Higher Spin 3d Gravity: Beyond AdS

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Work in Progress, MG, Grumiller, Perlmutter
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JHEP 1211 (2012) 099 [1209.2860] HA, MG, DG, RR, MR

Der Wissenschaftsfonds.

Outline

Motivation

- Black hole evaporation and unitarity
- Role of geometry in quantum gravity
- Microscopic description of black hole entropy?
- One theory in which semiclassical and quantum regime can both be understood?
- Holography beyond AdS?
- Null $WAdS_3$ as near-horizon extreme Kerr, dual to consistent chiral CFT?

Gravity in 3D with $\Lambda \leq 0$

$$I = \frac{1}{16\pi G_N} \int \sqrt{-g}\,(R+2)$$

- One dimensionless coupling G_N (in units of curvature radius)
- Vacuum solution: Anti de-Sitter space (AdS)
- Admits black holes with finite horizon size [BTZ92]
- (Boundary) Gravitons

First Order Formalism

- Dreibein e_μ^a and spin connection ω_μ^{ab} independent variables
- 3D trick: use invariant antisymmetric rank 3 symbol ϵ^{abc} to construct ω_μ^a
- Action becomes a difference of two $\mathfrak{sl}_2$ Chern-Simons theories [AT86, Wit88]

$$I = \frac{k}{4\pi}\left(\int \mathrm{tr}\left[\mathrm{A}\wedge \mathrm{dA} + \frac{2}{3}\mathrm{A}^3\right] - \int \mathrm{tr}\left[\bar{\mathrm{A}}\wedge \mathrm{d}\bar{\mathrm{A}} + \frac{2}{3}\bar{\mathrm{A}}^3\right]\right)$$

where $A = \frac{1}{2}(\omega + e), \bar{A} = \frac{1}{2}(\omega - e)$, $k = \frac{1}{4G_N}$

- $g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[(\mathrm{A}-\bar{\mathrm{A}})_\mu(\mathrm{A}-\bar{\mathrm{A}})_\nu\right]$

First Order Formalism: Gauge Symmetries

- Gauge symmetries

$$\delta_\epsilon A = d\epsilon + [A, \epsilon] \qquad \delta_{\bar\epsilon} \bar{A} = d\bar\epsilon + \left[\bar{A}, \bar\epsilon\right]$$

- $\epsilon - \bar\epsilon$ generate local translations (diffeomorphisms)
- $\epsilon + \bar\epsilon$ generate local Lorentz transformations

$$\delta_{\epsilon-\bar\epsilon} e = d(\epsilon - \bar\epsilon) + [\omega, \epsilon - \bar\epsilon] \qquad \delta_{\epsilon-\bar\epsilon}\omega = [e, \epsilon - \bar\epsilon]$$

$$\delta_{\epsilon+\bar\epsilon} e = [e, \epsilon + \bar\epsilon] \qquad \delta_{\epsilon+\bar\epsilon}\omega = d(\epsilon + \bar\epsilon) + [\omega, \epsilon + \bar\epsilon]$$

Canonical Analysis

- Imposing appropriate boundary conditions → gravity in asymptotically AdS_3

$$ds^2 = d\rho^2 + \left(e^{2\rho}\eta_{\mu\nu} + \mathcal{O}(1)\right) dx^\mu dx^\nu$$

- Canonical Analysis [BH86], originally in 2nd order formalism
- Later repeated in 1st order formalism, same result, but simpler to understand

Connection and Boundary Conditions

- Partially gauge fix $A = g^{-1} a g + g^{-1} dg$, $\bar{A} = g \bar{a} g^{-1} + g dg^{-1}$ with $g = e^{\rho L_0}$
- Split connection into background, state-dependent fluctuations, and state-independent (subleading in ρ) fluctuations $a = \hat{a}^{(0)}(t,\phi) + a^{(0)}(t,\phi) + a^{(1)}(\rho, t, \phi)$
- It is necessary [convenient] for $\hat{a}^{(0)}$ [$a^{(0)}$] to satisfy the asymptotic equations of motion $F = 0 = \overline{F}$

Boundary Conditions for Asymptotically AdS$_3$

$$\hat{a}^{(0)} = L_1 dx^+ \qquad \hat{\bar{a}}^{(0)} = -L_{-1} dx^-$$

$$a^{(0)} = \mathcal{L}(x^+) L_{-1} dx^+ \qquad \bar{a}^{(0)} = \overline{\mathcal{L}}(x^-) L_1 dx^-$$

$$a^{(1)} = \mathcal{O}(e^{-\rho}) \qquad \bar{a}^{(1)} = \mathcal{O}(e^{-\rho})$$

Canonical Analysis of CS Theories I: Hamiltonian

- Convenient to use a $2+1$ decomposition

$$I_{CS}[A] = \frac{k}{4\pi}\int_{\mathbb{R}} dt \int_{\mathcal{D}} d^2x \epsilon^{ij}\kappa_{ab}\left(\dot{A}_i^a A_j^b + A_0^a F_{ij}^b\right)$$

- Canonical momenta π_a^μ generate primary constraints φ_a^μ

$$\varphi_a^0 := \pi_a^0 \approx 0 \qquad\qquad \varphi_a^i := \pi_a^i - \frac{k}{4\pi}\epsilon^{ij}\kappa_{ab}A_j^b \approx 0$$

- Total Hamiltonian density $\mathcal{H}_T = -\frac{k}{4\pi}\epsilon^{ij}\kappa_{ab}A_0^a F_{ij}^b + u_\mu^a \varphi_a^\mu$
- Conservation of the primary constraints $\dot{\varphi}_a^\mu = \{\varphi_a^\mu, \mathcal{H}_T\} \approx 0$ leads to secondary constraints

$$\mathcal{K}_a := -\frac{k}{4\pi}\epsilon^{ij}\kappa_{ab}F_{ij}^b \approx 0 \qquad\qquad D_i A_0^a - u_i^a \approx 0$$

Canonical Analysis of CS Theories II: Charges

- Let $\overline{\mathcal{K}}_a = \mathcal{K}_a - D_i\varphi^i_a$. Then total Hamiltonian density expressed as sum of constraints

$$\mathcal{H}_T = A^a_0 \overline{\mathcal{K}}_a + u^a_0 \varphi^0_a$$

- φ^0_a, $\overline{\mathcal{K}}_a$ are first class, φ^i_a are second class
- Construct gauge generators via Castellani's algorithm.

$$\widetilde{\mathcal{G}}\left[\epsilon\right] = \int_{\mathcal{D}} d^2x \left(D_0\epsilon^a \pi^0_a + \epsilon^a \overline{\mathcal{K}}_a\right)$$

- Demanding functional differentiability determines the charges $\delta\mathcal{G}\left[\epsilon\right] = \delta\widetilde{\mathcal{G}}\left[\epsilon\right] + \delta\mathcal{Q}\left[\epsilon\right]$

$$\delta\mathcal{Q}\left[\epsilon\right] = \frac{k}{2\pi}\oint_{\partial\mathcal{D}} d\phi \operatorname{tr}\left(\epsilon\delta \mathrm{A}_\phi\right)$$

Asymptotic Symmetry Algebra

- Poisson Bracket Algebra (barred sector is identical)

$$\{\mathcal{L}(\theta), \mathcal{L}(\theta')\} = \delta(\theta - \theta')\mathcal{L}'(\theta') - 2\delta'(\theta - \theta')\mathcal{L}(\theta') - \frac{k}{4\pi}\delta^{(3)}(\theta - \theta')$$

- Quantizing $\rightarrow$ 2 copies of Virasoro Algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{k}{2}(n^3 - n)\delta_{n+m,0}$$

- $c_L = c_R = 6k$
- Boundary gravitons are descendents of the vacuum created by $L_{-n_1} \cdots L_{-n_m} |0\rangle$ with $n_i > 1$

Dual CFT and Unitary Models

Requirements for Unitary CFT (partial list)

- Central charge $c > 0$
- Modular invariant partition function

- $c = 1/2$ Ising Model
- $c = 7/10$ Tricitical Ising Model
- Only examples, no unitary semiclassical models ($c \gg 1$) [CGH+12]

Need an alternative theory with more allowed values of c

Generalization to Higher Spins

- Enlarge $\mathfrak{sl}_2$ to $\mathfrak{sl}_N$ (or other gauge group containing $\mathfrak{sl}_2$)
- Choice of embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_N$ determines other field content
- Spins of other field content given by weight under gravitational $\mathfrak{sl}_2$ action
- Number of embeddings grows exponentially with N
- Typical choice: Principal embedding, integer spins $2 \ldots N$

$$g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[(\mathrm{A} - \bar{\mathrm{A}})_\mu (\mathrm{A} - \bar{\mathrm{A}})_\nu\right]$$

$$\phi_{\mu\nu\rho} = \frac{1}{3!}\mathrm{tr}\left[(\mathrm{A} - \bar{\mathrm{A}})_{(\mu} (\mathrm{A} - \bar{\mathrm{A}})_\nu (\mathrm{A} - \bar{\mathrm{A}})_{\rho)}\right]$$

$$\vdots$$

Asymptotic Symmetry Algebra: Principal Embedding

- Boundary conditions

$$\hat{a}^{(0)} = L_1 dx^+$$

$$a^{(0)} = \left(\mathcal{L}(x^+)L_{-1} + \sum_{n=2}^{N-1} \mathcal{W}_n(x^+)W^n_{-n}\right) dx^+$$

$$a^{(1)} = \mathcal{O}(e^{-\rho})$$

$$\hat{\bar{a}}^{(0)} = -L_{-1} dx^-$$

$$\bar{a}^{(0)} = \left(\overline{\mathcal{L}}(x^-)L_1 + \sum_{n=2}^{N-1} \overline{\mathcal{W}}_n(x^-)W^n_n\right) dx^-$$

$$\bar{a}^{(1)} = \mathcal{O}(e^{-\rho})$$

- ASA: two copies of W_n algebra with central charges $c_L = c_R = 6k$

Unitary Representations: Principal Embedding

- Possible unitary representations again (partially) classified by [CGH^{+}12]
- $c = 4/5$ Potts Model
- $c = 6/7$ Tricritical Potts Model
- $c = 2\frac{N-1}{N+2}, N \in \{5, 6, 7, 8\}$ Parafermions

Again, no semi-classical limit $c \gg 1$ allowed. What about non-Principal embeddings?

No-Go Theorem

- No-Go Theorem for Unitary representations in the limit $k \to \infty$ [CHLJ12]
- All non-principal embeddings include a singlet, which leads to a Kac-Moody algebra as part of the ASA

$$[J_n, J_m] = \kappa n \delta_{n+m,0} + \cdots$$

- Unitarity requires $\kappa \geq 0$
- Unitary also requires central charge $c \geq 0$ for Virasoro algebra (from $\mathfrak{sl}_2$)

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$

- In the limit $|c| \to \infty$, $\mathrm{sign}(c) = -\mathrm{sign}(\kappa)$.

No-Go Guides The Way

- Work at finite (but possibly large) central charge c
- Next-to-principal ($W_N^{(2)}$) higher spin gravity
- For a given value of N, allows a discrete spectrum of unitary represenations
- Central charge c ranges from $\mathcal{O}(1)$ to $\mathcal{O}(N/4)$
- For AdS boundary conditions, ASA is two copies of Feigin-Semikhatov algebra $W_N^{(2)}$
- For Lobachevsky boundary conditions, ASA is $W_N^{(2)} \times \hat{\mathfrak{u}}(1)$

Boundary Conditions for $\mathfrak{sl}_3$

- Use non-principal embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_3$
- Generators $L_{-1,0,1}$, $G^{\pm}_{\pm\frac{1}{2}}$, S
- AdS Boundary Conditions

$$\hat{a}^{(0)} = -\frac{1}{4} L_1 dx^+$$

$$a^{(0)} = \frac{2\pi}{k}\left(\mathcal{J}(x^+)S + \mathcal{G}^{\pm}(x^+)G^{\pm}_{-\frac{1}{2}} + \mathcal{L}(x^+)L_{-1}\right) dx^+$$

$$\hat{\bar{a}}^{(0)} = -L_{-1} dx^-$$

$$\bar{a}^{(0)} = \frac{2\pi}{k}\left(\overline{\mathcal{J}}(x^-)S + \overline{\mathcal{G}}^{\pm}(x^-)G^{\pm}_{\frac{1}{2}} + \overline{\mathcal{L}}(x^-)L_1\right) dx^-$$

$$a^{(1)} = \mathcal{O}\left(e^{-\rho}\right) = \bar{a}^{(1)}$$

Example: Polyakov-Bershadsky Algebra $W_3^{(2)}$

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0} \\
[J_n, J_m] &= \kappa n \delta_{n+m,0} \\
[J_n, L_m] &= nJ_{n+m} \qquad \left[J_n, G_m^{\pm}\right] = \pm G_{n+m}^{\pm} \\
\left[G_n^+, G_m^-\right] &= \frac{\lambda}{2}(n^2 - \frac{1}{4})\delta_{n+m,0} + \dots \\
\left[L_n, G_m^{\pm}\right] &= \left(\frac{n}{2} - m\right) G_{n+m}^{\pm}
\end{aligned}
$$

- Level $\kappa = \frac{2k+3}{3}$
- Central charge $c = 25 - \frac{24}{k+3} - 6(k+3)$
- $G^{\pm}$ central term $\lambda = (k+1)(2k+3)$

Example: Polyakov-Bershadsky Algebra $W_3^{(2)}$

- Like $\mathcal{N} = 2$ Superconformal Algebra, but with commuting $G^\pm$
- Leads to negative norm descendents of the vacuum if $G^\pm$ generate physical states
- $G^\pm$ must be null for unitary theory $\Rightarrow \lambda = 0$
- No other restrictions on unitarity

Unitary Representations of $W_3^{(2)}$

$c = 0$ trivial theory

$c = 1$ theory of $\hat{\mathfrak{u}}(1)$ current algebra

Feigin-Semikhatov Algebra $W_N^{(2)}$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$
$$[J_n, J_m] = \kappa n \delta_{n+m,0}$$
$$[J_n, L_m] = nJ_{n+m} \qquad \left[J_n, G_m^{\pm}\right] = \pm G_{n+m}^{\pm}$$
$$\left[G_n^{+}, G_m^{-}\right] = \lambda f(n)\delta_{n+m,0} + \dots$$
$$\left[L_n, G_m^{\pm}\right] = \left(n(\frac{N}{2} - 1) - m\right) G_{n+m}^{\pm}$$

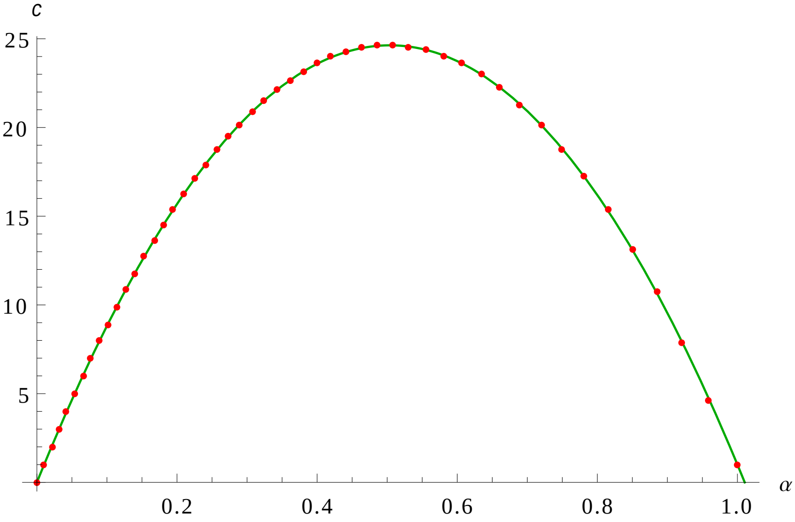
- $G^{\pm}$ central term

$$\lambda = \prod_{m=1}^{N-1} (m(N+k-1)-1)$$

Unitarity requires $\lambda = 0$

Unitary Represenations of $W_{100}^{(2)}$

Allowed values of central charge c for $N = 100$



Unitary Represenations of $W_N^{(2)}$

- Demaning unitarity $\rightarrow$ Newton's constant automatically quantized
- Critical values $\alpha = \frac{\hat{N}}{N-\hat{N}-1}$ where $\hat{N} \in \mathbb{N}, \hat{N} \leq \frac{N-1}{2}$
- Allowed values of central charge (let $m = N - 2\hat{N} - 1$)

$$c(\hat{N}, m) - 1 = (\hat{N} - 1)\left(1 - \frac{\hat{N}(\hat{N}+1)}{(m+\hat{N})(m+\hat{N}+1)}\right)$$

- Exactly values of central charge $W_{\hat{N}}$ minimal models, up to shift by 1 due to $\hat{\mathfrak{u}}(1)$ current algebra
- Small α quantum regime with $c \sim \mathcal{O}(1)$
- Intermediate α semiclassical regime with $c \sim \mathcal{O}(\frac{N}{4})$
- $\alpha \sim \mathcal{O}(1)$ dual quantum regime with $c \sim \mathcal{O}(1)$

Lobachevsky Boundary Conditions

- Background metric

$$ds^2 = d\rho^2 + dt^2 + e^{2\rho} d\phi^2$$

- Boundary Conditions for non-principal embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_3$

$$\hat{a}^{(0)} = -\frac{1}{4} L_1 d\phi$$

$$a^{(0)} = \frac{2\pi}{k} \left(\mathcal{J}(\phi) S + \mathcal{G}^{\pm}(\phi) G^{\pm}_{-\frac{1}{2}} + \mathcal{L}(\phi) L_{-1} \right) d\phi$$

$$\hat{\bar{a}}^{(0)} = -L_{-1} d\phi + \sqrt{3} S dt$$

$$\bar{a}^{(0)} = \frac{2\pi}{k} \overline{\mathcal{J}}(\phi) S d\phi$$

$$a^{(1)} = \mathcal{O}\left(e^{-\rho}\right) = \bar{a}^{(1)}$$

Asymptotic Symmetry Algebra

- Unbarred sector is Polyakov-Bershadsky Algebra $W_3^{(2)}$
- Barred sector is $\hat{\mathfrak{u}}(1)$ current algebra at level κ
- Straightforward generalization to $W_N^{(2)} \times \hat{\mathfrak{u}}(1)$

Null Warped AdS

- Metric

$$ds^2 = d\rho^2 + 2e^{2\rho}dtd\phi + e^{4\rho}d\phi^2$$

- Near-horizon geometry of extreme Kerr
- Conjectured to be dual to chiral CFT
- Previously known constructions required artificial truncations, non-unitarity, or fine-tuning

Spin 3 Boundary Conditions

- Work in principal embedding (generators $L_{-1,0,1}$, $W_{-2,\dots,2}$)
- Boundary Conditions

$$\hat{a}^{(0)} = (L_1 - W_2)d\phi$$

$$a^{(0)} = \left(\ell_{-1}(\phi)L_{-1} + \sum_{n=-2}^{1} w_n(\phi)W_n\right) d\phi$$

$$a^{(1)} = \mathcal{O}(e^{-\rho})$$

$$\hat{\bar{a}}^{(0)} = W_{-2}d\phi + L_{-1}dt$$

$$\bar{a}^{(0)} = \beta(\phi)L_{-1}d\phi$$

$$\bar{a}^{(1)} = \mathcal{O}(e^{-\rho})$$

Asymptotic Symmetry Algebra

- Asymptotic charges

$$\mathcal{Q}\left[\epsilon\right] = \oint d\phi \left(\mathcal{L}\epsilon_{L_1} + \mathcal{W}^2\epsilon_{W_2} + \mathcal{W}^1\epsilon_{W_1} + \mathcal{W}^0\epsilon_{W_0}\right)$$
$$\overline{\mathcal{Q}}\left[\bar{\epsilon}\right] = 0$$

- w_1, β are pure gauge
- Barred sector has vanishing asymptotic charge, so theory is chiral
- ASA is a single copy of the Polyakov-Bershadsky Algebra $W_3^{(2)}$

Open Issues and Ongoing Research

- Existence of modular invariant partition function for $W_N^{(2)}$ gravity?
- Do other non-principal embeddings admit unitary representations?
- Extension of Null Warped construction to spin greater than 3 (WIP)
- Holonomy analysis of Null Warped black holes (WIP)
- Gauge invariance vs. Geometry

Thank You

References

A. Achucarro and P.K. Townsend.
A Chern-Simons Action for Three-Dimensional anti-De Sitter Supergravity Theories.
Phys.Lett., B180:89, 1986.

J. David Brown and M. Henneaux.
Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity.
Commun.Math.Phys., 104:207–226, 1986.

Maximo Banados, Claudio Teitelboim, and Jorge Zanelli.
The Black hole in three-dimensional space-time.
Phys.Rev.Lett., 69:1849–1851, 1992.

Alejandra Castro, Matthias R. Gaberdiel, Thomas Hartman, Alexander Maloney, and Roberto Volpato.
The Gravity Dual of the Ising Model.
Phys.Rev., D85:024032, 2012.

Alejandra Castro, Eliot Hijano, and Arnaud Lepage-Jutier.
Unitarity Bounds in AdS3 Higher Spin Gravity.
JHEP, 1206:001, 2012.

Edward Witten.
(2+1)-Dimensional Gravity as an Exactly Soluble System.
Nucl.Phys., B311:46, 1988.