INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Topological Interactions at the LHC and a Generalized Laudau-Yang Theorem

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Keung, Low, JS, arXiv:0806.2864

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Keung, Low, JS, arXiv:080X.XXXX

Outline

- Motivation.
- Generalized Landau-Yang theorem.
- Angular distributons.
- Measurements at the LHC.
- Summary and Outlook.

Z’

- Topological interactions are interactions which are independent of the space-time metric.
- They are coming from anomalies of the UV physics which involves several gauge bosons or Goldstones.
- Topological physics BSM typically involves at least one extra gauge boson (Z’).

So, let’s start our topic with topological interactions that involves a Z’ particle. ....

What is Z' ?

Z' is a massive, neutral (no electromagnetic charge, anti-Z' = Z') , spin-one particle with its mass ranging from TeV to GUT scale.

Many extensions of SM predicts a Z' particle.

- As a massive gauge boson, its mass are generated by:
  - symmetry breaking of the extended gauge group.
  - compactification of extra spatial dimensions.

Z'@LHC

- Z' in the "moose" (with extended gauge group) models

deconstruction

- Z' in extra dimensions models.

GUT, Little Higgs, TC, ETC, Topcolor, etc.

Finding a Z' and measuring its properties is very important at the LHC!

ADD, RS, UED, Higgsless, etc.

Anomalies

Anomalies are powerful tools to probe the UV physics
Its presence is irrelevent to the detailed dynamics
of the theory (topological properties).

- Topological interactions may present in TeV.
  - In strongly coupled theory: Techicolor model, composite Higgs model.
  - WZW term in the nonlinear sigma model based on G/H. CS term in 5D theory (holographic dual)
  - Just heavy (TeV) exotic fermions in the loop, or Green-Schwarz mechanism to cancel the mixed anomalies. (Stringy motivated Intersection brane model).

Anomalies@LHC

- However, those topological interactions are always more than one loop suppressed. $(\frac{1}{48\pi^2} \sim 0.00211)$
- They might be completely overwhelmed by other kind of interactions, QCD radiations at the LHC.
- Even we have discoveried such interactions? How can we know the interactions we have measured are topological?

Discrete symmetries

In contrast to the regular interactions, the Lorentz index in the topological interactions are always contracted through the antisymmetric tensor. $\epsilon_{\mu\nu\rho\sigma}$

- The antisymmetric tensor in 4D violate P and T.
- So the discremination becomes how to determine the discrete symmetry of the operators at the LHC!

Anomalies@LHC

- We choose the three gauge boson couplings to study as they exsit in all cases and contain fewer particles. Then the anomalous operators are CP even and regular couplings are CP odd.
- In order to know the discrete symmetries of the coupling, one may need to know the gauge boson polarization, which requires to further decay the gauge bosons into light fermions.

Z' --->ZZ--> 4l

- We consider the Z' decay into two on-shell Zs.
  - The bose symmetry greatly simplified the form of the couplings (only 2), comparing to Z'-Z-gamma (4) and Z'-W-W (7).
  - The Z' might be produced in the cascade decay channel of some heavy particles instead of singly produced. We need a method that is independent of the Z' production mechanism.
- We consider the 4l final states in our measurements.
  - They are very clean channels and our measurements based on azimuthal angle really require high energy resolution.
  - The 4l final state is well studied in the $H \rightarrow ZZ \rightarrow 4l$

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The Landau-Yang theorem

The Landau-Yang theorem: A massive spin-one particle can never decay into two on-shell photons.

L. D. Landau, Dokl. Akad. Nawk., USSR 60, 207 (1948)

C. N. Yang, Phys. Rev. 77, 242 (1950)

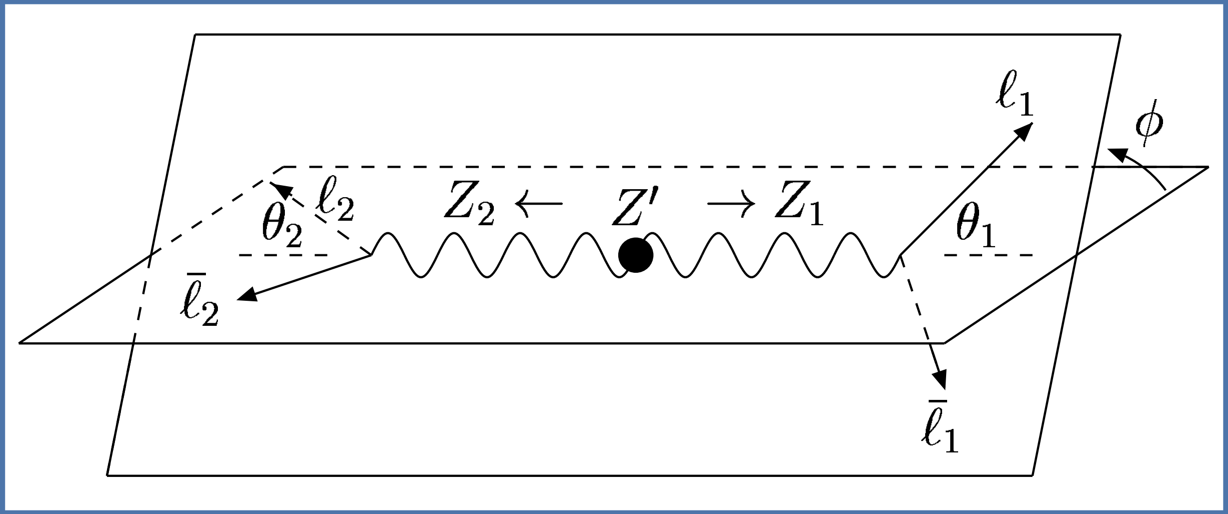
Notice that it doesn't apply to two on-shell gluons because of the additional color d.o.f.

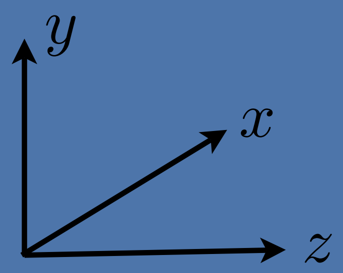
Our arguments:

For a massive spin-one particle (Z') decaying into two identical on-shell massive spin-one particles (Z),

- There are only two independent helicity amplitudes, which are from CP odd and CP even operators respectively.
- The differential cross section depends on the kinematics solely through a phase shift in the azimuthal angle between the two Z decay planes.

The Setup





In the Z' rest frame

$$\epsilon_0^{(1)} = \gamma(\beta, 0, 0, 1)$$

$$\epsilon_0^{(2)} = \gamma(-\beta, 0, 0, 1)$$

$$\epsilon_\pm^{(1)} = (0, \mp 1, -i, 0)/\sqrt{2} = \epsilon_\mp^{(2)}$$

The "+, - , 0" stands for the Z helciity.

Notice that we choose both the longitudinal polarization of Z to be along the z axis.

The Landau-Yang theorem

- We consider three symmetry transformations:
  - R^{ψ} : rotation around the z axis by an angle (angular momentum conservation along the z)
  - R^{ξ} : rotation around the x axis by π (Bose symmetry)
  - P : space inversion (parity)

The Landau-Yang theorem

Helicity amplitude $\mathcal{M}_{\kappa,\lambda_1\lambda_2}$ in $Z'(\kappa) \to Z_1(\lambda_1)Z_2(\lambda_2)$:

Spin-projection of Z' along the z axis.

The Z helicity.

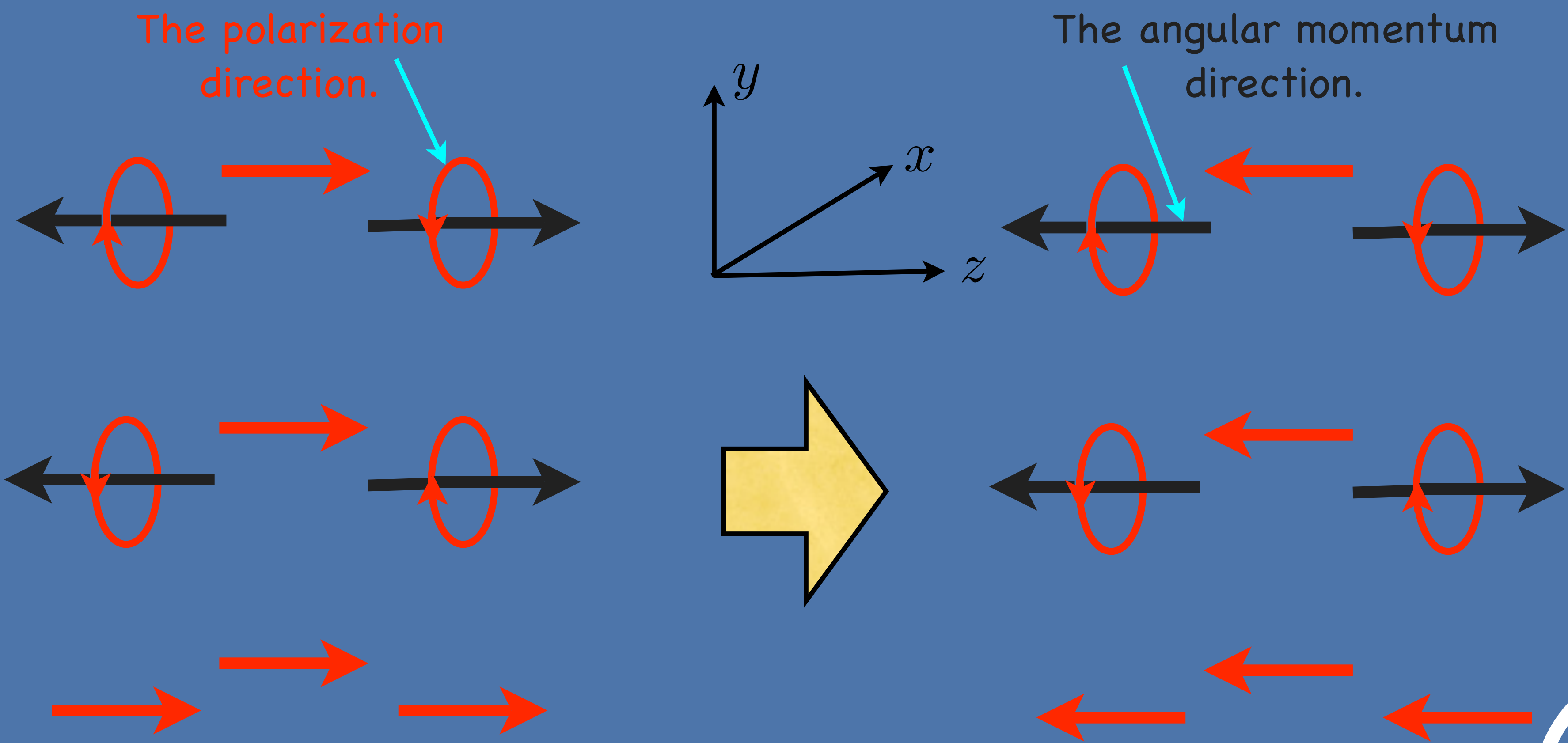
As a convention, we define

$$\epsilon_0^{(Z')} = (0,0,0,1) \qquad \epsilon_\pm^{(Z')} = \epsilon_\pm^{(1)}$$

The angular momentum conservation (R^ψ) along the z axis tells us that $\kappa = \lambda_1 - \lambda_2$

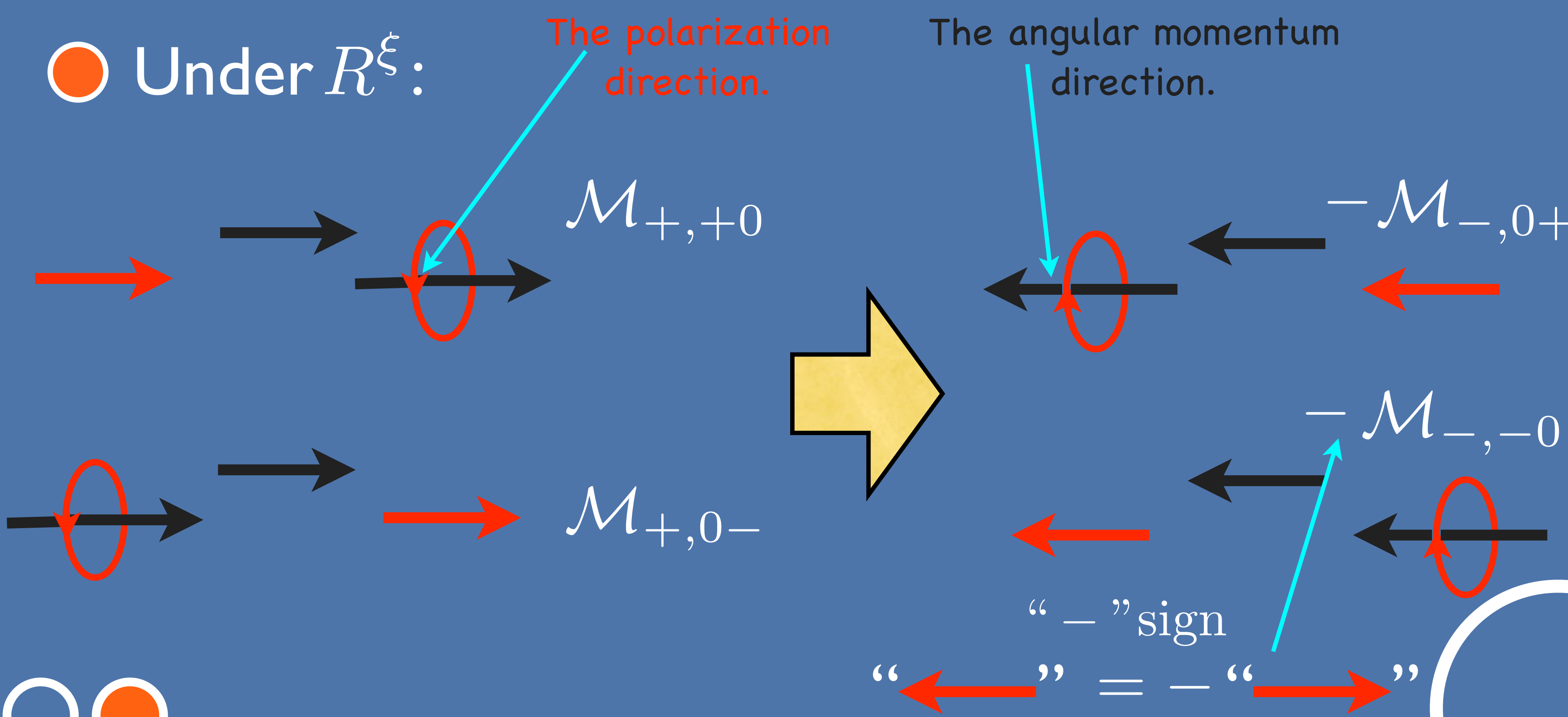
The Landau-Yang theorem

- Under R^{ξ}: it forbids $\mathcal{M}_{0,++}$ $\mathcal{M}_{0,--}$ and $\mathcal{M}_{0,00}$



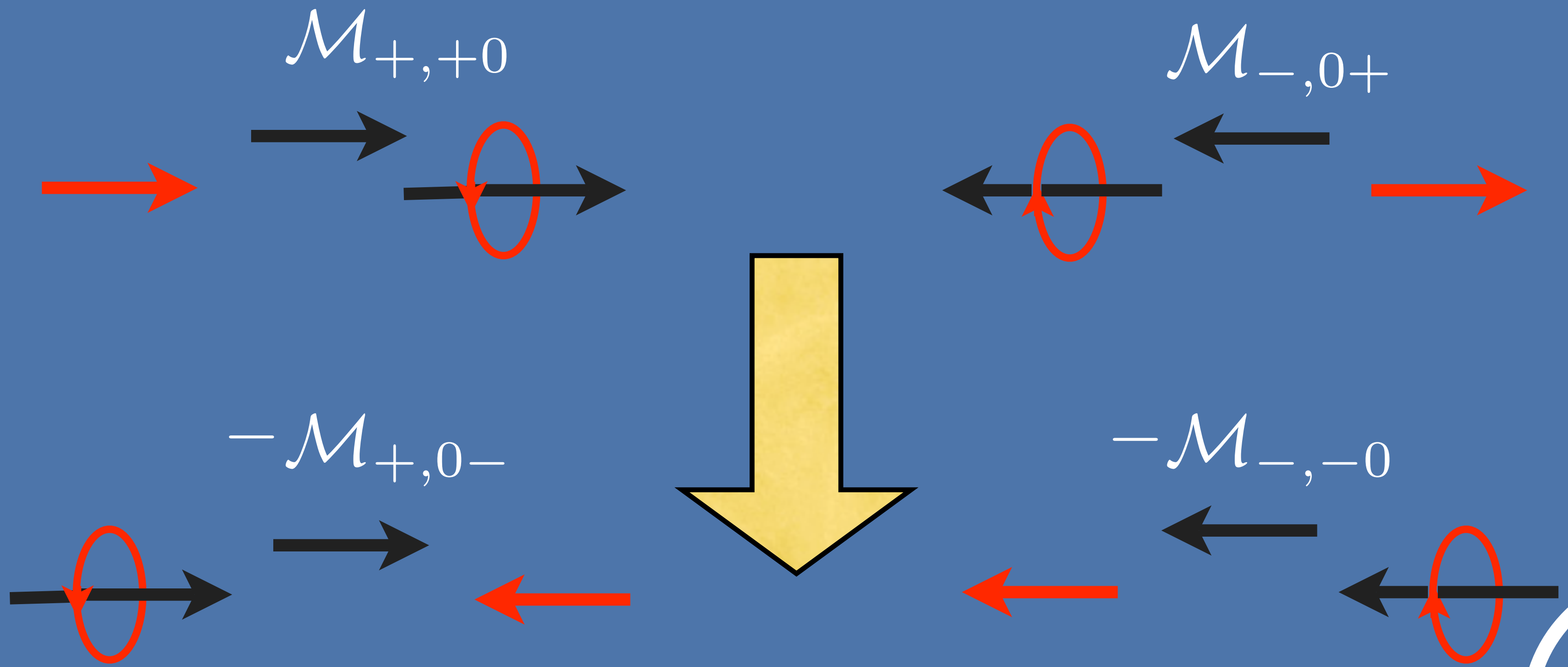
The Landau-Yang theorem

- There remain only four nonvanishing amplitudes
- Under R^ξ:



The Landau-Yang theorem

- Under space inversion (P):



The Landau-Yang theorem

- In summary, under R^ξ and P :

$$\begin{aligned} \mathcal{R}^\xi \quad &: \quad \mathcal{M}_{+,+0} \leftrightarrow -\mathcal{M}_{-,0+}, \quad \mathcal{M}_{+,0-} \leftrightarrow -\mathcal{M}_{-,-0}; \\ P \quad &: \quad \mathcal{M}_{+,+0} \leftrightarrow -\mathcal{M}_{+,0-}, \quad \mathcal{M}_{-,-0} \leftrightarrow -\mathcal{M}_{-,0+}. \end{aligned}$$

- So there are two independent helicity amplitudes :
  - All P odd, CP even operators contribute to the real amplitude. (anomulous coupling)
  - All P even, CP odd operators contribute to the imaginary amplitude. (regular coupling)

The Landau-Yang theorem

- So we parametrize the amplitudes as:

$$\begin{aligned}\mathcal{M}_{+,+0} &= A + i\,B = Ce^{i\delta} = -\mathcal{M}_{-,0+},\\ \mathcal{M}_{+,0-} &= A - i\,B = Ce^{-i\delta} = -\mathcal{M}_{-,-0}.\end{aligned}$$

- Except for an overall nomalization, everthing is embeded into the phase δ

$$\delta = \tan^{-1}(B/A)$$

which is the relatively strength of the CP odd and CP even amplitudes.

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- **Angular distributons.**
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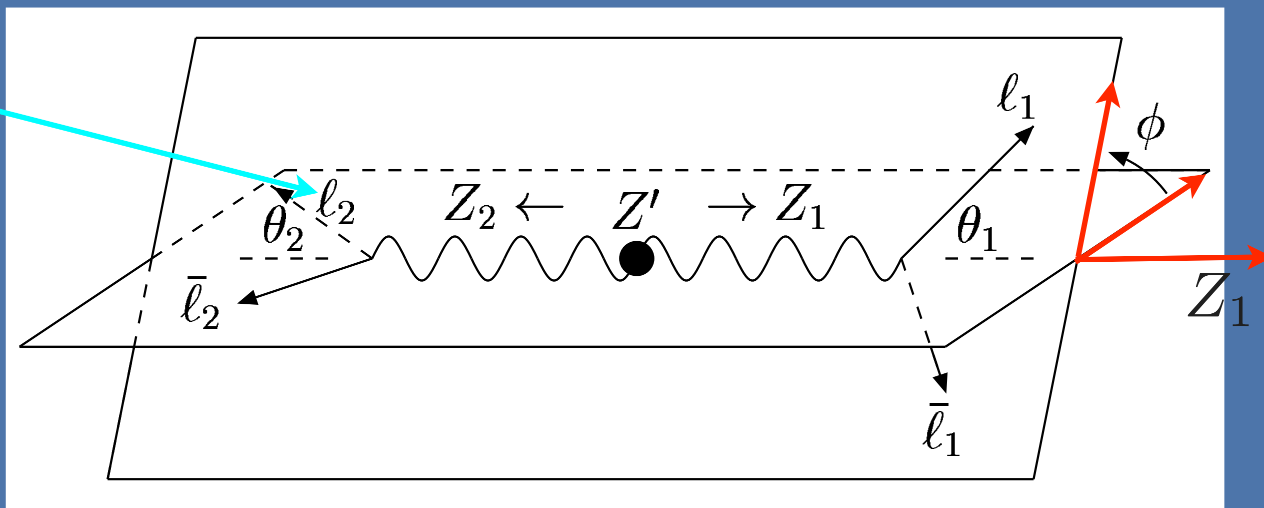
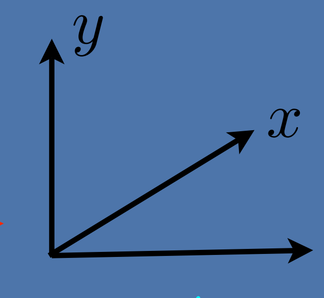
Angular Distributions

Negative charged





It doesn't matter which Z is the Z_1 as long as you pick up one.

The system are described by three angles $(\theta_1, \theta_2, \phi)$

- The azimuthal angle $\phi \in [0, 2\pi]$ is defined from half plane that contains l_2 to the one that contains l_1 and the cross product is parrell to Z_1 direction
- The polar angle $\theta \in [0, \pi]$ is the angle between the lepton and Z moving direction in the Z rest frame

Angular Distributions

We can even know how δ enters into the angular distributions without specific calculations

spin-projection of Z_1

$$\mathcal{M} = \mathcal{M}_0 \mathcal{M}_1(\theta_1, \phi) \mathcal{M}_1(\theta_2, 0)$$

The azimuthal angle dependence is $e^{im_1\phi}$

Consider $\mathcal{M}_{+,\lambda_1,\lambda_2}$

$$\left|a_1 \mathcal{M}_{+,+0} e^{i\phi} + a_2 \mathcal{M}_{+,0-}\right|^2 \sim \left|a_1 e^{i(\phi+2\delta)} + a_2\right|^2$$

Z decay amplitude

$$\frac{dN}{N d\phi} \sim c_1 + c_2 \cos(\phi + 2\delta)$$

Angular Distributions

Now we turn to specific couplings at dim-4 level:

$$O_{CPV} = f_4 Z'_\mu (\partial_\nu Z^\mu) Z^\nu , \quad O_A = f_5 \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu (\partial_\rho Z_\sigma)$$

Both operators are motivated at the 1-loop level, and their sizes are comparable if both exsit.

For the decay $Z'(q_1+q_2,\mu) \to Z(q_1,\alpha) Z(q_2,\beta)$

The form factor is

$$\Gamma^{\mu\alpha\beta}_{Z'\to Z_1 Z_2} = i f_4 (q_2^\alpha g^{\mu\beta} + q_1^\beta g^{\mu\alpha}) + i f_5 \epsilon^{\mu\alpha\beta\rho}(q_1 - q_2)_\rho .$$

The helcity amplitudes are

$$\begin{aligned} \mathcal{M}_{+,+0} &= -\mathcal{M}_{-,0+} = R(-f_5\beta + i f_4) \, , \\ \mathcal{M}_{+,0-} &= -\mathcal{M}_{-,-0} = R(-f_5\beta - i f_4) \end{aligned}$$

$$\beta^2 = 1 - 4m_Z^2/m_{Z'}^2$$

$$R = \frac{\beta m_{Z'}^2}{2m_Z}$$

$$\delta = \tan^{-1}(-f_4/f_5\beta)$$

Angular Distributions

The differential cross section could be obtained from summing over the different helicity states.

The Z helicity. chirality of the leptons

$$\sum_{\kappa,h_1,h_2}\left|\sum_{\lambda_1,\lambda_2}\mathcal{M}_{\kappa,\lambda_1\lambda_2}\, g_{h_1}\, f^{h_1}_{\lambda_1}(\theta_1,\phi)\, g_{h_2}\, f^{h_2}_{\lambda_2}(\theta_2,0)\right|^2$$

Spin-projection of Z′ along the z axis.

coupling between leptons of chirality h and Z

$$f^h_m(\bar\theta,\bar\phi) = (1+mh\cos\bar\theta)\frac{e^{im\bar\phi}}{2} \quad m=\pm$$

spin-one rotation matrix

$$f^h_0(\bar\theta,\bar\phi) = \frac{h}{\sqrt{2}}\sin\bar\theta$$

Angular Distributions

The normalized angular distribution is

$$\frac{8\pi dN}{N d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8}\Big[1 - \cos^2\theta_1\cos^2\theta_2$$
$$- \cos\theta_1\cos\theta_2\sin\theta_2\sin\theta_1\cos(\phi + 2\delta)$$
$$+ \frac{(g_L^2 - g_R^2)^2}{(g_L^2 + g_R^2)^2}\sin\theta_1\sin\theta_2\cos(\phi + 2\delta)\Big] \; .$$

- All coefficients are completely fixed by the symmetry!

The kinematical variables only enters into the angular dependence through phase δ

$$\beta^2 = 1 - 4m_Z^2/m_{Z'}^2$$
$$\delta = \tan^{-1}(-f_4/f_5\beta)$$

Angular Distributions

Integrating over the polar angles, the ϕ dependence is highly suppressed by the partial $\hat{C}$ symmetry $g_L \approx -g_R$ for leptonic decays, so we only integrate over the polar anglars

$\cos\theta_1 \cos\theta_2 > 0$ or < 0

$$\frac{2\pi dN_\pm}{N d\phi} = \frac{1}{2}\Big[1 \mp \frac{1}{8}\cos(\phi + 2\delta) \cancel{+\frac{9\pi^2}{128}\frac{(g_L^2 - g_R^2)^2}{(g_L^2 + g_R^2)^2}\cos(\phi + 2\delta)}\Big].$$

$\delta = 0 \quad \mathcal{O}_A$ only

$\delta = \pi/2 \quad \mathcal{O}_{CPV}$ only

$N_\pm$ stands for $N(\cos\theta_1 \cos\theta_2 \gtrless 0)$

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Measurements at the LHC

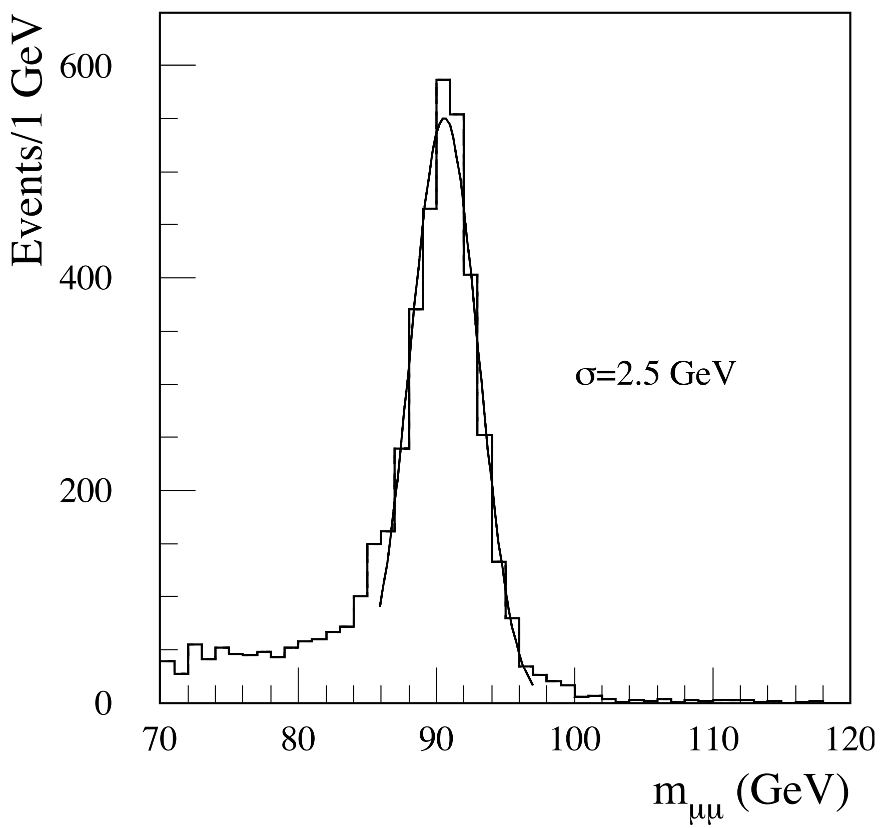
- Before we talked about the measurements, we may ask in what kind of models, it is possbile to discovery and discriminate the topological interactions at the LHC?
  - Since the topological interactions are always very small, if we don't want it to suppress the overall cross section (number of signals), the only place it exists is in the Z' decay vertex where the BR is not small.
  - Actually, quite a large number of interesting models does have such properties. For instance, little higgs model with anomalous T-parity where the lightest Z' only decay through topological interactions.

Measurements at the LHC

- The discovery and discrimination strategy:
  - We first have to find a resonance (5 σ CL) reconstructed from two identical Zs.
  - We have to make sure that the resonance is spin-one (Z').
  - From the azimuthal angular dependence, we can discriminate the anomalous coupling from the regular one (3 σ CL).

Measurements at the LHC

We first fix our Z' mass to be 240GeV.



ATLAS - Z mass resolution in $Z \to \mu^+\mu^-$

The Z' decay width is always very small, typically 1eV (large Z'->ZZ BR), so the cuts on Z' invariant mass window is always dominated by the detector energy resolution.

$$\frac{\sigma}{E} \sim \frac{0.2}{\sqrt{E}} + 0.01$$ *l in PGS4*

We expect the $\sigma_{Z'} \sim \sqrt{2}\sigma_Z$

A realistic simulation based on PGS4 shows that we can choose the cuts:

$$234 \text{ GeV} < \mathrm{m_{ZZ}} < 246 \text{ GeV}$$

Measurements at the LHC

A realistic simulation based on PGS4 shows that we can choose the cuts:

$$234 \text{ GeV} < \text{m}_{\text{ZZ}} < 246 \text{ GeV}$$

After the cuts on m_{ZZ}, the SM backgroud will be reduced to 79fb from 15pb.

The branching ratio for Z decays leptonically is 6.7%, and assuming the luminosity for LHC is 100 fb^{-1}

Requring the significance to be 5,

number of signals

$$\frac{S}{\sqrt{B}} = 5$$

number of backgrounds

, the ZZ production from Z' decay should be at last 67fb.

Measurements at the LHC

The spin of the resonance could be determined from the angular distributions. For instance, the azimuthal angle ϕ distribution for a scalar decay has a $\cos(2\phi + 2\delta)$ dependence.

D. Chang, W.Y. Keung and I. Phillips, Phys. Rev. D 48, 3225 (1993)

V. D. Berger et. al., Phys. Rev. D 49, 79 (1994)

C. P. Buszello *et. al.*, Eur. Phys. J. C 32, 209 (2004)

Since it is easier to determine the spin of the resonance (requrie less statistics of the signals) and they have been discussed in various references before. I will directly jump to the discremination.

If we include the SM bc, and assume it has a flat distribution, the expected disitribution becomes

$$n_{\pm}(\phi) \equiv \frac{dN_{\pm}}{d\phi} = \frac{N}{4\pi}\left[1 \mp \frac{1}{8}\frac{S}{S+B}\cos(\phi + 2\delta)\right].$$

Measurements at the LHC

We can estimate the required production rate for Z' in order to discriminate the operators from a simple counting.

We define a "up-down" asymmetry in the absence of bc.

$$\mathcal{A}_{ud} = \left(\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2} \right) \frac{n_+(\phi) - n_-(\phi)}{N} d\phi = -\frac{\cos(2\delta)}{4\pi} .$$

If we want to discremiante the two cases at the 99.7% CL

$$O_A \text{ only } (\delta = 0),\ \mathcal{A}_{\rm ud} = -1/4\pi$$

$$O_{CPV} \text{ only } (\delta = \pi/2),\ \mathcal{A}_{\rm ud} = 1/4\pi$$

For the asymmetric events $S_A = \mathcal{A}_{ud} \times S$

$$\frac{|S_A(\delta = 0) - S_A(\delta = \pi/2)|}{\sqrt{S+B}} = \frac{S}{2\pi\sqrt{S+B}} = 3 .$$

Measurements at the LHC

Then the required production rate of the Z boson from Z' decay is 0.9 pb for a 240 GeV Z'

Now we turn to a typical parameter space (without any tuning of the parameter) in the littest Higgs model with anomalous T-parity as a benchmark senario.

The Z' is the B_H

$$f = 1.5\mathrm{TeV}$$

$$m_{B_H} = \frac{g'}{\sqrt{5}} f = 240\mathrm{GeV}$$

$$\mathrm{BR}(\mathrm{Z}' \to \mathrm{ZZ}) = 1/3$$

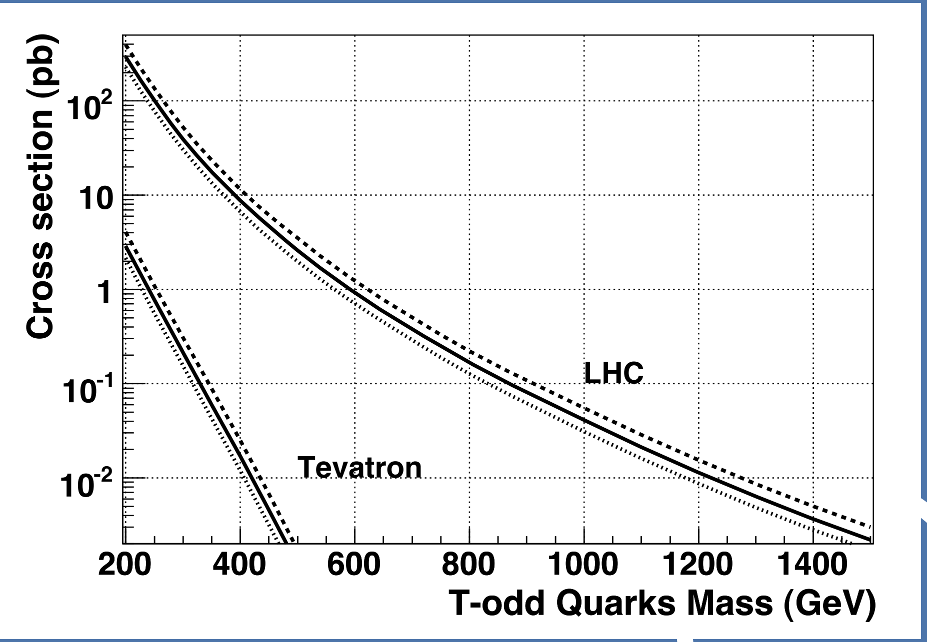
In order to discriminate the Z'-Z-Z vertex, the required production for pair-produced Z' is 1.3pb

Measurements at the LHC

The domiante Z' production channel is coming from the heavy T-odd quark decay.

For one single T-quark,

M. S. Carena et. al., Phys. Rev. D 75, 091701 (2007)



Considering six flavors, then even with a 750GeV T-quark mass (with the corresponding Yukawa coupling $\kappa = 0.5$)

We could discovery and discremiante the topological interactions at the LHC at 99.7% CL!!!

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Summary

- We study the decay of a Z' boson into two on-shell Zs by extending the Landau-Yang theorem. We find:
  - There are two independent helicity amplitudes (CP odd/even)
  - All kinematics are embeded through a phase shift in the azimuthal angle dependence between the two Z decay plane.
- Looking at the leptonic decay channel $Z' \to ZZ \to 4l$ (Golden channel to discover heavy higgs $h \to ZZ \to 4l$), we could disentangle the topological interactions (CP even) from the regular one (CP odd) at the LHC.

Outlook

There are still some intriguing questions that I can' help to talk here......................

- Measuring the N_c (free parameter of the theory)

Nc is the hyper-color number for underlying preon force (substructures for Higgs and other particles)

$$f_5 \propto \frac{N_c}{48\pi^2} \qquad O_A = f_5 \epsilon^{\mu\nu\rho\sigma} Z'_\mu Z_\nu (\partial_\rho Z_\sigma)$$

However, one can never measure the strength of O_A directly.

Decay width from Z' topological decay is only 1eV!

Decay width for heavy higgs is 1~ 2 GeV!

Outlook

It would be interesting if we find the phase shift δ between 0 to $\pi/2$ where both O_A and O_{CPV} exsit.

The operator O_{CPV} is coming from the scalar loops and one can calculate its strength by observing the scalars and measuring their couplings to the gauge bosons.

Why we can't measure the number of charged perons or heavy fermions???

confinement none decoupling effects

Measuring δ , the relative strength

Caculate the O_{CPV} strength

Nc !

Outlook

- Anomalies are extremely powerful to probe the symmetry breaking pattern of the underlying theory.

One famous example is the QCD, we know from t'Hooft anomaly matching condition that it must have the chiral symmetry breaking.

The mixed anomalies $SU(3)_L \times SU(3)_R \times U(1)_V$ at the hadron level and the quark level doesn't have a solution to match!

Now there is another example for models beyond SM!

Outlook

The Z' interactions from the WZW term in any extension of SM based on the sigma model G/H arise from the following two gauge invariant operators

$$\mathcal{O}_1 = \frac{iK_1}{F^2}\epsilon^{\mu\nu\rho\sigma}\tilde{B}_\mu[H^\dagger F^W_{\nu\rho}(D_\sigma H) - (D_\nu H^\dagger)F^W_{\rho\sigma}H],$$
$$\mathcal{O}_2 = \frac{iK_2}{F^2}\epsilon^{\mu\nu\rho\sigma}\tilde{B}_\mu F^B_{\nu\rho}[H^\dagger(D_\sigma H) - (D_\sigma H^\dagger)H]$$

How SM electroweak gauge group is embeded into the G/H completely determine the coefficients K_1 and K_2 .

The magic is: $Z' - Z - \gamma$ interaction is coming from both operators, and their contributions may cancel each other!

Outlook

Examples:

Littlest higgs model based on SU(5)/SO(5) has no $Z' - Z - \gamma$

Little higgs model based on minimal moose SU(3)*SU(3)/SU(3) has $Z' - Z - \gamma$ with sizable strength.

If we discovery the topological Z'-Z-Z coupling at the LHC, then whether the topological $Z' - Z - \gamma$ coupling exsits will tell us both the underlying symmetry structure and breaking pattern!

Outlook

- Another application for the techiques here is to consider non standard higgs (scalar) decay............
- Different opertors that decay the higgs will contribute to different helicity amplitudes which will affect the angular distributions.
- Because of different ratio of the helicty amplitudes that involves tranverse, longitude Z gauge bosons. The high energy behavior of the decay is quite different.
- If CP odd operators exsit, there is also a phase shift in the azimuthal angle distributions.