Local probes of horizons and singularities in AdS

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Outline:

1. Local bulk operators in AdS/CFT
2. Horizons and singularities

$\Big\} N = \infty$

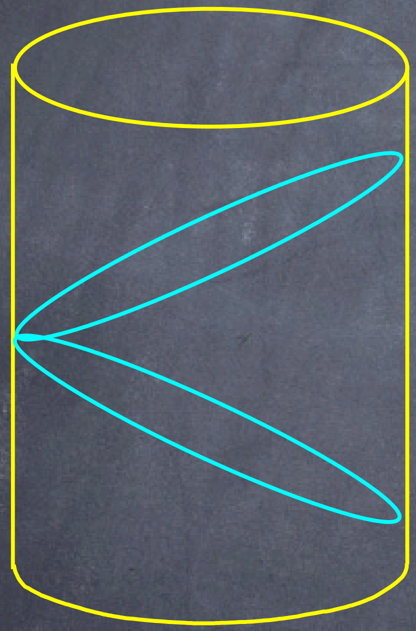
3. Timescales in CFT at finite N
4. Horizons and singularities at finite N?

Based on a series of papers with Hamilton, Lifschytz and Lowe, plus some work in progress.

hep-th/0506118, 0606141, 0612053, 0710.4334

Constructing local operators

AdS can be visualized as an infinite cylinder.



$\mathrm{AdS}_D\,/\,\mathrm{CFT}_d$

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 + |dX|^2 + dZ^2\right)$$

Poincaré coordinates: boundary at $Z = 0$, horizon at $Z = \infty$

Consider a scalar field of mass m in AdS, with normalizeable fall-off near the boundary.

$$\phi(T,X,Z) \sim Z^{\Delta}\phi_0(T,X) \quad \text{as } Z \to 0$$

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2 R^2}$$

The boundary field ϕ_0 is dual to an operator of dimension Δ in the CFT.

$$\phi_0(T,X)_{\mathrm{SUGRA}} \leftrightarrow \mathcal{O}(T,X)_{\mathrm{CFT}}$$

Can we reconstruct the bulk field ϕ given its boundary behavior ϕ_0 ?

For now we'll study this in the semiclassical limit

$$\ell_s,\, \ell_P \to 0 \qquad \text{in the bulk}$$
$$N,\, \lambda \to \infty \qquad \text{on the boundary}$$

The basic idea is to represent

$$\phi(T,X,Z) = \int dT' dX'\, K(T',X'|T,X,Z)\phi_0(T',X')$$

Banks, Douglas, Horowitz, Martinec
Balasubramanian, Giddings, Lawrence
Bena

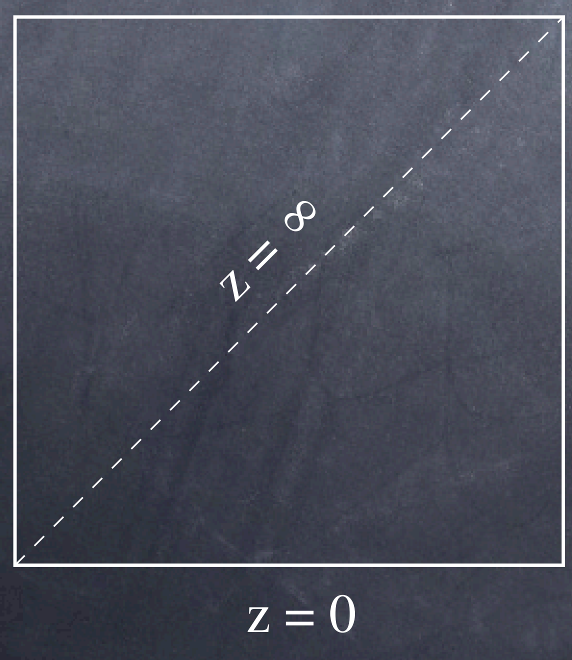
However this is not a standard Cauchy problem.
Neither existence nor uniqueness is guaranteed.

A cure for these problems - at least in a pure AdS background - is to Wick rotate to de Sitter space.

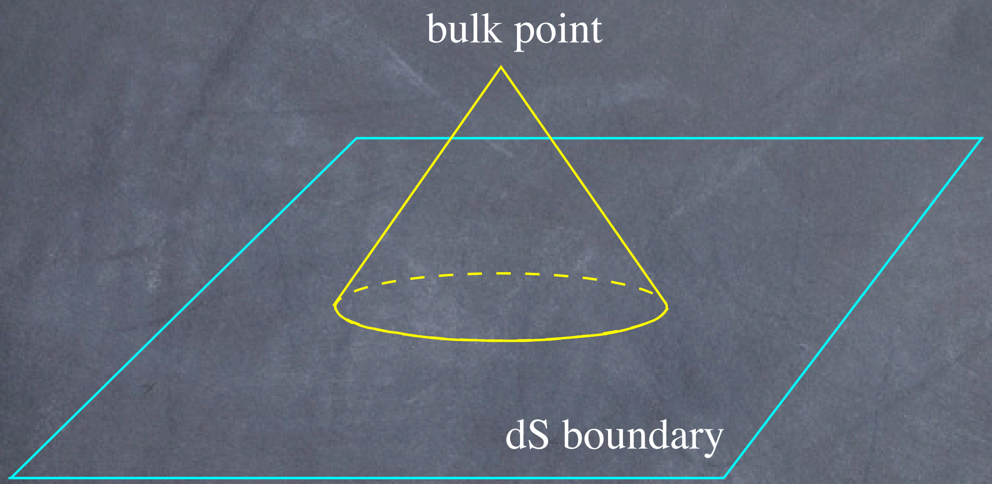
$$\text{set } X = iY$$

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 - dY^2 + dZ^2\right)$$

This is de Sitter space in flat FRW coordinates, with $Z =$ conformal time.



We now have a standard Cauchy problem, and can solve for the bulk field in terms of data on the past boundary.



Explicit expressions are pretty simple.

$$\phi(T, X, Z) = \text{const.} \int dT' dY' \left(\frac{Z^2 - T'^2 - |Y'|^2}{Z}\right)^{\Delta - d} \phi_0(T + T', X + iY')$$

smear in: real time

imaginary space

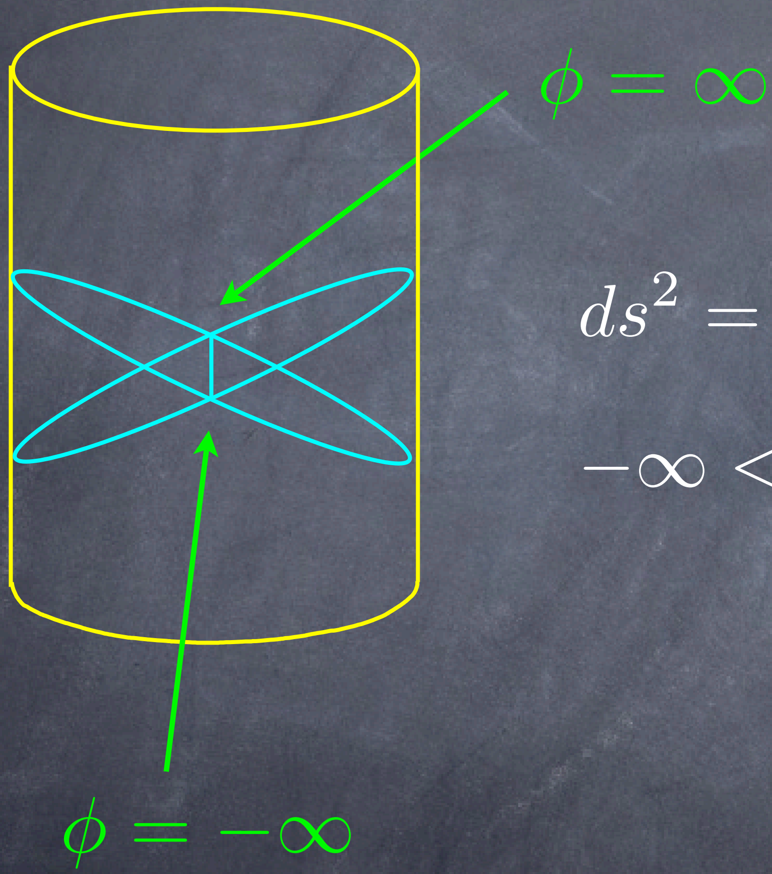
In the semiclassical limit this lets us reproduce bulk correlators.

$$\langle \phi_1 \phi_2 \rangle_{\rm SUGRA} = \int K(x_1'|x_1, Z_1) K(x_2'|x_2, Z_2) \langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{\rm CFT}$$

This works just because ϕ_0 and $\mathcal{O}$ have identical correlators.

Semiclassical horizons and singularities

AdS3 can also be described in Rindler coordinates

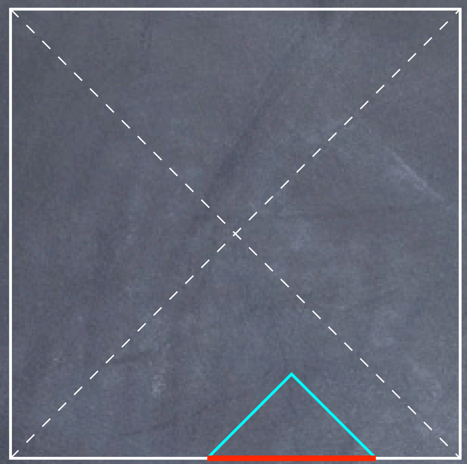


$$ds^2 = -\frac{r^2 - r_0^2}{R^2} dt^2 + \frac{R^2}{r^2 - r_0^2} dr^2 + r^2 d\phi^2$$

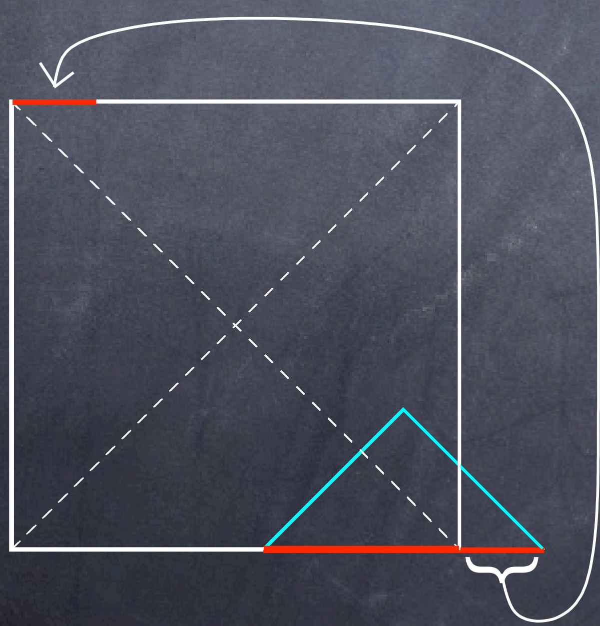
$$-\infty < \phi < \infty$$

Identify $\phi \approx \phi + 2\pi$ to make a BTZ black hole.

Wick rotating $\phi = iy$ again takes us to de Sitter space, now in static coordinates.



For points in the right Rindler wedge, we get a compact smearing function on the right Rindler boundary.



For points inside the horizon, the smearing function extends out of the right Rindler wedge. Use the antipodal map to move it to the other boundary.

So the region inside the horizon can be described by using both copies of the thermofield-doubled CFT.

Maldacena
Kraus, Ooguri, Shenker
Fidkowski, Hubeny, Kleban & Shenker
Festuccia & Liu

As the bulk point approaches the (future, past) horizon the smearing function extends to $(t = +\infty, t = -\infty)$ on the boundary. That's our signal of a horizon.

What about a singularity?

Semiclassically to make a BTZ black hole we just identify $\phi \approx \phi + 2\pi$. But this doesn't change the smearing functions at all (if the boundary field is periodic, so is the bulk field).

We can make boundary correlators periodic with an image sum.

Lifschytz & Ortiz

$$\langle \phi_0 \phi_0 \rangle_{\mathrm{BTZ}} = \sum_{n=-\infty}^{\infty} \langle \phi_0(t,\phi)\phi_0(t',\phi'+2\pi n)\rangle_{\mathrm{AdS}}$$

But $r = 0$ is a fixed point of the isometry

$$\phi \to \phi + \mathrm{const.}$$

So the image sum diverges at $r = 0$, and we get a divergent correlator - both from the bulk and boundary points of view.

More precisely as $r \to 0, r' \to \infty$

$$\langle \phi\phi \rangle_{BTZ} \sim \sum_n \frac{1}{\left(\frac{r}{r_0}\cosh(\phi + 2\pi n) + \sinh t\right)^{\Delta}}$$

The image sum is cut off at $n \sim \log(r_0/r)$ and the correlator diverges logarithmically as $r \to 0$.

Behavior at finite N

The whole program seems to crash at finite N. For bulk points inside the horizon the smearing functions grow on the boundary as $t \to \infty$.

$$K(t, \phi|\cdot) \sim e^{(\Delta - d)t}$$

In the semiclassical limit this is okay, because boundary correlators decay exponentially.

$$\langle \mathcal{O}\mathcal{O} \rangle \sim e^{-\Delta t}$$

But at finite N, correlators do not decay at late times.

How do correlators behave?

For a classical chaotic system phase space volume increases exponentially with time.

Ropotenko

$$\Gamma(t) \sim \Gamma(0)e^{ht} \qquad h = \mathrm{KS\ entropy}$$

The entropy increases linearly with time.

$$S(t) = S(0) + ht$$

In the quantum theory this means an initial excitation gets spread among Γ possible states, so

$$\langle \phi(t)\phi(0) \rangle \sim e^{-ht}$$

That's the typical behavior of a thermal system.

In a black hole background we expect correlators to decay on a timescale set by the quasinormal frequencies $\omega_n \sim T_H$. Birmingham, Sachs, Solodukhin

=> identify KS entropy $h \sim T_H$

How long can the entropy keep increasing? Until the system reaches its equilibrium thermal entropy, at time $t \sim S/h \sim \beta S$.

At that point the system realizes it has a finite phase space and correlators stop decaying.

This timescale $t \sim \beta S$ is much shorter than the recurrence time $t_r \sim \beta e^S$ after which the system returns to its initial state.

But it does seem to match the time at which the saddle point approximation breaks down in AdS.

$$\text{AdS black hole:} \quad \langle \phi\phi \rangle \sim e^{-t/\beta}$$

$$\text{thermal AdS:} \quad \langle \phi\phi \rangle \sim e^{-S} \times \mathcal{O}(1)$$

The thermal AdS saddle point starts to compete after $t \sim \beta S$.

Maldacena

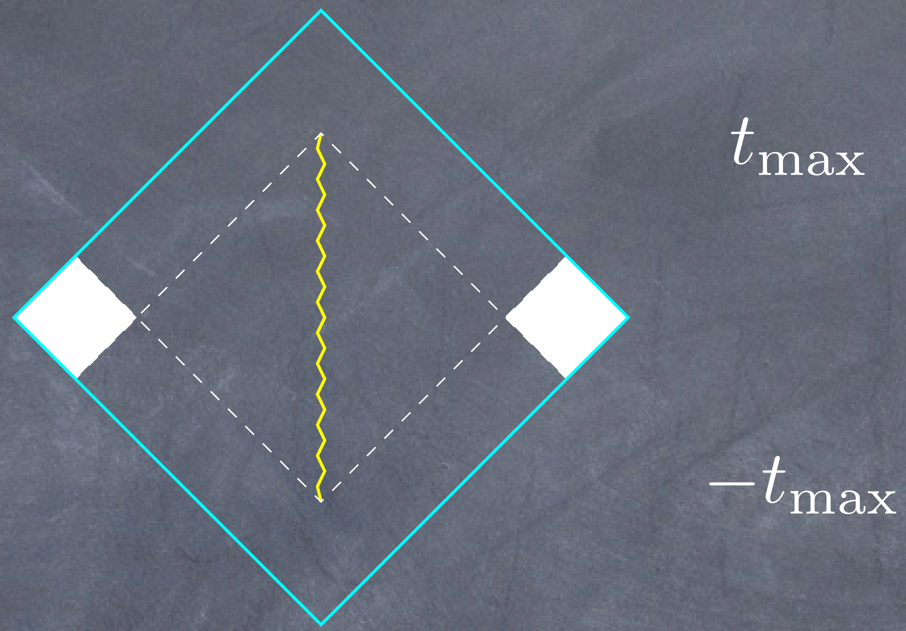
What does this mean for us?

These late-time tails make correlators of smeared operators ill-defined inside the horizon.

$$K(t, \phi|\cdot) \sim e^{(\Delta - d)t} \quad \text{as } t \to \infty$$

But when is the tail really important? Suppose we just set the boundary correlator to zero for $t > t_{\max} = \beta S$. Equivalently suppose we declare that the smearing functions vanish for $t > t_{\max}$. When would that make a difference in our bulk correlators?

We can study this in the semiclassical limit. With a cutoff at $t_{\max}$ the smearing function looks like



The white squares are spacelike separated from the support of the smearing function. This is like putting a cutoff at $\phi \approx t_{\max}$.

But remember for $r > 0$ there was already a cutoff on the image sum, at

$$n \sim \phi \sim \log(r_0/r)$$

So the additional cutoff at $\phi \approx t_{\rm max}$ is unimportant for

$$t_{\rm max} > \log(r_0/r)$$
$$r > r_0 e^{-t_{\rm max}}$$

With our previous estimate for $t_{\rm max}$ this suggests that one can approach the singularity, getting as close as $r_0 e^{-\beta S}$, before noticing deviations from semiclassical behavior. Interpretation?

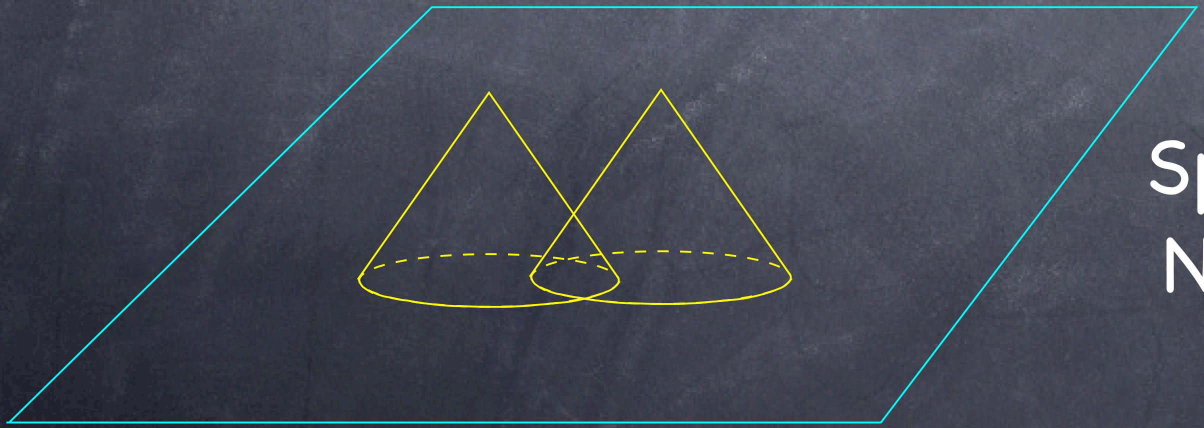
Conclusions

- In the semiclassical limit one can define local operators in the bulk of AdS.
- Their correlators are well-defined at the horizon and diverge at the singularity.
- At finite N this breaks down, since CFT correlators stop decaying after a time $\sim \beta S$.
- This seems to generate a new length scale on the gravity side: $r_0 e^{-\beta S}$.

Bulk locality? (semiclassical limit)

Recall that in the semiclassical limit, correlation functions of smeared boundary operators exactly reproduce bulk correlators.

Corrolary: bulk operators commute at spacelike separation => corresponding smeared operators commute, even though they overlap!



Special property of large N limit (commutators are c-numbers)

Bulk locality and holography at finite N?

What we've done so far is exact in the semiclassical limit (statement about wave equations in AdS).

What about finite N? No guarantees, but we could just use the same smearing functions at finite N. For example in $N=4$ Yang-Mills

$$\Phi(T,X,Z) = \int dT' d^3X' \, K(T',X'|T,X,Z) \, \mathrm{Tr}\, F^2$$

makes sense at any N.

In fact this seems singled out by the symmetries.

AdS-invariant distance:

$$\sigma = \frac{Z^2 + Z'^2 + \Delta X^2 - \Delta T^2}{2ZZ'}$$

$$K = \lim_{Z' \to 0} (\sigma Z')^{\Delta - d}$$

Note that

$$ds^2 = \frac{R^2}{Z^2}\left(-dT^2 + dX^2 + dZ^2\right)$$

isometry $\quad (T, X, Z) \to \lambda(T, X, Z)$

scale transformation on boundary,

Z has conformal weight -1

So K transforms covariantly with weight $d - \Delta$

That's exactly what we need for $\int KO$ to behave like a bulk scalar.

=> our smearing functions are the unique covariant way to map a primary operator in the CFT to a bulk scalar field

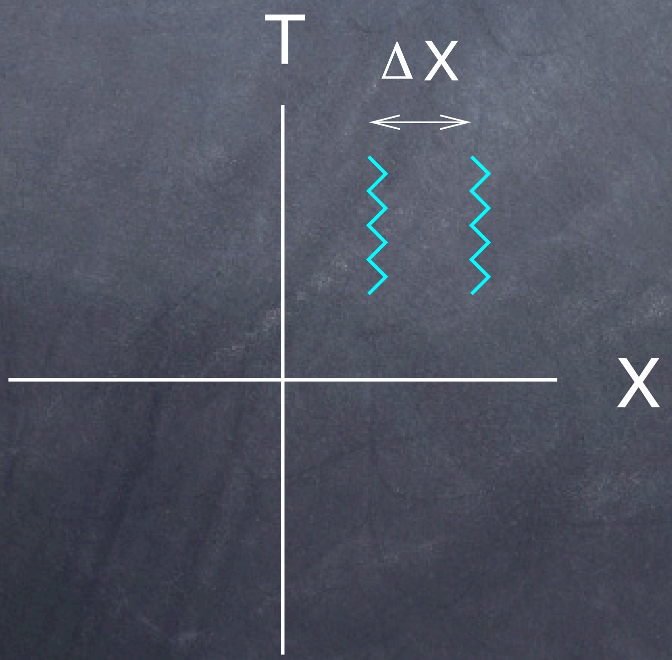
This seems very strange - we've defined an infinite number of bulk operators. Seems incompatible with holography.

When do these operators commute?

Work on a fixed-T hypersurface, with operators placed at some radial position Z.

Smearing functions have extent $\Delta T = Z$

Will be spacelike separated on boundary provided bulk operators have longitudinal separation $\Delta X > Z$



So we expect

- $1/Z^{d-1}$ commuting operators per coordinate area on the boundary

With a redshift factor R/Z this means

- $1/R^{d-1}$ commuting operators per proper area in the bulk

One commuting operator per AdS radius of curvature. Locality breaks down on the AdS scale!

How is this compatible with holography?

holographic bound $S < N^2/Z^{d-1}$
(entropy per longitudinal coordinate area)

The bound is saturated if we have N^2 commuting operators in the CFT. Seems reasonable - NxN matrices, central charge $\sim N^2$

What about quasi-local bulk physics at finite N? The commuting operators we can build from $\mathrm{Tr}\, F^2$ aren't enough to describe a local bulk dilaton on distances less than an AdS radius!

Our best guess: there are other operators which don’t commute, but whose commutators are small enough at low energies that they can be ignored.

(matrix elements of commutator between low-energy states is small)

I don’t have an explicit construction.