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Relaxing the constraint on the number of messengers in a low-energy gauge mediation

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Abstract

We propose a mechanism for relaxing a constraint on the number of messengers in low-energy gauge mediation models. The Landau pole problem for the standard-model gauge coupling constants in the low-energy gauge mediation can be circumvented by using our mechanism. An essential ingredient is a large positive anomalous dimension of messenger fields given by a large Yukawa coupling in an conformal field theory at high energies. The positive anomalous dimension reduces the contribution of the messengers to the beta function of the standard-model gauge couplings.

1 Introduction

The low-energy-scale gauge mediation with the gravitino mass $m_{3/2} < O(10)$ eV is very attractive, since it does not suffer from any cosmological gravitino problem [1]. In such a low-energy gauge mediation, the messengers have their masses of the order $10^2 - 10^3$ TeV. If the number of messengers, $N_{\rm mess}$, is large, the gauge coupling constants of the standard model easily blow up below the GUT scale, i.e. the gauge coupling constants hit Landau poles below the GUT scale. The requirement of the perturbative unification of the gauge coupling constants, thus, leads to a constraint on the number of messengers. It is known [2] that $N_{\rm mess} < 5$ for the messengers being $\mathbf{5} + \mathbf{5}^*$ of $SU(5)_{\rm GUT}$ if the masses of messengers are smaller than about 10^3 TeV.

The above constraint becomes more severe if one considers strongly interacting messengers in direct (see [3] and references therein) or semi-direct [4, 5] gauge mediation models, for instance. This is because the messengers receive most likely negative anomalous dimensions from the hidden strong gauge interactions and hence the standard-model gauge couplings run faster (see Section 2).

In this paper we point out that it is not always the case if the theory is embedded into a conformal field theory at high energies. We show several examples where hidden sector interactions induce even positive large anomalous dimensions for messengers. In those example models one may have $N_{\rm mess} \geq 5$ without ruining the perturbative unification. A crucial ingredient is an introduction of a large Yukawa coupling of the messengers to some other hidden sector fields.

2 Relaxing the constraint on $N_{\rm mess}$

In this section we describe our mechanism for relaxing the constraint on the number of messengers. In supersymmetric (SUSY) gauge theories, the β function of a gauge coupling is exactly given by the so-called NSVZ β function [6],

$$\beta(g) = \mu \frac{\partial}{\partial \mu} g^2 = -\frac{g^4}{8\pi^2} \frac{3t(A) - \sum_i (1-\gamma_i) t(i)}{1 - t(A) g^2/8\pi^2}, \tag{1}$$

where $t(A)$ and $t(i)$ are the Dynkin indices for the adjoint representation and the representation of matter fields i, γ_i an anomalous dimension of matter i, and μ a renormalization scale. Let us consider the β functions of the standard-model (SM) gauge couplings. From the β function (1), we can see that the effective messenger number contributing to the SM β functions is give by

$$N_{\rm mess}^{\rm eff} \equiv \sum_{i\in {\rm mess}} (1-\gamma_i) t_{\rm GUT}(i), \tag{2}$$

where the sum is taken over messenger fields, and $t_{\rm GUT}(i)$ the Dynkin indices for the GUT gauge group $SU(5)_{\rm GUT}$. Here, we have assumed that the messengers form a complete representation of the $SU(5)_{\rm GUT}$.

Now let us suppose that the messengers are charged under some hidden gauge group (with the gauge coupling g) and have a Yukawa interaction (with the Yukawa coupling λ) with some hidden matters. Then, the anomalous dimension of the messengers is given by, at the one-loop level,

$$\gamma \sim -\frac{g^2}{8\pi^2} + \frac{|\lambda|^2}{8\pi^2}. \tag{3}$$

(Here, we have neglected the contributions from the SM gauge interactions.) Then, from Eqs. (2) and (3), we can see that the hidden gauge interaction increases the effective messenger number, $N_{\rm mess}^{\rm eff}$, while the hidden Yukawa interaction decreases it.

In direct (see [3] and references therein) or semi-direct [4, 5] gauge mediation models and also in composite messenger models (e.g. [7]), the messenger fields are supposed to be charged under hidden gauge groups. Thus, the messenger fields have negative anomalous dimensions and the effective messenger number increases. Thus, the Landau pole problem discussed in the Introduction becomes more severe (for a more quantitative discussion, see Appendix A). However, if we introduce large Yukawa interactions in the messenger sector, the anomalous dimensions of the messengers can become positive and we can decrease the effective messenger number, $N_{\rm mess}^{\rm eff}$. For this mechanism to be efficient, it is desirable that the hidden gauge theory of the messengers is embedded into a conformal field theory, because the messengers can have large positive anomalous dimensions over a wide range of energy scales. (Otherwise, the large Yukawa coupling hits its own Landau pole below the GUT scale.)

In the rest of this section we give example models which realize the above mechanism. We will see that the models have direct applications to low-energy gauge mediation models in the next section.

The models are based on an $SU(N_C)$ hidden gauge group. We first introduce N_Q messenger quarks and anti-quarks, Q^i_α and $\tilde{Q}^\alpha_i$, with $i = 1, \cdots, N_Q$ and $\alpha = 1, \cdots, N_C$. The messengers Q^i_α and $\tilde{Q}^\alpha_i$ transform as fundamental and anti-fundamental representations of $SU(N_C)$, respectively. We restrict our discussion to the case of $N_Q = 5$, for simplicity, and assume that the quarks Q^i_α and anti-quarks $\tilde{Q}^\alpha_i$ transform as $\mathbf{5}^*$ and $\mathbf{5}$ of $SU(5)_{\rm GUT}$, respectively. We introduce a mass term $m_Q Q^i_\alpha \tilde{Q}^\alpha_i$ for the messengers Q^i_α and $\tilde{Q}^\alpha_i$. Notice that N_C is identified with the number of the messengers, $N_{\rm mess}$. The generalization to other gauge theories such as $SP(N_C)$ or $SO(N_C)$ is straightforward and hence we do not discuss it in this paper.

To embed the theory into a conformal field theory giving the messengers positive anomalous dimensions, we introduce N_P pairs of quarks and anti-quarks, P^p_α and $\tilde{P}^\alpha_p$ with $p = 1, \cdots, N_P$, and an adjoint quark chiral multiplet, A^α_β with $\alpha,\ \beta = 1, \cdots, N_C$. We introduce their mass terms,

$$W_{\rm mass} = m_P P^p_\alpha \tilde{P}^\alpha_p + m_A A^\alpha_\beta A^\beta_\alpha. \tag{4}$$

We assume, $m_P, m_A > m_Q$, for the additional quarks, P^p_α and $\tilde{P}^\alpha_p$, and the adjoint quark A^α_β to decouple from the strong dynamics at the messenger mass scale. We also introduce a Yukawa coupling,

$$W_{\rm Yukawa} = \sqrt{2}\lambda Q^i_\alpha A^\alpha_\beta \tilde{Q}^\beta_i. \tag{5}$$

The introduction of the Yukawa coupling is important for our mechanism to work, as explained above.

We find that the theory has a infrared conformal fixed point for a given appropriate value of N_C and that of N_P. We show, in Appendix B, the detailed determination of the infrared fixed points and of the anomalous dimensions of the messenger fields. We give the obtained anomalous dimensions γ_Q of the messenger fields Q and $\tilde{Q}$ and the effective messenger numbers $N^{\rm eff}_{\rm mess} = (1 - \gamma_Q) N_C$ for various sets of $(N_C,\ N_P)$ in Table 1. We see that the models have the effective messenger numbers $N^{\rm eff}_{\rm mess} < 5$ for many sets of $(N_C,\ N_P)$, even if the actual messenger number $N_{\rm mess} = N_C \geq 5$.

	$N_P = 2$	$N_P = 3$	$N_P = 4$	$N_P = 5$	$N_P = 6$
$N_C = 5$	0.303 (3.48)	0.156 (4.22)	0.062 (4.69)	$\times$	$\times$
$N_C = 6$	0.452 (3.29)	0.300 (4.20)	0.191 (4.85)	0.110 (5.34)	0.048 (5.71)
$N_C = 7$	$\times$	0.411 (4.13)	0.301 (4.90)	0.214 (5.50)	0.144 (5.99)
$N_C = 8$	$\times$	0.494 (4.05)	0.388 (4.89)	0.302 (5.58)	0.230 (6.16)
$N_C = 9$	$\times$	$\times$	0.458 (4.88)	0.375 (5.63)	0.303 (6.27)

Table 1: The anomalous dimensions γ_Q of messenger fields Q, $\tilde{Q}$ and the effective messenger numbers $N_{\rm mess}^{\rm eff}$ (in parentheses) at the conformal fixed points. Models marked with $\times$ do not have a desirable fixed point.

3 Applications to low-energy gauge mediation models

In this section we discuss applications of our mechanism to various gauge mediation models. The applications have three categories, (I) application to direct gauge mediation, (II) that to semi-direct gauge mediation and (III) that to composite messenger models. We consider a representative model for each categories to illustrate our mechanism discussed in Section 2.

3.1 Direct gauge mediation

Let us consider direct gauge mediation models [8, 9, 10, 11, 12] in which a subgroup of the flavor symmetry of the Intriligator-Seiberg-Shih (ISS) model [13] is gauged by $SU(5)_{\rm GUT}$. The model is based on the $SU(N_C)$ gauge theory with N_F pairs of quarks Q^I_α and anti-quarks $\tilde{Q}^\alpha_I$. Here, I and α run from $I = 1$ to $I = N_F$ and from $\alpha = 1$ to $\alpha = N_C$, respectively. We assume, for simplicity, that they have a common mass

$$W = mQ^I_\alpha \tilde{Q}^\alpha_I. \tag{6}$$

We have a global flavor symmetry $SU(N_F)_F$.

If the numbers of color and flavor satisfy the relation $N_C + 1 \leq N_F < \frac{3}{2}N_C$, this theory has a weakly coupled dual magnetic description at low energies. The dual magnetic theory is described in terms of mesons Φ^I_J and dual quarks φ^a_I, $\tilde{\varphi}^I_a$. Here, $a = 1, \cdots, \tilde{N}_C$ is the index of a dual gauge group $SU(\tilde{N}_C = N_F - N_C)_{\rm mag}$. The superpotential of this theory is given by

$$W = h\varphi_I^a \Phi_J^I \tilde{\varphi}_a^J - h\mu^2 \Phi_I^I. \tag{7}$$

Without a loss of generality, the Yukawa coupling constant h and the dimension one parameter μ can be taken to be real and positive.

At the tree level, the equation of motion of Φ gives the F-term of Φ,

$$-(F_\Phi^\dagger)_I^J = h\varphi_I^a \tilde{\varphi}_a^J - h\mu^2 \delta_I^J. \tag{8}$$

The right hand side of this equation cannot be zero, since the rank of the matrix $\varphi_I^a \tilde{\varphi}_a^J$ is no greater than $N_F - N_C$ and the unit matrix δ_I^J has rank N_F ($> N_F - N_C$). Thus some components of $(F_\Phi^\dagger)_I^J$ are non-zero and SUSY is broken. If non-perturbative effects of $SU(\tilde{N}_C)_{\rm mag}$ are taken into account, however, SUSY is dynamically restored [13]. So the SUSY breaking vacua are metastable.

Around the SUSY-breaking local minima of the potential, the fields $\varphi, \tilde{\varphi}$ and Φ can be expanded like

$$\varphi_I^a = \begin{pmatrix} \mu\delta_p^a + \delta\chi_p^a & \delta\rho_i^a \end{pmatrix}, \quad \tilde{\varphi}_a^I = \begin{pmatrix} \mu\delta_a^p + \delta\tilde{\chi}_a^p \\ \delta\tilde{\rho}_a^i \end{pmatrix}, \quad \Phi_J^I = \begin{pmatrix} \delta Y_q^p & \delta\tilde{Z}_j^p \\ \delta Z_q^i & \delta\Phi_j^i \end{pmatrix}, \tag{9}$$

where $p = I$ for $1 \le I \le N_F - N_C$ and $i = I$ for $N_F - N_C + 1 \le I \le N_F$. These vevs break the global flavor symmetry $SU(N_F)_F$ down to $SU(N_F - N_C)_F \times SU(N_C)_F$. To make this model a direct gauge mediation model, we embed the $SU(5)_{\rm GUT}$ gauge group into a subgroup of $SU(N_C)_F$ or $SU(N_F - N_C)_F$. Let us consider the theory above the mass scale μ for each case [8].

1. If $SU(5)_{\rm GUT} \subset SU(N_C)_F$, in the magnetic theory, fields charged under $SU(5)_{\rm GUT}$ are (a part of) $\delta\rho$, $\delta\tilde{\rho}$, δZ, $\delta\tilde{Z}$ in the (anti-)fundamental representation of $SU(N_C)_F$ and $\delta\Phi$ in the adjoint representation of $SU(N_C)_F$. Then, the contribution to the β function of the $SU(5)_{\rm GUT}$ gauge coupling is given by $N_{\rm mess}^{\rm (mag)} = 2(N_F - N_C) + N_C = 2N_F - N_C$ [1]. (Note that the adjoint representation of $SU(N_C)_F$ decomposes into an adjoint representation of $SU(5)_{\rm GUT}$, $N_C - 5$ flavors of fundamental and anti-fundamental representations of $SU(5)_{\rm GUT}$, and some singlets.) In the electric theory

[1] $N_{\rm mess}$ is not equal to the one contributing to the gaugino and sfermion soft masses. In this paper we are defining $N_{\rm mess}$ only by the contribution to the gauge coupling β function.

$N_{\text{mess}}^{(\text{ele})} = N_C$. From the inequalities $N_C \geq 5$ and $N_C + 1 \leq N_F < \frac{3}{2}N_C$, we obtain $N_{\text{mess}}^{(\text{mag})} \geq 7$ and $N_{\text{mess}}^{(\text{ele})} \geq 5$.

2. If $SU(5)_{\text{GUT}} \subset SU(N_F - N_C)_F$, in the magnetic theory, fields charged under $SU(5)_{\text{GUT}}$ are (a part of) $\delta\chi$, $\delta\tilde{\chi}$, δZ, $\delta\tilde{Z}$ in the (anti-)fundamental representation of $SU(N_F - N_C)_F$ and δY in the adjoint representation of $SU(N_F - N_C)_F$. (This counting is applicable above the mass scale μ. Below μ, the SM gauge group is in the diagonal subgroup of $SU(N_F - N_C)_F \times SU(\tilde{N}_C)_{\text{mag}}$.) Then, the contribution to the β function of the $SU(5)_{\text{GUT}}$ gauge coupling is given by $N_{\text{mess}}^{(\text{mag})} = (N_F - N_C) + N_C + (N_F - N_C) = 2N_F - N_C$. In the electric theory $N_{\text{mess}}^{(\text{ele})} = N_C$. From the inequalities $N_F - N_C \geq 5$ and $N_C + 1 \leq N_F < \frac{3}{2}N_C$, we obtain $N_{\text{mess}}^{(\text{mag})} > 20$ and $N_{\text{mess}}^{(\text{ele})} > 10$.

In the case $SU(5)_{\text{GUT}} \subset SU(N_F - N_C)_F$, N_{mess} is too large, so we concentrate on the case $SU(5)_{\text{GUT}} \subset SU(N_C)_F$. In this model, SUSY breaking is mediated to the MSSM sector by the fields $\delta\rho$, $\delta\tilde{\rho}$, δZ, $\delta\tilde{Z}$. The superpotential becomes

$$\begin{aligned} W &= h\varphi_I^a \Phi_J^I \tilde{\varphi}_a^J - h\mu^2 \Phi_I^I \\ &= h\mu(\delta\rho_i^a \delta Z_a^i + \delta\tilde{\rho}_a^i \delta\tilde{Z}_i^a) + h\rho_i^a \Phi_j^i \tilde{\rho}_a^j - h\mu^2 \Phi_i^i + \cdots, \end{aligned} \tag{10}$$

where dots represent terms irrelevant for the gauge mediation. Φ_i^i has a non-vanishing F-term, and then $\delta\rho$ and $\delta\tilde{\rho}$ have SUSY-breaking masses. This is the type of gauge mediation studied in Ref. [14]. R-symmetry breaking is rather non-trivial in this model and one has to consider some modification of the theory. See [8, 9, 10, 11, 12] for details.

Even in the case $SU(5)_{\text{GUT}} \subset SU(N_C)_F$, $N_{\text{mess}} \geq 5$ in both the electric and magnetic theory. In fact, the messenger number N_{mess} is smaller in the electric theory than in the magnetic theory, which was considered as a solution to the Landau pole problem in Ref. [15]. However, the analyses of Ref. [2] suggest that the two-loop effects from the MSSM sector make it difficult to maintain the perturbative GUT unification. Furthermore, the messenger fields are charged under the strong hidden gauge group $SU(N_C)$ in the electric theory [2]. Thus as explained in Section 2, this model suffers from the severe

[2] In the magnetic theory, $SU(5)_{\text{GUT}}$ charged fields have both the $SU(\tilde{N}_C)_{\text{mag}}$ gauge interaction (if $\tilde{N}_C \geq 2$) and the Yukawa interaction in the superpotential (7). Then it is non-trivial whether the total effect of these interactions decreases or increases the effective messenger number, compared with the naive messenger number $N_{\text{mess}}^{\text{mag}}$.

Landau pole problem when the messenger mass scale is of order 10^5 GeV. Notice that such a small mass $\sim 10^5$ GeV for the messenger is required in the models [9, 10, 11], since MSSM gaugino masses vanish at the leading order of SUSY-breaking scale (see also Ref. [16]). Thus the SUSY-breaking scale and the messenger mass scale must be comparable, of order 10^5 GeV.

We now consider a modification of the model to avoid the Landau pole problem. In the electric theory, we add a chiral field A^α_β which transforms in the adjoint representation of $SU(N_C)$ gauge group. We introduce the new terms in the superpotential

$$W \supset \sqrt{2}\lambda Q^i_\alpha A^\alpha_\beta \tilde{Q}^\beta_i + m_A A^\alpha_\beta A^\beta_\alpha, \tag{11}$$

where $i = N_F - N_C + 1, \cdots, N_F$. This is in fact the model considered in Section 2, with the identification $P^p_\alpha|_{\text{Section 2}} = Q^p_\alpha \quad (p = 1, \cdots, N_F - N_C)$, $N_Q|_{\text{Section 2}} = N_C$ and $N_P|_{\text{Section 2}} = N_F - N_C$. We take $N_C = 5$ and $2 \leq N_F - N_C \leq 4$ in the following discussion.

The dynamics of the model is as follows. At high energies, we assume that the theory is near the conformal fixed point. Then, as discussed in Section 2, the effective messenger number $N^{\text{eff}}_{\text{mess}}$ is smaller than 5 (see Table 1). Below the mass scale m_A, the adjoint field A decouples from the dynamics, and the theory exits from the conformal fixed point and the confinement occurs. Then at the low energies the model can be described by the weakly coupled magnetic theory. SUSY is broken as in the ISS model, and the direct gauge mediation works.

However, the low energy theory is not completely the same as the original ISS model. Integration of the adjoint field A generates a superpotential

$$W \supset -\frac{\lambda^2}{2m_A}\left[(Q^i_\alpha \tilde{Q}^\alpha_j)(Q^j_\beta \tilde{Q}^\beta_i) - \frac{1}{N_C}(Q^i_\alpha \tilde{Q}^\alpha_i)(Q^j_\beta \tilde{Q}^\beta_j)\right] = -\frac{\lambda^2\Lambda^2}{2m_A}\left[\delta\Phi^i_j \delta\Phi^j_i - \frac{1}{N_C}\delta\Phi^i_i \delta\Phi^j_j\right], \tag{12}$$

where Λ is the confinement scale of the electric theory defined by $Q^i_\alpha \tilde{Q}^\alpha_j = \Lambda \Phi^i_j$. When $N_C = N_Q|_{\text{Section 2}} = 5$, this term gives mass to the traceless part of $\delta\Phi$, i.e. the part which transforms in the adjoint representation of $SU(N_C)_F = SU(5)_{\text{GUT}}$. The traceless part of $\delta\Phi$ does not take part in SUSY breaking and gauge mediation, but this field contributes to the β functions of the SM gauge coupling constants. So below the mass scale $\lambda^2\Lambda^2/m_A$, the "messenger number" contributing to the β function is $N_{\text{mess}} = 2(N_F - N_C)$. Thus, if we take $N_F - N_C = 2$, the messenger number $N_{\rm mess}$ is smaller than 5 for the energy scale below $\lambda^2\Lambda^2/m_A$ in the magnetic theory. On the other hand, above the scale m_A, the theory is electric and $N_{\rm mess}^{\rm eff}$ is small because of the mechanism of Section 2. Furthermore, Λ is roughly related to m_A by the equation $\Lambda \sim m_A \exp(-8\pi^2/(3N_C - N_F)g_*^2)$, where g_* is the gauge coupling constant of $SU(N_C)$ at the fixed point. Then, if the fixed point is strongly coupled, which is the case in the present model, m_A and $\lambda^2\Lambda^2/m_A$ are of the same order. Thus the dangerous energy scale between $\lambda^2\Lambda^2/m_A$ and m_A is narrow and the perturbative unification of the SM gauge couplings is maintained.

In the case $2 < N_F - N_C \leq 4$, $N_{\rm mess}$ is larger than 5 in the magnetic theory. However, if we take m_A sufficiently low to ensure that the theory is in the electric theory over a wide range of energy scale, the Landau pole may be avoided. For example, consider the case $N_F - N_C = 3$. For simplicity, we use the following approximation; we approximate $\lambda^2\Lambda^2/m_A \sim m_A$, and above the scale m_A, $N_{\rm mess}^{\rm eff(ele)} = 4.2$ taken from Table 1 and below the scale m_A, $N_{\rm mess}^{\rm eff(mag)} = 2(N_F - N_C) = 6$. Furthermore, we suppose that all the MSSM sparticles have masses not far from the Z boson mass, $m_Z \simeq 91$ GeV. Then, the SM QCD coupling constant g_3 at the GUT scale $M_{\rm GUT} \sim 10^{16}$ GeV is , at the one-loop level, given by

$$\begin{aligned}\frac{8\pi^2}{g_3^2(M_{\rm GUT})} &\simeq \frac{8\pi^2}{g_3^2(m_Z)} + 3\log\left(\frac{M_{\rm mess}}{m_Z}\right) + (3 - N_{\rm mess}^{\rm eff(mag)})\log\left(\frac{m_A}{M_{\rm mess}}\right) \\ &\quad + (3 - N_{\rm mess}^{\rm eff(ele)})\log\left(\frac{M_{\rm GUT}}{m_A}\right), \qquad (13)\end{aligned}$$

where $M_{\rm mess}$ is the messenger mass scale which we take $M_{\rm mess} \sim 10^5$ GeV for a low-scale gauge mediation, and $8\pi^2/g_3^2(m_Z) \simeq 53.2$ from experiments. Requiring $g_3^2(M_{\rm GUT})/4\pi \lesssim 1$, we obtain the constraint on m_A as

$$m_A \lesssim 10^8 \text{ GeV}. \qquad (14)$$

Thus, m_A can be taken much larger than $M_{\rm mess} \sim 10^5$ GeV.

3.2 Semi-direct gauge mediation

We consider a SUSY-breaking model based on an $SU(5)_{\rm hid}$ gauge symmetry. It is known [17] that the SUSY is broken when we introduce only two matter multiplets, V^α and $X_{\alpha\beta}$ in the representations $\mathbf{5}^*$ and $\mathbf{10}$ of $SU(5)_{\rm hid}$, respectively. We now introduce N_F pairs of fundamental quarks Q^i_α and anti-quarks $\tilde{Q}^\alpha_i$. Here, $i = 1, \cdots, N_F$ and $\alpha = 1, \cdots, 5$ and they belong to $(\mathbf{N_F^*}, \mathbf{5})$ and $(\mathbf{N_F}, \mathbf{5}^*)$ representations of the $SU(N_F)_F \times SU(5)_{\rm hid}$, respectively. We introduce a common bare mass term for the messengers, for simplicity,

$$W = m_Q Q^i_\alpha \tilde{Q}^\alpha_i. \tag{15}$$

We gauge a subgroup of $SU(N_F)$ under the GUT gauge group [18]. This is the set up for the semi-direct gauge mediation in the $SU(5)_{\rm hid}$ SUSY-breaking model [3] . The bi-fundamental messenger fields Q and $\tilde{Q}$ link the $SU(5)_{\rm hid}$ hidden gauge sector and the MSSM sector, thus SUSY-breaking is mediated to the MSSM sector. In particular, when $N_F \geq 6$, it can be shown that the theory has a conformal fixed point above the mass scale m_Q. Then, after the decoupling of the messengers Q and $\tilde{Q}$, the theory exits from the conformal fixed point and the SUSY breaking occurs. This is a conformal gauge mediation model proposed in Ref. [18]. However, we only impose $N_F \geq 5$ in this paper.

In the above model, the messenger fields are charged under the strong $SU(5)_{\rm hid}$ gauge group. Thus, as discussed in Section 2, the effective messenger number $N^{\rm eff}_{\rm mess}$ is larger than 5. Thus this theory suffers from the Landau pole problem. To avoid the problem, we introduce a chiral field A^α_β transforming in the adjoint representation of $SU(5)_{\rm hid}$. We introduce a superpotential,

$$W \supset \sqrt{2}\lambda Q^i_\alpha A^\alpha_\beta \tilde{Q}^\beta_i + m_A A^\alpha_\beta A^\beta_\alpha, \tag{16}$$

for the mechanism explained in Section 2 to work. In fact, this model is not the same as the example model described in Section 2, but we can study the conformal fixed point of this theory by using a-maximization technique explained in Appendix B. The result is listed in Table 2. One can see that there is no Landau pole problem for $N_F = 5,\ 6$ and 7.

[3]In semi-direct gauge mediation, a messenger number is not necessarily larger than or equal to 5. For example, in the models of Ref. [5], the messenger number is minimally 2, so there is no Landau pole problem. However, in semi-direct gauge mediation, sparticle masses (especially gaugino masses) are suppressed by hidden sector loops [18, 5], so if one wants to bulid a model in which the gravitino is very light ($< \mathcal{O}(10)$ eV), the hidden sector gauge theory should be strongly coupled. Because the gauge theory should be strongly coupled even when we add messenger fields, the hidden sector gauge group should be somehow large, as in the above $SU(5)_{\rm hid}$ model. Then the model may suffer from the Landau pole problem.

N_F	γ_Q	γ_A	γ_V	γ_X	$N_{\rm mess}^{\rm eff}$
5	0.264	−0.528	−0.656	−0.901	3.68
6	0.160	−0.320	−0.527	−0.731	4.20
7	0.063	−0.126	−0.308	−0.438	4.69

Table 2: The anomalous dimensions of the fields of the model. γ_Q is the anomalous dimension of Q and $\tilde{Q}$. γ_A, γ_V and γ_X are the anomalous dimensions of A, V and X respectively. The fact that the anomalous dimensions of Q and $\tilde{Q}$ are the same is not obvious because the model is chiral, but a-maximization shows that is the case. $N_{\rm mess}^{\rm eff}$ is defined by $N_{\rm mess}^{\rm eff} = 5(1-\gamma_Q)$.

The dynamics of the model is as follows. We take $m_A > m_Q$. Then above the mass m_A, we assume that the theory is near the conformal fixed point. Below the threshold of A, the $SU(5)_{\rm hid}$ gauge coupling becomes larger, and it blows up (when $N_F = 5$) or goes to another fixed point discussed above (when $N_F \geq 6$). In any case, after the decoupling of the messenger fields Q and $\tilde{Q}$, SUSY is broken [19, 17].

In fact, the low energy theory after the decoupling of A is not the same as the original semi-direct gauge mediation of Ref. [18]. As in Eq. (12) of the previous subsection, the integration of A generates a superpotential

$$W \supset -\frac{\lambda^2}{2m_A}\left[(Q^i_\alpha \tilde{Q}^\alpha_j)(Q^j_\beta \tilde{Q}^\beta_i) - \frac{1}{5}(Q^i_\alpha \tilde{Q}^\alpha_i)(Q^j_\beta \tilde{Q}^\beta_j)\right]. \tag{17}$$

The presence of this superpotential is very interesting, since this term explicitly breaks R symmetry of the original semi-direct gauge mediation model, which may be useful for a generation of gaugino masses [20]. On the other hand, however, this term generates SUSY preserving vacua at $\left\langle Q\tilde{Q}\right\rangle \sim m_A m_Q/\lambda^2$, making the SUSY breaking vacuum at $\left\langle Q\tilde{Q}\right\rangle = 0$ metastable.

3.3 Composite messenger model

We consider a strong $SU(5)_{\rm hid}$ gauge theory with 5 pairs of fundamental quarks Q^i_α and anti-quarks $\tilde{Q}^\alpha_i$. Here, i and α run from 1 to 5 and they belong to $(\mathbf{5}^*, \mathbf{5})$ and $(\mathbf{5}, \mathbf{5}^*)$ representations of the $SU(5)_{\rm GUT} \times SU(5)_{\rm hid}$, respectively. Those 5 pairs of quarks play a role of messengers. We take a superpotential for the messengers,

$$W = hXQ^i_\alpha \tilde{Q}^\alpha_i, \tag{18}$$

where $X = M + F\theta^2$ is a SUSY-breaking spurion field. This model is the so-called minimal gauge mediation (see [3] and references therein), aside from the fact that the messengers are charged under the strong gauge group $SU(5)_{\rm hid}$.

The reason that we introduce the $SU(5)_{\rm hid}$ gauge interaction is to confine the messenger quarks and anti-quarks, Q and $\tilde{Q}$, forming composite fields. One of the composite states can be a candidate for the dark matter of the universe [7]. In fact we have, at low energies, mesons

$$M^i_j = Q^i_\alpha \tilde{Q}^\alpha_j, \tag{19}$$

and baryons

$$B = \det Q, \quad \tilde{B} = \det \tilde{Q}, \tag{20}$$

with the constraint

$$\det M - B\tilde{B} = \Lambda^{10}, \tag{21}$$

where Λ is the dynamical scale of $SU(5)_{\rm hid}$. The baryon B and anti-baryon $\tilde{B}$ are long-lived, since we have an approximate baryon number conservation [7]. Then, they can be a candidate for the dark matter.

Above the energy scale Λ, the messenger number is 5, and because the messengers are charged under the strong gauge group, the effective messenger number $N^{\rm eff}_{\rm mess}$ is larger than 5 as explained in Section 2. Below the energy scale Λ, the traceless part of M^i_j transforms in the adjoint representation of $SU(5)_{\rm GUT}$, so the messenger number (in the definition of this paper) is also 5. Thus this model suffers from the Landau pole problem.

For the mechanism of Section 2 to work, we introduce additional N_P flavors of quarks P^p_α and $\tilde{P}^\alpha_p$ in the representation $\mathbf{5}$ and $\mathbf{5}^*$ of $SU(5)_{\rm hid}$, respectively. Here p is the flavor index, $p = 1, \cdots, N_P$. We further introduce an adjoint field A^α_β of $SU(5)_{\rm hid}$, and introcuce a superpotential

$$W \supset \sqrt{2}\lambda Q^i_\alpha A^\alpha_\beta \tilde{Q}^\beta_i + m_P P^p_\alpha \tilde{P}^\alpha_p + m_A A^\alpha_\beta A^\beta_\alpha, \tag{22}$$

for the additional fields, as in Section 2. Then the messenger model becomes the same as the model in Section 2. The effective messenger number above the mass m_P and m_A

is given in Table 1 with N_C equal to 5. We see that the Landau pole problem can be avoided by taking m_P and m_A appropriately small.

The dynamics of the model is as follows. We assume that m_P and m_A are of the same order, $m_P \sim m_A$, for simplicity, and the theory is near the conformal fixed point above the threshold of these fields. After the decoupling of P, $\tilde{P}$ and A, the $SU(5)_{\rm hid}$ gauge coupling becomes strong and the gauge theory confines the color degrees of freedom, making composite fields described above.

However, we have to take care of the following point. When the theory is on the conformal fixed point, the Yukawa coupling in Eq. (18) becomes smaller as we lower the renormalization scale. Suppose that the theory is on the conformal fixed point from the energy scale M_* down to m_* ($\sim m_A \sim m_P$). Then, neglecting all effects other than the fixed point dynamics, the Yukawa coupling h at the scale m_* is

$$h|_{m_*} \sim \left(\frac{m_*}{M_*}\right)^{\gamma_Q} h|_{M_*}. \tag{23}$$

We show that the requirement $m_{3/2} < 16$ eV [1] leads to a constraint on N_P. For the messenger quarks not to be tachyonic [4], the SUSY-breaking scale F must satisfy $hF < (hM)^2$. Then, the gaugino mass is constrained as

$$M_{\tilde{g}} \simeq n\frac{\alpha}{4\pi}\frac{hF}{hM} < n\frac{\alpha}{4\pi}\sqrt{hF}, \tag{24}$$

where α is the SM gauge coupling fine structure constant corresponding to the gaugino $\tilde{g}$, and n is a "messenger number" contributing to the gaugino masses (in the present model, $n = 5$). Then, the gravitino mass is constrained as

$$m_{3/2} = \frac{F}{\sqrt{3}M_{Pl}} > \frac{\left(4\pi\alpha^{-1}n^{-1}M_{\tilde{g}}\right)^2}{\sqrt{3}hM_{Pl}} = 16 \text{ eV}\left(\frac{3.4\times 10^{-3}}{h}\right)\left(\frac{\alpha^{-1}}{60}\frac{M_{\tilde{g}}}{100 \text{ GeV}}\right)^2, \tag{25}$$

where $M_{Pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Note the dependence $1/h$ of this lower bound. Thus, to achieve the light gravitino mass, h should not be too small, and thus γ_Q should not be too large. In particular, the model with $N_P = 2$ may not be favored, although the effective messenger number $N_{\rm mess}^{\rm eff}$ is the smallest in this case.

[4]The messenger fields are strongly coupled in the present model, but here we pretend as if they can be treated as weakly coupled fundamental quarks, for simplicity. In fact, it is known that the gaugino masses are not so affected by strong interactions, due to the gaugino screening mechanism [21].

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Appendix A Effective messenger number in asymptotically free gauge theories

In this appendix we study how strong gauge interactions make the Landau pole problem of the SM gauge coupling severe. Suppose that messenger fields transform under the representation $r + \bar{r}$ of some strong gauge group G. At the one-loop level, the gauge coupling g of the gauge group G is given by

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g_0^2} + b\log(\mu/M_0), \tag{A.1}$$

where b is the coefficient of the one-loop β function, and g_0 is the gauge coupling at the scale M_0. We define $t \equiv \log(\mu/M_0)$ and $g^2(\mu)/8\pi^2 \equiv h_g(\mu)$ for simplicity. Then the above equation is rewritten as

$$h_g(t) = (h_{g0}^{-1} + bt)^{-1}. \tag{A.2}$$

The anomalous dimension of the messenger fields is, at the one-loop level, given by

$$\gamma(t) = -2C_2(r)h_g(t) = -2C_2(r)(h_{g0}^{-1} + bt)^{-1}, \tag{A.3}$$

where $C_2(r)$ is the quardratic Casimir of the representation r.

Taking into account Eq. (1), we define the averaged value of the anomalous dimension between $\mu = M_0$ and $\mu = M_1$ as

$$\tilde{\gamma} \equiv \frac{1}{t_1}\int_0^{t_1} dt\gamma(t) = -\frac{2C_2(r)}{b}\frac{\log(1 + bh_{g0}t_1)}{t_1}, \tag{A.4}$$

where $t_1 = \log(M_1/M_0)$. Then the averaged effective messenger number is

$$\tilde{N}_{\rm mess}^{\rm eff} = (1 - \tilde{\gamma})N_{\rm mess}, \tag{A.5}$$

where $N_{\rm mess}$ is "the tree level value" of the messenger number.

For example, consider the case $G = SU(5)$, $r = \mathbf{5}$, $b = 3 \cdot 5 - 5 = 10$, $M_0 = M_{\rm mess} \sim 10^6$ GeV, and $M_1 = M_{\rm GUT} \sim 10^{16}$ GeV. If we further assume that the coupling is very strong at M_0, e.g. $\gamma(\mu = M_0) \simeq -1$, we obtain

$$\tilde{\gamma} \simeq -0.080, \qquad \tilde{N}^{\rm eff}_{\rm mess} = (1 - \tilde{\gamma})5 \simeq 5.40. \tag{A.6}$$

The SM gauge couplings receive a contribution $\tilde{N}^{\rm eff}_{\rm mess} \log(M_{\rm GUT}/M_{\rm mess})$ from the messenger fields. Defining $M'_{\rm eff}$ by the equation

$$\tilde{N}^{\rm eff}_{\rm mess} \log(M_{\rm GUT}/M_{\rm mess}) = N_{\rm mess} \log(M_{\rm GUT}/M'_{\rm mess}), \tag{A.7}$$

we obtain

$$\frac{M_{\rm mess}}{M'_{\rm mess}} = \left(\frac{M_{\rm GUT}}{M_{\rm mess}}\right)^{\frac{\tilde{N}^{\rm eff}_{\rm mess}}{N_{\rm mess}} - 1} \sim 6. \tag{A.8}$$

The lower bound on the messenger mass scale becomes larger by this factor due to strong gauge interactions.

Appendix B Details on conformal fixed point

In this appendix, we describe how to find a conformal fixed point and compute anomalous dimensions of matter fields, taking the model of Section 2 as an example. We do not restrict the number of flavors of Q, N_Q, equal to 5 in this appendix.

Let us first discuss the existence of the infrared fixed point in the model in Section 2. For a time being we consider only the theory where the perturbative calculation for the β function is reliable. At the perturbative level, one can discuss the existence of a conformal fixed point by explicitly considering the renormalization group equations, as was first done in Ref. [22]. The NSVZ β function of the gauge coupling g and the β function of the Yukawa coupling λ of the model in Section 2 are given by

$$\beta_g = \mu \frac{\partial}{\partial \mu} g^2 = -\frac{g^4}{8\pi^2} \frac{3N_C - (1-\gamma_A)N_C - (1-\gamma_Q)N_Q - (1-\gamma_P)N_P}{1 - N_C g^2/8\pi^2}, \tag{B.1}$$

$$\beta_\lambda = \mu \frac{\partial}{\partial \mu} \lambda^2 = (\gamma_A + 2\gamma_Q)\lambda^2, \tag{B.2}$$

where we have taken λ to be real without a loss of generality, and γ_A, γ_Q and γ_P are the anomalous dimensions of A, Q, and P respectively. At the one-loop level, they are given by [5]

$$\gamma_A \simeq \frac{N_Q\lambda^2 - 2N_C g^2}{8\pi^2}, \qquad \gamma_Q \simeq \frac{N_C^2-1}{N_C}\frac{\lambda^2 - g^2}{8\pi^2}, \qquad \gamma_P \simeq -\frac{N_C^2-1}{N_C}\frac{g^2}{8\pi^2}. \tag{B.3}$$

By taking a large N limit, $N_C,\ N_Q,\ N_P \gg 1$ with $N_Q/N_C \sim \mathcal{O}(1)$, $N_P/N_C \sim \mathcal{O}(1)$ and $n \equiv 3N_C - N_C - N_Q - N_P \sim \mathcal{O}(1)$, one can find a solution to the equations $\beta_g = 0,\ \beta_\lambda = 0$ with the couplings g and λ being very small (Banks-Zaks like fixed point). The result is

$$\frac{\lambda^2}{8\pi^2} \simeq \frac{4n}{(2N_C - N_Q)^2 + N_P(2N_C + N_Q)}, \qquad \frac{g^2}{8\pi^2} \simeq \frac{N_Q + 2N_C}{4N_C}\frac{\lambda^2}{8\pi^2}, \tag{B.4}$$

and

$$\gamma_Q \simeq \frac{(2N_C - N_Q)n}{(2N_C - N_Q)^2 + N_P(2N_C + N_Q)}, \quad \gamma_P \simeq -\frac{(2N_C + N_Q)n}{(2N_C - N_Q)^2 + N_P(2N_C + N_Q)}, \tag{B.5}$$

with $\gamma_A = -2\gamma_Q$. It is also easy to check that the fixed point is infrared stable. The coupling constants (or more precisely, 't Hooft couplings) of the theory are small, so we can trust perturbative calculation. Thus we consider that the existence of a conformal fixed point in the above limit is established, and the fixed point values of the anomalous dimensions are given by Eq. (B.5).

However, we are interested in the case where the coupling is strong, so that the anomalous dimension γ_Q is quite large and any perturbative calculation is not reliable at all. A very astonishing fact of supersymmetric conformal field theory is that anomalous dimensions of fundamental fields can be determined exactly, even in strongly coupled theories. The general method is called a-maximization [24]. In $\mathcal{N} = 1$ superconformal field theories, there is an R symmetry which appears in superconformal algebra (which is an extension of ordinary supersymmetry algebra). In some theories there may be a unique anomaly free R symmetry, and if the theories are in the conformal window [25], the R symmetry must be the one which appears in the superconformal algebra. However, in general there is a family of anomaly free R symmetries (as in the model of Section 2; see below), and

[5] See e.g. Section 5.5 of Ref. [23]. Note that our convention for the anomalous dimension is larger than that used in Ref. [23] by factor 2.

	A	$Q,\ \tilde{Q}$	$P,\ \tilde{P}$
R	$-2x$	$1+x$	$1+N_P^{-1}(2N_C-N_Q)x$

Table 3: R charges of the fields. x is a parameter that parametrize the ambiguity of the definition of R symmetry.

we cannot determine from symmetry argument alone which R symmetry is the superconformal one [6]. According to Ref. [24], the superconformal R symmetry is the one which (locally) maximizes the following combination of t' Hooft anomalies,

$$\sum_i [3(R_i-1)^3-(R_i-1)], \tag{B.6}$$

where the sum is taken over fermions of a theory and R_i-1 is the R charge of the fermion in chiral field i. This condition determines the R charge R_i of the chiral field i. Furthermore, the scaling dimension D_i and the anomalous dimension γ_i of chiral field i is related to the R charge R_i by the equation

$$1+\frac{\gamma_i}{2}=D_i=\frac{3}{2}R_i. \tag{B.7}$$

The first equality in Eq. (B.7) is almost the definition of the anomalous dimension in conformal field theory. For the second equality, see e.g. [26]. From Eq. (B.7), we can determine the anomalous dimension γ_i from the R charge R_i.

Let us apply the above method to the model of Section 2. We, here, neglect all masses for the fields, and assume that the model is in the conformal window for certain values of N_C, N_Q and N_P. The R charges of the fields are shown in Table 3. We have imposed that Q and $\tilde{Q}$ (P and $\tilde{P}$) have the same R charge. Even then, the R charges of the fields are not uniquely determined. We have parametrized the ambiguity of R charges by x. Then, we define the following function of x,

$$\begin{aligned} a(x) &\equiv \sum_i [3(R_i-1)^3-(R_i-1)] \\ &= (N_C^2-1)[3(-2x-1)^3-(-2x-1)]+2N_CN_Q[3x^3-x] \\ &\quad +2N_CN_P[3(N_P^{-1}(2N_C-N_Q)x)^3-(N_P^{-1}(2N_C-N_Q)x)]. \end{aligned} \tag{B.8}$$

[6] In this paper we do not consider the case where the superconformal R symmetry is an accidental symmetry of a low energy theory.

	$N_P=2$	$N_P=3$	$N_P=4$	$N_P=5$	$N_P=6$
$N_C=5$	0.273	0.143	0.059	$\times$	$\times$
$N_C=6$	0.422	0.280	0.179	0.104	0.046
$N_C=7$	$\times$	0.391	0.287	0.205	0.138
$N_C=8$	$\times$	0.478	0.376	0.292	0.223
$N_C=9$	$\times$	$\times$	0.448	0.366	0.296

Table 4: The values of γ_Q obtained in Eq. (B.5) by perturbative calculation. This table should be compared with Table 1, where the exact value of γ_Q is listed. N_Q is taken to be 5.

Then, the condition for the local maximization of $a(x)$ is given by

$$\frac{\partial a(x)}{\partial x} = 0, \quad \frac{\partial^2 a(x)}{\partial x^2} < 0. \tag{B.9}$$

Solving these equations is quite straightforward. Using the solution for x, the anomalous dimesion of e.g. Q is given by $\gamma_Q = 3R_Q - 2 = 3x + 1$. The value of γ_Q is listed in Table 1.

As a check, we list the one-loop value of γ_Q obtained in Eq. (B.5) in Table 4. Note that the agreement between Table 1 and 4 is quite good, and becomes better as the coupling becomes weaker.

For what values of (N_C, N_Q, N_P) the model has a conformal fixed point is a rather non-trivial question. In conformal field theory, it is known that all gauge invariant operators of a theory have scaling dimensions greater than or equal to 1 [27]. As a criterion of the existence of a conformal fixed point, we require that all gauge invariant chiral (primary) operators have scaling dimensions greater than or equal to 1. Such a criterion was first used in Ref. [25] to find a conformal fixed point in SUSY QCD. Especially, in the case of the present models, we have imposed that the scaling dimensions of gauge invariant chiral operators $\mathrm{tr}\, A^2$ and $P^a \tilde{P}_b$ satisfy the conditions

$$D_{\mathrm{tr}\, A^2} = 2\left(1 + \frac{\gamma_A}{2}\right) \geq 1, \tag{B.10}$$

$$D_{P\tilde{P}} = 2\left(1 + \frac{\gamma_P}{2}\right) \geq 1. \tag{B.11}$$

In this paper we assume that if these conditions are satisfied, the model has a conformal fixed point.

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