# Three-dimensional Black Holes, Einstein and Non-Einstein

Wei Li

IPMU, Tokyo University, Japan

Quantum Black Hole, IPMU, Sep 13 2008

# Reference

1. Chiral Gravity in Three Dimensions

   with W. Song and A. Strominger

   arXiv:0801.4566

2. Warped $AdS_3$ Black Holes

   with D. Anninos, M. Padi, W. Song and A. Strominger

   arXiv:0807.3040

## Overview

- 3D pure Einstein gravity
- ECFT dual

## Topologically Massive Gravity (with negative $\Lambda$)

- Instability at generic $\mu\ell$
- Chiral gravity at $\mu\ell = 1$
- Symmetry enhancement?

## New Vacua of TMG

- Warped $AdS_3$ vacua
- Warped black holes
- Black hole thermodynamics and conjecture for CFT

## Summary.

# Outline

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  - 3D pure Einstein gravity
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- Topologically Massive Gravity (with negative $\Lambda$)
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  - Warped black holes
  - Black hole thermodynamics and conjecture for CFT
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# 3D pure Einstein gravity

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  1. DOF counting:

$$\begin{aligned} & \text{Spatial metric} + \text{Momenta} - \text{Diffeo} - \text{Bianchi} \\ = & \; 3 + 3 - 3 - 3 \\ = & \; 0 \end{aligned}$$

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- $\exists$ BTZ black holes (*when* $\Lambda < 0$)

  $\Longrightarrow$ **Non-trivial quantum mechanically**.
  - Microscopic origin of BTZ black hole entropy?

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  - Use $AdS_3/CFT_2$

$$\Lambda = -\frac{1}{\ell^2} \qquad \ell \;:\; AdS_3 \text{ radius.}$$

# ECFT dual

*Witten 2008*

- *Assuming holomorphic factorization*, 3D **pure** gravity

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is conjectured to be dual to Extremal-CFT

$$(c_L, c_R) = (24k, 24k) \qquad \text{with} \quad k = \frac{\ell}{16G} \in \mathbb{Z}$$

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- Partition function from ECFT counts BTZ entropy.

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  Deform pure gravity...

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- Add a gravitational Chern-Simons term

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\begin{aligned}
I &= \frac{1}{16\pi G}[\int d^3x\sqrt{|g|}(R-2\Lambda)+\frac{1}{\mu}I_{CS}] \\
I_{cs} &= -\frac{1}{2}\int \mathrm{Tr}(\Gamma\wedge d\Gamma+\frac{2}{3}\Gamma\wedge\Gamma\wedge\Gamma)
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☞ One DOF allows more structures

- When $\Lambda = 0$, TMG allows black holes

*Ait Moussa+Clement+Leygnac 2003*

- When $\Lambda < 0$, TMG allows warped $AdS_3$ and black holes (maybe even more).

*Anninos+WL+Padi+Song+Strominger 2008*

# Topologically Massive Gravity

- The action:

$$I = \frac{1}{16\pi G}[\int d^3x\sqrt{|g|}(R-2\Lambda)+\frac{1}{\mu}I_{CS}]$$

- EOM:

$$\mathcal{G}_{\mu\nu}+\frac{1}{\mu}C_{\mu\nu}=0$$

  - $\mathcal{G}_{\mu\nu}$ : c.c.-modified Einstein tensor

$$\mathcal{G}_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}$$

  - $C_{\mu\nu}$ : Cotton tensor (Weyl tensor vanishes identically in 3D.)

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- All solutions of Einstein gravity are also solutions of TMG.

# $AdS_3$ vacuum

- TMG has an $AdS_3$ vacuum

$$ds^2 = \ell^2(-\cosh^2\rho d\tau^2 + \sinh^2\rho d\phi^2 + d\rho^2)$$

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  - Need to specify boundary condition.

# Choosing boundary conditions

1. Criteria : **As weak as possible while keeping charges finite**
   - Not too restrictive (to allow non-trivial configuration).
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$$\mathrm{ASG} \equiv \frac{\text{Allowed Symmetry Transformations}}{\text{Trivial Symmtry Transformations}}$$

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3. Physical states are in representation of ASG (annihilated by trivial symmetries.)

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$\Longrightarrow$ Critical point at $\boldsymbol{\mu\ell = 1}$

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1. Unitarity $\Longrightarrow c \geq 0 \quad \Longrightarrow \boldsymbol{\mu\ell \geq 1}$
2. $\mathbf{c_L = 0}$ at $\mu\ell = 1$.

# BTZ black holes in Einstein gravity

- The only black holes in 3D pure Einstein gravity

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{N(r)^2} + r^2(d\phi + N^\phi(r)dt)^2$$

where

$$N(r)^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2} \,, \qquad N^\phi(r) = \pm\frac{r_+ r_-}{\ell r^2}$$

$r_\pm$ : outer and inner horizon.

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$\Longrightarrow$ Upper bound on $j$: $|j| \leq \ell m$

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- Also solutions of TMG
- Different conserved charges when measured in TMG (*CS term gives additional surface term*) *Kraus+Larsen 2005*

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- Also solutions of TMG
- Different conserved charges when measured in TMG (*CS term gives additional surface term*) *Kraus+Larsen 2005*

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2. $\mathbf{M = J}$ (BTZ becomes right moving!) at $\mu\ell = 1$.

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*WL+Song+Strominger 2008*

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   $\Longrightarrow$ Massive graviton becomes pure gauge in the bulk.

# Energy of massive graviton

- Energy of massive graviton:

$$E_M \sim -\frac{1}{\mu}(\mu^2 - \frac{1}{\ell^2})$$

| Branch | $\mu\ell < 1$ | $\mu\ell = 1$ | $\mu\ell > 1$ |
|---|---|---|---|
| **Massive** | $+$ | **0** | $-$ |

  - $E_M \geq 0 \quad \Longrightarrow \quad \boldsymbol{\mu\ell \leq 1}$

# TMG summary

- Summary of TMG so far

| | $\mu\ell < 1$ | $\mu\ell = 1$ | $\mu\ell > 1$ |
|---|---|---|---|
| $(c_L, c_R)$ **of CFT** | $(-,+)$ | $(0, \frac{3\ell}{G})$ | $(+,+)$ |
| $(h, \bar{h})$ **of massive graviton** | $(+,-)$ | $(2,0)$ | $(+,+)$ |
| **Energy of BTZ BH** | $-$ or $+$ | $0$ or $+$ | $+$ |
| **Energy of massive graviton** | $+$ | $0$ | $-$ |

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- TMG with $\Lambda = -1/\ell^2$ is **unstable** for generic $\mu$.
- **Exception :** $\mu\ell = 1$.

# Proof of chiral gravity at $\mu\ell = 1$

*Strominger 2008*

- ASG for **Brown-Henneaux** b.c.

$$\begin{aligned}\zeta &= [\epsilon^+ + \frac{e^{-2\rho}}{2}\partial_-^2\epsilon^- + \mathcal{O}(e^{-4\rho})]\partial_+ \\ &+ [\epsilon^- + \frac{e^{-2\rho}}{2}\partial_+^2\epsilon^+ + \mathcal{O}(e^{-4\rho})]\partial_- \\ &+ [\partial_+\epsilon^+ + \partial_-\epsilon^- + \mathcal{O}(e^{-2\rho})]\partial_\rho\end{aligned}$$

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  - $\epsilon^-(x^-)$ and $\epsilon^+(x^+)$ parameterize the left and right diffeomorphism.

- Diffeomorphism generator:

$$Q[\zeta] = \int_{\partial\Sigma} \sqrt{\sigma} u^i T_{ij} \zeta^j$$

$T_{ij}$ : **boundary stress tensor**.

# Proof of chiral gravity at $\mu\ell = 1$, cont.

- Boundary stress tensor:

$$T = \frac{1}{8\pi G\ell} \begin{pmatrix} (1 + \frac{1}{\mu\ell})h_{++} & -h_{+-} \\ -h_{+-} & (1 - \frac{1}{\mu\ell})h_{--} \end{pmatrix}$$

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  - **All left-moving diffeo become trivial.**
  - **Left-moving DOF become pure gauge.**

# Logarithmic mode at chiral point

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  1. Cannot be gauged away.
  2. Violates the Brown-Henneaux boundary conditions logarithmically.

# New boundary condition?

*Strominger, Grumiller+Johansson 2008*

- Brown-Henneaux b.c. of $AdS_3$:

$$\begin{aligned} h_{\rho\rho}, h_{\rho+}, h_{\rho-} &\sim \mathcal{O}(e^{-2\rho}) \\ h_{++}, h_{+-}, h_{--} &\sim \mathcal{O}(1) \end{aligned}$$

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- To accommodate $\psi^{new}$, relaxed b.c. logarithmically (*but only for left components*):

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  - **Important:** cannot relax entire b.c. logarithmically, stress tensor would diverge for generic solutions.

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  - Needs to check whether it is finite and chiral.
- Possible to relax b.c. even more? (with $\epsilon^{-}$ trivial)

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- $\exists$ a new mode that *preserves Brown-Henneaux b.c.*

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- Is it chiral $(E - J = 0)$ ?

# Open problems

1. $\exists$ other consistent $AdS_3$ boundary condition?
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2. ASG enhancement at chiral point?

# Outline

Overview

3D pure Einstein gravity

ECFT dual

Topologically Massive Gravity (with negative $\Lambda$)

Instability at generic $\mu\ell$

Chiral gravity at $\mu\ell = 1$

Symmetry enhancement?

New Vacua of TMG

Warped $AdS_3$ vacua

Warped black holes

Black hole thermodynamics and conjecture for CFT

Summary.

# Motivation

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  - All are perturbatively unstable — except at $\mu\ell = 1$.
- **Question:** $\exists$ **stable** vacua at generic $\mu$?

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- Spacelike fibration
- Timelike fibration

# Constructing warped $AdS_3$, cont.

- Varying size of $S^1$ fiber gives warped $AdS_3$.

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- New scale and warping factor
  1. TMG modified length $L = \ell \cdot \frac{6}{\sqrt{\mu^2 \ell^2 + 27}}$ $\rightarrow \ell$ at $\mu\ell = 3$.
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- Any warped $AdS_3$ at $\mu\ell = 3$?

# Null warped $AdS_3$

- $\exists$ null warped $AdS_3$ at $\mu\ell = 3$.
  - Null warping: $ds^2_{null} = \ell^2[\frac{du^2}{u^2} + \frac{dx^+ dx^-}{u^2} \pm \underbrace{(\frac{dx^-}{u^2})^2}_{S^1}]$

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☞ Solution of TMG *only at* $\mu\ell = 3$.

# A summary of results

*Anninos+WL+Padi+Song+Strominger 2008*

- ∃ Six types of warped $AdS_3$ as TMG vacua (two for every value of $\mu$):

| | Timelike | Null | Spacelike |
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| $\mu\ell > 3$ | Timelike Stretched | – | Spacelike Stretched |
| $\mu\ell = 3$ | $AdS_3$ | Null warped | $AdS_3$ |
| $\mu\ell < 3$ | Timelike squashed | – | Spacelike squashed |

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☞ Critical point: $\mu\ell = 3$

# Stability of warped $AdS_3$

- To understand the stability of the warped $AdS_3$ vacua, need to
  1. Determine appropriate boundary conditions for warped $AdS_3$
  2. Solve for the linearized spectrum

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  1. Determine appropriate boundary conditions for warped $AdS_3$
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- Do not know whether or when warped $AdS_3$ are perturbatively stable — yet.

# Quotienting procedure

- BTZ black holes are quotients of $AdS_3$:
  - Identifying points $\mathcal{P}$ under action of $\xi = T_L J_L + T_R J_R$:

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- Quotienting warped $AdS_3$ gives warped black holes.

# Summary of warped black holes

*Anninos+WL+Padi+Song+Strominger 2008*

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| | Timelike | Null | Spacelike |
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| $\mu\ell > 3$ | self-dual solutions | – | Spacelike stretched BHs<br>self-dual solutions |
| $\mu\ell = 3$ | BTZ | Null warped BHs | BTZ |
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1. Spacelike stretched black holes. *Bouchareb+Clement 2007*
   - ☞ Reduces to BTZ at $\mu\ell = 3$.
2. Null warped Black holes
3. Self-dual solutions

# Thermodynamics of spacelike-stretched black holes

1. $T_{L/R}$ are given by coefficients of quotienting direction $\xi$

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3. $c_{L/R}$ are independent of $r_\pm$

$$\Longrightarrow \quad \begin{cases} c_L = \dfrac{L}{G}\cdot\alpha \\ c_R = \dfrac{L}{G}\cdot(\alpha + \dfrac{1}{\alpha}) \end{cases}$$

# Conjecture for CFT

- Bulk isometry $U(1)_L \times SL(2,\mathbb{R})_R$ is enhanced at the boundary into $\mathrm{Vir} \times \overline{\mathrm{Vir}}$ with $(c_L = \frac{L}{G} \cdot \alpha, c_R = \frac{L}{G} \cdot (\alpha + \frac{1}{\alpha}))$

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- Open problems
  1. Derive ASG
  2. Compute conserved charges microscopically.
     - Conserved charges (as related to $E_{L/R} = \frac{\pi^2\ell}{6} c_{L/R} T^2_{L/R}$ ):

$$\mathcal{M}^{ADT} = \frac{1}{G}\sqrt{\frac{2\ell E_L}{3c_L}} \qquad \mathcal{J}^{ADT} = \ell(E_L - E_R)$$

☞ Require knowing more than just $c_{L/R}$.

# Connection to other systems

1. Self-dual quotient of spacelike-warped $AdS_3$ appears as constant-$\theta$-slice of extremal Kerr.

   *Guica+Hartman+Song+Strominger, in progress*

   - ☞ Left CFT counts Kerr entropy

2. Null-warped $AdS_3$ as dual to cold atom. *Son 2008*

   *Maldacena+Martelli+Tachikawa, Adams+Balasubramanian+McGreevy 2008*

# Open problems

- Embed warped black holes into string theory, find dual CFT.

# Outline

Overview
- 3D pure Einstein gravity
- ECFT dual

Topologically Massive Gravity (with negative $\Lambda$)
- Instability at generic $\mu\ell$
- Chiral gravity at $\mu\ell = 1$
- Symmetry enhancement?

New Vacua of TMG
- Warped $AdS_3$ vacua
- Warped black holes
- Black hole thermodynamics and conjecture for CFT

Summary.

# Summary

1. Quantizing even **3D pure** gravity is non-trivial.
2. TMG with $\mu\ell = 1$ is chiral
3. New vacua and black holes in TMG need microscopic description.

# Open problems

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  - ☞ Holomorphic factorization?
    - If yes, how to explain non-geometric states?
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- Chiral gravity
  - ☞ Any other consistent $AdS_3$ boundary condition?
  - ☞ Symmetry enhancement at chiral point?
- Warped $AdS_3$ and black holes.
  - ☞ Is warped $AdS_3$ stable?
  - ☞ Find dual CFT of various warped black holes.

THANK YOU !