# SUSY exclusive analyses 

Giacomo Polesello

INFN, Sezione di Pavia

## Introduction

Tommaso's talk: how LHC experiment will try to discover RP conserving SUSY A certain number of generic assumptions:

- Detection through discovery of squark and gluino production
- Squark and gluino decay to jets + some kind of $S U(2) \times U(1)$ gaugino/higgsino
- Mass difference between squark/gluino and gauginos with dominant BR such as to yield high $p_{T}$ jets. More or less guaranteed in case of gluino accessible and gaugino mass unification
- Gauginos will decay into "something" and finally into an invisible LSP

Searches are therefore: 2 to 4 jets, depending on relation between gluino and squark masses $+\mathbb{E}_{T}+$ "something"

Examples of "something": nothing, 1,2,3 leptons $(e, \mu) \tau$ (hadronic), $b$-jets, $Z, h$ Generic variables: $P_{T} / \eta$ of ingredients + estimator of mass of system. Canonically: $M_{\text {eff }}=\Sigma_{i}\left|p_{T(i)}\right|+E_{T}^{\text {miss }}$

## How generic?

Typically reach shown on mSUGRA plane (to fix the "something"), but shown to cover other $\tilde{\chi}_{1}^{0}$ LSP scenarios e.g NUHM (ATLAS Tokyo, N. Kanaya, S. Asai) Will also e.g. cover most cases in GMSB (gravitino LSP)

- NLSP is $\tilde{\chi}_{1}^{0} \rightarrow$. If long lived as for mSUGRA (checked as well by ATLAS Tokyo group). If short lived: add photons to the "something" If medium lived (decay inside the detector), discovery OK, need care to figure out photons
- NLSP is slepton/stau. If short lived OK, additional leptons in the "something".

If long-lived need detector-specific studies
Specific searches for cases where assumption of accessible squarks/gluino breaks:

- light stops
- direct gaugino/higgsino search in 3-lepton channel
- long lived heavy particles (staus or R-hadrons)

Also cases with very degenerate spectra need attention (see talk this week)

## Light stops

Search for direct $\tilde{t}_{1} \tilde{t}_{1}$ production in models with stop lighter than top Model proposed in Les Houches 2005, compatible with CDM and baryogenesis:
$m(\tilde{q})=m(\tilde{\ell})=10 \mathrm{TeV}$, and $M_{1}: M_{2}=\alpha_{1}: \alpha_{2}$
$M_{1}=60.5 \mathrm{GeV} \quad \mu=400 \mathrm{GeV} \quad \tan \beta=7 \quad M_{3}=950 \mathrm{GeV}$
$m\left(Q_{3}\right)=1500 \mathrm{GeV} \quad m\left(\tilde{t}_{R}\right)=0 \mathrm{GeV} \quad m\left(\tilde{b}_{R}\right)=1000 \mathrm{GeV} \quad A_{t}=-642.8 \mathrm{GeV}$
Masses: $m\left(\tilde{t}_{1}\right)=137 \mathrm{GeV}, m\left(\tilde{\chi}_{1}^{ \pm}\right)=111 \mathrm{GeV}, m\left(\tilde{\chi}_{1}^{0}\right)=58 \mathrm{GeV}$.
$B R\left(\tilde{t}_{1} \rightarrow b \tilde{\chi}_{1}^{ \pm} \rightarrow b W^{*} \tilde{\chi}_{1}^{0}\right)=100 \% \Rightarrow$ Signature as for $t \bar{t}:$
$2 b$-jets, $\mathbb{H}_{T}$ and either $2 \ell(e, \mu)(4.8 \% \mathrm{BR})$ or $1 \ell+2$ jets $(29 \% \mathrm{BR})$
$\sigma\left(p p \rightarrow \tilde{t}_{1} \tilde{t} 1\right)=412 \mathrm{pb}($ CTEQ5L NLO Prospino) Consider semi-leptonic channel for analysis

Perform ATLFAST analysis on $\sim 2 \mathrm{fb}^{-1}$. ATLAS full simulation study ongoing
Consider backgrounds from $t \bar{t}$ and from Wbbjj production
Case where $m\left(\tilde{t}_{1}\right)<m\left(\tilde{\chi}_{1}^{ \pm}\right)$much more difficult: either $\tilde{t}_{1} \rightarrow c \tilde{\chi} 01$ or 4-body decay. First one impossible, second one still to be studied (MC generator)

## Event selection

- One and only one isolated lepton $(e, \mu), p_{T}^{l}>20 \mathrm{GeV}$.
- At least four jets $P_{T}\left(J_{1}, J_{2}\right)>35 \mathrm{GeV}, P_{T}\left(J_{3}, J_{4}\right)>25 \mathrm{GeV}, E_{T}^{\text {miss }}>20 \mathrm{GeV}$.
- Exactly two $b$-tagged jets with $p_{T}>20 \mathrm{GeV}$. Assume $\epsilon_{b}=60 \%, R_{j}=100$

Consider $m(j j)_{\text {min }}$ minimum invariant mass of any non-b jets in event with $p_{T}>25 \mathrm{GeV}$ for signal and ttbar. $m(j j)_{\min }<m\left(\chi^{ \pm}\right)-m\left(\tilde{\chi}_{1}^{0}\right)$ for signal



By requiring $m(j j)<60 \mathrm{GeV}$, Achieve $\mathrm{S} / \mathrm{B}=1 / 10$ for signal eff. of $\sim 0.24 \%$

## The $m(b j j)_{\text {min }}$ variable

Define $m(b j j)_{\text {min }}$ as the minimum invariant mass for the combination a $b$-tagged jet and the two non-b jets.

The end-point of the $m(b j j)_{\text {min }}$ distribution should be $m\left(\tilde{t}_{1}\right)-m\left(\tilde{\chi}_{1}^{0}\right)=79 \mathrm{GeV}$ for stop and 175 GeV for top



Clear shoulder from stop, need precise determination of shape for top to see it An equivalent variable $\left(m(b l)_{\min }\right)$ can be built on the lepton side

## Data-driven background subtraction

Define a pure top control sample: require that on the lepton side $m(l b \nu)$ compatible with top mass within 15 GeV .

Reduce signal contamination by requiring $m(l b)_{\min }>60 \mathrm{GeV}$
No signal in sample for $m(b j j)_{\min }>80 \mathrm{GeV}$, can use this region for normalisation
Control sample (points with errors) matches well distribution for top



After subtracting $m(b j j)_{\text {min }}$ for normalised top control sample from $m(b j j)_{\text {min }}$ for "data", reproduce the distribution for signal

## SUSY mass scale from inclusive analysis

Start from multijet $+\mathbb{E}_{T}$ signature.
Simple variable sensitive to sparticle mass scale:

$$
M_{\mathrm{eff}}=\sum_{i}\left|p_{T(i)}\right|+E_{T}^{\mathrm{miss}}
$$

where $p_{T(i)}$ is the transverse momentum of jet $i$
 $M_{\text {eff }}$ distribution for signal (red) and background (brown) $\left(m S U G R A m_{0}=100 \mathrm{GeV}, m_{1} / 2=300 \mathrm{GeV}, \tan \beta=10\right.$, $A=0, \mu>0$ )

A cut on $M_{\text {eff }}$ allows to separate the signal from SM background

The $M_{\text {eff }}$ distribution shows a peak which moves with the SUSY mass scale.

Define the SUSY mass scale as:

$$
M_{\text {susy }}^{\mathrm{eff}}=\left(M_{\text {susy }}-\frac{M_{\chi}^{2}}{M_{\text {susy }}}\right) \text {, with } M_{\text {SusY }} \equiv \frac{\Sigma_{i} M_{i} \sigma_{i}}{\Sigma_{i} \sigma_{i}}
$$




Estimate $M_{\text {eff }}$ peak by a gaussian fit to background-subtracted signal distributions Test the correlation of $M_{\text {eff }}$ with $M_{\text {susy }}^{\text {eff }}$ on random sets: mSUGRA and MSSM Excellent correlation in mSUGRA, less good for MSSM

Can one think of a variable (on $x$ or $y$ ) which gives better correlation for MSSM?


Evaluate uncertainty in mass scale from spread in correlation plots.

- $10 \mathrm{fb}^{-1}$ - stars
- $100 \mathrm{fb}^{-1}$ - open circles
- $1000 \mathrm{fb}^{-1}$ - filled circles
$\sim 10 \%$ precision on SUSY mass scale for one year at high luminosity

Needs to be updated with more precise background estimates and their systematic uncertainties

## What might we know after inclusive analyses?

Assume we have a MSSM-like SUSY model with $m_{\tilde{q}} \sim m_{\tilde{g}} \sim 600 \mathrm{GeV}$
Observe excesses in $\mathscr{H}_{T}+j$ jets inclusive, and in some of the $\mathscr{H}_{T}+j e t s+$
"something" channels. Null results in specialised searches

- Undetectable particles in the final state: $\mathbb{E}_{T}$. Stableor ling-lived?
- Primary particles with mass $\sim 600 \mathrm{GeV}$ ( $M_{\text {eff }}$ study)
- Assigning spin hypotheses to produced sparticles can get an idea of couplings (exp. difficulty: need some assumption on gaugino spectrum to evaluate selection efficiency)
- Many more things depending on the excesses observed for the different "something". Examples:
- Excess of of same-sign lepton pairs: some of the primary particles are Majorana
- See same number of leptons and muons: lepton flavour $\sim$ conserved in first two generations
- .............


## How can we use it?

Too little information to zoom into a model
Probably with guess the composition of the produced primary particles
One can exclude detailed implementations of model


Ex. in mSUGRA for each point one has different inclusive signatures, and one can compare observed and predicted relative rates

Already quite a few theoretical attempts in this direction, e.g. LHC Olympics
However, more detailed info can be extracted from the data

## What kind of info for establishing SUSY?

Long lists of requests. Need to demonstrate that:

- Every particle has a superpartner
- Their spin differ by $1 / 2$
- Their gauge quantum numbers are the same
- Their couplings are identical
- Mass relations predicted by SUSY hold

Available observables:

- Sparticle masses,
- Production cross-sections,
- BR's of cascade decays
- Angular decay distributions

Measurements of observables depends on detail of model and requires development of ad-hoc techniques. Over last ten years strategy based on detailed MC study of reasonable candidate models

## What path from the observbles to the model?

The problem is the presence of a very complex spectroscopy due to long decay chains, with crowded final states.

Many concurrent signatures obscuring each other

## General strategy:

- Select signatures identifying well defined decay chains
- Extract constraints on masses, couplings, spin from decay kinematics/rates
- Try to match emerging pattern to template models, SUSY or anything else
- Having adjusted template models to measurements, try to find additional signatures to discriminate different options

Most of work done on sparticle mass measurement
Show today some recent ATLAS full simulation work on some cases providing "easy" mass constraints

Discuss thereafter the broader landscape of mass measurement techniques

The easiest mass constraint: cascade of two two-body decays The problem: R-parity conservation $\Rightarrow$ two undetected LSP's per event $\Rightarrow$ no mass peaks, but invariant mass distributions can give constraints

For single 2-body decay $a \rightarrow b+c$, in $a$ rest frame

$$
|\vec{p}|^{2}=\left[m_{b}^{2}, m_{a}^{2}, m_{c}^{2}\right] \text { where }[x, y, z] \equiv \frac{x^{2}+y^{2}+z^{2}-2(x y+x z+y z)}{4 y}
$$

In rest frame of $b: \quad m_{p q}^{2}=m_{p}^{2}+m_{q}^{2}+2\left(E_{p} E_{q}-\left|\overrightarrow{p_{p}}\right|\left|\overrightarrow{p_{q}}\right| \cos \theta\right)$
Take the maximum $(\cos \theta=-1)$ and $p$ and $q$ massless (quarks or leptons)

$$
\left(m_{p, q}^{\max }\right)^{2}=4|\vec{p}||\vec{q}|=4 \sqrt{\left[0, m_{b}^{2}, m_{a}^{2}\right]} \sqrt{\left[0, m_{b}^{2}, m_{c}^{2}\right]}
$$

Substitute formula for $[x, y, z]: \quad\left(m_{p q}^{\max }\right)^{2}=\frac{\left(m_{c}^{2}-m_{b}^{2}\right)\left(m_{b}^{2}-m_{a}^{2}\right)}{m_{a}^{2}}$

## Invariant mass distribution

If spin of intermediate particle $b$ is zero, the decay distribution is:

$$
\frac{d P}{d \cos \theta}=\frac{1}{2}
$$

Where $\cos \theta$ is the angle between visible $p$ and $q$ in $b$ rest frame.
If $p, q$ are massless: $m_{p q}^{2}=2\left|\overrightarrow{p_{p}}\right|\left|\overrightarrow{p_{q}}\right|(1-\cos \theta) \quad$ and $\quad\left(m_{p q}^{\max }\right)^{2}=4\left|\overrightarrow{p_{p}}\right|\left|\overrightarrow{p_{q}}\right|$
Define the dimensionless variable:

$$
\hat{m}^{2}=\frac{m_{p q}^{2}}{\left(m_{p q}^{\max }\right)^{2}}=\frac{1}{2}(1-\cos \theta)=\sin ^{2} \frac{\theta}{2}
$$

By a changement of variable:

$$
\frac{d P}{d \hat{m}}=2 \hat{m}
$$



Show examples of ATLAS full simulation analyses where this distribution observable

## The lepton-lepton edge

ATLAS Point SU1:
$m_{0}=70 \mathrm{GeV}, m_{1 / 2}=350 \mathrm{GeV} A=0 \mathrm{GeV}, \tan \beta=10, \mu>0$
Both $\ell_{R}$ and $\ell_{L}$ lighter than $\tilde{\chi}_{2}^{0}$
Require two OSSF (Opposite-sign Same
Flavour leptons), jets and $\mathbb{E}_{T}$
Plot the flavour-subtracted invariant mass
OS-OF flavour-correlated signal from

$$
\begin{array}{rll}
\tilde{q}_{L} \rightarrow & \tilde{\chi}_{2}^{0} & q \\
& \longrightarrow & \tilde{\ell}_{R(L)}^{ \pm} \quad \ell^{\mp} \\
& & \longrightarrow \tilde{\chi}_{1}^{0} \quad \ell^{ \pm}
\end{array}
$$

(Flavour subtracted)


Can clearly observe after flavour subtraction the two edges for the two different slepton helicities

## Lepton-lepton edge: fast discovery

ATLAS Point SU3: bulk Point, SPS1a family $m_{0}=100 \mathrm{GeV}, m_{1 / 2}=300 \mathrm{GeV} A=-300 \mathrm{GeV}$, $\tan \beta=6, \mu>0$

Canonical decay $\tilde{\chi}_{2}^{0} \rightarrow \ell^{ \pm} \tilde{\ell}_{R}^{F} \rightarrow \ell^{ \pm} \ell^{\mp} \tilde{\chi}_{1}^{0}$
Signal visible with $1 \mathrm{fb}^{-1}$
Perhaps best chance for early discovery


ATLAS Point SU4: low mass

$$
m_{0}=200 \mathrm{GeV}, m_{1 / 2}=160 \mathrm{GeV} A=-400 \mathrm{GeV}
$$

$\tan \beta=10, \mu>0, \quad \sigma=262 \mathrm{pb}$
In this case 3-body decay of $\tilde{\chi}_{2}^{0}$ :

$$
\operatorname{BR}\left(\tilde{\chi}_{2}^{0} \rightarrow \ell^{ \pm} \ell^{\mp} \tilde{\chi}_{1}^{0}\right)=6.2 \%(\ell=e, \mu)
$$

## The lepton-jet sneutrino edge

Always Point SU1, study the decay $\tilde{q}_{L} \rightarrow q \tilde{\chi}_{1}^{ \pm} q \ell^{ \pm} \tilde{\nu}_{l} \rightarrow q \ell^{ \pm} \nu_{l} \tilde{\chi}_{1}^{0}$
Lepton-Jet invariant mass should present an edge depending on $m\left(\tilde{\chi}_{1}^{ \pm}\right)$, $m\left(\tilde{\nu}_{\ell}\right) m\left(\tilde{\chi}_{1}^{0}\right)$
Very difficult to find the correct jet-lepton pairing $\rightarrow$ combinatorial
Hard cut on leading and second leading jet and $\mathbb{E}_{T}: P_{T}(j)>200 \mathrm{GeV}, \mathbb{E}_{T}>200 \mathrm{GeV}$
Exclude events for which $60<M_{T}\left(\ell, E_{T}^{\text {miss }}\right)<100 \mathrm{GeV}$ to reduce top background

Mixed-event technique to estimate combinatorial: randomly pairwith lepton jets from a different event satisfying same event selection

After subtraction of 'mixed event' sample recover triangular shape. End-point in expected position

Technique works in this case, need to study how general is the result


## The top-bottom edge

Work on Point SU4: $\operatorname{BR}\left(\tilde{g} \rightarrow \tilde{t}_{1} t\right)=42 \%, \sigma(\tilde{g} \tilde{g}+\tilde{g} \tilde{q}) \sim 165 \mathrm{pb}$
Study decay chain $\tilde{g} \rightarrow \tilde{t}_{1} t \rightarrow t b \tilde{\chi}_{1}^{ \pm}$
Channel previously studied by Hisano, Kawagoe, Nojiri in fast sim

Reconstruct fully hadronic top, and subtract $j j b$ combinatorial using sidebands


For this very low mass point, edge in principle visible with very little statistics In practice need really good understanding of detector to attack this channel

## Model independent mass determination

Shown realistic examples in which kinematic edges observed
Conforting to see that full simulation studies at low statistics tell us that these features may be observed beyond ATLFAST (no real life, though....). One step further is needed: extract from event kinematics absolute values for some of the masses.

Three families of techniques (and variations) proposed:

- Edge method (Many contributions: e.g. Paige, Hinchliffe, Lester, Miller, Osland)
- Mass relation method (Kawagoe, GP, Nojiri, Chen et al.)
- $M_{T 2}$ kink method (Lester, Barr, Cho et al.) method very recent and interesting: do not need long chains.

Significant recent advance, I will only comment on first two, as we have dedicated talk tomorrow on $M_{T 2}$ by one of proponents

## The edge method

With two decays only single mass combination $\Rightarrow$ only one edge constraint Consider longer chain. Key result (Paige, Hinchliffe):

If a chain of at least three two-body decays can be isolated, can measure masses and momenta of involved particles in model-independent way.

Example: full reconstruction of squark decays in models with light $\tilde{\ell}_{R}\left(m_{\tilde{\ell}_{R}}<m_{\tilde{\chi}_{2}^{0}}\right)$ :


Three visible particles: 4 invariant mass combinations: (12), (13), (23), (123)
For first three minimum value is zero: only $M_{\max }$ constraint. For fourth both $M_{\max }$ and $M_{\min }$ constraint: total 5 constraints

Complete results for $\tilde{q}_{L} \rightarrow \tilde{\ell} \ell$ decay chain: (Allanach et al. hep-ph/0007009)

$$
l^{+} l^{-} \operatorname{edge} \quad\left(m_{l l}^{\max }\right)^{2}=(\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi}) / \tilde{l}
$$

$$
l^{+} l^{-} q \operatorname{edge}\left(m_{M l_{q}}^{\max }\right)^{2}=(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi}) / \tilde{\xi}
$$



$l_{\text {near }}^{ \pm} q$ edge $\quad\left(m_{l_{\text {neara }}}^{\max }\right)^{2}=(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{l}) / \tilde{\xi}$
$l_{\text {farl }}^{ \pm} q \operatorname{edge} \quad\left(m_{\operatorname{la} q(y)}^{\max }\right)^{2}=(\tilde{q}-\tilde{\xi})(\tilde{l}-\tilde{\chi}) / \tilde{l}$

With $\tilde{\chi}=m_{\tilde{\chi_{0}},}^{2}, \quad \tilde{l}=m_{\tilde{I}_{R}}^{2}, \quad \tilde{\xi}=m_{\hat{\chi}}^{2}, \quad \tilde{q}=m_{\tilde{q}}^{2}$

## Example: Point SPS1a

## Snowmass Point 1

$m_{0}=100 \mathrm{GeV}, m_{1 / 2}=250 \mathrm{GeV} A=-100 \mathrm{GeV}$,
$\tan \beta=10, \mu>0$
Total cross-section: $\sim 50 \mathrm{pb}, \mathrm{BR}\left(\tilde{\chi}_{2}^{0} \rightarrow \tilde{\ell}_{R} \ell\right)=12.6 \%$ SPA: similar point, compatible with WMAP: $m_{0}=70 \mathrm{GeV}, m_{1 / 2}=250 \mathrm{GeV} A=-300 \mathrm{GeV}$,
$\tan \beta=10, \mu>0$



Lepton-lepton edge
Select events with high jet multiplicity and $\mathscr{F}_{T}$
Require two opposite-sign same-flavour $e, \mu$ (OSSF)
SUSY background: uncorrelated $\tilde{\chi}_{1}^{ \pm}$decays
Subtract SUSY and SM background via flavour correlation:
$e^{+} e^{-}+\mu^{+} \mu^{-}-e^{ \pm} \mu^{\mp}$
Fit to sharp edge + Gaussian smearing




Distributions fall ~linearly to end point. Shapes modified by resolutions and backgrounds, recently progress in using full shape

Statistical uncertainty from linear fit at the \% level

5 edge constraints: generate MC experiments as sets of edge measurements normal distributed according to estimated errors

For each set solve numerically system of equations for sparticle masses.

## Results and limitations of Edge method

Strong correlation, kinematic constraints of the form $\left(m_{a}^{2}-m_{b}^{2}\right)\left(m_{b}^{2}-m_{c}^{2}\right) / m_{b}^{2}$, measure mass differences rather than absolute scale

Key contribution to scale measurement from threshold, most difficult one to measure

Error for $\tilde{\chi}_{1}^{0}, \tilde{\chi}_{2}^{0}, \tilde{\ell}_{R}$ masses $\sim 5 \mathrm{GeV}$
for $\tilde{q}_{L}$ mass $\sim 9 \mathrm{GeV}\left(300 \mathrm{fb}^{-1}\right)$
Mass differences measured to $\sim 250 \mathrm{MeV}$


Limitations of method based on kinematic edges:

- Only events near end-point are used: loss of information
- Different edge formulas depending on mass hierarchy: multiple solutions
- Need good statistics to observe end-point
- Unknown systematics from shape of edge distribution


## Mass relation method

Consider a chain of 4 two-body decays: e.g

$$
\tilde{g} \rightarrow \tilde{q} q_{2} \rightarrow \tilde{\chi}_{2}^{0} q_{1} q_{2} \rightarrow \tilde{\ell} q_{1} q_{2} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} q_{1} q_{2} \ell_{1} \ell_{2}
$$

for each event 5 constraints from mass-shell conditions of 5 sparticles:

$$
\begin{align*}
m_{\tilde{\chi}_{1}^{0}}^{2} & =p_{\tilde{\chi}_{1}^{0}}^{2}, \\
m_{\tilde{\ell}}^{2} & =\left(p_{\tilde{\chi}_{1}^{0}}+p_{\ell_{1}}\right)^{2}, \\
m_{\tilde{\chi}_{2}^{0}}^{2} & =\left(p_{\tilde{\chi}_{1}^{0}}+p_{\ell_{1}}+p_{\ell_{2}}\right)^{2}, \\
m_{\tilde{b}}^{2} & =\left(p_{\tilde{\chi}_{1}^{0}}+p_{\ell_{1}}+p_{\ell_{2}}+p_{b_{1}}\right)^{2}, \\
m_{\tilde{g}}^{2} & =\left(p_{\tilde{\chi}_{1}^{0}}+p_{\ell_{1}}+p_{\ell_{2}}+p_{b_{1}}+p_{b_{2}}\right)^{2} . \tag{1}
\end{align*}
$$

9 Unknowns: 4-mom of $\tilde{\chi}_{1}^{0}$ (different event by event) +5 masses (common)
For each event solve the system by eliminating the $\tilde{\chi}_{1}^{0} 4$-momentum
Solution is quadratic form in the space of sparticle masses:

$$
f\left(m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{\chi}_{2}^{0}}, m_{\tilde{\ell}_{R}}, m_{\tilde{\chi}_{1}^{0}}\right)=0
$$

Coefficients of quadratic form are functions of 4-momenta $q_{1}, q_{2}, \ell_{1}, \ell_{2}$

Intersection of 5 quadratic forms: point in 5-dim mass space 5 events enough in principle to measure masses of 5 sparticles

Simplified version for visualization Consider simple case in which all the sparticle masses are known except 2 : $m_{\tilde{g}}, m_{\tilde{q}}$

Quadratic form is a parabola in $\left(m_{\tilde{g}}, m_{\tilde{q}}\right)$ plane With two events have two parabolas Intersection of two parabolas gives two points, measurement of masses with twofold ambiguity


If larger number of events used, multiple solutions creduced to one
Need to develop a viable technique for finding maximally populated point in 5 -dim space in situation where momenta of measured quarks are smeared by detector Exploratory exercise: assume 3 lowest masses already measured, e.g. using the Edge method

## Practical application

Apply technique to measurement of gluino and sbottom mass in SPS1a

- Life easier than for light squarks: $b$-jets in chain minimize combinatorial BG
- Life harder as $b$ jets measured with worse resolution in detector
- $\tilde{g}$ decays to two $\tilde{b}: \tilde{b}_{1}, \tilde{b}_{2}$ and $m_{\tilde{b}_{2}}-m_{\tilde{b}_{1}} \sim 35 \mathrm{GeV}$ : benchmark sensitivity of method

Take into account smearing of measurement of momenta of $b$-partons: represent each event not as parabola, but as a probability density function in the $\left(m_{\tilde{g}}, m_{\tilde{b}_{1}}\right)$ plane: $\mathcal{L}\left(m_{\tilde{g}}, m_{\tilde{b}_{1}}\right)$ Main ingredient: knowledge of the response function of ATLAS detector to $b$ partons


Examples of $\mathcal{L}$ functions in $\left(m_{\tilde{g}}, m_{\tilde{g}}-m_{\tilde{b}_{1}}\right)$ plane for 3 random events

Combine likelihoods for all the events as:

$$
\log \mathcal{L}_{\text {comb }}\left(m_{\tilde{g}}, m_{\tilde{b}}\right) \equiv \sum_{\text {events }} \log \mathcal{L}\left(m_{\tilde{g}}, m_{\tilde{b}}\right)
$$



Good combined measurement of $m_{\tilde{g}}$ and $m_{\tilde{b}}$ even for low statistics $\tan \beta=20$ case Search for maximum probability indeed rejects multiple solutions

Possibly shoulder from second sbottom
We did not develop detailed estimator for final error on masses
Need anyway to feed in errors on assumed sparticle masses

## Variations on mass relation method

Can constrain masses if at least one constraint left over after 4-momenta of all invisible particles $\tilde{\chi}_{1}^{0}$ eliminated from system of constraints.

- 1 leg: $4 \tilde{\chi}_{1}^{0}$ unknowns $\Rightarrow$ need 5 constraints: 4 step chain

Example: $\tilde{g} \rightarrow \tilde{q} q_{2} \rightarrow \tilde{\chi}_{2}^{0} q_{1} q_{2} \rightarrow \tilde{\ell} q_{1} q_{2} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} q_{1} q_{2} \ell_{1} \ell_{2}$.

- 2 legs: $4 \times 2 \tilde{\chi}_{1}^{0}$ unknowns minus $2 E_{T}^{\text {miss }}$ constraint:
$p_{x}\left(\tilde{\chi}_{1}^{0}(1)\right)+p_{x}\left(\tilde{\chi}_{1}^{0}(2)\right)=E_{x}^{\text {miss }} \quad p_{y}\left(\tilde{\chi}_{1}^{0}(1)\right)+p_{y}\left(\tilde{\chi}_{1}^{0}(2)\right)=E_{y}^{\text {miss }}$,
$\Rightarrow$ need 7 constraints: 2 steps on one side, 3 steps on the other side
Example: side 1: $\tilde{q} \rightarrow \tilde{\chi}_{2}^{0} q_{1} \rightarrow \tilde{\ell} q_{1} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} q_{1} \ell_{1} \ell_{2}$ side 2: $\tilde{\chi}_{2}^{0} \rightarrow \tilde{\ell} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} \ell_{1} \ell_{2}$
Two-leg approaches:
Cheng et al.: Consider two legs with two steps ( 6 event unknowns and 6 constraints)
Assuming a given value for sparticle masses can calculate 4-momenta of two $\tilde{\chi}_{1}^{0}$.
Solution for all the events for limited region in 3-dim mass space (broad constraint on masses)
By scanning on ( $\mathrm{m} 1, \mathrm{~m} 2, \mathrm{~m} 3$ ) mass space, point in which sudden drop of number of events with solution happens is estimator of solution


## Mass relation on two legs (preliminary!)

Work in progress: consider two legs with three steps each:

$$
\tilde{q} \rightarrow \tilde{\chi}_{2}^{0} q_{1} \rightarrow \tilde{\ell} q_{1} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} q_{1} \ell_{1} \ell_{2} . \quad 6 \text { event unknowns/ } 8 \text { constraints }
$$

- Take SPS1a. Select events with 4 reconstructed leptons in parametrized smearing. No cuts.
- Preliminarly, assume that visible particle can be assigned unique position in chain
- Assume as unknowns $m(\tilde{q}), m\left(\tilde{\chi}_{2}^{0}\right), m\left(\tilde{\ell}_{R}\right)$. Calculate $m\left(\tilde{\chi}_{1}^{0}\right)$ from $m\left(\tilde{\chi}_{2}^{0}\right), m\left(\tilde{\ell}_{R}\right)$ and $\ell \ell$ edge
- Scan on ( $\left.m\left(\tilde{\chi}_{2}^{0}\right), m\left(\tilde{\ell}_{R}\right)\right)$ grid. For each event and leg calculate $m(\tilde{q})$ from on-shell constraints.
- $N$ solutions $(0,2,4)$ per legs $\Rightarrow$ for each event and leg $N$ surfaces in $\left(m\left(\tilde{\chi}_{2}^{0}\right), m\left(\tilde{\ell}_{R}\right), m(\tilde{q})\right)$ space


Represent surfaces as hits in
3-d histograms
Sum all histograms for statistics of one experiment $m\left(\tilde{q}_{L}\right)=540 \mathrm{GeV}, m\left(\tilde{\chi}_{2}^{0}\right)=$ $176 \mathrm{GeV}, m\left(\tilde{\ell}_{R}\right)=143 \mathrm{GeV}$

For each experiment take bin with maximum occupancy as measure of masses
Take $\sim 100$ events per $300 \mathrm{fb}^{-1}$ experiment
Perform exercise for $\sim 900$ experiments (forced generation)
Plot measured values for each experiment


Poor precision, factor almost two worse than edge method

Need to perform proper analysis to assess correct number of events with unique particle assignation

Used smeared jets and leptons, but no correction applied for the smearing, pessimitic approach

Test likelihood as was done for single-leg exercise, should give more stable results

What if one tries to add event-by-event constraints to the edge method?

## Hybrid method

Consider again two legs with three steps each: (Tovey, G.P., Nojiri)

$$
\tilde{q} \rightarrow \tilde{\chi}_{2}^{0} q_{1} \rightarrow \tilde{\ell}_{1} \ell_{2} \rightarrow \tilde{\chi}_{1}^{0} q_{1} \ell_{1} \ell_{2}
$$

Assume that all the 5 edges from edge method are measured in the experiment ("expt")
Based on 4-momenta of 4 leptons and 2 jets, for each point in the $\left(m(\tilde{q}), m\left(\tilde{\chi}_{2}^{0}\right), m\left(\tilde{\ell}_{R}\right), m\left(\tilde{\chi}_{1}^{0}\right)\right)$ space and for each event can calculate:

- expected value of edges ("evt") (Lester et al. formulas)
- expected value of $E_{x}^{\text {miss }}, E_{y}^{\text {miss }}$ from solved $\tilde{\chi}_{1}^{0}$ momenta (on-shell mass constraints)

Extimate 4 unknown masses for each event by minimizing $\chi^{2}$ :

$$
\left.\left.\begin{array}{rl}
\chi^{2} & =\left(\frac{m(l l)_{e v t}^{m a x}}{\sigma_{m(l l}^{m a x}}-m(l l)_{e x p t}^{m a x}\right.
\end{array}\right)^{2}\right)
$$

Mass estimate for experiment is average value of event-wise estimates

Take SPS1a, require 4 leptons (2 OSSF pairs), $\notin T^{\boldsymbol{E}}, 2$ jets, $M_{e f f}$
Reject events where more than one assignment of jets and leptons consistent with edges
Final statistics is 38 events for $100 \mathrm{fb}^{-1}$
By building many MC experiments estimate precision on masses:


| State | End-Point Method |  | Mixed Method |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Mean | Error | Mean | Error |
| $\tilde{\chi}_{1}^{0}$ | 96.5 | 8.0 | 95.8 | 5.3 |
| $\tilde{\ell}_{R}$ | 143.3 | 7.9 | 142.2 | 5.4 |
| $\tilde{\chi}_{2}^{0}$ | 177.2 | 7.7 | 176.4 | 5.3 |
| $\tilde{q}_{L}$ | 540.4 | 12.6 | 540.7 | 8.5 |
| Gain 30\% in mass precision by addin |  |  |  |  |

$\sigma\left(m\left(\tilde{q}_{L}\right)-m\left(\tilde{\chi}_{2}^{0}\right)=4 \mathrm{GeV}\right.$, comparable to squark width $(5 \mathrm{GeV})$ and $\tilde{u}_{L}-\tilde{d}_{L}$ mass difference ( 6 GeV )

## Where do we go from masses?

The mass measurements do not depend a priori on a special choice of the model For instance, we can state that in the data appear the decays:


Where we know the masses of $a, b, c, d, e$, and we might conjecture that $a, b, d$ appearing in both decays are the same having the same masses

So we have a mass hierarchy, some of the decays related these particles and, perhaps, the relative rates

Having decay chains help restricting the possibilities, if one imposes some conservations, e.g. charges or quantum numbers

Model dependence enters when we try to give a name to the particles, and match them to a template decay chain

Among the models proposed to solve the hierarchy problem, various options providing a full spectrum of new particles, with cascade decays:

- Universal extra-dimensions: first KK excitation of each of the SM fields
- Little Higgs with $T$ parity

Special feature of SUSY: if one identifies the heavy partners through their quantum numbers, the spins of all of them are wrong by $1 / 2$

Next step is investigating if exploiting the identified chains one can obtain information on the sparticle spins

## Conclusions

If SUSY discovery long path to understand the nature of the involved signal Main focus is in ensuring discovery, combining very inclusive signatures to more exclusive ones

We do not forget however to work on a strategy for going beyond discovery Work ongoing in ATLAS to verify in full simulation that the kinematic features of SUSY events can indeed be observed in real life

Results mostly still preliminary as we are finalizing work on a dedicated ATLAS note Recently new ideas on how to build on kinematic features to reconstruct masses in model-independent way

Better and better nderstanding of measurements on chains with two invisible particles at the end

