Holographic non-local operators

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Motivation

1. Order parameters
   new phases in gauge theories

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2. S-duality
   strong coupling physics
   - dynamical aspects of S-duality

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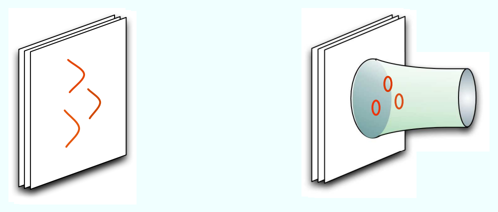
   new phases in gauge theories
2. S-duality
   strong coupling physics
   - dynamical aspects of S-duality
3. gauge/gravity duality
   emergent space-time
   - need to know all gaiuge-inv operators and their duals
   - non-trivial tests of the duality
   - strong coupling regime of gauge theories

Introduction

gauge/gravity duality

[Maldacena. Gubser-Klebanov-Polyakov, Witten]



$d = 4,\ \mathcal{N} = 4,\ U(N_c)$ super Yang-Mills $\equiv$ Type IIB strings on $AdS_5 \times S^5$ with RR flux N_c

$$g_{YM}^2 = 4\pi g_s, \qquad L^4 = 4\pi g_s N_c l_s^4$$

Three realizations of non-local operators

1. gauge theory description
   $\lambda \equiv g_{YM}^2 N_c << 1$; perturbative regime
   Any value of λ; matrix model
2. probe brane description
   $N >> 1, \lambda >> 1$ and the number of branes is small
3. supergravity description
   $N >> 1, \lambda >> 1$ and the number of branes is large
   $\rightarrow$ branes are replaced by fluxes supported on non-trivial cycles

Contents

- Surface Operators
  - definition in gauge theory
  - gravity dual description
  - correlation functions in gauge and gravity theory
- Wilson Loop
  - matrix model and correlation functions
  - gravity dual description and correlation functions

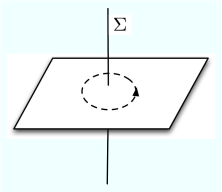
Surface Operator

Surface operator in gauge theory [Gukov-Witten]

A surface operator is defined by a co-dimension 2 singularity in the path integral (non-Abelian vortex).

Characterized by a surface Σ and a conjugacy class U of the gauge group G (Aharonov-Bohm phase).

$$U \equiv P \exp i \oint A$$
$$U \to gUg^{-1}$$



Surface operator with $\Sigma = \mathbb{R}^2$ in $\mathbb{R}^4$

- the gauge field

$$A = \begin{pmatrix} \alpha_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \alpha_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_M \otimes 1_{N_M} \end{pmatrix} d\psi$$

In the presence of the surface operator, the gauge group is broken

$$U(N) \to \prod_{l=1}^{M} U(N_l)$$

$$\exp\left(i \sum_{l=1}^{M} \eta_l \int_{\Sigma} \mathrm{Tr} F_l \right), \ \eta = \begin{pmatrix} \eta_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \eta_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \eta_M \otimes 1_{N_M} \end{pmatrix}$$

α_l, and η_l take values on a circle with unit radius

- the scalar field

$$\Phi = \frac{1}{\sqrt{2}z}\begin{pmatrix} \beta_1 + i\gamma_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \beta_2 + i\gamma_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta_M + i\gamma_M \otimes 1_{N_M} \end{pmatrix}$$

So a surface operator is labeled by 4M parameters $(\alpha_I, \eta_I, \beta_I, \gamma_I)$.

Symmetry: half BPS, $SO(2,2) \times SO(2)_a \times SO(4)$.

S-duality

Complex coupling constant in $\mathcal{N}=4$, $U(N_c)$ SYM is

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}.$$

This theory is invariant under $SL(2,Z)$ transformation,

$$\tau \to \frac{a\tau+b}{c\tau+d}, \quad \text{where } \mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z).$$

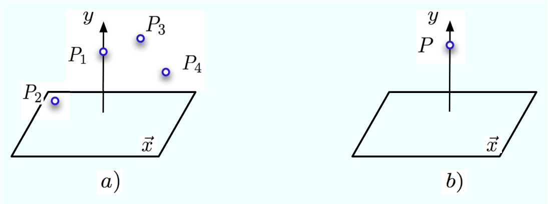
A surface operator transforms under the S-duality [Gukov-Witten] ,

$$\begin{aligned} (\beta_I, \gamma_I) &\to |c\tau+d|(\beta_I,\gamma_I) \\ (\alpha_I, \eta_I) &\to (\alpha_I,\eta_I)\mathcal{M}^{-1}. \end{aligned}$$

Bubbling surface operator [Lin-Maldacena, Gomis-Matsuura]

Symmetry: half BPS, $SO(2,2) \times SO(2)_a \times SO(4)$.

$$ds^2 = y\sqrt{\frac{2z+1}{2z-1}}ds^2_{AdS_3} + y\sqrt{\frac{2z-1}{2z+1}}d\Omega_3 + \frac{2y}{\sqrt{4z^2-1}}(d\chi+V)^2 + \frac{\sqrt{4z^2-1}}{2y}(dy^2+dx_i dx_i)$$



$a)$ general solution $\qquad$ $b)$ $AdS_5 \times S^5$

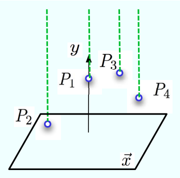
Boundary conditions: $(S^3 \to 0$ at $y=0)$ and $(S^1 \to 0$ at $P_i)$

Holographic data [Gomis-Matsuura]

- The scalar fields

$$(\beta_I, \gamma_I) = \frac{\vec{x}_I}{2\pi l_s^2}, \quad N_I = \frac{y_I^2}{4\pi l_p^2}$$

- The gauge field



$$S^1 \to 0 \text{ at } P_i \text{ and } \partial(AdS_5) = AdS_3 \times S^1$$

Need to specify the holonomy of the two forms.

$$\alpha_I = -\int_{D_I} \frac{B_{NS}}{2\pi}, \quad \eta_I = \int_{D_I} \frac{B_R}{2\pi}$$

Correlation functions in the gauge theory

[Drukker-Gomis-Matsuura]

- VEV

$$< \mathcal{O}_\Sigma > \;=\; \exp\left(-\mathcal{S}\right)\big|_{\text{background field}} = 1$$

- Correlation functions

$$\frac{\langle \mathcal{O} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \mathcal{O}\big|_{\text{background field}}$$

Stress-energy tensor

$$\frac{\langle T_{\mu\nu} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = h\frac{\eta_{\mu\nu}}{r^4}, \quad \frac{\langle T_{ij} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = \frac{h}{r^4}[4n_i n_j - 3\delta_{ij}], \quad \frac{\langle T_{\mu j} \cdot \mathcal{O}_\Sigma \rangle}{\langle \mathcal{O}_\Sigma \rangle} = 0$$

$$h = -\frac{2}{3g_{YM}^2}\sum_{l=1}^{M} N_l(\beta_l^2 + \gamma_l^2)$$

Correlation functions (Bubbling geometry)

Holographic Renormalization method [Skenderis-Taylor] ,

- construct gauge-inv quantities or fix gauge
- Kaluza-Klein reduction

$$\Psi_{5d} = \psi_{10d} + \mathcal{K}\psi_{10d}\psi_{10d} + \cdots$$

- Expand 5d dual fields from the boundary in FG coordinate

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}(g_{(0)ij} + \cdots + z^n g_{(n)ij})dx^i dx^j$$

Correlation functions (Bubbling geometry)

[Drukker-Gomis-Matsuura]

Stress-energy tensor

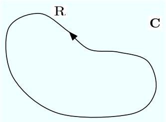
$$h = -\frac{2}{3g_{YM}^2}\sum_{I=1}^{M} N_I\left((\beta_I^2+\gamma_I^2)+\frac{\lambda}{4\pi^2}\frac{N-N_I}{2N}\right)$$

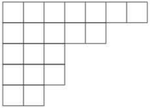
- Leading terms precisely agree with those in the semi-classical analysis
- The loop corrections truncate at order $\lambda^{(\Delta-|k|)/2}$

Wilson Loop

Wilson Loop in gauge theory

$$W_R(\theta, a) \equiv \frac{1}{\dim R} \mathrm{Tr}_R \mathrm{P} \exp \oint \left(iA + \phi^i \theta^i |\dot{x}| ds \right)$$



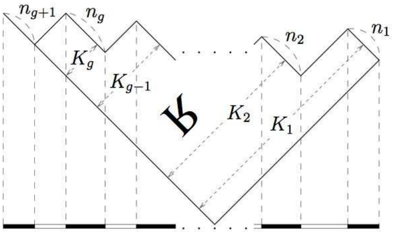


Hermitian matrix model [Erickson-Semenoff-Zarembo, Drukker-Gross, Pestun]

$$\langle W_R \rangle = \int [d\xi] e^{-\frac{2N}{\lambda} \mathrm{Tr} \xi^2} \frac{1}{\dim R} \mathrm{Tr}_R (e^{\xi})$$

Correlation functions (matrix model)

[Okuda-Trancanelli, Gomis-Matsuura-Okuda-Trancanelli]

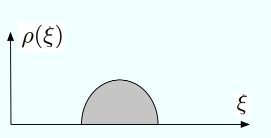


Supergravity regime :

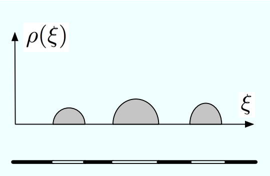
$$\lambda >> 1, \qquad g_{YM}^2 n_I = \mathcal{O}(\lambda), \qquad g_{YM}^2 (K_I - K_{I+1}) = \mathcal{O}(\lambda^{1/2})$$

Correlation functions (matrix model)

black-white regions $\leftrightarrow$ eigenvalue distribution



without Wilson loop



genus two

Correlation functions (matrix model)

[Gomis-Matsuura-Okuda-Trancanelli]

CPO: $\mathcal{O}_J = \frac{(8\pi^2)^{J/2}}{\lambda^{J/2}\sqrt{J}} \mathrm{Tr} Z^J$ where $Z \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}}$.

$$\frac{\langle W_R(\theta, a)\mathcal{O}_J(x)\rangle}{\langle W_R(\theta, a)\rangle} = \Xi_{R,J}\frac{1}{2^{J/2}}\frac{1}{\tilde{r}^J}$$

$$\Xi_{R,2} \quad = \quad \sqrt{2}\frac{N}{\lambda}\Delta\rho_2$$

where the moment of the eigenvalue distribution is

$$\langle \xi^n \rangle \equiv \rho_n \equiv \int d\xi \rho(\xi)\xi^n$$

and $\Delta\rho_2 = \rho_2 - \rho_2^0$.

Correlation functions (matrix model)

stress tensor

$$\langle W_R^{\text{line}}(\theta)\, T_{44}(x)\rangle = \frac{h_W}{l^4}\,, \qquad \langle W_R^{\text{line}}(\theta)\, T_{4a}(x)\rangle = 0\,,$$

$$\langle W_R^{\text{line}}(\theta)\, T_{ab}(x)\rangle = -h_W \frac{\delta_{ab} - 2n_a n_b}{l^4}$$

- Ward Identity
  the dim 2 CPO and the stress tensor are in the same multiplet

$$h_W = -\frac{N}{3\sqrt{2}\pi^2}\Xi_{R,2}$$

- S-duality
  $< W \cdot T_{\mu\nu} >$ at strong coupling $\leftarrow$ S-dual $\rightarrow$ $< T \cdot T_{\mu\nu} >$ at weak coupling
  Precise matching when cuts are widely separated

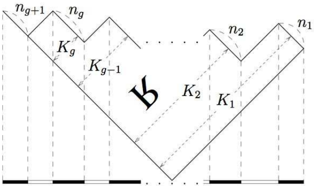
Bubbling Wilson Operator

[Yamaguchi, Lunin, D'Hoker-Estes-Gutperle]

Symmetry: half BPS, $SO(2,1) \times SO(3) \times SO(5)$

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + f_4^2 ds^2_{S^4} + 4\rho^2(dx^2 + dy^2)$$

Boundary conditions



corresponding Young diagram

$S^2 \to 0$ on the white regions and $S^4 \to 0$ on the black regions

Correlation functions (Bubbling geometry)

By applying the holographic renormalization method to the bubbling Wilson operator, we calculate the CPOs and the stress tensor.

The results from the bubbling geometry precisely agree with those from the gauge theory.

non-trivial test of AdS/CFT

Summary + Open Questions

- We have seen that correlation functions remarkably agree in different regime of the gauge theory.

- Matrix model description for the surface operator
- dynamical aspects of S-duality
- order parameter
- other non-local operators