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Deconfinement transition as black hole formation by the condensation of QCD strings

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We argue that the deconfinement transition of large-N Yang-Mills theory is the condensation of very long and self-intersecting chromo-electric flux strings (QCD string), which is analogous to the formation of a black hole in string theory. We do this by using lattice gauge theory and matrix models. As evidence, we derive an analytic formula for the deconfinement temperature in the strong coupling limit and confirm it numerically. Dual gravity descriptions interpreted in this manner should make it possible to understand the details of the formation of black holes in terms of fundamental strings. We argue that very simple matrix models capture the essence of the formation of black holes.

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INTRODUCTION

In the gauge/gravity duality [1], the deconfinement transition of the gauge theory is dual to the formation of a black hole in the gravity side [2]. In this paper, we give an intuitive way of understanding this correspondence, without referring to a sophisticated dictionary of the duality. Our argument does not assume the dual gravity description, and hence it is applicable to generic gauge theories including QCD. We pay attention to the stringy degrees of freedom in gauge theory – the Wilson loops – and see how their behaviors change across the deconfinement transition. The authors believe that this result will be useful in understanding the formation and thermalization of the quark-gluon plasma in QCD, and the formation of a black hole in the gravitational picture, from a unified perspective.

Although our argument applies to rather generic gauge theories and matrix models, our initial motivation comes from a simple matrix model for the black hole [3, 4] to study the duality [2] between the deconfinement transition in 4d $\mathcal{N}=4$ super Yang-Mills theory on a three-sphere and the Hawking-Page transition [5] of a black hole in the AdS space. At the Hawking-Page transition, a small black hole whose Schwarzschild radius is of order AdS radius is formed. Such a small black hole can naturally be identified with a long and winding string [7, 8]. One of the authors (L. S.) has argued that it can be modeled by using a lattice gauge theory with a continuum time and a few spatial lattice sites, e.g. a tetrahedron (Fig. 1) sitting at the center of the AdS space, by identifying the string and chromo-electric flux string. Our result in this paper justifies this argument.

As a concrete example, let us consider pure $U(N)$ Yang-Mills theory. The Hilbert space is spanned by Wilson loops acting on the vacuum $|0\rangle$, $W_C W_{C'}\cdots|0\rangle$, where W_C represent the Wilson loop along a closed contour C.

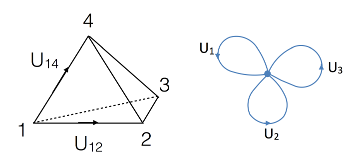

FIG. 1. Tetrahedron matrix model (left) and single-site model (Eguchi-Kawai model) with three links (right).

In the standard identification of the Feynman diagrams and the string world-sheet [6], the Wilson loop is naturally interpreted as the creation operator of the string. Therefore, $W_{C_1}W_{C_2}\cdots W_{C_k}|0\rangle$ is the state which consists of k strings, and $C_1, C_2, \cdots, C_k$ represent the shapes of the strings.

In the large-N limit, the energy of the string is approximated by its length. In the confinement phase, the energy is of order N^0 per unit volume, and hence a typical state is a finite-density gas of loops with finite length. In other words, it is gas of glue-balls. In this gas, two loops can intersect with each other and combined to form a longer string. Alternatively, when a loop intersects with itself, it can be split into two shorter loops. However such joining and splitting are suppressed at large-N.

In the deconfinement phase, the energy density is of order N^2. In this phase the loops necessarily intersect $O(N^2)$ times. Although the interaction at each intersection is $1/N$-suppressed, small interactions at many intersections accumulate to a non-negligible amount. As we will explain shortly, a typical state consists of finitely many very long and self-intersecting strings, whose lengths are of $O(N^2)$. In the string theory, it is natural to interpret such very long and self-intersecting strings as a black hole [7, 8]. In this sense, *the deconfinement transition can be understood as the formation of a 'black hole' through condensation of QCD strings.*

In this paper, we establish this picture by using lattice gauge theory and matrix models as concrete examples. Our interpretation does not rely on details of the theory and hence is applicable to any gauge theory. When it is applied to theories with gravity duals, the 'analogy' turns to an equivalence. That is to say that the 'QCD string' is naturally identified with the fundamental string, and their condensation is equivalent to the formation of an actual black hole in superstring theory. Therefore we believe our picture is useful for understanding the black hole from the degrees of freedom of the fundamental string.

QUANTITATIVE ARGUMENT ON LATTICE

Let us consider $(D+1)$-dimensional $U(N)$ Yang-Mills lattice gauge theory in the Hamiltonian formulation [9]. (We consider $D \geq 2$, which goes through a first-order deconfinement transition as the temperature is raised.) The time direction is continuous, and D-dimensional space is regularized by a D-dimensional square lattice. The Hamiltonian is given by the kinetic term (or electric term) K and the potential term (magnetic term) V respectively, $H = K + V$, where

$$K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}}\right)^2 \tag{1}$$

and

$$V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}})\right). \tag{2}$$

Here $U_{\mu}(\vec{x})$ is the unitary link variable connecting $\vec{x}$ and $\vec{x}+\hat{\mu}$, where $\hat{\mu}$ is the unit vector along μ-direction. $E^{\alpha}_{\mu,\vec{x}}$ is defined via its commutation relation which is given by,

$$\begin{aligned} &[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu}\delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}}, \\ &[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U^{\dagger}_{\nu,\vec{y}}] = 0. \end{aligned} \tag{3}$$

τ_{α} $(\alpha = 1, 2, \cdots, N^2)$ are generators of the $U(N)$ group, which are $N \times N$ Hermitian matrices normalized as $\sum_{\alpha=1}^{N^2} \tau^{\alpha}_{ij}\tau^{\alpha}_{kl} = \delta_{il}\delta_{jk}/N^2$. The vacuum $|0\rangle$ is annihilated by $E^{\alpha}_{\mu,\vec{x}}$, $E^{\alpha}_{\mu,\vec{x}}|0\rangle = 0$.

The Hilbert space consists of gauge-invariant states, in which Wilson loops acting on $|0\rangle$ form an over-complete basis. In the lattice gauge theory, the Wilson loop is the trace of the product of link variables along a contour C, $W_C = Tr\,(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}-\hat{\rho}})$. The Wilson loop sources a set of interacting gluons. The correspondence between the Feynman diagram expansion and the string world-sheet leads us to interpret the Wilson loop as the creation operator of the closed string.

Let us say the Wilson loop is not self-intersecting when each link variable appears only once. Similarly, two loops W_C and $W_{C'}$ do not intersect when they do not share the same link variable. (With this definition, even if two loops go through the same point $\vec{x}$, they do not necessarily 'intersect'. Although this definition may look strange at first glance, this is actually a natural generalization of the intersection between strings in continuum space, because two strings represented by the Wilson loops on the lattice can interact only when they share the same link, as we will see shortly.)

When the loop W_C does not self-intersect, the electric term K acts on the state $|W_C\rangle \equiv W_C|0\rangle$ as

$$K|W_C\rangle = \frac{\lambda N}{2} \sum_{\mu,\vec{x},\alpha} \left[E^{\alpha}_{\mu,\vec{x}}, \left[E^{\alpha}_{\mu,\vec{x}}, W_C\right]\right]|0\rangle = \frac{\lambda L}{2}|W_C\rangle. \tag{4}$$

where L is the length of the contour C in the lattice unit. In the same manner, for a multi-loop state $|W_C, W_{C'}, \cdots\rangle = W_C W_{C'} \cdots |0\rangle$, K acts as $K|W_C, W_{C'}, \cdots\rangle = \frac{\lambda}{2}(L + L' + \cdots)|W_C, W_{C'}, \cdots\rangle$ when there is no intersection.

When two loops W_C and $W_{C'}$ intersect once by sharing $U_{\mu,\vec{x}}$, they can be joined to form a longer loop as follows. Let us write W_C and $W_{C'}$ as $W_C = \mathrm{Tr}(U_{\mu,\vec{x}} M_C)$ and $W_{C'} = \mathrm{Tr}(U_{\mu,\vec{x}} M_{C'})$, where M_C and $M_{C'}$ are product of other link variables. Then,

$$\begin{aligned} &K|W_C, W_{C'}\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle \\ &\quad + \lambda N \sum_{\alpha} \mathrm{Tr}(\tau^{\alpha} U_{\mu,\vec{x}} M_C) \cdot \mathrm{Tr}(\tau^{\alpha} U_{\mu,\vec{x}} M_{C'})|0\rangle \\ &= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle + \frac{\lambda}{N}\mathrm{Tr}(U_{\mu,\vec{x}} M_C U_{\mu,\vec{x}} M_{C'})|0\rangle. \end{aligned} \tag{5}$$

The second term in (5) is a longer string whose length is $L + L'$. The state is then a superposition of a state containing two loops of length L and L' and a state of a single long string of length $L + L'$. In the same manner, a self-intersecting string can be split into two strings. If there are multiple intersections, such joinings and splittings take place at all intersections.

In this manner, the electric term K measures the total length of the strings, and also joins and splits strings. The magnetic term V is the plaquette, which is the smallest possible Wilson loop; this term adds a very short string (one plaquette) to the states.

Confinement phase as gas of free strings

In the confining phase, the energy density is of order N^0. Therefore, the length of the strings is also of order N^0. In this phase joining and splitting coming from the electric term is negligible at large-N, because there are only $O(N^0)$ intersections, and interaction at each intersection is $1/N$-suppressed. The confinement phase can therefore be understood as a gas of non-interacting strings. (The left of Fig. 2)

Deconfinement phase as black hole

In the deconfinement phase, the energy density is of order N^2. Therefore the total length of the strings is also of order N^2. The strings must intersect $O(N^2)$ times, and then the $1/N$-suppressed interaction at each intersection accumulates and becomes non-negligible (Fig. 2). Strings are joining, splitting, as well as changing their shapes very rapidly in this phase. This is true even in the strong coupling limit $\lambda = \infty$, where the magnetic term V vanishes. *The deconfinement phase is highly dynamical, unlike the common folklore stating that "the strong coupling limit is free string theory.*

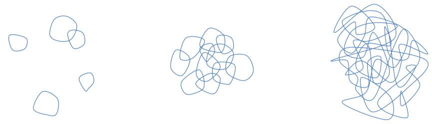

FIG. 2. The intuitive picture of the deconfinement transition in terms of QCD strings. In the confining phase, the strings do not intersect often, interaction between them is negligible, and the free-string picture is valid (left). As the system is heated up, more strings are excited, they intersect more often (middle). When the density of strings becomes large, the interaction becomes non-negligible and a long string is formed (right). This is the same as the formation of a black hole in string theory.

There are many string states with total length $L_{total} \sim N^2$. One extreme, are cases with configurations containing a single long string, while the other extreme, are states only containing the $O(N^2)$ shortest strings (plaquettes). A single long string can possess a lot of states; after $U_{\mu,\vec{x}}$, one can multiply any link variables originating from the point $\vec{x} + \hat{\mu}$ except $U^\dagger_{\mu,\vec{x}}$, and hence at each point there are $2D-1$ choices, and the entropy is roughly $\log((2D-1)^{L_{total}}) = L_{total}\log(2D-1) \sim N^2$. The states which consist of several long strings can carry the same amount of entropy. On the other hand, if we consider a bunch of very short strings, say a bunch of plaquetts, then the entropy is much smaller, because the state can be specified by the number of each type of plaquette. Therefore, it is natural to conclude that typical states contain a few long strings. In string theory, this is exactly how the black hole (or more precisely a D-dimensional black brane) is formed from the fundamental string.

In order to justify this picture, we derive the following analytic predictions, and then confirm them numerically. Let us consider the strong coupling limit ($\lambda = \infty$, $V = 0$). Then the energy of the deconfinement phase is proportional to the total length of the strings $L_{total}(T)$, which is an unknown function of the temperature T, because the electric term measures the length:

$$E = K = \frac{\lambda}{2} L_{total}(T). \tag{6}$$

The entropy S is also proportional to $L_{total}(T)$,

$$S = L_{total}\log(2D-1). \tag{7}$$

Therefore the free energy $F = E - TS$ is given by

$$F = L_{total}(T)\left(\frac{\lambda}{2} - T\log(2D-1)\right). \tag{8}$$

The free energy of the confinement phase is zero up to a $1/N$ correction, and hence the confinement phase is favored when (8) is positive and the deconfinement transition takes place when (8) crosses zero. Therefore, although we do not know the T-dependence of the length L_{total}, we can easily determine the critical temperature T_c,

$$T_c = \lambda/(2\log(2D-1)). \tag{9}$$

Note that this derivation is formally the same as the derivation of the Hagedorn temperature in free string theory on a lattice. See e.g. [10].

Matrix models

Strictly speaking, the $(D+1)$-dimensional Yang-Mills theory is analogous to a D-dimensional black brane rather than a black hole, because the condensation of the strings fills entire D-dimensional space. In order to describe a black hole (0-brane), let us consider the dimensionally reduced D-matrix model $H = K + V$, where $K = \frac{\lambda N}{2}\sum_\mu \sum_{\alpha=1}^{N^2}\left(E^\alpha_\mu\right)^2$ is the kinetic term (or electric term) and $V = \frac{N}{\lambda}\sum_{\mu\neq\nu}\left(N - \mathrm{Tr}(U_\mu U_\nu U^\dagger_\mu U^\dagger_\nu)\right)$. Now the space is reduced to a single point, and D link variables are attached to that point (see Fig. 1 for the case of $D = 3$). This is the Eguchi-Kawai model [11] with a continuous time direction. At strong coupling the $U(1)^D$ center symmetry along the spatial directions, $U_\mu \to e^{i\theta_\mu}U_\mu$, is not spontaneously broken. Then this theory is known to be equivalent to the $(D+1)$-dimensional theory at large-N in the sense that translationally invariant observables in the latter, for example the energy density and entropy density, are reproduced from the former to the leading order in the $1/N$-expansion [11]. At weak coupling, this model is analogous to the bosonic part of the matrix model of M-theory [12], which is dual to the black zero-brane in type IIA supergravity in the 't Hooft large-N limit [13]. For $D \geq 2$, this theory exhibits the deconfinement transition, which is characterized by the non-vanishing expectation value of the Polyakov loop. At sufficiently strong coupling, the transition is of first order. In the deconfinement phase, the energy and the entropy is of order N^2, and typical state should be described by long, winding strings such as $Tr(U_1U_2U_1^\dagger U_1^\dagger U_2U_2U_1\cdots)$. All of these arguments parallel the case of the $(D+1)$-dimensional lattice, and (9) should hold in this case as well.

We can also consider other matrix models such as the tetrahedron model (Fig. 1). In this model, space consists of four points labelled by $1, 2, 3$, and 4. There are six link variables U_{mn} $(m \neq n)$ which satisfy $U_{mn}^\dagger = U_{nm}$. We have shown numerically that this model also possesses a first-order deconfinement transition. The long string can be described with a term such as $Tr(U_{12}U_{23}U_{31}U_{14}U_{42}\cdots)$. The entropy is $S = L_{total}\log 2$, and (9) is modified as $T_c = \lambda/(2\log 2)$.

NUMERICAL CONFIRMATION

In this section we confirm the analytic predictions by lattice Monte Carlo simulation. There exists a wide literature on Monte Carlo algorithms and a nice review can be found in [14]. Thanks to the Eguchi-Kawai equivalence [11], we do not have to study the $(D+1)$-dimensional YM theory, and hence we consider the matrix models. In order to study the thermodynamics, we consider the theory in Euclidean time, and compactify the time direction to a circle with circumference $\beta = 1/T$. The Lagrangian in Euclidean signature is given by $L = K + V$, where V is the same as before and $K = \frac{N}{2\lambda}\sum_{\vec{x}} \mathrm{Tr}\left(D_tU_{\vec{x}}\cdot(D_tU_{\vec{x}})^\dagger\right)$ with $D_tU_{\vec{x}} = \partial_tU_{\vec{x}} - i[A_t, U_{\vec{x}}]$. We regularize the model by introducing n_t lattice sites along the temporal direction. The lattice spacing a is given by $an_t = \beta$. The action is

$$S_{lattice} = -\frac{N}{2a\lambda}\sum_{\mu,t}\mathrm{Tr}\left(V_tU_{\mu,t+a}V_{\mu,t}^\dagger U_{\mu,t} + c.c.\right) + \frac{aN}{\lambda}\sum_{\mu\neq\nu,t}\left(N - \mathrm{Tr}(U_{\mu,t}U_{\nu,t}U_{\mu,t}^\dagger U_{\nu,t}^\dagger)\right) \quad (10)$$

where $t = a, 2a, \cdots, n_ta$, and $t = n_ta$ is identified to $t = 0$. V_t is the unitary link variable connecting t and $t+a$, and $U_{\mu,t}$ are spatial links at time t. The tetrahedron matrix model can be regularized in a similar manner with action

$$S_{tet} = -\frac{N}{2a\lambda}\sum_t\sum_{m<n}\left(\mathrm{Tr}(V_{m,t}U_{mn,t+a}V_{n,t}^\dagger U_{nm,t}) + c.c.\right) - \frac{aN}{\lambda}\sum_t\sum_{l<m<n}\left((N - \mathrm{Tr}(U_{lm,t}U_{mn,t}U_{nl,t})) + c.c.\right). \quad (11)$$

In the following we concentrate on the strong coupling limit, where the magnetic terms are omitted. The order parameter for the deconfinement transition is the Polyakov loop, $P = \frac{1}{N}\mathrm{Tr}(V_{t=a}V_{t=2a}\cdots V_{t=n_ta})$. For the tetrahedron matrix model the loop is $P_{tet} = \frac{1}{4N}\sum_{m=1}^4 \mathrm{Tr}(V_{m,t=a}V_{m,t=2a}\cdots V_{m,t=n_ta})$. The expectation value of P itself vanishes trivially due to the $U(1)$ phase. Therefore we consider the expectation value of the absolute value, $\langle|P|\rangle$. We performed hot and cold start simulations, in which the temperature is gradually decreased and increased, respectively. In Fig. 3, $\langle|P|\rangle$ in the tetrahedron model is plotted. We can see a clear hysteresis at $0.66 \leq (T/\lambda) \leq 0.74$, which means the transition is of first order as expected. The theoretically predicted critical temperature $(T_c/\lambda) = 1/(2\log 2) \simeq 0.721$ is in this range. We observed similar hystereses also in the Eguchi-Kawai model with various values of D. In the right panel of Fig. 3 we plot the temperature range of the hystereses for $D = 2, 3, \cdots, 10$. We can see that the theoretical predictions for T_c is consistent with the simulation results. Note that T_c goes close to the lower edge of the hysteresis as D becomes large; it would be interesting we can understand the mechanism behind it.

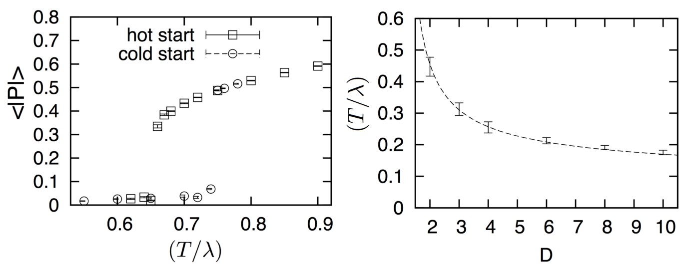

FIG. 3. (Left)The expectation value of the Polyakov loop in the tetrahedron matrix model in the strong coupling limit. There is a strong hysteresis around the theoretically predicted critical temperature $(T_c/\lambda) = 1/(2\log 2) \simeq 0.721$. $N = 64, n_t = 12$. The temperature range of the hystereses of the Eguchi-Kawai model. (Right)The dashed line is the analytic prediction for the critical temperature, $(T_c/\lambda) = 1/(2\log(2D-1))$. $N = 64, n_t = 16$.

CONCLUSION AND DISCUSSION

In this paper we have visualized the relationship between the deconfinement phase of the large-N Yang-Mills theory and a black hole, by paying attention to the stringy degrees of freedom (i.e. the Wilson loops) in Yang-Mills theory. The essence is contained already in the strong coupling limit of the lattice gauge theory. The common folklore that this theory describes free strings is only correct in the confinement phase. The deconfinement phase is dominated by very dynamical self-intersecting strings which should be understood as a black hole, unlike the free case.

In the gauge theories with dual gravity descriptions, the Wilson loops are naturally identified with the fundamental strings. Therefore, the condensation of the

Wilson loops argued in this paper is equivalent to the condensation of fundamental strings and formation of an actual black hole. By following the time evolution of the loops, it should be possible to see how a black hole forms, thermalizes, and eventually evaporates. As we have seen, qualitative features are common even in the strong coupling limit of simple matrix models, and hence they should serve as good toy models for a black hole.

Among various possible applications of this work is the fast scrambling conjecture [15]. It has been argued that a black hole is the fastest scrambler of information, in other words, it exhibits the fastest thermalization in nature. Due to the gauge/gravity duality, the large-N gauge theory is conjectured to scramble the information as fast as a black hole because they are equivalent. The intuitive picture discussed in this paper should be useful for understanding the microscopic mechanism of fast scrambling from gauge theory. Clearly, a huge number of simultaneous interactions at various intersections should be the essence of fast scrambling. As we have mentioned above, it would be possible to show fast scrambling by studying strong coupling lattice gauge theory or matrix models. The understanding of fast scrambling in gauge theory is interesting from the point of view of the quantum information theory, and even more, would be useful in understanding the very fast thermalization of RHIC fireball [16]. We hope to report progress in this direction in near future.

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