

Properties of Baryons in Holographic QCD

Shigeki Sugimoto (IPMU)

Based on

arXiv:0806.3122 K.Hashimoto, T.Sakai and S.S.
hep-th/0701280 H.Hata, T.Sakai, S.S. and S.Yamato

Closely related works:

arXiv:0807.0033 K.Y.Kim and I.Zahed
arXiv:0803.0180 H.Hata, M.Murata and S.Yamato
hep-th/0701276, arXiv:0705.2632, arXiv:0710.4615, ...
D.K.Hong, M.Rho, H.U.Yee and P.Yi

Part I

- ① **Introduction**
- ② **Brief summary of the model**

----- 5 minutes break -----

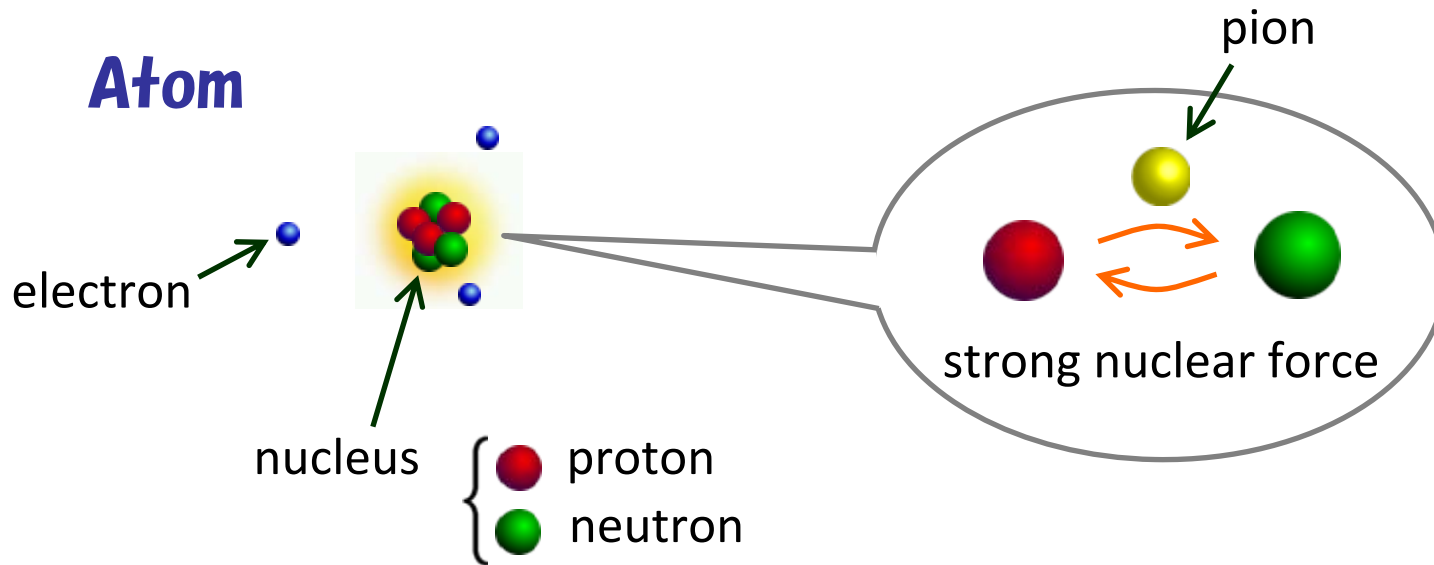
Part II











- ③ **Baryons as instantons**

⋮

1 Introduction

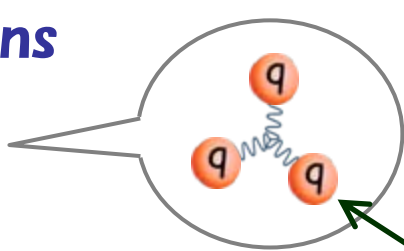
• Baryons and Mesons



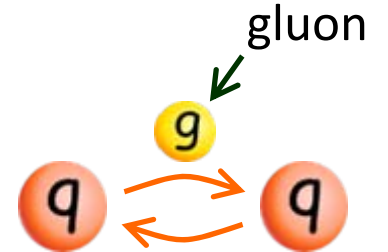
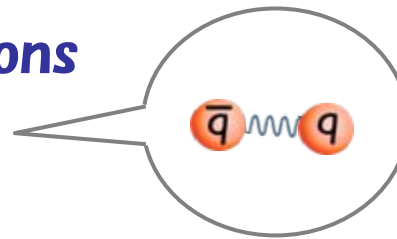
hadron	{	baryon						etc.
			p	n	Λ	Σ	Δ	
	{	meson						etc.
			π	K	η	ρ	ω	

• Quark model and QCD

Baryons



Mesons



Strong interaction

QCD

$SU(N_c)$ gauge theory ($N_c = 3$)

A_μ



(gluon)

ψ^i



(quark)

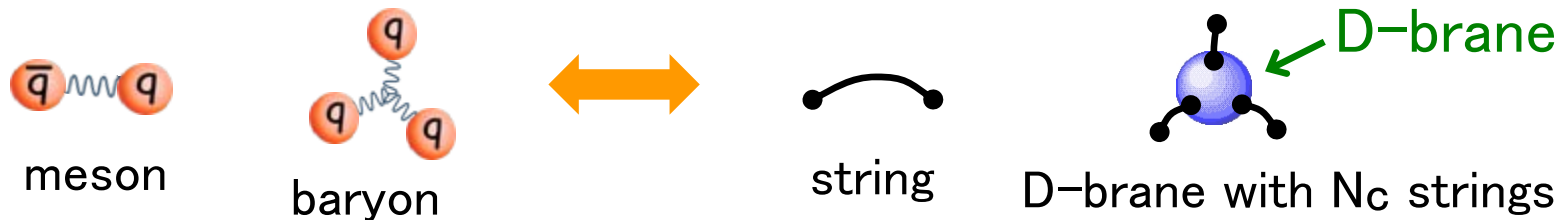
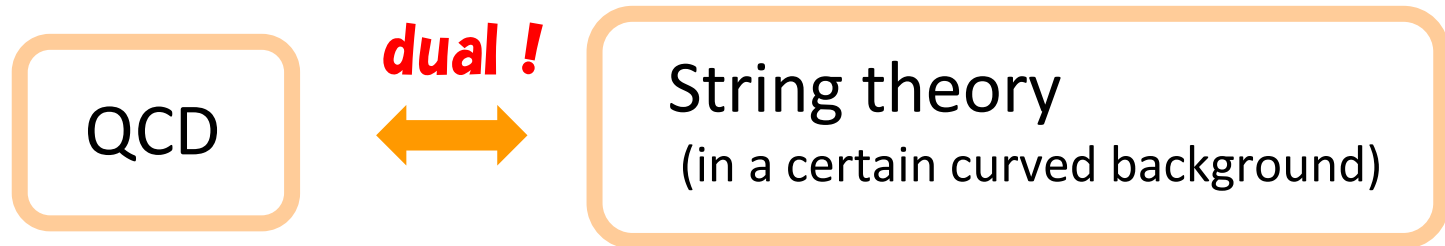
$i = 1, 2, \dots, N_f$

(u d s ...)

- In this talk, we mainly consider light quarks: (u d) ($N_f = 2$)

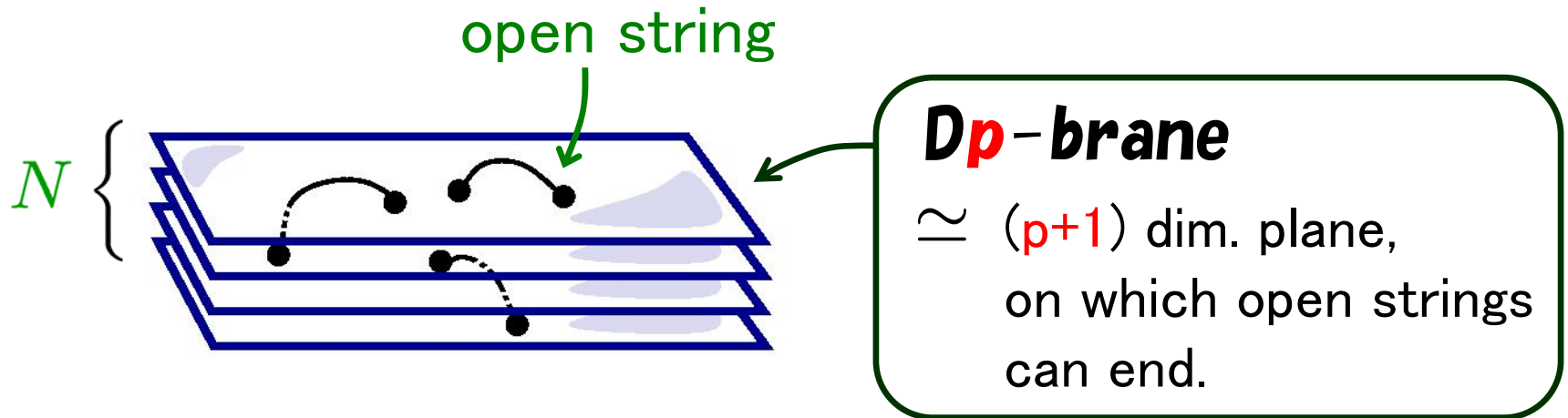
Claim :

Hadrons can be described by **string theory**
without using quarks !



2 Brief summary of the model

• D-brane and Gauge theory



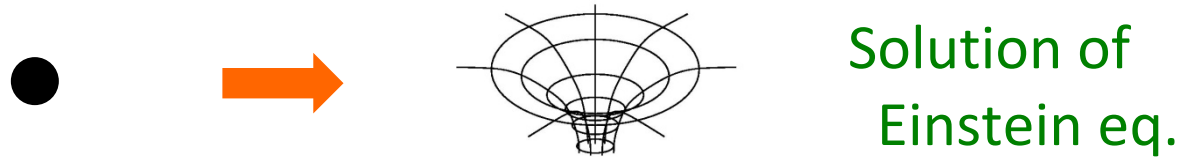
massless mode

$$\begin{array}{ccc} a \text{---} b & \longrightarrow & (A_\mu)^a_b \text{ etc.} \\ a, b = 1 \sim N & & U(N) \text{ gauge field} \end{array}$$

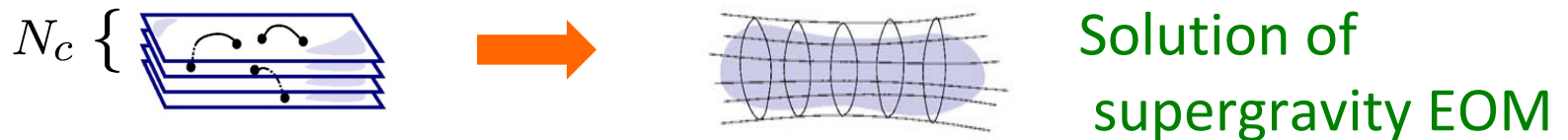
\Rightarrow $(p+1)$ dim. $U(N)$ gauge theory
is realized on the Dp -brane.

● *Gauge/string duality*

- In general relativity, a (heavy) particle is represented as a curved background



- In string theory, a D-brane system is represented as a curved background



$U(N_c)$ gauge theory \longleftrightarrow String theory in this background
dual

Recently, we proposed

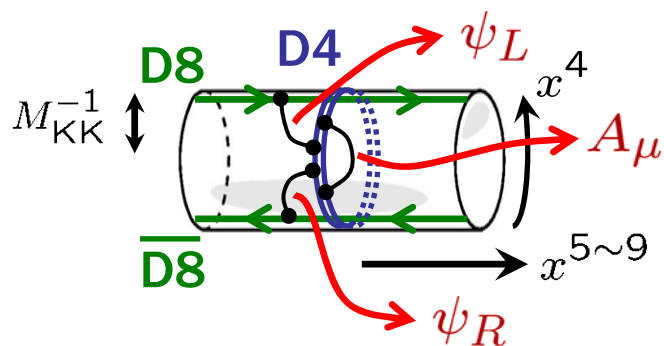
[Sakai-S.S. 2004]

Type IIA string theory
in Witten's D4 background
+ N_f Probe D8-branes
(assuming $N_c \gg N_f$)

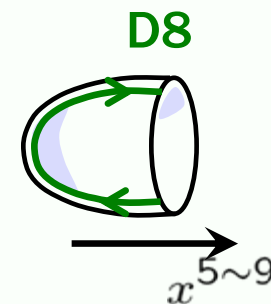
$\xleftrightarrow{\text{dual}}$ 4 dim QCD with
 N_f massless quarks
 \uparrow
at low energy

N_c $\overbrace{N_f \text{ pairs}}$
D4-D8- $\overline{\text{D8}}$ system
on S^1 (with ~~SUSY~~ b.c.)

QCD with N_f massless quarks
(at low energy)



String theory
in the D4 background
+ N_f probe D8-branes
(assuming $N_c \gg N_f$)



$\xleftrightarrow{\text{dual}}$

● Hadrons in the model

The topology of the D4 background is

$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

$x^\mu \quad (y, z)$

D8-branes are extended along $(x^\mu, z) \times S^4$

- Closed strings



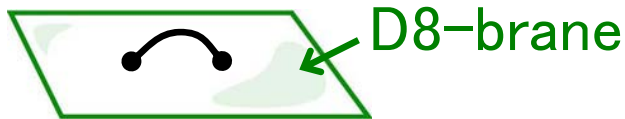
→ glueballs

[Csaki-Ooguri-Oz-Terning 1998,
Koch-Jevicki-Mihailescu-Nunes 1998,
A.Hashimoto-Oz 1998,
Brower-Mathur-Tan 2000, etc etc]

- Open strings on D8

→ mesons

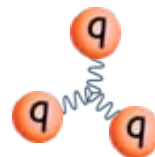
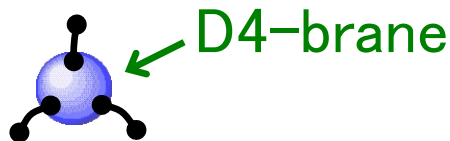
[Sakai-S.S. 2004,2005]



- D4 wrapped on S^4

→ baryons

[Witten, Gross-Ooguri 1998]

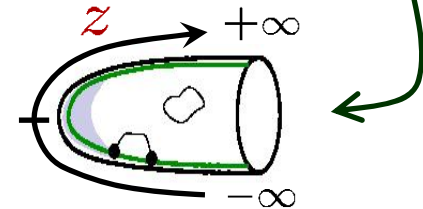


• The effective theory of mesons (open strings)

N_f D8-branes extended along $(x^\mu, z) \times S^4 \subset \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$

Low energy

9 dim $U(N_f)$ gauge theory



Reducing S^4 (Here we only consider $SO(5)$ invariant states)

5 dim $U(N_f)$ YM-CS theory

$A_\mu(x^\nu, z), A_z(x^\nu, z) \quad \mu, \nu = 0 \sim 3$
5 dim gauge field

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$$S_{\text{YM}} = \kappa \int d^4x dz \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \quad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c$ $h(z) = (1 + z^2)^{-1/3}$ $k(z) = 1 + z^2$ (CS5-form)

($M_{\text{KK}} = 1$ unit)

[See also, Son-Stephanov 2003]

● 5 dim YM-CS theory = 4 dim meson theory

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z)$$

complete sets

Chosen to diagonalize
kinetic & mass terms
of $B_\mu^{(n)}, \varphi^{(n)}$

$$\varphi^{(0)} \sim \text{pion} \quad B_\mu^{(1)} \sim \rho \text{ meson} \quad B_\mu^{(2)} \sim a_1 \text{ meson} \quad \dots$$



$$S_{5\text{dim}}(A) = S_{4\text{dim}}(\pi, \rho, a_1, \rho', a'_1, \dots)$$

● Reproduces old phenomenological models

- Vector meson dominance [Gell-Mann-Zachariasen 1961, Sakurai 1969]
- Gell-Mann Sharp Wagner model [Gell-Mann -Sharp-Wagner 1962]
- Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]

● Masses and couplings roughly agree with experiments.

● Quantitative tests

[Sakai-S.S. 2004, 2005]

(Our model vs Experiment)

Meson mass

mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

coupling

↑
input ($M_{KK} \simeq 949$ MeV)

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{KK}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{KK}^2$	0.164 GeV ²	0.121 GeV ²
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{KK}$	4.63 GeV	2.8 ~ 4.2 GeV

● Outline of Part II

Baryon = D4-brane wrapped on S^4

= **“instanton”** on D8-brane

Gauge config. with $\frac{1}{8\pi^2} \int_{\Sigma_4} \text{Tr } F \wedge F \neq 0$

4 dim space $\Sigma_4 \ni (\vec{x}, z)$

Quantization

Baryon spectrum, charge radii,
magnetic moments, form factor, etc.

5 minutes break



Part I

- ✓ 1 Introduction
- ✓ 2 Brief summary of the model

----- 5 minutes break -----

Part II

- 3 Baryons as instantons
- 4 Quantization
- 5 Currents
- 6 Exploration
- 7 Conclusion

3

Baryons as instantons

Baryon as wrapped D4-brane

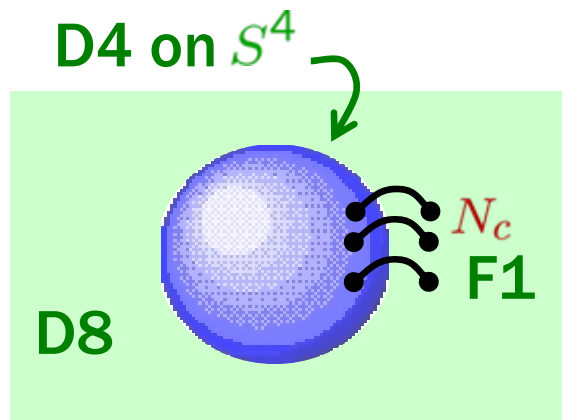
- Baryons in the AdS/CFT context are constructed by wrapped D-branes

[Witten 1998, Gross-Ooguri 1998]

In our case,

$$\text{Baryon} \simeq \text{D4-brane wrapped on the } S^4$$

- RR flux $\frac{1}{2\pi} \int_{S^4} dC_3 = N_c$ forces N_c F-strings to be attached on it.



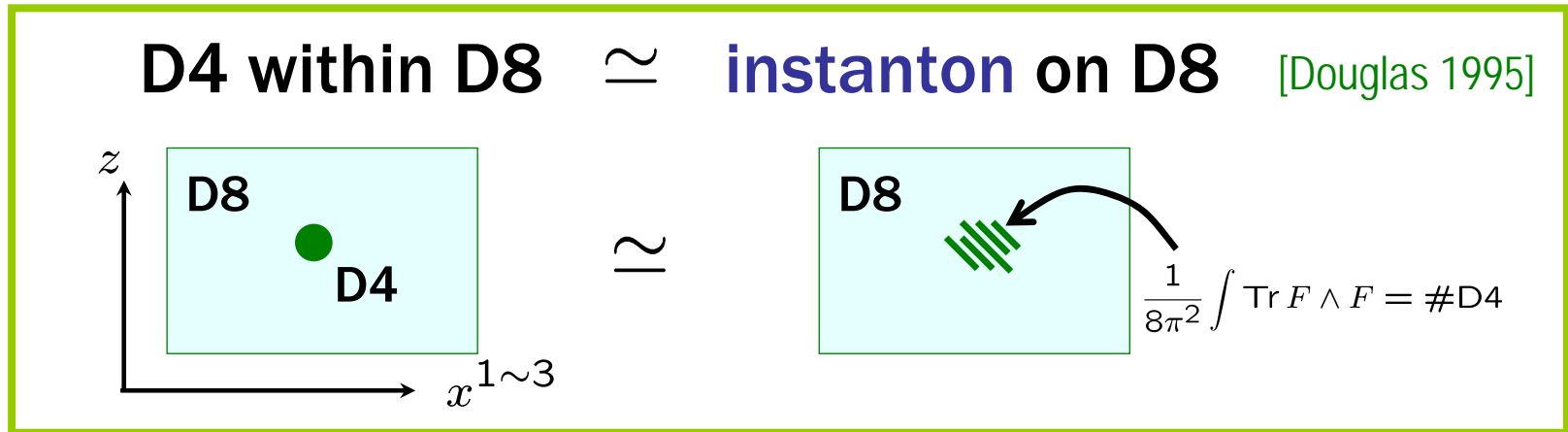
➡ Bound state of N_c quarks

➡ Baryon

Baryon mass (\propto vol. of S^4) is generated by the geometry!

• *Baryon as instanton*

- In our model, the wrapped D4 can be embedded in D8.



baryon #

Instanton #

$$N_B = \frac{1}{8\pi^2} \int \text{tr } F \wedge F$$

● **Prototype : Skyrme model**

- In 1961, Skyrme proposed

Baryons are solitons (**Skymion**) in a **pion** effective theory.

Pion field: $U(\vec{x}) \in U(N_f) \quad (\vec{x} \in \mathbb{R}^3) \quad \text{with} \quad U(\infty) = 1$

➡ classified by $\pi_3(U(N_f)) \simeq \mathbb{Z} \quad \leftarrow \# \text{ **baryon**}$

- In 1983, Adkins-Nappi-Witten (ANW)

succeeded to calculate the static properties of baryons
(mean square radii, magnetic moment, axial coupling, etc.)
by quantizing the collective modes of the Skymion.

➡ Roughly agree with the experimental data!

Q. Can we apply the idea of ANW to our system?

● **Classical solution** (We concentrate on the $N_f = 2$ case.)

- The instanton solution for the Yang-Mills action

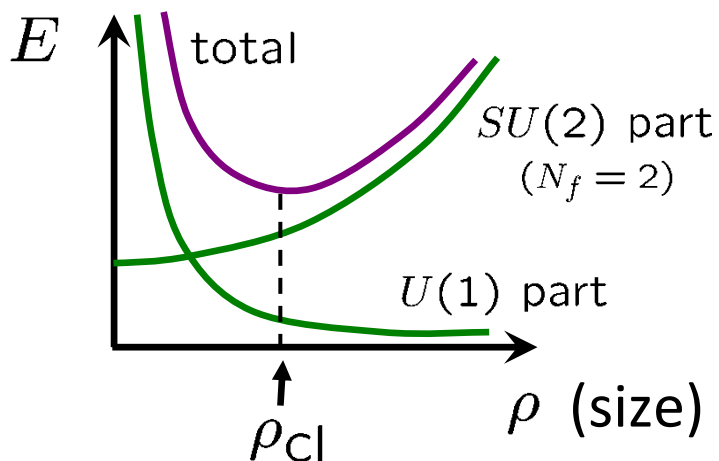
$$S_{\text{YM}} = \kappa \int d^4x dz \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right)$$

shrinks to **zero size** !

- The Chern-Simons term makes it larger ← $U(1)$ part

$$S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A) = \frac{N_c}{16\pi^2} \int d^4x dz \underbrace{A_0^{U(1)} \epsilon^{ijk} \text{Tr} F_{ij} F_{kz}}_{\substack{\uparrow \\ \text{Non-zero for instanton}}} + \dots$$

→ source of the $U(1)$ charge



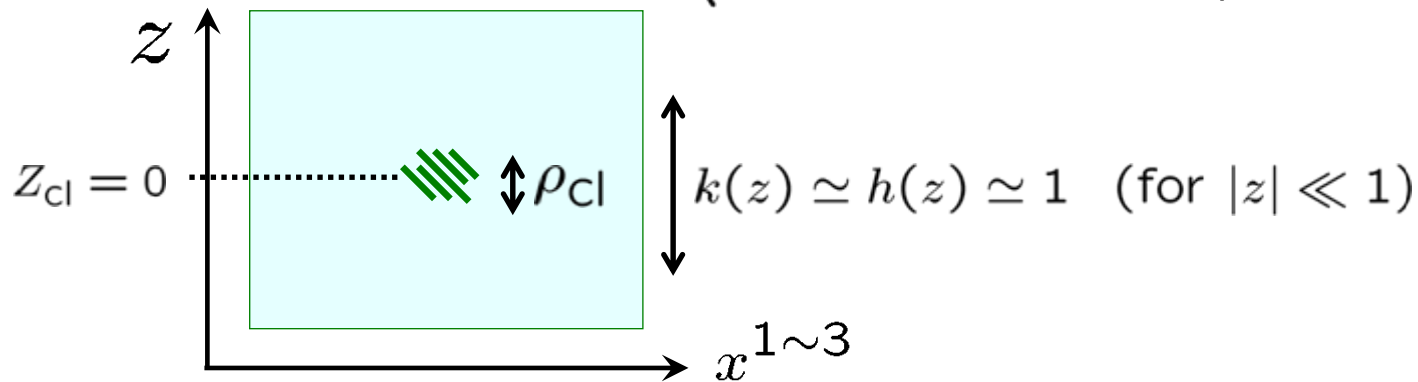
→ Stabilized at $\rho_{\text{CI}}^2 = \frac{N_c}{8\pi^2 \kappa} \sqrt{\frac{6}{5}}$

[Hong-Rho-Yee-Yi 2007]

[Hata-Sakai-S.S.-Yamato 2007]

● Note that $\rho_{\text{cl}} \sim \mathcal{O}(\lambda^{-1/2})$

If λ is large enough, the 5 dim space-time can be approximated by the flat space-time. $\left(\begin{array}{l} \text{The effect of the non-trivial } z\text{-dependence} \\ \text{is taken into account perturbatively.} \end{array} \right)$



➔ The leading order classical solution is the **BPST instanton** with $\rho = \rho_{\text{cl}}$ and $Z = Z_{\text{cl}} = 0$

$$A_M^{\text{cl}} = -i \frac{\xi^2}{\xi^2 + \rho^2} g \partial_M g^{-1} \quad g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi}$$

$$\xi = \sqrt{(\vec{x} - \vec{X})^2 + (z - Z)^2}$$

ρ : size (\vec{X}, Z) : position of the instanton

4 Quantization

[Hata-Sakai-S.S.-Yamato 2007]

- Consider a slowly moving (rotating) baryon configuration.

moduli space approximation method :

Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ ($\alpha = 1, 2, \dots, \dim \mathcal{M}$)

$$A_M(t, x) \sim A_M^{\text{cl}}(x; X^\alpha(t))$$

↑
time

$S_{5\text{dim}}$ \rightarrow Quantum Mechanics for $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}}_{\text{position}}, \underbrace{Z}_{\text{size}}, \underbrace{\rho}_{\text{SU(2) orientation}})\} \times SU(2)/\mathbf{Z}_2 \quad \mathbf{Z}_2 : a \rightarrow -a$$

$$\rightarrow L_{\text{QM}} = \frac{G_{\alpha\beta}}{2} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \quad U(X^\alpha) = 8\pi^2 \kappa \left(1 + \left(\frac{\rho^2}{6} + \frac{3^6 \pi^2}{5 \lambda^2 \rho^2} + \frac{Z^2}{3} \right) + \dots \right)$$

Note (\vec{X}, a) : genuine moduli (the same as in the Skyrme model)

(ρ, Z) : new degrees of freedom, added since they are light compared with the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics, we obtain the baryon states

→ Generalization of Adkins-Nappi-Witten including **vector mesons** and **ρ, Z modes**

We can construct baryon states for

$$n, p, \Delta(1232), N(1440), N(1530), \dots$$

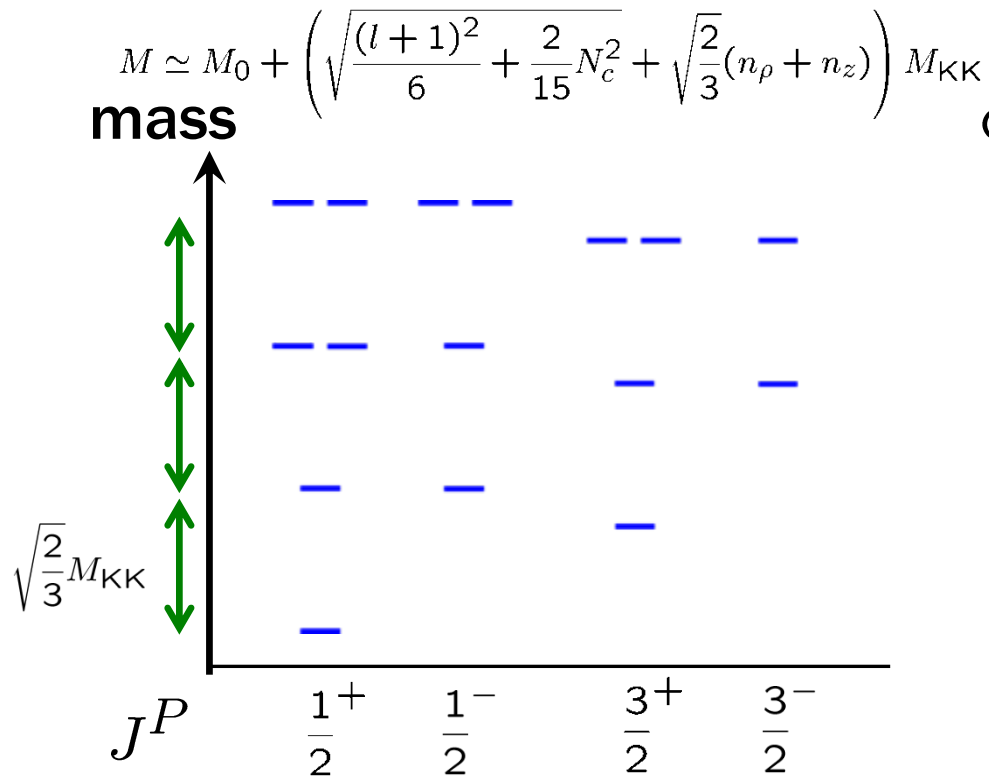
Example **Nucleon wave function:**

$$\psi(\vec{X}, \mathbf{a}, \rho, Z) \propto e^{i\vec{p} \cdot \vec{X}} R(\rho) \psi_Z(Z) T(\mathbf{a})$$

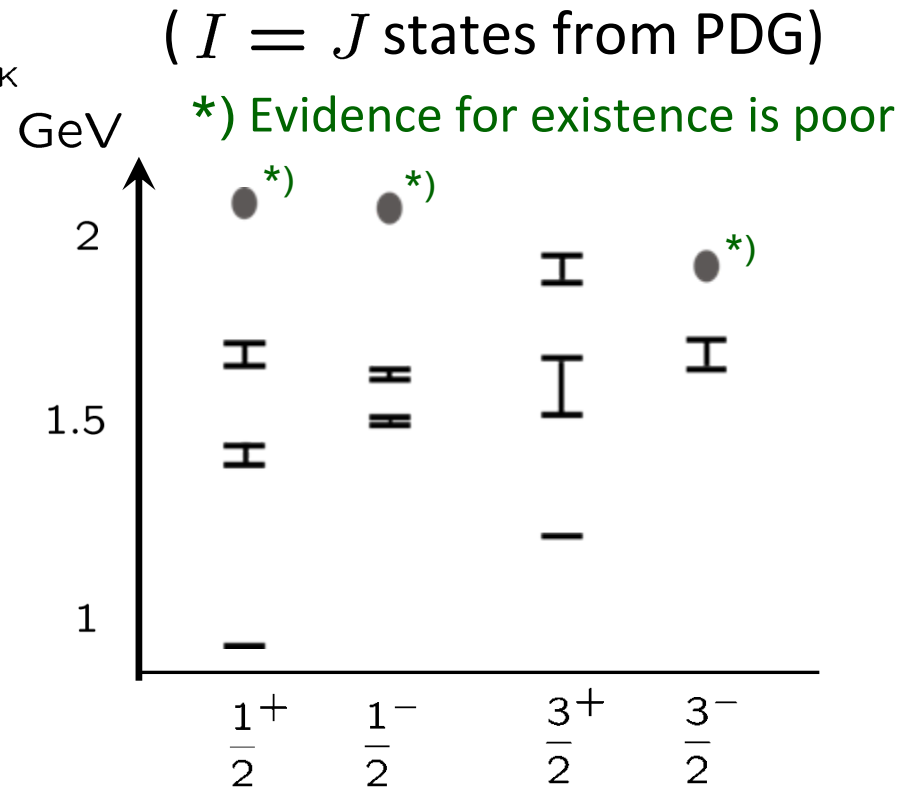
$$\left(\begin{array}{ll} R(\rho) = \rho^{\tilde{l}} e^{-A\rho^2} & \tilde{l} = -1 + 2\sqrt{1 + N_c^2/5} \\ \psi_Z(Z) = e^{-AZ^2} & A = \frac{8\pi^2\kappa}{\sqrt{6}} \\ T(\mathbf{a}) = a_1 + ia_2 \text{ for } |p \uparrow\rangle \text{ etc.} & \end{array} \right)$$

● Baryon spectrum

Theory



Experiment



- Note:
- We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known.
 - $M_{\text{KK}} \simeq 949$ MeV (fixed by ρ -meson mass) is a bit too large. It looks better if M_{KK} were around 500 MeV.

5 Currents

[Hashimoto-Sakai-S.S.2008]

[See also, Hata-Murata-Yamato 2008]

- Chiral symmetry

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\text{gauge}} (A_{L\mu}(x), A_{R\mu}(x))$$

- Interpreted as

$$A_{L\mu}(x) = \lim_{z \rightarrow +\infty} A_\mu(x, z) \quad A_{R\mu}(x) = \lim_{z \rightarrow -\infty} A_\mu(x, z)$$

$$\Rightarrow S_{5 \text{ dim}}|_{\mathcal{O}(A_L, A_R)} = - \int d^4x \left(A_{L\mu}^a J_L^{a\mu} + A_{R\mu}^a J_R^{a\mu} \right)$$

with

$$J_{L\mu} = -\kappa (k(z) F_{\mu z}) \Big|_{z=+\infty} \quad J_{R\mu} = +\kappa (k(z) F_{\mu z}) \Big|_{z=-\infty}$$

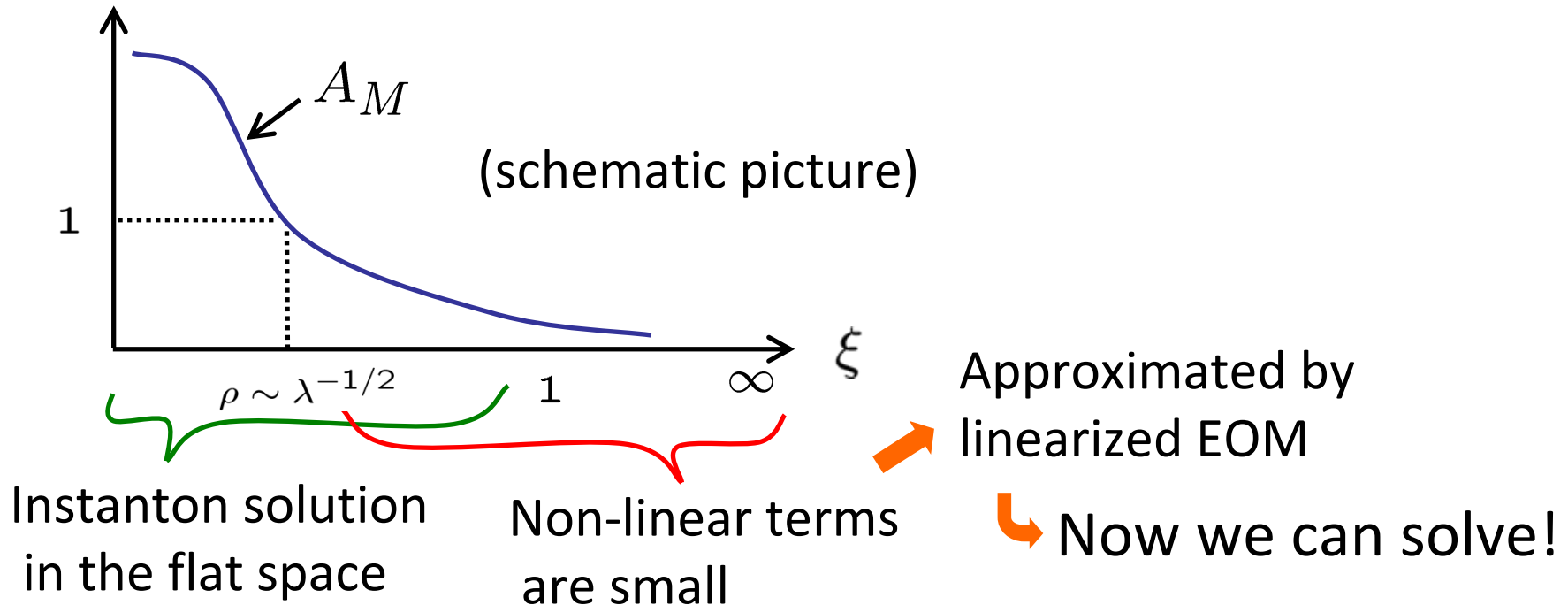
- vector and axial vector currents

$$J_V^\mu \equiv J_L^\mu + J_R^\mu = -\kappa \left[k(z) F^{\mu z} \right]_{z=-\infty}^{z=+\infty}$$

$$J_A^\mu \equiv J_L^\mu - J_R^\mu = -\kappa \left[\psi_0(z) k(z) F^{\mu z} \right]_{z=-\infty}^{z=+\infty} \quad (\psi_0(\pm\infty) = \pm 1)$$

● How to calculate

- We need to know how $F_{\mu z}(x, z)$ behaves at $z \rightarrow \pm\infty$
 - ➔ We cannot use the solution in the flat space.
- The EOM are complicated non-linear equations.
 - ➔ difficult to solve exactly.
- We use the following trick to calculate the currents.



6 Exploration

[Hashimoto-Sakai-S.S.2008]

[See also, Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008,
Kim-Zahed 2008]

Now we are ready to calculate various physical quantities

But, don't trust too much !

- λ may not be large enough.
- Higher derivative terms may contribute.
- $N_c = 3$ is not large enough.
- The model deviates from real QCD at high energy $\sim M_{KK}$
- We use $M_{KK} \simeq 949$ MeV (value consistent with ρ meson mass)
But we know this is too large to fit the baryon mass differences.

• Baryon number current

$$J_B^\mu = -\frac{2}{N_c} \kappa \left[k(z) F_{U(1)}^{\mu z} \right]_{z=-\infty}^{z=+\infty}$$

← U(1) part of the U(2) gauge field



$$J_B^0 \simeq \left[k(z) \partial_z G \right]_{z=-\infty}^{z=+\infty} \quad J_B^i \simeq -\frac{J^j}{16\pi^2 \kappa} \epsilon^{ijk} \partial_k J_B^0 + \dots$$

$$\left(\begin{array}{ll} G : \text{Green's function} & (h(z)\partial_i^2 + \partial_z k(z)\partial)G = \delta^3(\vec{x} - \vec{X})\delta(z - Z) \\ J^j : \text{Spin operator} & J^j = -i4\pi^2 \kappa \rho^2 \text{tr}(\tau^j \mathbf{a}^{-1} \dot{\mathbf{a}}) \end{array} \right)$$

Note: $k(z) \sim z^2$, $\partial_z G \sim 1/z^2$ at $z \rightarrow \pm\infty$

→ J_B^μ is non-zero, finite.

• Isoscalar mean square radius

$$\langle r^2 \rangle_{I=0} = \int d^3x r^2 J_B^0 \simeq (0.742 \text{ fm})^2$$

Numerical estimate using $M_{\text{KK}} \simeq 949 \text{ MeV}$
(fixed by ρ -meson mass)

$$\left(\text{cf. } \langle r^2 \rangle_{I=0}^{1/2} |_{\text{exp}} = 0.806 \text{ fm}, \langle r^2 \rangle_{I=0}^{1/2} |_{\text{ANW}} = 0.59 \text{ fm} \right)$$

Results for the Skyrme model
(Adkins-Nappi-Witten 1983)

● Isoscalar magnetic moment

$$\mu_{I=0}^i = \frac{1}{2} \epsilon^{ijk} \int d^3x x^j J_B^k \simeq \frac{J^i}{16\pi^2\kappa} \quad J_B^i \simeq -\frac{J^j}{16\pi^2\kappa} \epsilon^{ijk} \partial_k J_B^0 + \dots$$

For a spin up proton state $|p \uparrow\rangle$

$$\langle p \uparrow | \mu_{I=0}^i | p \uparrow \rangle = \frac{\delta^{i3}}{32\pi^2\kappa} \equiv \frac{g_{I=0}}{4M_N} \delta^{i3}$$

Isoscalar g-factor

Nucleon mass
($M_N \simeq 940 \text{ MeV}$)

$$\Rightarrow g_{I=0} = \frac{M_N}{8\pi^2\kappa M_{\text{KK}}} \simeq 1.68$$

$M_{\text{KK}} \simeq 949 \text{ MeV}$, $\kappa \simeq 0.00745$
(fixed by m_ρ) (fixed by f_π)

$$\left(\text{cf. } g_{I=0}|_{\text{exp}} \simeq 1.76, g_{I=0}|_{\text{ANW}} = 1.11 \right)$$

Summary of the results

	our result	exp.	ANW
$\langle r^2 \rangle_{I=0}^{1/2}$	0.742 fm	0.806 fm	0.59 fm
$\langle r^2 \rangle_{I=1}^{1/2}$	0.742 fm	0.939 fm	∞ ✖
$\langle r^2 \rangle_A^{1/2}$	0.537 fm	0.674 fm	—
$g_{I=0}$	1.68	1.76	1.11
$g_{I=1}$	7.03	9.41	6.38
g_A	0.734	1.27	0.61

✖ pion loop contribution is log divergent in the chiral limit.
Our calculation corresponds to the tree level in ChPT.

- We can also evaluate these for excited baryons such as $\Delta(1232)$, $N(1440)$, $N(1535)$, ...

● Form factors

Dirac form factor

Pauli form factor

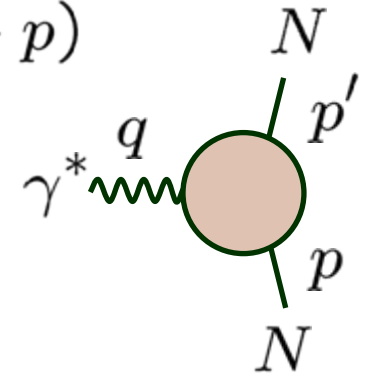
$$\langle N, \vec{p}' | J_{\text{em}}^\mu(0) | N, \vec{p} \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_N} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p, s)$$

Breit frame: $\vec{p}' = -\vec{p} = \vec{q}/2$

$$(q = p' - p)$$

$$\langle N, \vec{q}/2 | J_{\text{em}}^0(0) | N, -\vec{q}/2 \rangle = G_E(\vec{q}^2) \chi_{s'}^\dagger \chi_s$$

$$\langle N, \vec{q}/2 | J_{\text{em}}^i(0) | N, -\vec{q}/2 \rangle = \frac{i}{2m_N} G_M(\vec{q}^2) \chi_{s'}^\dagger (\vec{q} \times \vec{\sigma}) \chi_s$$



Sachs form factor

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2} F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

Electric form factor

Magnetic form factor

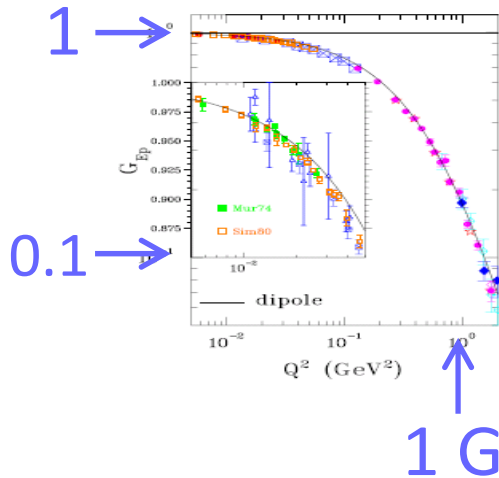
● Dipole behavior

Experimental data suggest

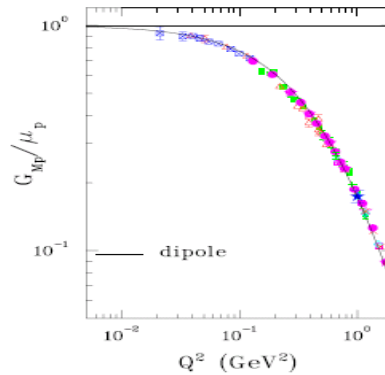
dipole ($\Lambda \simeq 0.71 \text{ GeV}^2$)

$$G_E^p(Q^2) \simeq \frac{1}{\mu_p} G_M^p(Q^2) \simeq \frac{1}{\mu_n} G_M^n(Q^2) \simeq \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \quad G_E^n(Q^2) \simeq 0$$

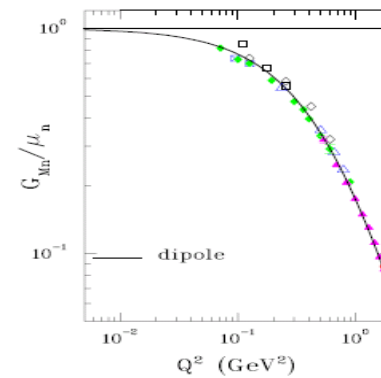
G_E^p



G_M^p / μ_p

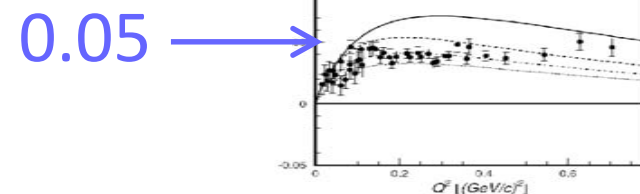


G_M^n / μ_n



— : dipole
dots : data

G_E^n

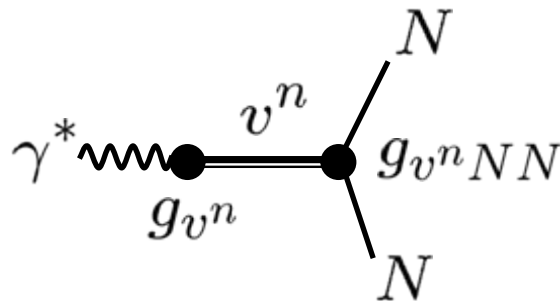


● Our result

$$G_E^p(Q^2) = \frac{1}{\mu_p} G_M^p(Q^2) = \frac{1}{\mu_n} G_M^n(Q^2) = \sum_{n \geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2} \quad G_E^n(Q^2) = 0$$

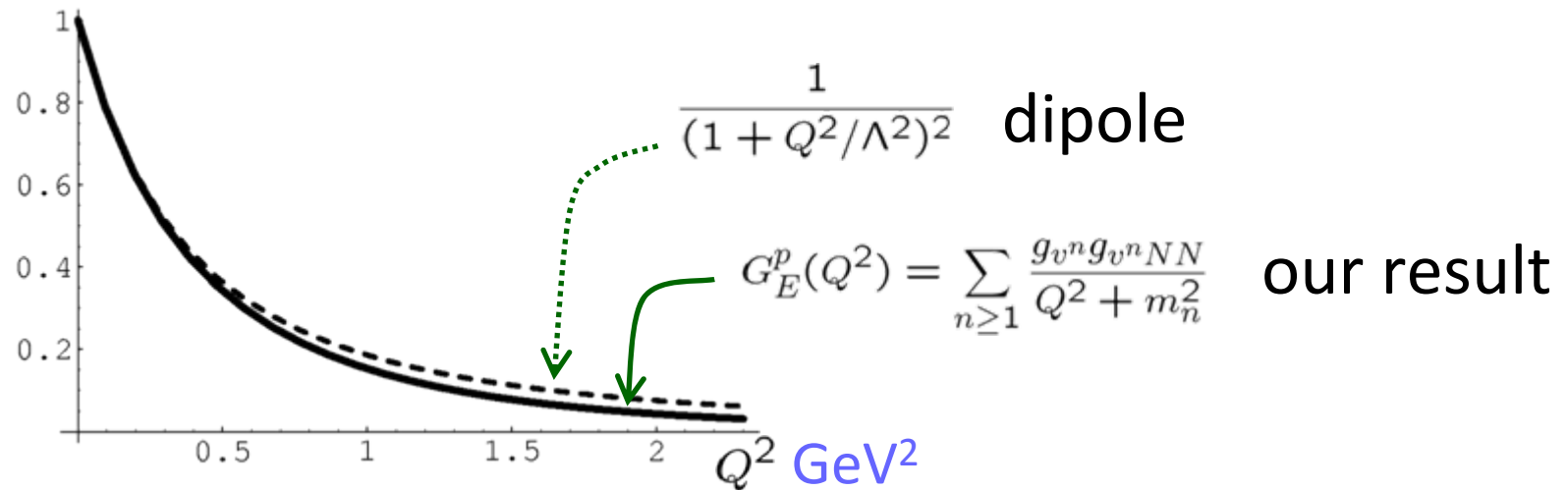
with $g_{v^n} = -2\kappa(k(z)\partial_z \psi_{2n-1}) \Big|_{z=+\infty}$

$$g_{v^n NN} = \langle \psi_{2n-1}(Z) \rangle$$



Vector meson dominance

- Can this be compatible with dipole?



- Taylor expansion

$$\left(\begin{array}{l} \text{✖ here we use the approximation} \\ g_{v^n} NN = \langle \psi_{2n-1}(Z) \rangle \simeq \psi_{2n-1}(0) \end{array} \right)$$

$$G_E^p(Q^2) \simeq 1 - 2.38Q^2 + 4.02(Q^2)^2 - 6.20(Q^2)^3 + 9.35(Q^2)^4 - 14.0(Q^2)^5 + \dots$$

$$\frac{1}{(1 + Q^2/\Lambda^2)^2} \simeq 1 - 2.38Q^2 + 4.24(Q^2)^2 - 6.71(Q^2)^3 + 9.97(Q^2)^4 - 14.2(Q^2)^5 + \dots$$

$$\text{with } \Lambda^2 = 0.758 \text{ GeV}^2$$

$$(M_{KK} = 1 \text{ unit})$$

5 Conclusion

- We proposed a new method to analyze static properties of baryons.
- Our model automatically includes the contributions from various massive vector and axial-vector mesons.
- Compared with the similar analysis in the Skyrme model (ANW), the agreement with the experimental values are improved in most of the cases.
- But, we should keep in mind that our analysis is still very crude and there are a lot of ambiguities remain unsolved.