Can supersymmetry breaking lead to electroweak symmetry breaking via formation of scalar bound states?

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If the particle recently discovered at the LHC is the lightest Higgs boson, it is heavy enough to raise a concern as to whether low-energy supersymmetry as in, for example, the minimal supersymmetric standard model (MSSM), is consistent with the 125 GeV mass of the Higgs boson. A number of solutions have been proposed to relax the MSSM upper bound on the mass of the lightest Higgs boson. Here we explore the possibility that the trilinear supersymmetry breaking terms can be large enough for the formation of bound states of squarks via the Higgs boson exchange. Such bound states can appear in the low-energy effective theory as additional Higgs bosons, and they can mix with the fundamental Higgs boson. Furthermore, supersymmetry breaking can trigger electroweak symmetry breaking by formation of such bound states with non-zero vacuum expectation values. In the resulting vacuum, the usual relation between the gauge couplings and the Higgs self-coupling does not apply, and there is no reason to expect the same upper bound on the mass of the lightest Higgs boson. We explore the bound states using the Bethe-Salpeter (BS) equation whose lowestorder kernel is a one-loop crossed box graph, and calculate the minimal value of the tri-linear coupling required for the formation of the bound states using a variational approach. The result is about 2.5 times the value at which one-loop corrections in the MSSM give a maximum Higgs mass.

The recently discovered 125 GeV boson [1, 2] is widely thought to be the Higgs boson. But with a mass outside the range associated with the predictions of the simplest supersymmetric models, combined with the lack of evidence for superpartners [3], it has encouraged a number of efforts to reconcile low-energy supersymmetry with a relatively heavy Higgs boson (see, e.g., Refs. [4–6]). The models usually assume heavy masses for the superpartners, as well as some novel features, for example, strong couplings in the supersymmetry breaking sector [6]. While many models with gauge-mediated supersymmetry breaking predict a small tri-linear supersymmetry breaking coupling A, a large value of such coupling is, in fact, helpful in raising the range of the Higgs boson masses toward 125 GeV [4, 7]. Large tri-linear terms can appear in gauge-mediated supersymmetry breaking models, albeit some fine-tuning of parameters may be required in a realistic model [4].

However, if the tri-linear couplings are large, the low-energy realization of supersymmetry may differ dramatically from the usual set of predictions. It has been pointed out [8, 9] that the exchange of the (lighter) Higgs boson between (heavier) squarks can lead to formation of bound states, resonances, and a new strongly coupled realization of the minimal supersymmetric standard model (MSSM). Here we reconsider this possibility and will focus, in particular, on the possibility that supersymmetry breaking may trigger electroweak symmetry breaking via formation of relativistic squark bound states having the quantum gauge numbers of the Higgs boson. Such new states can mix with the Higgs boson, they can acquire a vacuum expectation value (VEV), and the resulting multi-Higgs low-energy effective theory may have a very different appearance from the usual weakly coupled MSSM. At the same time, the ultraviolet behavior of the theory is preserved, and supersymmetry provides the usual solution to the hierarchy problem. The difference is in the low-energy effective theory, which contains different degrees of freedom: fewer squarks, and more Higgs bosons, whose VEVs produce a more complicated vacuum. In this vacuum, the usual MSSM relations between the gauge couplings and the scalar self-coupling do not hold, and, therefore, there is no reason for the upper bound on the lightest Higgs boson to be the same as in the usual version of MSSM.

Let us consider a simplified version of MSSM, in which we will focus only on the third generation of squarks and will assume that only one tri-linear term is large:

$$\mathcal{L} = A_t (\tilde{t}_L^{\dagger} \cdot \phi) \tilde{t}_R + h.c. \tag{1}$$

where \tilde{t}_L is the Y = 1/3 stop doublet under $SU(2)_L$, \tilde{t}_R the Y = 4/3 stop singlet, and ϕ the Y = -1 Higgs doublet. We omit writing ϕ^4 terms. For simplicity we assume the squarks have a common mass $M \sim$ a few TeV, considerably larger than the Higgs mass m.

We have suppressed the SU(3) indices in Eq. 1, and will concentrate on the color-singlet bound state. This is the only bound state that can have a mixing with the fundamental Higgs boson, and as discussed below, we expect there to be a range of parameters in which this bound state has a non-zero VEV, while all the SU(3) non-singlet bound states (which can form through the Higgs exchange as well) have a zero VEV. This case corresponds to the standard-model-like multi-Higgs vacuum consistent with the data. We seek a CP+ scalar doublet with Y = 1 (the quantum number of one of the Higgs fields) arising as a $(\tilde{t}_R \tilde{t}_L^{\dagger})$ bound state described by a Euclidean BS equation as shown in Fig. 1.



FIG. 1. The Bethe-Salpeter equation in "vertex" form. The double line represents the bound state Φ , of momentum P, and the single lines represent the constituent squarks, of mass M.

This BS equation is in vertex form, where the internal lines represent propagators and the usual BS wave function $\Psi(P,p)$ is related to Q(P,p) by

$$\left[\left(\frac{P}{2}+p\right)^2+M^2\right]\left[\left(\frac{P}{2}-p\right)^2+M^2\right]\Psi(P,p)=Q(P,p).$$
(2)

It is useful to state the BS equation in this form because it is closely related to a gap equation whose non-trivial solution yields an estimate of how large the coupling in the kernel K must be to yield a bound-state Higgs that mimics the Higgs field ϕ . This bound-state Higgs with symmetry breaking mixes L and R stops and contributes to their mass difference.

Let the line labeled (P/2) + p represent an outgoing \tilde{t}_R , and the line labeled (P/2) - p represent an outgoing \tilde{t}_L^{\dagger} . A few minutes of drawing Feynman graphs shows that the lowest-order kernel must be a crossed box graph, as shown in Fig. 2.

FIG. 2. The lowest-order kernel.

The lines labeled L, R are the stops, and the dashed lines are the Higgs fields of the MSSM. The Euclidean BS equation is

$$Q(P,p) = \frac{A_t^4}{(2\pi)^4} \int d^4k \, Q(P,k) \frac{1}{\left[\left(\frac{1}{2}P+k\right)^2 + M^2\right] \left[\left(\frac{1}{2}P-k\right)^2 + M^2\right]} K(P,p,k). \tag{3}$$

We have omitted writing an SU(2) spinor index on Q and a corresponding factor δ_{ij} on the kernel.

The complications of this one-loop kernel have prevented us from studying this BS equation at general momentum P, which would furnish a relation between the values of P for which the equation is solvable and the coupling A_t . Instead, we look for the value of A_t at which P = 0, corresponding to having four degenerate massless bound states. This total of four massless states corresponds to the two complex elements of the bound-state SU(2) spinor. Alternatively, these states can also be considered as states of a broken $SU(2) \times U(1)$ theory: a zero-mass composite Higgs boson, plus three zero-mass Nambu-Goldstone bosons. We are interested in Higgs bosons whose mass is comparable to m, the lightest Higgs boson mass in the MSSM. By hypothesis this is close to 125 GeV, while the squark mass M is much larger. Thus, it should be a reasonable approximation to consider the bound-state Higgs as having zero mass and study the P = 0 BS equation. In addition, this P = 0 BS equation can be tackled with decent quantitative accuracy. From now on we use the notation K(p, k) = K(k, p) for the original kernel at P = 0, whose bound states can only occur for specific values of A_t that are eigenvalues of the BS equation.

In the general case with $P \neq 0$ the BS wave function $\Psi(P,p)$ is the Fourier transform of the matrix element

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$$\psi(X,x) = \langle 0|T(\tilde{t}_L^{\mathsf{T}}(x_1)\tilde{t}_R(x_2))|P\rangle \tag{4}$$



FIG. 3. Diagrams for the effective potential quadratic in δM^2 (weights not shown).

with P conjugate to the center-of-mass coordinate $X = (1/2)(x_1 + x_2)$ and p conjugate to $x_1 - x_2$; the state $|P\rangle$ is the bound state. At P = 0 this looks like a vacuum-to-vacuum propagator $\Delta_{LR}(x_1 - x_2)$, but one must be careful about what the vacuum means. Just as in superconductivity the true vacuum is a non-perturbative construct quite different from the bare vacuum; in our case the true vacuum has matrix elements connecting L and R stops. This connection comes from a symmetry-breaking order parameter that is a mass splitting $\delta M^2(p)$ found in this LRpropagator, vanishing in the symmetric case, that mixes L and R squarks. This is analogous to the $\langle \psi \psi \rangle$ propagator of superconductivity [11]. To lowest order in this order parameter the diagonal propagators of the $\tilde{t}_{L,R}$ fields are just that already shown in the BS equation:

$$\Delta_{LL}(p) = \Delta_{RR}(p) = \frac{1}{p^2 + M^2} \tag{5}$$

while the LR mixing propagator is

$$\Delta_{LR}(p) = \frac{1}{p^2 + M^2} \delta M^2(p^2) \frac{1}{p^2 + M^2}.$$
(6)

Away from the symmetry-breaking threshold, one would expect that for larger values of A_t the mass of the Higgs particle moves away from zero, while the Nambu-Goldstone fields remain massless. This can proceed (for example, see [10]) through a tachyonic solution to the BS equation, much as in the stabilization of a Mexican-hat potential where the stable non-perturbative vacuum yields a condensate and a Higgs boson of normal mass. As for the Nambu-Goldstone bosons, since the pioneering work of Nambu [11] we know that these massless Nambu-Goldstone excitations occur as a consequence of a non-trivial solution to a gap equation, an integral equation whose solution is a symmetrybreaking order parameter. This gap equation is essentially the BS equation at P = 0. (See [12] for a proof of this Nambu-Goldstone mechanism in gauge theories such as QCD.) We express the dynamics of symmetry breaking through the usual two-particle irreducible (2PI) effective potential Γ [13], in which Γ is a functional of $\delta M^2(p^2)$. In this first investigation of the bound-state Higgs we ignore a number of interesting phenomena, including the VEV of the elementary Higgs fields and their possible mixing with the bound-state Higgs, so the effective potential (in the notation of [13]) is

$$\Gamma = \frac{1}{2} \operatorname{Tr} \{ \ln G + [1 - GG_0^{-1}] \} + 2 \operatorname{PI} \text{ graphs}, \tag{7}$$

where the trace is over space-time as well as other relevant indices, such as particle type, G is the exact propagator, and G_0 is the free propagator (when relevant; the term in square brackets is omitted for the LR propagator). The extrema of Γ as the G are varied yield the Schwinger-Dyson equations of the theory.

To lowest order in δM^2 the effective action is given by the diagrams shown in Fig. 3, which give for Γ the expression

$$\Gamma = \frac{1}{2} \int d^4 p \rho(p^2) [\delta M^2(p^2)]^2 - \frac{A_t^4}{2(2\pi)^4} \int d^4 p \int d^4 k \rho(p^2) \delta M^2(p^2) K(p,k) \rho(k^2) \delta M^2(k^2)$$
(8)

where

$$\rho(k^2) = \frac{1}{(k^2 + M^2)^2}.$$
(9)

Variation of the quadratic terms in Γ with respect to δM^2 yields

$$\delta M(p)^2 = \frac{A_t^4}{(2\pi)^4} \int d^4k \frac{1}{k^2 + M^2} \delta M(k)^2 \frac{1}{k^2 + M^2} K(p,k).$$
(10)

This equation is analogous to standard gap equations for chiral symmetry breaking. Just as for chiral gap equations it is in fact the original BS equation, in vertex form, at P = 0, illustrating as before [11, 12] the necessary existence of composite Nambu-Goldstone bosons when symmetries are broken without elementary Higgs fields. It differs from chiral symmetry breaking gap equations because the kernel is well-behaved in the UV and there are no UV divergences. The kernel falls like $1/p^4$ (modulo logarithms) at large momentum, implying the same falloff for δM^2 , and Eq. (10) is finite.

To analyze Eq. (10) we need to analyze first the kernel K(p,k). This kernel has the form

$$K(p,k) = \frac{1}{(2\pi)^4} \int d^4l \, \frac{1}{[l^2 + M^2][(p+l)^2 + m^2][(k+l-p)^2 + M^2][(k+l)^2 + m^2]}$$

$$= \frac{1}{16\pi^2} \int \prod dx_i \delta\left(1 - \sum x_i\right) \frac{1}{D^2}$$
(11)

with

$$D = k^{2}(x_{1}x_{2} + x_{3}x_{4}) + p^{2}(x_{1}x_{4} + x_{2}x_{3}) + (p+k)^{2}x_{2}x_{4} + (p-k)^{2}x_{1}x_{3} + M^{2}(x_{2} + x_{4}) + m^{2}(x_{1} + x_{3}).$$
(12)

Now suppose that $M^2 \gg m^2$, in which case x_2, x_4 have to be small compared to the other Feynman parameters. So write $x_2 = \lambda x, x_4 = \lambda(1-x)$, with new integration variables running from 0 to 1. The integral over λ will be dominated by small λ , so we can drop this variable judiciously. Then approximately

$$\prod dx_i \delta\left(1 - \sum x_i\right) = \lambda d\lambda dx dx_1 dx_3 \delta(1 - x_1 - x_3)$$
(13)

and

$$D = x_1 x_3 (p-k)^2 + m^2 (x_1 + x_3) + \lambda [ak^2 + (1-a)p^2 + M^2]$$
(14)

where

$$a = x_1 x + x_3 (1 - x), \ 0 \le a \le 1, \tag{15}$$

and we dropped a term $\sim \lambda^2$ in D.

Now do the integral over λ explicitly, with the result

$$K(p,k) \approx \frac{1}{16\pi^2} \int dx_1 dx_3 dx \delta(1-x_1-x_3) \left\{ \frac{1}{A^2} \ln\left[\frac{A+B}{B}\right] - \frac{1}{A(A+B)} \right\}$$
(16)

where

$$A = [ak^{2} + (1-a)p^{2} + M^{2}], B = x_{1}x_{3}(p-k)^{2} + m^{2}.$$
(17)

We now show that at large $k^2 \gg p^2$, $K \sim (1/k^4) \ln k^2$ and so vanishes rapidly. This means that the integral in the gap equation also vanishes rapidly, as we will soon see. Since B does not depend on x the integral over x can be done, yielding

$$K \to \frac{\ln k^2}{8\pi^2 k^4},\tag{18}$$

a result that we can get exactly at p = 0.

The next step is to reduce the gap equation to a one-dimensional equation by integrating over the angles of k. In the gap equation, because the angle between the four-momenta appears only in logarithms or in a term parametrically small with respect to M^2 , we make the approximation

$$\int d\Omega_k F[(p-k)^2] \approx 2\pi^2 [\theta(p^2-k^2)F(p^2) + \theta(k^2-p^2)F(k^2)].$$
(19)

This is exactly true for $F = 1/(p-k)^2$ or for constant F, and is acceptable for the logarithmic functions we encounter. After projecting out the s-wave, to lowest (quadratic) order in δM^2 the relevant part of Γ is

$$\Gamma = \frac{1}{2} \int dp^2 \, p^2 \, \rho(p^2) [\delta M^2(p^2)]^2 - \frac{A_t^4}{2(2\pi)^4} \int dp^2 \, p^2 \, \int dk^2 \, k^2 \, \rho(p^2) \delta M^2(p^2) \hat{K}(p^2, k^2) \rho(k^2) \delta M^2(k^2) \tag{20}$$

where \hat{K} is the s-wave projection of the kernel.

Variation of this equation yields the s-wave projection of Eq. (10), which becomes a standard one-dimensional homogeneous Fredholm integral equation with a discrete spectrum of eigenvalues A_t^4 . We seek the lowest eigenvalue by inserting a trial function into Eq. (20) and doing the integrals numerically, including an approximation to the integral over x_1 in the kernel (see Eq. (13)), for various values of the Higgs-to-stop mass ratio m/M.

We motivate our trial functions for the crossed-graph kernel of interest from known exact results [14] for the BS equation with the massless kernel

$$K \sim \frac{1}{(p-k)^2}.\tag{21}$$

For the vertex form of the BS equation, given in Eq. (3), the lowest eigenfunction at P = 0 is

$$Q(0,p) \equiv \delta M^2(p^2) \sim \frac{1}{p^2 + M^2}.$$
(22)

Naturally, the class of trial functions of this form with M^2 replaced by a variational parameter μ^2 yields the exact result. In the present problem the asymptotic behavior is different, so we choose as a zeroth-order trial function

$$\delta M_0^2(p^2) \sim \frac{1}{p^4 + \mu^4}.$$
 (23)

We have studied other trial functions, such as $1/(p^2 + \mu^2)^2$, with similar results. We improve this first variational estimate by using δM_0^2 as input to the right-hand side of Eq. (20), numerically calculating a new output δM_1^2 . We made a simple but accurate fit to δM_1^2 , amounting to adding a term $\sim p^2$ to the denominator of Eq. (23). Then we used the average $\delta M_2^2 \equiv (1/2)(\delta M_0^2 + \delta M_1^2)$ as a trial function, and calculated the output again. This yields excellent agreement between the new input and output, as shown in Fig. 4, for the specific value m/M = 0.05.



FIG. 4. A comparison of the input and output using the second-order eigenfunction δM_2^2 , as a function of p^2/M^2 , calculated numerically as described in the text. In the case of the exact solution, the two curves would be identical.

At this elementary Higgs mass the critical coupling resulting from our numerical calculations is

$$\frac{A_t}{M} \approx 15.14. \tag{24}$$

This estimate is about 2.5 times the value of $A_t/M = 6$ that maximizes the lightest Higgs mass in an approximate one-loop calculation [4]. But this is not the final verdict, since mixing of the bound Higgs with the MSSM Higgs and other possible bound states need investigation, the results of which we will report later.

Also of interest is the needed critical coupling for various masses of the elementary Higgs field. This is shown in Fig. 5. As expected, the critical coupling increases with increasing Higgs mass.



FIG. 5. Behavior of the critical coupling as a function of the elementary Higgs mass.

While it appears plausible that supersymmetry breaking in the MSSM can trigger electroweak symmetry breaking via the formation of bound states with non-zero VEVs, further work is needed before one can build a realistic model and compare its predictions with the data. In addition to the color-singlet states, the same trilinear scalar interactions can cause colored bound states to form. The viability of the model depends on its ability to produce a standard-modellike vacuum with broken $SU(2) \times U(1)$ but unbroken SU(3), in which case the colored bound states are turned into color singlets by strings attached to gluons or quarks. It is well known that the MSSM, in its traditional realization, has a number of dangerous color and charge breaking minima, although cosmological evolution favors the vacuum with unbroken SU(3) even in some cases where it is not the global minimum of the potential [15]. In our case, one must now re-examine the same issue taking into account a number of new effective degrees of freedom. While the full analysis is obviously complicated and the results will inevitably be model-dependent, there is one feature of the color-singlet states that sets them apart from the rest. The color-singlet states can have a mixing with the fundamental Higgs bosons via the same coupling as that which enters the BS equation. The mass matrix in the bound-state-Higgs basis has both diagonal terms and off-diagonal terms. In contrast, the colored bound states can only have diagonal terms. As the BS coupling increases, the mass squared of each bound state decreases, as discussed above. Since the scalar exchange forces are essentially color-blind, the bound states with different SU(3) properties can have similar binding energies. However, thanks to the off-diagonal terms, the colorless bound states can develop a VEV simultaneously with the Higgs boson for some value of the trilinear coupling for which the diagonal terms are still positive. This possibility leads to an appropriate standard vacuum.

The electroweak precision measurements, and, in particular, the ρ parameter, should place constraints on any strongly coupled model built along the lines we discussed. These constraints imply a lower bound on the mass of the squarks. It is possible, and, in fact, likely, that the squark masses would have to be of the order of 5 - 10 TeV for the model to be consistent with the precision measurements. This forces the A term to be correspondingly larger, and there may appear to be a small hierarchy between the supersymmetry breaking scale and the electroweak scale. This hierarchy may impose some degree of fine-tuning on a realistic model based on strongly coupled broken supersymmetry. This is a likely potential drawback of the otherwise very appealing scenario, in which the scale of the electroweak symmetry breaking is determined by the breaking of supersymmetry. However, the class of models we discuss still possesses a robust solution to the big hierarchy problem: above the scale of the bound states, the MSSM exists in its usual incarnation, and supersymmetry stabilizes the scales in the usual way.

We have examined the possibility of electroweak symmetry breaking by the formation of bound states of squarks via the Higgs boson exchange, which bound states can mix with the fundamental Higgs bosons and can acquire VEVs simultaneously with these fundamental bosons. This scenario is clearly different from the widely discussed technicolor models [16], walking technicolor [17], and the models in which supersymmetry and technicolor are combined [18]. Our scenario has potential to relate the scales of supersymmetry breaking and electroweak breaking in a new way, but the applications to realistic models requires a more detailed analysis. The next step that we plan to undertake is to investigate the mixing of the elementary and composite Higgs bosons and the resulting symmetry-breaking patterns. These results will be presented elsewhere.

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