(2008 9/16 @IPMU)

Hawking Radiation and Gravitational Anomalies

Satoshi Iso

KEK
&
Sokendai (The Graduate university for advanced studies)

S.I , H.Umetsu, F. Wilczek PRL 96 (2006) 151302
PR D74 (2006) 044017
S.I , T.Morita, H. Umetsu JHEP 0704 (2007) 068
PR D75 (2007) 124004
PR D76 (2007) 064015
NP B799 (2008) 60
PR D77 (2008) 045007

and discussions with Sen Zhang（張森）

Can we probe quantum space-time with blackholes ?

Black hole physics remind us of the history of (quantum) statistical physics in the beginning of 20th century

Planck solved the problem of black body radiation.
---> energy quanta

Einstein studied the Brownian motion to establish the existence of atoms.
---> fluctuation-dissipation theorem

Now we may be facing at a similar situation:

Hint: Blackhole thermodynamics

Hawking temperature: $T_H = \frac{\hbar \kappa}{2\pi}$

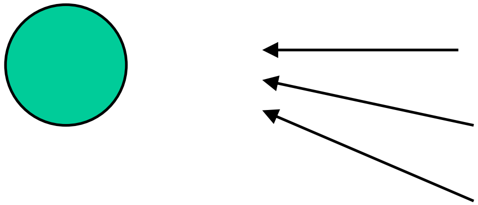
quantum effect

Entropy of BH: $S_{BH} = \frac{A}{4\hbar G} = \frac{A}{4 l_{PL}^2}$

	equilibrium	nonequilibrium (dynamical)
macro	Thermodynamics Wald formula Hawking radiation	information loss? thermalization ? fluid/gravity
micro	Strominger-Vafa Fuzzball ?	matrix ??

Our understanding is yet restricted
within linear response.

Macroscopic vs Microscopic



various
different models

robustness
universality

No hair theorem

The most important property of the thermodynamics is
Robustness.

Microscopic details are not relevant to the thermodynamics.
Universality

Hawking radiation is characterized by the thermal distribution with very few parameters. (no hair theorem)

- Can there be other parameters in the radiation spectrum besides T, Q, J ? ~ representation of W-infinity alg.
- Is the thermal distribution distorted if we consider radiation from dynamical black holes ?
  (radiation from evaporating black holes ?)

Ordinarily these problems are discussed using the Bogoliubov transformation method for a particular wave packet.

We will discuss these issues (robustness and universality) using a field theory method; gravitational anomaly.

Plan of the talk

1. Hawking radiation and gravitational anomalies

   charge flux ← gauge anomaly at horizon

   energy flux ← gravitational anomaly

   Fluxes are universally deterimined.

2. Demystification of the anomaly method

   near horizon conformality

   Fundamental Underlying Relation (FUR)

3. Generalizations to Higher spin currents

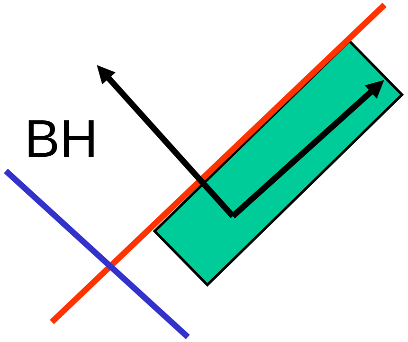
   All higher-spin fluxes can be determined.

   Is the radiation from dynamical black holes distorted ?

4. Discussions

1. Anomaly method

[Robinson Wilczek (05)] ,[S.I. Umetsu Wilczek(06)]



(a) Near horizon d=2 conformal field theory

Outgoing modes = right moving
Ingoing modes = left moving

(b) Classically ingoing modes are decoupled once they fall in the horizon.

→ So we first neglect ingoing modes near the horizon.

The effective theory becomes chiral
in the two-dimensional sense.

Quantum mechanically, gauge and gravitational anomalies
= breakdown of gauge and general coordinate invariance

(c) But the underlying theory is NOT anomalous.

Anomalies must be cancelled by quantum effects of the
classically irrelevant ingoing modes.

$\Rrightarrow$ flux of Hawking radiation

Set up: Reissner-Nordstrom black hole

Reissner-Nordström black hole

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2, \qquad A = -\frac{Q}{r}dt,$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2} \qquad r_\pm = M \pm \sqrt{M^2 - Q^2}$$

surface gravity $\kappa \equiv \frac{1}{2}(\partial_r f)|_{r_+}$

tortoise coordinate

$$dr_* = \frac{dr}{f(r)} \longrightarrow \begin{array}{ll} r_* \sim r & r \to \infty \\ r_* \sim ln(\frac{r}{r_+} - 1) & r \sim r_+ \end{array}$$

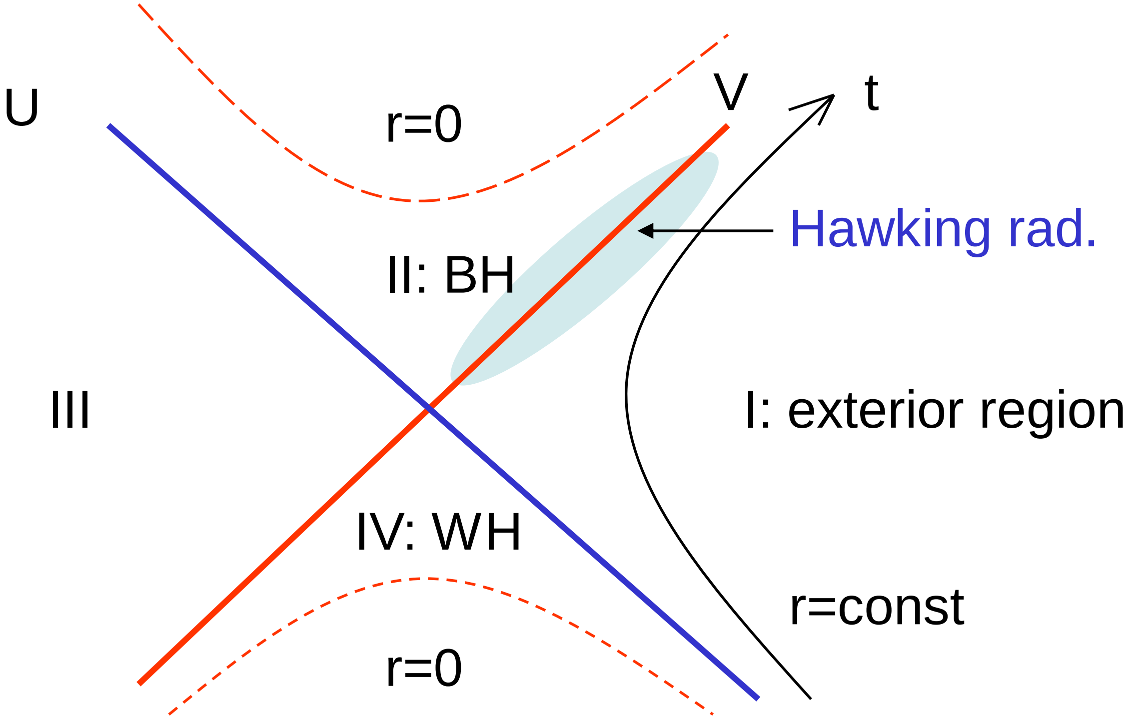
$$ds^2 = f(r)(dt^2 - dr_*^2)$$

Kruskal coordinates U,V : regular coordinates around horizon

$$U = -e^{-u/4M} < 0 \qquad \text{where} \qquad u = t - r_* \quad v = t + r_*$$

$$V = e^{v/4M} > 0 \qquad\qquad r_* = r + 2M \ln(r - 2M)$$

$$ds^2 = \frac{-32M^3 e^{-r/2M}}{r} dU dV$$



U=0, V=0 at horizon

U=0 future horizon
V=0 past horizon

Charged scalar field

$$S = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\left(\partial_\mu + ieA_\mu\right)\phi^*\left(\partial_\nu - ieA_\nu\right)\phi - m^2\phi^*\phi + V(\phi)\right].$$

Partial wave decomposition: $\phi = \sum \phi_{lm}(t,r)Y_{lm}(\Omega)$

$$\begin{aligned} S &= \sum_{l,m}\int dtdr_*\; r^2(r_*)\Big[|\left(\partial_t - ieA_t\right)\phi_{lm}|^2 + |\partial_{r_*}\phi_{lm}|^2 \\ &\qquad + f(r(r_*))\left(-m^2|\phi_{lm}|^2 + \frac{l(l+1)}{r^2}|\phi_{lm}|^2 + V(\phi_{lm})\right)\Big]. \end{aligned}$$

Near horizon, potential term $l(l+1)/r^2$, mass term and interaction terms are suppressed.
$\Longrightarrow$ Each partial wave mode behaves as $d = 2$ massless free field in $(r - t)$ section.

cf. Carlip has emphasized the near conformal symmetry.

Note; higher derivative terms are more enhanced near horizon!

$$\partial_r = \frac{\partial_{r_*}}{f(r)}$$ (1/M suppression)

We first study charge and energy fluxes:

Hawking radiation from RN BH.

Planck distribution with a chemical potential

$$J^{(\pm)}(w) = \frac{1}{e^{\beta(w \pm c)} + 1}$$

for fermions

$$c = eQ/r_+$$

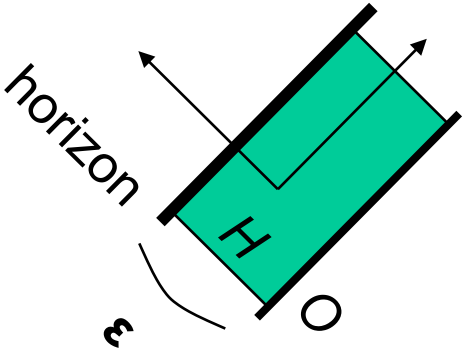
e: charge of emanated particles
Q: charge of BH

Fluxes of current and EM tensor are given by

$$J^r \;=\; e\int_0^\infty \frac{dw}{2\pi}\left(J^-(w) - J^{(+)}(w)\right) = \frac{e^2Q}{2\pi r_+}$$

$$T^r_t \;=\; \int_0^\infty \frac{dw}{2\pi}\,w\left(J^-(w) + J^{(+)}(w)\right) \;=\; \frac{e^2Q^2}{4\pi r_+^2} + \frac{\pi}{12\beta^2}.$$

Gauge current and gauge anomaly



If we neglect ingoing modes in region H $r \in [r_+, r_+ + \epsilon]$
the theory becomes chiral there.

↓

Gauge current has anomaly in region H.

$$\nabla_\mu J^\mu = \pm \frac{e^2}{4\pi\sqrt{-g}} \epsilon^{\mu\nu} \partial_\mu A_\nu$$

consistent current

+: left moving

- : right moving

We can define a covariant current by

$$\tilde{J}^\mu = J^\mu \mp \frac{e^2}{4\pi\sqrt{-g}} A_\lambda \epsilon^{\lambda\mu}$$

which satisfies

$$\nabla_\mu \tilde{J}^\mu = \pm \frac{e^2}{4\pi\sqrt{-g}} \epsilon_{\mu\nu} F^{\mu\nu}$$

In region *O*, $\partial_r J^r_{(o)} = 0$

In near horizon region H, $\partial_r J^r_{(H)} = \frac{e^2}{4\pi}\partial_r A_t$ consistent current

$$\boxed{\begin{aligned} J^r_{(o)} &= c_o, \\ J^r_{(H)} &= c_H + \frac{e^2}{4\pi}(A_t(r) - A_t(r_+)) \end{aligned}}$$

c_o = current at infinity

c_H = value of consistent current at horizon

c_o c_H are integration constants.

Current is written as a sum of two regions.

$$J^\mu = J^\mu_{(o)}\Theta_+(r) + J^\mu_{(H)}H(r)$$

where $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ $H(r) = 1 - \Theta_+(r)$

Variation of the effective action under gauge tr.

$$-\delta W = \int d^2x \sqrt{-g_{(2)}}\lambda\nabla_\mu J^\mu_{(2)}$$

Using anomaly eq.

$$-\delta W = \int d^2x \lambda \left[\delta(r-r_+-\epsilon)\left(J^r_o - J^r_H + \frac{e^2}{4\pi}A_t\right) + \partial_r\left(\frac{e^2}{4\pi}A_t H\right)\right]$$

Impose δW + δW'=0

W' = contribution from ingoing modes (WZ term)

↑

⇓

cancelled by WZ term

$$c_o = c_H - \frac{e^2}{4\pi}A_t(r_+)$$

- Determination of c_H

Covariant current should vanish at horizon. ⟷ regurality

Otherwise physical quantities diverge at (future) horizon.

$$\tilde{J}^r|_{r_H} = 0 \longrightarrow c_o = -\frac{e^2}{2\pi}A_t(r_+) = \frac{e^2 Q}{2\pi r_+}$$

$$\tilde{J}^r = J^r + \frac{e^2}{4\pi}A_t(r)H(r)$$

Reproduces the correct Hawking flux

EM tensor and Gravitational anomaly

Effective d=2 theory contains background of
graviton, gauge potential and dilaton.

Under diffeo. they transform

$$\delta g^{\mu\nu} = -(\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu)$$

$$\delta A_\mu = \xi^\nu \partial_\nu A_\mu + \partial_\mu \xi^\nu A_\nu$$

$$\delta\sigma = \xi^\mu \partial_\mu \sigma$$

Ward id. for the partition function $Z = \int \mathcal{D}\phi \exp(iS)$

$$-i\int d^n x \left[\delta g^{\mu\nu}(x) \underset{\substack{\uparrow \\ T_{\mu\nu}}}{\frac{\delta}{\delta g^{\mu\nu}(x)}} + \delta A_\mu(x) \underset{\substack{\uparrow \\ J^\mu}}{\frac{\delta}{\delta A_\mu(x)}} + \delta\sigma(x)\frac{\delta}{\delta\sigma(x)}\right] Z[g_{\mu\nu}, A_\mu, \sigma] \text{ =anomaly}$$

Gravitational anomaly (w/o gauge and dilaton backgrounds)

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} = \mathcal{A}_\nu$$

consistent current

$$\nabla_\mu \tilde{T}^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon_{\mu\nu}\partial^\mu R = \tilde{\mathcal{A}}_\nu$$

covariant current

In the presence of gauge and gravitational anomaly, Ward id. becomes

$$\nabla_\mu T^\mu{}_\nu = F_{\mu\nu}J^\mu + A_\nu\nabla_\mu J^\mu - \partial_\nu(e^\sigma)\mathcal{L} + \mathcal{A}_\nu$$

non-universal

gauge anomaly

gravitational anomaly

Solve $\nu = t$ component of Ward.id.

(1) In region *O*

$$\partial_r T^r_{t(o)} = F_{rt} J^r_{(o)}$$

(2) In region *H*

(near horizon)

$$\partial_r T^r_{t(H)} = \underbrace{F_{rt} J^r_{(H)} + A_t \nabla_\mu J^\mu_{(H)}}_{F_{rt} \tilde{J}^r_{(H)}} + \partial_r N^r_t$$

Using

$$J^r_{(o)} = c_o \qquad \tilde{J}^r_{(H)} = c_o + \tfrac{e^2}{2\pi} A_t(r)$$

$$T^r_{t(o)} = a_o + c_o A_t(r)$$

$$T^r_{t(H)} = a_H + \int_{r_+}^{r} dr \partial_r \left(c_o A_t + \frac{e^2}{4\pi} A_t^2 + N^r_t \right)$$

Anomaly is
total divertgent!

Variation of effective action under diffeo.

$$\int d^2x \sqrt{-g_{(2)}}\, \xi^t \nabla_\mu T^\mu_t$$

$$= \int d^2x\ \xi^t \left[c_o \partial_r A_t(r) + \partial_r \left(\frac{e^2}{4\pi} A_t^2 + N_t^r \right) \quad + \quad \left(T^r_{t\ (o)} - T^r_{t\ (H)} + \frac{e^2}{4\pi} A_t^2 + N_t^r \right) \delta(r - r_+ - \epsilon)(\right]$$

(1) (2) (3)

(1) classical effect of background electric field

(2) cancelled by induced WZ term of ingoing modes

(3) Coefficient must vanish.

$$a_o = a_H + \frac{e^2}{4\pi} A_t^2(r_+) - N_t^r(r_+)$$

Determination of a_H

Regurality: covariant current must vanish at horizon.

↓

since

$$\tilde{T}^r_t = T^r_t + \frac{1}{192\pi}(ff'' - 2(f')^2)$$

we can determine $a_H = \kappa^2/24\pi = 2N^r_t(r_+)$

and therefore flux at infinity is given by

$$a_o = \frac{e^2Q^2}{4\pi r_+^2} + N^r_t(r_+) = \frac{e^2Q^2}{4\pi r_+^2} + \frac{\pi}{12\beta^2}$$

Reproduces the flux of Hawking radiation

This method can be easily applied to any black holes

- rotating (Kerr, Kerr-Newman, Myers-Perry, …)
- black rings
- acoustic black holes
- dynamical black holes (Vaidya)
- etc.

2. Demystification : Conformality (Birrell Davis)

In d=2, current and EM tensor can be solved explicitly
(generalization of Christensen and Fulling method)

U(1) current

$$\boxed{\begin{aligned}\nabla_\mu J^\mu &= 0 \\ \nabla_\mu J^{5\mu} &= \frac{1}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}\end{aligned}} \quad \longleftrightarrow \quad \boxed{\begin{aligned}\partial_v\left(J_u - \frac{1}{\pi}A_u\right) &= 0 \\ \partial_u\left(J_v - \frac{1}{\pi}A_v\right) &= 0\end{aligned}}$$

We can define $j(u) \equiv J_u - \frac{1}{\pi}A_u \qquad \tilde{j}(v) \equiv J_v - \frac{1}{\pi}A_v$

Regularity at horizon → $j(u) = -\frac{e^2}{\pi}A_u(r_+)$

$J_U = J_u/(-\kappa U)$

Hence flux can be obtained as $J^r = J_u - J_v = -\frac{e^2}{\pi}A_u(r_+) = \frac{e^2 Q}{2\pi r_+}$

Similarly

EM tensor $\boxed{\nabla_\mu T^\mu{}_\nu \;=\; F_{\mu\nu} J^\mu \qquad T^\mu{}_\mu \;=\; \frac{c}{24\pi} R}$ can be solved as

$$t(u) \equiv T_{uu} - \frac{c}{24\pi}\left(\partial_u^2 \varphi - \frac{1}{2}(\partial_u \varphi)^2\right) - \frac{1}{\pi} A_u^2 - 2A_u j(u)$$

and the regularity condition at horizon determines the holomorphic part t(u) as

$$t(u) = \frac{c}{192\pi}\left(f'(r_+)\right)^2 + \frac{e^2}{\pi} A_u^2(r_+)$$

Then the EM flux at infinity is given by

$$T_t^r \longrightarrow \frac{c}{192\pi}\left(f'(r_+)\right)^2 + \frac{e^2}{4\pi} A_t^2(r_+)$$

If left and right central charges are different,

$$\nabla_\mu J^\mu = -\frac{(c_R - c_L)}{2}\frac{\hbar}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu} \qquad \nabla_\mu J^{5\mu} = \frac{(c_R + c_L)}{2}\frac{\hbar}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$

$$\parallel$$

$$\boxed{J_u = j(u) + \frac{c_R\hbar}{\pi}A_u,} \qquad J_v = \tilde{j}(v) + \frac{c_L\hbar}{\pi}A_v$$

$$\nabla^\mu T_{\mu\nu} = F_{\mu\nu}J^\mu - \frac{\hbar}{48\pi}\frac{c_R - c_L}{2}\epsilon_{\mu\nu}\nabla^\mu R \qquad T^\mu{}_\mu = \frac{\hbar}{24\pi}\frac{c_L + c_R}{2}R$$

$$\parallel$$

$$\boxed{t(u) = T_{uu} - 2A_u j(u) - \frac{c_R\hbar}{\pi}A_u^2 - \frac{c_R\hbar}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right)}$$

In deriving the Hawking flux, only the right-handed modes (outgoing modes) are relevant.
This is why the gauge and gravitational anomalies can determine the flux of the Hawking radiation.

The fundamental underlying relation (FUR) in the anomaly method is

$$\boxed{T_{uu}(u,v) = \frac{c}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + T_{uu}^{(conf)}(u)}$$

↓

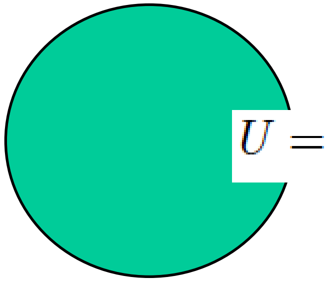
Can we generalize it to higher spins to obtain the full thermal flux?

YES

but let’s first consider a simpler but restricted method.

Simplification

In stationary blackholes, we know the conformal tr from Kruskal to Schwarzschild coordinates.



$U = -exp(-\kappa u)$ → u= t-r

Near the horizon,
Kruskal coordinate
Is the regular coordinate.

At infinity, u gives the asymptotic coordinate.

Two coodinates are connected by conformal tr.

It is restricted to cases where the tr. is conformal.

For Vaidya (dynamical), it is not conformal $\kappa = \kappa(v)$

Schwarzschild BH

light-like coordinate $u \longrightarrow$ Kruskal coordinate U: $U = -e^{-\kappa u}$

$$T^{(conf)}_{UU}(U) = \frac{1}{\kappa^2 U^2}\left(T^{(conf)}_{uu}(u) + \frac{c}{24\pi}\{U, u\}\right)$$

Boundary conditions corresponding to the Unruh vacuum

- Regularity condition
  Physical quantities in the Kruskal coordinate system should be regular at the horizon.

$$\Longrightarrow \qquad \langle T^{(conf)}_{UU}\rangle\Big|_{U=0} : \text{finite}$$

With a further boundary condition of no ingoing flux from infinity

$$\langle T^{(conf)}_{vv}\rangle \xrightarrow{r\to\infty} 0$$

The asymptotic flux is determined by the value of the Schwarzian derivative:

$$\langle T^r_t \rangle = \langle T_{uu} \rangle - \langle T_{vv} \rangle \quad \xrightarrow{r\to\infty} \quad -\frac{c}{24\pi}\{U,u\} = \frac{c}{48\pi}\kappa^2$$

$$\Longleftrightarrow \langle T^r_t \rangle = \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega}{e^{\beta\omega} \mp 1} = \begin{cases} \kappa^2/48\pi & \text{for boson, } (c=1) \\ \\ \kappa^2/96\pi & \text{for fermion, } (c=1/2) \end{cases}$$

This analysis can be easily generalized to radiation of
charged particles from RN black holes
by incorporating gauge transformations.

[3.] Higher-spin currents of Hawking radiation

Energy flux

$$\langle T^r_t \rangle\Big|_{r\to\infty} = \frac{1}{48\pi}\kappa^2 \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega}{e^{\beta\omega}\mp 1}$$

We have used two methods
(response to conformal transf. & anomaly method)
to calculate the fluxes of U(1) current and enegy.

↓

We can generalize them to higher-spins and reproduce thermal distribution.

$$\langle \text{Higher-spin current} \rangle\Big|_{r\to\infty} \quad \Longleftrightarrow \quad \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^{2n-1}}{e^{\beta\omega}\mp 1}$$

S.I. Morita Umetsu (07)

(1) Schwarzian derivative method

- In conformally flat space, there are conserved traceless symmetric tensors.

e.g massless real scalar field

$$\begin{aligned} J_{\mu\nu\rho\sigma} \quad \propto \quad & (8\ \partial_\mu\phi\partial_\nu\partial_\rho\partial_\sigma\phi - 12\partial_\mu\partial_\nu\phi\partial_\rho\partial_\sigma\phi - 4\ g_{\mu\nu}\partial^\lambda\phi\partial_\lambda\partial_\rho\partial_\sigma\phi \\ & +8\ g_{\mu\nu}\partial^\lambda\partial_\rho\phi\partial_\lambda\partial_\sigma\phi - g_{\mu\nu}g_{\rho\sigma}\partial^\lambda\partial^\tau\phi\partial_\lambda\partial_\tau\phi) + \text{symm.} \end{aligned}$$

In general, we can construct even-rank currents.

↓

Holomorphic part of the current is

$$J^{(conf)}_{uuuu} = -\frac{2}{5} : \partial_u\phi\partial_u^3\phi : +\frac{3}{5} : \partial_u^2\phi\partial_u^2\phi :$$

Schwarzian derivative of 4-th rank current can be calculated by using the point-splitting regularization.

$$: \partial_u \phi \partial_u^3 \phi(u) := \lim_{\epsilon \to 0} \left(\partial_u \phi(u + \epsilon/2) \partial_u^3 \phi(u - \epsilon/2) + \frac{3}{2\pi\epsilon^4} \right)$$

Under $u \to w(u)$: $\quad \partial_u \phi(u) = \partial_u w(u) \partial_w \phi^{(w)}(w(u))$

$$\begin{aligned} : \partial_u \phi \partial_u^3 \phi(u) : \quad = \quad & w' w''' : \partial \phi^{(w)} \partial \phi^{(w)}(w) : + 3(w')^2 w'' : \partial \phi^{(w)} \partial^2 \phi^{(w)}(w) : \\ & + (w')^4 : \partial \phi^{(w)} \partial^3 \phi^{(w)}(w) : - \frac{1}{480\pi} \{w, u\}_{(1,3)} \end{aligned}$$

generalization of the Schwarzian derivative:

$$\{w, u\}_{(1,3)} = 6 \frac{w^{'''''}}{w'} - 20 \left(\frac{w'''}{w'} \right)^2 - 45 \left(\frac{w''}{w'} \right)^4 + 90 \frac{(w'')^2 w'''}{(w')^3} - 30 \frac{w^{''''} w''}{(w')^2}$$

In the case of BH : $w(u) = U = -e^{-\kappa u}$

$$
\begin{aligned}
:\partial_u\phi\partial_u^3\phi(u): \quad &= \quad \kappa^4 U^2 :\partial_U\phi^{(U)}\partial_U\phi^{(U)}: +3\kappa^4 U^3 :\partial_U\phi^{(U)}\partial_U^2\phi^{(U)}: \\
&\quad +\kappa^4 U^4 :\partial_U\phi^{(U)}\partial_U^3\phi^{(U)}: -\frac{1}{480\pi}\kappa^4
\end{aligned}
$$

$\boxed{\text{Regularity condition at } U = 0}$

$$\Longrightarrow \qquad -\langle :\partial_u\phi\partial_u^3\phi(u):\rangle = \frac{1}{480\pi}\kappa^4 \qquad \longleftarrow \text{ Flux at the infinity}$$

Note: $:\partial_u^2\phi\partial_u^2\phi:$ and $:-\partial_u\phi\partial_u^3\phi(u):$ give the same contribution to the flux.

$$\downarrow$$

Reproduce the 3rd moment of the Planck distribution

$$\langle J_{uuuu}^{(conf)}\rangle = \frac{1}{480\pi}\kappa^4 \qquad \Longleftrightarrow \qquad \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega^3}{e^{\beta\omega}-1} = \frac{1}{480\pi}\kappa^4$$

General higher-spin currents

Instead of considering each higher-spins, it is more convenient to study the generating function

$$:\partial_u\phi(u)\partial_u\phi(u+a):=\sum_{n=0}^{\infty}\frac{a^n}{n!}:\partial_u\phi(u)\partial_u^{n+1}\phi(u):$$

Transformation : $u \longrightarrow w(u)$

$$:\partial_u\phi(u)\partial_u\phi(u+a):=\partial_u w(u)\partial_u w(u+a):\partial_w\phi^{(w)}(w(u))\partial_w\phi^{(w)}(w(u+a)):+A_b(w,u)$$

$A_b(w,u)$ is a generating function of the Schwarzian derivatives.

$$A_b(w,u)=-\frac{1}{4\pi}\frac{\partial_u w(u)\partial_u w(u+a)}{[w(u)-w(u+a)]^2}+\frac{1}{4\pi a^2}$$

In the case of BH : $w(u) = U = -e^{\kappa u}$

$$: \partial_U \phi^{(U)}(U(u)) \partial_U \phi^{(U)}(U(u+a)) := e^{\kappa a} \left(\frac{1}{\kappa U} \right)^2 [: \partial_u \phi_u \partial_u \phi(u+a) : - A_b(U,u)]$$

Regularity condition at the horizon

$$\begin{aligned} \langle : \partial_u \phi(u) \partial_u \phi(u+a) : \rangle &= A_b(U,u) \\ &= -\frac{\kappa^2}{16\pi} \frac{1}{\sinh^2 \frac{\kappa a}{2}} + \frac{1}{4\pi a^2} = \sum_{n=0}^{\infty} (-1)^n \frac{B_{n+1} \kappa^{2(n+1)}}{8\pi(n+1)} \frac{a^{2n}}{(2n)!} \end{aligned}$$

B_n : Bernoulli number

Hawking flux corresponding to $2n$-th rank current

$$\langle : (-1)^{n-1} \partial_u \phi \partial_u^{2n-1} \phi(u) : \rangle = \frac{B_n}{8\pi n} \kappa^{2n} \qquad \Longleftrightarrow \qquad \int_0^\infty \frac{d\omega}{2\pi} \frac{\omega^{2n-1}}{e^{\beta\omega} - 1}$$

Reproduce the $(2n-1)$-th moment of the Planck distribution

Physical meaning of $A_b(U,u)$

$A_b(U,u)$ can be written as

$$A_b(U,u) = \int_0^\infty \frac{d\omega}{2\pi}\frac{\omega}{e^{\beta\omega}-1}\cos(a\omega)$$

This is the temperature dependent part of thermal Green function

$$\langle T\phi(x)\phi(y)\rangle_\beta = \int \frac{d^2k}{(2\pi)^2}\left(\frac{i}{k^2+i\epsilon} + 2\pi\delta(k^2)\frac{1}{e^{\beta|\omega|}-1}\right)e^{-ik(x-y)}$$

tortoise coordinate $\longrightarrow$ Kruskal coordinate : $U=-e^{-\kappa u}$

$\Longleftrightarrow$ zero temperature $\longrightarrow$ finite temperature with $\beta = 2\pi/\kappa$

Similar analysis can be applied to radiation of fermions or charged particles.

(2) Anomaly method (FUR) for higher-spin currents

SI, Morita, Umetsu (07)

Gravitational anomaly can be generalized to higher-spin currents.

$$\nabla^\mu T_{\mu\nu} = F_{\mu\nu} J^\mu + \frac{1}{96\pi} \epsilon_{\mu\nu} \partial^\mu R$$

↓

Examples of gauge anomalies

spin 3

mixing with lower rank currents

$$\nabla_\mu J^{(3)\mu}{}_{\nu\rho} = \boxed{\begin{array}{c} -F_{\nu\mu} J^{(2)\mu}{}_\rho - F_{\rho\mu} J^{(2)\mu}{}_\nu \\ -\frac{1}{16} \nabla_\nu \left(R J^{(1)}_\rho \right) - \frac{1}{16} \nabla_\rho \left(R J^{(1)}_\nu \right) + \frac{1}{16} g_{\nu\rho} \nabla_\mu \left(R J^{(1)\mu} \right) \end{array}}$$

$$\boxed{+ \frac{\hbar}{96\pi} \left(\epsilon_{\nu\sigma} \nabla^\sigma \nabla_\mu F^\mu{}_\rho + \epsilon_{\rho\sigma} \nabla^\sigma \nabla_\mu F^\mu{}_\nu - g_{\nu\rho} \epsilon_{\alpha\sigma} \nabla^\sigma \nabla_\mu F^{\mu\alpha} \right)}$$

anomalous part

rhs is a total derivative.

spin 4

$$\nabla^\mu J^{(4)}_{\mu\nu\rho\sigma} = \boxed{\begin{gathered} F_{\mu\nu} J^{(3)\mu}{}_{\rho\sigma} + F_{\mu\rho} J^{(3)\mu}{}_{\sigma\nu} + F_{\mu\sigma} J^{(3)\mu}{}_{\nu\rho} \\ -\tfrac{1}{8} R \left(\nabla_\nu J^{(2)}_{\rho\sigma} + \nabla_\rho J^{(2)}_{\sigma\nu} + \nabla_\sigma J^{(2)}_{\nu\rho} \right) \\ -\tfrac{1}{6} \left(J^{(2)}_{\nu\rho} \nabla_\sigma R + J^{(2)}_{\rho\sigma} \nabla_\nu R + J^{(2)}_{\sigma\nu} \nabla_\rho R \right) \\ -\tfrac{1}{24} \Big(J^{(1)}_\nu \nabla_\rho \nabla_\mu F^\mu{}_\sigma + J^{(1)}_\rho \nabla_\sigma \nabla_\mu F^\mu{}_\nu + J^{(1)}_\sigma \nabla_\nu \nabla_\mu F^\mu{}_\rho \\ + J^{(1)}_\rho \nabla_\nu \nabla_\mu F^\mu{}_\sigma + J^{(1)}_\nu \nabla_\sigma \nabla_\mu F^\mu{}_\rho + J^{(1)}_\sigma \nabla_\rho \nabla_\mu F^\mu{}_\nu \Big) \end{gathered}}$$

$$\boxed{-\frac{\hbar}{1920\pi} \left(\epsilon_{\nu\alpha} \nabla^\alpha \nabla_\rho \nabla_\sigma R + \epsilon_{\rho\alpha} \nabla^\alpha \nabla_\sigma \nabla_\nu R + \epsilon_{\sigma\alpha} \nabla^\alpha \nabla_\nu \nabla_\rho R \right)}$$

anomaly

$$-\tfrac{1}{4} \left(g_{\nu\rho} \hat{C}_\sigma + g_{\rho\sigma} \hat{C}_\nu + g_{\sigma\nu} \hat{C}_\rho \right)$$

$$\hat{C}_\nu \equiv \boxed{-\tfrac{1}{4} R \nabla_\rho J^{(2)\rho}{}_\nu - \tfrac{1}{3} J^{(2)\rho}{}_\nu \nabla_\rho R - \tfrac{1}{12} \left(J^{(1)\rho} \nabla_\rho \nabla_\mu F^\mu{}_\nu + J^{(1)}_\rho \nabla_\nu \nabla_\mu F^{\mu\rho} \right)}$$

$$\boxed{-\frac{\hbar}{1920\pi} \left(\epsilon_{\nu\alpha} \nabla^\alpha \nabla_\rho \nabla^\rho R + 2\epsilon_{\rho\alpha} \nabla^\alpha \nabla^\rho \nabla_\nu R \right)}$$

Brief sketch of the derivations of these anomaly equations

(step 1) Regularize the higher-spin currents covariantly on the light-cone (v=fixed) using "geodesic distance".

(step 2) Define conformal fields and regularize the associated holomorphic currents.

(step 3) Compare these two currents and obtain the relations between (u…u) component of the covariant higher-spin currents and holomorphic higher-spin currents.

$J_{u\ldots u}(u,v) = J^{(conf)}_{u\ldots u}(u) +$ background dependent terms (u,v)

Fundamental Underlying Relations

Spin 2

$$\boxed{t(u)} \equiv T_{uu} - \boxed{\frac{c}{24\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right)} - \frac{1}{\pi}A_u^2 - 2A_u j(u)$$

Spin 3

$$J^{(3)}_{uuu} = \boxed{j^{(3)}(u)} + 4A_u j^{(2)}(u) + \left(\frac{1}{4}\left(\partial_u^2\varphi - (\partial_u\varphi)^2\right) + 4A_u^2\right) j^{(1)}(u) + \frac{1}{4}\partial_u\varphi\partial_u j^{(1)}(u)$$

$$\boxed{+\frac{\hbar}{4\pi}\left(A_u\left(\partial_u^2\varphi - (\partial_u\varphi)^2\right) + \partial_u\varphi\partial_u A_u - \frac{1}{3}\partial_u^2 A_u + \frac{16}{3}A_u^3\right)}$$

Spin 4

$$J^{(4)}_{uuuu} = \boxed{j^{(4)}(u)} + 6A_u j^{(3)}(u) + \frac{3}{4}\partial_u\varphi\partial_u j^{(2)}(u) + \left[\frac{1}{4}\left(4\partial_u^2\varphi - 5(\partial_u\varphi)^2\right) + 12A_u^2\right] j^{(2)}(u)$$

$$+ \frac{3}{2}A_u\partial_u\varphi\partial_u j^{(1)}(u) + \left[2A_u\left(\partial_u^2\varphi - \frac{5}{4}(\partial_u\varphi)^2\right) + \frac{3}{2}\partial_u A_u\partial_u\varphi - \frac{1}{2}\partial_u^2 A_u + 8A_u^3\right] j^{(1)}(u)$$

$$\boxed{\begin{aligned} &- \frac{\hbar}{2\pi}A_u(\partial_u - 2\partial_u\varphi)(\partial_u - \partial_u\varphi)A_u + \frac{\hbar}{2\pi}A_u^2\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + \frac{2\hbar}{\pi}A_u^4 \\ &- \frac{\hbar}{160\pi}(\partial_u - 3\partial_u\varphi)(\partial_u - 2\partial_u\varphi)\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right) + \frac{7\hbar}{480\pi}\left(\partial_u^2\varphi - \frac{1}{2}(\partial_u\varphi)^2\right)^2 . \end{aligned}}$$

(step 4) Fully covariantize the anomaly equations.

→ gauge anomaly equations $(c_R - c_L)$

&

trace anomaly equations $(c_R + c_L)$

examples of trace anomalies:

$$J^{(3)\mu}{}_{\mu\nu} = \frac{\hbar}{12\pi}\frac{c_L + c_R}{2}\nabla_\mu F^\mu{}_\nu$$

$$J^{(4)\mu}{}_{\mu\nu\rho} = -\frac{\hbar}{160\pi}\frac{c_L + c_R}{2}\nabla_\nu\nabla_\rho R + g_{\nu\rho}\frac{c_L + c_R}{2}\left[\frac{\hbar}{160\pi}\nabla^2 R + \frac{\hbar}{24\pi}\left(\tilde{F}^2 + \frac{11}{60}R^2\right)\right]$$

Hawking fluxes for higher-spin currents

Anomalies are again total derivatives and the fluxes are written only in terms of the information at the horizon.

(This is equivalent to the existence of holomorphic currents constructed from the original currents and backgrounds.)

$$c_O^{(3)} = -\frac{\kappa^2}{24\pi} A_t(r_+) - \frac{1}{6\pi} A_t(r_+)^3 = \frac{\kappa^2}{24\pi}\frac{Q}{r_+} + \frac{1}{6\pi}\left(\frac{Q}{r_+}\right)^3$$

$$c_O^{(4)} = \frac{7\kappa^4}{1920\pi} + \frac{\kappa^2}{16\pi}\left(\frac{Q}{r_+}\right)^2 + \frac{1}{8\pi}\left(\frac{Q}{r_+}\right)^4$$

$$\sum_{n=0}^{\infty} \frac{(2i\alpha)^n}{n!} c_O^{(n+1)} = -\frac{i\hbar}{4\pi\alpha}\frac{\alpha\kappa e^{-\frac{2i\alpha Q}{r_+}}}{\sinh(\alpha\kappa)} + \frac{i\hbar}{4\pi\alpha}$$

[4] Summary and Discussions

Two methods to calculate Hawking radiation are shown.

(1) To see the response to a conformal transformations
from Kruskal U to Schwarzschild u.

This can be generalized to arbitrary higher-spins.
→ generalization of the Schwarzian derivative

(2) Anomaly method - - wider applicability than method (1)

We have obtained higher-spin generalizations.

Universality of Hawking radiation is assured by the fact that these anomalies are total derivatives.

Boundary effects !

instanton → index　　　Hawking radiation → horizon

Discussions:

(1) Classification of radiation

Are there fields with the same central charge but different higher spin anomalies ?

⟶ If so, violation of no hair theorem.

Are higher spin anomalies cohomologically trivial?

Recently Bonora et.al. (0808.2360) have shown that spin 4 anomaly is cohomologically trivial, and can be absorbed by a redefinition of the currents.

→ W-infinity algebra does not seem to give any new hair.

(2) Nontrivial CFT ?

Classification of Hawking radiation

boson	Planck distribution
fermion	Fermi Dirac distribution

Other nontrivial CFT near horizon ?

Nonderivative interactions are suppressed near the horizon.
Higher derivatives terms are more enhanced.
(string theory ?)

(3) How much robust ?

Is the thermal spectrum modified for evaporating blackholes?

The answer seems NO.

Quite generally (including some dynamical black holes),

$$\partial_u^{(n)} \varphi|_{horizon} = 0 \qquad n > 1$$

and the only nonvanishing quantity in the fundamental underlying relation is the sufrace gravity

$$\partial_u \varphi|_{horizon} \quad \sim \quad \kappa = \frac{1}{2}\partial_r f(r_H)$$

$$\downarrow$$

Thermal spectrum is very robust. (only 1-parameter and the coefficients are fixed by the FURs)

Future problems

- back reaction ?
  higher derivative terms → nonlocality ?

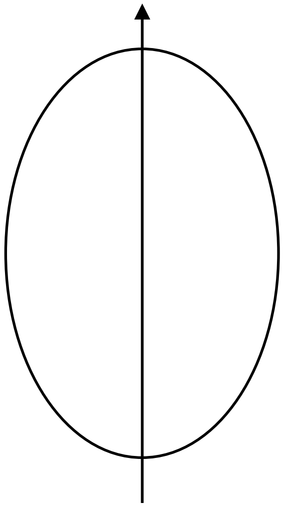
- Evaluate fluxes for evaporating black hole ?
  At the final stage of the evaporation, do fluxes diverge?

- black hole entropy
  asymptotic Virasoro and near horizon Virasoro ?
  (Carlip)

Umetsu, Wilczek SI (07)

Rotating black holes (Kerr, Kerr-Newman)

Kerr=axial symmetric



diffeo in axial direction $\xrightarrow{\text{KK}}$ U(1) gauge symmetry in d=2

a part of metric → background electric field

partial wave with *m* → charge *m*

Near horizon, each partial wave is decoupled and can be treated as free massless d=2 field.

$$S=\int dtdr\;(r^2+a^2)\phi^*_{lm}\left[\frac{r^2+a^2}{\Delta}\left(\partial_t+\frac{iam}{r^2+a^2}\right)^2-\partial_r\frac{\Delta}{r^2+a^2}\partial_r\right]\phi_{lm}.$$

dilaton $\Phi=r^2+a^2,$

metric $g_{tt}=f(r),\qquad g_{rr}=-\frac{1}{f(r)},\qquad g_{rt}=0,$

gauge potential $A_t=-\frac{a}{r^2+a^2},\qquad A_r=0.$

U(1) charge of ϕ_{lm} is *m*.

Results

Flux of angular momentum

$$c_o = -\frac{m^2}{2\pi} A_t(r_+) = \frac{m^2 a}{2\pi(r_+^2 + a^2)}.$$

Flux of energy

$$a_o = \frac{m^2 a^2}{4\pi(r_+^2 + a^2)^2} + N_t^r(r_+) = \frac{m^2\Omega^2}{4\pi} + \frac{\pi}{12\beta^2}$$

where

$$\Omega = \frac{a}{r_+^2 + a^2}.$$

(angular velocity at horizon)