Fuzzy Ring From M2-brane Giant Torus

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Based on arXiv: 0808.2691

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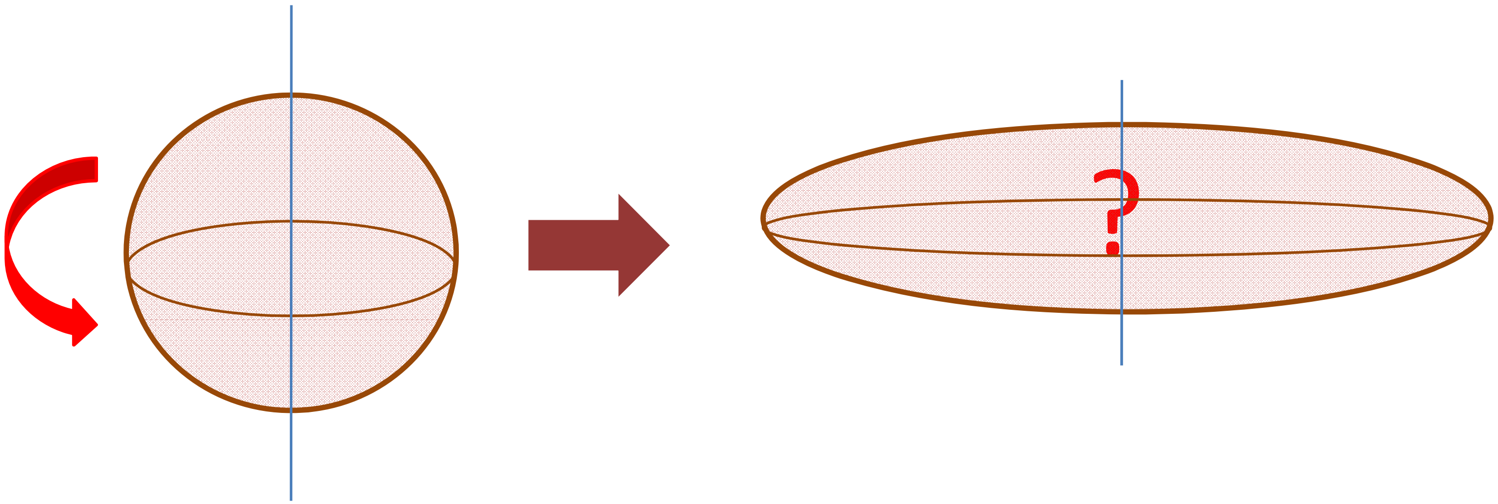
① Introduction

Two motivations of this work, which look different at first sight:

(i) Can we rotate a fuzzy sphere (dielectric D2-brane) ?

D-brane in the low energy limit ⟺ (?) **a rigid body**



(ii) New BPS states in the CFT3 and BPS Black objects in AdS4

(also some insights in AdS5/CFT4 ?)

Non-spinning Giant Gravitions ⟷ R-charged small black holes in AdS (superstar [Myers-Tafjord 01'])

Spinning Giant Gravitons ⟷ R-charged Kerr black holes in AdS? or (small) black rings in AdS?

AdS4 case See e.g. Chong-Cvetic-Lu-Pope 04'

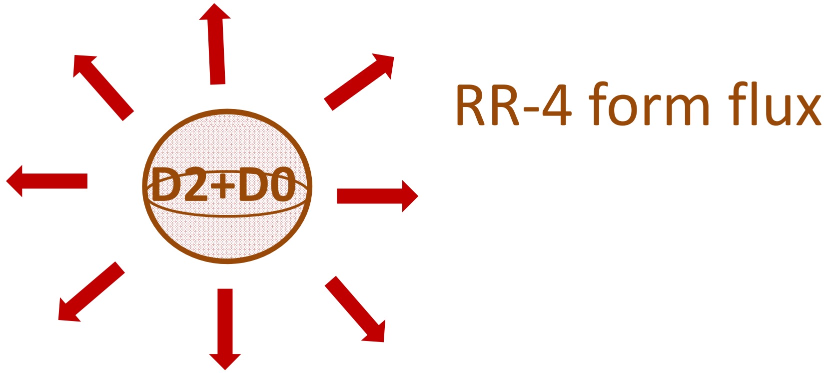
This has not been completely understood at present.
Recent arguments in AdS5:
Kunduri-Lucietti-Reall 06' 07'
Caldarelli-Emparan-Rodriguez 08'

Giant Gravitons in AdS/CFT

$$E << N \quad \Rightarrow \quad \text{Gravitons} \quad \Leftrightarrow \quad \mathrm{Tr}[\Phi^{\mathrm{E}}]$$

$$E \approx O(N) \quad \Rightarrow \begin{cases} \text{Giant Gravitons} & \Leftrightarrow \quad \chi_{\mathrm{Asym}}(\Phi) \\ (\text{D-branes in } S^m) & \\ & \\ \text{Dual Giant Gravitons} & \Leftrightarrow \quad \chi_{\mathrm{Sym}}(\Phi) \\ (\text{D-branes in } AdS_n) & \end{cases}$$

Let us consider how to rotate a dielectric D2-brane.

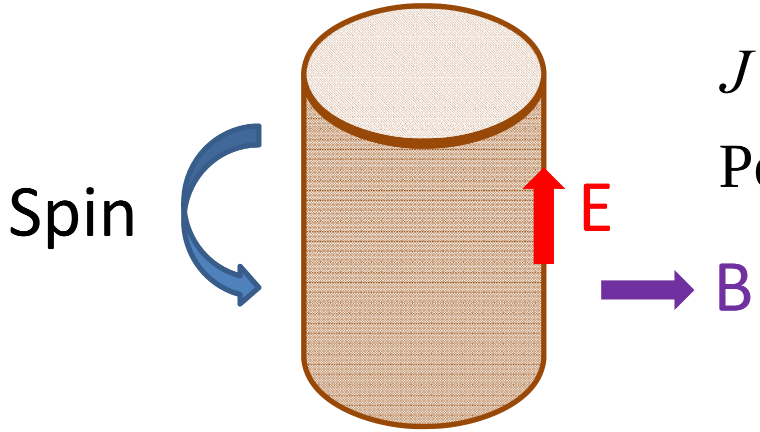


Since it is approximated by a rigid body in the low energy limit, we should be able to give it a spin.

Naively , this leads to an immediate puzzle:

D-branes are reparametrization invariant and it is not clear how to describe any spins (or angular momentum) on branes.

However, if we remember for example, supertubes, we can excite both the electric flux and magnetic flux on the brane and produce a non-zero angular momentum !



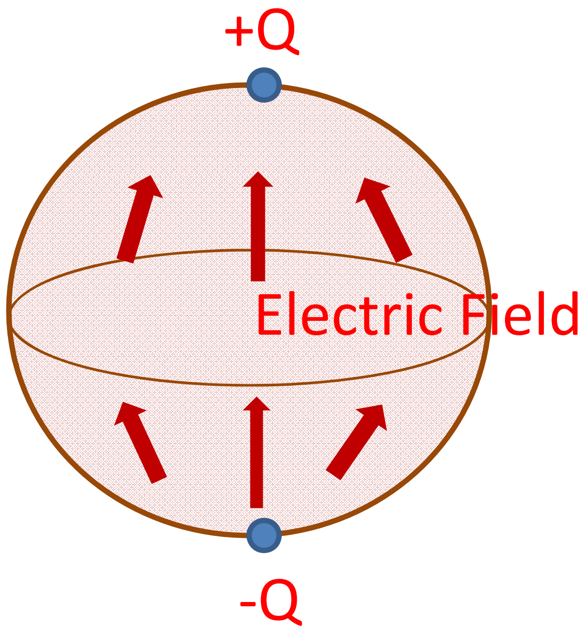
$J \sim E \times B$

Poynting vector

D2+D0+F1

But, if we apply this to our problem, we encounter the second puzzle !

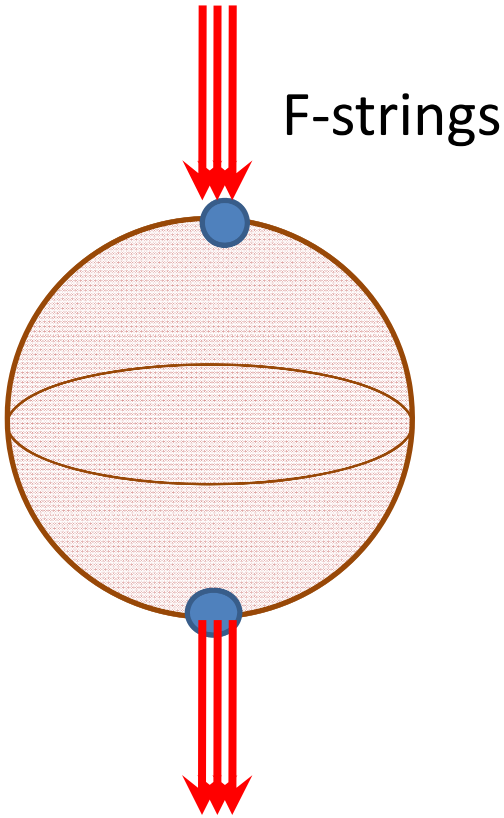
The Gauss law requires extra sources of the electric charge !
(~ F-string charge)

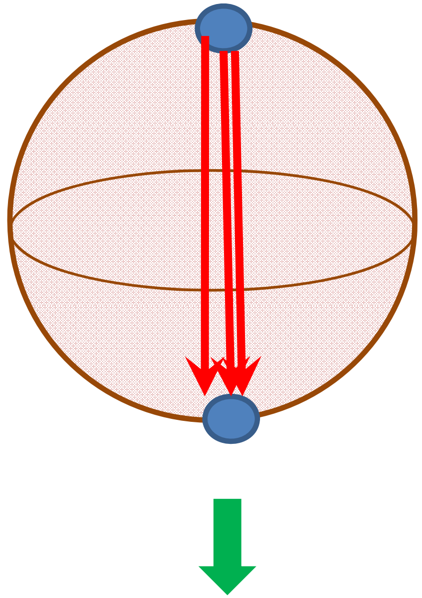


Does this mean that the spinning fuzzy sphere is impossible?
The answer is Yes and No.

Indeed, we can attach the F-strings on the fuzzy sphere.

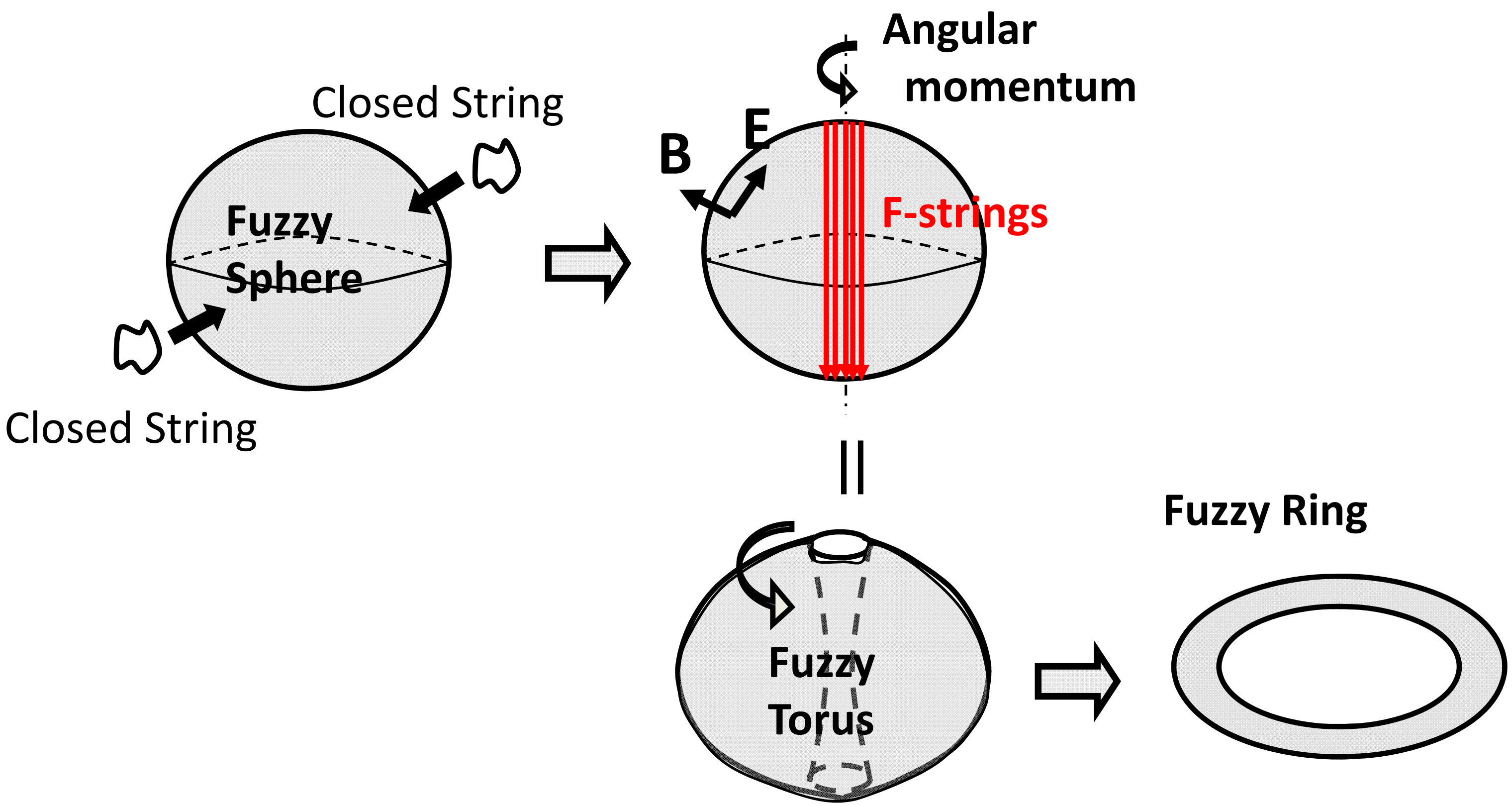
There are two possibilities:





Finite energy configuration

Interestingly , this intuitive argument tells us that the topology of the dielectric brane should be changed into a torus !



In fact, we will construct BPS configurations of the dielectric D2-brane with the torus world-volume in IIA string on AdS4 × CP3.

It will be amusing to note that this topology change from S2 to T2 by adding the spin looks analogous to the transition from a black hole to a black ring.

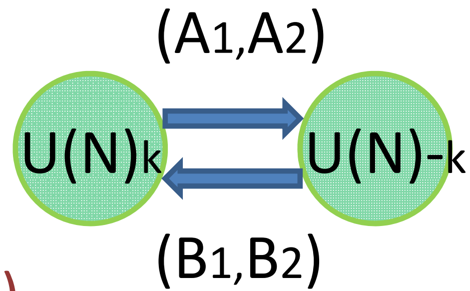
Contents

② M2-branes and AdS4/CFT3

Recently, the CFT dual of AdS4 × S7, which corresponds to the multiple M2-branes , has been proposed by Aharony-Bergman-Jafferis-Maldacena arXiv:0806.1218.

ABJM Theory

3D N=6 Chern-Simons gauge theory
with the gauge group U(N) × U(N)
level k -k



(k=1 ,2 N=8 SUSY → BLG theory for SU(2))

(2-1) M-theory on $AdS_4 \times S^7/Z_k$ and IIA String on $AdS_4 \times CP^3$

The ABJM theory at level k is conjectured to be dual to the M-theory on $AdS_4 \times S^7/Z_k$.

$$ds_M^2 = \frac{R^2}{4}[ds_{AdS4}^2 + d\Omega_7^2], \qquad R = l_p (2^5 \pi^2 N k)^{1/6}$$

$$ds_{AdS4}^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$F^{(4)} = -\frac{3R^3}{8} r^2 \sin\theta \, dt \wedge dr \wedge d\theta \wedge d\varphi.$$

When k is large, we can reduce this to type IIA string on $AdS_4 \times CP^3$.

$$S^7: \ |z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 = 1,$$

$$\Rightarrow \quad z_i = \mu_i \, e^{i\xi_i}, \ \textit{where}$$

$$(\mu_1, \mu_2, \mu_3, \mu_4)$$

$$= (\sin\alpha, \cos\alpha \sin\beta, \cos\alpha \cos\beta \sin\gamma, \cos\alpha \cos\beta \cos\gamma)$$

We define the coordinates as follows:

$$x^0 = t, \ x^1 = r, \ x^2 = \theta, \ x^3 = \varphi, \ x^4 = \alpha, \ x^5 = \beta,$$

$$x^6 = \gamma, \ x^7 = \xi_1, \ x^8 = \xi_2, \ x^9 = \xi_3, x^{10} = \xi_4.$$

Z_k orbifold: $z_i \to z_i e^{\frac{2\pi i}{k}}$, $\qquad \xi_{\mathrm{i}} \to \xi_i + \frac{2\pi}{k}$

32 Killing Spinors in AdS4 × S7

$$\delta\Psi_\mu = D_\mu \varepsilon - \tfrac{1}{288}(\Gamma_\mu^{\nu\lambda\rho\sigma} - 8\delta_\mu^\nu \Gamma^{\lambda\rho\sigma})F_{\nu\lambda\rho\sigma}\varepsilon = 0$$

$$\Downarrow$$

$$\varepsilon = e^{\frac{\alpha}{2}\hat{\gamma}\gamma_4} e^{\frac{\beta}{2}\hat{\gamma}\gamma_5} e^{\frac{\gamma}{2}\hat{\gamma}\gamma_6} e^{\frac{\xi 1}{2}\gamma_{47}} e^{\frac{\xi 2}{2}\gamma_{58}} e^{\frac{\xi 3}{2}\gamma_{69}} e^{\frac{\xi 4}{2}\hat{\gamma}\gamma_{10}} e^{\frac{\rho}{2}\hat{\gamma}\gamma_1} e^{\frac{t}{2}\hat{\gamma}\gamma_0} e^{\frac{\theta}{2}\gamma_{12}} e^{\frac{\varphi}{2}\gamma_{23}} \varepsilon_0$$

where $\hat{\gamma} = \gamma^{0123}$ and $\sinh\rho \equiv r$.

The 11D spinor ε_0 should have only to satisfy

$$\varepsilon_0 = \gamma_{012345678910}\varepsilon_0. \quad \Rightarrow \exists 32\ \text{SUSYs}$$

24 Killing Spinors in AdS4 × S7/Zk (k>2)

We define $s_i = \pm 1$ as follows:

$$\gamma_{47}\varepsilon_0 = is_1\varepsilon_0, \quad \gamma_{58}\varepsilon_0 = is_2\varepsilon_0, \quad \gamma_{69}\varepsilon_0 = is_3\varepsilon_0, \quad \hat{\gamma}\gamma_{10}\varepsilon_0 = is_4\varepsilon_0,$$

where they satisfy $s_1 s_2 s_3 s_4 = 1.$

The orbifold twist acts as $\varepsilon \to \varepsilon\, e^{\frac{\pi i}{k}(s_1+s_2+s_3+s_4)}.$

Therefore the following combinations survive the projection:

$$(s_1, s_2, s_3, s_4) = \cancel{(+,+,+,+)}, \cancel{(-,-,-,-)},$$

$$\boxed{\begin{array}{l}(+,+,-,-), (+,-,+,-), (+,-,-,+) \\ (-,+,+,-), (-,-,+,+), (-,+,-,+).\end{array}}$$

$\Rightarrow \exists \frac{3}{4}$ SUSY = 24 SUSYs $\Leftrightarrow$ 3D N = 6 SCFT

Assuming k large, let us reduce the M-theory background to that of the type IIA theory.

We take the 11th coordinate to be the compactified direction

$$y = \tfrac{1}{4}(\xi_1 + \xi_2 + \xi_3 + \xi_4), \qquad y \sim y + \tfrac{2\pi}{k} \ .$$

The seven sphere metric can be rewritten as follows

$$\mathrm{d}\Omega_{\mathrm{S7}}^2 = ds_{CP3}^2 + (d\tilde{y} + C^{(1)})^2, \qquad (\tilde{y} \equiv ky)$$

$$F^{(2)} = dC^{(1)} = 2kJ, \qquad (J = \text{Kähler form of CP3}).$$

In this way, we obtain IIA string on AdS4 × CP3:

$$ds^2_{IIA} = \tilde{R}^2 (ds^2_{AdS4} + 4ds^2_{CP3}),$$

$$F^{(2)} = dC^{(1)} = 2kJ,$$

$$C^{(3)} = -\frac{k\tilde{R}^2}{2} r^3 \sin\theta \, dt \wedge d\theta \wedge d\varphi$$

$$e^{2\phi} = \frac{4\tilde{R}^2}{k^2}, \qquad \tilde{R}^2 = \frac{R^3}{4k} = \pi\sqrt{\frac{2N}{k}} \quad .$$

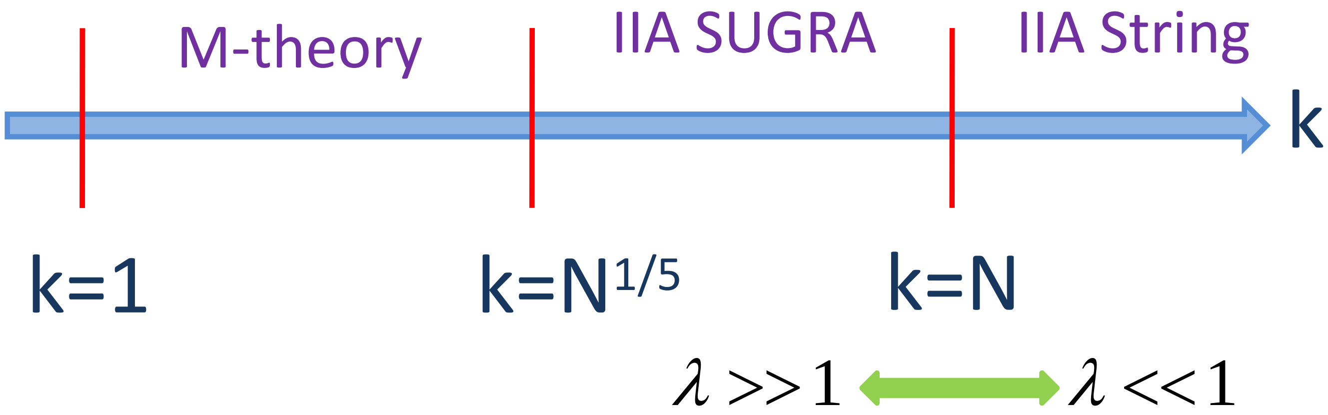
⇕

$3D \quad N = 6 \quad U(N)_k \times U(N)_{-k}$ Chern-Simons Theory

(2-2) ABJM 't Hooft Limit

The 't Hooft Limit of the ABJM theory is defined by

$$\text{taking } N \to \infty \quad \text{with } \lambda \equiv \frac{N}{k} \quad \text{kept finite.}$$



$$\text{Notice } g_s \sim \left(\frac{N}{k^5}\right)^{1/4} \quad \text{and} \quad \tilde{R} \sim \sqrt{\frac{N}{k}} \quad .$$

(2-3) Penrose Limit of ABJM Theory

[Nishioka-T, Arxiv:0806.3391,
Gaiotto-Giombi-Yi, Arxiv:0806.4589]

In the AdS5 × S5, the analysis of Penrose limit offers us a very important check of AdS/CFT for non-BPS sectors.

However, we will notice that the situation of the Penrose limit in the ABJM/AdS4 duality is rather different and becomes a bit complicated as we will explain briefly.
(In other words, AdS5 × S5 is unusually simple !)

We can show that the Penrose limit of IIA string on AdS4 × CP3 becomes a plane-wave with 24 susys and we can quantize the IIA string theory in this background.

Now consider the following BMN operator:

$$\mathrm{O}_n = \frac{1}{\sqrt{2J}} \sum_{l=0}^{J} e^{\frac{2\pi i}{J} n l} \mathrm{Tr}[(A_1 B_1)^l A_1 B_2 (A_1 B_1)^{J\text{-}l} A_1 B_2].$$

$$\text{The R-charge } J \text{ is defined such that}$$

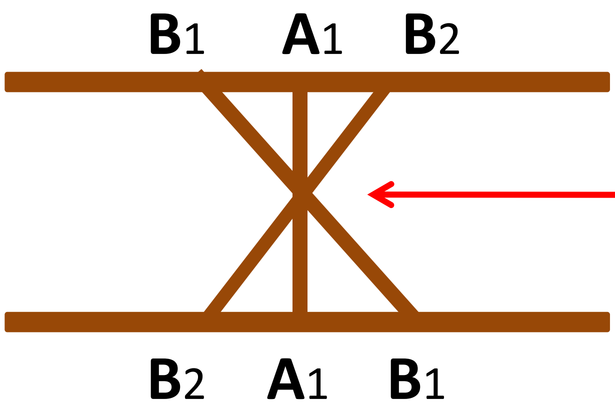
$$J(A_1) = J(B_1) = \frac{1}{2}, \quad J(A_2) = J(B_2) = 0.$$

The IIA string theory on pp-wave predicts

$$(\Delta - J)_{IIA} = \sqrt{\frac{1}{4} + \frac{2\pi^2 n^2}{J^2} \cdot \frac{N}{k}} \approx \frac{1}{2} + \frac{2\pi^2 N n^2}{kJ^2} + \cdots$$

On the other hand, the two loop calculation in ABJM leads to

$$(\Delta - J)_{ABJM} = \frac{1}{2} + \frac{4\pi^2 N^2 n^2}{k^2 J^2} + \cdots$$



6 - point interaction

$$\frac{16\pi^2}{k^2}\mathrm{Tr}[B_2 A_1 B_1 \overline{B}_2 \overline{A}_1 \overline{B}_1] + \mathrm{h.c.}$$

This `mismatch' will be due to the `violation of BMN scaling' as opposed to the AdS5/N=4SYM. Instead, we expect

$$\delta(\Delta - J)_{ABJM} = f(\lambda)\frac{n^2}{J^2} + O(J^{-4}),$$

$$\text{with} \quad \begin{cases} f(\lambda) \to 2\pi^2\lambda & (\lambda \to \infty) \\ f(\lambda) \to 4\pi^2\lambda^2 & (\lambda \to 0) \end{cases}.$$

This expectation is consistent with the recent proposal of all order Bethe ansatz [Gromov-Vieira, ArXiv:0807.0777].

Magnon Energy with momentum p : $\Delta = \sqrt{\frac{1}{4} + h(\lambda)\sin^2\frac{p}{2}}$.

③ Dual Giant Gravitons /Fuzzy Spheres Relation

A dual giant graviton is a M2-brane wrapped on the S^2 in AdS4

[Mcgreevy-Susskind-Toumbas 00' , Hashimoto-Hirano- Itzhaki 00']

$$ds^2_{AdS4} = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$\text{such that } (\sigma_0, \sigma_1, \sigma_2) = (t, \theta, \varphi).$$

It is rotating along the y-direction in S^7

$$\xi_1 = \xi_2 = \xi_3 = \xi_4 = y(t).$$

The M2-brane action reads

$$S = -T_2 \int d\sigma^3 \sqrt{-\det(G)} - T_2 \int C^{(3)} \qquad \left(T_2 \equiv \tfrac{1}{4\pi^2}\right)$$

$$= -\tfrac{\pi}{2} R^3 T_2 \int dt \left[r^2 \sqrt{1 + r^2 - 4\dot{y}^2} - r^3 \right],$$

$$H = P_y \dot{y} - L = \tfrac{\pi}{2} R^3 T \left[\sqrt{(1+r^2)\left(r^4 + \frac{p_y^2}{(\pi R^3 T_2)^2} \right)} - r^3 \right] \; .$$

By minimizing the energy we have two solutions :

(i) $r = 0$ (small graviton)

(ii) $r = \dfrac{P_y}{\pi R^3 T_2}$, $y(t) = \dfrac{t}{2}$ (dual giant graviton)

Now let us take the Z_k orbifold and reduce the y-direction down to type IIA string theory.

The y-momentum becomes the D0-brane charge.

Therefore, the dual giant is reduced to a fuzzy sphere (i.e. dielectric D2-brane) in AdS4 × CP3.

Indeed the action of a D2-brane with a gauge flux reads

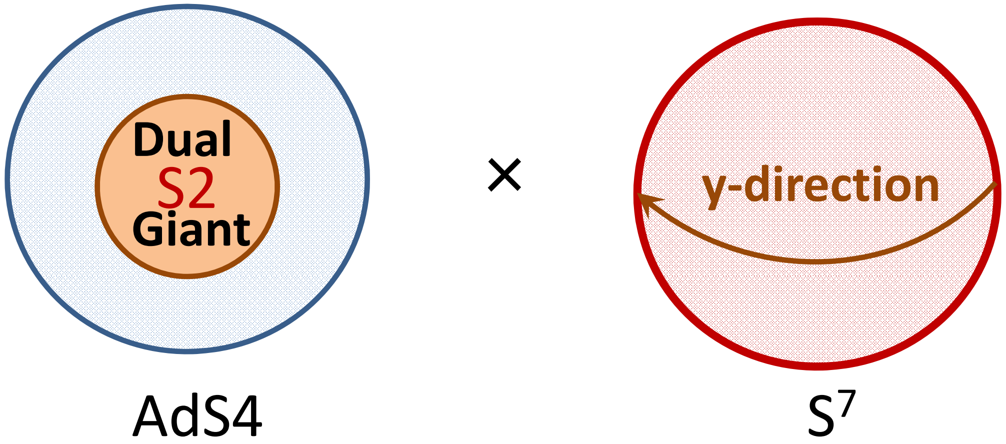
$$S = -T_2 \int d\sigma^3 e^{-\phi} \sqrt{-\det(G + 2\pi F)} - T_2 \int C^{(3)}$$
$$= -2\pi T_2 k \tilde{R}^2 \int dt \left[\sqrt{(r^2+1)(r^4 + \pi^2 M^2 \tilde{R}^{-4})} - r^3 \right],$$

where the gauge flux is quantized $F = \frac{M}{2} \sin\theta \, d\theta \wedge d\varphi.$

Its energy satisfies the BPS bound

$$E = \frac{P_y}{2} = \frac{1}{2}(J_1 + J_2 + J_3 + J_4).$$

$$S^7 \to SO(8) \supset U(1)^4 = (J_1, J_2, J_3, J_4).$$



By minimizing the energy, we get the BPS fuzzy sphere solution

$$r = \frac{\pi M}{\tilde{R}^2} = M\sqrt{\frac{k}{2N}}, \qquad (P_y = kM \in kZ)$$

$$E = \frac{kM}{2}.$$

This offers us the first example of fuzzy spheres in AdS spaces.

(cf. pp-wave b.g. -> BMN matrix model, Flat space -> RR-flux is not allowed)

The dual operator in ABJM theory will be a non-singlet operator with kM scalar fields $\left[(Y_I)^k\right]^M$.

Remember in the level k N=6 Chern-Simons theory, the Gauss law requires $J_{Baryon} \in kZ$.

④ Spinning Dual Giant Gravitons and Fuzzy Rings

(4-1) Spinning Solution Ansatz

So far we have considered non-spinning dual giants i.e. S=0.

$$\underbrace{AdS_4}_{S} \times \underbrace{S^7}_{(J_1, J_2, J_3, J_4)}$$

In AdS4 × S7, the non-spinning dual giants become either 1/2, 1/4 or 1/8 BPS states.

(for AdS5 × S5 see G.Mandal-N. Suryanarayana , hep-th/0606088)

On the other hand, the spinning dual giants are either

1/4, 1/8, 1/16 BPS states. (cf. AdS5 × S5 : Mikhailov hep-th/0010206

Kim-Lee hep-th/0607085

Ashok-Suryanarayana /0808.2042)

→ Dual to macroscopic BPS black holes ?

(The counting microstates of BPS BH even in AdS5 has not still done.)

We are especially interested in the axially symmetric M2-brane (or stationary solution as in the Kerr BHs or Black rings).

$$\text{Ansatz}: \quad r = r(\theta), \qquad y = w\,\varphi + \omega\, t, \qquad (kw \in Z).$$
$$\mu_{\mathrm{i}} = \text{const.} \propto J_i,$$

Spinning = F-String charge

$$S = -\tfrac{1}{8} R^3 T_2 \int dt\, d\theta\, d\varphi\, L,$$

where

$$L = \sqrt{\left(\frac{1}{1+r^2}\left(\frac{dr}{d\theta}\right)^2 + r^2\right)\left(r^2(1+r^2)\sin^2\theta + 4w^2(1+r^2) - 4\omega^2 r^2 \sin^2\theta\right)}$$
$$- r^3 \sin\theta \qquad .$$

(4-2) Supersymmetry Condition

A given M2-brane preserves a part of the bulk supersymmetries specified by the projection $(\Gamma+1)\varepsilon=0.$

Position dependent

$$\Gamma=\frac{1}{3!\sqrt{-G}}\varepsilon^{ijk}\partial_i X^\mu \partial_j X^\nu \partial_k X^\rho \Gamma_{\mu\nu\rho} \quad .$$

When μ_i are generic, we need to require

$$(\gamma_{47}-\gamma_{10}\hat{\gamma})\varepsilon_0=0, \qquad (\gamma_{58}-\gamma_{10}\hat{\gamma})\varepsilon_0=0, \qquad (\gamma_{69}-\gamma_{10}\hat{\gamma})\varepsilon_0=0.$$

Notice that only two of these conditions are independent.

$\Leftrightarrow\ s_1=s_2=s_3=s_4 \qquad$ (but always $s_1 s_2 s_3 s_4=1$).

The other conditions are summarized as follows:

$$(2\omega - \gamma_{010} - 2w\gamma_1)\varepsilon_0 = 0,$$

$$\left(\gamma_1\rho' - 2w\cosh\rho\cos\theta\gamma_{010}\sqrt{s}\right)\varepsilon_0 = 0,$$

where we defined

$$s = \frac{\rho'^2 + \sinh^2\rho}{\sinh^2\rho\cosh^2\rho\sin^2\theta + 4w^2\cosh^2\rho - 4\omega^2\sinh^2\rho\sin^2\theta}.$$

Remember also $\mathrm{r} = \sinh\rho.$

In summary, we find the following BPS equation:

$$\omega = \eta_1 w + \frac{\eta_2}{2},$$

$$\frac{dr}{d\theta} = 2\eta_1\eta_2 w \cdot \frac{r(1+r^2)}{r^2 - 2\eta_1\eta_2 w} \cdot \frac{\cos\theta}{\sin\theta},$$

where $\eta_1 = \pm 1$ and $\eta_2 = \pm 1$ are defined by

$$(\gamma_1 - \eta_1)\varepsilon_0 = 0, \qquad (\gamma_{010} - \eta_2)\varepsilon_0 = 0.$$

⇨ Generically, this M2-brane becomes 1/16 BPS.

However, for a particular choice of μ_i, it enhances to a 1/4 BPS or 1/8 BPS state.

Bogomolnyi Argument

$$L=\sqrt{\left(\frac{1}{1+r^2}\left(\frac{dr}{d\theta}\right)^2+r^2\right)(r^2(1+r^2)\sin^2\theta+4w^2(1+r^2)-4\omega^2r^2\sin^2\theta)}-r^3\sin\theta$$

$$=\sqrt{\left(2w\frac{d(r\cos\theta)}{d\theta}+\eta r^3\sin\theta\right)^2+\frac{(r^2+2w)^2\sin^2\theta}{1+r^2}\left(\frac{dr}{d\theta}-\frac{2\eta wr(1+r^2)\cos\theta}{(r^2-2\eta w)\sin\theta}\right)^2}$$

$$-r^3\sin\theta$$

$$\underbrace{=}_{BPS}\pm 2\eta w\frac{d(r\cos\theta)}{d\theta}\Rightarrow \text{Total Derivative!}$$

Preserved Supersymmetries

$$\text{Case A}: \ J_2 = J_3 = J_4 = 0 \ \text{ or perm.}$$
$$\text{Case B}: \ J_3 = J_4 = 0 \ \text{ or perm.}$$

SUSY of non-spinning dual Giant in $AdS_4 \times S^7 / Z_k$

k	Total SUSY	Case A	Case B	Generic
1,2	32	16	8	4
$3,4,\cdots$	24	12	4	0

⇨ **Fuzzy Sphere**

SUSY of spinning dual Giant in $AdS_4 \times S^7 / Z_k$

k	Total SUSY	Case A	Case B	Generic
1,2	32	8	4	2
$3,4,\cdots$	24	6	2	0

⇨ **Fuzzy Torus (Ring)**

(4-3) The analytical profile of the BPS solution

Now, it is straightforward to analytically solve the BPS solution

$$r\sin\theta = A\cdot(1+r^2)^{\frac{1}{2}+\frac{1}{4\eta w}}, \qquad (0\le\theta\le\pi)$$

$$\text{where} \quad A=\text{constant}, \quad \eta=\pm 1.$$

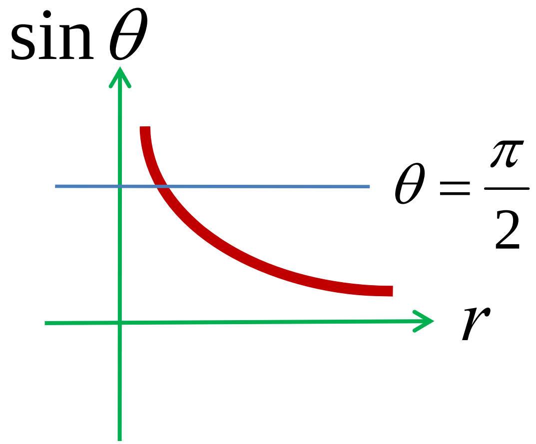
The behavior of this solution does radically change depending whether ηw is positve or negative.

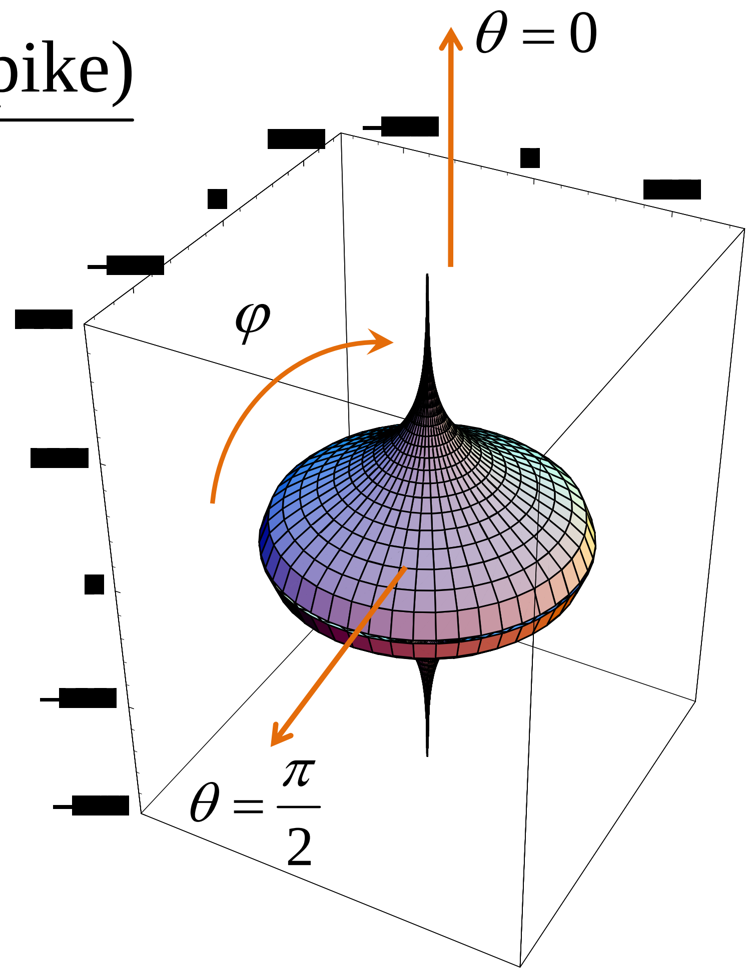
The energy can be computed as follows:

$$E=\frac{\eta_1}{2}(2S+J_1+J_2+J_3+J_4)+\int d\theta d\varphi\, L,$$

$$\text{with the condition} \quad S=w(J_1+J_2+J_3+J_4).$$

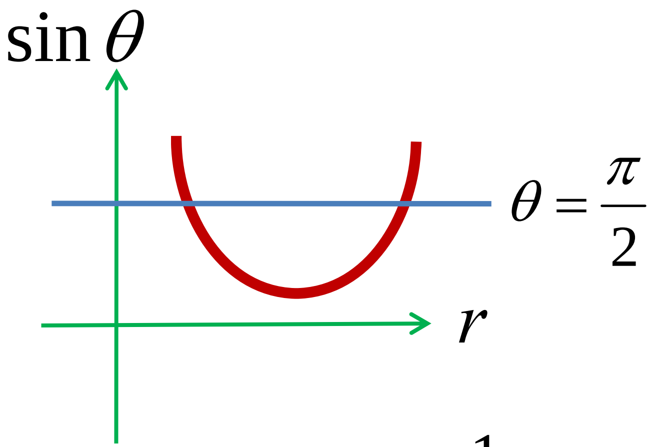
Case 1 : $\eta w < 0$ (Giant Spike)

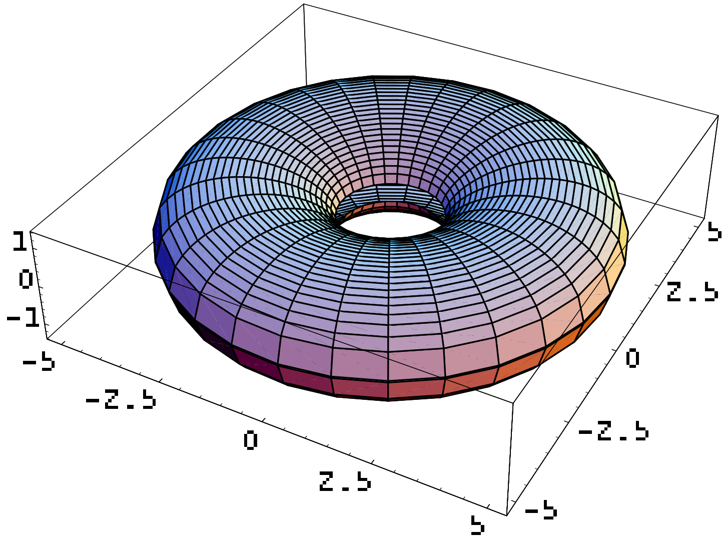




$$E = S + \frac{1}{2}(J_1 + J_2 + J_3 + J_4) + \underbrace{\pi w R^3 T_2 (r_{\max} - r_{\min})}_{\text{Spike energy} \\ = \text{F-string tension}}.$$

Case 2 : $\eta w > 0$ (Giant Torus)





$$E = S + \frac{1}{2}(J_1 + J_2 + J_3 + J_4)$$
$$= \omega(J_1 + J_2 + J_3 + J_4).$$

Taking Z_k Orbifold is straightforward.

Note that $J_1 + J_2 + J_3 + J_4 \in kZ, \quad w \in \frac{Z}{k}.$

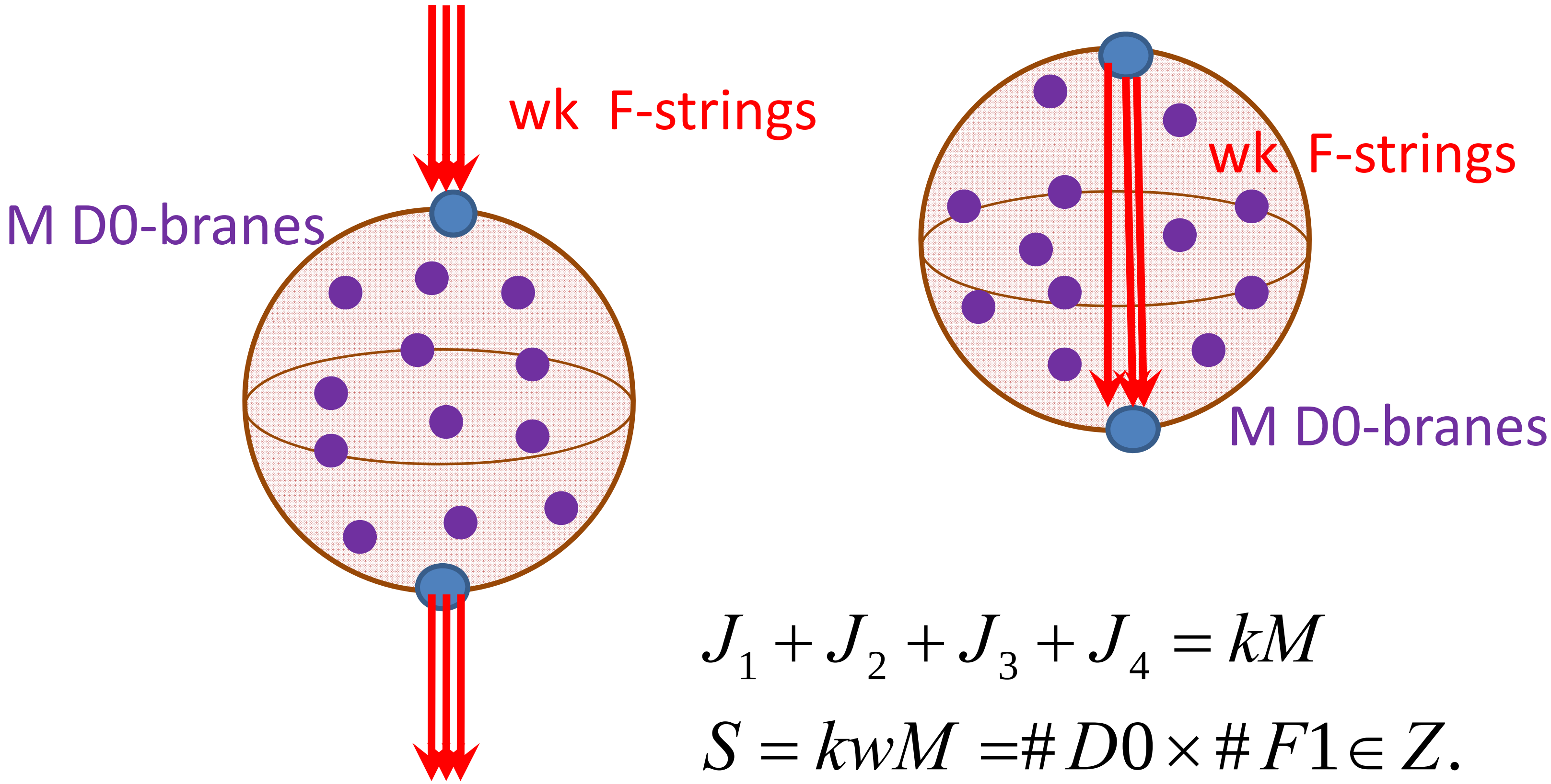
(4-4) Fuzzy Rings in IIA String on AdS4 × CP3

We can reduce the M2-brane action to the D2-brane one:

$$S_{DBI-D2} = -T_2 \int dx^3 e^{-\phi} \sqrt{-\det(G_{IIA} + 2\pi F)}$$
$$= -T_2 \int dx^3 \left[e^{-\phi} \sqrt{-\det(G_{IIA} + e^{2\phi} a_i a_j)} + \pi \varepsilon^{ijk} a_i F_{jk} \right]$$
$$= -T_2 \int dx^3 \sqrt{-\det(G_M + e^{\frac{4}{3}\phi} \partial_i \tilde{y} \partial_i \tilde{y})} = S_{M2}$$
$$EOM \ \rightarrow \ \partial_i \tilde{y} = a_i \quad (\tilde{y} = ky) \ .$$

$$\Rightarrow \begin{cases} F_{t\theta} = -w\sqrt{\dfrac{2N}{k}} \cdot \dfrac{r(1+r^2)}{|r^2 - 2\eta w| \sin\theta} & \rightarrow \ \mathrm{F-string} \\ F_{\theta\varphi} = \omega\sqrt{\dfrac{2N}{k}} \cdot \dfrac{r^3 \sin\theta}{|r^2 - 2\eta w|} & \rightarrow \ \mathrm{D0-brane} \end{cases},$$

In this way, we reproduced the speculated two possibilities:

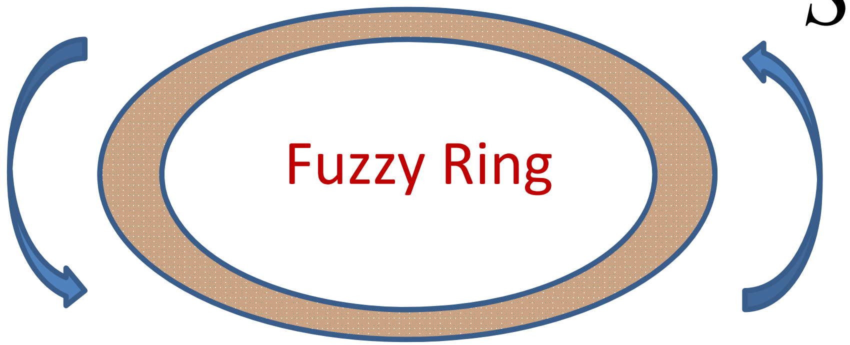


$$J_1 + J_2 + J_3 + J_4 = kM$$

$$S = kwM = \# D0 \times \# F1 \in Z.$$

It will be very interesting to observe that for a large angular momentum, we obtain a ring-like BPS (or non-BPS) object in the AdS4 spacetime.

$$S >> J_i$$



This may suggest an existence of (BPS) small black rings in AdS4........ [4D small BH cf. Iizuka-Shigemori 07']

A similar story also seems to be true for AdS5 case.

⑤ Conclusions and Discussions

- We showed that BPS fuzzy spheres exist in IIA string on AdS4 × CP3, which lifts to a dual giant graviton in AdS4 × S7.

  In the ABJM theory, they are dual to non-singlet operators.

- We constructed fuzzy torus (or ring) solutions in AdS4 × CP3.

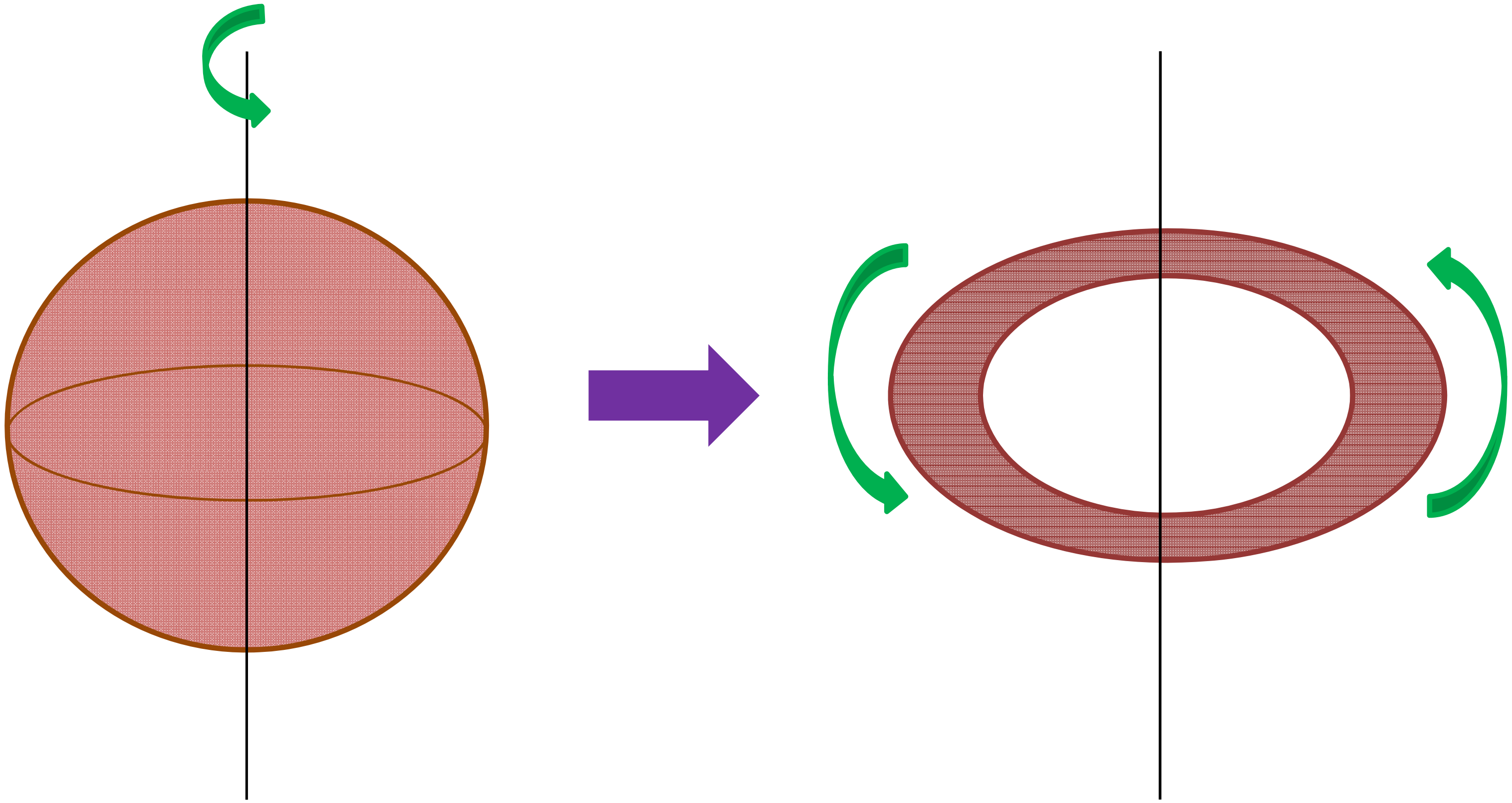
  Its lift to M-theory is given by spinning dual giant gravitons.

  Some of the fuzzy torus becomes 1/4 BPS and

  some even become non-BPS.

The main lesson: Topology Change due to spinning effect

(cf. Black hole/Black Ring transition)



Future Problems:

(1) Backreacted supergravtity solutions ?

(2) Entropy Counting: Can we describe the 1/16 BPS BH in AdS4
or small black rings ?

(3) A similar ring D3-brane solutions in AdS5 × S5 ?
Any macrocopic BPS black rings in AdS5 ?
[But, No macroscopic BPS black ring ?
: Kunduri-Lucietti-Reall arXiv:0705.4214]

(4) Precise identifications of operators in ABJM theory dual to
giant gravitons (M2,M5-branes).