SISSA 48/2009/EP

Majorana Phases and Leptogenesis in See-Saw Models with A_4 Symmetry

C. Hagedorn

SISSA and INFN-Sezione di Trieste, Trieste I-34014, Italy
*E-mail:*hagedorn@sissa.it

E. Molinaro

SISSA and INFN-Sezione di Trieste, Trieste I-34014, Italy
*E-mail:*molinaro@sissa.it

S. T. Petcov *

SISSA and INFN-Sezione di Trieste, Trieste I-34014, Italy;
IPMU, University of Tokyo, Tokyo, Japan

ABSTRACT: The related issues of Majorana CP violation in the lepton sector and leptogenesis are investigated in detail in two rather generic supersymmetric models with type I see-saw mechanism of neutrino mass generation and A_4 flavour symmetry, which naturally lead at leading order to tri-bimaximal neutrino mixing. The neutrino sector in this class of models is described at leading order by just two real parameters and one phase. This leads, in particular, to significant low energy constraints on the Majorana phases α_{21} and α_{31} in the PMNS matrix, which play the role of leptogenesis CP violating parameters in the generation of the baryon asymmetry of the Universe. We find that it is possible to generate the correct size and sign of the baryon asymmetry in both A_4 models. The sign of the baryon asymmetry is directly related to the signs of $\sin\alpha_{21}$ and/or $\sin\alpha_{31}$.

*Also at: INRNE, Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria.

Contents

1. Introduction

The presence of two large and one small mixing angles in the lepton sector [1, 2],

$$\sin^2\theta_{12} = 0.304^{+0.066}_{-0.054}\ ,\ \ \sin^2\theta_{23} = 0.50^{+0.17}_{-0.14}\ ,\ \ \sin^2\theta_{13} < 0.056\quad (3\sigma)\ , \tag{1.1}$$

suggests a pattern of neutrino mixing which is remarkably similar to the so called "tri-bimaximal" (TB) one [3]. Indeed, in the case of TB mixing, the solar and atmospheric neutrino mixing angles θ_{12} and θ_{23} have values very close to, or coinciding with, the best fit ones in eq. (1.1), determined in global analyses of neutrino oscillation data,

$$\sin^2\theta_{12} = 1/3\ ,\ \ \sin^2\theta_{23} = 1/2\quad . \tag{1.2}$$

The TB mixing scheme predicts also that $\theta_{13} = 0$. Correspondingly, the neutrino mixing matrix is given by

$$U_\nu = U_{TB}\,\mathrm{diag}\left(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}\right) \tag{1.3}$$

where

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix} \tag{1.4}$$

and α_{21} and α_{31} are Majorana CP violating phases [4, 5].

TB mixing can appear naturally in models using the tetrahedral group A_4 as flavor symmetry [6]. [1] The three generations of left-handed leptons and right-handed (RH) neutrinos are unified into a triplet representation of the A_4 group, whereas the right-handed charged leptons are A_4-singlets. To be concrete, in the following we focus on two models which represent a class of A_4 models, namely the models of Altarelli-Feruglio (AF) [8] and Altarelli-Meloni (AM) [9]. Both models are based on the Standard Model (SM) gauge symmetry group and are supersymmetric. Additional degrees of freedom, flavons, are introduced in order to appropriately break the A_4 flavor symmetry at high energies. Both models have in common that they predict at leading order (LO) a diagonal mass matrix for charged leptons and lead to exact TB mixing in the neutrino sector. The mass matrix of the RH neutrinos contains only two complex parameters X, Z. Light neutrino masses are generated through the type I see-saw mechanism. All low energy observables are expressed

[1] It can also be derived from models with an S_4 flavor symmetry [7].

through only three independent quantities: $\alpha = |3Z/X|$, the relative phase ϕ between X and Z, and the absolute scale of the light neutrino masses. The latter is a combination of the unique neutrino Yukawa coupling and $|X|$ which determines the scale of RH neutrino masses. The main difference at LO between the AF and AM models is in the generation of the charged lepton mass hierarchy: in [8] an additional Froggatt-Nielsen (FN) symmetry $U(1)_{FN}$ is invoked, whereas in [9] the hierarchy between the masses of the charged leptons arises through multi-flavon insertions [2]. As a result the next-to-leading (NLO) corrections in these models are different. The expansion parameter in the A_4 models of interest is the vacuum expectation value (VEV) of a generic flavon field divided by the cut-off scale Λ of the theory. Its typical size is $\lambda_c^2 \approx 0.04$ with $\lambda_c \approx 0.22$ being of the size of the Cabibbo angle.

The fact that the properties of light as well as heavy neutrinos are essentially fixed by the three parameters α, ϕ and $|X|$ leads to strong constraints. We will be interested, in particular, in the dependence of the Majorana CP violating phases in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [10], which are relevant in leptogenesis and neutrinoless double beta decay, on α and ϕ. Actually, α and ϕ, as was shown in [9, 11], are related through the ratio $r = \Delta m^2_{\odot}/|\Delta m^2_{\rm A}| = (m_2^2 - m_1^2)/|m_3^2 - m_1^2|$, where $\Delta m^2_{\odot}$ and $\Delta m^2_{\rm A}$ are neutrino mass squared differences driving the solar and the dominant atmospheric neutrino oscillations. As a consequence, the Majorana CP violating phases depend effectively only on α. The related constraints on the neutrino mass spectrum have been studied in [9,11,12].

In the present article we investigate in detail the generation of baryon asymmetry of the Universe Y_B within the AF and AM models. Although our analysis is done for these two specific models, it has generic features which are common to models based on A_4 flavour symmetry. In the class of models under discussion the neutrino masses arise via the type I see-saw mechanism. Correspondingly, one can implement the leptogenesis scenario of matter-antimatter asymmetry generation. As is well known, in this scenario the baryon asymmetry is produced through the out of equilibrium CP violating decays of the RH neutrinos ν_i^c (and their SUSY partners - RH sneutrinos $\tilde{\nu}_i^c$) in the early Universe [13, 14]. The observed value of Y_B to be reproduced, given as the ratio between the net baryon number density and entropy density, reads [15]

$$Y_B \equiv \left.\frac{n_B - n_{\bar{B}}}{s}\right|_0 = (8.77 \pm 0.24) \times 10^{-11} \tag{1.5}$$

where the subscript "0" refers to the present epoch.

As has been discussed in [16], the CP asymmetries ϵ_i, originating in the decays of the RH neutrinos and sneutrinos ν_i^c and $\tilde{\nu}_i^c$ and relevant for the generation of the baryon asymmetry of the Universe, vanish at LO in the class of models under discussion. Thus, $\epsilon_i \neq 0$ are generated due to the NLO corrections. These are, as already mentioned, different in the AF and AM models, so that the results for ϵ_i differ in the two models.

[2]This is due to a different choice in the AM model of the vacuum alignment of the flavon, φ_T, slightly different transformation properties of the right-handed charged leptons under A_4 and the presence of a different additional cyclic symmetry.

We calculate the CP asymmetries ϵ_i in the AF and AM types of models and discuss the dependence of Y_B on the parameter α. This is done in versions of the two models in which the RH neutrinos have masses in the range $M_i \sim (10^{11} \div 10^{13})$ GeV. As discussed in [9], the natural range of RH neutrino masses in the AF and AM models is $M_i \sim (10^{14} \div 10^{15})$ GeV. Such large values of the RH neutrino masses can lead [17] in SUSY theories with see-saw mechanism to conflict with the existing stringent experimental upper limits on the rates of lepton flavour violating (LFV) decays and reactions [3], like $\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- + (A,Z) \to e^- + (A,Z)$, etc. The investigation of the LFV processes in the class of AF and AM models is beyond the scope of the present study. However, in order to avoid potential problems related to the issue of LFV, we work in versions of the AF and AM models in which the scale of the RH neutrino masses is lower than in the original AF and AM models. This is achieved by minimally extending the AF and AM models through an additional Z_2 symmetry, which enables us to appropriately suppress the neutrino Yukawa couplings. This in turn allows to lower the scale of RH neutrino masses down to $(10^{11} \div 10^{13})$ GeV.

We perform the analysis of baryon asymmetry generation in the so-called "one flavour" approximation. The latter is valid as long as the masses of the RH neutrinos satisfy [18, 19] $M_i \gtrsim 5 \times 10^{11}(1 + \tan^2\beta)$ GeV, where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets present in the SUSY extensions of the Standard Model. The "one flavour" approximation condition is satisfied for, e.g. $\tan\beta \sim 3$ and $M_i \sim 10^{13}$ GeV. Actually in the models we consider relatively small values of $\tan\beta$ are preferable [8]. Further, with masses of the RH neutrinos in the range of $(10^{11} \div 10^{13})$ GeV one can safely neglect the effects of the $\Delta L = 2$ wash-out processes in leptogenesis [20]. This allows us to use simple analytic approximations in the calculation of the relevant efficiency factors η_{ii}. We perform the calculation of the baryon asymmetry for the two types of light neutrino mass spectrum - with normal and inverted ordering. Both types of spectrum are allowed in the class of models considered.

We find that it is possible to generate the correct size and sign of the baryon asymmetry Y_B in the versions of both the AF and AM models we discuss. The sign of Y_B is uniquely determined by the sign of $\sin\phi$, since all other factors in Y_B have a definite sign. Interestingly, in the low energy observables only $\cos\phi$ is present, so that both $\sin\phi \lesseqgtr 0$ are compatible with the low energy data. Conversely speaking, taking into account the sign of the baryon asymmetry Y_B we are able to fix uniquely also the sign of $\sin\phi$, which is otherwise undetermined through the low energy data.

Leptogenesis is not studied for the first time in the class of models of interest. However, our work overlaps little with the already existing publications on the subject. In [21] the CP asymmetries and Y_B were also calculated. This is done, however, not within the context of a self-consistent model since instead of computing the NLO corrections, the authors introduce ad hoc random perturbations in the Dirac neutrino mass matrix. In [16] only the CP asymmetries are calculated within the AF model without discussing the washout effects which can change the results for Y_B. In [9] results for the CP asymmetries are

[3] The indicated problem typically arises if the SUSY particles have masses in the range of few to several hundred GeV, accessible to the LHC experiments [17].

also given, but the washout effects are not taken into account. Finally, in [22] a highly degenerate spectrum of masses of RH neutrinos is considered and the baryon asymmetry is produced via resonant leptogenesis.

The paper is organized as follows: in Section 2 we give a short introduction to the AF and the AM models and discuss the changes due to adding the Z_2 symmetry. We also give the expressions for the NLO corrections relevant for the calculation of the baryon asymmetry Y_B. In Section 3 we discuss the light and heavy Majorana neutrino mass spectra. We study the Majorana phases and their dependence on the parameter α. Our analysis of leptogenesis in both models for light neutrino mass spectrum with normal ordering (NO) and inverted ordering (IO) is given in Section 4. We summarize the results of the present work in Section 5. The two appendices contain details on the flavon superpotential and the generation of an appropriate VEV for the additional flavon field ζ, present in the models considered by us.

2. Variants of the Two A_4 Models

In this section we recapitulate the main features of the AF [8] and the AM model [9]. We supplement them with an additional Z_2 symmetry to appropriately suppress the Dirac Yukawa couplings of the neutrinos and to lower the mass scale of the RH neutrinos. We explicitly check that changes in the models connected to the Z_2 extension do not affect the LO results for the lepton masses and mixings and also only slightly affect the NLO results.

For an introduction to the group theory of A_4 we refer to [8, 9], whose choice of generators for the A_4 representations we follow.

2.1 Altarelli-Feruglio Type Model

In this model the flavor symmetry A_4 is accompanied by the cyclic group Z_3 and the Froggatt-Nielsen symmetry $U(1)_{FN}$. We add, as mentioned, a further Z_2 symmetry to suppress the Dirac couplings of the neutrinos. By assuming that the RH neutrinos acquire a sign under Z_2, the renormalizable coupling becomes forbidden [4]. To allow a Yukawa coupling for neutrinos at all we introduce a new flavon ζ which only transforms under Z_2. We call the VEV of ζ $\langle\zeta\rangle = z$ in the following and assume that $z \approx \lambda_c^2\Lambda$ as all other flavon VEVs. Clearly, the Majorana mass terms of the RH neutrinos remain untouched, at LO. The symmetries and particle content of the AF variant are as given in table 1. At LO neutrino masses are generated by the terms[5]

$$y_\nu(\nu^c l)h_u\zeta/\Lambda + a\xi(\nu^c\nu^c) + b\,(\nu^c\nu^c\varphi_S) \tag{2.1}$$

with $(\cdots)$ denoting the contraction to an A_4-invariant. Thus, the LO terms in the neutrino sector are the same as in the original AF model, apart from the suppression of the Dirac coupling. Also the LO result that the charged lepton mass matrix is diagonal, is not

[4] Alternatively, one could also let h_u instead of ν^c transform under the Z_2 symmetry to forbid the Dirac Yukawa coupling at the renormalizable level.

[5] The field $\tilde{\xi}$ does not have a VEV at LO and thus is not relevant at this level.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ_T	φ_S	$\xi,\,\tilde{\xi}$	ζ
A_4	3	1	$1''$	$1'$	3	1	3	3	1	1
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	1
Z_2	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$	$-$

Table 1: Particle content of the AF variant. Here we display the transformation properties of lepton superfields, Minimal Supersymmetric SM (MSSM) Higgs and flavons under the flavor group $A_4 \times Z_3 \times Z_2$. l denotes the three lepton doublets, e^c, μ^c and τ^c are the three $SU(2)_L$ singlets and ν^c are the three RH neutrinos forming an A_4-triplet. Apart from ν^c and the flavon ζ all fields are neutral under the additional Z_2 symmetry. Note that ω is the third root of unity, i.e. $\omega = \mathrm{e}^{\frac{2\pi i}{3}}$. To accommodate the charged lepton mass hierarchy the existence a $U(1)_{FN}$, under which only the RH charged leptons are charged, is assumed. The $U(1)_{FN}$ is broken by an FN field θ only charged under $U(1)_{FN}$ with charge -1. Additionally, the model contains a $U(1)_R$ symmetry relevant for the alignment of the vacuum.

changed compared to the original model. The mass matrices of the neutrinos are of the form (m_D is given in the right-left basis)

$$m_D = y_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{z}{\Lambda} v_u \quad \text{and} \quad m_M = \begin{pmatrix} au + 2bv_S & -bv_S & -bv_S \\ -bv_S & 2bv_S & au - bv_S \\ -bv_S & au - bv_S & 2bv_S \end{pmatrix} \tag{2.2}$$

with $v_u = \langle h_u \rangle$, $\langle \xi \rangle = u$ and $\langle \varphi_{Si} \rangle = v_S$ according to the alignment in [8]. The light neutrino mass matrix is given by the type I see-saw term

$$m_\nu = -m_D^T m_M^{-1} m_D \tag{2.3}$$

and has the generic size $\lambda_c^2 v_u^2/\Lambda$. At the same time, the effective dimension-5 operator $l h_u l h_u/\Lambda$, [6] which can also contribute to the light neutrino masses, is only invariant under the flavor group, if it involves two flavons of the type φ_S and ξ ($\tilde{\xi}$). Thus, its contribution to the light neutrinos masses scales as $\lambda_c^4 v_u^2/\Lambda$, which is always subdominant compared to the type I see-saw contribution. The size of the contribution from the effective dimension-5 operator is actually of the same size as possible NLO corrections to the type I see-saw term.

Considering the NLO corrections note that these involve for the Dirac neutrino mass matrix either the two flavon combination $\varphi_T \zeta$ or the shift of the vacuum of ζ. The first type gives rise to two different terms

$$y_A (\nu^c l)_{3_S} \varphi_T h_u \zeta/\Lambda^2 + y_B (\nu^c l)_{3_A} \varphi_T h_u \zeta/\Lambda^2 \tag{2.4}$$

with $(\cdots)_{3_{S\,(A)}}$ standing for the (anti-)symmetric triplet of the product $\nu^c l$. The correction due to the shift in $\langle \zeta \rangle$ can be simply absorbed into a redefinition of the coupling y_ν. Thus,

[6] We make the "conservative" assumption that all non-renormalizable operators are suppressed by the same cutoff scale Λ.

using the alignment of φ_T as given in [8], the structure of the NLO corrections to m_D is the same as in the original model

$$\delta m_D = \begin{pmatrix} 2y_A & 0 & 0 \\ 0 & 0 & -y_A - y_B \\ 0 & -y_A + y_B & 0 \end{pmatrix} \frac{v_T z}{\Lambda^2} v_u \; . \tag{2.5}$$

The NLO corrections to the Majorana mass matrix of the RH neutrinos are exactly the same as in the original AF model, i.e.

$$\begin{aligned} &a\,\delta\xi(\nu^c\nu^c) + \tilde{a}\,\delta\tilde{\xi}(\nu^c\nu^c) + b\,(\nu^c\nu^c\delta\varphi_S) \\ &+ x_A\,(\nu^c\nu^c)(\varphi_S\varphi_T)/\Lambda + x_B\,(\nu^c\nu^c)'(\varphi_S\varphi_T)''/\Lambda + x_C\,(\nu^c\nu^c)''(\varphi_S\varphi_T)'/\Lambda \\ &+ x_D\,(\nu^c\nu^c)_{3_S}(\varphi_S\varphi_T)_{3_S}/\Lambda + x_E\,(\nu^c\nu^c)_{3_S}(\varphi_S\varphi_T)_{3_A}/\Lambda + x_F\,(\nu^c\nu^c)_{3_S}\varphi_T\xi/\Lambda + x_G\,(\nu^c\nu^c)_{3_S}\varphi_T\tilde{\xi}/\Lambda \end{aligned} \tag{2.6}$$

where $\delta\varphi_S$, $\delta\xi$ and $\delta\tilde{\xi}$ indicate the shifted vacua of the flavons φ_S, ξ and $\tilde{\xi}$. Taking into account the possibility of absorbing these corrections partly into the LO result, they give rise to four independent additional contributions to m_M which can be effectively parametrized as

$$\delta m_M = \begin{pmatrix} 2\tilde{x}_D & \tilde{x}_A & \tilde{x}_B - \tilde{x}_C \\ \tilde{x}_A & \tilde{x}_B + 2\tilde{x}_C & -\tilde{x}_D \\ \tilde{x}_B - \tilde{x}_C & -\tilde{x}_D & \tilde{x}_A \end{pmatrix} \lambda_c^4 \Lambda \; . \tag{2.7}$$

Compared to these, NLO corrections involving the new flavon ζ are suppressed, since invariance under the Z_2 symmetry requires always an even number of ζ fields and invariance under the Z_3 at least one field of the type φ_S, ξ or $\tilde{\xi}$. The NLO corrections to the charged lepton masses are also the same as in the original model and effects involving ζ can only arise at the level of three flavons.

As we show in appendix A, the VEV of the flavon ζ is naturally also of the order $\lambda_c^2\Lambda$ as the VEVs of the other flavons and the shift of its VEV is of the size δVEV $\sim \lambda_c^2$VEV. We also calculate its effect on the vacuum alignment of the other flavons and show that the results achieved in the original AF model, especially the alignment at LO, remain unchanged.

2.2 Altarelli-Meloni Type Model

The AM model, proposed in [9], possesses as flavor symmetry $A_4 \times Z_4$. To this we add a Z_2 symmetry under which only the Higgs field h_u and the new flavon ζ transform. Compared to the original model, we change the transformation properties of h_u into $1''$ under A_4 and it transforms now trivially under Z_4. The flavon ζ is a $1'$ under A_4 and acquires a phase i under Z_4. The transformation properties of leptonic superfields, MSSM Higgs and flavons can be found in table 2. The Dirac neutrino coupling is at LO given by

$$y_\nu(\nu^c l)h_u\zeta/\Lambda \; , \tag{2.8}$$

which leads to the same Dirac mass matrix m_D as in the original model, suppressed by the factor λ_c^2 for $z/\Lambda \approx \lambda_c^2$. The Majorana mass terms for the RH neutrinos remain unaffected

Field	l	e^c	μ^c	τ^c	ν^c	h_d	h_u	φ_T	ξ'	φ_S	ξ	ζ
A_4	3	1	1	1	3	1	$1''$	3	$1'$	3	1	$1'$
Z_4	i	1	i	-1	-1	1	1	i	i	1	1	i
Z_2	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$	$-$

Table 2: Particle content of the AM variant. Transformation properties of lepton superfields, MSSM Higgs and flavons under the flavor group $A_4 \times Z_4 \times Z_2$ are shown. The nomenclature is as in table 1. Apart from h_u and the flavon ζ all fields are neutral under the additional Z_2 symmetry. Apart from $A_4 \times Z_4 \times Z_2$, the model also contains a $U(1)_R$ symmetry relevant for the alignment of the vacuum, similar to the AF variant.

by the changes of the model, at LO,

$$M(\nu^c\nu^c) + a\xi(\nu^c\nu^c) + b\,(\nu^c\nu^c\varphi_S) \tag{2.9}$$

so that the contribution from the type I see-saw to the light neutrino masses arises from

$$m_D = y_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{z}{\Lambda} v_u \quad \text{and} \quad m_M = \begin{pmatrix} M + au + 2bv_S & -bv_S & -bv_S \\ -bv_S & 2bv_S & M + au - bv_S \\ -bv_S & M + au - bv_S & 2bv_S \end{pmatrix} \tag{2.10}$$

Thereby the flavon alignment given in [9] is used. Equation (2.10) leads to exact TB mixing in the neutrino sector. For $M \approx \lambda_c^2 \Lambda$, as argued in [9], we find the generic size of the light neutrino masses to be $\lambda_c^2 v_u^2/\Lambda$. The effective dimension-5 operator $lh_u lh_u/\Lambda$ arises in our variant only at the two flavon level

$$\begin{aligned}&(\varphi_T\varphi_T)''(ll)h_u^2/\Lambda^3 + (\varphi_T\varphi_T)'(ll)'h_u^2/\Lambda^3 + (\varphi_T\varphi_T)(ll)''h_u^2/\Lambda^3 + ((\varphi_T\varphi_T)_{3_S}(ll)_{3_S})''h_u^2/\Lambda^3 \\ &+(\xi')^2(ll)h_u^2/\Lambda^3 + \xi'(\varphi_T ll)'h_u^2/\Lambda^3 + \zeta^2(ll)h_u^2/\Lambda^3\end{aligned} \tag{2.11}$$

where we omit order one couplings. Thus, its contributions to the light neutrino masses are of order $\lambda_c^4 v_u^2/\Lambda$, i.e. of the same size as the expected NLO corrections to the type I see-saw contribution, and hence subdominant.

The effect of the introduction of the Z_2 symmetry and the new field ζ on the charged lepton sector is the following: an insertion of three flavons, two of which are ζ, gives a new LO contribution to the electron mass

$$\zeta^2(e^c l\varphi_T)'h_d/\Lambda^3\,. \tag{2.12}$$

Using the same alignment as in [9], its contribution resembles the one from the operator with ξ' instead of ζ and thus gives also a non-vanishing term in the (11) entry of the charged lepton mass matrix. The latter is of the same size as those already encountered in the original version of the AM model. Therefore in the variant of the AM model we are considering the charged lepton mass matrix is also diagonal at LO and the correct hierarchy among the charged lepton masses is predicted.

At NLO, the Dirac couplings of the neutrinos are

$$y_\nu(\nu^c l)\delta\zeta h_u/\Lambda + y_A(\nu^c l)\xi\zeta h_u/\Lambda^2 + y_B(\nu^c l)_{3_S}\varphi_S\zeta h_u/\Lambda^2 + y_C(\nu^c l)_{3_A}\varphi_S\zeta h_u/\Lambda^2 \,. \quad (2.13)$$

The first two contributions can be absorbed into the LO coupling y_ν. Compared to the original model, the other corrections are of the same type and generate the same structure

$$\delta m_D = \begin{pmatrix} 2y_B & -y_B - y_C & -y_B + y_C \\ -y_B + y_C & 2y_B & -y_B - y_C \\ -y_B - y_C & -y_B + y_C & 2y_B \end{pmatrix} \frac{v_S z}{\Lambda^2} v_u \,. \quad (2.14)$$

Note that actually the contribution associated to the coupling y_B is still compatible with TB mixing so that only y_C can lead to deviations from the TB mixing pattern. As we will see in section 4.2, for this reason also the CP asymmetries only depend on the coupling y_C.

All effects to the Majorana mass matrix of the RH neutrinos involving ζ are negligible, since we always need at minimum two fields ζ and additionally have to balance the Z_4 charge of the operator. Thus, the NLO corrections are only those already present in the original model

$$\begin{aligned} &x_A\,(\nu^c\nu^c)\xi^2/\Lambda + x_B\,(\nu^c\nu^c)(\varphi_S\varphi_S)/\Lambda + x_C(\nu^c\nu^c)_{3_S}(\varphi_S\varphi_S)_{3_S}/\Lambda + x_D(\nu^c\nu^c)_{3_S}\varphi_S\xi/\Lambda \\ &+ x_E\,(\nu^c\nu^c)'(\varphi_S\varphi_S)''/\Lambda + x_F\,(\nu^c\nu^c)''(\varphi_S\varphi_S)'/\Lambda \,. \end{aligned} \quad (2.15)$$

The first four contributions can be absorbed into the LO result (or vanish). We do not mention effects from shifts in the vacua of φ_S and ξ, since these effects can in this model also be absorbed into the LO result. The new structures at NLO lead to δm_M of the form

$$\delta m_M = 3 \begin{pmatrix} 0 & x_E & x_F \\ x_E & x_F & 0 \\ x_F & 0 & x_E \end{pmatrix} \frac{v_S^2}{\Lambda} \,. \quad (2.16)$$

For the charged leptons, additional NLO corrections to the muon and the electron mass arise from three and four flavon insertions, respectively, involving the field ζ. The operator

$$\zeta^2(\mu^c l\varphi_S)' h_d/\Lambda^3 \quad (2.17)$$

corrects the muon mass. This type of subleading contribution already exists in the original model such that no new structures are introduced. The NLO corrections to the electron mass are induced through the operator $\zeta^2(e^c l\varphi_T)' h_d/\Lambda^3$, if the shifts in the vacua are taken into account, as well as through the four flavon operators

$$\zeta^2(e^c l(\varphi_T\varphi_S)_{3_S})' h_d/\Lambda^4 + \zeta^2(e^c l(\varphi_T\varphi_S)_{3_A})' h_d/\Lambda^4 + \zeta^2\xi(e^c l\varphi_T)' h_d/\Lambda^4 + \zeta^2\xi'(e^c l\varphi_S) h_d/\Lambda^4 \,. \quad (2.18)$$

All structures arising from these corrections are already generated by the NLO corrections present in the original model so that the analysis given in [9] for the NLO corrections is still valid in the constructed variant.

In appendix B we discuss how to give a VEV of the desired size to the field ζ, the shift of this VEV from NLO corrections as well as the effects of ζ on the flavon superpotential of the original model at LO and NLO.

3. Neutrino Masses and CP Violating Phases in the A_4 Models

The models discussed in the previous section have in common that the Majorana mass matrix of RH neutrinos is of the form

$$m_M = \begin{pmatrix} X+2Z & -Z & -Z \\ -Z & 2Z & X-Z \\ -Z & X-Z & 2Z \end{pmatrix} \tag{3.1}$$

and the neutrino Dirac mass matrix reads

$$m_D = y_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{z}{\Lambda} v_u \tag{3.2}$$

The symmetry of this class of models implies that, at leading order, the neutrino part of the Lagrangian depends only on few parameters: X, Z and y_ν. These parameters are, in general, complex numbers. One can set y_ν real by performing a global phase transformation of the lepton doublet fields. As we will see, CP violating phases, which enter in the CP asymmetries of the RH neutrino decays, are functions of the relative phase between X and Z. The see-saw mechanism for the neutrino mass generation implies that the full parameter space of the neutrino sector can be constrained significantly by the low energy data.

The RH neutrino mass matrix (3.1) is diagonalized by an orthogonal matrix U_{TB}, given in (1.4):

$$\mathrm{diag}(M_1 e^{i\varphi_1}, M_2 e^{i\varphi_2}, M_3 e^{i\varphi_3}) = U_{TB}^T m_M U_{TB}\,, \tag{3.3}$$

where

$$M_1 = |X+3Z| \equiv |X|\,|1+\alpha e^{i\phi}|, \quad \varphi_1 = \arg(X+3Z) \tag{3.4}$$

$$M_2 = |X|, \quad \varphi_2 = \arg(X) \tag{3.5}$$

$$M_3 = |X-3Z| \equiv |X|\,|1-\alpha e^{i\phi}|, \quad \varphi_3 = \arg(3Z-X). \tag{3.6}$$

Here $\alpha \equiv |3Z/X|$ and $\phi \equiv \arg(Z) - \arg(X)$.

A light neutrino Majorana mass term is generated after electroweak symmetry breaking via the type I see-saw mechanism:

$$m_\nu = -m_D^T\, m_M^{-1}\, m_D = U^* \mathrm{diag}\,(m_1, m_2, m_3)\, U^\dagger \tag{3.7}$$

where

$$U = i\, U_{TB}\, \mathrm{diag}\left(e^{i\varphi_1/2}, e^{i\varphi_2/2}, e^{i\varphi_3/2}\right) \tag{3.8}$$

and $m_{1,2,3}$ are the light neutrino masses,

$$m_i \equiv \frac{(y_\nu)^2 v_u^2}{M_i} \left(\frac{z}{\Lambda}\right)^2, \quad i = 1,2,3 \tag{3.9}$$

The i in eq. (3.8) correspond to a unphysical common phase and we will ignore it in what follows. We observe also that one of the phases φ_k, say φ_1, can be considered as a common

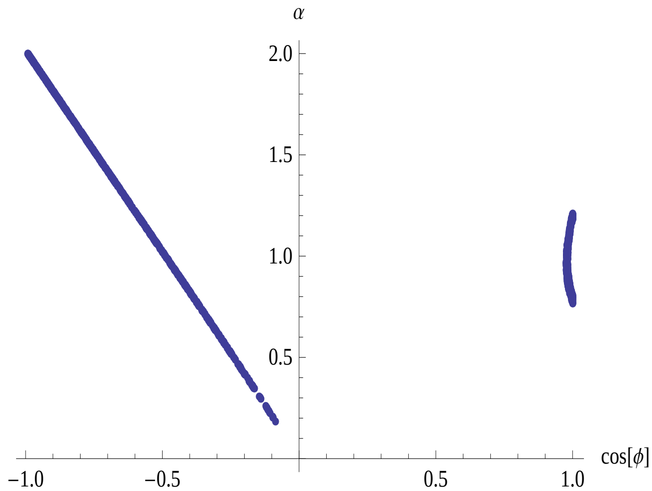

Figure 1: The correlation between the real parameter α and the phase ϕ, which appear in the RH neutrino Majorana mass matrix. The figure is obtained by using the 3σ range of the parameter r given in eq. (3.10). See text for details.

phase of the neutrino mixing matrix, and thus has no physical relevance. In the following we always set $\varphi_1 = 0$.

The parameters $|X|$, α and ϕ defined in (3.4)-(3.6), which determine the RH neutrino mass matrix (3.1), can be constrained by the neutrino oscillation data. More specifically, we have for the ratio [9]:

$$r \equiv \frac{\Delta m^2_{\odot}}{|\Delta m^2_{\rm A}|} \;=\; \frac{(1+\alpha^2 - 2\,\alpha\,\cos\phi)(\alpha + 2\cos\phi)}{4\,|\cos\phi|}\,, \tag{3.10}$$

where $\Delta m^2_{\odot} = \Delta m^2_{21} \equiv m^2_2 - m^2_1 > 0$ and $|\Delta m^2_{\rm A}| = |\Delta m^2_{31}| \cong |\Delta m^2_{32}|$ are the $\nu-$mass squared differences responsible respectively for solar and atmospheric neutrino oscillations. Since the value of r is fixed by the data, this relation implies a strong correlation between the values of the parameters α and $\cos\phi$. Let us note that the sign of $\sin\phi$ cannot be constrained by the low energy data. As we will see later, the sign of $\sin\phi$ is fixed by the sign of the baryon asymmetry of the Universe, computed in the leptogenesis scenario.

At 3σ, the following experimental constraints must be satisfied [1]:

$$\begin{aligned} \Delta m^2_{\odot} &> 0 \\ |\Delta m^2_{\rm A}| &= (2.41 \pm 0.34) \times 10^{-3}\ {\rm eV}^2 \\ r &= 0.032 \pm 0.006\ . \end{aligned} \tag{3.11}$$

In Fig. 1 we show the correlation between α and $\cos\phi$, following from (3.10) taking into account (3.11). Depending on the sign of $\cos\phi$, the parameter space is divided into two physically distinctive parts: $\cos\phi > 0$ corresponds to light neutrino mass spectrum with normal ordering (NO), whereas for $\cos\phi < 0$ one obtains neutrino mass spectrum with inverted ordering (IO) .

The main difference between the models we are discussing and the original models reported in [8] and [9] is in the mass scale of the RH neutrino fields. In the models considered here the predicted RH neutrino masses are always rescaled by the additional factor $(\lambda_c^2)^2 \sim 10^{-3}$. Depending on the value of the neutrino Yukawa coupling y_ν, in our model the lightest RH Majorana neutrino mass can be in the range from $(10^{11} \div 10^{12})$ GeV, and up to 10^{13} GeV for a neutrino Yukawa coupling $y_\nu \sim \mathcal{O}(1)$.

For neutrino mass spectrum with NO, the RH neutrino masses show approximately the following partial hierarchy [16]: $M_1 \approx 2M_2 \approx 10M_3$. The lightest neutrino mass m_1, compatible with the experimental constraints given in (3.11), takes values in the interval $3.8 \times 10^{-3}\mathrm{eV} \lesssim m_1 \lesssim 6.9 \times 10^{-3}\mathrm{eV}$. This implies that the light neutrino mass spectrum is with partial hierarchy [7]. For the sum of the neutrino masses we have:

$$6.25 \times 10^{-2}\ \mathrm{eV} \lesssim m_1 + m_2 + m_3 \lesssim 6.76 \times 10^{-2}\ \mathrm{eV}\,. \tag{3.12}$$

In the case of IO spectrum, the overall range of variability of the lightest neutrino mass, m_3, is the following: $0.02\,\mathrm{eV} \lesssim m_3 \leq 0.50\,\mathrm{eV}$, where only the lower bound follows from the low energy constraints. The upper bound was chosen by us to be compatible with the "conservative" cosmological upper limit on the sum of the neutrino masses [24, 25]. Thus, the light neutrino mass spectrum can be with partial hierarchy or quasidegenerate. If the spectrum is with partial hierarchy (i.e. $0.02\,\mathrm{eV} \lesssim m_3 < 0.10$ eV), for the RH Majorana neutrino masses, to a good approximation, we have: $M_1 \cong M_2 \cong M_3/3$. Quasidegenerate light neutrino mass spectrum implies that, up to corrections $\sim \mathcal{O}(r)$, one has $M_1 \cong M_2 \cong M_3$. The sum of the light neutrino masses in the case of IO spectrum is predicted to satisfy:

$$m_1 + m_2 + m_3 \gtrsim 0.125\ \mathrm{eV}\,. \tag{3.13}$$

We give below the expressions for the lightest neutrino mass in the NO and IO spectrum as functions of α and r. Recall that for fixed value of r, all the parameter space and the associated phenomenology is characterized by the parameter α. In the numerical examples reported in the following, we always use the best fit value of the ratio r: $r = 0.032$. By expanding with respect to r, we get for the square of the lightest neutrino mass:

$$m_1^2 \;=\; \Delta m_{\mathrm{A}}^2 r \left(\frac{1}{1+2\alpha^2} + \frac{2(1+\alpha^2)r}{(1+2\alpha^2)^3} \right)\,, \quad \text{NO spectrum}\,; \tag{3.14}$$

$$m_3^2 \;= |\Delta m_{\mathrm{A}}^2| \left(\frac{1}{2\alpha^2} + \frac{(1+\alpha^2)r}{\alpha^2(1+2\alpha^2)} \right)\,, \quad \text{IO spectrum}\,. \tag{3.15}$$

For $\alpha = 1$ the expression for m_1^2 reduces to the one obtained in [9].

In the class of models we are considering, the three light neutrino masses obey the general sum rule (valid for both types of spectrum) [9, 11]:

$$\frac{e^{i\varphi_3}}{m_3} \;=\; \frac{1}{m_1} - \frac{2e^{i\varphi_2}}{m_2} \tag{3.16}$$

[7] This was noticed also in [23].

This equation implies a strong correlation between the neutrino masses and the Majorana phases arising from the RH neutrino mass matrix. The Majorana phases are responsible for CP violation in leptogenesis and therefore we will discuss them in detail in the following subsection.

3.1 The Majorana CP Violating Phases

In the following, we use the standard parametrization of the PMNS matrix (see, e.g. [26, 27]):

$$U_{\rm PMNS} \;=\; \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} {\rm diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}) \tag{3.17}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\theta_{ij} \in [0, \pi/2]$, $\delta \in [0, 2\pi]$ is the Dirac CP violating phase and α_{21} and α_{31} are the two Majorana CP violating phases, $\alpha_{21,31} \in [0, 2\pi]$. From the see-saw mass formula (3.7) one can read directly the form of the neutrino mixing matrix that arises at leading order in perturbation theory. Taking into account the standard parametrization (3.17) and (1.4), the PMNS matrix is indeed:

$$U_{\rm PMNS} = {\rm diag}(1, 1, -1) U_{TB}\, {\rm diag}(1, e^{i\varphi_2/2}, e^{i\varphi_3/2}) \tag{3.18}$$

From eqs. (3.17) and (3.18) we identify the "low energy" Majorana phases as

$$\alpha_{21} = \varphi_2 \tag{3.19}$$

$$\alpha_{31} = \varphi_3 \tag{3.20}$$

We remark that, at this order of perturbation theory, the CHOOZ mixing angle, θ_{13}, is always zero as a consequence of the TB form of the neutrino mixing matrix, imposed by the broken A_4 discrete symmetry.

In the models under discussion the Majorana phases α_{21} and α_{31} can also be constrained by using the neutrino oscillation data [8]. After some algebraic manipulation, we arrive at the following relations between the CP violating phases α_{21} and α_{31} and the parameters α and ϕ of the model:

$$\tan\alpha_{21} \;=\; -\,\frac{\alpha\sin\phi}{1+\alpha\cos\phi} \tag{3.21}$$

$$\tan\alpha_{31} \;=\; 2\,\frac{\alpha\sin\phi}{\alpha^2-1}\,, \tag{3.22}$$

where α and $\cos\phi$ satisfy eq. (3.10).

[8] Let us recall that the probabilities of oscillations involving the flavour neutrinos do not depend on the Majorana phases of the PMNS matrix [4,28]. Thus, the Majorana phases cannot be *directly* constrained by the neutrino oscillation data.

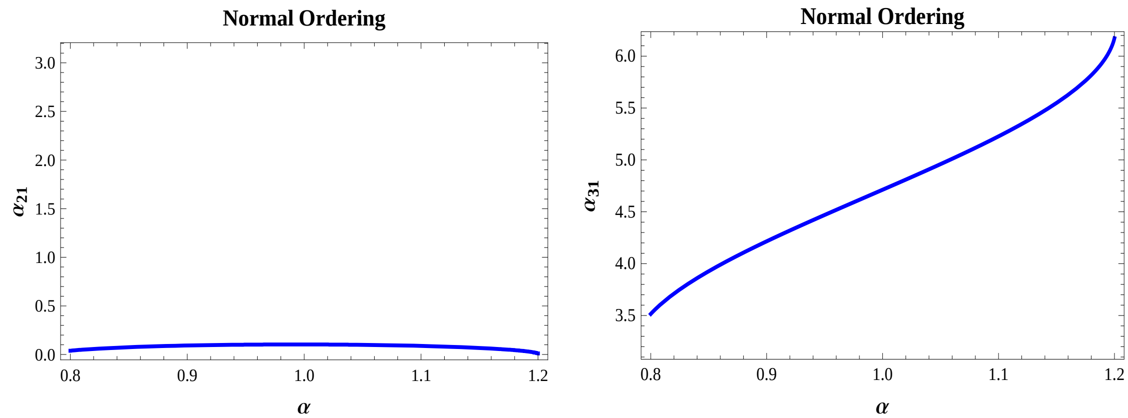

Figure 2: The Majorana phases α_{21} and α_{31} in the case of a light neutrino mass spectrum with normal ordering. The parameter r is set to its best fit value, $r = 0.032$. The solutions of equations (3.21) and (3.22) shown in the figure correspond to $\sin\phi < 0$. See text for details

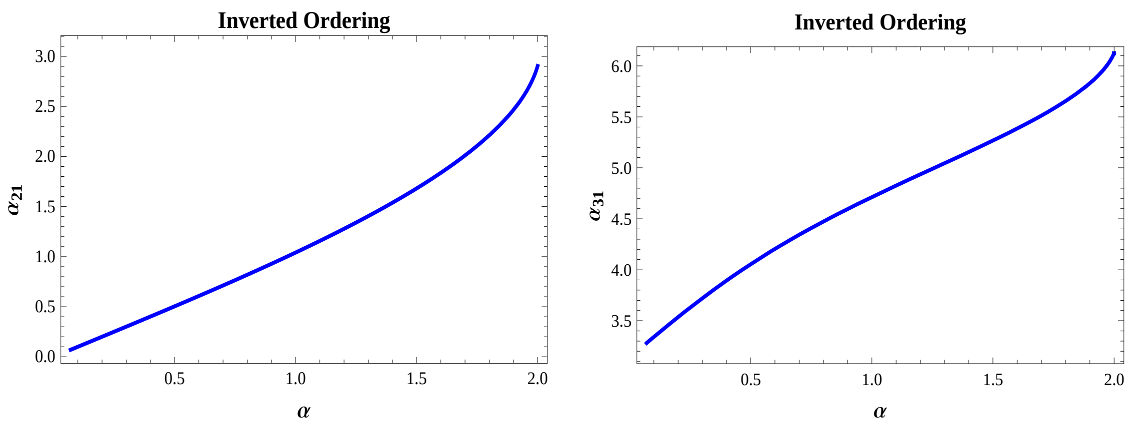

Figure 3: The same as in Fig. 2, but for a light neutrino mass spectrum with inverted ordering.

In the case of NO spectrum, we have $\phi = 0 \pm \varepsilon, 2\pi \pm \varepsilon$, with $\varepsilon < 0.2$ and $0.8 \lesssim \alpha \lesssim 1.2$ (see Fig. 1). If $\varepsilon \cong 0$, the two CP violating phases become unphysical. No CP violation is possible in this case. As regards the IO light neutrino mass spectrum, it is easy to show that [9] $2\cos\phi \approx -\alpha$. The correction, $\delta_\alpha(\alpha)$, which appears in the right-hand side of this equation, is given by

$$\delta_\alpha(\alpha) \;=\; \frac{2\alpha r}{1+2\alpha^2}\left(1 - \frac{2(1+\alpha^2)r}{(1+2\alpha^2)^2}\right) \tag{3.23}$$

For light neutrino mass spectrum with inverted ordering, the parameter α varies in the interval $0.07 \lesssim \alpha \lesssim 2$, where the lower limit of α comes from the indicative upper bound on the absolute neutrino mass scale used by us, $m_{1,2,3} \lesssim 0.5$ eV.

In Figs 2 and 3 we show the behavior of the Majorana phases α_{21} and α_{31} as functions of α, for the NO and IO mass spectrum, respectively. We choose $\sin\phi < 0$ in (3.21) and (3.22). As we will see, this is dictated by reproducing correctly the sign of the baryon asymmetry. On the other hand, the relative sign of $\sin\alpha_{21}$ and $\sin\alpha_{31}$ is fixed by the requirement that the sum rule in eq. (3.16) is satisfied. In the case of NO spectrum, the phase α_{21} is close to zero. The maximum value of α_{21} is obtained for $\alpha \cong 1$. At $\alpha = 1$ we have approximately $\alpha_{21} = \sqrt{r/3} \cong 0.1$. The other Majorana phase α_{31} can assume large CP violating values. The largest $|\sin\alpha_{31}|$ is reached for $\alpha = 1$: at this value of α we have $\sin\alpha_{31} = -1$.

If the light neutrino mass spectrum is with IO, both phases can have large CP violating values. We get $\sin\alpha_{21} = 1$ and $\sin\alpha_{31} = -1$ for $\alpha \approx \sqrt{2}$ and $\alpha = 1$, respectively.

The Majorana phase α_{21} can be probed, in principle, in the next generation of experiments searching for neutrinoless double beta decay [29]. Below we give the expression of the effective Majorana mass m_{ee} predicted in the class of models we are considering in the cases of neutrino mass spectrum with normal and inverted ordering [26]:

$$m_{ee} \cong \left| m_1 \cos^2\theta_{12} + \sqrt{m_1^2 + \Delta m_\odot^2}\, \sin^2\theta_{12} e^{i\alpha_{21}} \right| , \quad \text{NO}\,; \tag{3.24}$$

$$m_{ee} \cong \sqrt{m_3^2 + |\Delta m_{\rm A}^2|}\, \left|\cos^2\theta_{12} + e^{i\alpha_{21}} \sin^2\theta_{12}\right| , \quad \text{IO}\,. \tag{3.25}$$

We recall that in the class of models under discussion, $\sin^2\theta_{12} = 1/3$, $\cos^2\theta_{12} = 2/3$ and a non-zero value of θ_{13} arises only due to the NLO corrections. As a consequence, the predicted value of θ_{13} is relatively small, $\theta_{13} \sim \mathcal{O}(\lambda_c^2 \sim 0.04)$. Thus, the terms $\sim \sin^2\theta_{13}$ in m_{ee} give a negligible contribution. Further, since the Majorana phase $\alpha_{21} \cong 0$ (see Fig. 2), the two terms in the expression for m_{ee} in the case of NO spectrum add up. As a consequence, we have:

$$m_{ee} \cong \left| \frac{2}{3}\, m_1 + \frac{1}{3}\, \sqrt{m_1^2 + \Delta m_\odot^2} \right| , \quad \text{NO}\,, \tag{3.26}$$

where 3.8×10^{-3} eV $\lesssim m_1 \lesssim 6.9 \times 10^{-3}$ eV.

We show in Fig. 4 the effective Majorana mass m_{ee} and the lightest neutrino mass m_1 versus α, for the NO spectrum. In this case, m_{ee} takes values in the interval: 6.5×10^{-3} eV $\lesssim m_{ee} \lesssim 7.5 \times 10^{-3}$ eV. Similar conclusion has been reached in [23]. In what concerns the IO spectrum, the full range of variability of the effective Majorana mass, compatible with neutrino oscillation data, is predicted to be

$$\frac{1}{3}\, \sqrt{m_3^2 + |\Delta m_{\rm A}^2|} \lesssim m_{ee} \lesssim \sqrt{m_3^2 + |\Delta m_{\rm A}^2|}\,, \text{ with } \; m_3 \gtrsim 0.02 \text{ eV}\,, \;\text{IO}\,. \tag{3.27}$$

For $m_3 \gtrsim 0.02$ eV, this implies $m_{ee} \gtrsim 0.018$ eV (see also [9]).

4. Leptogenesis

In this section we compute the baryon asymmetry within the AF and AM type models defined in Section 2. As we have already noticed earlier, leptogenesis cannot be realized if we take into account only the leading order contribution to the neutrino superpotential. In

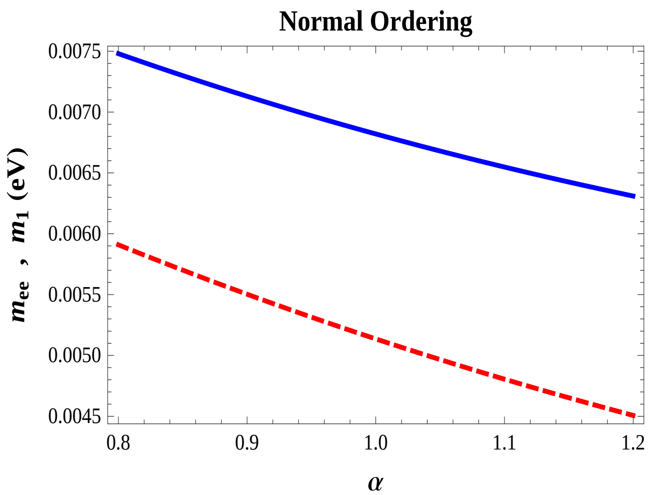

Figure 4: The effective Majorana mass m_{ee} (blue continuous line) and lightest neutrino mass m_1 (red dashed line) in the case of a light neutrino mass spectrum with normal ordering. In both cases $\Delta m^2_{\rm A}$ and r are fixed to their best fit values.

order to generate a sufficiently large CP asymmetry, higher order corrections to the Dirac mass matrix of neutrinos must be taken into account. The RH neutrino mass spectrum in this class of models is not strongly hierarchical. Consequently, the standard thermal leptogenesis scenario in which the relevant lepton CP violating asymmetry is generated in the decays of the lightest RH (s)neutrino only is not applicable and one has to take into account the contribution from the out of equilibrium decays of the heavier RH (s)neutrinos. The lepton asymmetry thus produced in the decays of all heavy RH (s)neutrinos ν^c_i ($\tilde{\nu}^c_i$), $i = 1, 2, 3$, is converted into a baryon number by sphaleron interactions. The neutrino and sneutrino CP asymmetry ϵ_i, which are equal for lepton and slepton final states, is the following [30]:

$$\epsilon_i \;=\; \frac{1}{8\pi v_u^2} \sum_{j \neq i} \frac{{\rm Im}[(\hat{m}_D \hat{m}_D^\dagger)^2_{ji}]}{(\hat{m}_D \hat{m}_D^\dagger)_{ii}} \, f\left(\frac{m_i}{m_j}\right) \;, \tag{4.1}$$

where

$$\hat{m}_D = U^\dagger m_D \tag{4.2}$$

is the neutrino Dirac mass matrix in the mass eigenstate basis of RH neutrinos, and m_i, $i = 1, 2, 3$ are the light neutrino masses. The matrix U and the masses m_i coincide with those given in eqs. (3.8) and (3.9), respectively. The loop function $f(m_i/m_j)$ is defined as

$$f(x) \;\equiv\; -x\left(\frac{2}{x^2-1} + \log\left(1+\frac{1}{x^2}\right)\right) \tag{4.3}$$

This function depends strongly on the hierarchy of light neutrino masses. It can lead to a strong enhancement of the CP asymmetries if the light neutrino masses m_i and m_j are nearly degenerate. As we have seen earlier, the neutrinos can be quasidegenerate in mass in the case of IO spectrum. In this case we have to a good approximation $f(m_i/m_j) \cong -f(m_j/m_i)$.

We recall that in the case of IO spectrum, the lightest two heavy Majorana (s)neutrinos, $\nu^c_{1,2}$ ($\tilde{\nu}^c_{1,2}$), have very close masses. However, the conditions for resonant leptogenesis [31] are not satisfied in the models under consideration. Indeed, in all the region of the relevant parameter space we have $0.2 \lesssim \alpha \lesssim 2$. Correspondingly, the relative mass difference of the two heavy Majorana (s)neutrinos in question is

$$\left|\frac{M_2 - M_1}{M_1}\right| = 1 - \frac{m_1}{m_2} \cong (2 \div 14) \times 10^{-3} \gg \max\left|\frac{(\hat{m}_D \hat{m}_D^\dagger)_{12}}{16\pi^2 v_u^2}\right| \approx \frac{\lambda_c^6}{\pi^2} \approx 10^{-5} \tag{4.4}$$

Under the above condition, the CP asymmetries for each (s)neutrino decay can be computed in perturbation theory as the interference between the tree level and one loop diagrams (see, e.g. [32, 33]).

The general expression for the baryon asymmetry [34], in which each RH (s)neutrino gives a non-negligible contribution, can be cast in the following form:

$$Y_B \equiv \frac{n_B - \bar{n}_B}{s} \;=\; -1.48 \times 10^{-3} \sum_{i,j=1}^{3} \epsilon_i \eta_{ij} \tag{4.5}$$

where η_{ij} is an efficiency factor that accounts for the effects of washout due to the $\Delta L = 1$ interactions of ν^c_j and $\tilde{\nu}^c_j$ of the asymmetry Y_{l_i}, generated in the decays $\nu^c_i \to l_i\, h_u$, $\tilde{l}_i\, \tilde{h}_u$ and $\tilde{\nu}^c_i \to \tilde{l}_i\, h_u$, $l_i\, \tilde{h}_u$. They take into account also the decoherence effects on l_i caused by the ν^c_j and $\tilde{\nu}^c_j$ ($j \neq i$) interactions. We refer in the following discussion only to the leptons number densities. The same considerations apply for the interactions involving slepton states.

The computation of the efficiency factors in the models under discussion is considerably simplified [35, 36] (see also [37]). This is due to the fact that to leading order, the heavy Majorana neutrinos ν^c_1, ν^c_2 and ν^c_3, as can be shown, couple to orthogonal leptonic states. As a consequence, the Boltzmann evolutions of the three lepton CP violating asymmetries, associated with the indicated three orthogonal leptonic states, are practically independent. Taking into account the above considerations, one can compute the total baryon asymmetry as an incoherent sum of the contributions arising from decays of each of the three heavy RH neutrinos:

$$Y_B \approx \sum_{i=1}^{3} Y_{Bi}\ , \tag{4.6}$$

where

$$Y_{Bi} \;\equiv\; -1.48 \times 10^{-3} \epsilon_i\, \eta_{ii}\ . \tag{4.7}$$

In the class of models considered the RH neutrino mass scale is set below 10^{14} GeV, preventing possible washout effects from $\Delta L = 2$ scattering processes. In this case, the efficiency factors η_{ii} can be expressed only in terms of the washout mass parameters $\widetilde{m}_i$ [20]:

$$\eta_{ii} \;=\; \left(\frac{3.3 \times 10^{-3}\,\mathrm{eV}}{\widetilde{m}_i} + \left(\frac{\widetilde{m}_i}{0.55 \times 10^{-3}\,\mathrm{eV}}\right)^{1.16}\right)^{-1}\ , \tag{4.8}$$

where

$$\widetilde{m}_i \;\equiv\; \frac{(\hat{m}_D \hat{m}_D^\dagger)_{ii}}{M_i}\;. \tag{4.9}$$

Here $\hat{m}_D$ is the neutrino Dirac mass matrix in the basis in which the Majorana mass matrix of RH neutrinos is diagonal with real eigenvalues (see eq. (4.2).

4.1 Leptogenesis in the Variant of AF Model

In this Section we compute the baryon asymmetry for the AF type model, in the one-flavor leptogenesis regime.

In the basis in which the RH Majorana neutrino mass term given in (2.2) is diagonal, the relevant matrix that enters into the expression of the leptogenesis CP asymmetries (4.1) is given by

$$\begin{aligned}\hat{m}_D\hat{m}_D^\dagger \;=\; & \mathbf{1}\left(\frac{z}{\Lambda}\right)^2 y_\nu^2 v_u^2 \\ & + \begin{pmatrix} 2\,\mathrm{Re}(y_A) & 2\sqrt{2}e^{i\frac{\alpha_{21}}{2}}\,\mathrm{Re}(y_A) & \frac{2}{\sqrt{3}}e^{i\frac{\alpha_{31}}{2}}\,\mathrm{Re}(y_B) \\ 2\sqrt{2}e^{-i\frac{\alpha_{21}}{2}}\,\mathrm{Re}(y_A) & 0 & -2\sqrt{\frac{2}{3}}e^{i\frac{\alpha_{31}-\alpha_{21}}{2}}\,\mathrm{Re}(y_B) \\ \frac{2}{\sqrt{3}}e^{-i\frac{\alpha_{31}}{2}}\,\mathrm{Re}(y_B) & -2\sqrt{\frac{2}{3}}e^{i\frac{\alpha_{21}-\alpha_{31}}{2}}\,\mathrm{Re}(y_B) & -2\,\mathrm{Re}(y_A) \end{pmatrix}\left(\frac{v_T}{\Lambda}\right)\left(\frac{z}{\Lambda}\right)^2 y_\nu v_u^2\end{aligned} \tag{4.10}$$

where y_A and y_B are the higher order (complex) Yukawa couplings defined in (2.4) and (2.5). We can take all flavon VEVs real without loss of generality.

The CP asymmetries ϵ_k $(k=1,2,3)$ can be written in the following way:

$$\epsilon_1 = -\frac{1}{6\pi}\left(\frac{z}{\Lambda}\right)^2\left(\frac{v_T}{\Lambda}\right)^2\left(6f(m_1/m_2)\sin\alpha_{21}\,\mathrm{Re}(y_A)^2 + f(m_1/m_3)\sin\alpha_{31}\,\mathrm{Re}(y_B)^2\right) \tag{4.11}$$

$$\epsilon_2 = \frac{1}{3\pi}\left(\frac{z}{\Lambda}\right)^2\left(\frac{v_T}{\Lambda}\right)^2\left(3f(m_2/m_1)\sin\alpha_{21}\,\mathrm{Re}(y_A)^2 + f(m_2/m_3)\sin(\alpha_{21}-\alpha_{31})\,\mathrm{Re}(y_B)^2\right) \tag{4.12}$$

$$\epsilon_3 = \frac{1}{6\pi}\left(\frac{z}{\Lambda}\right)^2\left(\frac{v_T}{\Lambda}\right)^2\left(2f(m_3/m_2)\sin(\alpha_{31}-\alpha_{21}) + f(m_3/m_1)\sin\alpha_{31}\right)\,\mathrm{Re}(y_B)^2 \tag{4.13}$$

where m_k are the LO neutrino masses and $z/\Lambda \approx v_T/\Lambda \approx \lambda_c^2$. Thus, in the model under consideration we have

$$|\epsilon_k| \;\propto\; \lambda_c^8 \;\approx\; 6\times 10^{-6}\,,\quad k=1,2,3\;. \tag{4.14}$$

This is the order of magnitude we expect for the CP asymmetry if we require successful leptogenesis. Depending on the loop factor $f(m_i/m_j)$ (eq. 4.3) and the values of the Majorana phases, the CP asymmetry can be enhanced or suppressed.

The washout mass parameters, associated to each of the three lepton asymmetries, are given by:

$$\widetilde{m}_1 \;=\; m_1(1+\mathcal{O}(\lambda_c^2)) \tag{4.15}$$

$$\widetilde{m}_2 \;=\; m_2(1+\mathcal{O}(\lambda_c^2)) \tag{4.16}$$

$$\widetilde{m}_3 \;=\; m_3(1+\mathcal{O}(\lambda_c^2)) \tag{4.17}$$

We see that the washout mass parameters, to a good approximation, coincide with the neutrino masses.

Results for NO Spectrum

We study the baryon asymmetry in the region of the parameter space corresponding to a neutrino mass spectrum with normal ordering: $0.8 \lesssim \alpha \lesssim 1.2$. The lightest RH Majorana neutrino in this scenario is ν_3^c. The Majorana phases, that provide the requisite CP violation for a successful leptogenesis, are solutions of equations (3.21) and (3.22) corresponding to $\sin\phi < 0$. The dependence on α of each of the two CP violating phases is shown in Fig. 2. We recall that only the solutions corresponding to $\sin\phi < 0$ give the correct sign of the total baryon asymmetry.

We show in Fig. 5, left panel, the dependence of the baryon asymmetry Y_B on the parameter α. The individual contributions to Y_B from the decays of each of the three RH Majorana neutrinos are also shown. The term Y_{B3}, originating from the lightest RH neutrino decays, is suppressed by largest washout effects, with respect to Y_{B1} and Y_{B2} (see (4.17)).

The contribution to the total baryon asymmetry given by Y_{B1} shows an interplay between two independent terms proportional to y_A and y_B, respectively. These two terms have always the same signs and are of the same order of magnitude. The suppression due to the Majorana phase $\alpha_{21} \lesssim 0.1$ of the term proportional to y_A is compensated by the enhancement due to the loop factor: we find that $6f(m_1/m_2)/f(m_1/m_3) \cong -(8\div 20)$. The same considerations apply to Y_{B2}. Now $\sin\alpha_{21}$ and $\sin(\alpha_{21}-\alpha_{31})$ have the same sign and the ratio of the corresponding loop factors is approximately $3f(m_2/m_1)/f(m_2/m_3) \cong (20\div 30)$.

In conclusion, in the case of NO light neutrino mass spectrum, each of the two Majorana phases α_{21} and α_{31}, having values within the ranges allowed by neutrino oscillation data (see Fig. 2), can provide the CP violation which is required in order to have successful leptogenesis. Even in the case in which the term proportional to $\sin\alpha_{31}$ in the CP asymmetries is strongly suppressed (which corresponds to the case of strong fine-tuning of $y_B \ll 1$), successful baryogenesis can be naturally realised for values of the Majorana phase $\alpha_{21} \approx (0.04 \div 0.10)$ and a moderately large neutrino Yukawa coupling $y_A \sim (2.5 \div 3.0)$.

Results for IO Spectrum

We now study in detail the region of the parameter space for which the neutrino mass spectrum is with inverted ordering and is hierarchical. This scenario is realized for $0.2 < \alpha \lesssim 2$. In the following, we report the behavior of the baryon asymmetry in all the interval of variability of α, compatible with an IO neutrino spectrum and for which the computation of the CP asymmetry can be done in perturbation theory. Thus, the results we show for $0.07 < \alpha \lesssim 0.2$ should be valid provided the renormalisation group (RG) effects [38] are sufficiently small in the indicated region.

In Fig. 5, right panel, we plot the different contributions to the baryon asymmetry, as we have done previously for the normal hierarchical mass spectrum.

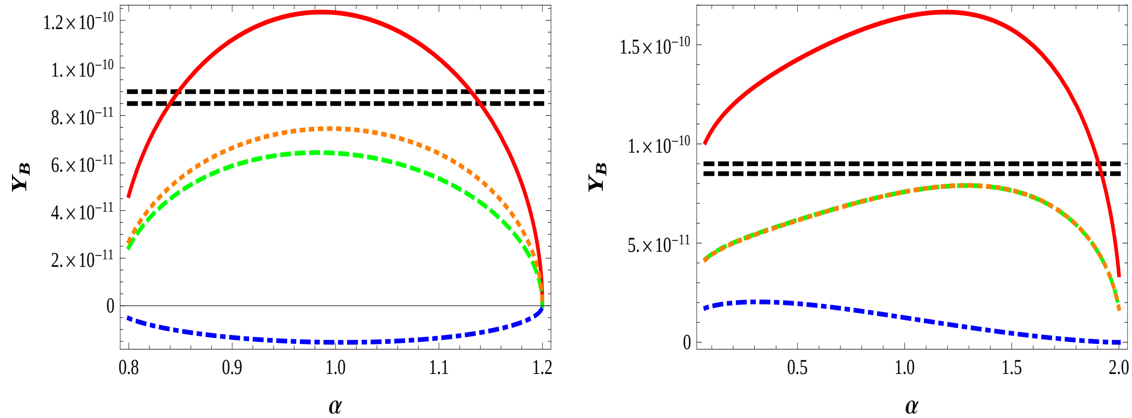

Figure 5: AF type model: baryon asymmetry versus α in the cases of neutrino mass spectrum with normal (left panel) and inverted (right panel) ordering. In each plot we show: i) the total baryon asymmetry Y_B (red continuous curve), ii) Y_{B1} (green dashed curve), iii) Y_{B2} (orange dotted curve) and iv) Y_{B3} (blue dot-dashed curve). On the right panel, the lines corresponding to Y_{B1} and Y_{B2} overlap. In both cases $\sin\phi < 0$ and $\Delta m^2_{\rm A}$ and r are fixed at their best fit values. The results shown in the left (right) panel correspond to $y_A = 2.5$ and $y_B = 3$ ($y_A = 0.4$ and $y_B = 2$). The horizontal dashed lines represent the allowed range of the observed value of Y_B, $Y_B \in [8.5, 9] \times 10^{-11}$.

The Majorana CP violating phases which enter into the expressions for the CP asymmetries are reported in Fig. 3. The solutions of equations (3.21) and (3.22) corresponding to $\sin\phi < 0$ must be used also in this case in order to obtain the correct sign of the baryon asymmetry. Now ν^c_3 is the heaviest RH Majorana neutrino and the washout effects for the CP asymmetry generated in the decays of this state are less strong since they are controlled to LO by the lightest neutrino mass m_3: $\widetilde{m}_3 = m_3$. We note, however, that also in this scenario the contribution of the term Y_{B3} in Y_B is always much smaller than the contribution of the other two terms Y_{B1} and Y_{B2}. This is a consequence of the strong enhancement in the self energy part of the loop function that enters into the expressions for Y_{B1} and Y_{B2}. Indeed, if the spectrum is inverted hierarchical, we have $f(m_1/m_2) \cong -f(m_2/m_1) \approx 50 f(m_3, m_{1,2})$. For this reason the CP violating phase α_{31} gives, in general, a subdominant contribution in the CP asymmetries ϵ_1 and ϵ_2 when the Yukawa couplings y_A and y_B are of the same order of magnitude. This conclusion is valid even in the region of the parameter space where $\alpha_{31} \approx 3\pi/2$.

The analysis of all the parameter space defined by α, compatible with low energy neutrino oscillation data, in the AF type model, show that in both the normal and inverted patterns of light neutrino masses, the Majorana phases can provide enough CP violation in order to have successful leptogenesis, even in the case in which only one of the phases α_{21} and α_{31}, effectively, contributes in the generation of the CP asymmetry.

4.2 Leptogenesis in the Variant of AM Model

In this section we study the generation of the baryon asymmetry of the Universe in the variant of the AM model considered by us. We work in the one-flavour leptogenesis ap-

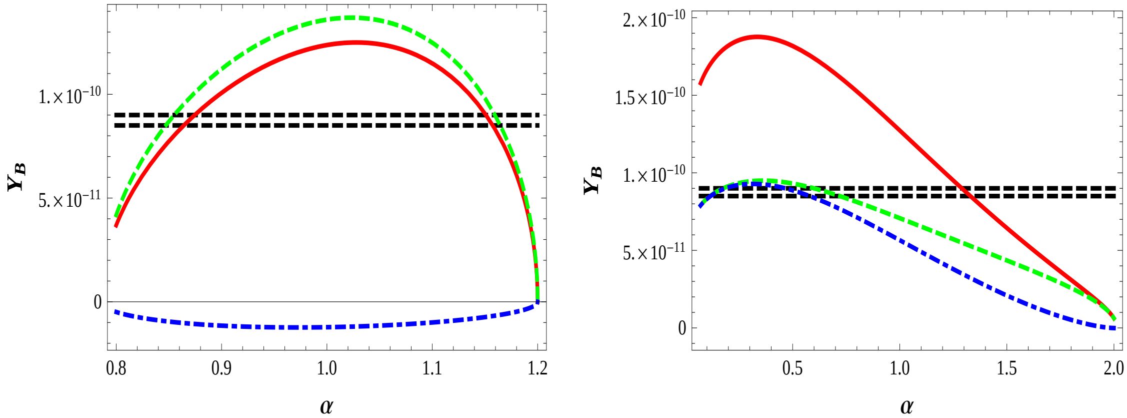

Figure 6: The same as in Fig. 5 but for the AM type model. The Yukawa coupling y_C is set to $y_C = 2$ in both the figures.

proximation. The quantity relevant for the calculation of the CP asymmetries in this case is:

$$\begin{aligned} \hat{m}_D \hat{m}_D^\dagger &= \mathbf{1} \left(\frac{z}{\Lambda}\right)^2 y_\nu^2 v_u^2 \\ &+ \begin{pmatrix} 6\,\mathrm{Re}(y_B) & 0 & 2\sqrt{3} e^{i\frac{\alpha_{31}}{2}}\,\mathrm{Re}(y_C) \\ 0 & 0 & 0 \\ 2\sqrt{3} e^{-i\frac{\alpha_{31}}{2}}\,\mathrm{Re}(y_C) & 0 & -6\,\mathrm{Re}(y_B) \end{pmatrix} \left(\frac{v_S}{\Lambda}\right) \left(\frac{z}{\Lambda}\right)^2 y_\nu v_u^2 \end{aligned} \tag{4.18}$$

where the y_B and y_C are defined in eqs. (2.13) and (2.14). Again we can choose all flavon VEVs to be real without loss of generality.

The CP asymmetries in this model are given by

$$\epsilon_1 = -\frac{3}{2\pi} \left(\frac{z}{\Lambda}\right)^2 \left(\frac{v_S}{\Lambda}\right)^2 f(m_1/m_3) \sin(\alpha_{31})\,\mathrm{Re}(y_C)^2 \tag{4.19}$$

$$\epsilon_2 = 0 \tag{4.20}$$

$$\epsilon_3 = \frac{3}{2\pi} \left(\frac{z}{\Lambda}\right)^2 \left(\frac{v_S}{\Lambda}\right)^2 f(m_3/m_1) \sin(\alpha_{31})\,\mathrm{Re}(y_C)^2 \tag{4.21}$$

where $m_{1,3}$ are again the LO neutrino masses (see eq. (3.9)). The leptogenesis CP violating phase now coincides with the Majorana phase α_{31}. Moreover, the CP asymmetries $\epsilon_{1,3} \neq 0$ are controlled by only one parameter, y_C, of the matrix of neutrino Yukawa couplings (2.14), the reason being that only this parameter breaks the TB form of the latter.

As we see, the heavy RH Majorana neutrino ν_2^c "decouples": the CP violating lepton asymmetry is produced in the out of equilibrium decays of the heavy Majorana neutrinos ν_1^c and ν_3^c alone. This constitutes a major difference with the variant of the AF model, analyzed in the preceding subsection. After the lepton asymmetries are converted into a baryon asymmetry by sphaleron processes, the final matter-antimatter asymmetry of the

Universe can be estimated as:

$$Y_B \equiv Y_{B1} + Y_{B3} \tag{4.22}$$

where Y_{Bi}, for $i = 1, 3$, are given in eq. (4.7). The LO washout mass parameters $\widetilde{m}_{1,3}$ are the same as in the variant of the AF model:

$$\widetilde{m}_1 = m_1(1 + \mathcal{O}(\lambda_c^2)) \tag{4.23}$$

$$\widetilde{m}_3 = m_3(1 + \mathcal{O}(\lambda_c^2)) \tag{4.24}$$

In Fig. 6 we show the dependence of the baryon asymmetry on the parameter α in the cases of neutrino mass spectrum with normal and inverted ordering. Both types of spectrum are allowed in the model considered. The ranges of possible values of the Majorana phase α_{31} which provides the correct sign of the baryon asymmetry are shown for the NO and IO spectra in Figs. 2 and 3 (they are the same as for the AF type model).

We observe that, as in the variant of the AF model, the suppression of the term Y_{B3} with respect to Y_{B1} in the case of NO spectrum is due to the relatively larger washout effects in the generation of the asymmetry ϵ_3. The maximum of the total baryon asymmetry Y_B is reached for $\alpha \approx 1$ where the CP violating Majorana phase $\alpha_{31} \cong 3\pi/2$. (see Fig. 2, right panel).

In what concerns the IO spectrum, the two terms Y_{B1} and Y_{B3} enter with the same sign in the total baryon asymmetry and are of the same order of magnitude. The enhancement of the asymmetry for $\alpha < 0.7$ is explained by the increase of the loop function $f(m_1/m_3) \cong -f(m_3/m_1)$ in the region of quasi-degenerate light neutrino mass spectrum.

In this class of models, successful leptogenesis can be naturally realized for both types of spectrum - NO and IO, for an effective Yukawa coupling $y_C \gtrsim 1.5$.

5. Summary

In the present work we studied the related issues of Majorana CP violating phases and leptogenesis in variants of two prominent (and rather generic) supersymmetric A_4 models [8,9] which naturally lead at leading order (LO) to tri-bimaximal (TB) mixing in the lepton sector. The pattern of neutrino mixing suggested by the existing neutrino oscillation data is remarkably similar to the TB one. Both models are supersymmetric and employ the type I see-saw mechanism of neutrino mass generation. They predict at LO a diagonal mass matrix for charged leptons and lead to exact TB mixing in the neutrino sector. The mass matrix of the RH neutrinos contains only two complex parameters X, Z. All low energy observables are expressed through only three independent quantities: the real parameter $\alpha = |3Z/X|$, the relative phase ϕ between X and Z, and the absolute scale of the light neutrino masses. The latter is a combination of the neutrino Yukawa coupling and the parameter $|X|$ which determines the scale of RH neutrino masses.

The main difference between the original models and those considered by us is in the scale of RH neutrino masses. In the original models this scale is around $(10^{14} \div 10^{15})$ GeV. In order to avoid possible potential problems with LFV processes we consider versions of both models in which the scale of RH neutrino masses is lower, namely, is in the range of $(10^{11} \div 10^{13})$ GeV. This is achieved by imposing an additional Z_2 symmetry capable of suppressing sufficiently the neutrino Yukawa couplings. As a consequence, the mass scale of the RH neutrinos is lowered as well. We discussed in detail the flavon superpotential in the modified models of interest. The results obtained at leading order and next to leading order (NLO) in the original models are still valid in the extensions we consider.

The two Majorana phases of the PMNS matrix, α_{21} and α_{31}, effectively play the role of leptogenesis CP violating parameters in the generation of the baryon asymmetry. In the models considered both the phases α_{21} and α_{31} and the ratio $r \equiv \Delta m^2_{\odot}/|\Delta m^2_{\rm A}|$ are functions of the two parameters α and ϕ. We analyzed in detail the dependence of the two "low energy" Majorana phases α_{21} and α_{31} on α and ϕ. In contrast to the low energy observables, like neutrino masses and the effective Majorana mass in neutrinoless double beta decay, m_{ee}, we show that these phases depend both on $\sin\phi$ and $\cos\phi$, and not only on $\cos\phi$. We show also that the sign of the baryon asymmetry Y_B uniquely determines the sign of $\sin\phi$, which has to be negative: $\sin\phi < 0$.

In the case of neutrino mass spectrum with normal ordering (NO), α_{21} is shown to be small, $\alpha_{21} \lesssim 0.1$. In the types of models considered $\sin^2\theta_{13}$ is also predicted to be small, $\sin^2\theta_{13} \sim 10^{-3}$. As a consequence, the contributions of the terms $\propto \sin^2\theta_{13}$ in m_{ee} are strongly suppressed. The lightest neutrino mass is predicted to lie in the interval $(3.8 \div 6.9) \times 10^{-3}$ eV, thus the neutrino mass spectrum is with partial hierarchy. The effective Majorana mass m_{ee} has a relatively large value, $m_{ee} \sim 7 \times 10^{-3}$ eV. We note that if α_{21} had a value close to π, one would have $m_{ee} \ll 10^{-3}$ eV. Depending on α, the phase α_{31} can take large CP violating values. For light neutrino mass spectrum with inverted ordering (IO), the Majorana CP phases α_{21} and α_{31} vary (for $\sin\phi < 0$) between 0 and π and π and 2π, respectively.

Throughout this study we have neglected renormalization group effects on neutrino masses and mixings which can be large for a quasi-degenerate (QD) light neutrino mass spectrum. A QD spectrum can arise in the models considered if $\Delta m^2_{\rm A} < 0$ (i.e. the spectrum is with inverted ordering) and $\alpha \lesssim 0.2$. However, this corresponds only to a small portion of the parameter space of the models.

As has been already discussed in the literature, in the models of interest leptogenesis is not possible at LO: the corresponding CP asymmetries ϵ_i vanish. Thus, the inclusion of NLO effects is crucial for the generation of the baryon asymmetry Y_B. More precisely, the NLO effects correcting the neutrino Dirac mass matrix m_D give rise to non-vanishing ϵ_i and therefore - to non-vanishing Y_B. Due to this fact the CP asymmetries are naturally of the order of 10^{-6} (independent of the precise value of the loop function). Further, although the AF and AM type models lead to the same results at LO, they differ at NLO so that the CP asymmetries generated in the two models are different.

We find that it is possible to generate the correct size and sign of the baryon asymmetry Y_B in the versions of both the AF and AM models we discuss. The study of leptogenesis was performed in the framework of the one flavor approximation and by using analytic formulae for the relevant efficiency factors η_{ii}. Since the mass spectrum of the RH neutrinos is generically not strongly hierarchical, the decays of all three RH (s)neutrinos contribute to the generation of the baryon asymmetry. We find that the correct magnitude as well as the correct sign of the baryon asymmetry Y_B can be easily obtained in the AF and AM type models for most values of the parameter α and natural values of the NLO couplings. As already mentioned, the sign of Y_B uniquely fixes the sign of $\sin\phi$. The latter cannot be determined by low energy observables since they exhibit only a $\cos\phi$-dependence.

To conclude, the results of our detailed study show that SUSY models with A_4 flavour symmetry and type I see-saw mechanism of neutrino mass generation, which gives rise to tri-bimaximal mixing and Majorana CP violation in the lepton sector, can account also successfully for the observed baryon asymmetry of the Universe.

Acknowledgements

We thank Werner Rodejohann and Yasutaka Takanishi for discussions. C.H. would like to thank the Max-Planck-Institut für Kernphysik for kind hospitality during the last stages of the work. This work was supported in part by the Italian INFN under the program "Fisica Astroparticellare" and by the European Network "UniverseNet" (MRTN-CT-2006-035863).

A. Flavon Superpotential in the AF Type Model

In the construction of the flavon superpotential we closely follow [8] and introduce an additional $U(1)_R$ symmetry under which driving fields have charge +2, superfields containing SM fermions +1 and flavons, $h_{u,d}$ and FN field(s) are uncharged. To give a VEV of order $\lambda_c^2\Lambda$ to ζ we introduce a new driving field ζ_0 which is a singlet under all symmetries of the model, apart from carrying a $U(1)_R$ charge +2. The terms contributing to the flavon superpotential containing ζ_0 at LO read [9]

$$w_d^\zeta = M_\zeta^2 \zeta_0 + g_a \zeta_0 \zeta^2 + g_b \zeta_0 (\varphi_T \varphi_T) \; . \tag{A.1}$$

Analogously to the original model, we demand a vanishing $F-$ term of ζ_0

$$M_\zeta^2 + g_a \zeta^2 + g_b (\varphi_{T1}^2 + 2\varphi_{T2}\varphi_{T3}) = 0 \; . \tag{A.2}$$

At the same time, the field ζ does not couple to the other driving fields, $\varphi_0^T \sim (3,1)$, $\varphi_0^S \sim (3,\omega^2)$ and $\xi_0 \sim (1,\omega^2)$ under (A_4, Z_3), in the model at LO. Thus, their $F-$terms read as in [8]. We find as solution

$$z^2 = -\frac{1}{g_a}\left(M_\zeta^2 + g_b v_T^2\right) \tag{A.3}$$

and the same results for the VEVs of φ_T, φ_S, ξ and $\tilde{\xi}$ as in [8]. For the mass parameter M_ζ being of order $\lambda_c^2\Lambda$ the VEV z is also of order $\lambda_c^2\Lambda$.

Concerning the NLO contributions stemming from ζ to the alignment of the flavons φ_T, φ_S, ξ and $\tilde{\xi}$ we find just one term

$$\frac{t_z}{\Lambda}\zeta^2\left(\varphi_0^T \varphi_T\right) \tag{A.4}$$

which gives an additional contribution

$$\frac{3t_z}{2gg_a}\left(g_b + \frac{M_\zeta^2}{v_T^2}\right)\frac{v_T^2}{\Lambda} \tag{A.5}$$

to the shift δv_{T1} of φ_T. Its size is $\lambda_c^4\Lambda$, as expected. Furthermore, the shifts $\delta v_{T2,3}$ remain unchanged and thus still equal. The shifts in the vacuum of φ_S and $\tilde{\xi}$ are also unchanged and the VEV of ξ is still a free parameter.

The NLO terms affecting w_d^ζ read

$$\Delta w_d^\zeta = \frac{1}{\Lambda}\sum_{i=1}^{8} z_i I_i^Z \tag{A.6}$$

with

$$\begin{aligned} &I_1^Z = \zeta_0(\varphi_T\varphi_T\varphi_T)\;, && I_2^Z = \zeta_0(\varphi_S\varphi_S\varphi_S)\;, && I_3^Z = \zeta_0\xi(\varphi_S\varphi_S)\;, && I_4^Z = \zeta_0\tilde{\xi}(\varphi_S\varphi_S)\;,\\ &I_5^Z = \zeta_0\xi^3\;, && I_6^Z = \zeta_0\xi^2\tilde{\xi}\;, && I_7^Z = \zeta_0\xi\tilde{\xi}^2\;, && I_8^Z = \zeta_0\tilde{\xi}^3\;. \end{aligned} \tag{A.7}$$

[9] Terms such as $\zeta_0 h_u h_d$ are not relevant, since we assume that the flavor symmetry is broken much above the electroweak scale.

The result for the shift in the VEV of ζ, $z+\delta z$, in the usual linear approximation, is

$$\delta z = \frac{g_b \tilde{g}_4}{2g\tilde{g}_3 g_a}\left(t_{11}+\frac{\tilde{g}_4^2}{3\tilde{g}_3^2}(t_6+t_7+t_8)\right)\frac{u^3}{z\Lambda} - \frac{3g_b t_z}{2gg_a^2}\left(g_b - \frac{g_a t_3}{t_z}+\frac{M_\zeta^2}{v_T^2}\right)\frac{v_T^3}{z\Lambda}$$
$$-\frac{1}{2g_a}\left(z_1\left(\frac{v_T^3}{u^3}\right)+\frac{\tilde{g}_4^2}{3\tilde{g}_3^2}z_3+z_5\right)\frac{u^3}{z\Lambda} \qquad \text{(A.8)}$$

with $g_4 = -\tilde{g}_4^2$ and $g_3 = 3\tilde{g}_3^2$ as introduced in [8]. This shift δz in $\langle\zeta\rangle$ is of order $\lambda_c^4\Lambda$. Additionally, we find that - unless some non-trivial relation among the couplings in the flavon superpotential is fulfilled - the VEVs of all driving fields vanish at the minimum.

B. Flavon Superpotential in the AM Type Model

In order to induce a VEV for the flavon ζ we add a driving field ζ_0 which transforms as $1'$ under A_4, with -1 under Z_4 and is invariant under the Z_2 symmetry. Since it is responsible for the vacuum alignment, its charge under the $U(1)_R$ symmetry is $+2$. The LO potential for ζ_0 is of the form

$$w_d^\zeta = g_a\zeta_0\zeta^2 + g_b\zeta_0(\varphi_T\varphi_T)'' + g_c\zeta_0(\xi')^2\ . \qquad \text{(B.1)}$$

From the F-term of ζ_0 we can derive

$$g_a\zeta^2 + g_b(\varphi_{T2}^2 + 2\varphi_{T1}\varphi_{T3}) + g_c(\xi')^2 = 0\ . \qquad \text{(B.2)}$$

Thus, z takes the value

$$z^2 = -\frac{1}{g_a}\left(g_b v_T^2 + g_c(u')^2\right) = -\frac{1}{g_a}\left(\frac{g_b h_1^2}{4h_2^2}+g_c\right)(u')^2 \qquad \text{(B.3)}$$

so that $z \propto u'$ holds in case of no accidental cancellations. u' is a free parameter in [9] which is taken to be of order $\lambda_c^2\Lambda$.

As one can check, the field ζ does not have renormalizable interactions with the driving fields, $\varphi_0^T \sim (3,-1)$, $\varphi_0^S \sim (3,1)$ and $\xi_0 \sim (1,1)$ under (A_4, Z_4), of the original model. Thus, the results for the vacuum alignment found in [9] still hold.

At NLO the field ζ contributes to the flavon superpotential of the original model through

$$\frac{1}{\Lambda}\zeta^2(\varphi_0^T\varphi_S)'\ , \qquad \text{(B.4)}$$

while it does not introduce any contribution at this level involving φ_0^S or ξ_0.

The NLO effects on the vacuum alignment of the field ζ stem from (order one coefficients are omitted)

$$\frac{1}{\Lambda}\zeta_0\zeta^2\xi + \frac{1}{\Lambda}\zeta_0(\varphi_T\varphi_T\varphi_S)'' + \frac{1}{\Lambda}\zeta_0\xi(\varphi_T\varphi_T)'' + \frac{1}{\Lambda}\zeta_0\xi'(\varphi_T\varphi_S)' + \frac{1}{\Lambda}\zeta_0\xi'\xi'\xi\ . \qquad \text{(B.5)}$$

Computing the effect of all NLO terms on the vacuum alignment one finds that still all shifts δv_{Si} are equal, i.e. the shifts do not change the structure of the vacuum, that the generic size of all shifts - for mass parameters and VEVs of order $\lambda_c^2\Lambda$ - is $\lambda_c^4\Lambda$ and the free parameter u' is still undetermined.

Eventually, we checked that all driving fields can have a vanishing VEV at the minimum.

References

[1] T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **10** (2008) 113011 [arXiv:0808.2016 [hep-ph]];

[2] A. Bandyopadhyay *et al.*, arXiv:0804.4857 [hep-ph]; G.L. Fogli *et al.*, Phys. Rev. D **78** (2008) 033010, and arXiv:0905.3549; M. C. Gonzalez-Garcia and M. Maltoni, Phys. Rept. **460** (2008) 1.

[3] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002); Phys. Lett. B **535**, 163 (2002); Z. Z. Xing, Phys. Lett. B **533**, 85 (2002); X. G. He and A. Zee, Phys. Lett. B **560**, 87 (2003); see also: L. Wolfenstein, Phys. Rev. D **18**, 958 (1978); Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. D **25**, 1895 (1982) [Erratum-ibid. D **29**, 2135 (1984)].

[4] S. M. Bilenky, J. Hošek and S. T. Petcov, Phys. Lett. B **94**, 495 (1980).

[5] J. Schechter and J.W.F. Valle, Phys. Rev. D **22** (1980) 2227; M. Doi *et al.*, Phys. Lett. B **102** (1981) 323.

[6] E. Ma and G. Rajasekaran, Phys. Rev. D **64** (2001) 113012 [arXiv:hep-ph/0106291]; K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552** (2003) 207 [arXiv:hep-ph/0206292]; M. Hirsch *et al.*, arXiv:hep-ph/0312244; Phys. Rev. D **69** (2004) 093006 [arXiv:hep-ph/0312265]; S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B **724** (2005) 423 [arXiv:hep-ph/0504181]; K. S. Babu and X. G. He, arXiv:hep-ph/0507217; X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **0604** (2006) 039 [arXiv:hep-ph/0601001]; S. F. King and M. Malinsky, Phys. Lett. B **645** (2007) 351 [arXiv:hep-ph/0610250]; F. Bazzocchi, S. Kaneko and S. Morisi, JHEP **0803** (2008) 063 [arXiv:0707.3032 [hep-ph]].

[7] C. S. Lam, Phys. Rev. Lett. **101** (2008) 121602 [arXiv:0804.2622 [hep-ph]]; C. S. Lam, Phys. Rev. D **78** (2008) 073015 [arXiv:0809.1185 [hep-ph]]; F. Bazzocchi, L. Merlo and S. Morisi, Nucl. Phys. B **816** (2009) 204 [arXiv:0901.2086 [hep-ph]]; C. S. Lam, arXiv:0907.2206 [hep-ph].

[8] G. Altarelli and F. Feruglio, Nucl. Phys. B **741** (2006) 215 [arXiv:hep-ph/0512103].

[9] G. Altarelli and D. Meloni, arXiv:0905.0620 [hep-ph].

[10] B. Pontecorvo, Zh. Eksp. Teor. Fiz. (JETP) **33** (1957) 549; *ibid.* **34** (1958) 247; *ibid.* **53** (1967) 1717; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870; B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53** (1967) 1717.

[11] G. Altarelli, F. Feruglio and C. Hagedorn, JHEP **0803** (2008) 052 [arXiv:0802.0090 [hep-ph]].

[12] Y. Lin, arXiv:0905.3534 [hep-ph].

[13] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45.

[14] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36.

[15] J. Dunkley *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 306 [arXiv:0803.0586 [astro-ph]].

[16] E. E. Jenkins and A. V. Manohar, Phys. Lett. B **668** (2008) 210 [arXiv:0807.4176 [hep-ph]].

[17] S. T. Petcov *et al.*, Nucl. Phys. B **739** (2006) 208; see also: S. T. Petcov *et al.*, Nucl. Phys. B **676** (2004) 453.

[18] S. Pascoli, S. T. Petcov and A. Riotto, Phys. Rev. D **75** (2007) 083511; Nucl. Phys. B (2007).

[19] S. Blanchet, P. Di Bari and G. G. Raffelt, JCAP **0703** (2007) 012.

[20] G. F. Giudice *et al.*, Nucl. Phys. B **685** (2004) 89 [arXiv:hep-ph/0310123].

[21] B. Adhikary and A. Ghosal, Phys. Rev. D **78** (2008) 073007 [arXiv:0803.3582 [hep-ph]].

[22] G. C. Branco, R. Gonzalez Felipe, M. N. Rebelo and H. Serodio, Phys. Rev. D **79** (2009) 093008 [arXiv:0904.3076 [hep-ph]].

[23] F. Bazzocchi, L. Merlo and S. Morisi, arXiv:0902.2849 [hep-ph].

[24] M. Fukugita *et al.*, Phys. Rev. D **74** (2006) 027302.

[25] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 330 [arXiv:0803.0547].

[26] S. M. Bilenky, S. Pascoli and S. T. Petcov, Phys. Rev. D **64** (2001) 053010 [arXiv:hep-ph/0102265].

[27] S. T. Petcov, Nucl. Phys. Proc. Suppl. **143** (2005) 159.

[28] P. Langacker *et al.*, Nucl. Phys. B **282** (1987) 589.

[29] F. Avignone, *Nucl. Phys. Proc. Suppl.* **143** (2005) 233; C. Aalseth *et al.*, hep-ph/0412300.

[30] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169 [arXiv:hep-ph/9605319]; M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B **345** (1995) 248 [Erratum-ibid. B **382** (1996) 447]; M. Flanz *et al.*, Phys. Lett. B **389** (1996) 693.

[31] A. Pilaftsis, Phys. Rev. D **56** (1997) 5431.

[32] W. Buchmuller and M. Plumacher, Phys. Lett. B **431** (1998) 354 [arXiv:hep-ph/9710460].

[33] S. Blanchet and P. Di Bari, JCAP **0606** (2006) 023 [arXiv:hep-ph/0603107].

[34] J.A. Harvey and M. Turner, Phys. Rev. D **42** (1990) 3344.

[35] G. Engelhard *et al.*, Phys. Rev. Lett. **99** (2007) 081802 [arXiv:hep-ph/0612187].

[36] G. Engelhard, Y. Grossman and Y. Nir, JHEP **0707** (2007) 029 [arXiv:hep-ph/0702151].

[37] H. B. Nielsen and Y. Takanishi, Nucl. Phys. B **636** (2002) 305 [arXiv:hep-ph/0204027].

[38] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B **316** (1993) 312 [arXiv:hep-ph/9306333]; K. S. Babu, C. N. Leung and J. T. Pantaleone, Phys. Lett. B **319** (1993) 191 [arXiv:hep-ph/9309223]; S. Antusch *et al.*, Phys. Lett. B **519** (2001) 238 [arXiv:hep-ph/0108005]; S. Antusch *et al.*, Nucl. Phys. B **674** (2003) 401 [arXiv:hep-ph/0305273]; S. Antusch *et al.*, JHEP **0503** (2005) 024 [arXiv:hep-ph/0501272]; see also, e.g. S.T. Petcov, T. Shindou and Y. Takanishi, Nucl. Phys. B **738** (2006) 219.