Simulating Quantum Black Holes and Quantum Universe

Sept.12-16, IPMU focus week

“Quantum Black Holes”

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- based on collaborations with

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0. Introduction

Simulating Quantum Black Holes

"quantum black hole"

or

microscopic description of black holes

requires

superstring theory

gauge/string duality ('98-)

strongly coupled gauge theory

Monte Carlo simulations

analogous to lattice QCD

- Anagnostopoulos-Hanada-J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]

CERN COURIER, March 2008

NEWS

KEK

Superstrings reveal the interior structure of a black hole

A research group at KEK has succeeded in calculating the state inside a black hole using computer simulations based on superstring theory. The calculations confirmed for the first time that the temperature dependence of the energy inside a black hole agrees with the power-law behaviour expected from calculations based on Stephen Hawking's theory of black-hole radiation. The result demonstrates that the behaviour of elementary particles as a collection of strings in superstring theory can explain thermodynamical properties of black holes.

In 1974, Stephen Hawking at Cambridge showed theoretically that black holes are not entirely black. A black hole in fact emits light and particles from its surface, so that it shrinks little by little. Since then, physicists have suspected that black holes should have a certain interior structure, but they have been unable to describe the state inside a black hole using general relativity, as the curvature of space–time becomes so large towards the centre of the hole that quantum effects make the theory no longer applicable. Superstring theory, however, offers the possibility of bringing together general relativity and quantum mechanics in a consistent manner, so many theoretical physicists have been investigating whether this theory can describe the interior of a black hole.

Jun Nishimura and colleagues at KEK established a method that efficiently treats the oscillation of elementary strings depending on their frequency. They used the Hitachi SR11000 model K1 supercomputer installed at KEK in March 2006 to calculate the thermodynamical behaviour of the collection of strings inside a black hole. The results showed that as the temperature decreased, the simulation reproduced behaviour of a black hole as predicted by Hawking's theory (figure 1).

This demonstrates that the mysterious thermodynamical properties of black holes can be explained by a collection of strings fluctuating inside. The result also indicates that superstring theory will develop further to play an important role in solving problems such as the evaporation of black holes and the state of the early universe.

The Hitachi SR11000 model K1 supercomputer calculated the interior structure of a black hole. It provides a peak performance of 2.15 teraflops peak. (Courtesy KEK.)

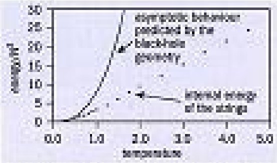


Fig. 1. Plot of the energy of the collection of strings against the temperature. The solid line represents the behaviour of the black hole predicted by Hawking's theory. The results agree in the lower temperature regime, where the calculation based on general relativity becomes valid.

Further reading

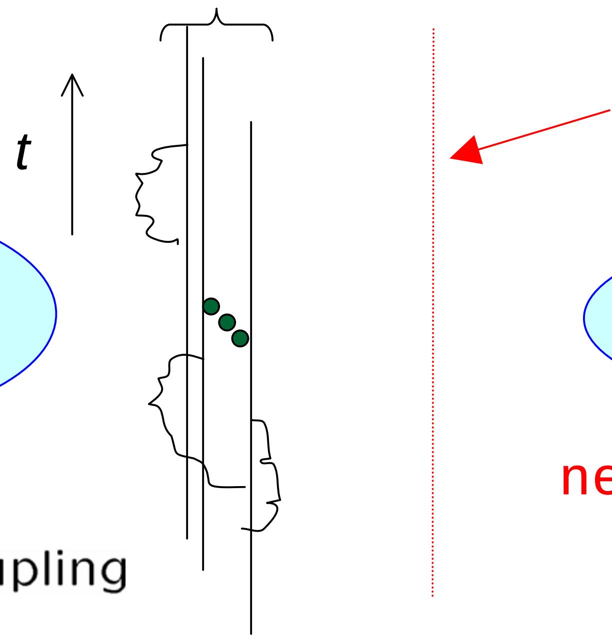
KN Anagnostopoulos *et al.* 2008 *Phys. Rev. Lett.* **100** 021601.

Gauge-gravity duality for D0-brane system

type IIA superstring

Itzhaki-Maldacena-Sonnenschein
-Yankielowicz ('98)

N D0 branes



horizon

1d U(*N*) SUSY gauge theory

at finite *T*

λ : 't Hooft coupling

black 0-brane solution in type IIA SUGRA

near-extremal black hole

In the decoupling limit, the D0 brane system describes the black hole microscopically.

large *N* and large λ ⟹ SUGRA description : valid

D0 brane effective theory

$$S = \frac{N}{\lambda} \int_0^\beta dt \ \mathrm{tr} \left\{ \frac{1}{2}(DX_i(t))^2 - \frac{1}{4}[X_i(t), X_j(t)]^2 + (\text{fermionic part}) \right\}$$

- black hole thermodynamics

Anagnostopoulos-Hanada-J.N.-Takeuchi ('08)

$$\frac{E}{N^2} = \frac{9}{14} \left\{ 4^{13} 15^2 \left(\frac{\pi}{7} \right)^{14} \right\}^{1/5} T^{14/5} \sim 7.41 \ T^{14/5}$$

- Schwarzschild radius from Wilson loop

Hanada-Miwa-J.N.-Takeuchi, in prep.

$$W \equiv \mathrm{tr}\, \mathcal{P} \exp \left[i \int_0^\beta dt \{ A(t) + i X_9(t) \} \right]$$

$$\sim \exp \left(\beta \frac{R_{\mathrm{Sch}}}{2\pi\alpha'} \right)$$

$$\frac{R_{\mathrm{Sch}}}{2\pi\alpha'} = \frac{1}{2\pi} \left\{ \frac{16\sqrt{15}\pi^{7/2}}{7} T \right\}^{2/5} \sim 1.89 \ T^{2/5}$$

Plan of the talk

0. Introduction

1. Simulating SUSY matrix QM with 16 supercharges

2. Dual gravity description and black hole thermodynamics

3. Schwarzschild radius from Wilson loop

4. Related on-going projects

   SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

   gauge/gravity duality for D=3,4 gauge theories

   "simulating quantum universe"

   4d universe from 10d(11d) space-time ?

5. Summary

1. Simulating SUSY QM with 16 supercharges

SUSY matrix QM with 16 supercharges

$$S_b = \frac{N}{\lambda} \int_0^\beta dt \; \mathrm{tr} \left\{ \frac{1}{2}(DX_i(t))^2 - \frac{1}{4}[X_i(t), X_j(t)]^2 \right\}$$

$$S_f = \frac{N}{\lambda} \int_0^\beta dt \; \mathrm{tr} \left\{ \frac{1}{2}\psi_\alpha D\psi_\alpha - \frac{1}{2}\psi_\alpha(\gamma_i)_{\alpha\beta}[X_i, \psi_\beta] \right\}$$

1d gauge theory $\quad D = \partial_t - i\,[A(t), \;\cdot\;]$

$$\begin{cases} X_j(t) \quad (j = 1, \cdots, 9) & \text{p.b.c.} \\ \psi_\alpha(t) \quad (\alpha = 1, \cdots, 16) & \text{anti p.b.c.} \end{cases}$$

$T = \beta^{-1}$ temperature

$\lambda = g^2 N$ 't Hooft coupling

$$\lambda_{\mathrm{eff}} = \frac{\lambda}{T^3}$$

$\lambda = 1$ (without loss of generality)

- low T ➡ strongly coupled — dual gravity description
- high T ➡ non-zero modes : weakly coupled (high T exp.)
  (zero modes : integrated non-perturbatively)

Kawahara-J.N.-Takeuchi,
JHEP 0712 (2007) 103, arXiv:0710.2188[hep-th]

Fourier-mode simulation (to respect SUSY maximally)

Hanada-J.N.-Takeuchi, PRL 99 (07) 161602 [arXiv:0706.1647]

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_{i,n} \mathrm{e}^{i\omega n t} \qquad \omega = \frac{2\pi}{\beta}$$

Note: Gauge symmetry can be fixed non-perturbatively in 1d.

- static diagonal gauge :

$$A(t) = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

$$S_{\mathrm{FP}} = -\sum_{a<b} 2 \ln \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- residual gauge symmetry : $g(t) = \mathrm{diag}(\mathrm{e}^{i\omega\nu_1 t}, \cdots, \mathrm{e}^{i\omega\nu_N t})$

$$\begin{cases} \tilde{X}^{ab}_{i,n} \mapsto \tilde{X}^{ab}_{i,n-\nu_a+\nu_b} \\ \alpha_a \mapsto \alpha_a + 2\pi\nu_a \end{cases} \qquad \begin{aligned} X_i &\mapsto g X_i g^\dagger \\ A &\mapsto g A g^\dagger + i g \partial_t g^\dagger \end{aligned}$$

should be fixed by imposing $-\pi < \alpha_a \leq \pi$

c.f.) lattice approach : Catterall-Wiseman, arXiv:0803.4273[hep-th]

2. Dual gravity description and black hole thermodynamics

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]

Dual gravity description

After taking the decoupling limit : $\alpha' \to 0$

$$U \equiv \frac{r}{\alpha'} \quad , \quad \lambda \equiv g_s N \alpha'^{-3/2} \quad \text{(fixed)}$$

$$f(U) \equiv \frac{U^{7/2}}{\sqrt{d_0 \lambda}} \left\{ 1 - \left(\frac{U_0}{U} \right)^7 \right\}$$

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + \sqrt{d_0 \lambda} U^{-3/2} d\Omega_{(8)}^2 \right\}$$

range of validity: $N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$

Black hole thermodynamics

Hawking temperature : $\frac{T}{\lambda^{1/3}} = \frac{7}{16\sqrt{15}\pi^{7/2}} \left(\frac{U_0}{\lambda^{1/3}} \right)^{5/2}$

Bekenstein-Hawking entropy : $S = \frac{1}{28\sqrt{15}\pi^{7/2}} N^2 \left(\frac{U_0}{\lambda^{1/3}} \right)^{9/2}$

$$\Rightarrow \quad \frac{1}{N^2} \frac{E}{\lambda^{1/3}} = \frac{9}{14} \left\{ 4^{13} 15^2 \left(\frac{\pi}{7} \right)^{14} \right\}^{1/5} \left(\frac{T}{\lambda^{1/3}} \right)^{14/5} \sim 7.41 \left(\frac{T}{\lambda^{1/3}} \right)^{14/5}$$

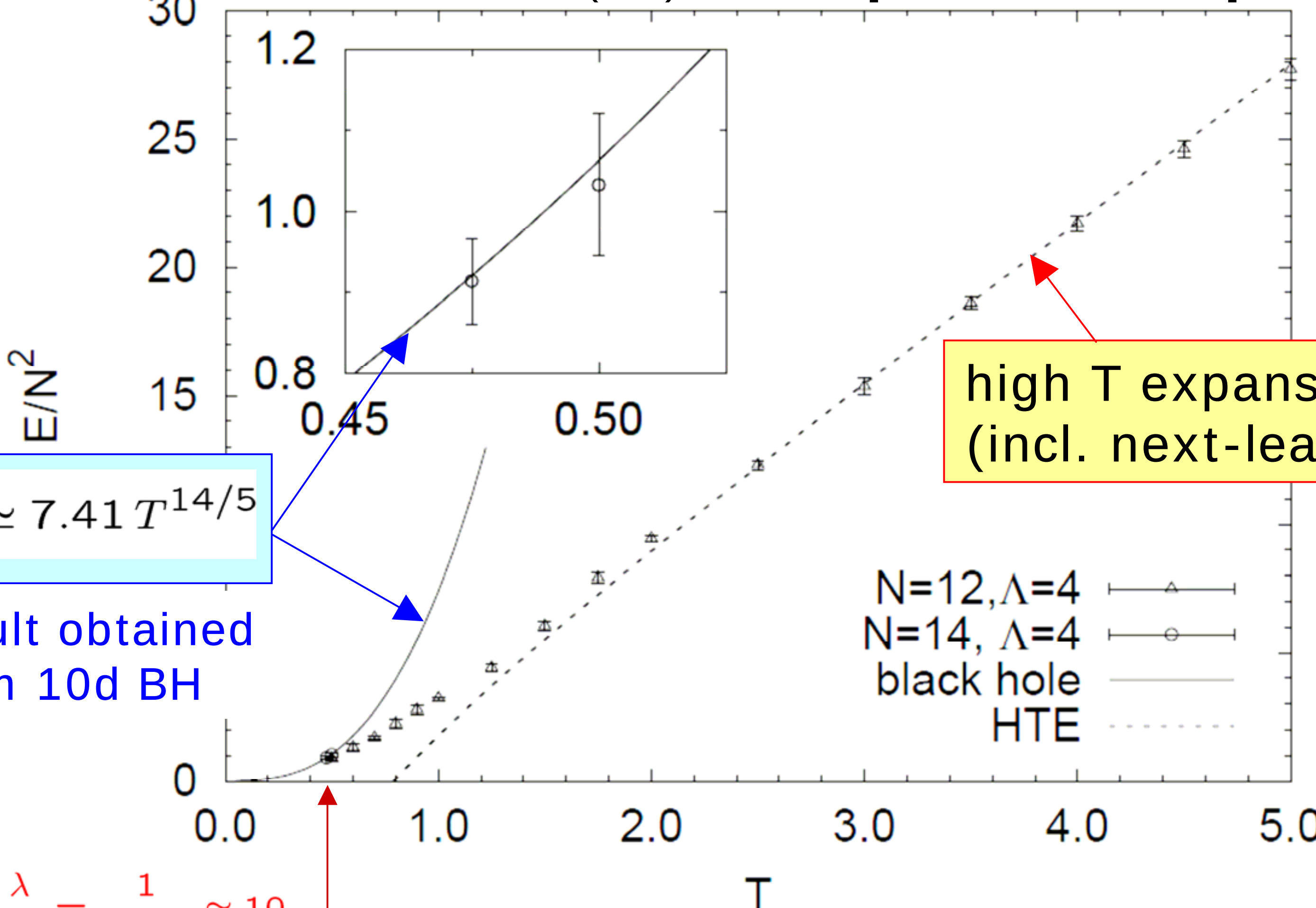
Klebanov-Tseytlin ('96)

Result: Internal energy

$$E = \frac{\partial}{\partial \beta}(\beta \mathcal{F})$$

free energy

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]



high T expansion (incl. next-leading order)

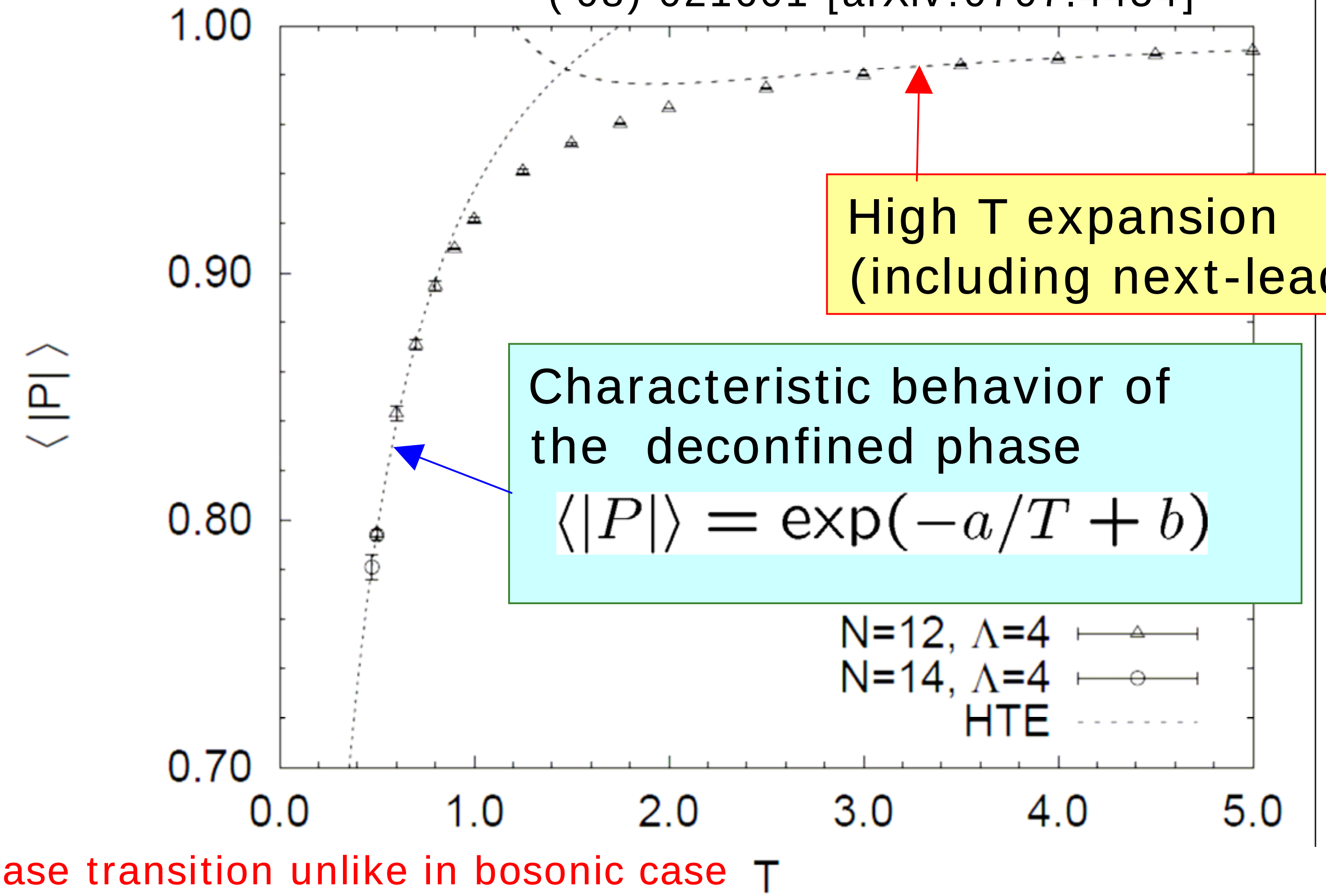
$$\frac{E}{N^2} \simeq 7.41\, T^{14/5}$$

result obtained from 10d BH

$$\lambda_{\text{eff}} = \frac{\lambda}{T^3} = \frac{1}{0.463^3} \approx 10$$

Result: Polyakov line

Anagnostopoulos-Hanada- J.N.-Takeuchi, PRL 100 ('08) 021601 [arXiv:0707.4454]



no phase transition unlike in bosonic case

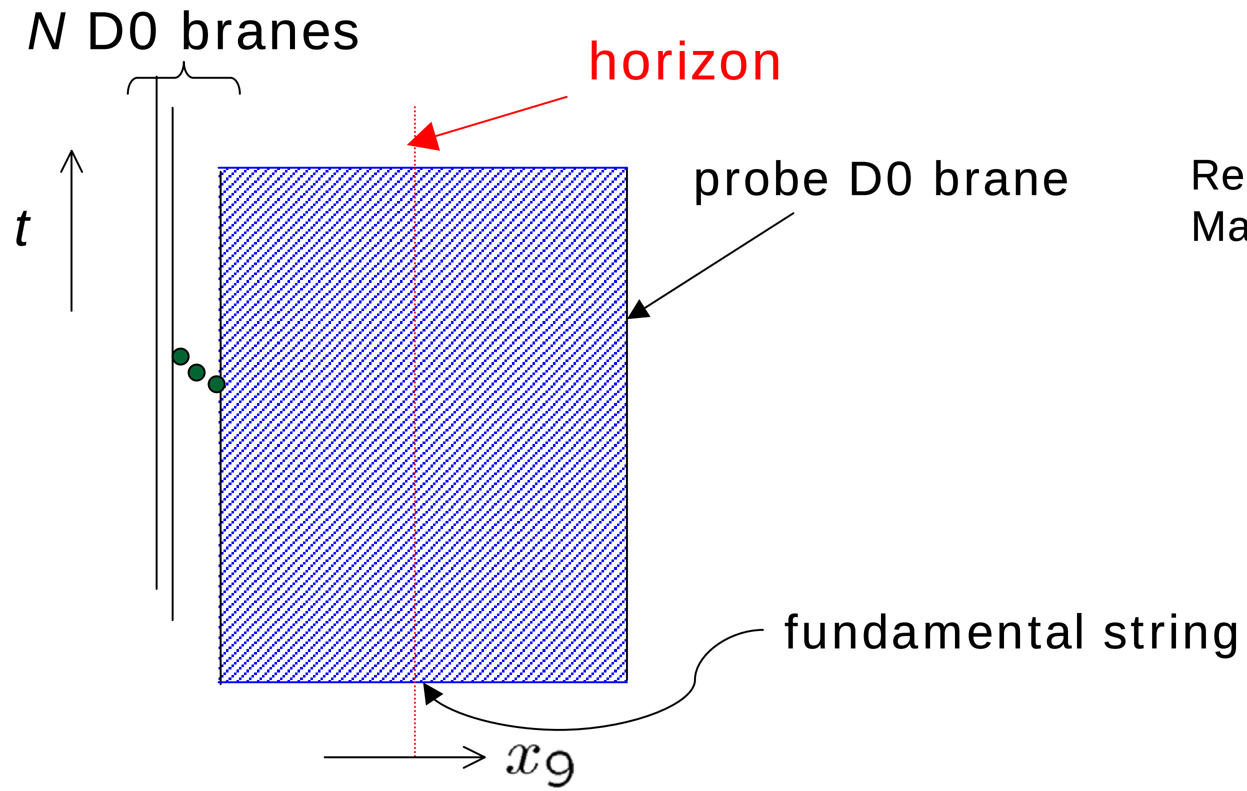
➡ consistent with analyses on the gravity size (Barbon et al., Aharony et al.)

3. Schwarzschild radius from Wilson loop

Hanada-Miwa-J.N.-Takeuchi, in prep.

Calculation of Wilson loop

Hanada-Miwa-J.N.-Takeuchi, in prep.



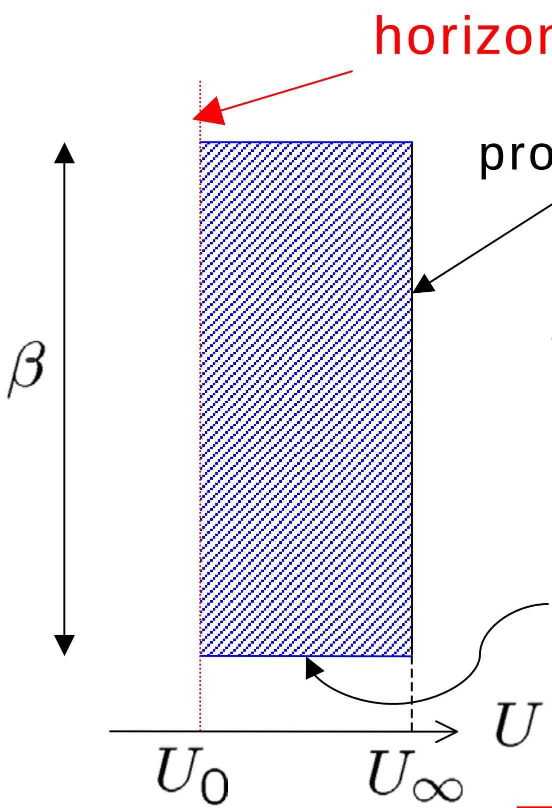
Rey-Yee ('98),
Maldacena ('98)

gauge theory side :

propagation of a test particle coupled to $A(t)$ and $X_9(t)$

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+iX_9(t)\}\right]$$

Calculation of Wilson loop (cont'd)



Replace N D0 branes by
the black 0-brane background

$$ds^2 = \alpha' \left\{ f(U)dt^2 + \frac{1}{f(U)}dU^2 + (S^8 \text{ part}) \right\}$$

<u>gravity theory side :</u>
propagation of the string
in the b.g. geometry

string action for the minimal surface :

$$S_{\text{string}} = \frac{1}{2\pi}(U_\infty - U_0)\,\beta$$

Calculation of Wilson loop (cont'd)

$$W\, e^{-M\beta} = e^{-S_{\rm string}}$$ at large N and large λ

perimeter-law suppression factor
due to propagation of a particle with mass *M*

$$S_{\rm string} = \frac{1}{2\pi}(U_\infty - U_0)\,\beta$$

$$\log W - M\beta = \frac{U_0}{2\pi}\beta - \frac{U_\infty}{2\pi}\beta$$

natural to identify

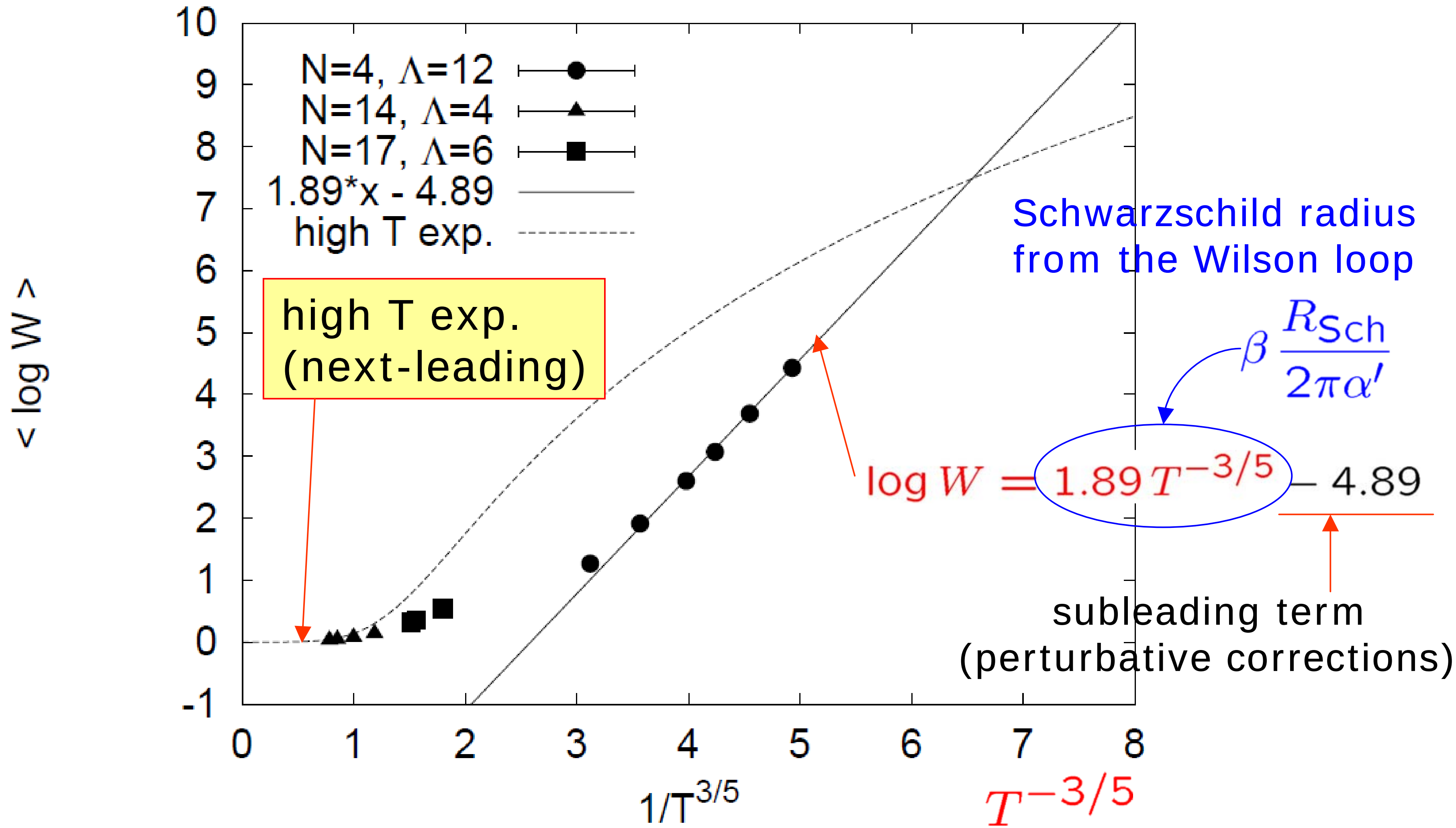
more sophisticated justification
a la Drukker-Gross-Ooguri ('99)

$$\frac{U_0}{2\pi} = \frac{R_{\rm Sch}}{2\pi\alpha'} = \frac{1}{2\pi}\left\{\frac{16\sqrt{15}\pi^{7/2}}{7}T\right\}^{2/5} \sim 1.89\ T^{2/5}$$

$$\log W = \beta\,\frac{R_{\rm Sch}}{2\pi\alpha'} \sim 1.89\,T^{-3/5}$$

Results: Wilson loop

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t)+iX_9(t)\}\right]$$



4. Related on-going projects

SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

4d universe from 10d(11d) space-time ?

SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM

Ishiki-Kim-J.N.-Tsuchiya, in prep.

respect SUSY maximally   c.f.) lattice approach

1) U(k) SYM on $R \times S^2$ in the planar limit

(dual geometry
Lin-Maldacena ('05))

Fourier mode cut off (for R)

fuzzy sphere (matrix regularization) (for S^2)

↑

plane wave matrix model

(matrix QM with mass terms & CS terms)

around k copies of the fuzzy sphere solution

$$X_a = L_a^{(n)} \otimes \mathbf{1}_k \quad (a = 1, 2, 3)$$

$k \to \infty$ limit removes fuzzyness

SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM (cont'd)

2) U(k) SYM on $R \times S^3$ in the planar limit

Fourier mode cut off (R)

S^1 fibered to S^2 (S^3)

thermodynamics at strong coupling from the gravity side (Witten '98)

↑

plane wave matrix model
around k copies of the multi-fuzzy-sphere solution

$$X_a = L_a \otimes \mathbf{1}_k \quad (a = 1, 2, 3)$$

Ishii-Ishiki-Shimasaki-Tsuchiya ('08)

$$L_a = \begin{pmatrix} L_a^{(n)} & & & \\ & L_a^{(n+1)} & & \\ & & \ddots & \\ & & & L_a^{(n+s)} \end{pmatrix}$$

$k \to \infty$ limit removes fuzzyness

Agreement at weak coupling

Ishiki-Kim-J.N.-Tsuchiya, in prep.

$$S_{\text{PWMM}} = N\int_0^\beta dt\,\mathrm{tr}\Big\{\frac{1}{2}(DX_i)^2 + \frac{1}{2}\mu^2(X_a)^2 + \frac{1}{8}\mu^2(X_m)^2 + \frac{1}{2}i\mu\epsilon_{abc}X_a[X_b,X_c] - \frac{1}{4}[X_i,X_j]^2 + \frac{1}{2}\psi^\dagger D\psi + \frac{3}{8}i\mu\psi^\dagger\gamma^{123}\psi - \frac{1}{2}\psi^\dagger\gamma^i[X_i,\psi]\Big\}$$

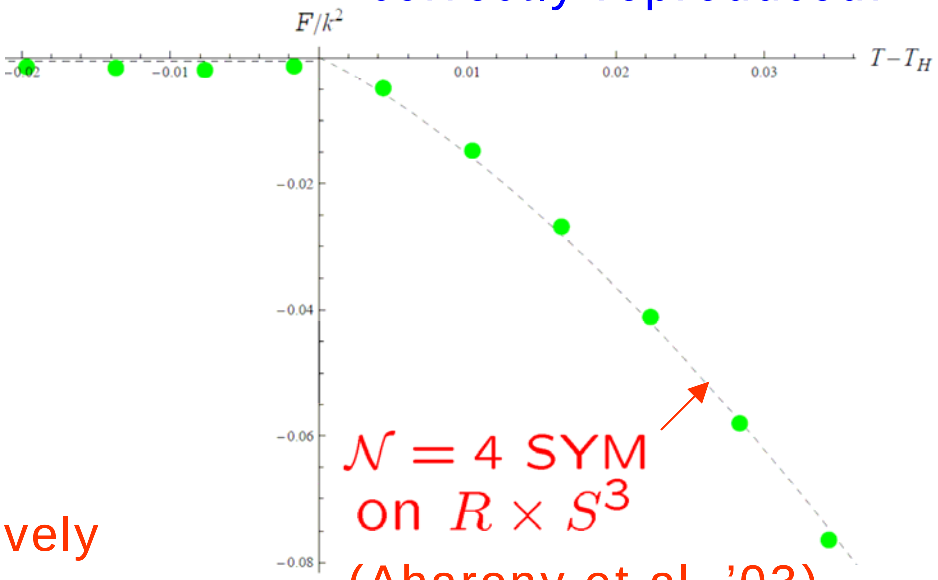
$$X_a = L_a \otimes \mathbf{1}_k \quad (a=1,2,3)$$

$$L_a = \begin{pmatrix} L_a^{(n)} & & & \\ & L_a^{(n+1)} & & \\ & & \ddots & \\ & & & L_a^{(n+s)} \end{pmatrix}$$

For $\mu \gg 1$ (weak coupling),

- fluctuations of X_i and ψ integrated out at 1-loop
- gauge field $A(t)$: moduli integrated non-perturbatively by MC sim.

deconfinement transition correctly reproduced!



$\mathcal{N}=4$ SYM on $R\times S^3$ (Aharony et al. '03)

Simulating Quantum Universe

Another interpretation of D0 brane quantum mechanics

(different large-N limit)

Matrix Theory

Banks-Fischler-Shenker-Susskind ('97)

microscopic description of M Theory

(c.f.) Matrix cosmology
Freedman-Gibbons-Schnabl (hep-th/0411119))

How does our 4d space-time appear
from 10d (11d) space-time ?

e.g.) SO(9) —SSB→ SO(3) ?

Gaussian expansion method (GEM) → application to matrix QM
Kabat-Lifschytz ('99)

BH thermodynamics
Kabat-Lifschytz-Lowe('00)

Aoyama-J.N.-Okubo-Takeuchi, in progress

Gaussian Expansion Method

$$F = -\log\left(\int d\phi\, \mathrm{e}^{-S}\right) \qquad S = \frac{1}{4} N \,\mathrm{tr}\phi^4$$

$$\tilde{S}(\epsilon, t) \equiv S_{\mathrm{G}}(t) + \epsilon\,\{S - S_{\mathrm{G}}(t)\} \qquad S_{\mathrm{G}}(t) = \frac{1}{2} N t\, \mathrm{tr}\phi^2$$

$$\tilde{F}(\epsilon, t) \equiv -\log\left(\int d\phi\, \mathrm{e}^{-\tilde{S}(\epsilon,t)}\right) \qquad F = \tilde{F}(1, t)$$

indep. of *t*

$$\tilde{F}(\epsilon, t) = \sum_{k=0}^{\infty} \epsilon^k f_k(t)$$

Truncate the series at $k = n$ and set $\epsilon = 1$.

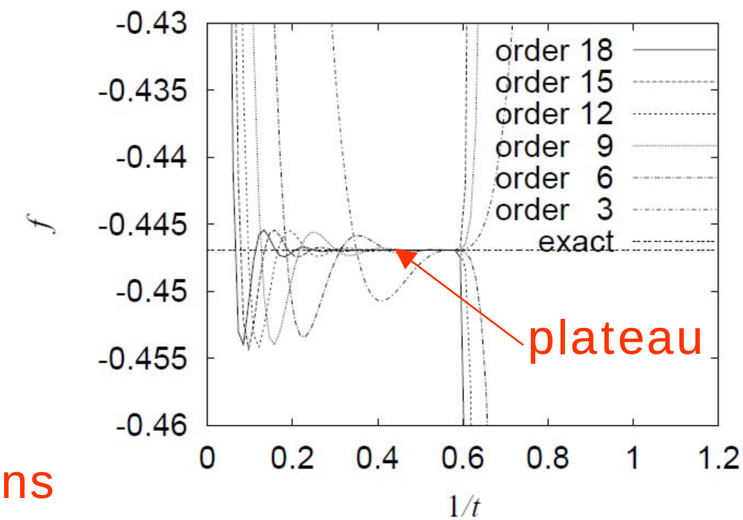
$$\tilde{F}_n(t) \equiv \sum_{k=0}^{n} f_k(t)$$

How to identify the plateau ?
if S_{G} contains many param.

self-consistency eq.:

$$\frac{\partial}{\partial t}\tilde{F}_n(t) = 0$$

Search for concentration of solutions



Dynamical generation of space-time in type IIB matrix model

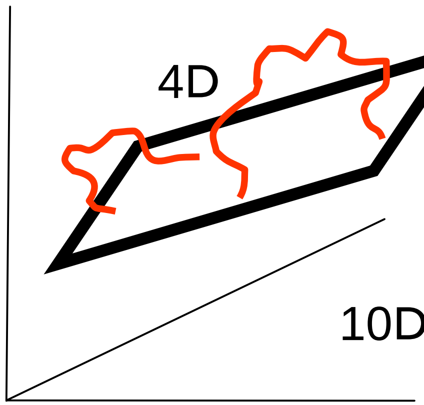
$$S = N\mathrm{tr}\left\{-\frac{1}{4}[X_\mu, X_\nu]^2 + \frac{1}{2}\psi_\alpha(\Gamma_\mu)_{\alpha\beta}[X_\mu, \psi_\beta]\right\}$$

(Ishibashi-Kawai-Kitazawa-Tsuchiya '96)

Gaussian expansion method

J.N.-Sugino ('01),
Kawai et al. ('01),...

Eigenvalue distribution of X_μ



Analogous studies
in the D=6 model (less SUSY)
Aoyama-J.N.-Okubo, in prep.

more systematic studies of SSB patterns

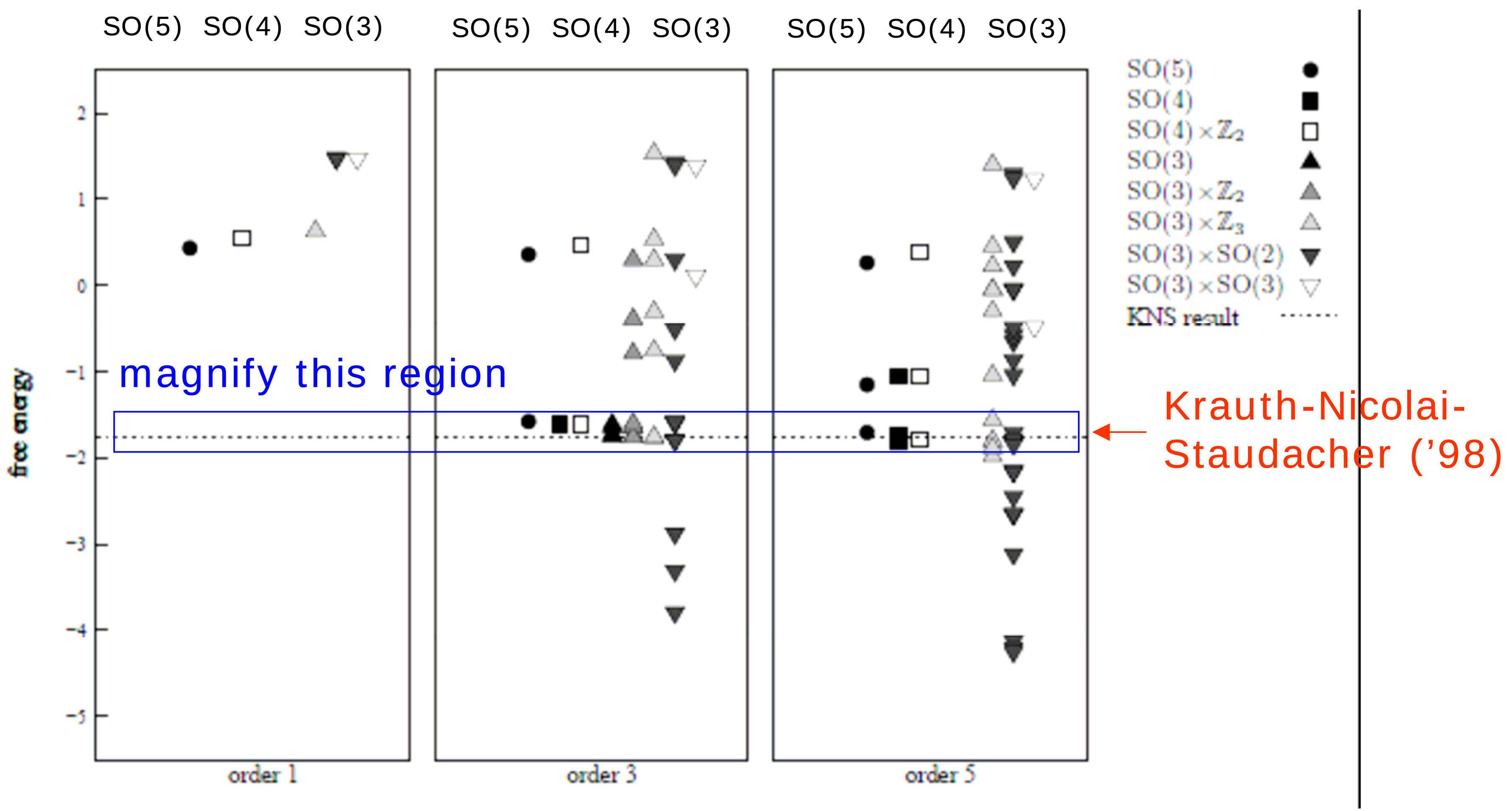
SO(6) → SO(3)
SSB

finite extra dimensions !

confirmation by MC sim.
Aoyama-Azuma-Hanada-J.N.,
in progress

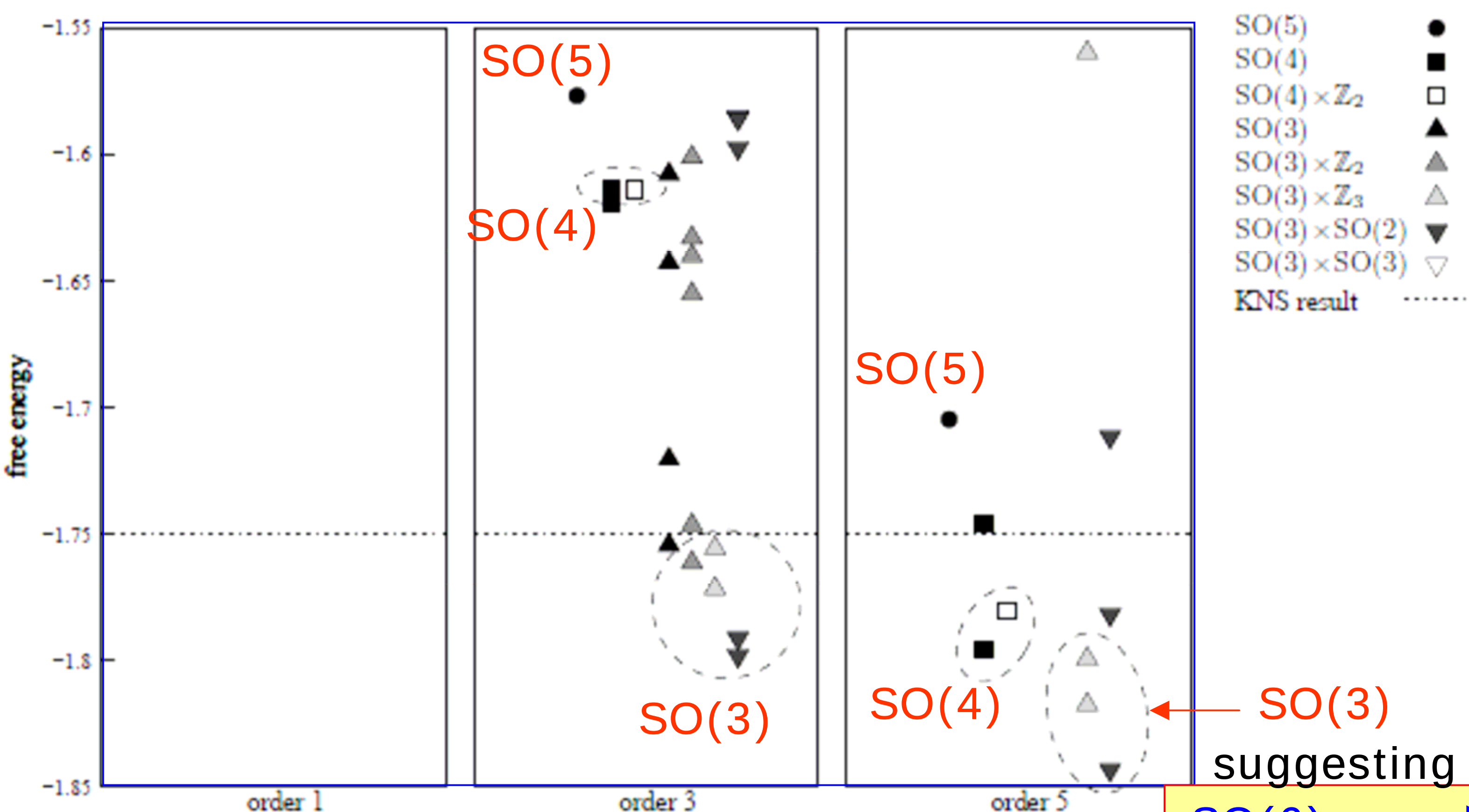
Results of GEM for the little IIB matrix model

Aoyama-J.N.-Okubo, in prep.



Results of GEM for the little IIB matrix model (cont'd)

concentration of solutions identified

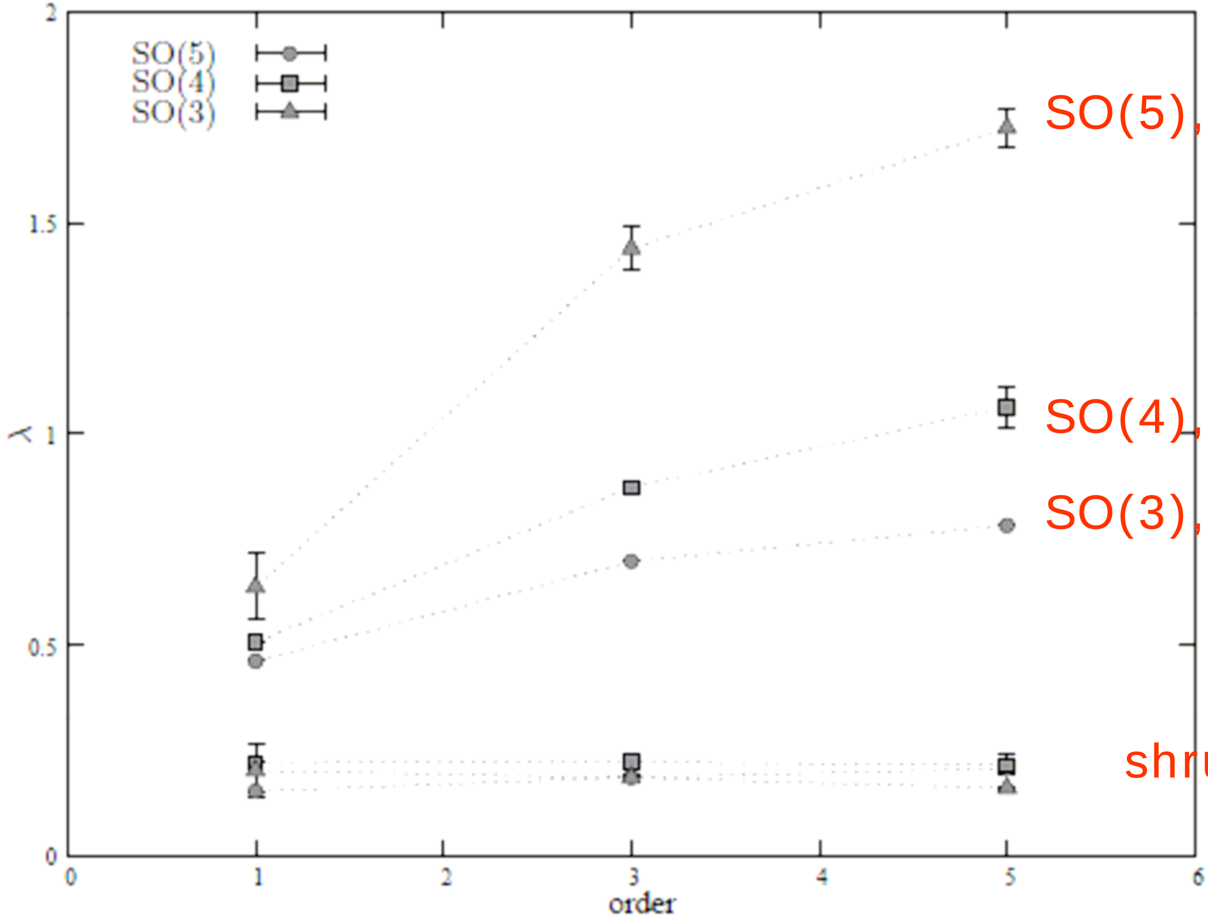


suggesting :

SO(6) → SO(3) (SSB)

Results of GEM for the little IIB matrix model (cont'd)

extent of the eigenvalue distribution
in the extended/shrunk direction



SO(5), extended

SO(4), extended

SO(3), extended

shrunk directions

↓

finite extra dimension ?!

5. Summary

Summary

- Monte Carlo studies of quantum black holes
  based on gauge/string duality

  Black hole thermodynamics (E v.s. T relation)
  T-dependence of the Schwarzschild radius
  reproduced from Wilson loop operator

- SYM on $R \times S^2$, $R \times S^3$ from SUSY matrix QM
  deconfinement transition reproduced at weak coupling

- Dynamical generation of 4d universe in matrix models ?
  finite extra dimensions suggested by GEM

geometry : emergent notion
matrices : fundamental d.o.f. of superstring theory

Monte Carlo sim. : an important tool to explore
the dynamical properties of matrix models