Low energy lessons for High energy physics

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Motivation

Issues in Quantum Gravity

- Information loss paradox
- Black hole microstates
- Allowed semi-classical geometries
- Cosmic censorship

Main theme

- Propose to discuss some of these issues using concepts from a low energy (condensed matter) perspective.
- Focus on simple models which can teach us some lessons about Quantum Gravity.
- Exploit the broad framework of the gauge-gravity correspondence.

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The information loss problem

- Can one establish concretely how the semi-classical computation of Hawking evaporation fails?
- Explicit model for unitary evolution, say from a dual perspective.

Black hole & microstates

- Are there good diagnostics for geometric characteristics of black holes, viz., singularities, horizons, etc?
- Can one tell apart black holes from microstates (say fuzzball geometries)?

Motivation

Restrictions on semi-classical geometries?

- Are there any restrictions on semi-classical geometries that are allowed in a full quantum theory of gravity?
- Can one postulate precise conditions on causal structures that are acceptable?

Cosmic censorship & Singularities

- Does the gauge-gravity correspondence provide a framework to prove cosmic censorship?
- Can one understand physics in the vicinity of spacelike singularities?
- Singularity resolution?

Quantum Quench: The problem

Sudden quenches

- Consider a quantum system with Hamiltonian $\mathrm{H}(\lambda)$ where λ is a control parameter.
- Lets take the Hamiltonian to be

$$\mathrm{H} = \begin{cases} \mathrm{H}(\lambda_0) & = \mathrm{H}_0 \ , \qquad \textbf{for} \ \ t < 0 \\ \mathrm{H}(\lambda_1) & = \mathrm{H}_1 \ , \qquad \textbf{for} \ \ t \geq 0 \end{cases}$$

  which we achieve by suddenly tuning λ – this the the quench.
- Start off with the system in the ground state $|\,\psi_0\,\rangle$ of H_0.
- $|\,\psi_0\,\rangle$ is not the ground state of H_1: rather it is an excited state.
- Physically relevant in studies of cold atoms, optical traps, etc..

Quantum Quench: Observables

Sudden quenches

- The main question concerns the evolution of observables: correlation functions of local observables in the state ψ_0 evolved now with H_1

$$\langle\, \mathcal{O} \,\rangle_0 = \langle\, \psi_0 \,|\, e^{-i\,H_1\,t}\, \mathcal{O}(\{x_i\})\, e^{i\,H_1\,t}\,|\,\psi_0\,\rangle$$

- In particular, we are interested in the asymptotic behaviour of the correlation functions at late times.
- Conventional intuition would tell us that:
  - ⋆ If $|\,\psi_0\,\rangle$ is a generic excited state, we should expect the system thermalize.
  - ⋆ In particular, the correlator should show signs of exponential decay at late times.
- Similarity with the issue of black hole formation: evolution of an highly excited, but pure state.

Boundary CFT and the quench problem

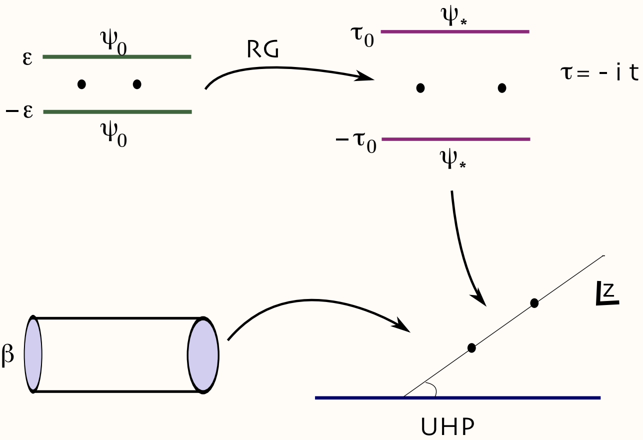
Quench & CFT

- To extract the asymptotic late-time behaviour of the correlator, one can use CFT techniques if H_1 corresponds to a critical point.

Calabrese & Cardy

- Basic idea: compute the correlation functions in Euclidean time with an appropriate $i\,\epsilon$ prescription with $|\,\psi_0\,\rangle$ providing the boundary conditions.
- To compute asymptotics, one can replace the state $|\,\psi_0\,\rangle$ by an effective state $|\,\psi_*\,\rangle$ which is RG invariant.
- Encode the distinction between $|\,\psi_0\,\rangle$ and $|\,\psi_*\,\rangle$ by a single parameter, the extrapolation length, τ_0.
- Claim: Correlation functions can be computed by looking at CFT correlators on a strip of width $2\,\tau_0$.

Boundary CFT and the quench problem



Boundary CFT and the quench problem

Asymptotics from CFTs

- One-point functions of generic operators relax to their ground state values exponentially.

$$\langle\, \mathcal{O}(t)\, \rangle_0 \propto e^{-\frac{\pi\,\Delta_{\mathcal{O}}}{2\,\tau_0}\, t}$$

- Relaxation time is set by the conformal dimension $\Delta_{\mathcal{O}}$.
- The connected two point function also damps out exponentially:

$$\langle\, \mathcal{O}(r,t)\, \mathcal{O}(0,t)\, \rangle_0^c \propto \begin{cases} 0 & \text{for } \; t < x/2 \\ e^{-c_1 x - c_2\, t} & \text{for } \; t > x/2 \end{cases}.$$

- Correlation functions share characteristics of thermal correlators, with effective temperature

$$T_{\text{eff}} = \frac{1}{4\,\tau_0}$$

Quench and information loss

Lessons for the information loss paradox

- The quench problem confirms our basic intuition: systems started off in a highly excited state tend to thermalize.
- Of course, thermalization is the effect of coarse-graining.
- Following the system with arbitrary precision will allow us to distinguish the precise state we are in.
- Focussing on the thermodynamic limit one sees non-unitary evolution. Festuccia & Liu; Izuka, Polchinski + Okuda

Quench and information loss

The Generalized Gibbs Ensemble

Conjecture: The asymptotic behaviour of the correlation function is given in terms of a stationary state governed by a Gibbs ensemble

$$\rho = \frac{1}{Z}\, e^{-\alpha_m\, Q_m}$$

where Q_m is the maximal set of commuting, linearly independent integrals of motion. Rigol, Djunko,Yurovsky, Olshanii

- The effective temperature is determined by α_m, which are fixed by the initial state condition.
- Does this imply that black holes are best thought of a mixed states governing thermal equilibrium?

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On microstates and geometries

Microstate perspective

- Black hole microstates correspond to smooth, horizon free 'geometries'. Mathur, Lunin, Bena, Warner, ⋯
- The microstate geometries differ from the black hole spacetime inside the horizon, being comprised of some spacetime foam.

Can correlation functions $\langle\, \mathcal{O}(x)\, \mathcal{O}(y)\, \rangle_*$ distinguish

- microstates from each other?
- individual microstates from thermal state?

On microstates and geometries

For a system with large entropy S:

- Classical degeneracy expected to be broken by quantum effects with

$$\Delta E \sim e^{-S}$$

Balasubramanian, Marolf, Rozali

- Expect e^{-S} to govern characteristic scales of deviations between microstates $\Longrightarrow$ resolving power of e^{-S}.
- The relevant time scale from analysis of correlation functions in the canonical and micro-canonical ensembles is the Poincaré time

$$t_{\text{dist}} \sim e^{S}$$

On microstates and geometries

Macrostate perspective

- Black holes are characterized by non-trivial causal structure.
- Microstate geometries do not have complicated causal structure; the spacetime foam can however act coherently.
- Two spacetime boundaries for eternal black holes versus one boundary for microstates.

On microstates and geometries

- AdS/CFT correspondence can be interpreted as an isomorphism between $\mathcal{H}_{\text{bulk}}$ and $\mathcal{H}_{\text{CFT}}$, for pure states and for density matrices.

$$|\text{pure}\rangle_{\text{bulk}} \quad \leftrightarrow \quad |\text{pure}\rangle_{\text{CFT}}$$

$$\rho_{\text{bulk}} \quad \leftrightarrow \quad \rho_{\text{CFT}}$$

  ⋆ Easy to construct geometries dual to density matrices in the field theory, like black hole or Wheeler bags of gold.

Frievogel, Hubeny, Maloney, Myers, MR, Shenker

- Having a dual semi-classical geometry is neither necessary nor guaranteed for most states in the CFT.

On microstates and geometries

- Eternal black holes in AdS correspond to the thermal density matrix.
- Should be able to tell apart the black hole from a microstate.
- Use the double boundary picture seriously – analytic continuation.

Balasubramanian, Czech, Hubeny, Larjo, MR, Simón

- We however have to first deal with influence of statistics on our notions of distinguishability of microstates: the bane of ensemble equivalence.

Basis states in the microcanonical ensemble

Consider the microcanonical ensemble at energy E, with energy resolution $\mathcal{O}(\Delta E)$. We can choose to parameterize the states by

- Energy eignestates:

$$\mathcal{M}_{\text{bas}} = \{ \ |s\rangle \ : \ H|s\rangle = e_s|s\rangle \quad ; \quad E \leq e_s \leq E + \Delta E \ \}$$

- Normalized superpositions of energy eigenstates:

$$\mathcal{M}_{\text{sup}} = \left\{ |\psi\rangle = \sum_s c_s^\psi |s\rangle \right\} \ , \qquad \sum_s |c_s^\psi|^2 = 1$$

Note that

$$\dim(\mathcal{M}_{\text{sup}}) = \dim(\mathcal{M}_{\text{bas}}) - 1 = e^S$$

Variances in the microcanonical ensemble

Consider some local operator $\mathcal{O}$ whose correlator we want to measure in the ensemble of microstates.

Entropic suppression

The variance in the ensemble of superpositions is diminished by a factor of e^{S} in comparison to the variance in the ensemble comprising of energy eigenstates:

$$\mathrm{Var}(\mathcal{O})_{\mathcal{M}_{\rm sup}} = \frac{1}{\mathrm{e}^{\mathrm{S}}+1}\,\mathrm{Var}(\mathcal{O})_{\mathcal{M}_{\rm bas}}$$

- $\mathcal{M}_{\rm sup}$ gives us the worst case scenario for distinguishing microstates.
- We need to defeat the exponential suppression in order to be able to distinguish the microstates (apart from the usual statistical suppression).

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Canonical versus microcanonical ensemble

- Calculation in the microcanonical ensemble in general is non-trivial.
- Compare the canonical expectations to get an estimate of how the variance behaves.
- This will certainly give us information about how the canonical ensemble differs from the microcanonical: distinguish pure states from mixed!

Two toy models

- Free chiral boson with $E \gg 1$ for statistics.
- D1-D5 system and the $M = 0$ BTZ black hole.

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Results from toy models

- In the free chiral boson case, we can can show that variances get large for

$$\tau_{\text{bas}} \sim \frac{\beta}{2}$$
$$\tau_{\text{sup}} \sim \frac{3\beta}{2}$$

- In the more interesting D1-D5 system which captures the physics of fractionation, variances get large for imaginary time

$$\tau_{\text{bas}} \sim \mathrm{S}$$
$$\tau_{\text{sup}} \sim \mathrm{S}^2$$

- In either case the time scales are much shorter than e^{S}.

Details of free boson | Details of D1-D5

Summary thus far

- Black hole geometries are best thought of as mixed states. Suggested by the
  - ⋆ Thermalization picture resulting from quantum quenches.
  - ⋆ Hydrodynamic description of interacting QFTs.
- It is possible to distinguish black holes from microstates, provided we wander off into the complex plane.
- Non-trivial causal structure is key to these complex excursions.

Causal structures

Restrictions on causal structure

- One important issue in the context of holography as realized by the gauge-gravity correspondence is the nature of allowed causal structures in the bulk spacetime.
- Does it suffice for semi-classical spacetimes, to be causal i.e. devoid of closed causal curves to allow for a dual description? Or does one need a more stringent criterion?

Hierarchy of causality conditions

The causality hierarchy

∃ a hierarchy of causality conditions which are inclusive:

$$\begin{aligned} \text{Causal} \quad &\Leftarrow \quad \text{Distinguishing} \Leftarrow \text{Strong causality} \\ &\Leftarrow \quad \text{Stable causality} \Leftarrow \text{Global hyperbolicity} \end{aligned}$$

▸ Causality conditions

Question

What is the minimum causality contrainst a spacetime must satisfy in order for it to admit a dual holographic description?
Naive answer: Stable causality.

Holographic duals for "cold atoms"

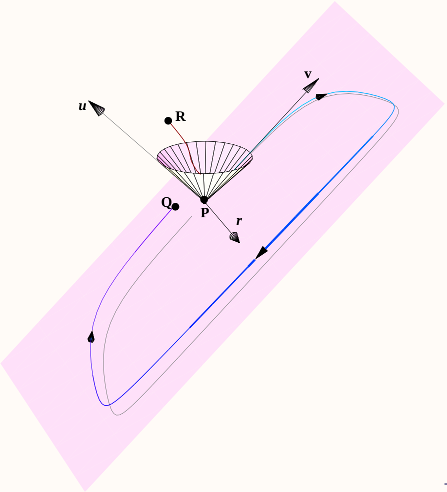
- Recently, spacetimes with Schrödinger group isometry have been discussed as potential playgrounds to understand physics of cold atoms. Son; Balasubramanian, McGreevy
- The dual spacetimes are of the form

$$\mathrm{ds}^2 = \mathrm{r}^2 \left(-2\,\mathrm{dudv} - \mathrm{r}^{2\nu}\,\mathrm{du}^2 + \mathrm{dx}^2\right) + \frac{\mathrm{dr}^2}{\mathrm{r}^2}$$

- $\nu = 1$ is the conformal case and $\nu = 2$ is realized for light-like non-commutative SYM.
- These spacetimes can be realized as near-horizon limits of D3-branes probing a Null Melvin geometry.

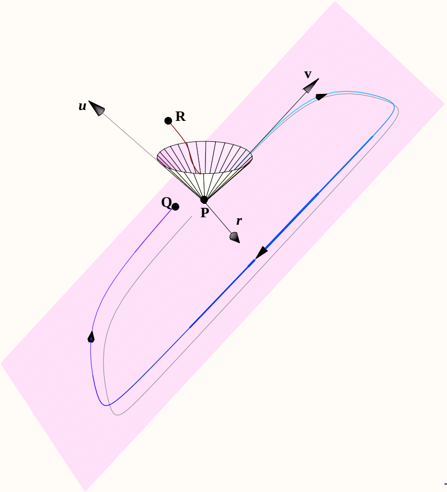
  Herzog, MR, Ross; Maldacena, Martelli, Tachikawa; Adams, Balasubramanian, McGreevy
- These spacetimes are non-distinguishing for $\nu \neq 0$.

Why is the spacetime non-distinguishing?



- The causal future of $p = (u_0, v_0, r_0, \vec{x}_0)$ is the set of points with $u > u_0$.
- So every point on a plane of constant u shares the same causal future.

Why is the spacetime non-distinguishing?



- Effectively, the spacetime has almost closed timelike curves.
- Nevertheless, we know what the field theory dual is and it is sensible!

Hubeny, MR, Ross

Moral from cold atoms

Non-relativistic symmetry and non-distinguishability

- The fact that the dual theory has non-relativistic invariance, necessitates that the bulk spacetime be non-distinguishing.
- Otherwise it would not be possible for a sensible bulk spacetime with local Lorentz invariance to be dual to a theory with Galilean symmetry.

Moral from cold atoms

Non-relativistic symmetry and non-distinguishability

- Note that the field theory (for $\nu = 1$) is $\mathcal{N} = 4$ SYM deformed by a (heterotic) star product

$$f \star g = e^{i \left(\mathcal{V}^f R^g - \mathcal{V}^g R^f \right)} f\, g$$

  where $\mathcal{V}$ is the v-momentum of the field and R refers to a global $U(1)_R$ charge.

- These theories provide a playground to explore interesting physics in strongly coupled non-relativistic CFTs, e.g. thermodynamics, transport coefficients, etc..
- At the same time they also provide important lesson for Quantum Gravity, viz., non-distinguishingness is 'acceptable'.

Summary

- Condensed matter systems provide a vast theoretical laboratory to study issues relevant for quantum gravity via gauge/gravity correspondence and its generalizations.
- Understanding the details of quantum quench phenomenon in greater generality should shed light on information loss.
- Analytic properties of correlators are a useful diagnostic to distinguish black holes (eternal) from the microstates.
- The increase in number of "AdS/CFT" examples provides us with new arenas to explore questions relevant for quantum gravity, such as nature of causal structure.

Open questions

- A precise formulation for the quench problem in $\mathcal{N} = 4$: In particular, understand the non-equilibrium physics of evolution from a generic high energy state.
- How does physics of relaxation feed into the issue of scrambling?
- Clean understanding of the spectrum at $E \sim \mathcal{O}(N^2)$: relevant for quench and also distinguishability of states.
- Manifestation/diagnostics of bulk causal structure in the field theory?

I. The free field results

- For a free chiral boson one can show that

$$\frac{\sqrt{\mathrm{var}(\mathcal{O}(\tau))_{\mathcal{M}_{\mathrm{bas}}}}}{\langle \mathcal{O}(\tau)\rangle_{\mathcal{M}_{\mathrm{bas}}}} \gg 1$$

  for Euclidean time scale τ:

$$\tau \sim \frac{\beta}{2}$$

- The entropic suppression in $\mathcal{M}_{\mathrm{sup}}$ makes its presence felt by increasing the relevant time scale:

$$\tau \sim \frac{3\beta}{2}$$

- The calculation is easy in the canonical ensemble as two-point functions are linear in the occupation numbers $\{N_n\}$.

I. The free field results

Moral from free field

- Simple probes (like $\mathrm{Tr}\left(X^{(i}X^{j)}\right)$ are able to distinguish microstates from thermal state at
$$\tau_{\text{dist}} \sim \beta \propto \frac{1}{S}$$
- Contrast this with usual Poincaré recurrence time $t_P \sim e^S$.

Some caveats

- The free field theory doesn't describe a black hole!
- Single chiral boson is incapable of encoding fractionation that is crucial to the picture of the microstate geometries.

◂ Back to toy models

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II. The D1-D5 system

- For the D1-D5 system at the orbifold point, we can calculate

$$\langle\{N_{n\mu}, N'_{n\mu}\}|\mathcal{A}^{\dagger}(t,\phi)\mathcal{A}(0,0)|\{N_{n\mu}, N'_{n\mu}\}\rangle$$

Balasubramanian, Kraus, Shigemori

- Like the free field case the correlation function is a linear function of the occupation numbers $\{N_{n\mu}, N'_{n\mu}\}$.
- Variances easy to estimate in the canonical ensemble from standard statistical distributions.

II. The D1-D5 system

Moral from fractionated free field

- Variances in the correlation function get large for

$$\tau \sim \log S$$

- Simple probes (like $\mathbf{Tr}\left(X^{(i}X^{j)}\right)$ are able to distinguish microstates in $\mathcal{M}_{\text{bas}}$ from thermal state at

$$\tau \sim S$$

- Folding in the exponential supression factor for $\mathcal{M}_{\text{sup}}$ we find that the relevant timescale is

$$\tau_{\text{sup}} \sim S^2$$

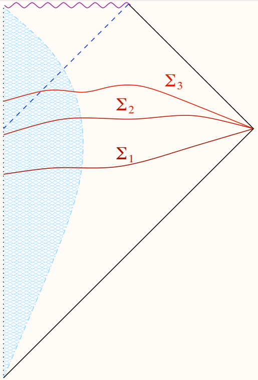
Back to toy models

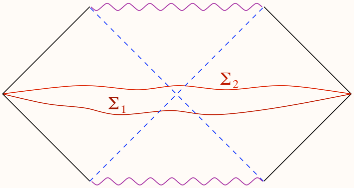
Causality conditions I: Top-Down

1. *Global hyperbolicity:* A spacetime is said to be globally hyperbolic if it admits a Cauchy surface.

Examples of globally hyperbolic spacetimes

Minkowski space, Schwarzschild black hole.





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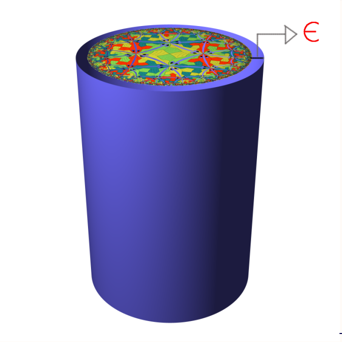
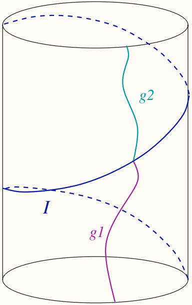
Non globally hyperbolic spacetimes

AdS, plane wave geometries.





Causality conditions I: Top-Down

1. *Global hyperbolicity:* A spacetime is said to be globally hyperbolic if it admits a Cauchy surface.
2. *Stable causality:* A stably causal spacetime is one that admits a time-function, i.e.,

$$\exists \text{ smooth } t : \mathcal{M} \to \mathrm{R}, \text{with } \|\nabla_a t\|^2 < 0 \text{ everywhere}$$

Examples

Minkowski space, AdS, plane wave spacetimes.

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3. *Strong causality:* For point $p \in \mathcal{M}$, causal curves passing close to p do not come arbitrarily close to being CCCs.

◂ Causal hierarchy

Causality conditions II: Bottom-Up

1. *Causal:* A causal spacetime is one which is devoid of closed causal curves.

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Not Examples

Gödel,Minkowski space with periodic time identification.

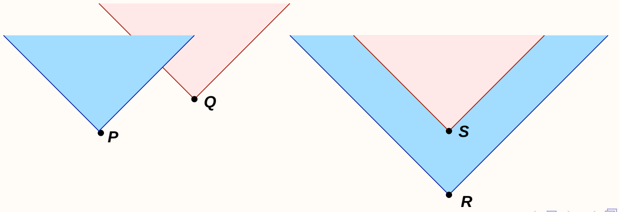
Causality conditions II: Bottom-Up

1. *Causal:* A causal spacetime is one which is devoid of closed causal curves.
2. *Distinguishing:* A spacetime is said to be distinguishing if we can distinguish points on the manifold $\mathcal{M}$ based on their causal sets. For $p, q \in \mathcal{M}$,

$$\mathcal{J}^{\pm}(p) = \mathcal{J}^{\pm}(q) \Rightarrow p = q$$

Examples of distinguishing spacetimes

Minkowski space, AdS, plane wave spacetimes.



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Examples of non-distinguishing spacetimes

A large class of pp-wave spacetimes are non-distinguishing.

Causal hierarchy