Black holes in fluid/gravity correspondence

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S Bhattacharyya, VH, S Minwalla, M Rangamani; arXiv:0712.2456

S Bhattacharyya, VH, R Loganayagam, G Mandal, S Minwalla, T Morita, M Rangamani, H Reall; arXiv:0803.2526

Veronika Hubeny Black holes in fluid/gravity correspondence

Key question of quantum gravity:

What is the fundamental nature of spacetime?

Invaluable tool in recent years: AdS/CFT correspondence



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 $\begin{array}{rcl} \mbox{string theory in } AdS \times S \\ & \leftrightarrow & \mbox{gauge theory on boundary} \\ \mbox{Schwarzschild-AdS black hole} \\ & \leftrightarrow & \mbox{(approximately) thermal state} \end{array}$

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Key question of quantum gravity:

What is the fundamental nature of spacetime?

Invaluable tool in recent years: AdS/CFT correspondence



Need to probe AdS/CFT dictionary further:

- Which CFT configurations have a spacetime description?
- What types of spacetime singularities are allowed?
- Probe spacetime dynamics.

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Current interesting questions in QCD:

- Explore universal properties of non-abelian plasmas
- Understand quark gluon plasma (RHIC data)

Any strongly interacting field theory admits an effective description in terms of fluid dynamics.

Important questions in fluid dynamics:

- Physics away from thermodynamic equilibrium
- Global regularity of Navier-Stokes equation
- Turbulence
- Causality issues (Israel-Stewart)

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- Motivation
- 2 Background
 - fluid dynamics
 - gravity
 - fluid/gravity map and DOF truncation
 - overview
- 3 Iterative construction of bulk $g_{\mu\nu}$ and boundary $T^{\mu\nu}$
 - Oth order
 - Ist order
- Analysis of 2nd order solution
 - boundary stress tensor and transport coefficients
 - bulk geometry and event horizon
- 5 Summary & Remarks

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Outline

References:

• Significant work over last 6 years in this subject pioneered by

Policastro, Son, Starinets

Herzog, Kovtun, Buchel

many, many others . . .

Inspiration for our work

Janik, Peschanski

Bhattacharyya, Lahiri, Loganayagam, Minwalla

• Related very interesting recent developments

Baier, Romatschke, Son, Starinets, Stephanov; Loganayagam

Van Raamsdonk; Natsuume, Okamura; Haack, Yarom

Dutta; Buchel et.al.

Bhattacharyya, Loganayagam, Minwalla, Nampuri, Trivedi, Wadia

Fluid dynamics

- Fluid dynamics is continuum effective description of any microscopic QFT valid when scales of variation are long compared to mean free path lmfp.
- The fluid description assumes that the system achieves local thermodynamic equilibrium.

Regime of validity: "long-wavelength approximation"

For local temperature of the fluid Tand scale of variation of the dynamical degrees of freedom L, local equilibrium demands:

$$LT \equiv \frac{1}{\epsilon} \gg 1$$

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Fluid dynamics

Dynamical degress of freedom:

- Local temperature T
- Fluid velocity u_{μ} (normalized $\eta^{\mu
 u} \, u_{\mu} \, u_{
 u} = -1$)
- Particle and charge densities ρ and q_i
- Pressure *P* and chemical potentials determined by equation of state
 - For conformal fluids $P = \frac{1}{d-1}\rho$
 - For convenience we will set all charges to zero
 - ρ can be expressed in terms of T

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 - ρ can be expressed in terms of T
- ⇒ This leaves *d* functions, $T(x^{\mu})$ and $u_{\nu}(x^{\mu})$, which specify our fluid configuration.

(x^{μ} are coordinates on the boundary spacetime on which the fluid lives.)

Conformal Fluid dynamics

The conformal fluid stress tensor $T^{\mu\nu}$

Encode all the fluid information by stress tensor $\mathcal{T}^{\mu
u}$, which is

- Traceless: $T^{\mu}_{\ \mu} = 0$
- Conserved: $\nabla_{\mu}T^{\mu\nu} = 0$

The conservation equation encapsulates the dynamical content of fluid dynamics.

Form of stress tensor is determined by symmetries, order by order in derivative expansion; fluid properties specified by finite # of undetermined coefficients.

In *d* dimensions:

$$T^{\mu\nu} = \alpha T^{d} (\eta^{\mu\nu} + d u^{\mu} u^{\nu}) + \pi^{\mu\nu}_{dissipative}$$

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Gravity in the bulk

- Consider any 2-derivative theory of 5-d gravity interacting with other fields with AdS_5 as a solution (e.g. *IIB* SUGRA on $AdS_5 \times S^5$)
- Solution space has a universal sub-sector: pure gravity with negative cosmological constant $(R_{AdS} = 1 \Rightarrow \Lambda = -6)$:

$$E_{MN} \equiv R_{MN} - \frac{1}{2} R g_{MN} + \Lambda g_{MN} = 0$$

- We will focus on this sub-sector in long-wavelength limit.
- Our solutions "tubewise" approximate black branes in AdS.

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A 'Chiral Lagrangian' for black branes

- Isometry group of AdS_5 is SO(4, 2).
- Distinguished subalgebra: Poincare + dilatations
 - *SO*(3) rotations and translations leave black brane invariant
 - dilatation + boosts generate a 4-parameter family of black brane solutions (specified by temperature *T* and velocity u_ν of the brane)
- Our construction promotes these to 'Goldstone fields' (collective coordinate fields) T(x^μ), u_ν(x^μ)
- We determine the effective dynamics for these fields order by order in boundary derivative expansion.

Validity of semi-classical gravity and DOF truncation

Gravity dual to field theory

• The boundary stress tensor is related to the normalizable modes of the gravitational field in AdS.

$$ds^{2} = \frac{dz^{2} + \left(\eta_{\mu\nu} + \alpha z^{d} T_{\mu\nu}\right) dw^{\mu} dw^{\nu}}{z^{2}}$$

• Conversely, to a given a boundary stress tensor $T^{\mu\nu}$ there corresponds an asymptotically AdS solution.

Degrees of freedom counting

- A boundary conformally invariant stress tensor has ^d(d+1)/2 - 1 degrees of freedom.
- ?: Can any such stress tensor give a regular bulk geometry?

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Regularity and dof truncation

- Claim: Regular solutions are given by stress tensors which are fluid dynamical.
- For pure gravity + cosmological constant, this is a reduction of degress of freedom, since fluid stress tensors have d dof (i.e. T and u_{μ}), rather than $\frac{d(d+1)}{2} 1$.
- Uniqueness: In fact, the gravity solutions thus constructed are the most general *regular* long-wavelength solutions to Einstein's equations (gravity & -ve cc).
- i.e. the solutions admit a regular event horizon which shields a curvature singularity.

Overview

Dynamical picture:

- Start with generic high energy initial conditions
- System quickly settles down to local thermodynamic equilibrium ⇔ non-uniform black brane in AdS (described by local velocity and temperature fields)
- Subsequent evolution: hydrodynamics \Leftrightarrow Einstein's equations
- Late time behaviour: relaxation to global equilibrium state ⇔ uniform black brane in AdS

Technical aspects:

- Long-wavelength regime of fluid dynamics: use perturbative expansion in boundary derivatives (exact in radial coordinate).
- We construct the stress-energy tensor $T^{\mu\nu}$ and corresponding metric $g_{\mu\nu}$ to second order in boundary derivative expansion.

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Technical aspects:

- Long-wavelength regime of fluid dynamics: use perturbative expansion in boundary derivatives (exact in radial coordinate).
- We construct the stress-energy tensor $T^{\mu\nu}$ and corresponding metric $g_{\mu\nu}$ to second order in boundary derivative expansion.
- This yields a map between fluid dynamics and gravity.

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Motivation Background Construction Solution Summary

Oth order 1st order

0th order: boosted Schwarzschild-AdS black hole

Start with the well-known stationary solution: (derivation)
 boosted Schwarzschild-AdS₅ black hole (w/ planar symmetry)

$$ds^2 = -2 \, u_\mu \, dx^\mu dr + r^2 \, \left(\eta_{\mu\nu} + [1 - f(r/\pi \, T)] \, u_\mu \, u_
u
ight) \, dx^\mu dx^
u \; ,$$

with $f(r) \equiv 1 - \frac{1}{r^4}$

- It is parameterized by 4 parameters: temperature T and boosts u_i.
- The bulk black hole is dual to a bdy perfect fluid with

$$T^{\mu\nu} = \pi^4 T^4 (\eta^{\mu\nu} + 4 u^{\mu} u^{\nu})$$

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Oth order 1st order

0th order: boosted Schwarzschild-AdS black hole

Causal structure of this solution is:



- spacelike singularity
- regular event horizon
 - timelike boundary (asymptotically AdS)

Deforming the 0th order solution

- Now promote u_{μ} and T to fields depending on the boundary coordinates. Call such a metric $g^{(0)}$.
- Note: $g^{(0)}$ does NOT satisfy the equations of motion:

$$E_{MN} \equiv R_{MN} - \frac{1}{2}g_{MN}R - 6\,g_{MN} = 0$$

• But starting from here we will construct an iterative solution.

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A perturbation scheme for gravity

Assume that the variation in local temperature and velocities are slow

$$rac{\partial_\mu \log T}{T} \sim \mathbb{O}\left(\epsilon
ight) \;, \qquad rac{\partial_\mu u}{T} \sim \mathbb{O}\left(\epsilon
ight)$$

 $\Rightarrow~$ In local patches the solution is like a boosted black brane.

Basic idea

The perturbative scheme is aimed at constructing a regular bulk solution, by patching together pieces of the uniform boosted brane.

Use ϵ as a book-keeping parameter (counting # of x^{μ} derivatives), and expand:

$$g = \sum_{k=0}^{\infty} \epsilon^k g^{(k)}$$
, $T = \sum_{k=0}^{\infty} \epsilon^k T^{(k)}$, $u = \sum_{k=0}^{\infty} \epsilon^k u^{(k)}$

A perturbation scheme for gravity

At a given order in the ϵ -expansion we find equations for $g^{(k)}$. These are ultra-local in the field theory directions and take the schematic form:

$$\mathbb{H}\left[g^{(0)}(u^{(0)}_{\mu},T^{(0)})
ight]g^{(k)}(x^{\mu})=s_{k}$$

- \mathbb{H} is a second order linear differential operator in r alone.
- s_k are regular source terms which are built out of $g^{(n)}$ with $n \le k 1$.

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A perturbation scheme for gravity

Importantly the equations of motion split up into two kinds:

- Constraint equations: $E_{r\mu} = 0$, which implement stress-tensor conservation (at one lower order).
- Dynamical equations: $E_{\mu\nu} = 0$ and $E_{rr} = 0$ allow determination of $g^{(k)}$.

We solve the dynamical equations

$$g^{(k)} = \operatorname{particular}(s_k) + \operatorname{homogeneous}(\mathbb{H})$$

subject to

- regularity in the interior
- asymptotically AdS boundary conditions

(coordinate choice for g)

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Oth order 1st order

Explicit solution to first order

Bulk metric:

$$ds^{2} = -2 u_{\mu} dx^{\mu} dr + r^{2} (\eta_{\mu\nu} + [1 - f(r/\pi T)] u_{\mu} u_{\nu}) dx^{\mu} dx^{\nu} + 2r \left[\frac{r}{\pi T} F(r/\pi T) \sigma_{\mu\nu} + \frac{1}{3} u_{\mu} u_{\nu} \partial_{\lambda} u^{\lambda} - \frac{1}{2} u^{\lambda} \partial_{\lambda} (u_{\nu} u_{\mu}) \right] dx^{\mu} dx^{\nu},$$

with

$$F(r) = \int_{r}^{\infty} dx \frac{x^{2} + x + 1}{x(x+1)(x^{2}+1)} = \frac{1}{4} \left[\ln \left(\frac{(1+r)^{2}(1+r^{2})}{r^{4}} \right) - 2 \arctan(r) + \pi \right]$$

Boundary stress tensor:

$$T^{\mu\nu} = \pi^4 T^4 \left(4 u^{\mu} u^{\nu} + \eta^{\mu\nu} \right) - 2 \pi^3 T^3 \sigma^{\mu\nu}$$

with $\sigma^{\mu\nu}$ = transverse traceless symmetric part of $\partial^{\mu}u^{\nu}$

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Viscosity/entropy ratio

Boundary stress tensor:

$$T^{\mu\nu} = \pi^4 T^4 (4 u^{\mu} u^{\nu} + \eta^{\mu\nu}) - 2 \pi^3 T^3 \sigma^{\mu\nu}.$$

Note: shear of the fluid is defined by

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta} \ \partial_{(\alpha}u_{\beta)} - \frac{1}{3} P^{\mu\nu} \partial_{\alpha}u^{\alpha}$$

where $P^{\mu\nu} = \eta^{\mu\nu} + u^{\mu} u^{\nu}$ is a co-moving spatial projector.

The coeff of $\sigma^{\mu\nu}$ gives the viscosity; here

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

in agreement with well-known results.

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The 4-dimensional conformal fluid from AdS₅

The stress tensor to second order

$$T^{\mu\nu} = (\pi T)^4 (\eta^{\mu\nu} + 4 u^{\mu} u^{\nu}) - 2 (\pi T)^3 \sigma^{\mu\nu} + (\pi T)^2 \left((\ln 2) T^{\mu\nu}_{2a} + 2 T^{\mu\nu}_{2b} + (2 - \ln 2) \left[\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e} \right] \right)$$

$$\begin{split} T_{2a}^{\mu\nu} &= \epsilon^{\alpha\beta\gamma(\mu} \, \sigma^{\nu)}_{\ \gamma} \, u_{\alpha} \, \ell_{\beta} \ , \qquad T_{2b}^{\mu\nu} = \sigma^{\mu\alpha} \sigma^{\nu}_{\ \alpha} - \frac{1}{3} \, P^{\mu\nu} \, \sigma^{\alpha\beta} \sigma_{\alpha\beta} \\ T_{2c}^{\mu\nu} &= \partial_{\alpha} u^{\alpha} \, \sigma^{\mu\nu} \ , \qquad T_{2d}^{\mu\nu} = \mathcal{D} u^{\mu} \, \mathcal{D} u^{\nu} - \frac{1}{3} \, P^{\mu\nu} \, \mathcal{D} u^{\alpha} \, \mathcal{D} u_{\alpha} \\ T_{2e}^{\mu\nu} &= P^{\mu\alpha} \, P^{\nu\beta} \, \mathcal{D} \left(\partial_{(\alpha} u_{\beta)} \right) - \frac{1}{3} \, P^{\mu\nu} \, P^{\alpha\beta} \, \mathcal{D} \left(\partial_{\alpha} u_{\beta} \right) \end{split}$$

with $\mathcal{D} = u^{\mu} \partial_{\mu}$ and $\ell_{\mu} = \epsilon_{\alpha\beta\gamma\mu} u^{\alpha} \partial^{\beta} u^{\gamma}$.

more compact definitions

The 4-dimensional conformal fluid from AdS₅

Fluid description of $\mathcal{N} = 4$ Super-Yang Mills: It is useful to write the second order stress tensor in a different basis of operators $\mathcal{T}_k^{\mu\nu}$:

$$T_{(2)}^{\mu\nu} = \tau_{\pi} \eta \, \mathfrak{T}_{1}^{\mu\nu} + \kappa \, \mathfrak{T}_{2}^{\mu\nu} + \lambda_{1} \, \mathfrak{T}_{3}^{\mu\nu} + \lambda_{2} \, \mathfrak{T}_{4}^{\mu\nu} + \lambda_{3} \, \mathfrak{T}_{5}^{\mu\nu}$$

which manifest the conformal properties.

Baier, Romatschke, Son, Starinets, Stephanov; Loganayagam The fluid parameters (shear viscosity, relaxation timescales, ...) are

$$\eta = \frac{N^2}{8\pi} \ (\pi T)^3$$

$$au_{\pi} = rac{2 - \ln 2}{\pi T} , \qquad \lambda_1 = rac{2 \eta}{\pi T} , \qquad \lambda_2 = rac{2 \eta \ln 2}{\pi T} , \qquad \lambda_3 = 0.$$

which agrees with the results of Baier, Romatschke, Son, Starinets, Stephanov. They also derive the curvature coupling term: $\kappa = \frac{\eta}{\pi T}$

The spacetime geometry dual to fluids

The bulk solution thus constructed is tubewise approximated by a black brane!



Bulk causal structure; in each "tube" metric approximates uniformly boosted Schwarzschild-AdS black hole.

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The event horizon

The background has a regular event horizon.

- One can determine the event horizon locally using the fact that the solution settles down at late times to an uniformly boosted black brane.
- The horizon location can be determined within the perturbation scheme

$$r = r_H(x) = \pi T(x) + \sum_{k=1}^{\infty} \epsilon^k r_{(k)}(x)$$

• In fact, $r_{(k)}(x)$ is determined algebraically by demanding that the surface given by $r = r_H(x)$ be null.

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Cartoon of the event horizon



Note:

• Horizon is null everywhere

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- Late time approach to uniform black brane
- Horizon area increases

The Entropy current

• Given a bulk geometry with a horizon we can determine the Bekenstein-Hawking entropy.

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• Bulk construction of entropy: using area-form A of spatial slices of the event horizon in Planck units.

Fluid entropy current

The area-form A on event horizon can be pulled back to the boundary to define a fluid entropy current J_S^{μ}

$$J_S = *_\eta A$$

with non-negative divergence

$$\partial_{\mu}J^{\mu}_{S} \geq 0$$

Entropy details

Τ^{μν} **g**_{μν}

Properties of Entropy current

• The bulk-boundary pull-back is facilitated by our coordinates: pull-back along radial ingoing geodesics (const *r*)

$$x^{\mu}(\mathcal{H}) \rightarrow x^{\mu}(\mathrm{bdy})$$

- Fluid entropy current consistent with second law and equations of motion has a 5 parameter ambiguity.
- Bulk construction of entropy current is ambiguous, but less so:

 ability to add total derivative terms without changing area
 pull-back is ambiguous to boundary diffeomorphisms. At second order this results in a two parameter ambiguity for Weyl covariant current with positive divergence.

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Summary

- ∃ a map between conformal fluid configurations on ℝ^{d-1,1}
 & regular asymp. AdS_{d+1} non-uniform black branes
 - \Rightarrow gain insight into *generic* behaviour of gravity
- Bulk spacetime solutions
 - naturally uphold Cosmic Censorship
 - imply a new variant of Uniqueness Theorem
- Long-wavelength regime of fluid dynamics allows this construction to any order in a boundary derivative expansion.
- This yields *local* determination of the event horizon.
- The solutions satisfy the Area increase theorem & corresponding entropy current satisfies the 2nd law.
- Recovered the well-known value of viscosity: $\frac{\eta}{s} = \frac{1}{4\pi}$
- Predicted second order fluid parameters $(au_{\pi}, \lambda_1, \lambda_2, \lambda_3)$

Horizon physics described by fluid dynamics...

Where does the fluid live?

- On the event horizon? (null hypersurface, defined globally...)
- On the dynamical horizon? Gourgoulhon & Jaramillo (spacelike hypersurface)
- On the stretched horizon? (a la Membrane Paradigm)

Membrane Paradigm

Thorne, Macdonald, Price

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Horizon interpreted as a fluid membrane with certain dissipative properties: (e.g. electrical conductivity, shear & bulk viscosity, etc.)

 On the spacetime boundary. (AdS/CFT) Fluid dynamics describes the full spacetime, not just horizon.

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Membrane Paradigm

Thorne, Macdonald, Price

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Horizon interpreted as a fluid membrane with certain dissipative properties: (e.g. electrical conductivity, shear & bulk viscosity, etc.)

• On the spacetime boundary. (AdS/CFT) Fluid dynamics describes the full spacetime, not just horizon.

Beyond the horizon...

Consequence of ultra-locality:

- The fluid configuration on the boundary determines the full radial form of the metric in the bulk.
- ullet \Rightarrow The fluid encodes the geometry past the horizon and
- it "knows" about the black hole singularity.
- (However, geometrically the near-singularity structure mimics that of the uniform black brane.)

Puzzles & future directions

- Role of non-long-wavelength bulk semiclassical solutions
- More detailed bulk analysis: horizon topology, nature of curvature singularity, Cosmic Censorship
- ∃ striking difference between turbulence in 3+1 and 2+1 nonrelativistic fluids (eg. inverse cascade)
 ?⇒ qualitative difference in gravitational dynamics (eg. equilibration time of AdS₄ vs. AdS₅ BHs) Van Raams
- Relation to the black hole Membrane Paradigm
- Generalisations: forced fluids, fluids on curved background, charged fluids, extremal fluids (superfluids), non-conformal fluids, non-relativistic fluids, ...
- Gravity dual of turbulence

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Van Raamsdonk

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Boosted Schwarzschild-AdS black hole

• Static Schwarzschild-AdS black hole in planar limit:

$$ds^{2} = r^{2} \left(-f(r) dt^{2} + \sum_{i} (dx^{i})^{2} \right) + \frac{dr^{2}}{r^{2} f(r)}$$

with $f(r) = 1 - \frac{r_+^4}{r^4}$. (\rightsquigarrow temperature $T = r_+/\pi$.)

• To avoid coordinate singularity at the horizon $r = r_+$, use ingoing coordinates: $v = t + r_*$ where $dr_* = \frac{dr}{r^2 f(r)}$:

$$ds^{2} = -r^{2} f(r) dv^{2} + 2 dv dr + r^{2} \sum_{i} (dx^{i})^{2}$$

• Now 'covariantize' by boosting: $v \to u_{\mu} x^{\mu}$, $x_i \to P_{i\mu} x^{\mu}$.

For dealing with regularity issues etc., it is simplest to work in an analog of ingoing Eddington-Finkelstein coordinates.

$${ds}^2=-2\,u_\mu(x)\, {\mathbb S}(r,x)\, {dr}\, {dx}^\mu+\chi_{\mu
u}(r,x)\, {dx}^\mu\, {dx}^
u$$

- The choice of coordinates is such that $x^{\mu} = constant$ are ingoing null geodesics.
- It is well adapted to discuss features of horizon, such as entropy in the fluid language.

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Details of first order computation:

• To solve the equations to first order we need to ensure conservation of the perfect fluid stress tensor

$$\partial_{\mu}T^{\mu\nu}_{(0)}=0$$

which needs to be solved only locally (at say $x^{\mu} = 0$).

• This can be used to eliminate derivatives of $T^{(0)}$ in terms of those of $u_i^{(0)}$.

$$\partial_{\nu}(\pi T^{(0)})^{-1} = \frac{1}{3} \partial_i u_i^{(0)}, \qquad \partial_i (\pi T^{(0)})^{-1} = \partial_{\nu} u_i^{(0)}$$

• Then we solve $\mathbb{H}g^{(1)} = s_1$ where the operators and sources are given as follows:

Computation at first order

The operator \mathbb{H} : Useful to decompose metric perturbations into SO(3) representations: scalars 1, vectors 3 and symmetric traceless tensors 5. For instance, we find:

$$\mathbb{H}_{3} \# = \frac{d}{dr} \left(\frac{1}{r^{3}} \frac{d}{dr} \# \right)$$
$$\mathbb{H}_{5} \# = \frac{d}{dr} \left(r^{5} f(r) \frac{d}{dr} \# \right)$$

The source terms: These differ at various orders in perturbation theory. At first order:

$$s_1^3 = -\frac{3}{r^2} \partial_v u_i^{(0)}$$
$$s_1^5 = -6 r^2 \sigma_{ij}^{(0)}$$

Veronika Hubeny Black holes in fluid/gravity correspondence

solution

Useful fluid velocity gradient quantities

Velocity field u^{μ} naturally decomposes spacetime \rightarrow space + time, w/ induced metric on spatial slices $P^{\mu\nu} \equiv g^{\mu\nu} + u^{\mu} u^{\nu}$.

We can decompose 4-velocity gradient $\nabla^{\nu} u^{\mu}$ as follows:

$$\nabla^{\nu} u^{\mu} = -a^{\mu} u^{\nu} + \sigma^{\mu\nu} + \omega^{\mu\nu} + \frac{1}{3} \theta P^{\mu\nu} ,$$

where expansion, acceleration, shear, and vorticity, are defined as:

$$\begin{split} \theta &= \nabla_{\mu} u^{\mu} = P^{\mu\nu} \nabla_{\mu} u_{\nu} \\ a^{\mu} &= u^{\nu} \nabla_{\nu} u^{\mu} \equiv \mathcal{D} u^{\mu} \\ \sigma^{\mu\nu} &= \nabla^{(\mu} u^{\nu)} + u^{(\mu} a^{\nu)} - \frac{1}{3} \theta P^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \nabla_{(\alpha} u_{\beta)} - \frac{1}{3} \theta P^{\mu\nu} \\ \omega^{\nu\mu} &= \nabla^{[\mu} u^{\nu]} + u^{[\mu} a^{\nu]} = P^{\mu\alpha} P^{\nu\beta} \nabla_{[\alpha} u_{\beta]} \end{split}$$

In terms of these, and adopting the notation (used in $_{\rm Baier\ et.al.})$ $A^{\langle\mu\nu\rangle}$ to denote the symmetric, transverse, traceless part of $A^{\mu\nu}$,

$$T_{2a}^{\mu\nu} = -2 \,\omega^{\rho\langle\mu} \,\sigma_{\rho}^{\nu}$$
$$T_{2b}^{\mu\nu} = \sigma^{\rho\langle\mu} \,\sigma_{\rho}^{\nu\rangle}$$
$$T_{2c}^{\mu\nu} = \theta \,\sigma^{\mu\nu}$$
$$T_{2d}^{\mu\nu} = \mathbf{a}^{\langle\mu} \,\mathbf{a}^{\nu\rangle}$$

and

$$\frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} = \langle \mathfrak{D}\sigma^{\mu\nu} \rangle + \frac{1}{3} \theta \sigma^{\mu\nu}$$

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Event horizon in Vaidya-AdS

Vaidya = spher. sym. black hole with ingoing null matter:

$$ds^2 = -\left(1 - \frac{2\,m(v)}{r}\right)\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

- suppose horizon is at $r = r_H(v)$
- normal $n = dr \dot{r} dv$ is null when

$$r_H(v) = 2 m(v) + 2 r_H(v) \dot{r}_H(v)$$

- Exact solution gives horizon *non-locally* in terms of m(v).
- But for m(v) slowly varying, $\dot{m}(v) = O(\epsilon)$, $m\ddot{m} = O(\epsilon^2)$, use ansatz

$$r_H = 2 m + a m \dot{m} + b m \dot{m}^2 + c m^2 \ddot{m} + \dots$$

• Iterative solution gives a = 8, b = 64, c = 32, ...

Expression for entropy current

The gravitational entropy current:

$$(4\pi \eta)^{-1} J_{S}^{\mu} = \left[1 + b^{2} \left(A_{1} \sigma_{\alpha\beta} \sigma^{\alpha\beta} + A_{2} \omega_{\alpha\beta} \omega^{\alpha\beta} + A_{3} \Re \right) \right] u^{\mu} + b^{2} \left[B_{1} \mathcal{D}_{\lambda} \sigma^{\mu\lambda} + B_{2} \mathcal{D}_{\lambda} \omega^{\mu\lambda} \right] + C_{1} b \ell^{\mu} + C_{2} b^{2} u^{\lambda} \mathcal{D}_{\lambda} \ell^{\mu} + \dots$$

with

$$A_{1} = \frac{1}{4} + \frac{\pi}{16} + \frac{\ln 2}{4}; \qquad A_{2} = -\frac{1}{8}; \qquad A_{3} = \frac{1}{8}$$
$$B_{1} = \frac{1}{4}; \qquad B_{2} = \frac{1}{2}$$
$$C_{1} = C_{2} = 0$$

Entropy current

Divergence of entropy current

Entropy current:

$$(4\pi \eta)^{-1} J_{S}^{\mu} = \left[1 + b^{2} \left(A_{1} \sigma_{\alpha\beta} \sigma^{\alpha\beta} + A_{2} \omega_{\alpha\beta} \omega^{\alpha\beta} + A_{3} \mathcal{R} \right) \right] u^{\mu} + b^{2} \left[B_{1} \mathcal{D}_{\lambda} \sigma^{\mu\lambda} + B_{2} \mathcal{D}_{\lambda} \omega^{\mu\lambda} \right] + C_{1} b \ell^{\mu} + C_{2} b^{2} u^{\lambda} \mathcal{D}_{\lambda} \ell^{\mu} + \dots$$

Divergence of entropy current:

$$4 G_{N}^{(5)} b^{3} \mathcal{D}_{\mu} J_{5}^{\mu} = \frac{b}{2} \left[\sigma_{\mu\nu} + b \left(2 A_{1} + 4 A_{3} - \frac{1}{2} + \frac{1}{4} \ln 2 \right) u^{\lambda} \mathcal{D}_{\lambda} \sigma^{\mu\nu} \right. \\ \left. + 4 b \left(A_{2} + A_{3} \right) \omega^{\mu\alpha} \omega_{\alpha}{}^{\nu} + b \left(4 A_{3} - \frac{1}{2} \right) (\sigma^{\mu\alpha} \sigma_{\alpha}{}^{\nu}) + b C_{2} \mathcal{D}^{\mu} \ell^{\nu} \right]^{2} \\ \left. + \left(B_{1} - 2A_{3} \right) b^{2} \mathcal{D}_{\mu} \mathcal{D}_{\lambda} \sigma^{\mu\lambda} + \left(C_{1} + C_{2} \right) b^{2} \ell_{\mu} \mathcal{D}_{\lambda} \sigma^{\mu\lambda} + \dots \right]^{2}$$

Non-negativity of divergence: $\mathcal{D}_{\mu} J_{S}^{\mu} \geq 0$ (when $\sigma^{\mu\nu} = 0$) demands

$$B_1 = 2 A_3 , \qquad C_1 + C_2 = 0$$