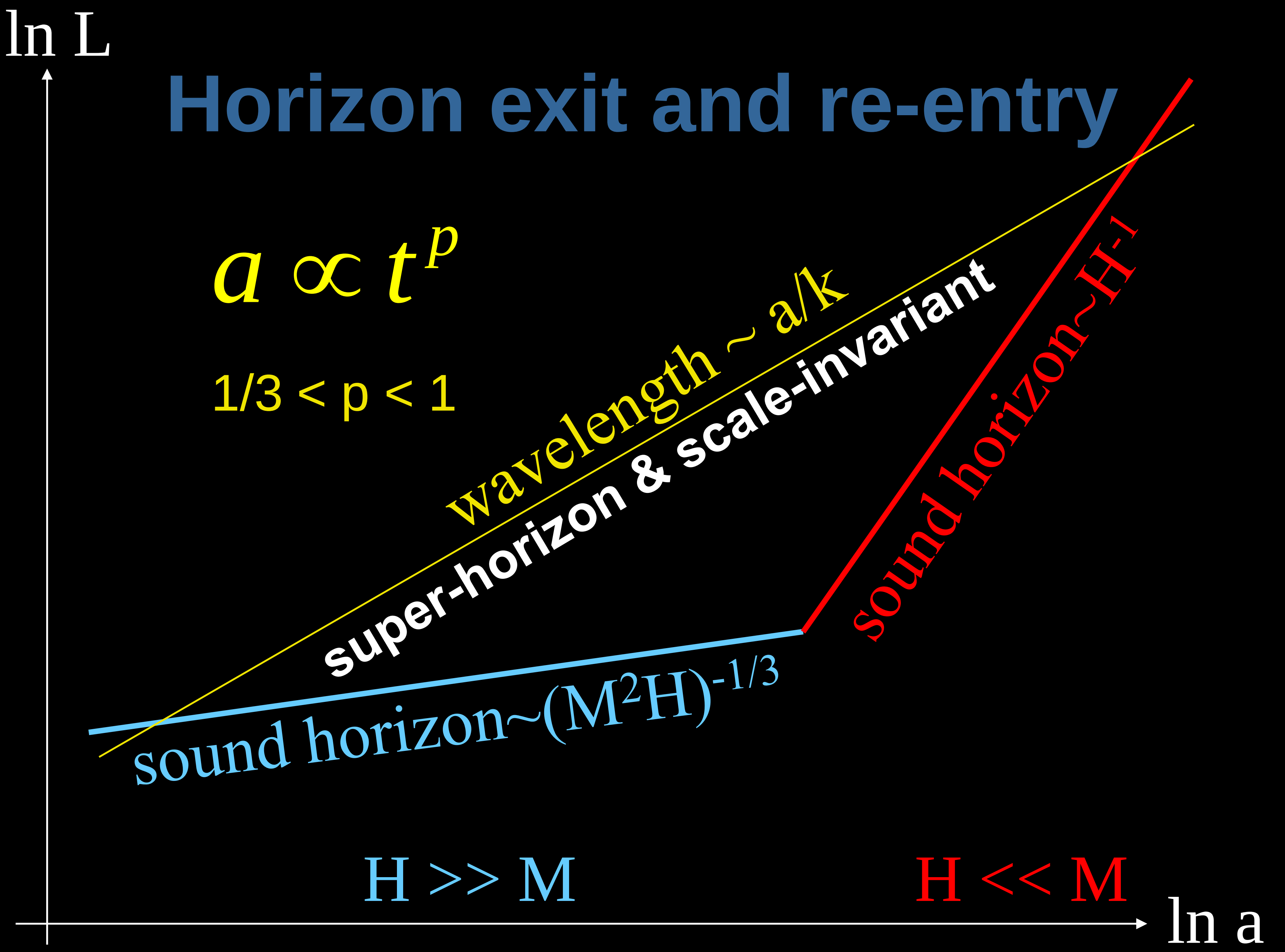
  $t \rightarrow b^3 t$ ($E \rightarrow b^{-3} E$)

  $x \rightarrow b\, x$

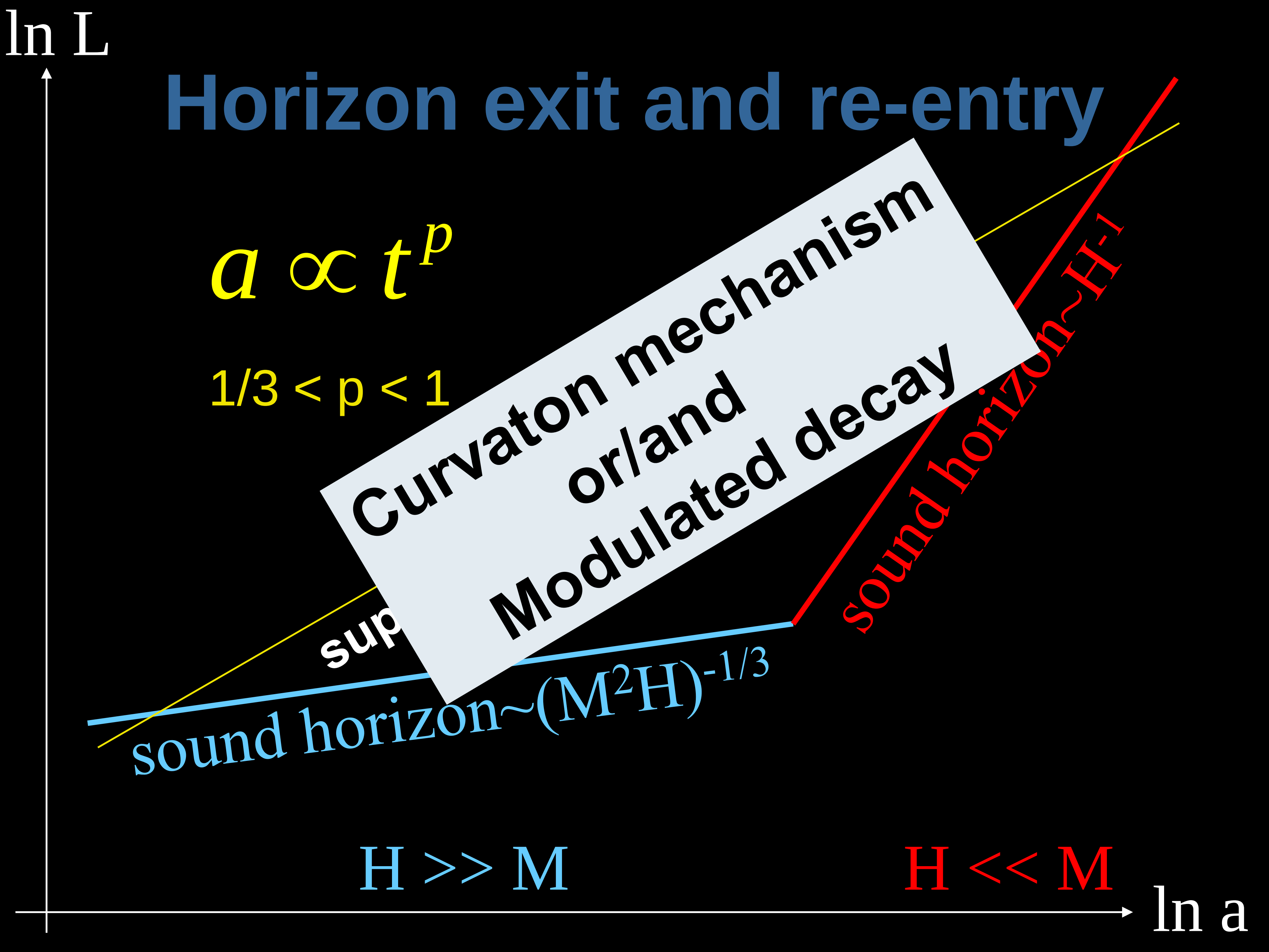
  $\phi \rightarrow b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$

  Scale-invariant fluctuations!

Horizon exit and re-entry



Horizon exit and re-entry



Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

Structure of GR

- 4D diffeomorphism → 4 constraints = 1 Hamiltonian + 3 momentum @ each time @ each point
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeormorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressueless dust.
- Microscopic lumps of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009])
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter" → Any astrophysical implications at collisions & cusps?

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- The lack of local Hamiltonian constraint leads to "dark matter" as an integration constant.

Open problems

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ?
- Embedding into an unified theory : can we get a common “limit of speed” ?
- Are there vDVZ discontinuity and Vainshtein effect? Unlike massive gravity case, Vainshtein effect can be trusted because of “UV completeness”. (work in progress with K.Izumi and K.Takahashi)
- How to setup initial condition for “dark matter”?
- Spectral tilt from anomalous dimension?
- Can we solve the flatness problem without inflation?
- …
- There are many things to do!

Spare slides

A free scalar field (I)

$$I = \frac{1}{2}\int dt d^3\vec{x} a^3 N\sqrt{q}\left[\frac{1}{N^2}\left(\partial_t\Phi - N^i\partial_i\Phi\right)^2 + \Phi\mathcal{O}\Phi\right]$$

$$\mathcal{O} = \underbrace{\frac{1}{M^4}\Delta^3}_{\text{UV: z=3}} - \frac{\lambda}{M^2}\Delta^2 + \underbrace{\Delta - m^2}_{\text{IR: z=1}}$$

FRW background with H >> M

$$I_{UV} = \frac{1}{2}\int d\eta d^3\vec{x}\left[a^2(\partial_\eta\delta\Phi)^2 + \frac{1}{M^4a^2}\delta\Phi(\delta^{ij}\partial_i\partial_j)^3\delta\Phi\right]$$

$$(\delta\Phi_1, \delta\Phi_2)_{KG} = -i\int d\vec{x}^3 a^2\left(\delta\Phi_1\partial_\eta\delta\Phi_2^* - \delta\Phi_2^*\partial_\eta\delta\Phi_1\right)$$

A free scalar field (II)

Normalized mode function

$$\phi_{\vec{k}} = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^3} \times 2^{-1/2} k^{-3/2} M \exp\left(-i\frac{k^3}{M^2}\int \frac{d\eta}{a^2}\right)$$

for $a \propto t^p, \quad p > 1/3$

$$\int^{\eta_\infty} \frac{d\eta}{a^2} = \int^{t_\infty} \frac{dt}{a^3}$$ converges

$\phi_{\vec{k}}$ initially oscillates and freezes @ $\omega^2 \sim H^2$

Power spectrum

$$\mathcal{P}^{1/2}_{\delta\Phi} = \sqrt{\frac{k^3}{2\pi^2}}\left|(2\pi)^3 \phi_{\vec{k}}\right| = \frac{M}{2\pi}$$

independent of H and scale-invariant!

FRW spacetime in GR

- $ds^2 = - dt^2 + a^2(t)\,(dx^2 + dy^2 + dz^2)$
- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- Hamiltonian constraint
  → Friedmann eq

  $$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} \rho_i$$

  E.o.m. of matter
  → conservation eq.

  $$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  is not independent
  but follows from the above n+1 eqs.

  $$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No "local" Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.
  $$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$
- Dynamical eq
  $$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$
  can be integrated to give Friedmann eq with "dark matter as integration constant"
  $$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

More general case

- General solution to the momentum constraint and dynamical eq.

$$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} + O(\lambda - 1)$$
$$+ \text{(higher curvature corrections)}$$
$$= 8\pi G_N \left(T_{\mu\nu} + \rho^{HL} n_\mu n_\nu \right)$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu} + O(\lambda - 1)$$
$$+ \text{(higher curvature corrections)}$$

Black holes with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame
Doran coordinate

- Relativistic star with $\lambda > 1$ and $\lambda \rightarrow 1$
work in progress with K.Izumi and K.Takahashi