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Englert–Brout–Higgs Machanism in Nonrelativistic Systems

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We study the general theory of Englert–Brout–Higgs (EBH) mechanism without assuming Lorentz invariance using effective Lagrangians. We clarified problems in previous studies, and present a concrete prescription that satisfies the requirement of the charge neutrality and continuity in the limit of vanishing gauge coupling constant for non-abelian gauge theories.

Introduction. —The discovery of the Higgs boson at the Large Hadron Collider marks a great triumph of unity of physics. The original idea emerged from the study of superconductivity and its theory by Bardeen, Cooper, and Schrieffer (BCS) [1]. After Anderson found that there are collective excitations in the gap region [2], Nambu first clarified that the BCS ground state is still consistent with gauge invariance [3], and introduced the concept of spontaneous symmetry breaking (SSB) into particle physics [4, 5]. Soon afterwards, Goldstone proved that the SSB leads to massless scalar particles called Nambu–Goldstone bosons (NGBs) [6]. Even though the original theorem did not apply to non-Lorentz-invariant systems, the present authors generalized the theorem so that it has real-life applications in condensed matter physics, atomic physics, nuclear physics, and astrophysics [7–9]. On the other hand, Englert, Brout [10], and Higgs [11] proposed the gauged symmetry with SSB to go around Goldstone's theorem because Nature does not appear to have a massless scalar boson. It is called Englert–Brout–Higgs (EBH) mechanism. It was finally discovered fifty years later. At the same time, the concept of the Higgs boson made a full circle back to condensed matter physics, becoming a hot subject of research (see, *e.g.*, Refs. [12, 13]).

Given this tremendous cross-pollination among different subareas in physics, it is natural to ask what the general theory of Higgs phenomenon is without relying on the Lorentz invariance. In particular, the non-abelian gauge theory developed in particle physics [14] is making inroads into condensed matter physics, such as spin liquid [15], multi-layer graphene [16, 17], ultracold atoms in optical lattices [18–20] etc. Therefore a specific question of great importance is: what is the general theory of the EBH mechanism in non-abelian gauge theories without Lorentz invariance?

In relativistic field theories, spontaneously broken gauge symmetries do not give rise to physical NGBs would-be NGBs are *eaten* by gauge fields and in turn gauge fields acquire the longitudinal component and a finite mass. This is how the EBH mechanism works. It turns out that the extension of this famous story to the general non-relativistic setup is not a trivial problem. In pioneering works [21, 22], Gusynin and his collaborators found that, in a gauged linear σ model at a finite charge density, gauge fields A^i_{μ} develop a finite expectation value due to the linear term in the Lagrangian $-A^{\mu}_{i}\langle j^{i}_{\mu}\rangle$, generically ending up with breaking the spatial rotation. Later, Hama and his collaborators [23] pointed out a subtlety of this result: the low-energy spectrum is discontinuous as a function of the gauge coupling e and the mass of the gauge bosons does not go to zero in the limit $e \to 0$ (see the appendix of Ref. [23]). In general, there is nothing wrong with discussing non-perturbative phenomena, but if we are interested in a perturbative, weak coupling physics, the low-energy spectrum should be continuous as a function of the gauge coupling. They traced the origin of the discontinuity to the non-zero charge density of the ground state. Having in mind a situation where the background of the system has exactly the opposite charge densities and the net charge densities of the entire system vanish, they subtracted the ground state charge densities from the Lagrangian, following the prescription called "charge neutrality" discussed in Refs. [24, 25], and found that the mass of the gauge bosons continuously vanishes as $e \to 0$ this time.

However, as we shall see below in detail, this seemingly reasonable prescription actually has many issues. The crucial flaw is that the low-energy excitation spectrum is still not continuous. In the setup they considered, there is a NGB with a quadratic dispersion as well as its accompanying gapped mode at e = 0, but both of them are absent in the limit $e \rightarrow 0$. Although we will review this pathological behavior below in details, the discontinuity in the long wavelength limit can be already seen in their original paper; see Fig. 2 of Ref. [23]. We claim that the origin of this sick behavior is explicit breaking of both local and global symmetry due to the naive subtraction of *non-Abelian* charge densities.

In this Letter, we develop the general theory of spontaneously broken non-abelian gauge theory without Lorentz invariance, given the puzzles above. In particular, the emergence of type-A and type-B NGBs not seen in Lorentz-invariant systems raises questions about the number of "eaten" degrees of freedom, as there are only half as many type-B NGBs as the broken gauge symmetries.

Main Issues. — It is clear that the basic idea of the EBH mechanism should carry over to non-Lorentzinvariant systems. We assume rotational invariance in d = 3 spatial dimensions throughout this Letter, and stick to the unitarity gauge to understand the physical degrees of freedom. In the limit of zero gauge coupling constant, we have two massless states for each generator of the gauge group. On the other hand, a massive spin-one boson at rest has three states of the same energy required by the rotational invariance. Their energy should go to zero together as the gauge coupling constant is switched off. Therefore, the three states of a massive vector boson should decompose into two massless gauge boson states and another massless state. The last state is said to be "eaten" by the gauge boson when it becomes massive. Namely, a gauge boson can only eat a gapless state and acquires a mass as well as an additional degree of freedom. So far it is the same as in Lorentz-invariant case.

There are two related issues that make this familiar story paradoxical. As the present authors showed [8], there are two types of NGBs in non-Lorentz-invariant systems. For type-A NGBs, there is one degree of freedom per each broken symmetry. Therefore, we expect that the EBH mechanism should work just fine. However for type-B NGBs, there is one gapless boson for two broken symmetries. The simplest example is the Heisenberg ferromagnet. SO(3) symmetry is broken to SO(2), namely two generators are spontaneously broken. We expect two gauge bosons to acquire mass. However, there is only one type-B NGB in the limit of zero gauge coupling, and hence there is only one state to be "eaten." In addition, type-B NGBs are always associated with nonzero charge density in the ground state. Namely spontaneously broken generators Q_1 and Q_2 excite one type-B NGB when

$$\langle 0|[Q_1, j_2^0]|0\rangle = i\epsilon_{123}\langle 0|j_3^0|0\rangle \neq 0, \ Q_i \equiv \int d^3x j_i^0(x).$$
 (1)

Non-zero charge density coupled to the gauge field means there is a term linear in the gauge field in the Lagrangian $\langle j_0^3 \rangle A_0^3$, namely a tadpole. Together with the mass term $m_A^2 \propto g^2 v^2$, it causes the expectation value $\langle A_0^3 \rangle \propto \langle j_0^3 \rangle / gv \to \infty$ in the zero coupling limit. Therefore, we do not expect a smooth limit of zero gauge coupling constant in the presence of the finite charge density in the ground state.

Therefore, we study these two questions together: degrees of freedom and charge density. We study a few examples and draw a general lesson.

Gauged U(2) Model. —Let us start with discussing the model introduced in Refs. [26, 27],

$$\mathcal{L} = (D_{\mu}\phi)^{2} + m^{2}|\phi|^{2} - \lambda|\phi|^{4} - \frac{1}{4}[(F_{\mu\nu})^{2} + (F_{\mu\nu}^{i})^{2}](2)$$

where ϕ is a two-component complex scaler field and $D_{\nu} = \partial_{\nu} + i(e_1A_{\nu} - \mu\delta_{\nu,0}) + ie_2A_{\nu}^i(\sigma_i/2)$ is the covariant derivative, with A_{μ} being the U(1) gauge field and A_{ν}^i (i = 1, 2, 3) being the SU(2) gauge field. The vacuum expectation value $\langle \phi \rangle = (0, \phi_0)^T [\phi_0^2 = (\mu^2 + m^2)/2\lambda]$ breaks the U(2) symmetry down to U'(1) symmetry that acts only the first component of ϕ . The ground state possesses a non-Abelian charge density $\langle [Q_1, j_2^0] \rangle = i\langle j_3^0 \rangle = -i\mu\phi_0^2 \neq 0$, in addition to U(1) charge density $\langle j^0 \rangle = 2\mu\phi_0^2$, combining two NGBs into one type-B NGB. When $e_1 = e_2 = 0$, the Lagrangian (2) describes four modes: one type-A NGB, one Higgs (amplitude) mode, one type-B NGB. Their dispersion relations are respectively given by (we set c = 1 in this section)

$$\omega_A^2 = m^2 + 3\mu^2 + k^2 - \sqrt{(m^2 + 3\mu^2)^2 + (2\mu k)^2}, \quad (3)$$

$$\omega_H^2 = m^2 + 3\mu^2 + k^2 + \sqrt{(m^2 + 3\mu^2)^2 + (2\mu k)^2}, \quad (4)$$

$$\omega_B = \sqrt{\mu^2 + k^2} - \mu, \quad \omega_{B'} = \sqrt{\mu^2 + k^2} + \mu. \tag{5}$$

Our interest is how these modes are affected by switching on the gauge couplings.

When we gauge only the U(1) part of the symmetry $(e_1 \neq 0 \text{ and } e_2 = 0)$, we fix the gauge in such a way that the second component of ϕ is real, *i.e.*, $\phi = (\pi, \phi_0 + h)$ with $h \in \mathbb{R}$ and $\pi \in \mathbb{C}$. Expanding the Lagrangian $\mathcal{L}_1 + \mathcal{L}_2$ to the quadratic order in fields and dropping total derivatives, we find

$$\mathcal{L} = [\partial_{\mu}\pi^{*}\partial^{\mu}\pi + i\mu(\pi^{*}\dot{\pi} - \dot{\pi}^{*}\pi)] + \frac{1}{2} \left[\dot{\vec{A}}_{T} \cdot \dot{\vec{A}}_{T} - (\vec{\nabla} \times \vec{A}_{T})^{2} - 2m_{1}^{2}\vec{A}_{T} \cdot \vec{A}_{T}\right] + \partial_{\mu}h\partial^{\mu}h - 2(\mu^{2} + m^{2})h^{2} + \frac{1}{2}(\vec{\nabla}A_{0})^{2} + m_{1}^{2}A_{0}^{2} + \frac{1}{2}\dot{A}_{L}^{2} - m_{1}^{2}A_{L}^{2} + \vec{\nabla}A_{0} \cdot \partial_{0}\vec{A}_{L} - 4m_{1}\mu hA_{0} - 2e_{1}\mu\phi_{0}^{2}A_{0}, \qquad (6)$$

where we defined $m_1 \equiv e_1\phi_0$ and decomposed \vec{A} into the longitudinal and transverse components with respect to $\vec{\nabla}$. Note that the last term is linear in A_0 , corresponding to the U(1) charge density $\langle j^0 \rangle = 2\mu\phi_0^2$. Assuming the existence of a background that neutralizes the net U(1) charge of the system, Ref. [24] suggested that one should add the background contribution $-e_1A_0j_{\rm br}^0 = 2e_1\mu\phi_0^2A_0$ to the Lagrangian to cancel this term.

It is then a standard exercise to derive the excitation spectrum in this phase. The first line of Eq. (6) gives $\omega_{B,B'}$ modes in Eq. (5), unaffected by the gauge coupling. The second line describes the gapped transverse gauge bosons with $\omega_T = \sqrt{2m_1^2 + k^2}$. Finally, the rest part describes the longitudinal gauge boson and the Higgs mode with

$$\omega_L^2 = 2m_1^2 + \tilde{m}^2 + k^2 - \sqrt{\tilde{m}^4 + 4\mu^2 k^2},\tag{7}$$

$$\omega_{H}^{\prime 2} = 2m_{1}^{2} + \tilde{m}^{2} + k^{2} + \sqrt{\tilde{m}^{4} + 4\mu^{2}k^{2}}, \qquad (8)$$

where $\tilde{m}^2 \equiv m^2 + 3\mu^2 - m_1^2$. Observe that ω_L and ω'_H continuously go back to ω_A and ω_H in Eqs. (3), (4) in the limit $e_1 \to 0$. It is thus clear that the type-A NGB is eaten by the longitudinal part of the gauge field, making it a physical mode with a mass (gap) $\sqrt{2m_1^2}$.

Next let us consider gauging only the SU(2) part of the symmetry $(e_1 = 0 \text{ and } e_2 \neq 0)$. This time, we fix the gauge by setting $\phi = (0, \phi_0 + h)$. The Lagrangian to the quadratic order in fields is then given by

$$\mathcal{L} = \frac{1}{2} \left[\dot{A}_T^3 \cdot \dot{A}_T^3 - (\vec{\nabla} \times \vec{A}_T^3)^2 - (m_2^2/2) \vec{A}_T^3 \cdot \vec{A}_T^3 \right] - \frac{1}{4} \left[F_{\mu\nu}^1 F^{1\mu\nu} + F_{\mu\nu}^2 F^{2\mu\nu} + m_2^2 (A_\mu^1 A^{1\mu} + A_\mu^2 A^{2\mu}) \right] + \partial_\mu h \partial^\mu h - 2(\mu^2 + m^2) h^2 + \frac{1}{2} (\vec{\nabla} A_0^3)^2 + \frac{m_2^2}{4} (A_0^3)^2 + \frac{1}{2} (\dot{A}_L^3)^2 - \frac{m_2^2}{4} (A_L^3)^2 + \vec{\nabla} A_0^3 \cdot \partial_0 \vec{A}_L^3 + 2m_2 \mu h A_0^3 + e_2 \mu \phi_0^2 A_0^3, \qquad (9)$$

where $m_2 = e_2\phi_0$. Again, the last term is linear in A_0^3 , corresponding to the non-Abelian charge density $\langle j_3^0 \rangle = -\mu\phi_0^2$. In the same spirit as the above U(1) case, Ref. [23] subtracted this term by assuming the opposite contribution from the background $-e_2A_0^3(j_{\rm bg})_3^0$ with $(j_{\rm bg})_3^0 = \mu\phi_0^2$. Then we notice that the last three lines of Eq. (9) is identical to those of Eq. (6) if we replace m_2 by $-2m_1$ and A_{μ}^3 by A_{μ} . Therefore, we do not have to repeat the calculation for this sector as we already know that it correctly describes the type-A NGB and the Higgs mode in the limit $e_2 \to 0$.

However, we now encounter a serious problem. The first and second lines of Eq. (9) describe gapped gauge bosons with the dispersion relation $\omega_{L,T} = \sqrt{(m_2^2/2) + k^2}$. Although the total number of the physical modes is conserved, the $\omega_{B,B'}$ modes at $e_2 = 0$ are missing in the limit of $e_2 \rightarrow 0$. Instead, there are two more gapless modes with the linear dispersion $\omega = k$ in this limit.

The Dynamics of the Background. —We attribute this sick behavior to the incorrect treatment of the background degrees of freedom. We subtracted the background charge densities from the Lagrangian in the form $-eA^i_\mu(j_{\rm bg})^\mu_i$. However, if we do not take into account the dynamics of the background and completely freeze it, the fixed expectation value $(j_{bg})_i^{\mu}$ can no longer transform under the symmetry transformation acting the internal index i. Hence this added term *explicitly* breaks the original symmetry. In the above example, adding $-(j_{\rm br})_3^0 A_0^3$ term explicitly breaks the U(2) symmetry down to the $U(1)^2$ symmetry generated by σ_3 and 1. This residual symmetry explains why $\omega_{A,H}$ modes were properly described in the limit $e_2 \rightarrow 0$ in this scheme, but because the symmetry related to the $\omega_{B,B'}$ modes were explicitly broken, it was no wonder why they were missing in the limit same limit.

Note that subtracting Abelian charge densities do not

explicitly break any symmetries, for A_{μ} changes by $\partial_{\mu}\chi$ and the Lagrangian thus changes only by a surface term $\partial_{\mu}(-\chi e j_{\rm bg}^{\mu})$. This is why above treatment of the gauged U(1) symmetry returned physical results.

If we fully take into account the dynamics of the background degrees of freedom, the added term $-eA^i_{\mu}(j_{\rm bg})^{\mu}_i$ together with other terms that are needed to describe the background dynamics can now perfectly respect the symmetry of the original system. However, by assumption, the background has non-zero charge densities $\langle (j_{bg})_i^0 \rangle$, which are supposed to cancel the charge densities of the original system. These non-Abelian charge densities imply spontaneous breaking of generators Q_a such that $\langle [Q_a, j_b^0] \rangle = i f_{ab}{}^i \langle j_i^0 \rangle \neq 0$ for $\exists b$. Thus the background itself must support NGBs as an inevitable consequence of the NG theorem. Then, the question is whether once take a decoupling limit, where interactions of the background to other parts of the system vanish. We argue that such a limit does not exist in the case of non-Abelian symmetries.

Decoupling Limit. —Let us first review an example of the proper decoupling limit in the case of the background Abelian charge densities to clarify the difficulty in the non-Abelian cases. Our example is a superconductor, which exhibits the EBH mechanism via the condensation of Cooper pairs of electrons. Clearly there is a charge density of the electrons that couple to the electromagnetic gauge field. However, there is the ion background with a positive charge to ensure the charge neutrality. For simplicity we assume an isotropic elastic medium as a model,

$$\mathcal{L}_{\rm ion} = \frac{n_0 M}{2} [\vec{u}^2 - c_L^2 (\vec{\nabla} \cdot \vec{u})^2 - c_T^2 (\vec{\nabla} \times \vec{u})^2] - e A_\mu j^\mu, \ (10)$$

where e, M, n are the charge, mass, and number density of the ions, $c_{L,T}$ are the phonon velocities, and $j^{\mu} = n_0(1 - \vec{\nabla} \cdot \vec{u}, \dot{\vec{u}}) + O(u^2)$ is the number current density of the ions. If we canonically normalize the displacement field \vec{u} so that the coefficient of $\dot{\vec{u}}^2$ term becomes 1/2, the coupling to the gauge field is suppressed by the factor of $M^{-1/2}$ per \vec{u} field. Therefore, in the limit $M \to \infty$, the ion dynamics should completely decouple from the rest of the system, while the charge density $\langle j^0 \rangle = n_0$ still plays its role of electrically neutralizing the system.

In the Supplemental Material [28], we discuss a nonrelativistic superfluid as another example of the background with a proper decoupling limit.

A Ferromagnet. —Let us now move on to the case of non-Abelian charge densities to see whether a similar decoupling limit exists. As the simplest example, we examine the gauged version of ferromagnets. To construct an effective Lagrangian with local SO(3) symmetry, it is useful to define the gauged Maurer-Cartan form $\Omega(\pi, A)$ via $\Omega^i_{\mu}T_i = -iU^{\dagger}(\partial_{\mu} + ieA^i_{\mu}T_i)U$, where T_i (i = x, y, z) is a matrix representation of SO(3) generators and $U(\pi) \equiv e^{i\pi^a T_a}$ (a = x, y). A local symmetry transformation $g \in G$ acts NG fields π^a and the gauge field $\mathcal{A}_{\mu} = A^i_{\mu} T_i$ as

$$gU(\pi) = U(\pi')h, \quad h \equiv e^{i\theta(g,\pi)T_z} \in H,$$
(11)

$$\mathcal{A}'_{\mu} = g\mathcal{A}_{\mu}g^{-1} - ie^{-1}g\partial_{\mu}g^{-1}, \qquad (12)$$

so that the Maurer-Cartan form transforms nicely:

$$(\omega^a_\mu)' = R(\theta)^a_{\ b} \omega^b_\mu, \quad (\omega^z_\mu)' = \omega^z_\mu - \partial_\mu \theta, \tag{13}$$

where $R(\theta)$ is the rotation matrix around the z axis by an angle $\theta(g, \pi)$.

The low-energy effective Lagrangian of a ferromagnet with the local SO(3) symmetry is then given by

$$\mathcal{L}_{\rm FM} = -m_z \omega_t^z - \frac{1}{2} g \vec{\omega}^a \cdot \vec{\omega}^a \qquad (14)$$

to the leading order in derivatives. Here, g is a positive constant and m_z is the magnetization. Using the transformation rule in Eq. (13), one can check that the Lagrangian changes just by a surface term $m_z \partial_t \theta(g, \pi)$ under a local symmetry transformation.

Expanding the Maurer-Cartan form ω_t^z in NG fields π^a , one finds

$$\omega_t^z = -\frac{\epsilon_{ab}}{2}\pi^a \dot{\pi}^b + e\left(A_t^z + \epsilon_{ab}\pi^a A_t^b - \frac{1}{2}\pi^{a^2} A_t^z\right) (15)$$

to the order of $O(\pi^3, \pi^2 A)$. Note that all interactions are dictated by the SO(3) symmetry, in contrast to the case of Abelian symmetries. Since the coupling constant e is common to all fields coupling to the non-Abelian gauge field, one cannot control e arbitrarily if one wishes to use this ferromagnet as a background of a system. One cannot tune m_z either since it should be fixed by the magnetization density of the main system. This argument proves that the coupling to the gauge field in the first term of Eq. (14) cannot be decoupled. Unfortunately, this is a general property of non-Abelian gauge theories — in the presence of non-Abelian charge densities, the dynamics of the background that is supposed to neutralize the charge densities of the whole system cannot be decoupled.

Two Copies of Ferromagnets. — This observation motivates us to explicitly include the dynamics of the background, at least when we are interested in the situation with non-Abelian charge densities. As a toy model, here we discuss two copies of ferromagnets, one with the magnetization m_z pointing in the positive z direction and the other one in the opposite direction with the equal magnitude. We assume $G = SO(3) \times SO(3)$ symmetry, which is broken down to $H = SO(2) \times SO(2)$ by magnetizations. In this case, we can safely gauge the diagonal SO(3) symmetry thanks to the cancelation of the magnetization. Note that, due to the non-Abelian nature of the SO(3) symmetry, gauging the vector part of SO(3) × SO(3) explicitly breaks the axial part of the global symmetry. Denoting the NG fields of the first ferromagnet by π^a and those of second one by Π^a (a = x, y), the linearized Lagrangian of the whole system reads

$$\mathcal{L} = -\frac{1}{2}g(\vec{\nabla}\pi^{a} - e\vec{A}^{a})^{2} - \frac{1}{2}G(\vec{\nabla}\Pi^{a} - e\vec{A}^{a})^{2} + m_{z}\epsilon_{ab}\left(\frac{1}{2}\Pi^{a}\dot{\Pi}^{b} - \frac{1}{2}\pi^{a}\dot{\pi}^{b} + e\Pi^{a}A_{t}^{b} - e\pi^{a}A_{t}^{b}\right) + \frac{\epsilon}{2}(\vec{\nabla}A_{t}^{i} + \partial_{t}\vec{A}^{i})^{2} - \frac{\epsilon v^{2}}{2}(\vec{\nabla}\times\vec{A}^{i})^{2},$$
(16)

where g, G are positive constants. ϵ is the electric permittivity v is the speed of light in the medium. When e = 0, the Lagrangian describes two type-B NGBs with

$$\omega_{B_1} = \frac{gk^2}{m_z} , \quad \omega_{B_2} = \frac{Gk^2}{m_z} .$$
 (17)

and $(d-1) \times 3 = 6$ states for transverse gauge bosons with $\omega_T = vk$. Since A^z_{μ} stays gapless regardless of the gauge coupling, we will not discuss them below. In the Supplemental Material [28], we discuss additional gapped modes in the presence of terms with second power in time derivatives so that our results can be compared to those in Ref. [29], but we find they do not change any of the discussions here.

For a finite coupling $e \neq 0$, we use the unitary gauge to set $\pi^a + \Pi^a = 0$. Then the Lagrangian is

$$\mathcal{L} = -\frac{1}{2}g(\vec{\nabla}\pi^{a} - e\vec{A}_{L}^{a})^{2} - \frac{1}{2}G(\vec{\nabla}\pi^{a} + e\vec{A}_{L}^{a})^{2} - 2em_{z}\epsilon_{ab}\pi^{a}A_{t}^{b} + \frac{\epsilon}{2}(\vec{\nabla}A_{t}^{a} + \partial_{t}\vec{A}_{L}^{a})^{2} + \frac{\epsilon}{2}(\partial_{t}\vec{A}_{T}^{a})^{2} - \frac{\epsilon v^{2}}{2}(\vec{\nabla}\times\vec{A}_{T}^{a})^{2} - \frac{g+G}{2}(e\vec{A}_{T}^{a})^{2}.(18)$$

The last line describes gapped transverse gauge bosons with

$$\omega_T = \sqrt{m_1^2 + (vk)^2}, \quad m_1^2 \equiv e^2(g+G)/\epsilon, \quad (19)$$

and the first three lines describes two longitudinal gauge bosons with

$$\omega_{L_{1,2}} = \sqrt{m_1^2 + \left[\frac{(g+G)k^2}{2m_z}\right]^2} \pm \frac{(g-G)k^2}{2m_z} .$$
(20)

In the limit of switching off the gauge coupling, the longitudinal modes $\omega_{L_{1,2}}$ smoothly go back to two type-B NGBs $\omega_{B_{1,2}}$ in Eq. (17). Therefore, the two type-B NGBs at e = 0 are "eaten" to become the two longitudinal gauge bosons at a finite coupling.

Therefore, we see clearly that we need two type-B NGBs to make both A^1 and A^2 massive, while gapped modes play no role, contrary to claims in Refs. [23, 29]. Given the necessity of a type-B NGB from the background, there is no decoupling limit where we can simply replace the background system by a current density.

Conclusion. —In this Letter, we proposed a concrete prescription to realize the EBH mechanism in non-Lorentz-invariant systems. The naive subtraction of background non-Abelian charge densities, as suggested in the literature, explicitly breaks the symmetry of the system and results in a discontinuity of the spectrum as a function of the gauge coupling. When we include the dynamics of the background to properly respect the symmetry, there is no limit where the background dynamics is decoupled from the other part of the system. We explicitly show that the gauged ferromagnet is consistent after fully taking into account the background dynamics, with a continuous limit of zero gauge coupling and correct numbers of "eaten" degrees of freedom.

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1. Gauged U(2) Model

Here we present the full derivation of Eqs. (7) and (8). The (h, A_0, A_L) sector of the Lagrangian (6) reads

$$L = \int d^3x dt \left\{ \left[\partial_\mu h \partial^\mu h - 2(\mu^2 + m^2)h^2 \right] + \left[\frac{1}{2} (\vec{\nabla}A_0)^2 + m_1^2 A_0^2 \right] + \left[\frac{1}{2} \dot{A}_L^2 - m_1^2 A_L^2 \right] + \vec{\nabla}A_0 \cdot \partial_0 \vec{A}_L - 4m_1 \mu h A_0 \right\}$$

$$= \int \frac{d^3k d\omega}{(2\pi)^4} \left\{ \left[\omega^2 - k^2 - 2(\mu^2 + m^2) \right] h^* h + \left(\frac{1}{2} k^2 + m_1^2 \right) A_0^* A_0 + \left(\frac{1}{2} \omega^2 - m_1^2 \right) A_L^* A_L - \frac{1}{2} k \omega (A_0^* A_L + A_L^* A_0) - 2m_1 \mu (h^* A_0 + A_0^* h) \right\}$$

$$= \int \frac{\mathrm{d}^3 k \mathrm{d}\omega}{(2\pi)^4} \left(h^*, A_0^*, A_L^*\right) M \begin{pmatrix} h\\ A_0\\ A_L \end{pmatrix},\tag{21}$$

where

$$M \equiv \begin{pmatrix} \omega^2 - k^2 - 2(\mu^2 + m^2) & -2m_1\mu & 0\\ -2m_1\mu & \frac{1}{2}k^2 + m_1^2 & -\frac{1}{2}k\omega\\ 0 & -\frac{1}{2}k\omega & \frac{1}{2}\omega^2 - m_1^2 \end{pmatrix},$$

$$(22)$$

$$\det M = \frac{m_1^2}{2} \left\{ \omega^4 - 2(m^2 + 3\mu^2 + m_1^2 + k^2)\omega^2 + \left[k^4 + 2(m^2 + \mu^2 + m_1^2)k^2 + 4m_1^2(m^2 + 3\mu^2)\right] \right\}.$$
 (23)

Dispersion relations, Eqs. (7) and (8), can be obtained by solving $\det M = 0$:

$$\omega^{2} = (m^{2} + 3\mu^{2} + m_{1}^{2} + k^{2}) \pm \sqrt{(m^{2} + 3\mu^{2} + m_{1}^{2} + k^{2})^{2} - [k^{4} + 2(m^{2} + \mu^{2} + m_{1}^{2})k^{2} + 4m_{1}^{2}(m^{2} + 3\mu^{2})]}$$

$$= 2m_{1}^{2} + (m^{2} + 3\mu^{2} - m_{1}^{2}) + k^{2} \pm \sqrt{(m^{2} + 3\mu^{2} - m_{1}^{2})^{2} + (2\mu k)^{2}}.$$
(24)

2. Two Copies of Ferromagnets

In this section, we discuss the two copies of ferromagnets, including additional terms with second power in time derivatives. The linearized Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\bar{g}(\dot{\pi}^{a} + eA_{t}^{a})^{2} - \frac{1}{2}g(\vec{\nabla}\pi^{a} - e\vec{A}^{a})^{2} + \frac{1}{2}\bar{G}(\dot{\Pi}^{a} + eA_{t}^{a})^{2} - \frac{1}{2}G(\vec{\nabla}\Pi^{a} - e\vec{A}^{a})^{2} + m_{z}\epsilon_{ab}\left(\frac{1}{2}\Pi^{a}\dot{\Pi}^{b} - \frac{1}{2}\pi^{a}\dot{\pi}^{b} + e\Pi^{a}A_{t}^{b} - e\pi^{a}A_{t}^{b}\right) + \frac{\epsilon}{2}(\vec{\nabla}A_{t}^{i} + \partial_{t}\vec{A}^{i})^{2} - \frac{\epsilon v^{2}}{2}(\vec{\nabla}\times\vec{A}^{i})^{2},$$
(25)

where g, \bar{g}, G, \bar{G} are positive constants. for the sake of brevity we set G = g and $\bar{G} = \bar{g}$, but one can work out without this simplification if one wishes.

When e = 0, the Lagrangian

$$\mathcal{L} = \frac{1}{2} \left[\bar{g} (\dot{\pi}^{a})^{2} - m_{z} \epsilon_{ab} \pi^{a} \dot{\pi}^{b} - g (\vec{\nabla} \pi^{a})^{2} \right] + \frac{1}{2} \left[\bar{g} (\dot{\Pi}^{a})^{2} + m_{z} \epsilon_{ab} \Pi^{a} \dot{\Pi}^{b} - g (\vec{\nabla} \Pi^{a})^{2} \right] + \frac{\epsilon}{2} \left[(\vec{\nabla} A_{t}^{i} + \partial_{t} \vec{A}^{i})^{2} - v^{2} (\vec{\nabla} \times \vec{A}^{i})^{2} \right],$$
(26)

describes two type-B NGBs with

$$\omega_B^2 = \frac{m_z^2}{2\bar{g}^2} \left(1 + 2\frac{g\bar{g}}{m_z^2} k^2 - \sqrt{1 + 4\frac{g\bar{g}}{m_z^2} k^2} \right) \simeq \frac{g^2 k^4}{m_z^2} + O(k^6), \tag{27}$$

two gapped modes with

$$\omega_G^2 = \frac{m_z^2}{2\bar{g}^2} \left(1 + 2\frac{g\bar{g}}{m_z^2} k^2 + \sqrt{1 + 4\frac{g\bar{g}}{m_z^2}} k^2 \right) \simeq \frac{m_z^2}{\bar{g}^2} + \frac{2g}{\bar{g}} k^2 + O(k^4), \tag{28}$$

and $(d-1) \times 3$ transverse gauge bosons with

$$\omega_T = vk. \tag{29}$$

Note that ω_G modes in general is beyond the scope of an effective Lagrangian, as they balance $O(\partial_t)$ term with $O(\partial_t^2)$ term; hence their gap may be beyond the cutoff scale and not belong to the effective Lagrangian. Especially, they should disappear in the limit $\bar{g} \to 0$. Nevertheless, here we assume the existence of the ω_G modes to see their fate after coupling to gauge fields. Since A^z_{μ} is decoupled from the other part of the system, regardless of the gauge coupling, we will not discuss them below.

For a finite coupling $e \neq 0$, we use the unitary gauge to set $\pi^a + \Pi^a = 0$. The Lagrangian is then decomposed into

$$\mathcal{L} = \left[\bar{g}(\dot{\pi}^a)^2 - g(\vec{\nabla}\pi^a)^2\right] + \left[\frac{\epsilon}{2}(\vec{\nabla}A^a_t)^2 + \bar{g}e^2(A^a_t)^2\right] + \left[\frac{\epsilon}{2}(\partial_t\vec{A}^a_L)^2 - ge^2(\vec{A}^a_L)^2\right] - 2em_z\epsilon_{ab}\pi^aA^b_t + \epsilon\vec{\nabla}A^a_t \cdot \partial_t\vec{A}^a_L$$
(30)

$$+\frac{\epsilon}{2}\left[(\partial_t \vec{A}_T^a)^2 - v^2(\vec{\nabla} \times \vec{A}_T^a)^2 - \frac{2e^2g}{\epsilon}(\vec{A}_T^a)^2\right].$$
(31)

The last line describes $(d-1) \times 2$ gapped transverse gauge bosons with

$$\omega_T' = \sqrt{(2e^2g/\epsilon)^2 + (vk)^2}.$$
(32)

The first two lines describes 1×2 gapped longitudinal gauge bosons. In the basis $(\pi^1, A_t^1, A_L^1, \pi^2, A_t^2, A_L^2)^T$, the quadratic Lagrangian in the Fourier space can be represented by the matrix

$$M = \begin{pmatrix} \bar{g}\omega^2 - gk^2 & 0 & 0 & 0 & -em_z & 0\\ 0 & \frac{\epsilon}{2}k^2 + \bar{g}e^2 & -\frac{\epsilon}{2}k\omega & em_z & 0 & 0\\ 0 & -\frac{\epsilon}{2}k\omega & \frac{\epsilon}{2}\omega^2 - ge^2 & 0 & 0 & 0\\ 0 & em_z & 0 & \bar{g}\omega^2 - gk^2 & 0 & 0\\ -em_z & 0 & 0 & 0 & \frac{\epsilon}{2}k^2 + \bar{g}e^2 & -\frac{\epsilon}{2}k\omega\\ 0 & 0 & 0 & 0 & -\frac{\epsilon}{2}k\omega & \frac{\epsilon}{2}\omega^2 - ge^2 \end{pmatrix}.$$
(33)

By solving $\det M = 0$, one gets

$$\omega_L^2 = \frac{m_z^2}{2\bar{g}^2} \left[\left(1 + \frac{e^2 g \bar{g}^2}{\epsilon m_z^2} \right) + 2 \frac{g \bar{g}}{m_z^2} k^2 - \sqrt{\left(1 - \frac{e^2 g \bar{g}^2}{\epsilon m_z^2} \right)^2 + 4 \frac{g \bar{g}}{m_z^2} k^2} \right] \\ \simeq (2e^2 g/\epsilon)^2 \left(1 - \frac{\frac{g \bar{g} k^2}{m_z^2}}{1 - \frac{2e^2 g \bar{g}^2}{\epsilon m_z^2}} \right) + \frac{\frac{g^2 k^4}{m_z^2}}{\left(1 - \frac{e^2 g \bar{g}^2}{\epsilon m_z^2} \right)^3} + O(k^6), \tag{34}$$

$$\omega_G'^2 = \frac{m_z^2}{2\bar{g}^2} \left[\left(1 + \frac{e^2 g \bar{g}^2}{\epsilon m_z^2} \right) + 2 \frac{g \bar{g}}{m_z^2} k^2 + \sqrt{\left(1 - \frac{e^2 g \bar{g}^2}{\epsilon m_z^2} \right)^2 + 4 \frac{g \bar{g}}{m_z^2} k^2} \right] \\ \simeq \frac{m_z^2}{\bar{g}^2} + \frac{g}{\bar{g}} \left(1 + \frac{1}{1 - \frac{2e^2 g \bar{g}^2}{\epsilon m_z^2}} \right) k^2 + O(k^4). \tag{35}$$

One can see that ω_L has the same gap $\sqrt{2e^2g/\epsilon}$ as the transverse components, as expected by the spatial rotational symmetry. This mode is continuously connected to the type-B NGB ω_B in the limit $e \to 0$. Also, ω'_G continuously goes back to ω_G in this limit.

3. Another Example of Background With a U(1) Charge

As an example of a background that does break an internal symmetry, let us take a non-relativistic superfluid of charged bosons, described by the Galilean invariant Lagrangian

$$\mathcal{L}_{\rm SF} = \frac{n_0}{2Mc_s^2} \left[Mc_s^2 - (\dot{\varphi} + eA_t) - \frac{(\vec{\nabla}\varphi - e\vec{A})^2}{2M} \right]^2.$$
(36)

Here M is the mass of the bosons and c_s is the speed of sound. One may worry that the superfluid phonon (NGB) messes up the system or the broken symmetry gives a mass to the U(1) gauge field by the standard EBH mechanism. However, redefining $\varphi' = \varphi \sqrt{n_0/Mc_s^2}$, we find

$$\mathcal{L}_{\rm SF} = \frac{1}{2} [(\dot{\varphi}')^2 - c_s^2 (\vec{\nabla} \varphi')^2] - e n_0 A_t + O(M^{-1/2}).$$
(37)

Therefore the superfluid dynamics is completely decoupled in the limit of $M \to \infty$. Especially, the Higgs mass $\sqrt{n_0 e^2/M}$ vanishes in this limit, while the U(1) charge n_0 remains finite.