Black Holes as Fast Scramblers

Yasuhiro Sekino
(Okayama Institute for Quantum Physics)

Based on

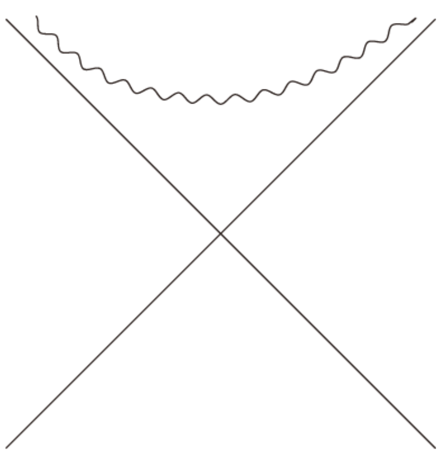
Y.S. and L. Susskind (Stanford),
0808.2096 [hep-th]

Black hole

- Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- Horizon at R=2GM. Near the horizon, Rindler space



$$\begin{aligned} ds^2 &= -\rho^2 d\omega^2 + d\rho^2 \\ &= -dX^+ dX^- \end{aligned}$$

$$(X^\pm = \pm \rho e^{\pm\omega})$$

Singularity at $X^+ X^- = R^2$

- Black hole behaves as an object with
  - Entropy: $$S = \frac{A}{4G} = \frac{\pi R^2}{G}$$

    First law (dM=T dS), Generalized second law
  - Temperature: $$T = \frac{1}{4\pi R}$$

    Emits Hawking radiations

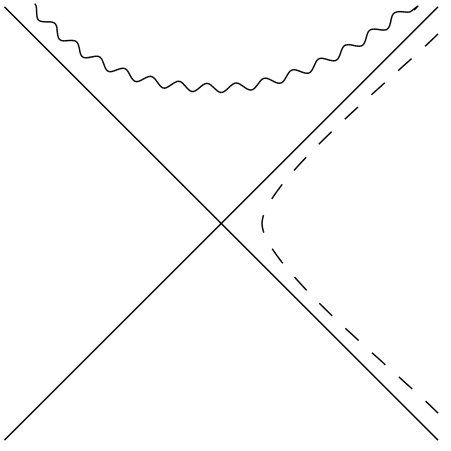
- Schwarzschild BH is unstable.
  - Negative specific heat
  - Evaporation time: $$\frac{dM}{dt} \sim T^2 \sim \frac{1}{M^2} \qquad \Rightarrow t_{\mathrm{evap}} \sim M^3$$

“Information puzzle”

- Naive application of local field theory on a global slice (and the semi-classical approx.)

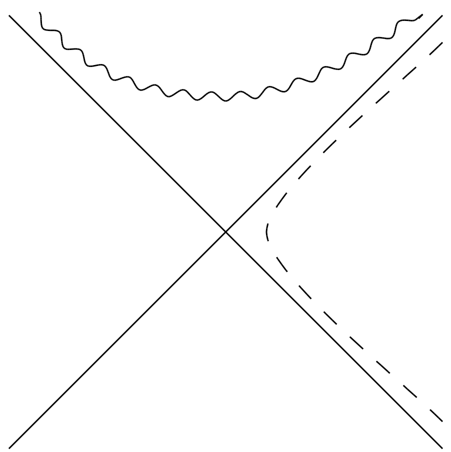
$$\mathcal{H}(\Sigma) = \mathcal{H}(\Sigma_{in}) \otimes \mathcal{H}(\Sigma_{out})$$

seems to suggest information loss (pure state evolving into mixed state).



Complementarity

- Consistent theory should be defined on a patch that a single observer can see.
  - Interior view: things fall into a BH
  - Exterior view: BH is a hot membrane on the “stretched horizon” which emits Hawking rad.

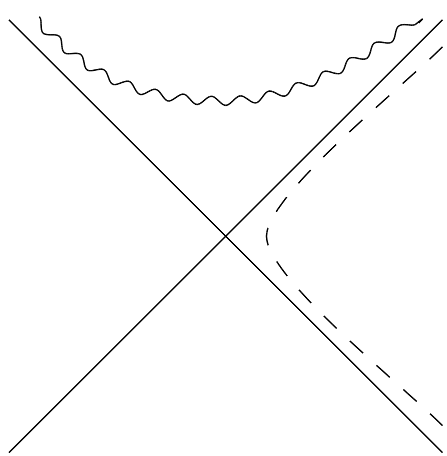


- These viewpoints are “complementary” (different descriptions of the same phenomenon).

Complementarity

<u>Hypothesis:</u>

- Hawking radiations should carry information on the quantum states that have fallen into the BH.
- Formation and evaporation of BH is a unitary process.

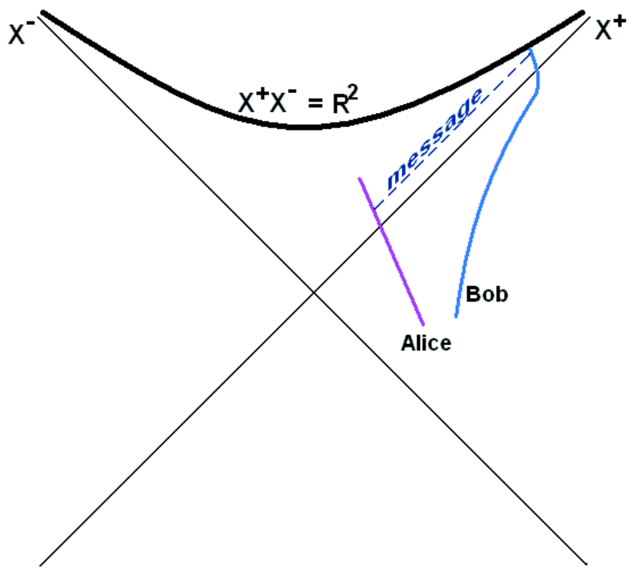


- This is supported by string theory (AdS/CFT, Matrix theory).

Potential trouble for complementarity

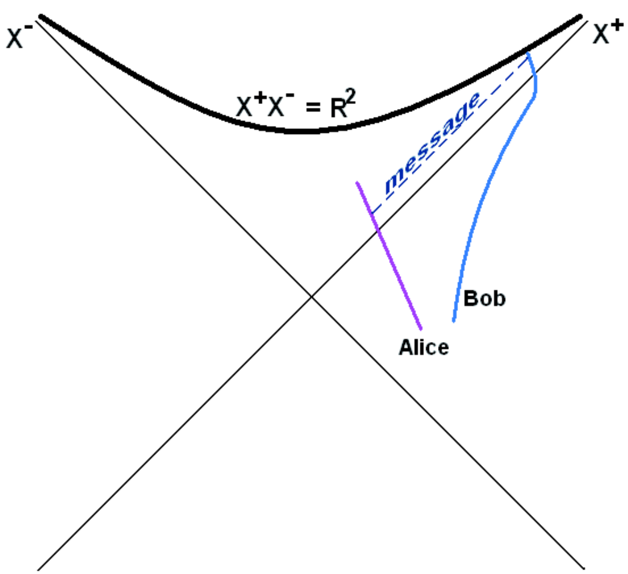
- Isn't a quantum state cloned?

  → There is no problem, if a single observer cannot copy a quantum state.



<u>Thought experiment (Susskind-Thorlacius, '94):</u>
Can Bob get two copies of Alice's quantum state?

- Bob stays at $\rho = R$
- After time $\omega_{\rm ret}$ he jumps into the BH. $(X^+ = R\exp(\omega_{\rm ret}))$
- He hits the horizon at $X^- < R\exp(-\omega_{\rm ret})$.
- If Alice wants that her message reaches Bob before he hits the horizon, she has to send it no later than $\Delta\tau = R\exp(-\omega_{\rm ret})$ after crossing the horizon.



- From the uncertainty principle, she cannot send it no sooner than $\Delta\tau > \frac{1}{M} = \frac{2G}{R}$
- Thus, cloning doesn't happen if $\omega_{\rm ret} > \log R$

Estimate for the retrieval time?

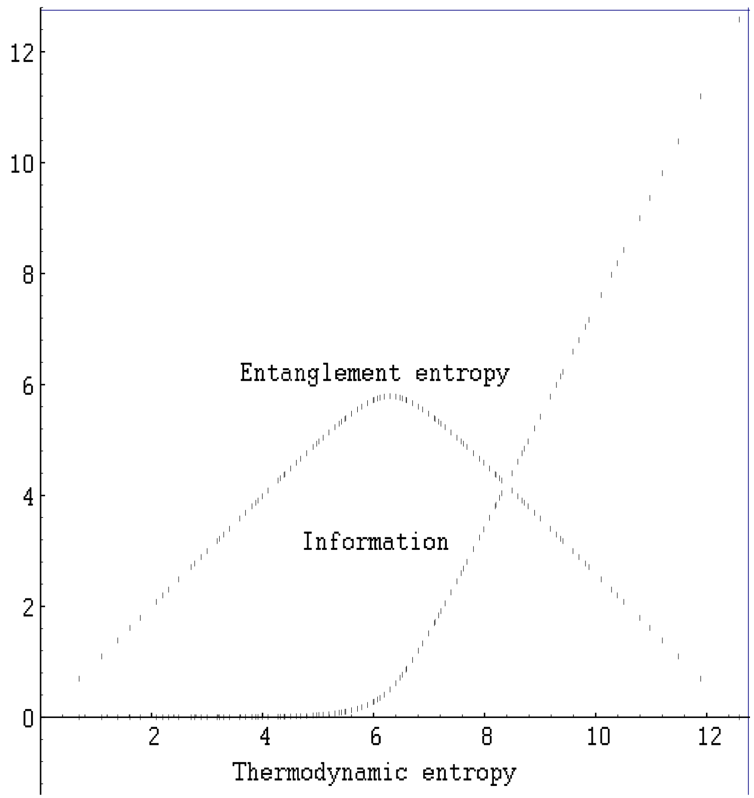
- Consider a system which consists of subsystems A and B. The whole system is in a pure state.
  - Wave function: $\psi(\alpha, \beta)$
  - Density matrix on A: $(\rho_A)_{\alpha\alpha'} = \sum_{\beta} \psi(\alpha, \beta)^* \psi(\alpha', \beta)$
  - Entanglement entropy: $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$
  - Information in a subsystem:

$$I_A = S_{\max} - S_A, \quad (S_{\max} = \dim(\mathcal{H}_A))$$

- How much info does a "typical" subsystem have?

Average info in a subsystem (Page, ‘93)

- Average over possible pure states
- When subsystem A is smaller than half the whole system, there is almost no info.



$$S_A = m - O(e^{2m-N})$$

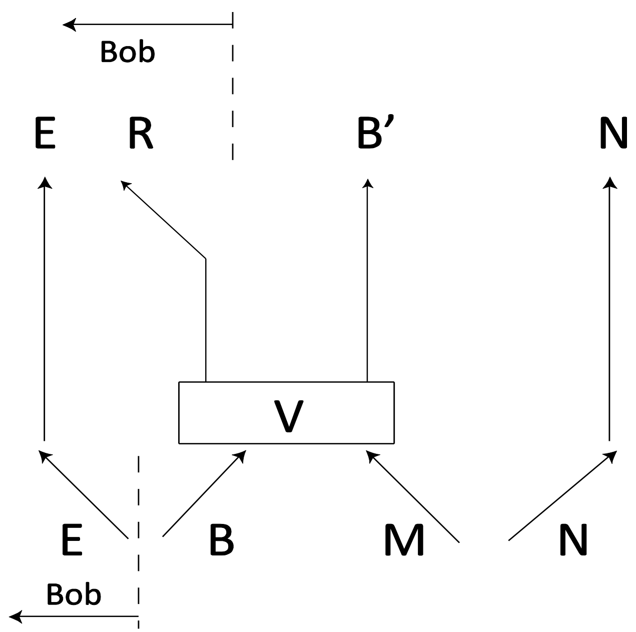
$$2^m = \dim(\mathcal{H}_A), \quad 2^N = \dim(\mathcal{H}_{\text{total}})$$

After “halfway point” of evaporation, information comes out.

This is late enough to prevent cloning.

Refined argument (Hayden-Preskill, '07)

- Assume Bob has collected all the Hawking radiation (since the BH has formed) before Alice jumps into BH. Namely,



E: previously emitted Hawking rad.

B: black hole (Assume dim(E)>dim(B))

M: Alice's message (k bits)

N: reference system (maximally entangled with M)

<u>V: unitary transformation on BM</u>

R: additionally emitted Hawking rad.

B': black hole after emitting R

Hayden and Preskill's result

- For typical unitary V, subsystems B' and N are almost decoupled when slightly more than k bits are emitted.

$$\int dV ||\rho^{NB'}(V) - \rho(V) \otimes \rho_{\max}^{B'}||_1^2 \leq 2^{-2(s-k)}$$

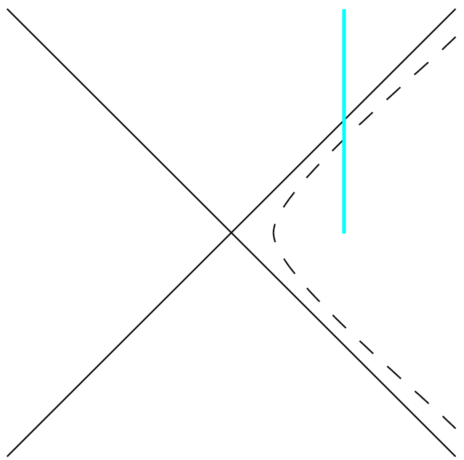
(s: bits in Hawking rad.)

- In other words, the system RE is maximally correlated with N (i.e. Bob has Alice's info) almost immediately after k bits of Hawking radiations are emitted.

- In the above argument, V is completely random on BM.
- The time scale relevant for info retrieval would be the time needed for M to be mixed ("scrambled") with B.

Estimate for the scrambling time

- Consider a charged particle falling into a BH.



Rindler space (near horizon)

$$\begin{aligned} ds^2 &= -\rho^2 d\omega^2 + d\rho^2 + dx_i^2 \\ &= -dt^2 + dz^2 + dx_i^2 \end{aligned}$$

$$(z = \rho \cosh \omega, \quad t = \rho \sinh \omega)$$

Electric field of the point particle

$$E_z = \frac{e(z - z_0)}{[(z - z_0)^2 + x_i^2]^{3/2}} = \frac{1}{\rho} E_\rho$$

- "Membrane paradigm": $\sigma = \frac{1}{4\pi\rho_0} E_\rho$

σ: surface charge density on stretched horizon

- At late time (in ω), surface charge is

$$E_\rho \sim \frac{e}{(\ell_s e^\omega)^2[1+(x_i e^{-\omega}/\ell_s)]^2}$$

  – charge has spread over $\Delta x \sim \ell_s e^\omega$

- Thus, time needed for perturbation to spread over the whole horizon (scrambling time) is

$$\omega_* = \log R/\ell_s \sim \log S$$

  – Note that this is fast. Usually, diffusion takes time $\omega_* \sim N^{2/d}$

Conjectures

1. Fastest scramblers in nature take time log N to scramble information over the whole system (N: # of d.o.f. of the whole system).

2. Black holes saturate the bound.

3. Matrix quantum mechanics (dual to the BH) saturates the bound.

D0-brane black hole

- Metric (in the "decoupling limit")

$$ds^2 = \alpha' \left[-\left(\frac{g_{\rm YM}^2 n}{U^7}\right)^{-1/2} \left(1 - \frac{U_0^7}{U^7}\right) dt^2 + \left(\frac{g_{\rm YM}^2 n}{U^7}\right)^{1/2} \left\{ \left(1 - \frac{U_0^7}{U^7}\right)^{-1} dU^2 + U^2 d\Omega_8^2 \right\} \right].$$

  – Charge (number of the D0-branes): n

  – Energy (mass above extremality), Entropy, temperature

$$E \sim \frac{U_0^7}{g_{\rm YM}^4}, \qquad S \sim \frac{n^2 U_0^{9/2}}{(g_{\rm YM}^2 n)^{3/2}}, \qquad T \sim \frac{U_0^{5/2}}{(g_{\rm YM}^2 n)^{1/2}},$$

  – String coupling:

$$e^{\phi} \sim \frac{1}{n} \left(\frac{g_{\rm YM}^2 n}{U^3}\right)^{7/4}$$

Scrambling time for D0 black hole

- Classical gravity analysis: valid when coupling and curvature are small at the horizon.

$$1 \ll (g_{\rm YM}^2 n/U_0^3) \ll n^{4/7}$$

- Scrambling time (in unit of inverse temperature)

$$\omega_* = C \log n, \quad (\text{since } R_s \sim (g_{\rm YM}^2 n/U_0^3)^{1/4} \ell_s)$$

- Remark:
  True causal structure (including back reaction) is believed to be similar to that for Schwarzschild BH.

Matrix theory

- (0+1) D SYM: Lagrangian is (schematically)

$$L = \mathrm{Tr}\sum_a \dot{X}^a \dot{X}^a - \mathrm{Tr}\sum_{ab}[X^a, X^b]^2$$

$$X^a : n \times n \text{ matrices, } N = n \times n\text{: total number of d.o.f.}$$

- Operator corresponding to the perturbation: $\mathrm{Tr}(X \cdots X)$ (having angular momentum $\ell \sim R_s \sim (g_{\mathrm{YM}}^2 n / U_0^3)^{1/4}$)

- Basic picture of scrambling: at each time step, # of bits "connected" to the perturbed bit grows by a factor of 4 (the # of bits in the interaction term).

Conclusions

- Conjecture:
  - Black holes are the fastest scramblers in nature.
  - Minimal scrambling time: log N
  - Matrix quantum mechanics saturates the bound.

- Problems for future work:
  - More precise definition of scrambling
  - Derivation of the bound including prefactor
  - Information retrieval from cosmological horizons?