# Effects of particle production during inflation

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**General Relativity** *and* **particle physics** *provide the tools to construct a succesful cosmological model*

The main features of the observed Universe are the following:

- Large scale isotropy and homogeneity
- Redshift of light emitted by distant objects according to Hubble law:

$$H_0 d_L = z + \frac{1}{2}(1 - q_0) z^2$$

- Large deviations from homogeneity on short scales
- CMB radiation
- Light elements abundance consistent with big-bang nucleosynthesis

## Horizon problem

The length corresponding to our present Hubble radius at the time of last-scattering was

$$\lambda_H(t_{\rm LS}) = R_H(t_0)\left(\frac{a_{\rm LS}}{a_0}\right) = R_H(t_0)\left(\frac{T_0}{T_{\rm LS}}\right).$$

During the matter-dominated period, the Hubble length has evolved according to :

$$H^2 \propto \rho_M \propto a^{-3} \propto T^3.$$

So that

$$H_{LS}^{-1} = R_H(t_0)\left(\frac{T_{LS}}{T_0}\right)^{-3/2} \ll R_H(t_0).$$

The length corresponding to our present Hubble radius was much larger that the horizon at that time :

$$\frac{\lambda_H^3(t_{LS})}{H_{LS}^{-3}} = \left(\frac{T_0}{T_{LS}}\right)^{-\frac{3}{2}} \approx 10^6. \tag{1}$$

The condition for solving the horizon problem is to have the **wave-length stretch faster** than the naive **horizon scale** set by $H^{-1}$, i.e.

$$\frac{d}{dt}(\frac{a}{H^{-1}}) = \frac{d}{dt}(\dot{a}) = \ddot{a} > 0 \tag{2}$$

Using Einstein equations for the FRW metric we can write:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \tag{3}$$

$$P < -\frac{1}{3}\rho \tag{4}$$

## Scalar field

A simple way to model inflation is to introduce a scalar field, called inflaton :

$$S = \int d^4x\,\sqrt{-g}\,\mathcal{L} = \int\, d^4x\,\sqrt{-g}\,\left[\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + V(\varphi)\right], \qquad (5)$$

where $\sqrt{-g} = a^3$ for the FRW metric (**??**).

Lagrange equations or conservation of energy-momentum tensor give:

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2\varphi}{a^2} + V'(\varphi) = 0, \qquad (6)$$

with $V'(\varphi) = (dV(\varphi)/d\varphi)$.

The energy-momentum tensor of the scalar field is

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\,\mathcal{L}.$$

from which we obtain the energy density $\rho_\varphi$ and pressure density $p_\varphi$

$$T_{00} = \rho_\varphi = \frac{\dot{\varphi}^2}{2} + V(\varphi) + \frac{(\nabla\varphi)^2}{2a^2}, \qquad (7)$$

$$T_{ii} = p_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi) - \frac{(\nabla\varphi)^2}{6a^2}. \qquad (8)$$

**Slow roll inflation**

Slow roll inflation is the stage of the inflaton evolution when it is slowly rolling down its potential, i.e. the second order time derivative term in eq.(6) is neglected :

$$3H\dot{\varphi} \approx -V^{'}(\varphi) \tag{9}$$

Using the conventions of Liddle (1994), it is convenient to use the definitions for the slow roll parameters given in terms of the Hubble function $H(\varphi)$ as follows:

$$\begin{aligned} \epsilon &\equiv \frac{M_p^2}{4\pi}\left(\frac{H'(\varphi)}{H(\varphi)}\right)^2 \\ \eta(\varphi) &\equiv \frac{M_p^2}{4\pi}\left(\frac{H''(\varphi)}{H(\varphi)}\right) \\ \sigma &\equiv \frac{M_p^2}{\pi}\left[\frac{1}{2}\left(\frac{H''}{H}\right) - \left(\frac{H'}{H}\right)^2\right] \end{aligned} \tag{10}$$

$${}^{\ell}\lambda_{\mathrm{H}} \equiv \left(\frac{M_p^2}{4\pi}\right)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{d^{(\ell+1)}H}{d\varphi^{(\ell+1)}}. \tag{11}$$

Here, the Hubble function is defined by the Hamilton-Jacobi equations

$$\dot{\varphi} = -\frac{M_p^2}{4\pi}\frac{dH}{d\varphi} \tag{12}$$

$$\left(\frac{dH}{d\varphi}\right)^2 - \frac{12\pi}{M_p^2}H^2 = \frac{-32\pi^2}{M_p^4}V(\varphi) \tag{13}$$

which has the advantage of having a readily solvable model of power law inflation.

It can be shown that

$$\epsilon \ll 1 \tag{14}$$

is a sufficient condition for inflation to occur. Since condition (14) is not necessary, in principle inflation could continue even after it is violated, but in in practice this regime should be very short.

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu} \tag{15}$$

$$G_{\mu\nu} = G^0_{\mu\nu} + \delta G_{\mu\nu} \tag{16}$$

$$T_{\mu\nu} = G_{\mu\nu} \tag{17}$$

$$\Downarrow \tag{18}$$

$$T^0_{\mu\nu} = G^0_{\mu\nu} \ , \ \delta T_{\mu\nu} = \delta G_{\mu\nu} \tag{19}$$

The most general perturbation to FRW metric scalar taking into account only scalar degrees of freedom can be written as:

$$\delta g_{\mu\nu} \ = \ a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2(\psi\delta_{ij} - E_{,ij}) \end{pmatrix} , \tag{20}$$

**Gauge transformation**

General relativity invariance under coordinate transformation recovered at infinitesimal level

$$x^\mu \;\rightarrow\; \tilde{x}^\mu = x^\mu + \xi^\mu \,. \tag{21}$$

The zero (time) component $\xi^0$ of $\xi^\mu$ leads to a scalar metric fluctuation. The spatial three vector $\xi^i$ can be decomposed

$$\xi^i \;=\; \xi^i_{tr} + \gamma^{ij}\xi_{,j} \tag{22}$$

where $\gamma^{ij}$ is the spatial background metric, the transverse part satosfies the condition $\nabla_i \xi^i_{tr} = 0$, $\xi$ is an appropriate scalar function, and only the non transverse part contributes to scalar perturbations.

These are the resulting gauge transformations for scalar perturbations:

$$\begin{aligned}
\tilde{\phi} \;&=\; \phi - \frac{a'}{a}\xi^0 - (\xi^0)' \\
\tilde{B} \;&=\; B + \xi^0 - \xi' \\
\tilde{E} \;&=\; E - \xi \\
\tilde{\psi} \;&=\; \psi + \frac{a'}{a}\xi^0 \,,
\end{aligned} \tag{23}$$

A convenient basis of gauge-invariant variables was introduced by

Bardeen :

$$\Phi = \phi + \frac{1}{a}[(B - E')a]' \tag{24}$$

$$\Psi = \psi - \frac{a'}{a}(B - E') \,. \tag{25}$$

Since $\Phi$ and $\Psi$ coincide with the corresponding metric perturbations $\phi$ and $\psi$ in longitudinal gauge, $E = B = 0$, it is possible to carry out calculations in this gauge and then use eq.(24,25) to obtain a gauge-invariant result.

In a similar way it is possible to introduce a gauge-invariant Einstein tensor and energy tensor $\delta G_{\nu}^{(gi)\,\mu}$

$$\begin{aligned}
\delta G_0^{(gi)\,0} &\equiv \delta G_0^0 + ({}^{(0)}G_0^{\prime\,0})(B - E') \\
\delta G_i^{(gi)\,0} &\equiv \delta G_i^0 + ({}^{(0)}G_i^0 - \frac{1}{3}{}^{(0)}G_k^k)(B - E')_{,i} \\
\delta G_j^{(gi)\,i} &\equiv \delta G_j^i + ({}^{(0)}G_j^{\prime\,i})(B - E') \,,
\end{aligned} \tag{26}$$

We can then derive the gauge-invariant equations for cosmological perturbations:

$$
\begin{aligned}
-3\mathcal{H}(\mathcal{H}\Phi + \Psi^{'}) + \nabla^2\Psi \;&=\; 4\pi G a^2 \delta T_0^{(gi)\,0} \\
(\mathcal{H}\Phi + \Psi^{'})_{,i} \;&=\; 4\pi G a^2 \delta T_i^{(gi)\,0} \\
[(2\mathcal{H}^{'} + \mathcal{H}^2)\Phi + \mathcal{H}\Phi^{'} + \Psi^{''} + 2\mathcal{H}\Psi^{'}]\delta^i_j & \\
+\frac{1}{2}\nabla^2 D\delta^i_j - \frac{1}{2}\gamma^{ik}D_{,kj} \;&=\; -4\pi G a^2 \delta T_j^{(gi)\,i}\,,
\end{aligned}
\tag{27}
$$

where $D \equiv \Phi - \Psi$ and $\mathcal{H} = a'/a$. In longitudinal gauge, then $\delta T_j^{(gi)\,i} = \delta T^i_j$, $\Phi = \phi$ and $\Psi = \psi$. If no anisotropic stress is present in the energy tensor, i.e. $\delta T^i_j = 0$ for $i \neq j$, then the two metric fluctuation variables coincide:

$$
\Phi \;=\; \Psi\,. \tag{28}
$$

This kind of simplification occurs both for scalar fields with a Klein-Gordon action, and in the case of an isotropic perfect fluid.

In the case of a single scalar field $\varphi$ with action

$$
S \;=\; \int d^4x\sqrt{-g}[\frac{1}{2}\varphi^{,\alpha}\varphi_{,\alpha} - V(\varphi)] \tag{29}
$$

and decomposing the field as

$$
\varphi(\mathbf{x},\eta) \;=\; \varphi_0(\eta) + \delta\varphi(\mathbf{x},\eta) \tag{30}
$$

in terms of classicall background $\varphi_0$ and quantum fluctuation $\delta\varphi(\mathbf{x},\eta)$, then in longitudinal gauge (27) we have:

$$\begin{aligned} \nabla^2\phi - 3\mathcal{H}\phi^{'} - (\mathcal{H}^{'} + 2\mathcal{H}^2)\phi &= 4\pi G(\varphi_0^{'}\delta\varphi^{'} + V^{'}a^2\delta\varphi) \\ \phi^{'} + \mathcal{H}\phi &= 4\pi G\varphi_0^{'}\delta\varphi \\ \phi^{''} + 3\mathcal{H}\phi^{'} + (\mathcal{H}^{'} + 2\mathcal{H}^2)\phi &= 4\pi G(\varphi_0^{'}\delta\varphi^{'} - V^{'}a^2\delta\varphi)\,, \end{aligned} \tag{31}$$

where $V^{'}$ denotes the derivative of $V$ with respect to $\varphi$. From these equations we can derive the equation for $\phi$:

$$\phi^{''} + 2\left(\mathcal{H} - \frac{\varphi_0^{''}}{\varphi_0^{'}}\right)\phi^{'} - \nabla^2\phi + 2\left(\mathcal{H}^{'} - \mathcal{H}\frac{\varphi_0^{''}}{\varphi_0^{'}}\right)\phi = 0\,. \tag{32}$$

**Quadratic action approach**

An alternative approach would be to calculate the quadratic action for the scalar field and then, after a rather cumbersome calculation to show that this is equivalent to a **scalar field** with **time dependent mass** on flat background:

$$S^{(2)} \;=\; \frac{1}{2}\int d^4x[v'^2 - v_{,i}v_{,i} + \frac{z''}{z}v^2]\,, \tag{33}$$

where

$$v \;=\; a[\delta\varphi + \frac{\varphi_0^{'}}{\mathcal{H}}\phi]\,, \tag{34}$$

with $\mathcal{H} = a'/a$, and

$$z \;=\; \frac{a\varphi_0^{'}}{\mathcal{H}}\,. \tag{35}$$

In both the cases of power law inflation and slow roll inflation, $\mathcal{H}$ and $\varphi_0^{'}$ are proportional and hence

$$z(\eta) \;\sim\; a(\eta)\,. \tag{36}$$

Note that the variable $v$ is related to the comoving curvature perturbation $\mathcal{R}$ :

$$\mathcal{R} \;=\; z^{-1}v \;=\; \phi + \delta\varphi\frac{\mathcal{H}}{\varphi_0^{'}} \tag{37}$$

$$v \;=\; z\mathcal{R}\,. \tag{38}$$

From the quadratic action (33) we get the equations:

$$v'' - \nabla^2 v - \frac{z''}{z} v = 0 \tag{39}$$

$$v_k'' + k^2 v_k - \frac{z''}{z} v_k = 0 \tag{40}$$

where $v_k$ is the k'th Fourier mode of $v$.

**Physical interpretation**

- Scalar metric perturbation constitute one **single physical degree of freedom** because of the **Bianchi identities, Einstein equations, and gauge invariance.**

- The field $v$ appearing in the quadratic action is a **gauge invariant** combination of the metric perturbation $\varphi$ and the inflaton quantum component $\delta\phi$, and each of them can be set individually to zero with an appropriate gauge choice.

**Power spectrum**

The quadratic action formalism shows that quantum field theory in Minkowsky space is enough to compute the power spectrum of cosmological perturbations. Expanding the operator $\hat{v}(\eta, \mathbf{x})$ in terms of plane waves

$$\hat{v}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[ v_k(\eta) \hat{a}_{\mathbf{k}} e^{i\mathbf{k}.\mathbf{x}} + v_k^*(\eta) \hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}.\mathbf{x}} \right] \tag{41}$$

The usual commutation relations for bosons are satisfied by the creation and annihilation operators:

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{l}}] = [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{l}}^\dagger] = 0, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{l}}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{l})\,. \tag{42}$$

The vacuum is defined as the state that is annihilated by all the $\hat{a}_{\mathbf{k}}$, i. e., $\hat{a}_{\mathbf{k}}|0\rangle = 0$.

On small scales ordinary quantum field theory should be reproduced, so in the limit that $k/aH \to \infty$, the modes should be plane waves of the form

$$v_k(\eta) \to \frac{1}{\sqrt{2k}} e^{-ik\eta}\,. \tag{43}$$

In the long wavelength regime where $k$ can be neglected, we have the growing mode solution

$$v_k \propto z \tag{44}$$

The curvature perturbation $\mathcal{R}$ can be expanded in a Fourier series

$$\mathcal{R} = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \mathcal{R}_{\mathbf{k}}(\eta) e^{i\mathbf{k}.\mathbf{x}} \tag{45}$$

The power spectrum $\mathcal{P}_\mathcal{R}(k)$ is defined in terms of the vacuum expectation value

$$\langle \mathcal{R}_\mathbf{k} \mathcal{R}_\mathbf{l}^* \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_\mathcal{R} \delta^{(3)}(\mathbf{k} - \mathbf{l}) \tag{46}$$

so that

$$\mathcal{P}_\mathcal{R}^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{v_k}{z} \right| . \tag{47}$$

For super-horizon modes, the growing mode of $v_k$ will dominate and the power spectrum will tend to a constant value.

It is possible to express $\frac{z''}{z}$ in terms of the slow roll parameters defined in the following sections as:

$$\frac{1}{z}\frac{d^2 z}{d\eta^2} = 2a^2 H^2 \left[ 1 + \epsilon - \frac{3}{2}\eta + \epsilon^2 - 2\epsilon\eta + \frac{1}{2}\eta^2 + \frac{1}{2}\xi^2 \right] , . \tag{48}$$

The power spectrum for a general slow roll potential is

$$P_\mathcal{R}^{1/2}(k) = [1 - (2C+1)\epsilon + C\eta] \left. \frac{2}{m_{\rm Pl}^2} \frac{H^2}{|H'|} \right|_{k=aH} , \tag{49}$$

where $C = -2 + \ln 2 + \gamma \simeq -0.73$ is a numerical constant, and $\gamma$ is the Euler constant from the expansion of the Gamma function.

## Context and previous results

**Inflation** is considered one of the most promising candidates to **explain** the statistical features of the observable Universe revealed by the detection of the cosmic microwave background **(CMBR)** and large scale structure surveys such as the Sloan Digital Sky Survey **(SDSS)**.

In the simplest models, inflation is an exponential expansion period of the Universe, driven by **scalar field**, called inflation, **slowly rolling** down its potential.

Various extensions have been proposed, and we will consider the **coupling** of the inflaton to another **massive scalar** field, which has been previously investigated by Chung (Phys. Rev. D **62**, 043508 (2000)).

They obtained just a **local peak** in the power spectrum for scales which leave the horizon around the time of particle production.

We study the problem both analytically and numerically, obtaining an intermediate result with **oscillations** of the power spectrum on small scales.

**The model**

$$V(\phi, \varphi) = \frac{1}{2} m_\phi^2 \phi^2 + V_0 + \frac{1}{2} N (m_\varphi - g\phi)^2 \varphi^2 \tag{50}$$

$$\tag{51}$$

with the $V_0$ **dominating** the other terms during the period of interest, so that the **evolution** of $H$ can be **neglected** in the calculation.

The equation for the inflaton can be written in the form

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} - gN(m_\varphi - g\phi)\langle\varphi^2\rangle = 0 \tag{52}$$

$$\langle\varphi^2\rangle \approx \frac{\mathcal{C}}{m_\varphi - g\phi} n_0 \left(\frac{a}{a_0}\right)^{-3} \theta(t - t_0) \tag{53}$$

$$n_0 = g^{3/2} \frac{|\dot{\phi}_0|^{3/2}}{(2\pi)^3} \tag{54}$$

where where $t_0$ is the time at which $\phi(t_0) = gm_\varphi$, when the effective mass of the $\varphi$ field is zero, and most of the **particles are produced**.

The two point function can be approximated with the expression above after renormalizing by substraction of its asymptotic past value, when no particle were produced .

## Previous calculation

They solved the **first order** differential equation for the inflaton

$$\frac{d\dot{\phi}}{dt} + 3H_*\dot{\phi} + (dV/d\phi)_* - N\lambda n_* \theta(t-t_*)\ \exp[-3H_*(t-t_*)] = 0 \quad (55)$$

and calculated the spectrum according to

$$\delta_k = \frac{H}{5\pi\dot{\phi}(t_k)} \quad (56)$$

## Just a peak in the power spectrum?

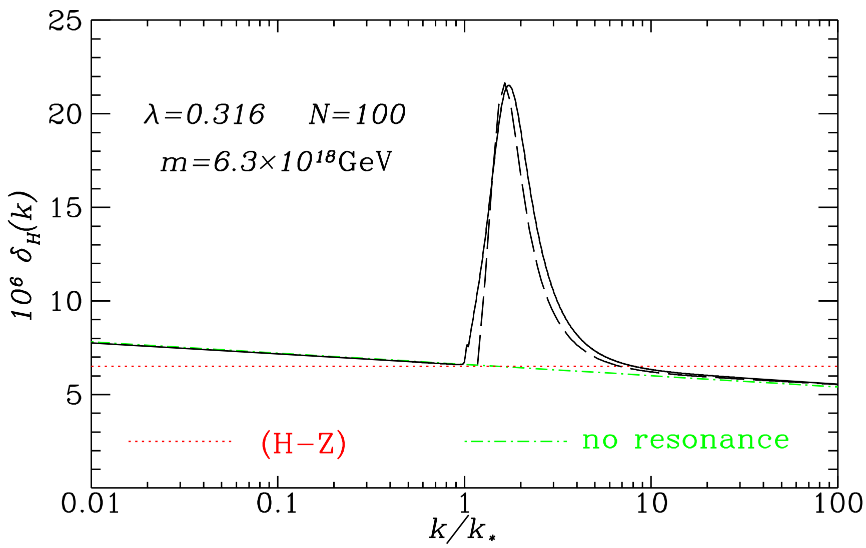


FIG. 1: Resonant particle production produces a peak in the perturbation spectrum as shown by the solid curve in the figure. Shown by a dashed curve is the analytic form of $\delta_H(k)$ in Eq. Finally, also shown is a Harrison–Zel'dovich spectrum and the spectrum in the inflation model without resonant particle creation.

We solved the equation for the curvature perturbation on co-moving hypersurfaces :

$$R'' + 2\frac{z'}{z}R' + k^2 R = 0 \tag{57}$$

and we calculated the power spectrum according to:

$$P_R^{1/2} = \sqrt{\frac{k^3}{2\pi^2}}|R_k| \tag{58}$$

$$z = \frac{a\dot{\phi}}{H} \tag{59}$$

.

**The calculation**

For ease of comparison, we will make the following choices for initial conditions and the masses :

$$a_0 = a(t_0) = 1 \; m_\phi = 10^{-6} m_{pl} \; m_\varphi = 2 m_p \tag{60}$$

$$g = 1 \quad \phi_0 = g m_\varphi \quad k_0 = (aH)_{t_0} \tag{61}$$

$$\phi(t_0) = \phi_0 \quad \eta_0 = \eta(t_0) \tag{62}$$

We have split the **calculation** of the power spectrum into **two parts**, corresponding to the modes **greater** and **smaller** than $k_0$.

$k > k_0$ In this case particle production takes place **before** the modes **leave** the **horizon**, and so the problem can be treated mathematically using **WKB**

$k < k_0$ For these modes the sudden change in the effective potential of the inflaton happens after horizon crossing, and we solved **numerically** the differential equation.

In order to check the accuracy of the **analytical** approximation, we solved **numerically** the equation also for $k > k_0$, obtaining a **good agreement** between the two.

# Evolution of $\dot{\phi}$ and $\ddot{\phi}$

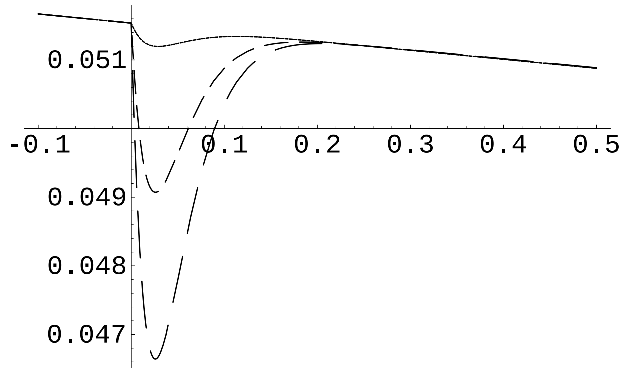


FIG. 2: $\frac{\dot{\phi(t)}}{m_p m_\phi}$ is plotted for $\frac{-0.1}{m_\phi} < t < \frac{0.5}{m_\phi}$.Solid line corresponds to N=1,small dashed to N=8, and long dashed to N=16.

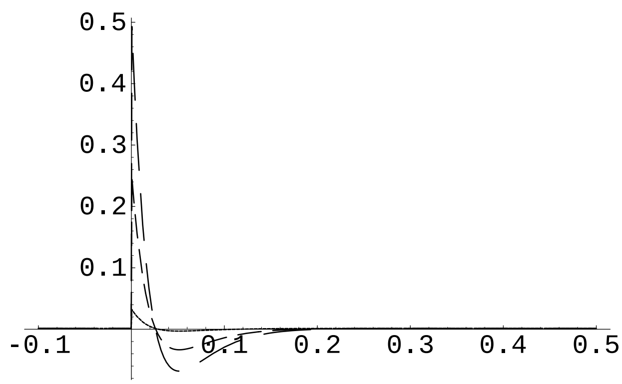


FIG. 3: $\frac{\phi(\ddot{t}_k)}{m_p m_\phi^2}$ is plotted for $\frac{-0.1}{m_\phi} < t < \frac{0.5}{m_\phi}$

**Spectrum calculation**

Setting $u = Rz$ in the equation for curvature perturbations we get

$$u'' + (k^2 - \frac{z''}{z})u = 0 \tag{63}$$

In order to solve analytically this equation before and after $\eta_0$, we use the following **aproximation**, which is quite good for sufficiently high k:

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}(1 - \frac{i}{k\eta}) \tag{64}$$

$$u_{\eta>\eta_0} = u_+ = \alpha_k v + \beta_k v^* \tag{65}$$

$$u_{\eta<\eta_0} = u_- = v \tag{66}$$

We used the the fact that $u_-$ and $u_+$ form an **approximate base** for the solutions of equation (63), in terms of which we decompose different modes **after** particle production.

Since we are modeling the particle production with a **step function**, this will give rise to a **Dirac delta** in the **third derivative** of the solution of the differential equation, which will produce a discontinuity in the equation of the cosmological perturbations at $\eta_0$:

$$\int_{\eta_0-\epsilon}^{\eta_0+\epsilon} u'' + (k^2 - \frac{z''}{z})u\,d\eta = 0 \quad D_0 = \int_{\eta_0-\epsilon}^{\eta_0+\epsilon} \frac{z''}{z}\,d\eta \qquad (67)$$

$$u'_+(\eta_0) = u'_-(\eta_0) + D_0 u_-(\eta_0) \quad u_+(\eta_0) = u_-(\eta_0) \qquad (68)$$

where a prime here denotes derivative respect to conformal time.

Since the only terms in the integral of $\frac{z''}{z}$ which will survive in the limit in which $\epsilon$ goes to zero are those proportional the Dirac delta coming from the third time derivative of $\phi$, we get:

$$D0 = [\ddot{\phi}_+(t_0) - \ddot{\phi}_-(t_0)]\frac{a_0^2}{\dot{\phi}_0} = N\,g\,n\,C\,\frac{a_0^2}{\dot{\phi}_0} \qquad (69)$$

The **matching conditions** (68) for $u$ and its derivative at the time of particle production $\eta_0$ will determine **different** $\alpha_k, \beta_k$ **for each mode**, which produce the **oscillatory** behavior of the spectrum for $k > k_0$.

Taking the appropriate limit of $u_+$ we get the following expression for the power spectrum:

$$P_k^{1/2} = \frac{H^2}{\dot{\phi}(t_k)} |\alpha_k - \beta_k|^2 \tag{70}$$

As it can be seen in the figures the **analytical** approximation is very **accurate** for **high k**, and starts to loose accuracy for those modes which leave the horizon around the time of particle production $\eta_0$, and later, since the approximation (64) for the curvature perturbation modes is not working anymore.

The oscillatory feature of the spectrum is in agreement with [JCAP **0309**, 008 (2003)] and other studies

For $k < k_0$ the spectrum should be **unaffected**, because of **super-horizon conservation of curvature perturbation**.

# Curvature perturbation spectrum

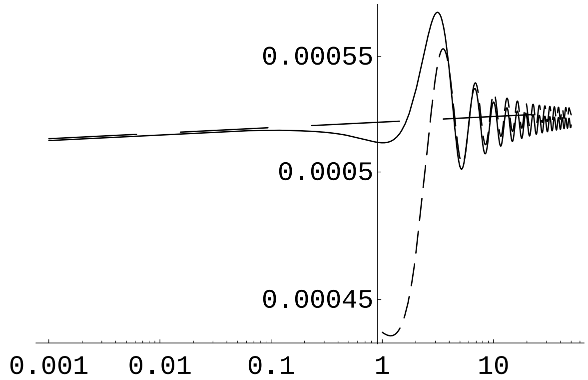


FIG. 4: $P_k$ is plotted for N=8 and $3\,10^{-2} < \frac{k}{a_0 H_0} < 50$. The solid line is the numerical result, the dashed is the analytical approximation, the long dashed is the spectrum in absence of coupling.

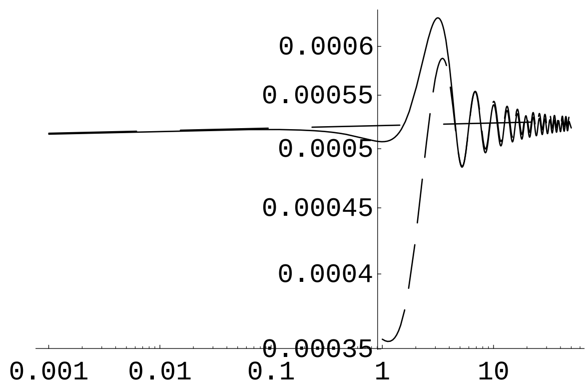


FIG. 5: $P_k$ is plotted for N=16 and $3\,10^{-2} < \frac{k}{a_0 H_0} < 50$.The solid line is the numerical result, the dashed is the analytical approximation, the long dashed is the spectrum in absence of coupling.

## Coupling strength and spectrum oscillation

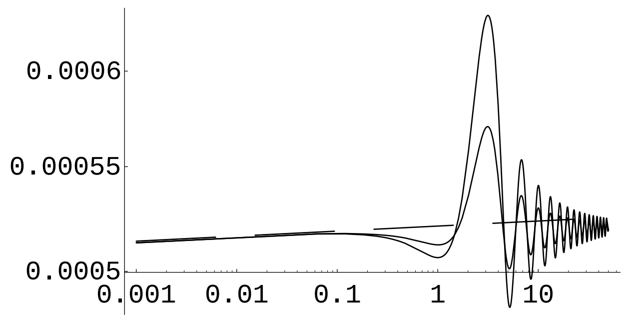


FIG. 6: $P_k$ is plotted for N=16 and N=8 and $3\,10^{-2} < \frac{k}{a_0 H_0} < 50$. The oscillation is larger for larger N, since this corresponds to a bigger discontinuity of the second derivative of the inflaton at the time of particle production. The long dashed line is the spectrum in absence of coupling.

**Could modes evolve after horizon exit?**

The only way modes could evolve after horizon crossing, impacting the spectrum for $k < k_0$, would be the presence of a **source term** in the equation for the curvature perturbation, which is **absent**, since all the **effect** is the time **evolution of** $z$. It is possible however that such a term would arise if **entropy perturbation** is taken into account because of the presence of **two fields**.

## Conclusion

We have studied the **impact** on the primordial curvature perturbation **spectrum** of the **coupling** of the inflaton to a scalar field.

We found the presence of an **oscillatory** behavior on **small scales**, for the modes which leave the horizon after the time of particle production.

We have also been able to find a very **good analytical approximation** for the evolution of **small scale modes**, which can be extended to the **general** case of a **sudden change of the inflaton potential**, leading to a temporary violations of the slow roll conditions.

The presence of such features in the observed **CMBR** spectrum could help to determine the **magnitude** and time extension of such periods of **slow-roll conditions violations**, even if they could **also** be due to **intermediate astrophysical processes** happening before and after recombinations, and not necessarely come from the primordial power spectrum.

In a more complete treatment of the two fields system it is possible that a source term due to **entropy perturbation** would arise, and that modes could evolve also after horizon exit. This problem will be investigated in a future work.

**Quantization of the $\phi$ field**

Working in the Heisenberg picture the $\varphi$ field can be canonically quantized and decomposed in terms of annihilation and creation operators

$$a^{3/2}\varphi = \frac{1}{(2\pi)^{3/2}} \int d^3k a_k \left(a^{3/2}\varphi_k\right)(t)^{-ikx} + a_k^+ \left(a^{3/2}\varphi_k\right)^* (t)^{+ikx}, \quad (71)$$

satisfying the usual commutation relations

$$[a_k, \hat{a}_{k'}] = [a_k^\dagger, a_{k'}^+] = 0, \qquad [a_k, \hat{a}_{k'}^+] = \delta^3(\vec{k} - \vec{k'}). \quad (72)$$

By construction $\varphi_k$ is a solution to the classical Klein–Gordon equation of a massive field with time-dependent mass

$$\left(a^{3/2}\varphi_k\right)^{\cdot\cdot} + \omega_k^2 \left(a^{3/2}\varphi_k\right) = 0, \quad (73)$$

$$\omega_k^2 = \frac{k^2}{a^2} + (m - g\phi)^2 - \frac{9}{4}H^2 - \frac{3}{4}\dot{H}. \quad (74)$$

Initially $\varphi_k$ is in the vacuum state of a heavy bosonic field

$$a^{3/2}\varphi_k = \frac{1}{\sqrt{2\omega_k}} \exp\left(-\imath \int^t \omega_k dt\right), \quad (75)$$

Generally $\varphi_k$ after particle production can be written like

$$a^{3/2}\varphi_k(t) = \frac{\alpha_k}{\sqrt{2\omega_k}} \exp\left(-i \int^t \omega_k dt\right) + \frac{\beta_k}{\sqrt{2\omega_k}} \exp\left(+i \int^t \omega_k dt\right). \quad (76)$$

Then initially $\alpha_k = 1$, $\beta_k = 0$, while at later times the effective mass $(m - g\phi)$ only changes slowly and equation (76) is a solution to equation (73) *with constant* $\alpha_k$, $\beta_k$.

Taking the quantum expectation value and subtracting the vacuum contribution to deal with ultraviolet divergences we get

$$
\langle\varphi^2\rangle = \frac{1}{2\pi^2 a^3}\int \frac{dk k^2}{\omega_k}\left[|\beta_k|^2 + Re\left(\alpha_k\beta_k^* \exp\left(-2\imath \int^t \omega_k \underset{\cdot}{t}\right)\right)\right] \simeq \frac{\mathcal{C}}{m - g\phi} n_0 \left(\frac{a}{a_0}\right)^{-3}, \quad (77)
$$

where $\mathcal{C} = 0.680$ is a numerical constant.