IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

The Weight of Vacuum Energy: Possible Observational Signatures

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Plan of the Talk

- **The Problem of the Cosmological Constant**
- **Quark Matter and Quark Stars**
- **Implications of the Weight of Vacuum Energy**
- **Do Quark Stars Really Exist?**

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Part I: The Problem of the Cosmological Constant

Semi-classical Gravity

- **Bare Cosmological Constant**
- **Vacuum Energy of Quantum Matter**

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$$

The r.h.s. is only defined after suitable regularization and renormalization!!!

$$\langle T_{\mu\nu} \rangle \sim A\,\frac{g_{\mu\nu}}{\sigma^2} + B\,\frac{G_{\mu\nu}}{\sigma} + (C_1 H^{(1)}_{\mu\nu} + C_2 H^{(2)}_{\mu\nu})\ \ln\,\sigma$$

$$\begin{aligned} H^{(1)}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}[\sqrt{-g}R^2] \\ &= 2\nabla_\nu\nabla_\mu R - 2g_{\mu\nu}\nabla_\rho\nabla^\rho R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu} \end{aligned}$$

$$\begin{aligned} H^{(2)}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}[\sqrt{-g}R_{\alpha\beta}R^{\alpha\beta}] = 2\nabla_\alpha\nabla_\nu R^\alpha_\mu - \nabla_\rho\nabla^\rho R_{\mu\nu} \\ &\quad -\frac{1}{2}g_{\mu\nu}\nabla_\rho\nabla^\rho R - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 2R^\rho_\mu R_{\rho\nu} \end{aligned}$$

Renormalized Einstein Equations

$$S_G = \frac{1}{16\pi G_0} \int d^4x \sqrt{-g} \left(R - 2\Lambda_0 + \alpha_0 R^2 + \beta_0 R_{\alpha\beta} R^{\alpha\beta} \right)$$

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \alpha_0 H^{(1)}_{\mu\nu} + \beta_0 H^{(2)}_{\mu\nu} = 8\pi G_0 \langle T_{\mu\nu} \rangle$$

- **At the end of the calculation, the result is independent of the regularization and renormalization procedures**

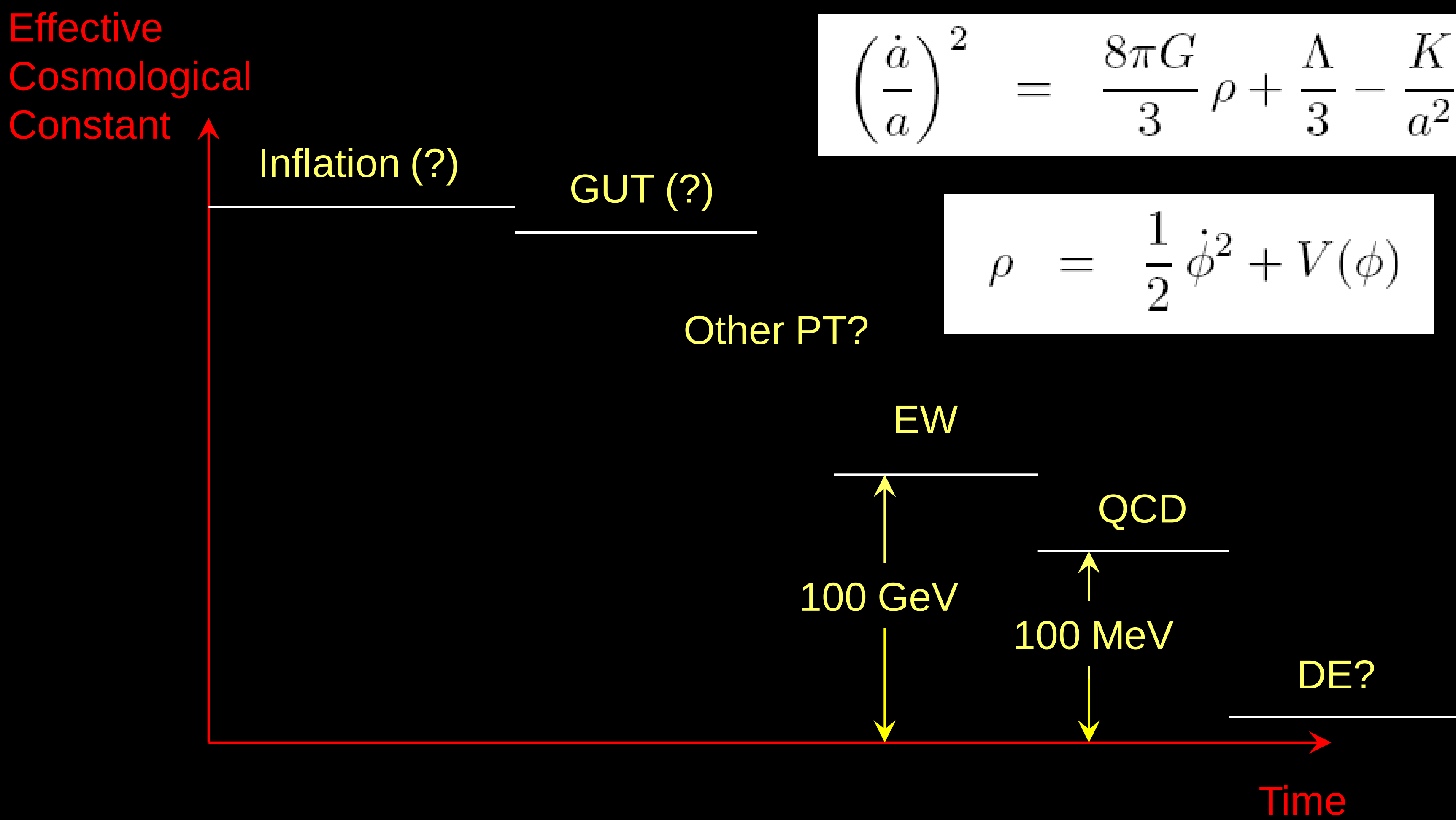
Vacuum Energy from Spontaneously Broken Symmetries

$$T_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi)g_{\mu\nu} - V(\phi)g_{\mu\nu}$$

$$\Rightarrow \quad T_{\mu\nu} = -V(\phi_0)g_{\mu\nu}$$

- **Electroweak Symmetry (Higgs)**
- **Chiral Symmetry (QCD)**
- **GUT?**

“Standard Picture” of the Universe



$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Solutions to the Puzzle

- **Special Initial Conditions: we live in a very peculiar Universe! (Anthropic Principle?)**

- **Special properties of the vacuum energy density (Unknown mechanism: compensating fields, violation of the EP, etc.)**

Part II: Quark Matter and Quark Stars

QCD Vacuum

- **QCD Lagrangian**

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu + g\, T_a \gamma^\mu A^a_\mu - m)\psi$$

$$\langle \hat{T}^\rho_\rho \rangle_{\rm ren} = \frac{\beta(g)}{2g} \langle \hat{G}^a_{\mu\nu} \hat{G}_a^{\mu\nu} \rangle_{\rm ren} + (1 + \gamma^m)\, m\, \langle \bar{\hat{\psi}} \hat{\psi} \rangle_{\rm ren}$$

$$\langle \bar{\hat{\psi}} \hat{\psi} \rangle_{\rm ren} = \mathcal{O}(\Lambda^3_{\rm QCD}) \qquad \langle \hat{G}^a_{\mu\nu} \hat{G}_a^{\mu\nu} \rangle_{\rm ren} = \mathcal{O}(\Lambda^4_{\rm QCD})$$

Gellmann-Renner-Oakes Relation

$$f_\pi^2 m_\pi^2 = m_q < \bar{\psi}_q \psi_q >$$

Quark Matter

- **QM Energy-Momentum Tensor**

$$T_{\mu\nu} = T_{\mu\nu}^{\text{Fermi gas}} + T_{\mu\nu}^{\text{vacuum}}$$

$$T_{\mu\nu}^{\text{vacuum}} = B g_{\mu\nu}$$

- **B is the energy density difference between the QCD vacua of the deconfined and confined phase**

$$\rho = A\, n^{4/3} + B$$

$$P = \frac{A}{3}\, n^{4/3} - B$$

- **In general B is NOT the Bag Constant we can deduce from Hadron Spectroscopy!**

Is Quark Matter Absolutely Stable?

- The introduction of a third flavor can reduce the energy of the system. But the mass of s is high…

$$\epsilon_2/n_{B2} \approx 934\mathrm{MeV}\, B_{145}^{1/4} \qquad \epsilon_3/n_{B3} \approx 829\mathrm{MeV}\, B_{145}^{1/4}$$

$$B_{145}^{1/4} \equiv B^{1/4}/145\mathrm{MeV}$$

- Energy per Baryon in Iron: 930 MeV
- $B^{1/4} < 144.4$ MeV
- Even if QM is metastable at zero pressure, it may be stabilized at high pressure within a compact star

Quark Stars: Equations

- **TOV Equations:**

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G_N[\rho(r) + P(r)][M_\mathrm{G}(r) + 4\pi r^3 P(r)]}{r[r - 2G_N M_\mathrm{G}(r)]}$$

$$\frac{\mathrm{d}M_\mathrm{G}}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

$$P(0) = \frac{A}{3} n_\mathrm{C}^{4/3} - B$$

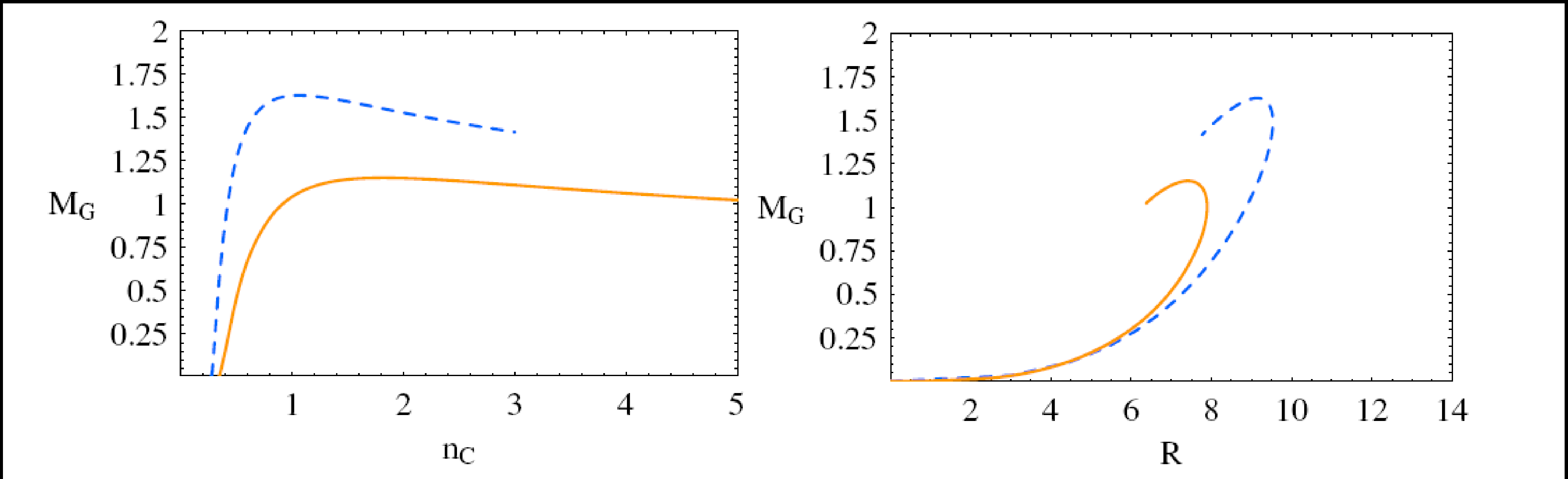
$$M_\mathrm{G}(0) = 0$$

$$P(R) = \frac{A}{3} n^{4/3}(R) - B = 0$$

Quark Stars: Results

$$M_{\max} = 1.63 \left(\frac{80 \text{ MeV fm}^{-3}}{B} \right)^{1/2} M_\odot$$

$$R(M_{\max}) = 9.15 \left(\frac{80 \text{ MeV fm}^{-3}}{B} \right)^{1/2} \text{km}$$



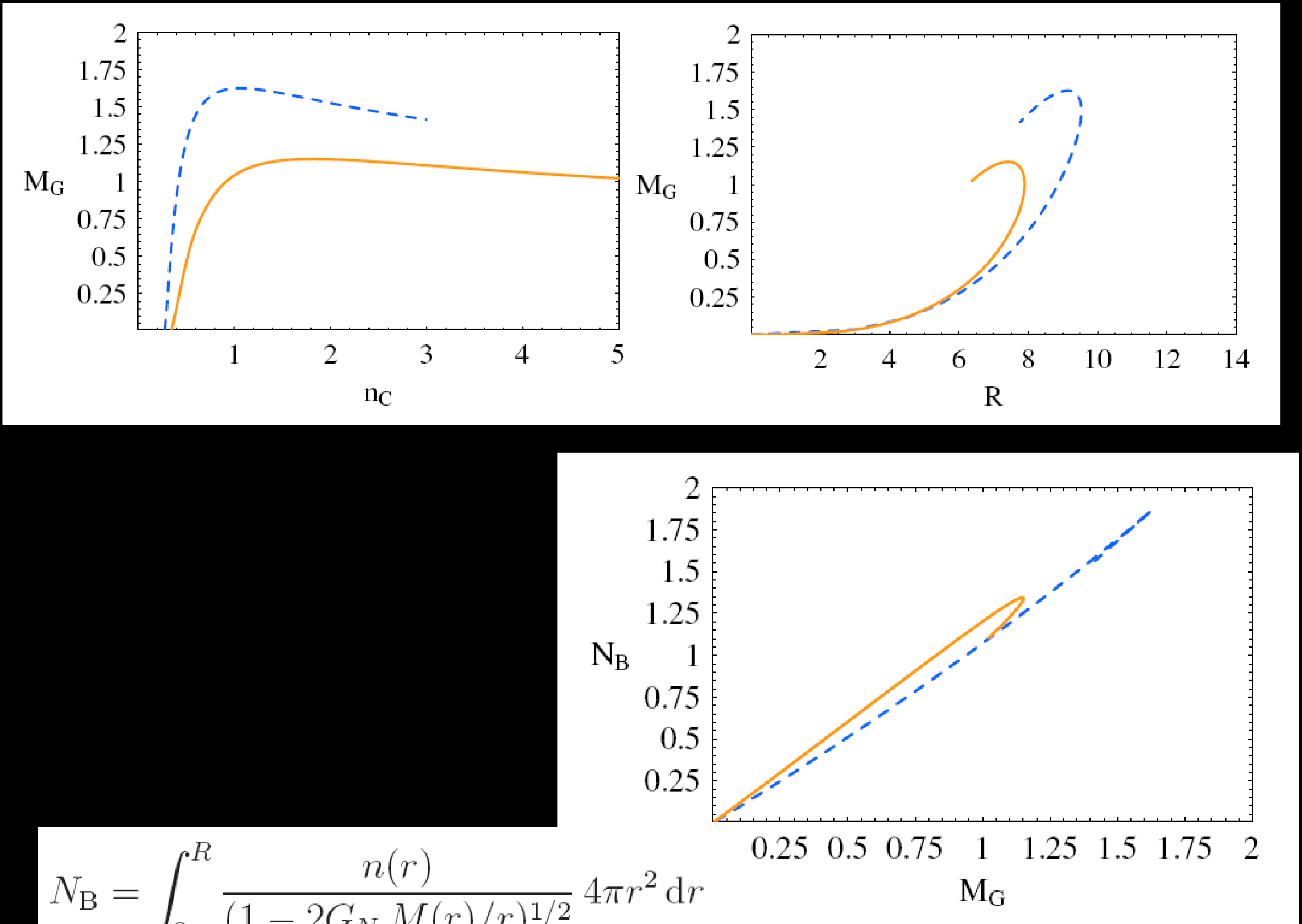
Part III: Implications of the Weight of Vacuum Energy

“New Equilibrium”

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G_N[\rho(r) + P(r)][M_{\mathrm{G}}(r) + 4\pi r^3 P(r)]}{r[r - 2G_N M_{\mathrm{G}}(r)]}$$

$$\frac{\mathrm{d}M_{\mathrm{G}}}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

B (MeV fm^{-3})	$M_{\mathrm{max}}/M_{\odot}$	$R(M_{\mathrm{max}})$ (km)	$N_{\mathrm{B}}(M_{\mathrm{max}})$ ($\times 10^{57}$)
60	1.33	8.57	1.66
80	1.15	7.42	1.34
100	1.03	6.63	1.13
120	0.94	6.06	0.99
140	0.87	5.60	0.88
160	0.81	5.24	0.80

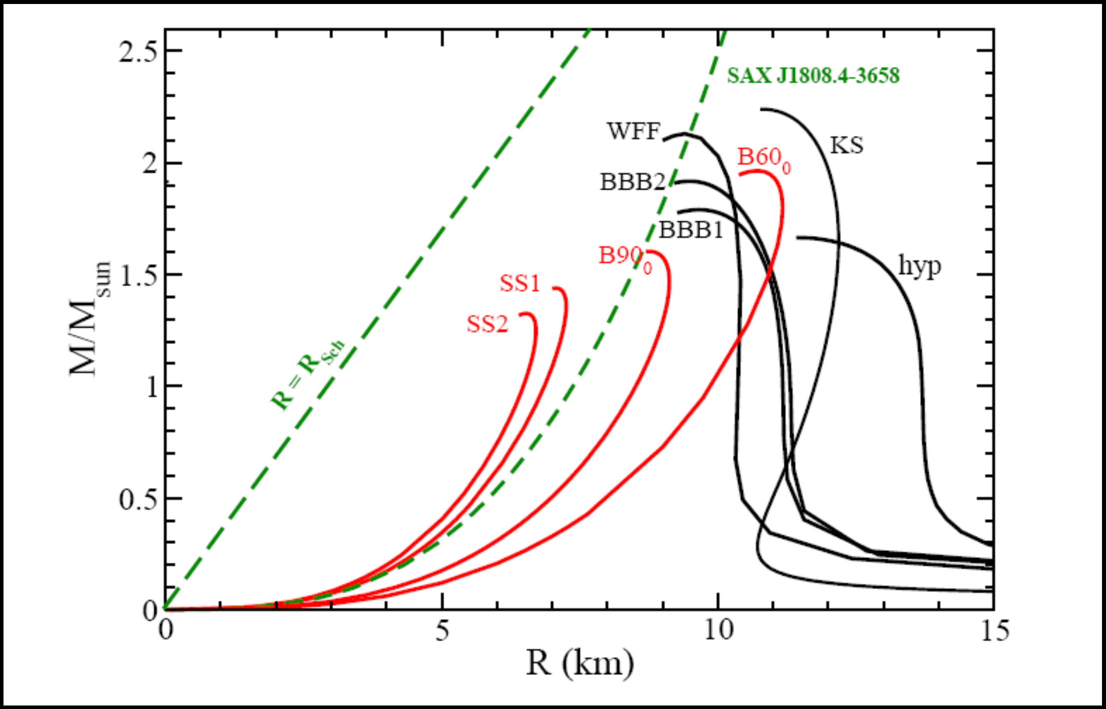


$$N_{\rm B} = \int_0^R \frac{n(r)}{(1 - 2G_N\, M(r)/r)^{1/2}}\, 4\pi r^2\, \mathrm{d}r$$

Part IV: Do Quark Stars Really Exist?

Relativistic Stars

- 3 possibilities: NS, HS or QS
- QS formation: proto-NS phase or accretion phase



How Can We Recognize a Quark Star?

- “Crusted” QS are expected to have emission properties similar to NS
- Stellar Cooling (thermal emission + NS age)
- Maximum Mass (NS-NS binary system: 1.2 – 1.4 Solar masses, Ter-5-I: mass > 1.68 Solar masses)
- Mass-Radius Relation
- Gravitational Waves? Neutrino Flux? …?

XTE J1739-285

- X-Ray Transient discovered at the end of 2006
- Burst oscillations at 1122 Hz
- If the burst oscillation frequency is equal to the stellar spin rate, we have discovered the first sub-millisecond compact stars
- In this case XTE J1739-285 would be a HS or QS!

Conclusions

- **Do we live in a very special Universe? Has the Vacuum Energy very peculiar (and unknown) properties?**
- **Can we test the two classes of solutions?**
- **The equilibrium configuration of Quark Stars depends on the QCD Vacuum Energy Density.**
- **Quark Stars may thus be our unique possibility of “weighting” the Vacuum!**