A NEW DIMENSION FOR THE ADS/CFT CORRESPONDENCE

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BASED ON 0808.2503

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OUTLINE

- ☐ BASICS OF ADS/CFT
- ☐ THE PUZZLE OF 3D
- ☐ CHERN-SIMONS + MATTER AND PERTURBATION THEORY
- ☐ N=6 SCFT IN 3D: THE ABJM MODEL
- ☐ MODULI SPACE AND MEMBRANES
- ☐ OPERATORS AND WILSON LOOPS.

BASICS OF ADS/CFT

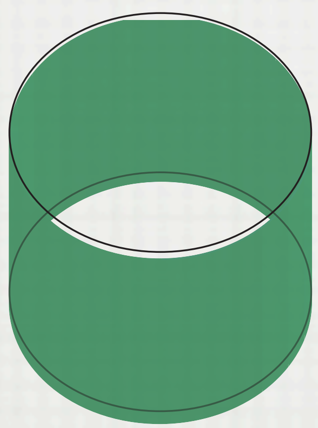
THE ADS/CFT CORRESPONDENCE IS A QUANTUM EQUIVALENCE BETWEEN TWO APPARENTLY DIFFERENT QUANTUM SYSTEMS

QUANTUM GRAVITY ON ASYMPTOTICALLY ADS GEOMETRIES IN D+1 DIMENSIONS

A CONFORMAL FIELD THEORY ON D-DIMENSIONS

☐ SIMPLEST EXAMPLE IS MALDACENA DUALITY '97.

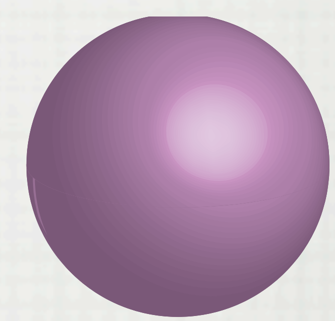
N=4 SYM on a sphere



Type IIB Superstring on

$$AdS_5 \times S^5$$

x



AdS/CFT is a strong weak-coupling duality for the 't Hooft coupling constant.

AdS	CFT
Isometries	Global symmetries
R^4	$g_{YM}^2 N$
Flux = N	Gauge group U(N)
Quantum State	Quantum State

THE FIELD THEORY CAN BE ANALYZED PERTURBATIVELY. ONE CAN TRY TO COMPARE QUANTITIES.

THE PUZZLE IN 3D

THERE ARE NATURAL SOLUTIONS OF M-THEORY

$$AdS_4 \times S^7$$

THE FIELD THEORY DUAL IS AN IR FIXED POINT WITH SO(8) SYMMETRY OF THE MAXIMALLY SUSY QUANTUM FIELD THEORY IN 3D

CONFORMALITY WAS ARGUED BY
SETHI, SUSSKIND; BANKS, SEIBERG '97
BEFORE THE ADS/CFT

- ONLY PARAMETER IS N: THERE IS NO ANALOG OF THE 'T HOOFT COUPLING CONSTANT.
- NEED TO EXPLAIN WHY A LARGE N THEORY IS NOT A THEORY OF STRINGS, BUT MEMBRANES.
- CAN'T DO PERTURBATION THEORY
- WEIRD SCALING OF THE SPECIFIC HEAT (FROM SUGRA)

$$E \sim N^{3/2} T^3$$

CHERN-SIMONS+MATTER +PERTURBATION THEORY

CAN ONE DO BETTER EXAMPLES IN 3D?

YES!

NEED TO ANALYZE WHAT THEORIES
ARE PERTURBATIVE AND CONFORMAL AND START
FROM THERE

DIMENSIONAL ANALYSIS

ENGINEERING DIMENSIONS OF TYPICAL FIELDS

$$\begin{cases} [\psi_\alpha] = 1 \\ [\phi] = 1/2 \\ [A_\mu] = 1 \end{cases}$$

THIS IS DETERMINED BY COVARIANT DERIVATIVE HAVING THE RIGHT SCALING.

- TO HAVE PERTURBATIVE SETUP WE NEED TO WRITE LAGRANGIAN WITH LOCAL POLYNOMIAL OPERATORS OF DIMENSION LESS THAN OR EQUAL TO THREE.

- WE ARE ALLOWED A FIRST ORDER ACTION FOR THE GAUGE FIELD: CHERN SIMONS LAGRANGIAN (NO PROPAGATING DEGREES OF FREEDOM). DEPENDS ON ONE INTEGER PARAMETER K (ALSO CALLED LEVEL)

- STANDARD KINETIC TERMS FOR BOSONS + FERMIONS.

- WE ARE ALLOWED INTERACTIONS OF THE FORM

$$\phi^6 \qquad \phi^2\psi^2$$

- THEN ADD SUPERSYMMETRY TO THE MIX.
- ONE CAN UNDERSTAND N=2 SUSY IN 3D (SAME SUPERFIELDS AS N=1 SUSY IN 4D).
- SUGGESTED JOHN SCHWARZ (HEP-TH 0411077)
- GAIOTTO, YIN (0704.3740): COMPUTATIONS OF BETA FUNCTIONS WHERE THEY FIND FIXED RG POINTS. COUPLINGS ARE RELATED TO THE LEVEL K

BAGGER-LAMBERT-GUSTAVSSON

- TRY TO GET A CS + FIELD CONTENT WITH MAXIMAL SUPERSYMMETRY
- NOT AN ORDINARY CHERN-SIMONS, BUT A NEW STRUCTURE BASED ON 3-ALGEBRAS RATHER THAN LIE ALGEBRAS
- ONE THEORY FOR 'GAUGE GROUP' SO(4) WITH REQUIRED SYMMETRIES.
- THERE IS ALSO A LEVEL K. WHAT DOES THIS DO?
- DESPITE A LOT OF WORK, THE FOG HAS NOT CLEARED

OBSERVATION OF M. VAN RAAMSDONK THAT SO(4) BAGGER-LAMBERT-GUSTAVSSON IS A MORE ORDINARY CHERN-SIMONS

$$SU_k(2) \times SU_{-k}(2)$$

ALSO THERE IS A COMPUTATION OF MODULI SPACE BY DISTLER-MUKHI-RAAMSDONK THAT GIVES

$$\mathbb{C}^8/D_k$$

SUGGESTS THAT THIS IS MODULI SPACE OF 2 BRANES ON A QUOTIENT SPACE.

ABJM (0806:1218)

AHARONY-BERGMAN-JAFFERIS-MALDACENA

INSTEAD OF MAXIMAL SUSY, JUST GENERALIZE OBSERVATIONS OF RAAMSDONK TO U(N)X U(N) AND TREAT IT LIKE A MORE ORDINARY GAUGE THEORY WITH N=4 SUSY IN 3D (SAME AS N=2 SUSY IN 4D).

$$U(N)_k \times U(N)_{-k}$$

$$A_{1,2} \in (N, \bar{N})$$

$$B_{1,2} \in (\bar{N}, N)$$

N=2 VECTOR MULTIPLET IN 4D, REDUCED TO 3D

$$V, a$$

CHERN SIMONS LAGRANGIAN MAKES ALL FIELDS IN MULTIPLET AUXILIARY.
THEY CAN BE INTEGRATED OUT.

$$\mathcal{S}_{\rm CS} = -iK \int d^3x\, d^4\theta \int_0^1 dt\ \mathrm{tr}\Big[\mathcal{V}\bar{D}^\alpha\Big(e^{t\mathcal{V}} D_\alpha e^{-t\mathcal{V}}\Big) - \hat{\mathcal{V}}\bar{D}^\alpha\Big(e^{t\hat{\mathcal{V}}} D_\alpha e^{-t\hat{\mathcal{V}}}\Big)\Big]$$

$$+K \int d^2\theta a^2$$

BENNA, KLEBANOV,
KLOSE, SMEDBACK 0806.1519

IF ONE DOES IT THIS WAY, ONE OBTAINS THE ABJM MODEL WITH MANIFEST N=4 SUSY. THE (A,B) ARE TWO HYPER-MULTIPLETS.

ONE NOTICES THAT AFTER ONE INTEGRATES THE CHIRAL PARTNER FIELD OF THE GAUGE FIELD, THAT ONE OBTAINS AN EFFECTIVE SUPERPOTENTIAL OF THE FORM

$$W \sim \frac{1}{k^2} \int d^2\theta \epsilon_{ij} \epsilon_{lm} (A_i B_l A_m B_n)$$

THE SUPERPOTENTIAL IS THE SAME AS THE ONE OF THE KLEBANOV-WITTEN THEORY

NOT SURPRISING: SUPERFIELD FORMALISM FOR CHIRAL PARTNER OF GAUGE FIELD IS JUST A MASS TERM

THE NEW EFFECTIVE SUPERPOTENTIAL HAS AN ENHANCED $SU(2) \times SU(2)$ GLOBAL SYMMETRY THAT DOES NOT COMMUTE WITH THE R-CHARGE

THIS MEANS THAT THE R-CHARGE SYMMETRY BECOMES $SU(4) \sim SO(6)$* SO IN THE END ONE HAS A THEORY WITH $N=6$ SUSY IN 4 DIMENSIONS

*SPINORS IN 3D ARE REAL

- ABJM: N=4 CHERN SIMONS PLUS MATTER.
- UNIQUE COUPLING CONSTANT K (LEVEL) AND ONE CAN ALSO CHANGE NUMBER OF COLORS N.
- EFFECTIVE T'HOOFT COUPLING N/K
- FOR LARGE K FIXED N, CAN DO PERTURBATION THEORY.
- CAN TAKE A 'T HOOFT LIMIT
- IN THE T'HOOFT LIMIT IT SHOULD BE SOME TYPE OF STRING THEORY.

ADS/CFT

GEOMETRY	ABJM MODEL
$AdS_4 \times S^7/\mathbb{Z}_k$	LEVEL K
FLUX=N ALSO EQUALS NUMBER OF BRANES	N
ISOMETRY= SU(4)	R-CHARGE SO(6)
QUANTUM STATE	QUANTUM STATE
$AdS_4 \times \mathbb{C}P^3$ STRING LIMIT	LARGE 'T HOOFT LIMIT

MODULI SPACE AND MEMBRANES

MODULI SPACE WAS COMPUTED IN ABJM, FOLLOWING DMV

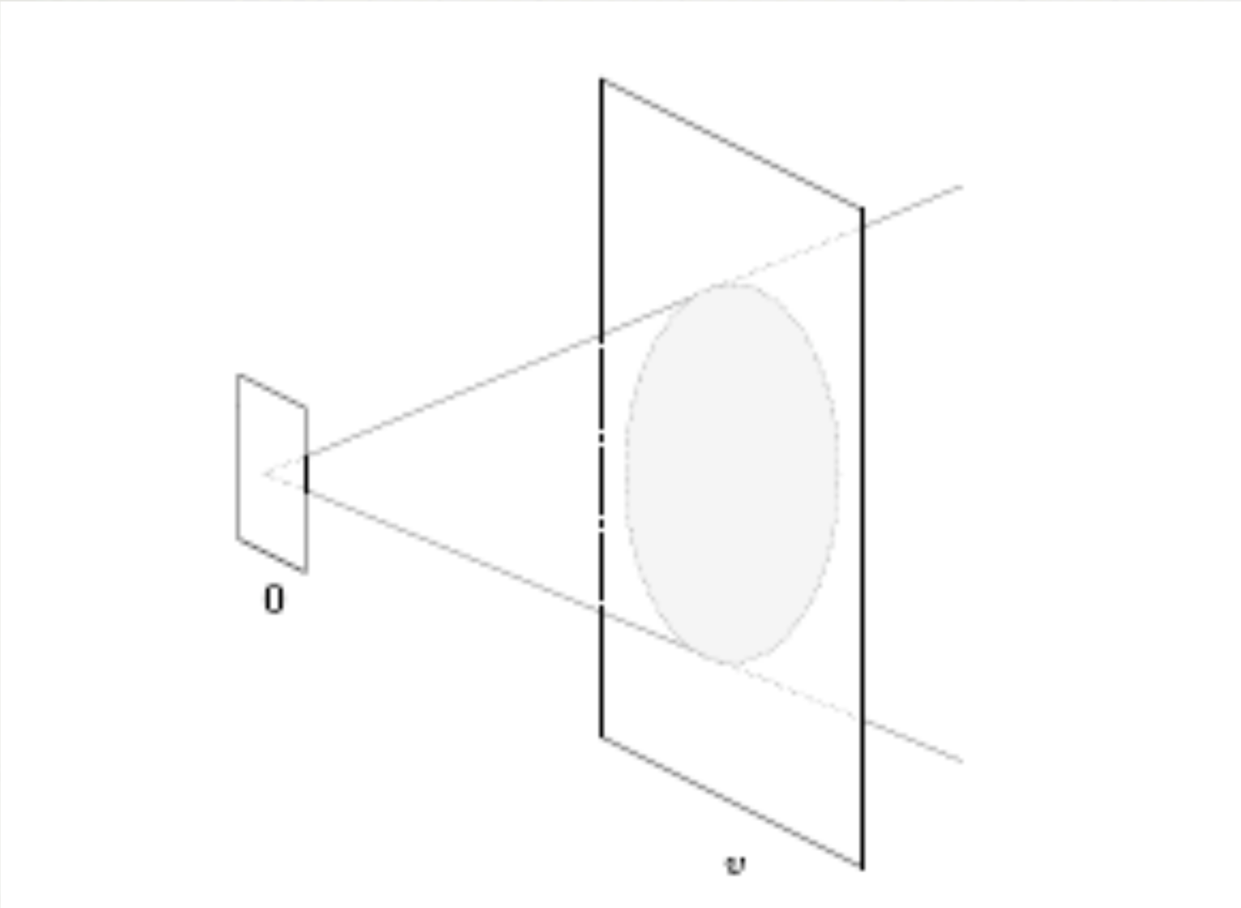
$$Sym^N(\mathbb{R}^8/\mathbb{Z}_k)$$

VEV'S OF A, B FIELDS ARE DIAGONAL MATRICES. THE DIMENSION IS NOT OBVIOUS.

- NAIVE OBSERVATIONS:

- POTENTIAL IS SCALARS TO SIXTH POWER, AND ALSO SCALAR SQUARED TIMES FERMION SQUARED.

- MASSES OF OFF-DIAGONAL MODES GROW LIKE DISTANCE SQUARED, RATHER THAN LINEAR IN DISTANCE



OFF-DIAGONAL MASSES ARE LIKE AREAS OF CONES.

THIS MEANS THAT THERE SHOULD BE A MEMBRANE INTERPRETATION.

ONE OF OUR COMPUTATIONS IS THE MASSES OF OFF-DIAGONAL MODES

$$m_{ij}^2 \sim \frac{4\pi^2}{k^2}\left((|\vec{x}_i|^2 + |\vec{x}_j|^2)^2 - 4|\vec{x}_i \cdot \vec{x}_j^*|^2\right)$$

THIS IS INVARIANT UNDER INDEPENDENT REPHASINGS OF THE COORDINATES OF THE TWO BRANES: THERE IS A SPECIAL CIRCLE THAT THE MEMBRANE BITS WRAP.

THIS IS ASSOCIATED TO THE CARTAN GAUGE SYMMETRY IN THE FIELD THEORY.

THE MEMBRANE BITS MUST WRAP THE HOPF FIBER OF THE SPHERE OVER THE CIRCLE IN ORDER FOR THE GEOMETRY TO MATCH THE FIELD THEORY CALCULATION.

LOCALITY ALONG THE HOPF FIBER IS HARD TO EXPLAIN BECAUSE ALL DEGREES OF FREEDOM WRAP IT (AS HARD AS IN MATRIX THEORY FOR THE LIGHTCONE COORDINATE)

LOCAL SCALAR OPERATORS

CAN WE UNDERSTAND EXCITED STATES AND NON-PERTURBATIVE EFFECTS?

YES.

WE NEED THE OPERATOR STATE CORRESPONDENCE TO DO THIS.

ASSUME YOU HAVE ADDED AN OPERATOR AT THE ORIGIN IN AN EUCLIDEAN CFT

$$ds^2 = r^2 \left(\frac{dr^2}{r^2} + d\Omega^2 \right)$$

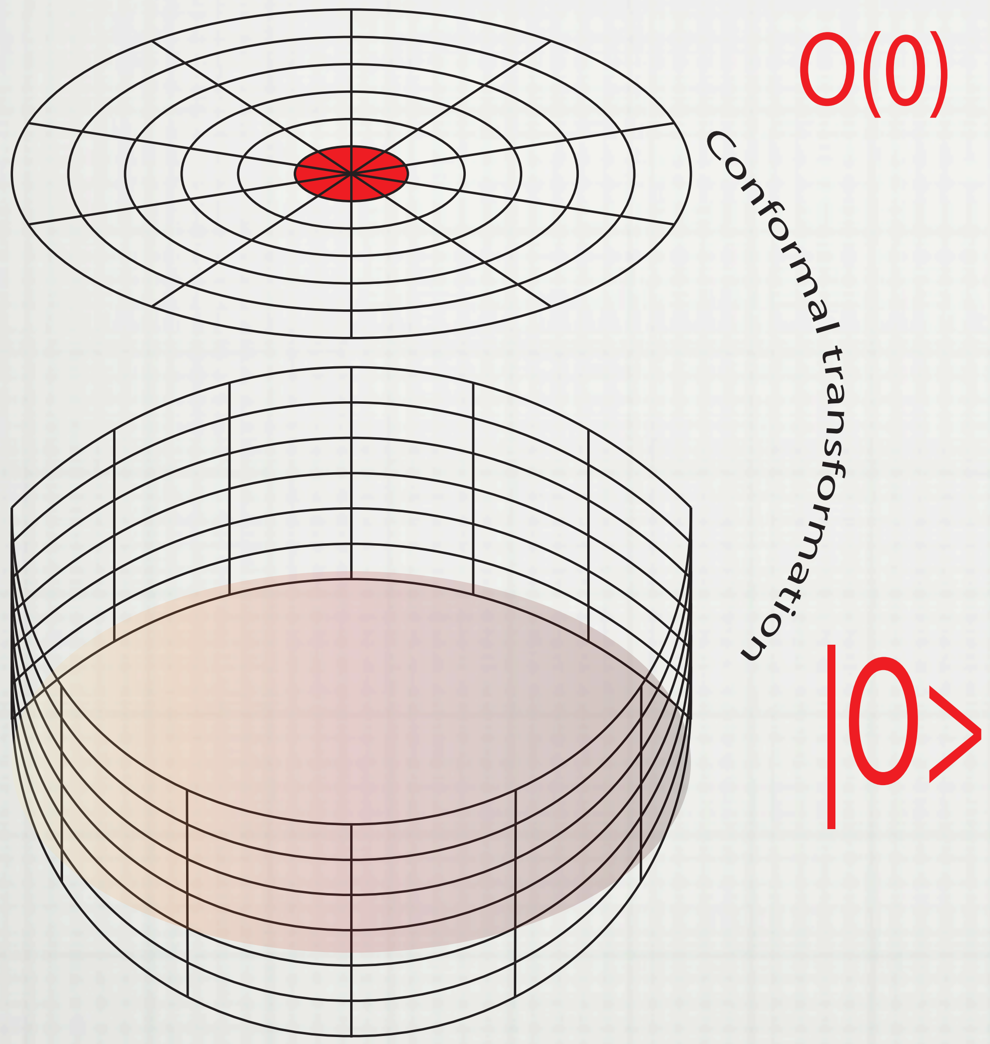
CONFORMALLY RESCALE TO REMOVE ORIGIN.

$$dt^2 + d\Omega^2$$

HOW DO WE KNOW WE INSERTED AN OPERATOR?

THE ORIGIN IS CHARACTERIZED NOW BY THE INFINITE 'PAST'. IT BECOMES A BOUNDARY CONDITION IN THE TIME COORDINATE

IN LORENTZIAN SYSTEMS
A TIME BOUNDARY CONDITION IS AN INITIAL CONDITION: A STATE IN THE THEORY.



$$\mathcal{O}(0) \sim |\mathcal{O}\rangle$$

Hamiltonian is scaling dimension

DICTIONARY BETWEEN STATES AND OPERATORS

STATES	OPERATORS
ANGULAR MOMENTUM	SPIN
ENERGY	DIMENSION
R-CHARGE	R-CHARGE

THE QUANTUM STATES CAN BE UNDERSTOOD IN A SEMICLASSICAL SETUP!

HAS THE ADVANTAGE THAT WE CAN DEAL WITH NON-PERTURBATIVE OPERATOR DEFECTS: THOSE THAT HAVE DIFFERENT TOPOLOGIES BUT SOLVE THE SAME CLASSICAL EQUATIONS.

CHIRAL RING STATES

SATURATES AN INEQUALITY BETWEEN ENERGY AND R-CHARGE

$$E \geq R$$

NECESSARILY HAVE SPIN ZERO (ROTATIONALLY INVARIANT CLASSICALLY)

CAN BE UNDERSTOOD IN THE CLASSICAL FIELD THEORY

GAUGE FIELD HAS TO BE COVARIANTLY CONSTANT ON SPHERE

$$\nabla_{\theta,\phi} F_{\theta\phi} = 0$$

CAN CHOOSE A GAUGE WHERE F ALONG SPHERE IS DIAGONAL

CONDITION IMPLIES THAT EIGENVALUES OF F ARE CONSTANT (THERE IS CONSTANT MAGNETIC FLUX ON THE CARTAN)

SUCH CONFIGURATIONS ARE SOLUTIONS OF 2-D YM AUTOMATICALLY (CLASSIFIED BY ATIYAH-BOTT)

FOR U(N), FLUX ON EACH EIGENVALUE IS QUANTIZED.

NO DYNAMICS HAS BEEN SOLVED YET!
JUST SPHERICALLY INVARIANT KINEMATICS.

SCALAR FIELDS ARE SENSITIVE TO DIFFERENCES IN FLUX BETWEEN THE TWO U(N) FLUXES.

CONDITION FOR SPHERICALLY INVARIANT CONFIGURATIONS IS THAT ONE CAN ONLY TURN ON VEVS BETWEEN CARTAN'S THAT HAVE SAME EIGENVALUE.

BPS INEQUALITY CAN BE THOUGHT OF AS AN INEQUALITY IN A POISSON MANIFOLD AT THE CLASSICAL LEVEL

SATURATING THE BPS INEQUALITY IS FINDING THE MINIMUM LOCUS OF A HAMILTONIAN FUNCTION

STANDARD ENERGY EVOLUTION = R-CHARGE EVOLUTION

SIMPLIFIED FIRST ORDER EQUATION!

BPS EQUATIONS OF MOTION

$$
\begin{aligned}
\dot{A} &= iA \\
\dot{B} &= iB
\end{aligned}
$$

CONDITIONS FOR EQUALITY

FIELDS NEED TO BE IN MODULI SPACE
OF VACUA

SPHERICAL SYMMETRY (MATCHED FLUXES)

NEED TO ALSO INCLUDE GAUSS' LAW FOR
GAUGE INVARIANCE.

EQUATIONS OF MOTION OF A_0

$$F_1 \sim k(A\bar{A} - \bar{B}B)$$

$$F_2 \sim (-k)(-\bar{A}A + B\bar{B})$$

REMEMBER F IS QUANTIZED CLASSICALLY
AND THAT RIGHT HAND SIDE SHOULD BE QUANTIZED
(GENERATORS OF LIE ALGEBRA TRANSFORMATIONS).
THIS IMPLIES K IS INTEGER.

F, A, B ARE DIAGONAL

THE WHOLE PROBLEM REDUCES
TO STUDYING MONOPOLE OPERATORS FOR
N COPIES OF THE U(1)XU(1) THEORY:
THE CARTAN DIRECTIONS

$$A^{n_1} B^{n_2}$$

$$n_1 - n_2 = k[\phi_1] = k[\phi_2]$$

THIS IS A SET OF HOLOMORPHIC FUNCTIONS ON

$$\mathbb{C}^4/\mathbb{Z}_k$$

FOR EACH EIGENVALUE.

THE PERMUTATION SYMMETRY OF EIGENVALUES SHOWS THAT ONE CAN ALSO REPRODUCE THE MODULI SPACE IN THIS WAY:

THE MODULI SPACE IS THEN DEFINED AS REPRESENTATIONS OF THE CHIRAL RING

WILSON LOOP OPERATORS

IN $\mathcal{N}=4$ SYM WILSON LOOPS GET DRESSED BY SCALAR FIELDS

ONE CAN TRY THE SAME IN ABJM, EXCEPT THAT SCALARS ARE NOT IN ADJOINT AND HAVE WRONG DIMENSION

COMPOSITES OF TWO SCALARS ARE IN THE ADJOINT AND HAVE RIGHT DIMENSION

$$[1/2] + [1/2] = 1$$

$$W = \frac{1}{N} \, \mathrm{Tr} P \exp \left(\oint iA + \frac{2\pi}{k} \omega_A^B \phi^A \bar{\phi}_B ds \right)$$

WE FOUND BY STUDYING HOW FIELDS CONTRIBUTE TO THE ACTION OF AN OFF-DIAGONAL MODE THAT

$$\omega_A^B \sim (-1, 1, 1, 1)$$

HAS BEEN STUDIED BY OTHER GROUPS.

CONDITION FOR SUSY IS THAT

$$\omega_A^B \sim (-1, -1, 1, 1)$$

DRUKKER, PLEFKA, YOUNG 0809.2787

CHEN, WU, 0809.2863

KLUSON, PANIGRAHI, 0809.3355

REY, SUYAMA, YAMAGUCHI, 0809.3786

OTHER DEVELOPMENTS

INTEGRABILITY: MINAHAN AND ZAREMBO

GIANT MAGNON SOLUTIONS: TOO MANY PEOPLE

WE GAVE A FORMAL PROOF IN FIELD THEORY OF GIANT MAGNON DISPERSION RELATION UP TO AN UNKNOWN FUNCTION OF COUPLING

USING SOME STRONG COUPLING TECHNIQUES

ALL KINDS OF TORIC COMPUTATION OF MODULI SPACES FOR OTHER THEORIES: MARTELLI AND SPARKS

MANY MORE: OVER 100 PAPERS TO DATE

OUTLOOK

- NEW EXAMPLE OF ADS/CFT
- LOTS OF STUFF TO EXPLORE IN VARIOUS LIMITS
- GEOMETRY OF ELEVEN DIMENSIONS IS MORE MYSTERIOUS THAN GEOMETRY OF 10 D IN N=4 SYM.
- NON-PERTURBATIVE OPERATORS ARE IMPORTANT TO DESCRIBE CHIRAL RING
- TECHNIQUE TO WRITE PAPERS: LOOK AT ANYTHING THAT WAS DONE IN N=4 SYM AND PORT IT OVER.