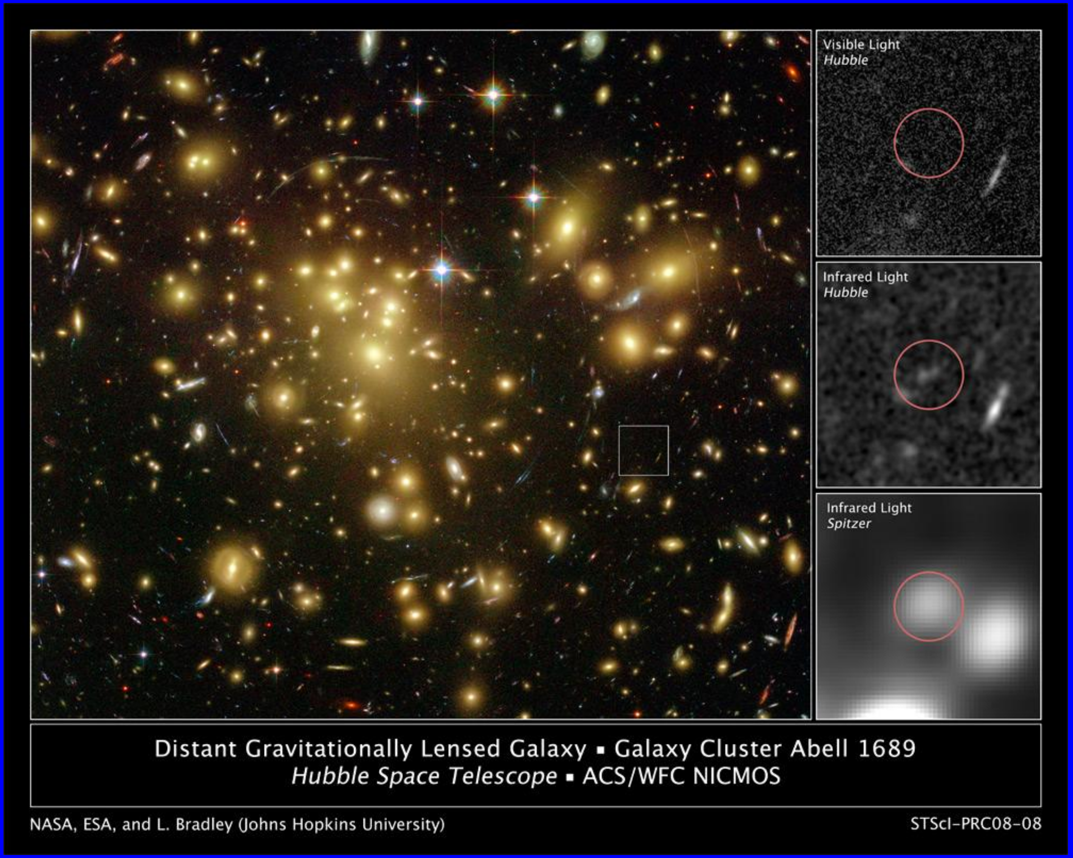
Where is the Information in Cluster Lenses?

Presented at the IPMU
August 21, 2008



Prof. Dave Goldberg
Drexel University Department of Physics

Outline

1. Lensing Basics
2. Why Clusters?
3. New Approaches to Lensing Reconstruction
4. Where is the Information in Cluster Lensing?

Lensing Basics

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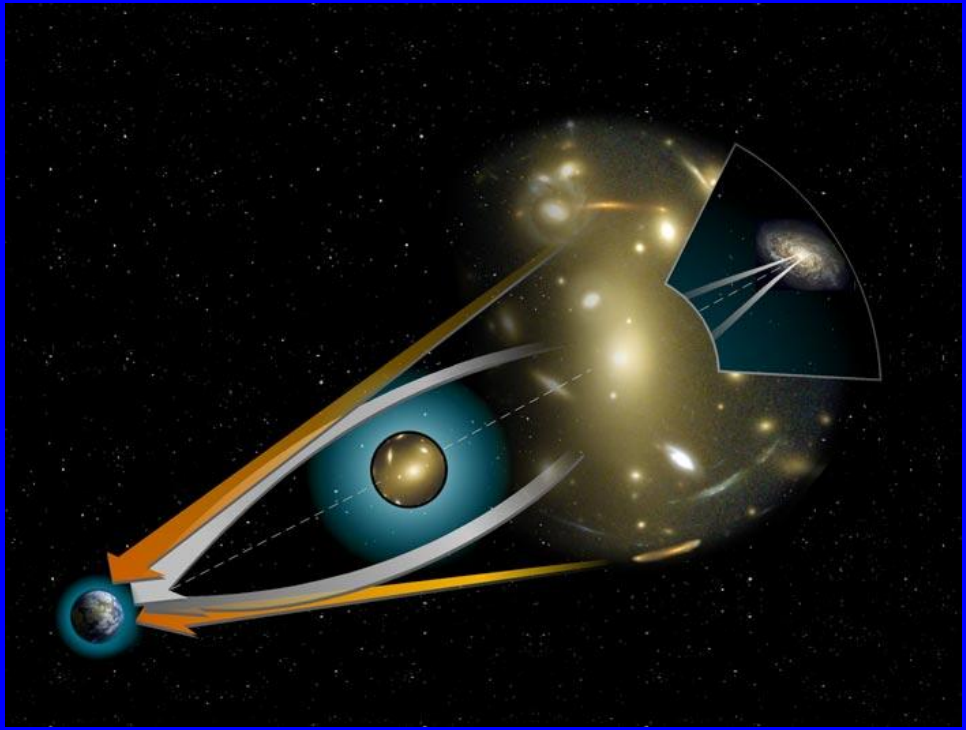


Lensing Basics: Derivatives of the Potential

Light takes the path that extremizes the travel time:

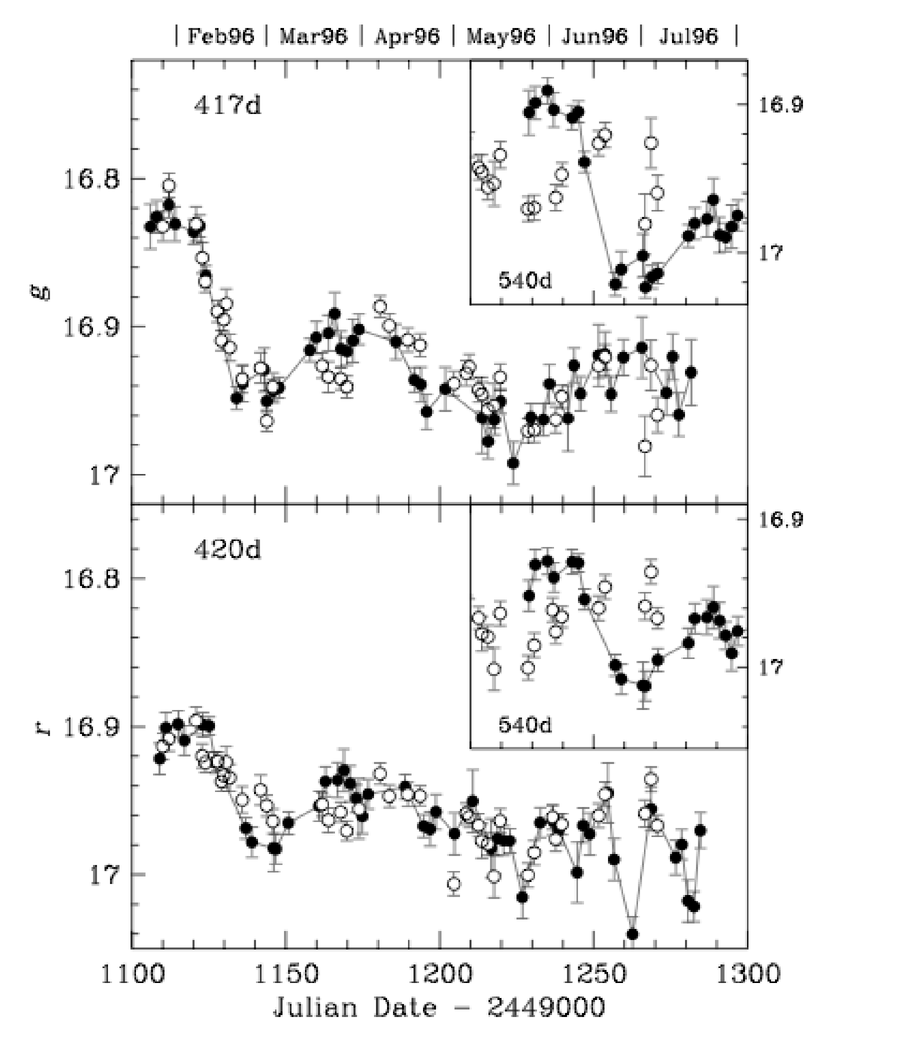
0) Time Delay

Geometric Relativistic

$$\tau = \left(\frac{1+z_D}{c}\right)\left(\frac{D_D D_S}{D_{DS}}\right)\left[\frac{1}{2}\left(\vec{\theta}-\vec{\beta}\right)^2 - \psi_{2D}(\vec{\theta})\right]$$

$$\psi_{2D} = \frac{D_{dS}}{D_S D_d}\int \frac{2\Phi_{3D}}{c^2}dl$$

Lensing Basics: Derivatives of the Potential



Side Note: since the light travel time is a probe of both geometry and mass, time delays between multiple images provide an absolute distance scale.

This can (and has) been used as a principle way of estimating H_0!

From Kundic et al. (1996), the time delay of images in 0957+561A,B

Lensing Basics: Derivatives of the Potential

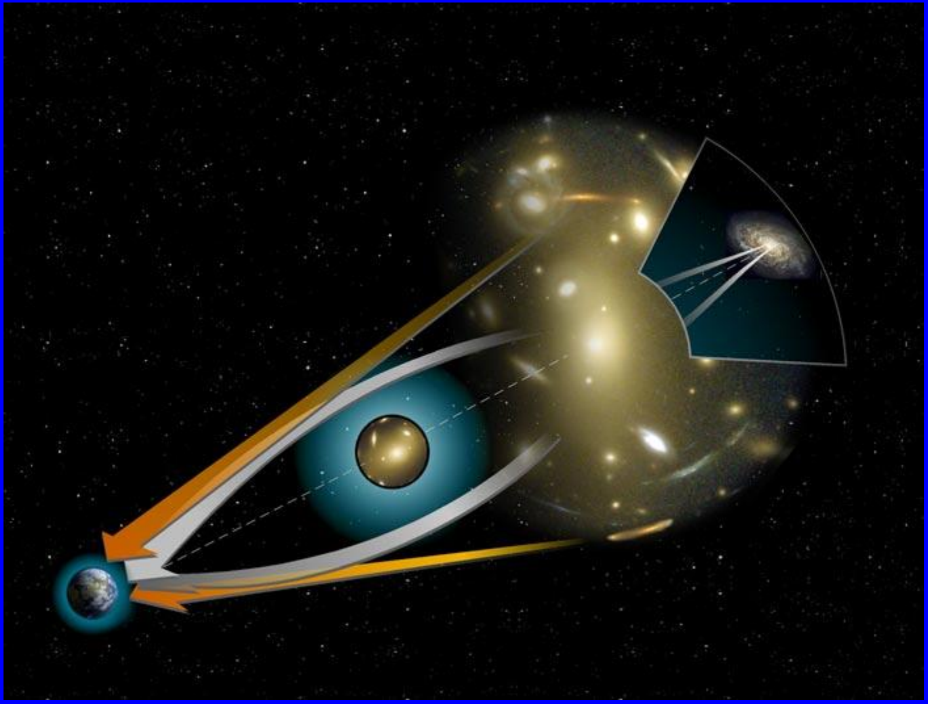
Light takes the path that extremizes the travel time:

1) Deflection

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\alpha_i(\vec{\theta}) = \psi_{,i}$$

Because there can be many *local* minima/maxima of light travel time, there can be more than one image!



Lensing Basics: Derivatives of the Potential

Light takes the path that extremizes the travel time:

2) Distortion (Convergence & Shear)

Convergence:

This is *the* signal we're looking for! It's a proxy for the density field, and expresses itself as magnification.

$$\kappa = \frac{\psi_{,11} + \psi_{,22}}{2}$$

$$\kappa = \frac{\Sigma(\vec{\theta})}{\Sigma_c}$$

Shear:

This is the normal observational signal in weak lensing

$$\begin{aligned} \gamma &= |\gamma| e^{2i\phi} \\ &= \gamma_1 + i\gamma_2 \end{aligned}$$

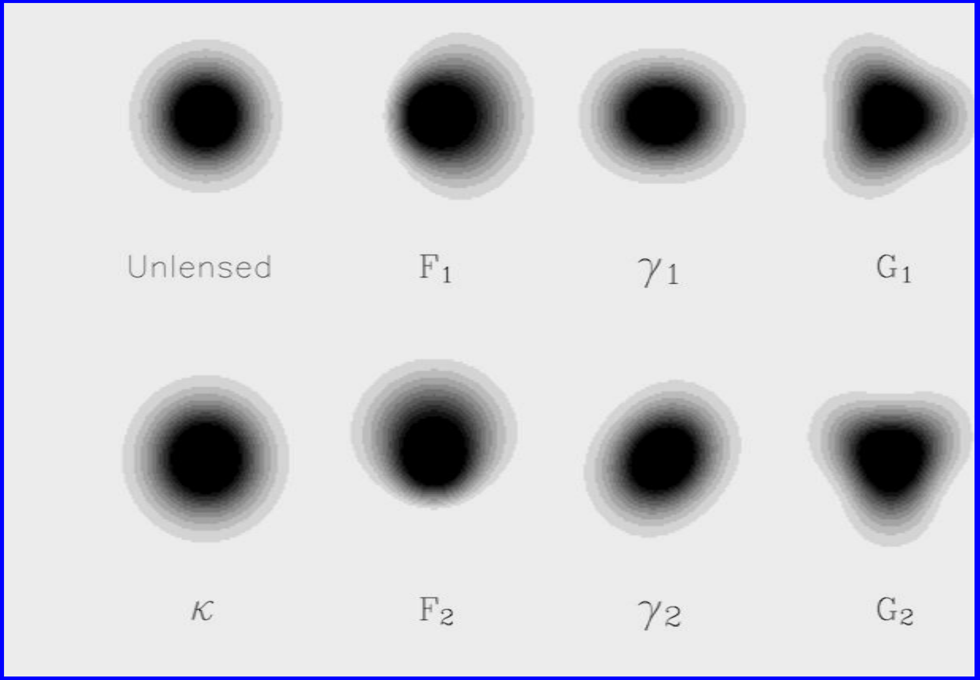
$$\gamma_1 = \frac{\psi_{,11} - \psi_{,22}}{2}$$

$$\gamma_2 = \psi_{,12}$$

Lensing Basics: Image Distortions

Each of the lensing "signals" produces a unique distortion of the image:

- Magnification
- Shearing
- Bananabess (Flexion)?



From Bacon, Goldberg, Rowe & Taylor (2005). The projection effects of "pure" lensing signals.

Why study clusters?

1. Lensing Basics
2. Why Clusters?
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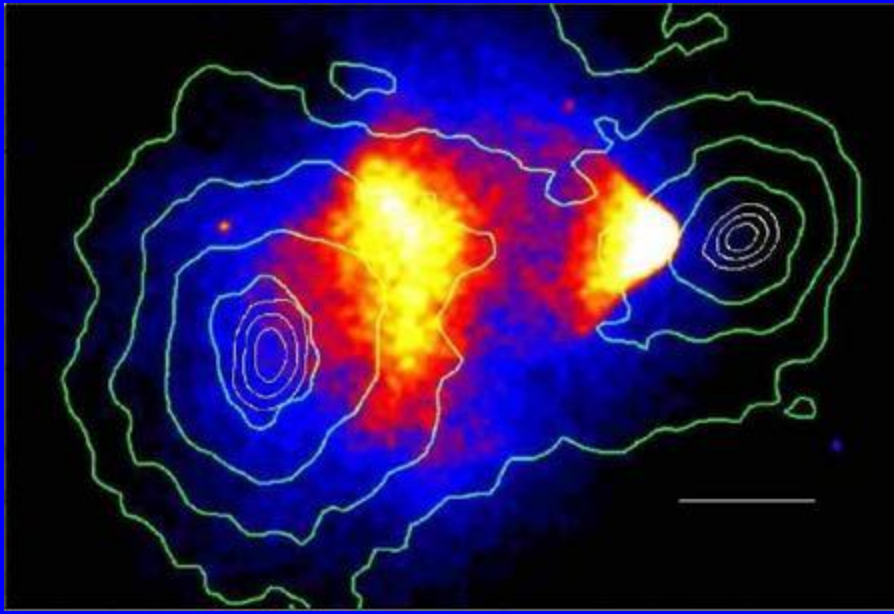


From Clowe et al. 2006, Deep Chandra Imaging of 1E 0657-56

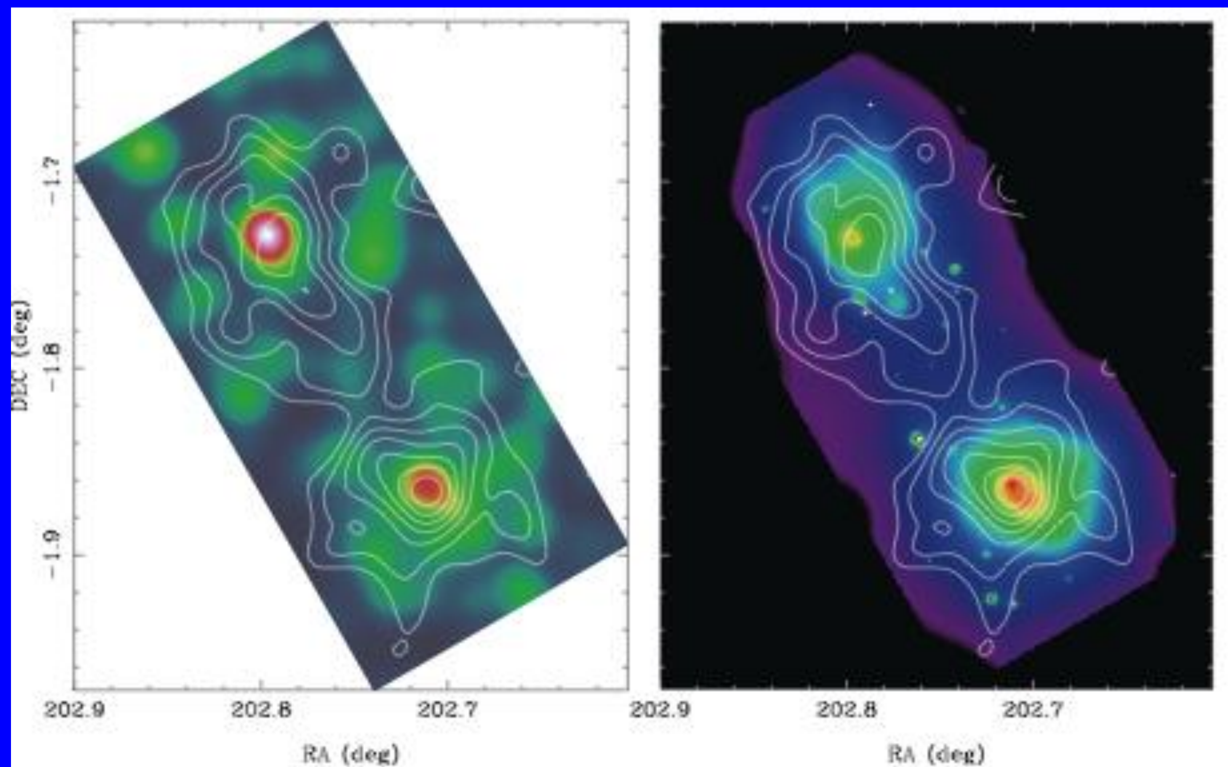
Why clusters: They probe very different structure

1) The Mass Function is a probe of Ω_0 and σ_8
2) The shape of cluster halos probe the gas distribution
3) Is the mass distribution at the center cuspy or core-like?
4) Clusters at various epochs probe the growth of structure
5) Do clusters contain lots of sub-halos? Is there significant substructure?
6) Are the halos highly elongated?

Why clusters: Recent Results

Okabe & Umetsu (2008) did a weak lensing of study of 7 nearby merging cluster systems using Subaru.

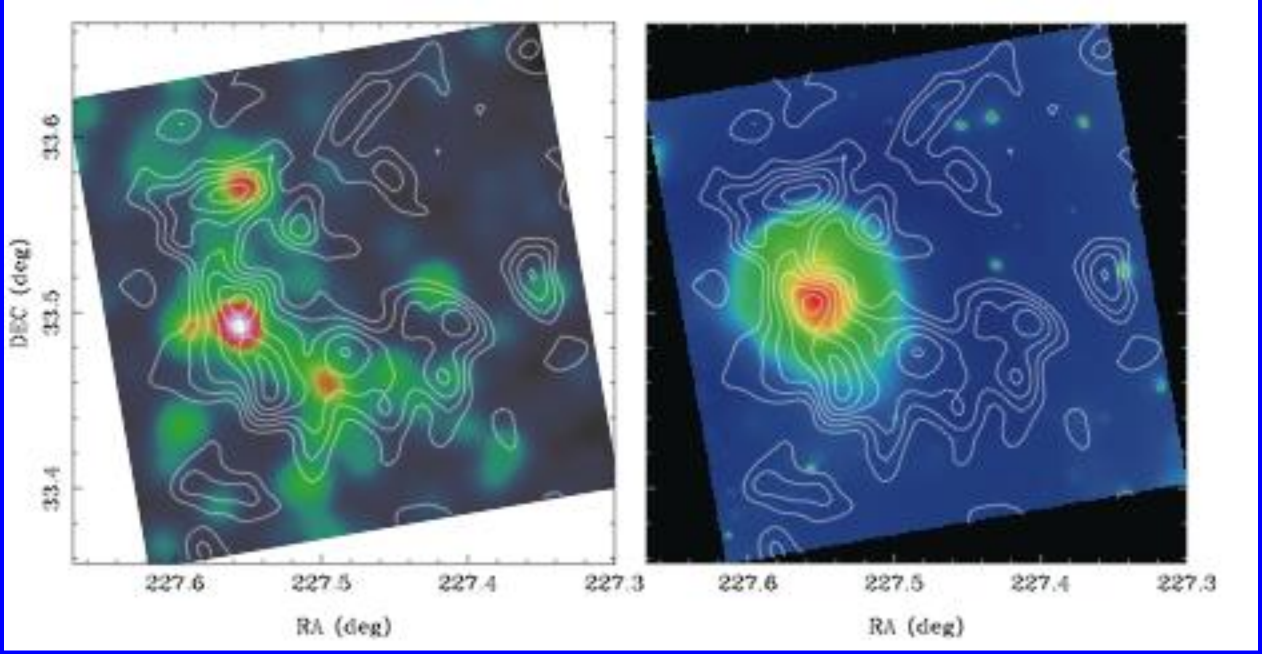
Optical X-ray



Okabe & Umetsu (2008), Fig. 6. A1750, a binary cluster at z=0.086, undergoing initial infall.

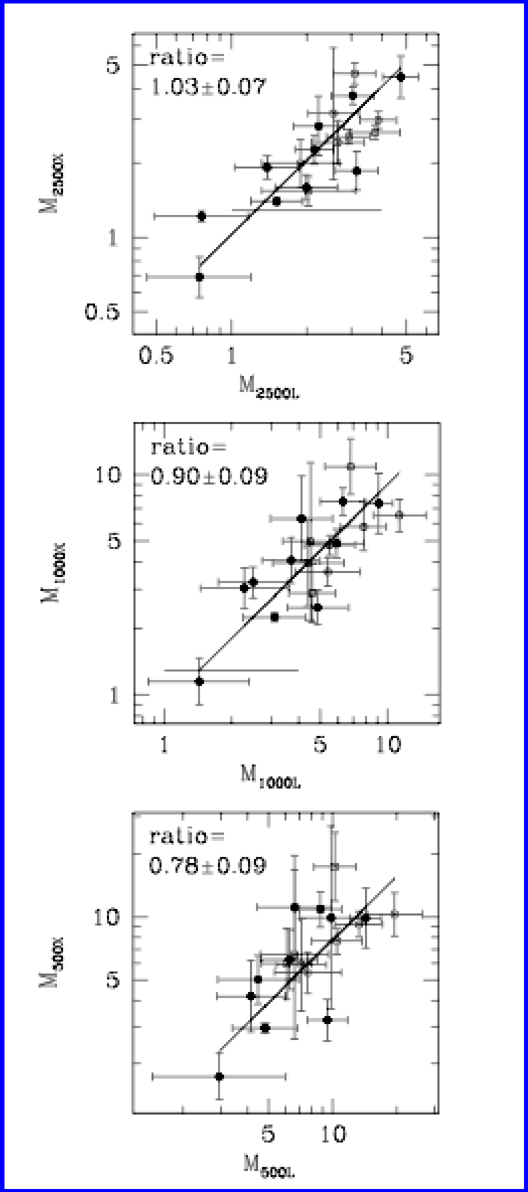
Why clusters: Recent Results

Optical X-ray



Okabe & Umetsu (2008), Fig. 9. A2034, at z=0.11. While there is only a single X-ray peak, there are genuine DM (and galaxy) secondary peaks.

Why clusters: Recent Results



Combined with X-ray data, we can also get insight into the relaxation of individual clusters

Mahdavi et al. (2008), Fig. 2. A composite of 18 clusters. The outer regions are presumably non-virialized, with M_X/M_lens < 1

Why clusters: Recent Results

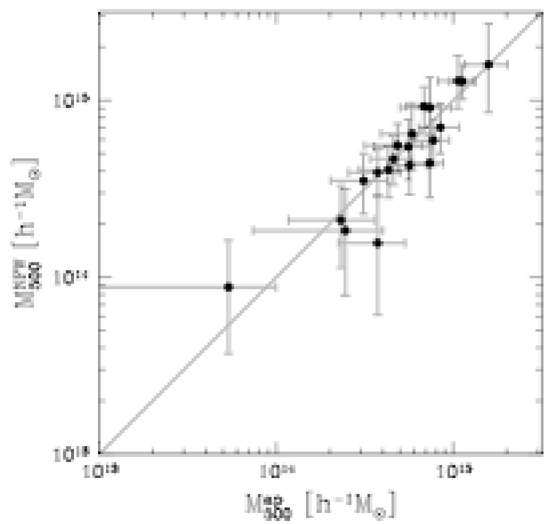


Figure 4. A comparison of M_{500} determined from an NFW fit to the data at radii $0.25 < r < 1h^{-1}$ Mpc and the value for M_{500} derived from the aperture mass method. The latter predominantly uses data at large radii, whereas the former is based on the lensing signal at small scales (note that the fitted range is smaller than for the values listed in Table 3). The two measurements are almost independent as the average r_{500} for the sample studied here is $870h^{-1}$kpc. The solid line corresponds to the line of equality.

Hoekstra (2007), Fig. 4. A composite of 20 X-ray clusters showing the fitted NFW profile mass with Lensing Aperture mass.

Clusters can be fit **very** well parametrically (by NFW profiles)!

$$\rho(r) \propto \frac{1}{r/r_{200}\,(r/r_{200} + 1/c)^2}$$

Why clusters: Recent Results

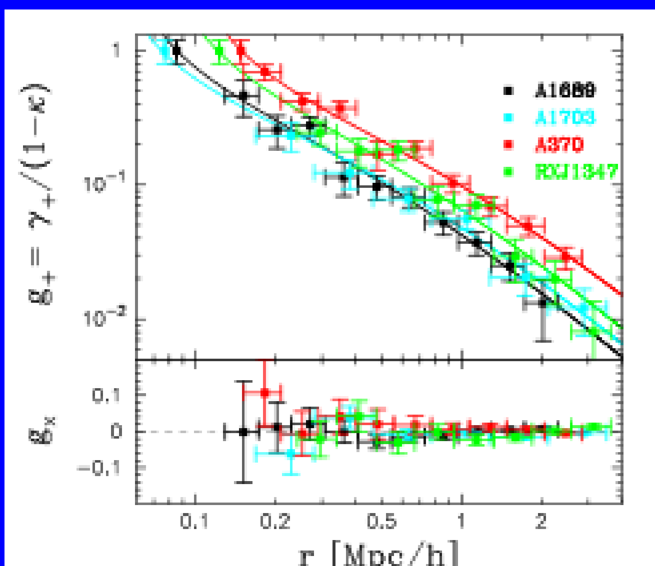
Interestingly, though, the reconstructed mass profiles produce much more:

- cuspy peaks (c_{VIR} ~ 10)

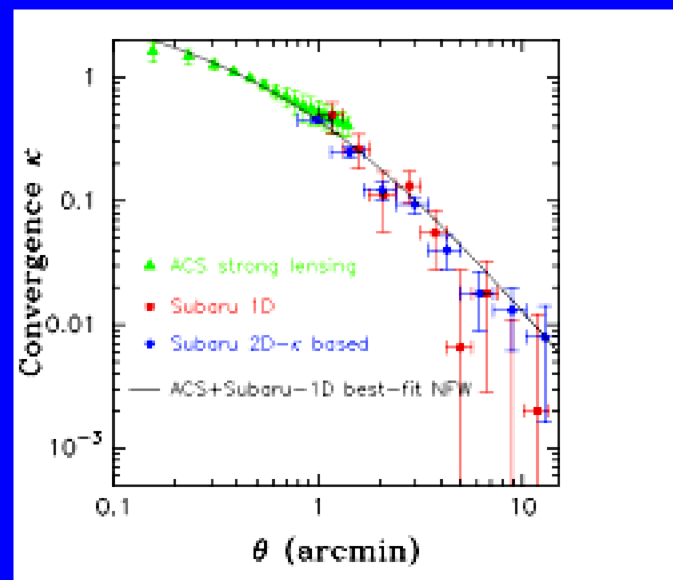
  compared to

□ΛCDM simulations (c_{VIR} ~5)

How do we reconcile this?



From Broadhurst et al. (2008) Fig. 2. The radial reduced shear profiles of 4 well studied clusters seem to follow a very similar profile.



From *Umetsu, Takada, & Broadhurst (2008) Fig 5.* The reconstructed mass profile of A1689 from Subaru and ACS data.

Why clusters: Remaining Questions

What do we do well?
1) Find the centers of mass
2) Compute radial profiles of clusters
3) Compute M/L ratios of galaxies in clusters

What do we still need work on?
1) Compute the ellipticity of clusters
2) Find substructure in clusters (beyond two giant merging peaks)
3) Sensitively probe the cores of clusters

New Techniques in Cluster Reconstruction: Flexion, HOLICs, PBL, and Strong+Weak

1. Lensing Basics
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New Techniques: Flexion

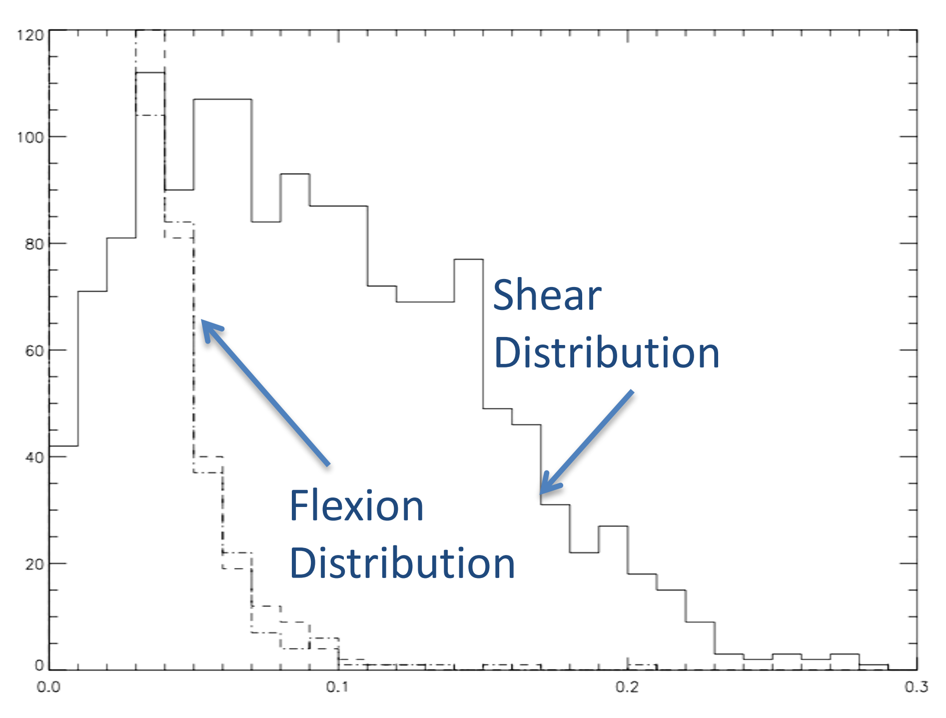
Traditional lensing inversions have assumed
a linear distortion field: ellipses lens to ellipses:

$$\underbrace{\beta_i = A_{ij}\theta_j}_{\text{Linear}} + \underbrace{\frac{1}{2}\partial_k A_{ij}\theta_j\theta_k}_{\text{2}^{\text{nd}}\text{ Order}}$$

The 2nd order effect can be important if the size of the image is comparable
to the characteristic scale of the lensing field.

New Techniques: Flexion

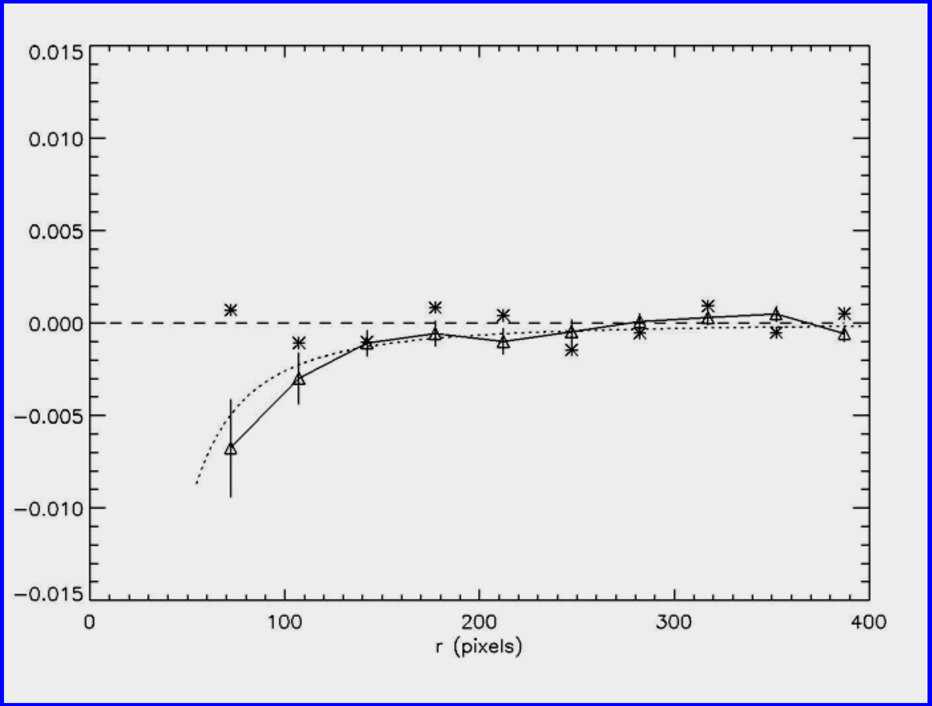
Because the *intrinsic* Flexion is so small, this signal can be powerful in a number of environments.



Courtesy of David Bacon. The distribution of Intrinsic galaxy shapes as measured from the GEMs survey.

New Techniques: Flexion

This signal is visible statistically in Galaxy-Galaxy lenses, and, as we will see, in clusters as well.



From Leonard, Goldberg, Massey & Haaga (2007), a galaxy-galaxy flexion measurement In A1689.

New Techniques: Flexion -> HOLICs

An important step forward came when Okura, Umetsu & Futamase (2006), expressed flexion in terms of (3rd) multipole moments of the image, which they termed:
Higher Order Lensing Image Characteristics – HOLICs
From their paper:

We first define the normalization factor ξ as

$$\xi \equiv Q_{1111} + 2Q_{1122} + Q_{2222} \tag{25}$$

with spin-0. Then, we define the following combinations of octopole moments as our HOLICs:

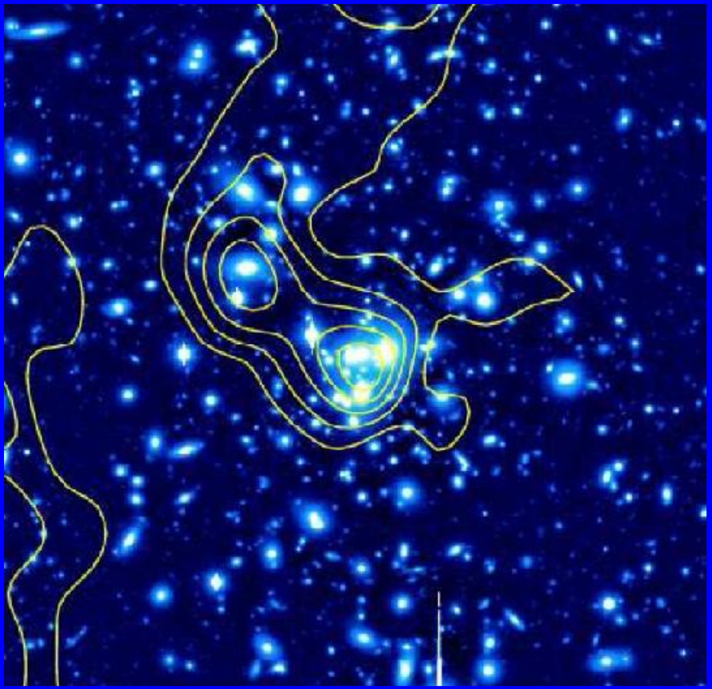
$$\zeta \equiv \frac{(Q_{111} + Q_{122}) + i(Q_{112} + Q_{222})}{\xi}, \tag{26}$$

$$\delta \equiv \frac{(Q_{111} - 3Q_{122}) + i(3Q_{112} - Q_{222})}{\xi}, \tag{27}$$

Main Flexion Signal
Secondary Flexion Signal

New Techniques: HOLICs

By carefully selecting only VERY high S/N images, the HOLICs team is able to reconstruct substructure in clusters. The A1689 reconstruction used only ~8 images!



HOLICs reconstruction of A1689 from Okura, Umetsu & Futamase (2007)

New Techniques: Strong+Weak United

In recent years, there's been a lot of interest in combining weak and strong lensing. One of the difficulties with weak lensing reconstructions is that weak lensing is **Very** noisy:

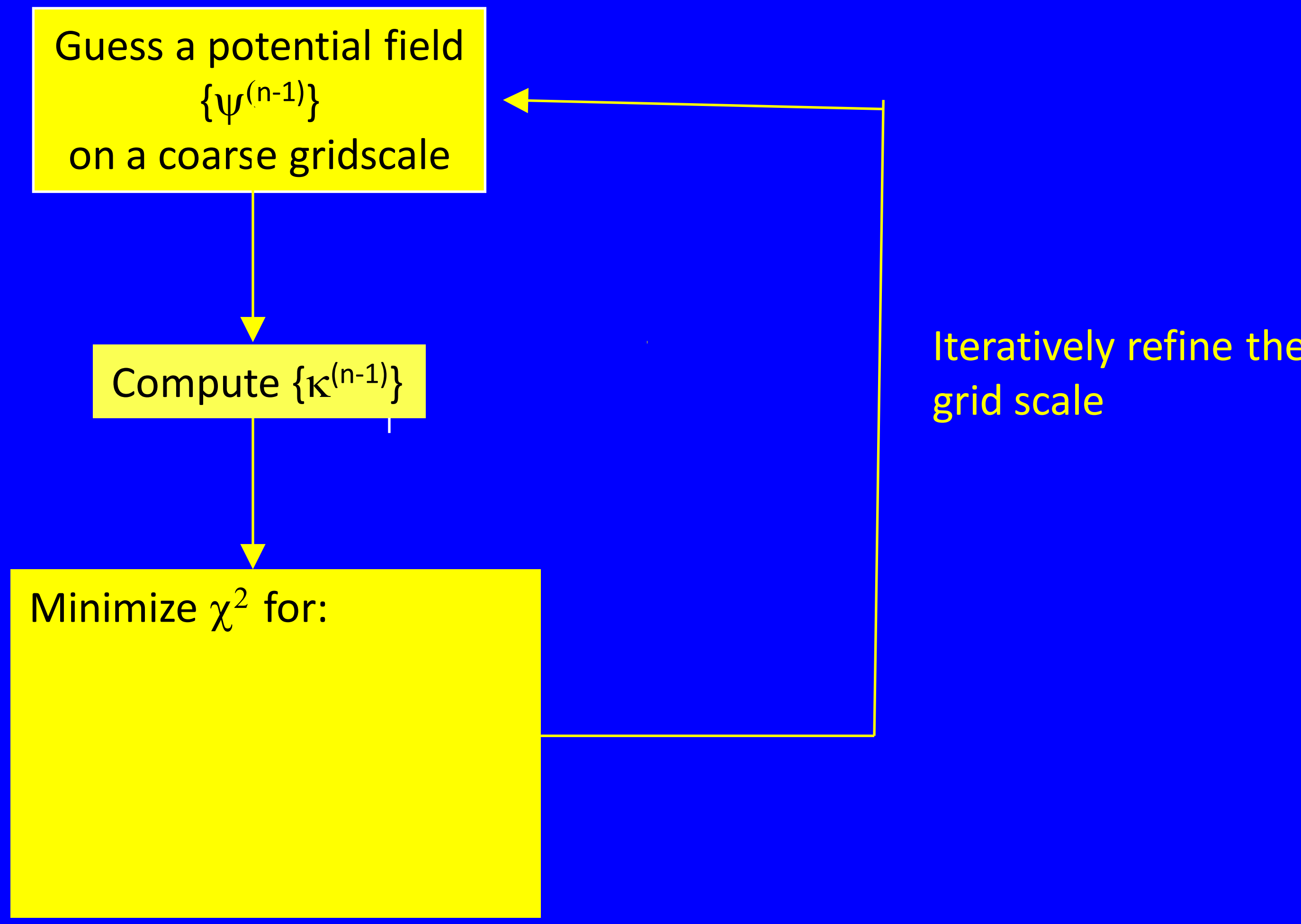
$$\sigma_\gamma \simeq \frac{0.3}{\sqrt{N}}$$

It can take ~100's of galaxies to produce a high S/N image, resulting in very Poor resolution.

New Techniques: Strong+Weak United

Step 1:

Reconstruct the field from Weak lensing only.



New Techniques: Strong+Weak United

Step 2:

Produce a likelihood function including a strong lensing signal.

$$\chi^2_{strong} = \Sigma \, \eta \left([\vec{\theta}^A - \vec{\theta}^B] - [\vec{\alpha}(\vec{\theta}^A) - \vec{\alpha}(\vec{\theta}^B)] \right)^2$$

This allows us to constrain the “critical curves” of a lensing field.

New Techniques: Strong+Weak United

One of the most important recent results has involved the "Bullet Cluster" in which two colliding clusters have:

- Dark Matter and Galaxies are well-aligned
- The gas, however, trails the DM.

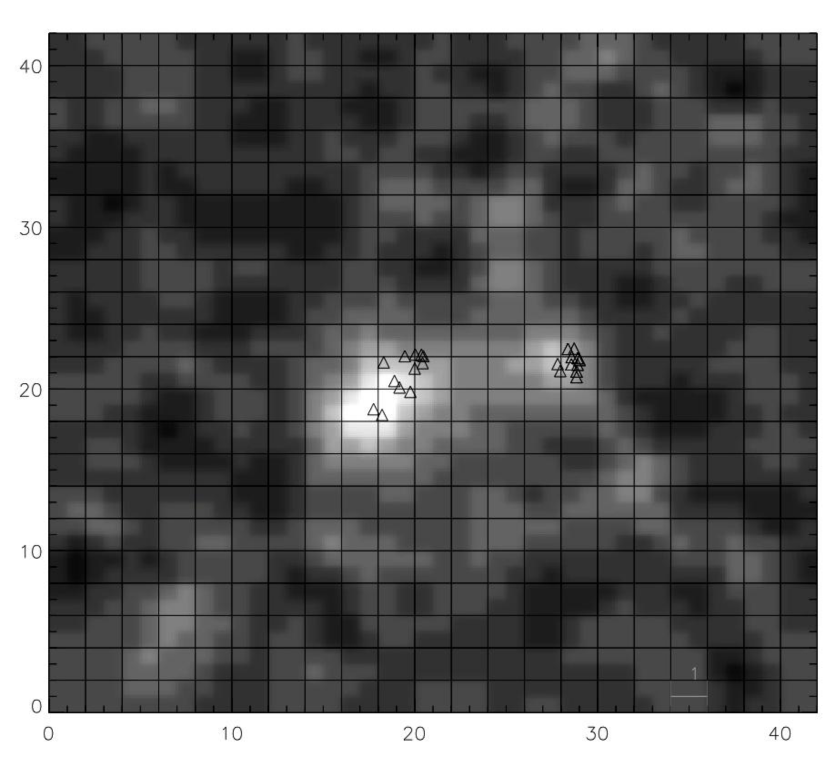
This is considered by many to be the first direct detection of Dark Matter!

The "Bullet Cluster," 1E0657-56, Bradac et al. (2006) astro-ph/0608408

New Techniques: Particle-Based Lensing

One of the limits of S+W reconstructions is that the information from the "strong" and "weak" signals are on vastly different scales.

Perhaps we should abandon grids all together!



Reconstruction of the Bullet Cluster. Gridcells indicate a typical weak-lensing Resolution scale. Triangles represent Multiple images of sources (strong lensing)

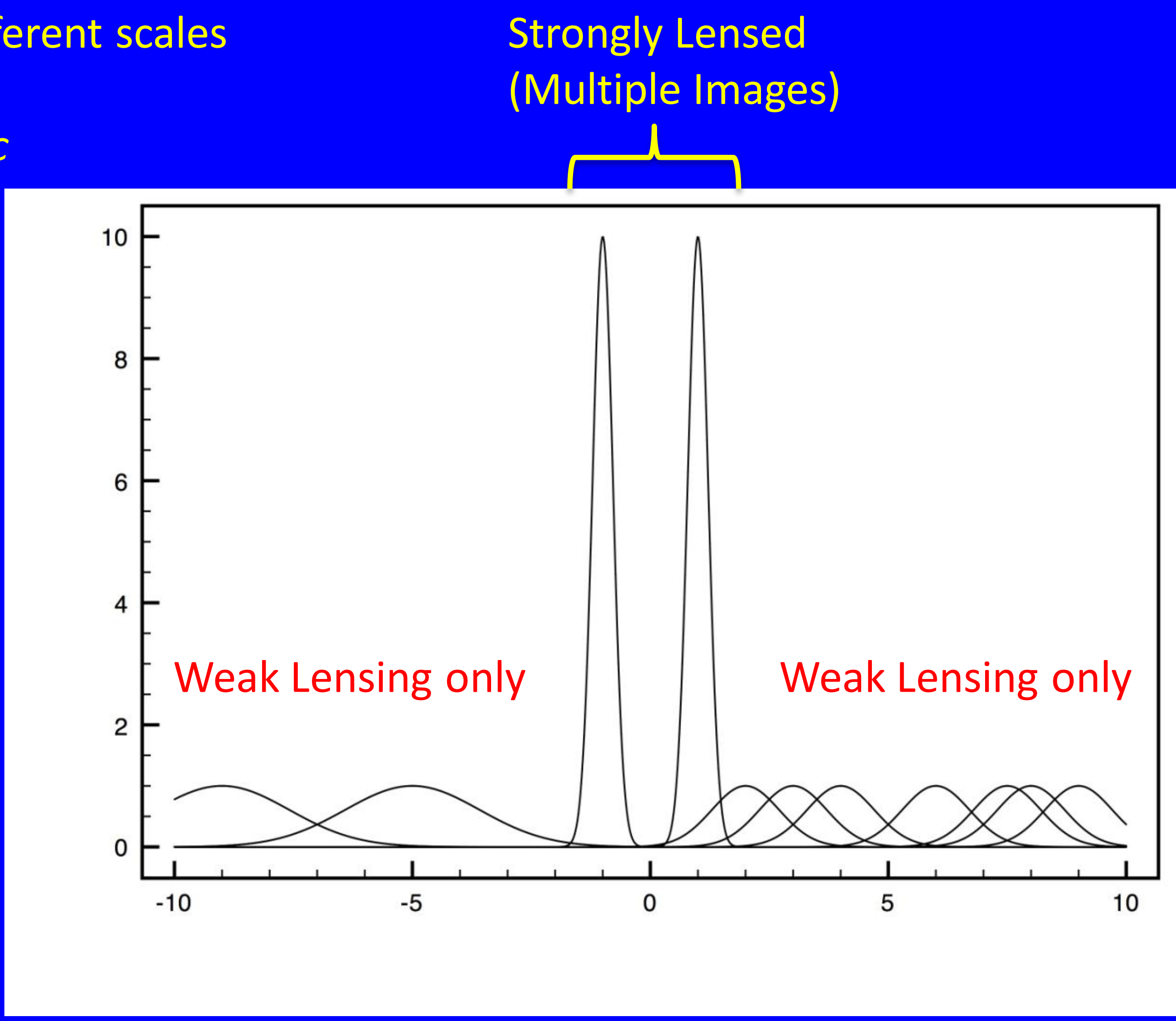
New Techniques: Particle-Based Lensing

Q: How can we reconcile the different scales for strong and weak without introducing *ad hoc* regularization?

A: Don't use grids!

We can take our lesson from simulations and use smoothed particle hydrodynamics to model the potential field.

The cartoon to the right illustrates a potential weighting scheme for a lens with variable density of information.



New Techniques: Particle-Based Lensing

We can treat individual source galaxies as discrete sources of a continuous field:

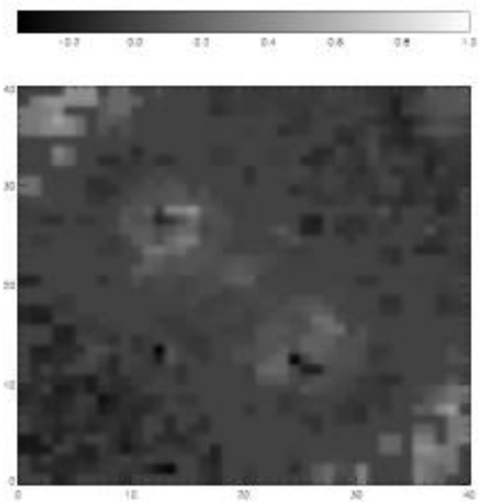
$$\psi(\vec{\theta}) = \psi_n + \theta_j \psi_{n,j} + \frac{1}{2}\theta_j \theta_k \psi_{n,jk} + \cdots$$

Where the derivatives are determined via χ^2 minimization, and where each source position has its own local smoothing kernel.

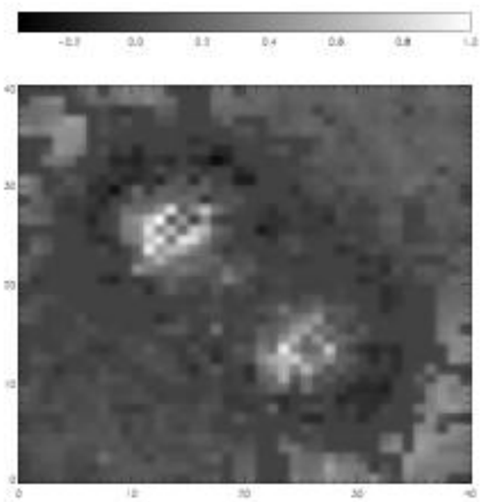
We can make kernels smaller in regions of higher information density, and larger elsewhere.

Even where there is only weak-lensing data, this allows us to resolve finer structure…

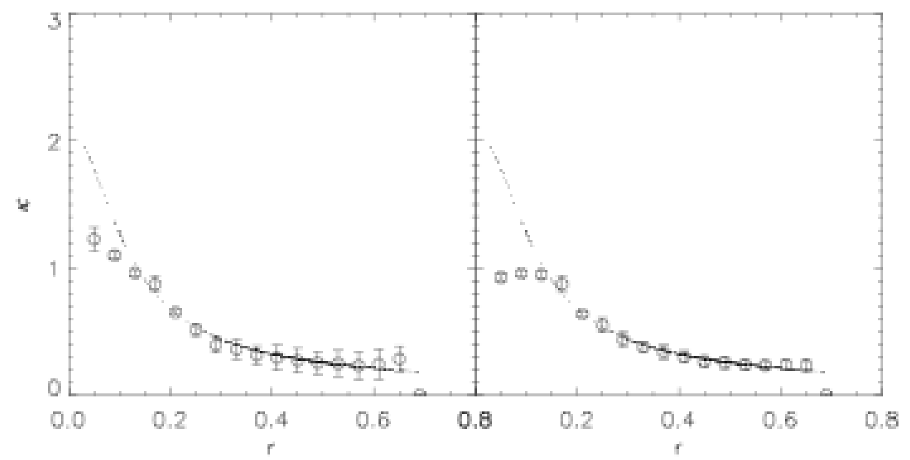
New Techniques: Particle-Based Lensing



PBL



Grid

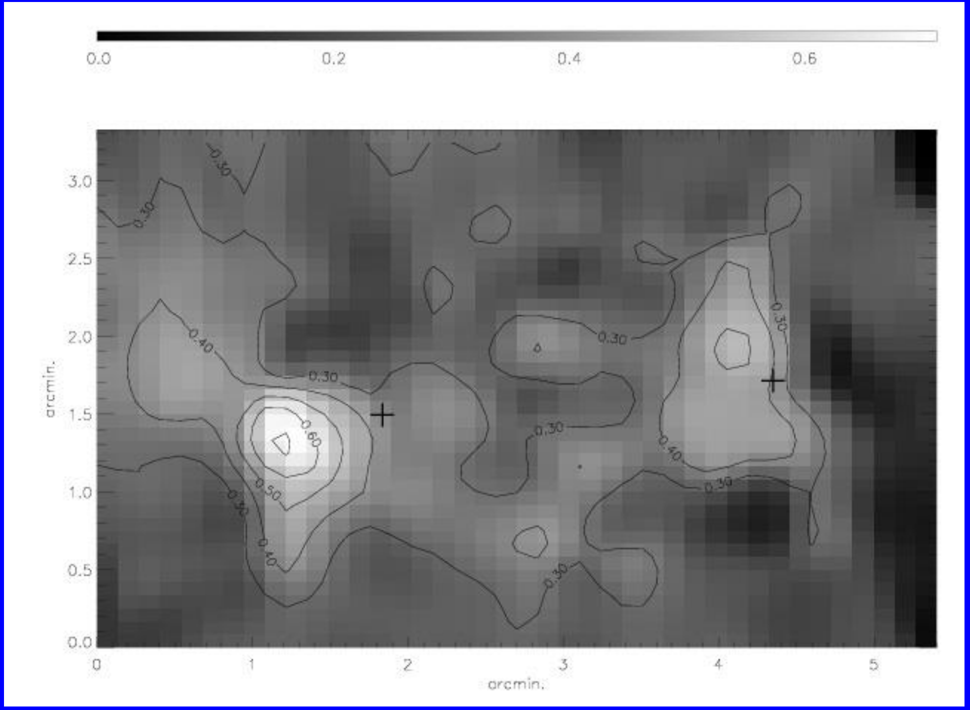


PBL (1 peak)   Grid (1 Peak)

From Deb, Goldberg & Ramdass, 2008
Reconstruction errors of a simulated one
And two peak systems

New Techniques: Particle-Based Lensing

Initial results have utilized weak lensing signals only.



From Deb, Goldberg & Ramdass (2008). Reconstruction of the Bullet Cluster using PBL Weak Lensing Signal ONLY!

New Techniques: Particle-Based Lensing

But wait! There are still more signals!
On the strong lensing side, we use:
0) Position Differences (2 constraints/pair)

But we don't use:

1) Flux ratios (1 constraint/pair)
2) Ellipticity Differences (2 constraints/pair)

Why not? Naively, we might expect to increase our S/N by:

$$\left(\frac{S}{N}\right)_{new} = \sqrt{\frac{5}{2}}\left(\frac{S}{N}\right)_{old} \simeq 1.6\left(\frac{S}{N}\right)_{old}$$

The Fisher Matrix and Clusters

1. Lensing Basics
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All of these reconstruction schemes involve smoothing data in order to reduce noise. However, over-smoothing washes out real structure.

Information: The Fisher Matrix

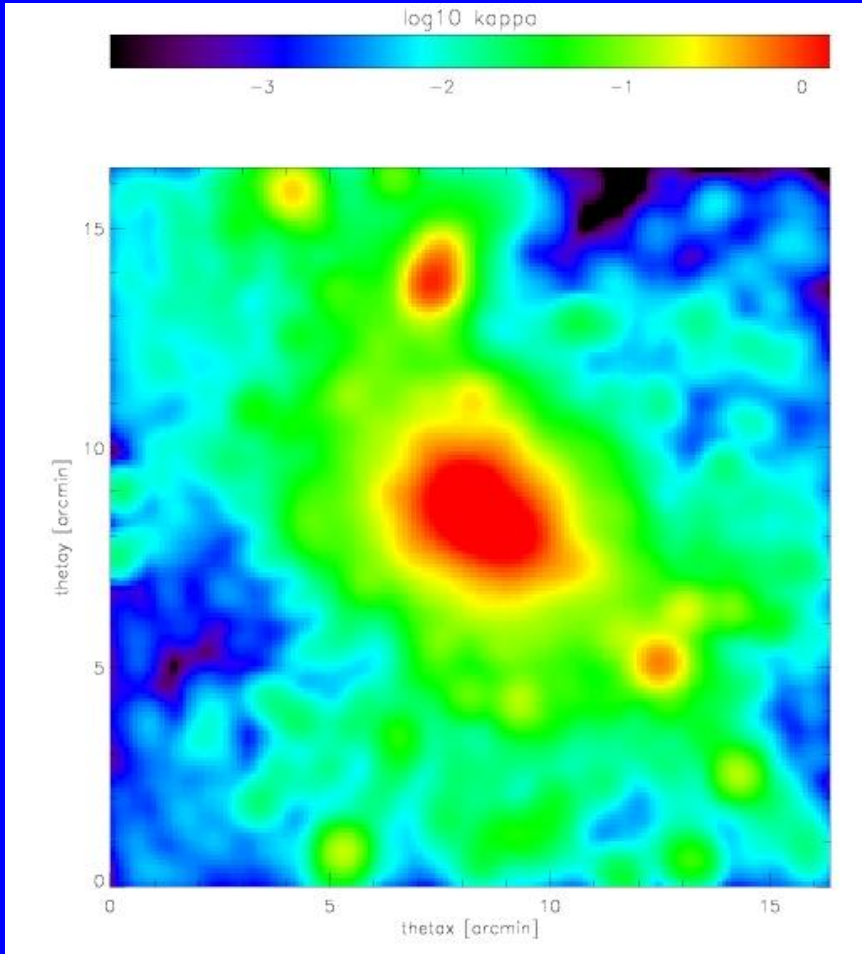
When we talk about "Information" what we really mean is the ability to resolve parameters using a measured likelihood function:

$$F_{ij} \equiv \langle \frac{\partial^2 \mathcal{L}}{\partial p_i \partial p_j} \rangle$$

where "p" can represent parameters in a parametrized mass fit, or, in our case, simply κ at different points in the mass map.

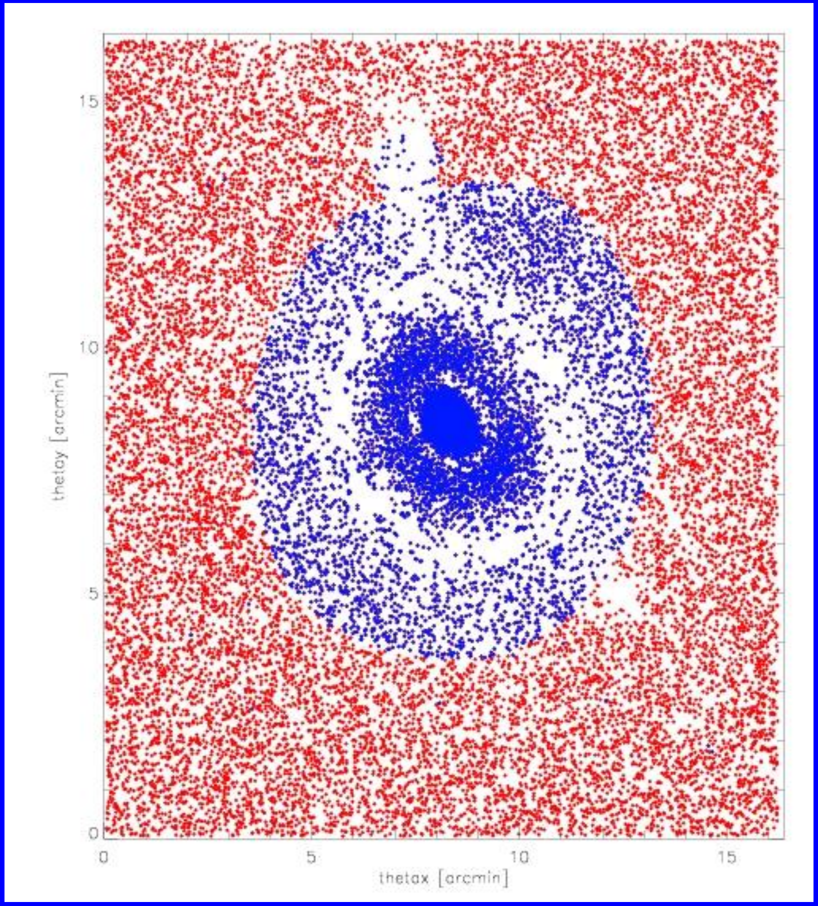
Information: Cluster Simulations

We simulated the lensing by a cluster from the Virgo Consortium GIF simulations.



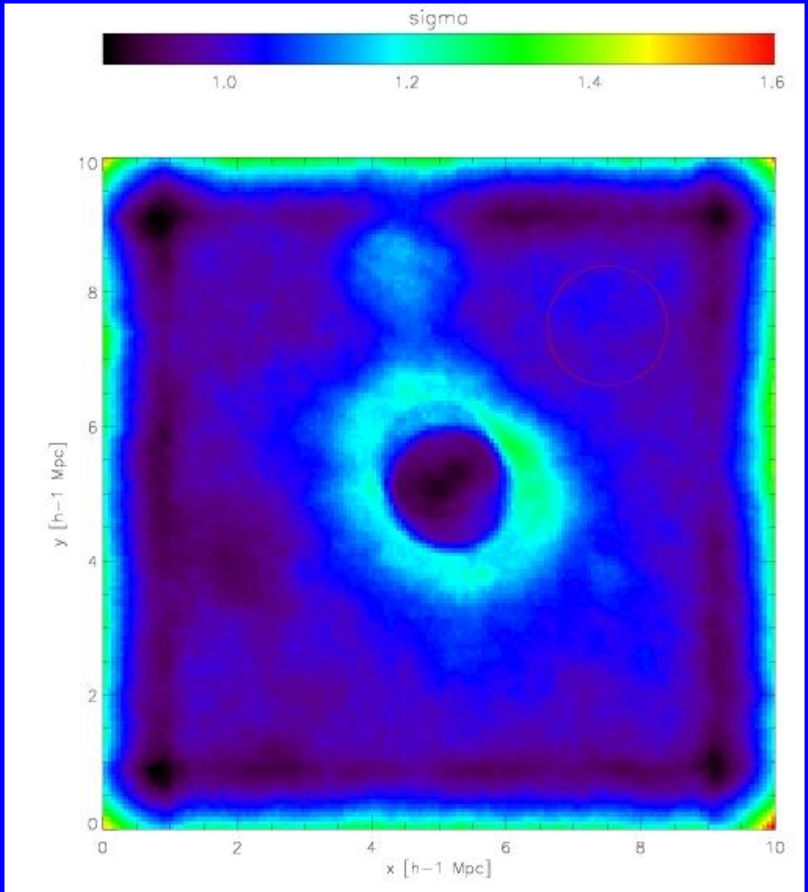
Information: Cluster Simulations

Taking a uniform prior distribution of background galaxies, they don't lens to a uniform distribution:



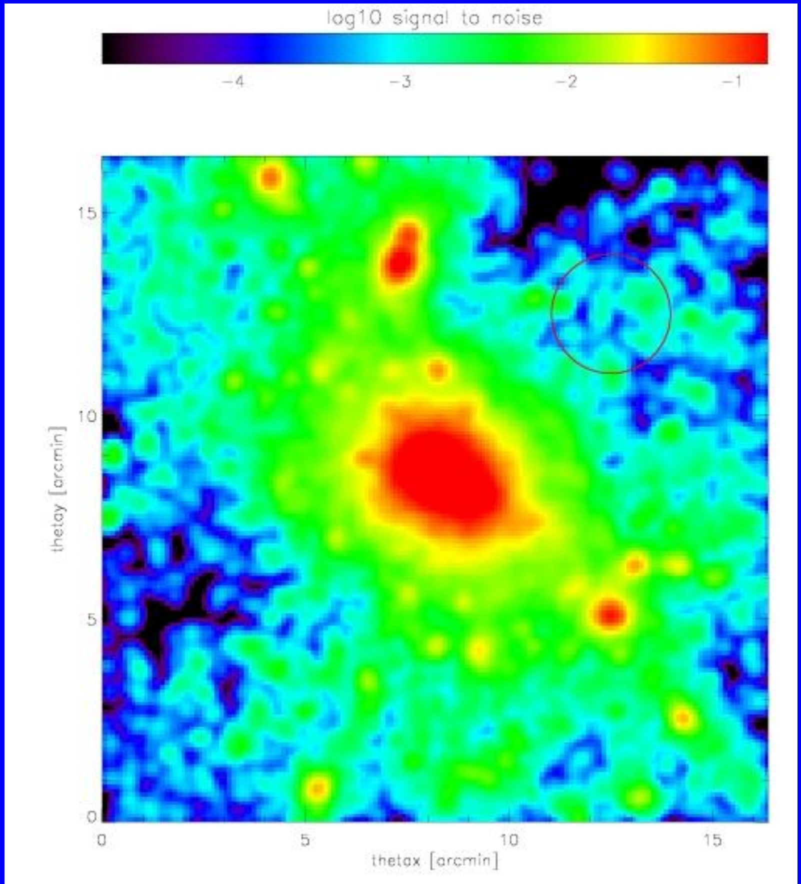
Information: Cluster Simulations

Taking a uniform prior distribution of background galaxies, and we don't end up with a uniform distribution of noise:



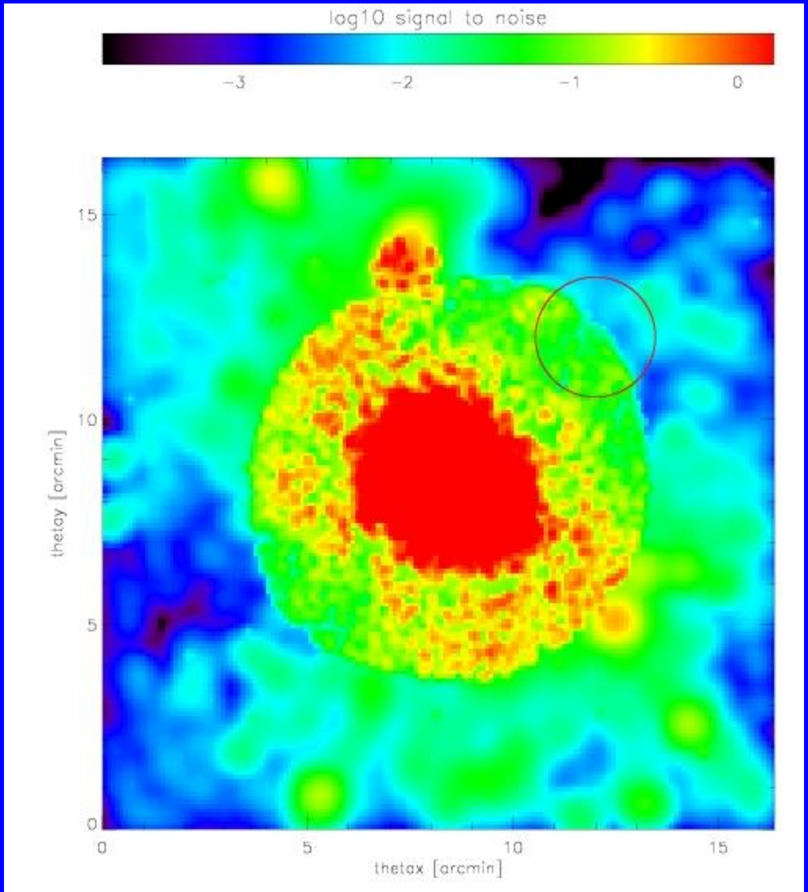
Information: Cluster Simulations

But most importantly, we don't end up with anything like a uniform signal/noise ratio:



Information: Cluster Simulations

We can take this a step further, and realize that we have a *very* variable signal to noise if we include both strong and weak lensing signals.



Information: The Bottom Line

This is precisely our point!

By identifying regions of high signal to noise, we can adjust the smoothing scale.
By identifying datapoints within high S/N regions, we can adjust the relative weighting.

This will allow unprecedented levels of cluster resolution!

Thank you!

Students:
- Sanghamitra Deb
- Jason Haaga
- Adrienne Leonard
- Vede Ramdass
- Alyssa Wilson

Collaborators:
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- Richard Massey
- Barnaby Rowe
- Andy Taylor

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- HST Archival Program

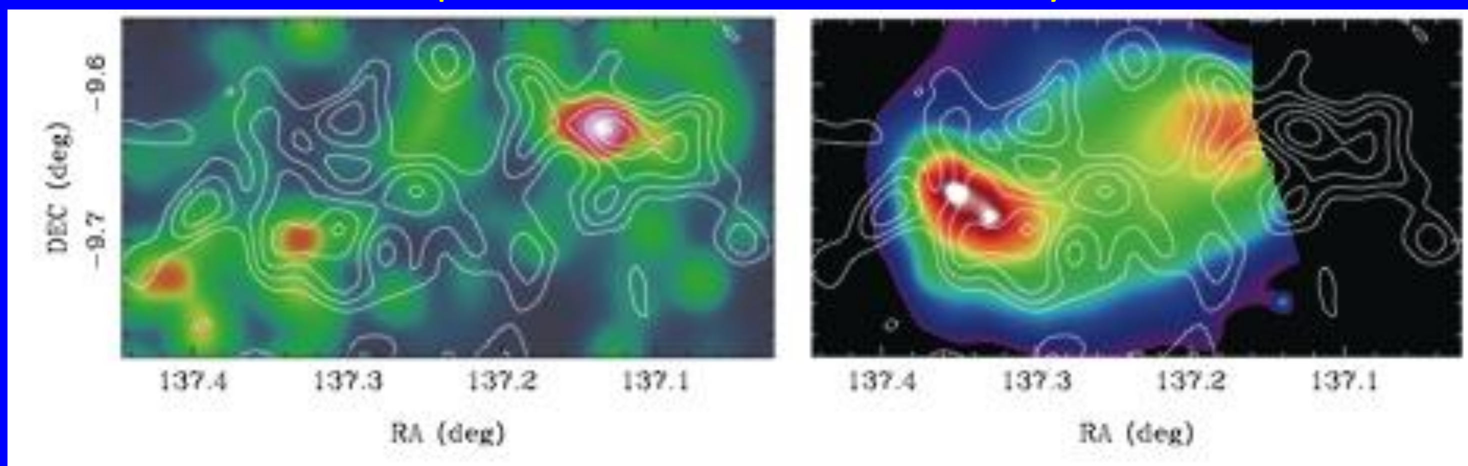
Special Thanks to the IPMU and Dr. Hitoshi Murayama and Dr. Masahiro Takada for hosting me

For Further Reading:

- *Reconstruction of Cluster Masses using Particle Based Lensing I: Application to Weak Lensing*, S Deb, DM Goldberg, & VJ Ramdass, 2008, accepted to ApJ, arxiv/0802.0004
- *Gravitational Shear, Flexion, and Strong Lensing in Abell 1689*, A Leonard, DM Goldberg, J Haaga & R Massey, 2007, ApJ 666, 51 astro-ph/0702242
- *Measuring Flexion*, DM Goldberg & A Leonard, 2006, ApJ 660, 1003, astro-ph/0607602
- *Weak Gravitational Flexion*, DJ Bacon, DM Goldberg, BTP Rowe, and AN Taylor, 2006, MNRAS 365, 414 astro-ph/0504478
- *Galaxy-Galaxy Flexion: Weak Lensing to Second Order*, DM Goldberg & DJ Bacon, 2005, Astrophys. J. 619, 741, astro-ph/0406376

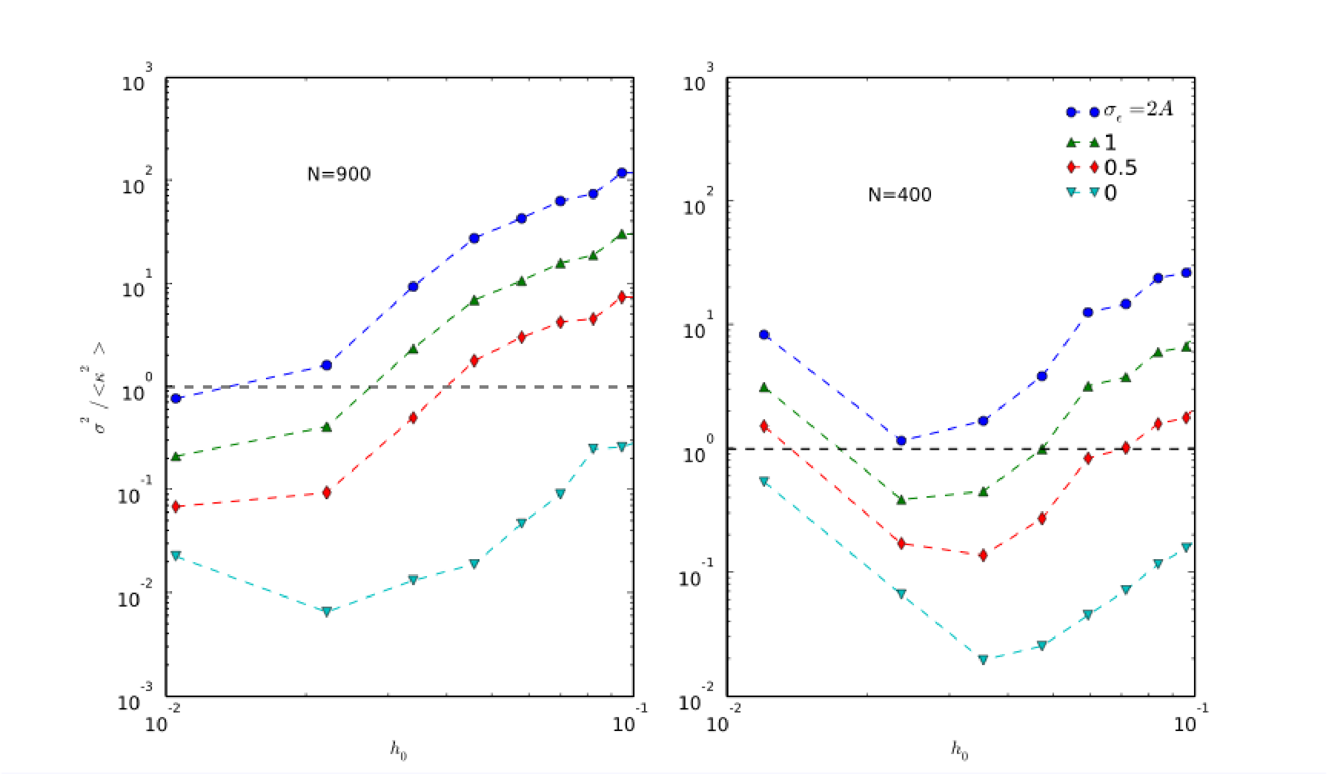
Why clusters: Recent Results

Optical X-ray



Okabe & Umetsu (2008), Fig. 5. A 754, a merging cluster at z=0.054. The main clumps have passed through the core perhaps once, and based on dynamical models, may be at or near turnaround.

Information: Weak Lensing Scaling



From Deb et al., 2009 (in prep). An optimal smoothing kernel can be chosen to optimize the tradeoff between resolution and shot noise.

New Techniques: Flexion -> HOLICs

In our original formulation, the process was highly unstable.

1) Decompose images into “Shapelets” (Hermite Polynomial Basis functions)
2) Express the flexion operator as a combination of step-up and step-down operators:

$$
\begin{aligned}
S^{(2)}_{11} &= \frac{1}{4\sqrt{2}}\Big[-2\hat{a}_1^3 + \hat{a}_1\left(4 - 2\hat{N} + 12 < xx >\right) + 8 < xy > \hat{a}_2 - 8 < xy > \hat{a}_2^\dagger + \hat{a}_1^\dagger\left(6 + 2\hat{N} - 12 < xx >\right) \\
&\quad + 2\hat{a}_1^{\dagger 3}\Big] \\
S^{(2)}_{12} &= \frac{1}{4\sqrt{2}}\Big[-8 < xy > \hat{a}_1 + 2\hat{a}_2^3 + \hat{a}_2\left(-4 + 2\hat{M} - 12 < yy >\right) + \hat{a}_2^\dagger\left(-6 - 2\hat{M} + 12 < yy >\right) \\
&\quad - 2\hat{a}_2^{\dagger 3} + 8 < xy > \hat{a}_1^\dagger\Big] \\
S^{(2)}_{21} &= \frac{1}{4\sqrt{2}}\Big[-3\hat{a}_1^2\hat{a}_2 - \hat{a}_1^2\hat{a}_2^\dagger + 12 < xy > \hat{a}_1 - \hat{a}_2^3 + \hat{a}_2\left(3 - 2\hat{N} - 1\hat{M} + 2 < xx > + 10 < yy >\right) \\
&\quad + \hat{a}_2^{\dagger 1}\left(6 + 2\hat{N} + \hat{M} - 2 < xx > - 10 < yy >\right) + \hat{a}_2^{\dagger 3} - 12 < xy > \hat{a}_1^\dagger + \hat{a}_1^{\dagger 2}\hat{a}_2 + 3\hat{a}_1^{\dagger 2}\hat{a}_2^\dagger\Big] \\
S^{(2)}_{22} &= \frac{1}{4\sqrt{2}}\Big[-\hat{a}_1^3 - 3\hat{a}_1\hat{a}_2^2 + \hat{a}_1\left(3 - \hat{N} - 2\hat{M} + 10 < xx > + 2 < yy >\right) + \hat{a}_1\hat{a}_2^{\dagger 2} + 12 < xy > \hat{a}_2 \\
&\quad - 12 < xy > \hat{a}_2^\dagger - \hat{a}_1^\dagger\hat{a}_2^2 + \hat{a}_1^\dagger\left(6 + \hat{N} + 2\hat{M} - 10 < xx > - 2 < yy >\right) + 3\hat{a}_1^\dagger\hat{a}_2^{\dagger 2} + \hat{a}_1^{\dagger 3}\Big] ,
\end{aligned}
$$

3) Find the derivatives of the shear which best fit the observed image.

$$f(\mathbf{x}) \simeq (1 + \kappa\hat{K} + \gamma_i\hat{S}_i^{(1)} + \gamma_{i,j}\hat{S}_{ij}^{(2)})f(\mathbf{x}')$$