Asymptotic safety: motivations and results

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Outline

The gospel according to de Witt

Expand

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gauge fix using a background gauge fixing condition e.g.

$$S_{GF}(\bar{g}, h) = \frac{1}{2}\int dx\sqrt{-\bar{g}}\,\bar{g}^{\mu\nu}\chi_\mu\chi_\nu\ ; \qquad \chi_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{1}{2}\bar{\nabla}_\mu h$$

Add ghost Lagrangian

$$S_{ghost}(\bar{g}, \bar{c}, c) = \int dx\sqrt{-\bar{g}}\,\bar{c}^\mu(-\delta^\nu_\mu\bar{\nabla}^2 - \bar{R}^\nu{}_\mu)c_\nu$$

Compute $\Gamma(\bar{g}, h)$.
Formalism preserves background gauge invariance:
$\delta_\epsilon\bar{g}_{\mu\nu} = \mathcal{L}_\epsilon\bar{g}_{\mu\nu}$, $\delta_\epsilon h_{\mu\nu} = \mathcal{L}_\epsilon h_{\mu\nu}$.

Issues

Non-renormalizable (Goroff and Sagnotti 1985)

- interaction strength grows like $\tilde{G} = Gk^2$
- violation of unitarity
- lack of predictivity

General EFT recipe

- fix the level of precision that is required in the calculation
- fix the ratio E/M. This determines the order of the expansion that will be required
- at the given order in the expansion, determine all the terms in the action that can contribute to the given process.
  By power counting there can only be finitely many.
- if the fundamental theory is known, their coefficients can in principle be calculated
- otherwise, measure them with as many experiments.
- use the EFT Lagrangian to compute the amplitudes to the desired precision

Chiral perturbation theory

Strong interactions at low energy described by chiral model

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6) \right]$$

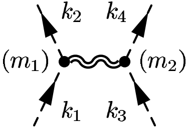
Expansion parameter $E/4\pi f_\pi$.
For low energy meson physics need f_π, ℓ_1, ℓ_2 and perhaps a few others. One loop in f_π, tree level in ℓ_1, ℓ_2.
Successfully describe a rich phenomenology.

Gravity

$$S = \int dx\sqrt{g}\left[2m_P^2\Lambda - m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu}R^{\mu\nu} + \ell_3 R^2 + O(\partial^6)\right]$$

m_P similar to f_π.

Newtonian potential



$$\sim \frac{Gm_1 m_2}{\mathrm{q}^2}$$

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{Gm_1 m_2}{\mathrm{q}^2} e^{i\mathrm{qr}} = -\frac{Gm_1 m_2}{r}$$

Leading quantum correction

For dimensional reasons the leading quantum correction is

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + \beta\frac{G\hbar}{r^2 c^3} + \dots\right]$$

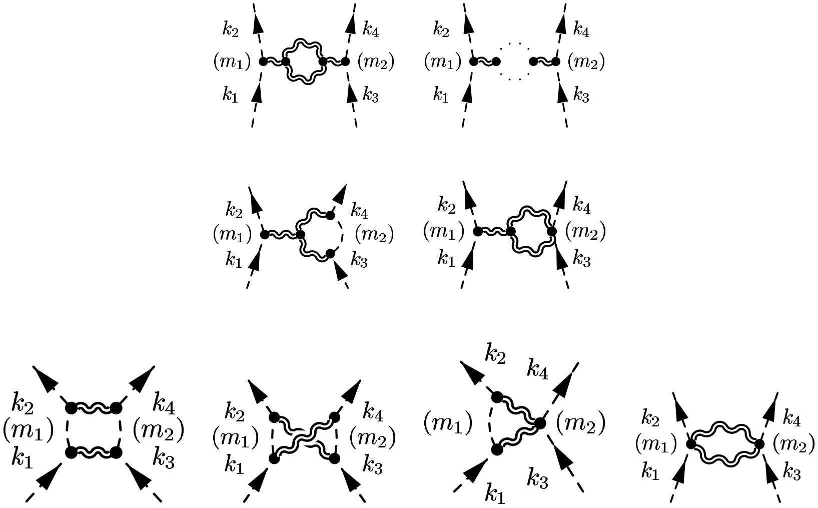
and

$$\int \frac{d^3q}{(2\pi)^3} \log\left(\frac{\mathrm{q}^2}{\mu^2}\right) e^{i\mathrm{qr}} = -\frac{1}{2\pi^2 r^3}$$

Clearly distinct from contributions of local counterterms that give analytic corrections to the amplitude. E.g.

$$\int \frac{d^3q}{(2\pi)^3} \ell e^{i\mathrm{qr}} = \ell\delta(r)$$

One loop graphs



(from Bjerrum Bohr, Donoghue, Holstein, 2002)

Evaluation

- J.F. Donoghue, P.R.L. 72, 2996 (1994); P.R.D50, 3874 (1994)
- H.W. Hamber, S. Liu, Phys. Lett. B357, 51 (1995)
- A. Akhundov, S. Bellucci, A. Shiekh, Phys. Lett. B395, 16 (1997)
- N.E.J. Bjerrum-Bohr (2002) Phys. Rev. D66, 084023
- I.B. Khriplovich, G.G. Kirilin (2002) J. Exp. Theor. Phys. 95, 981-986 (Zh. Eksp. Teor. Fiz. 95, 1139-1145 (2002))
- N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein Phys. Rev. D68, 084005; Erratum-ibid.D71, 069904 (2005)
- N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein Phys. Rev. D 67, 084033 (2003) [Erratum-ibid. D 71 (2005) 069903
- I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

A prediction of quantum gravity

$$V(r) = -\frac{Gm_1 m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2 c^3} + \dots\right]$$

EFT makes predictions!
Comes from non-local terms in the effective action, e.g.

$$\int dx\sqrt{g}\,[RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu}]$$

Local terms cannot be predicted

Lessons

- no clash between QM and GR
- experimentally indistinguishable from classical GR
- agrees with all experimental data
- to some extent, background independent
- open issues in the UV, IR, strong field

The issues of QG, and the RG solution

Try to extend beyond Planck scale

- interaction strength grows like $\tilde{G} = Gk^2$
- lack of predictivity

first problem solved if $\tilde{G} = G(k)k^2 \to \tilde{G}_*$

more generally, if

$$\Gamma_k(\bar{g}, h) = \sum_i \lambda_i(k)\mathcal{O}_i(\bar{g}, h)$$

require $\tilde{\lambda}_i \to \tilde{\lambda}_{i*}$, where $\tilde{\lambda}_i = k^{-d_i}\lambda_i$

the RG solution cont'd

Define a theory space (fields, symmetry, action functionals) parameterized by $\tilde{\lambda}_i$.

- RG trajectory is "renormalizable" or "asymptotically safe" if it flows to a FP in the UV
- The RG trajectories that flow into the FP for $k \to \infty$ form the UV critical surface S_{UV}
- Predictivity demands $\dim(S_{UV})$ is finite

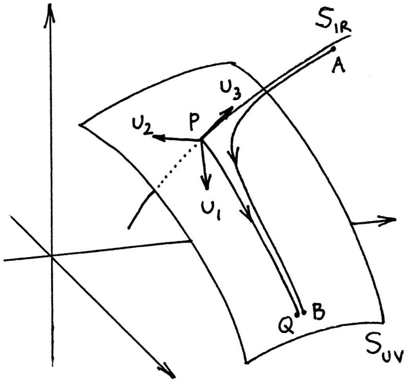
Examples

QCD:

- Gaußian Fixed Point at $\tilde{\lambda}_{i*} = 0$.
- $\tilde{\lambda}_i = k^{-d_i} \lambda_i$
- $\tilde{\beta}_i = \partial_t \tilde{\lambda}_i = -d_i \tilde{\lambda}_i + k^{-d_i} \beta_i$
- $M_{ij}\big|_* = \frac{\partial \tilde{\beta}_i}{\partial \tilde{\lambda}_j}\big|_* = -d_i \delta_{ij}$
- relevant couplings=renormalizable couplings

Non-renormalizable examples: 4-Fermi interactions in $d = 3$

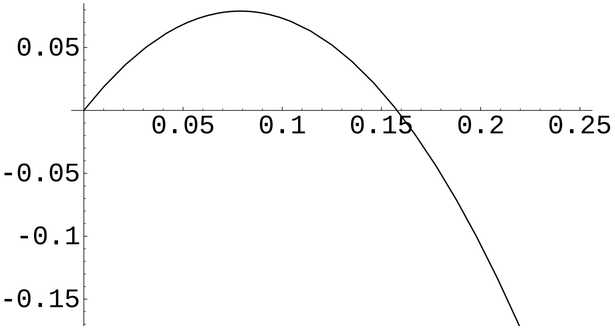
General picture



Gravity in $2+\epsilon$

$d-2=\epsilon$

$$\tilde{G} = Gk^{\epsilon}$$
$$\beta_{\tilde{G}} = \epsilon\tilde{G} - \frac{38}{3}\tilde{G}^2$$



Higher derivative gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} \left(C^2 - \frac{2\omega}{3} R^2 + 2\theta E \right) \right]$$

$$Z = \frac{1}{16\pi G}$$

K.S. Stelle, Phys. Rev. **D16**, 953 (1977).
J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).
E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).
I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).
G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).
A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006)
N. Ohta and R.P. arXiv:1308.3398

Beta functions I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1+\lambda_0\frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

Beta functions II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right] - \frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83 + 70\omega + 8\omega^2)/18\pi$

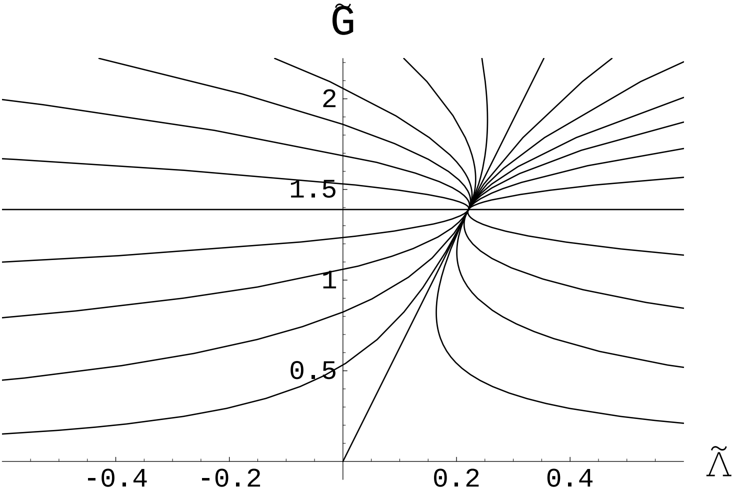
Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_*\tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - q_*\tilde{G}^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221 \ , \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389 \ .$$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



Topologically massive gravity

Action

$$S(g) = Z \int d^3x \sqrt{g} \left(2\Lambda - R + \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu} \Gamma^{\rho}_{\lambda\sigma} \left(\partial_\mu \Gamma^{\sigma}_{\nu\rho} + \frac{2}{3} \Gamma^{\sigma}_{\mu\tau} \Gamma^{\tau}_{\nu\rho}\right)\right)$$

$$Z = \frac{1}{16\pi G}$$

Dimensionless combinations of couplings

$$\nu = \mu G \; ; \qquad \tau = \Lambda G^2 \; ; \qquad \phi = \mu/\sqrt{|\Lambda|}$$

R.P., E. Sezgin, Class.Quant.Grav. 27 (2010) 155009, arXiv:1002.2640 [hep-th]

Recently extended to TM SUGRA: R.P., M. Perry, C. Pope, E. Sezgin, arXiv 1302.0868

Beta functions of

$$\beta_\nu = 0 \,,$$
$$\beta_{\tilde{G}} = \tilde{G} + B(\tilde{\mu})\tilde{G}^2 \,,$$
$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{2}\tilde{G}\left(A(\tilde{\mu},\tilde{\Lambda}) + 2B(\tilde{\mu})\tilde{\Lambda}\right)$$

Since $\nu = \mu G = \tilde{\mu}\tilde{G}$ is constant

can replace $\tilde{\mu}$ by $\nu/\tilde{G}$

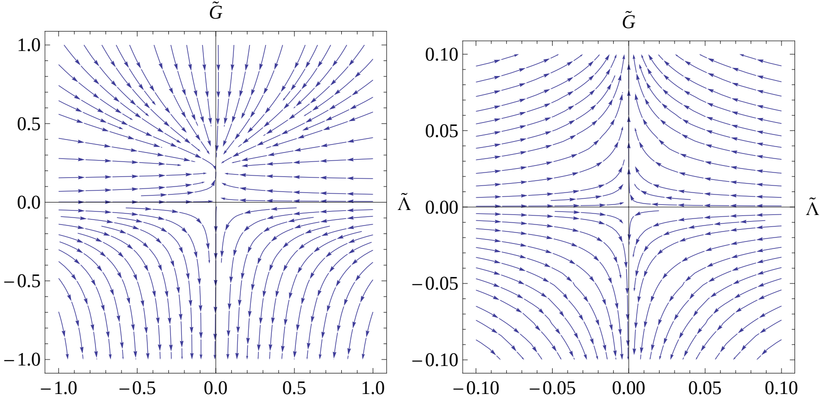


Figure : The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\alpha = 0$, $\nu = 5$. Right: enlargement of the region around the origin, showing the Gaussian FP. The beta functions become singular at $|\tilde{G}| = 1.9245$.

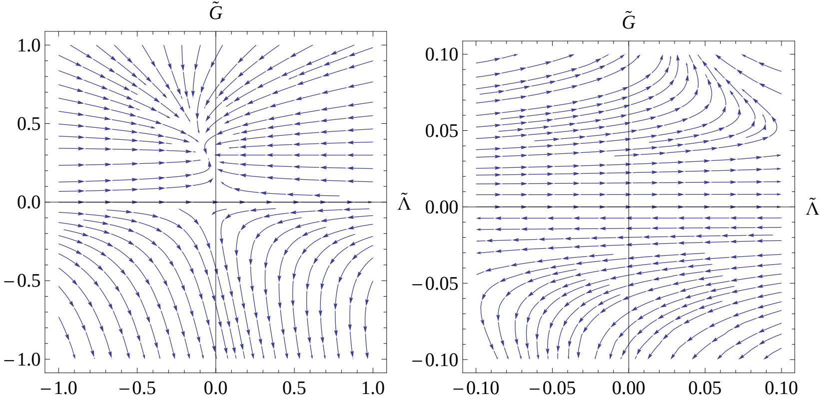


Figure : The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\alpha = 0$, $\nu = 0.1$. Right: enlargement of the region around the origin, showing that there is no Gaussian FP. The beta functions diverge on the $\tilde{\Lambda}$ axis.

Functional renormalization

Define $\Gamma_k(\phi)$. $\lim_{k\to 0} \Gamma_k(\phi) = \Gamma(\phi)$. It satisfies a FRGE

$$k\frac{d\Gamma_k(\phi)}{dk} = \beta(\phi)$$

The quantity β is UV and IR finite.

Use FRGE to calculate the effective action.

Application to gravity

FRGE can only be defined for $\Gamma_k(g_{\mu\nu}, h)$. Expand

$$\Gamma_k(g_{\mu\nu}, h) = \bar{\Gamma}_k(g_{\mu\nu}) + \Gamma_k^{(1)}(g_{\mu\nu}, h) + \Gamma_k^{(2)}(g_{\mu\nu}, h) + \dots$$

$$\bar{\Gamma}_k(g_{\mu\nu}) = \Gamma_k(g_{\mu\nu}, 0)$$

Single-field truncations: neglect $\Gamma_k^{(n)}(g_{\mu\nu}, h)$, $n > 0$.

[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. Percacci, Class. and Quantum Grav. **15** 3449 (1998)]

Einstein–Hilbert truncation I

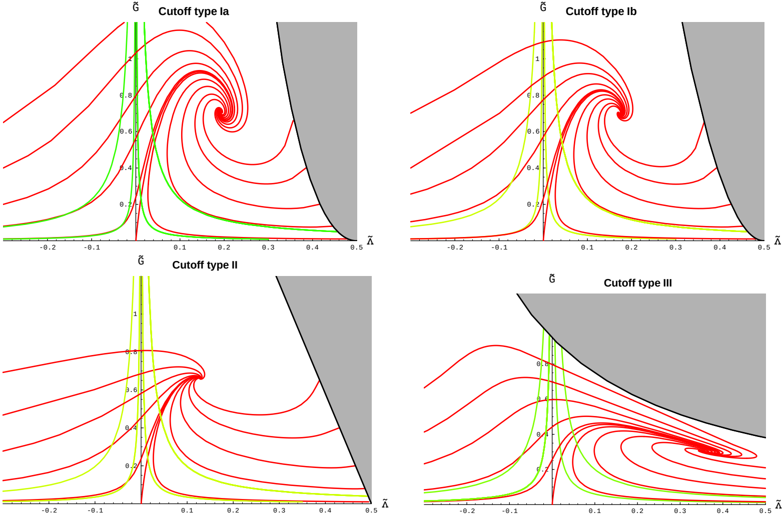
$$\Gamma_k(\bar{g},h) = S_{EH}(\bar{g}_{\mu\nu}+h) + S_{GF}(\bar{g},h) + S_{ghost}(\bar{g},\bar{C}^\mu,C_\nu)$$

$$S_{EH}(g_{\mu\nu}) = \int dx\sqrt{g}\,Z(2\Lambda - R)\ ;\quad Z = \frac{1}{16\pi G}$$

$$\beta_{\tilde\Lambda} = \frac{-2(1-2\tilde\Lambda)^2\tilde\Lambda + \frac{36-41\tilde\Lambda+42\tilde\Lambda^2-600\tilde\Lambda^3}{72\pi}\tilde G + \frac{467-572\tilde\Lambda}{288\pi^2}\tilde G^2}{(1-2\tilde\Lambda)^2 - \frac{29-9\tilde\Lambda}{72\pi}\tilde G}$$

$$\beta_{\tilde G} = \frac{2(1-2\tilde\Lambda)^2\tilde G - \frac{373-654\tilde\Lambda+600\tilde\Lambda^2}{72\pi}\tilde G^2}{(1-2\tilde\Lambda)^2 - \frac{29-9\tilde\Lambda}{72\pi}\tilde G}$$

Einstein–Hilbert truncation III



Fourth order gravity

- R^2 O.Lauscher, M. Reuter, Phys. Rev. D 66, 025026 (2002) arXiv:hep-th/0205062
- $R^2 + C^2$ D. Benedetti, P.F. Machado, F. Saueressig, Mod. Phys. Lett. A24, 2233-2241 (2009) arXiv:0901.2984 [hep-th] Nucl. Phys. B824, 168-191 (2010), arXiv:0902.4630 [hep-th]
- M. Niedermaier, Nucl. Phys. B833, 226-270 (2010)

Bi-field truncation I

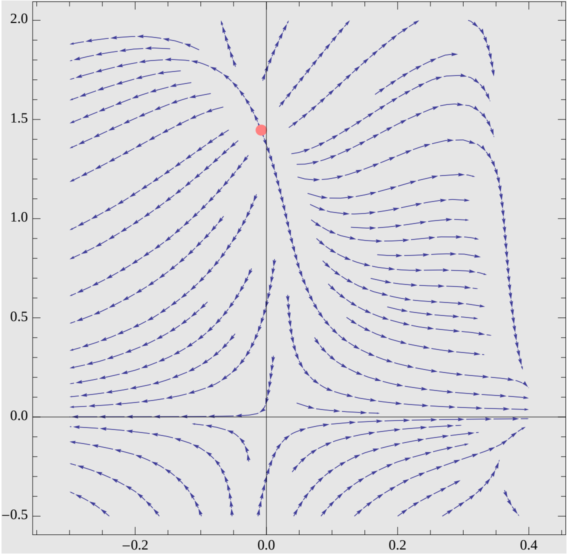
$$\Gamma_k(\bar{g}, h) = S_{EH}(\bar{g} + h) + S_{GF}(\bar{g}, h) + S_{ghost}(\bar{g}, \bar{c}, c)$$

$h_{\mu\nu} \to Z_h^{1/2} h_{\mu\nu}$, $c^\mu \to Z_c^{1/2} c^\mu$ and $c_\nu \to Z_c^{1/2} c_\nu$
Anomalous dimensions $\eta_h = -\frac{\partial_t Z_h}{Z_h}$ $\eta_c = -\frac{\partial_t Z_c}{Z_c}$
computed from $\langle h_{\mu\nu} h_{\rho\sigma} \rangle$, $\langle \bar{c}^\mu c_\nu \rangle$

FP at $\tilde{\Lambda} = -0.00812786$, $\tilde{G} = 1.44627$
scaling exponents -3.32286, -1.95403
anomalous dimensions $\eta_h = 0.0720708$, $\eta_c = -1.50325$.

A. Codello, G. d'Odorico, G. Pagani, arXiv:1304.4777 [gr-qc]

Bi-field truncation II



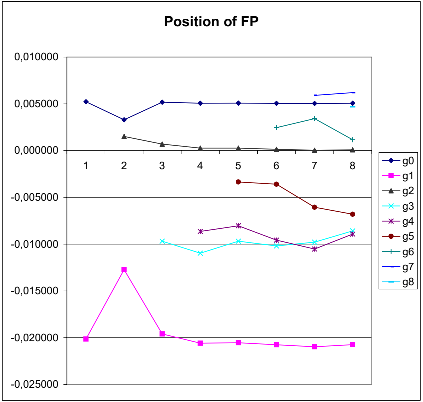
$f(R)$ gravity

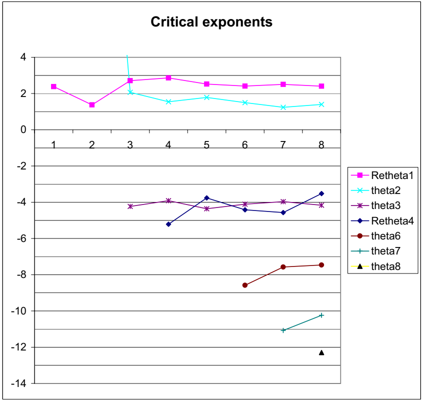
$$\Gamma_k(g_{\mu\nu}) = \int d^4x\,\sqrt{g}\,f(R)$$

$$f(R) = \sum_{i=0}^{n} g_i(k) R^i$$

n=6
A. Codello, R.P. and C. Rahmede Int.J.Mod.Phys.A23:143-150 arXiv:0705.1769 [hep-th];
n=8
A. Codello, R.P. and C. Rahmede Annals Phys. 324 414-469 (2009) arXiv: arXiv:0805.2909;
P.F. Machado, F. Saueressig, Phys. Rev. D arXiv: arXiv:0712.0445 [hep-th]
n=35
K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]
n=∞
Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]
Dario Benedetti, arXiv:1301.4422 [hep-th]

$f(R)$ gravity $n = 8$



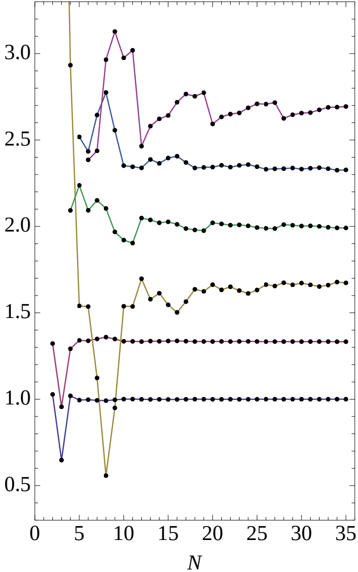


$f(R)$ gravity $n = 8$ predictions

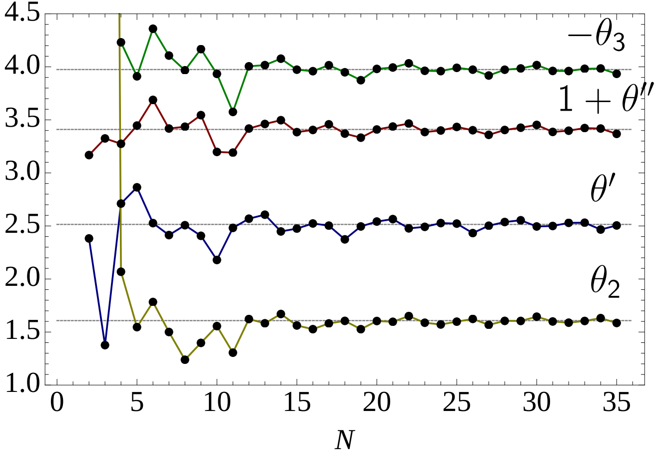
Critical surface:

$$\tilde{g}_3 = 0.00061243 + 0.06817374\,\tilde{g}_0 + 0.46351960\,\tilde{g}_1 + 0.89500872\,\tilde{g}_2$$
$$\tilde{g}_4 = -0.00916502 - 0.83651466\,\tilde{g}_0 - 0.20894019\,\tilde{g}_1 + 1.62075130\,\tilde{g}_2$$
$$\tilde{g}_5 = -0.01569175 - 1.23487788\,\tilde{g}_0 - 0.72544946\,\tilde{g}_1 + 1.01749695\,\tilde{g}_2$$
$$\tilde{g}_6 = -0.01271954 - 0.62264827\,\tilde{g}_0 - 0.82401181\,\tilde{g}_1 - 0.64680416\,\tilde{g}_2$$
$$\tilde{g}_7 = -0.00083040 + 0.81387198\,\tilde{g}_0 - 0.14843134\,\tilde{g}_1 - 2.01811163\,\tilde{g}_2$$
$$\tilde{g}_8 = 0.00905830 + 1.25429854\,\tilde{g}_0 + 0.50854002\,\tilde{g}_1 - 1.90116584\,\tilde{g}_2$$

$f(R)$ gravity $n = 35$



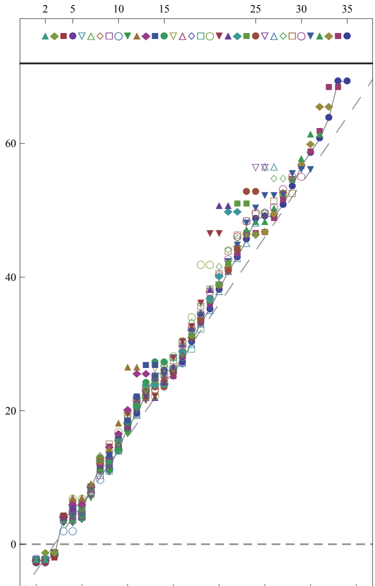
$f(R)$ gravity $n = 35$



n=35

K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]

$f(R)$ gravity $n = 35$



$f(R)$ gravity, functional treatment

Do not expand $f(R)$ but write flow equation for f

$$\partial_t \tilde{f}(\tilde{R}) = \beta(\tilde{f}, \tilde{f}', \tilde{f}'', \tilde{f}''')$$

where $\tilde{R} = R/k^2$, $\tilde{f} = f/k^4$.
For large $\tilde{R}$

$$\tilde{f}(\tilde{R}) = A\tilde{R}^2 \left(1 + \sum_{n>0} d_n \tilde{R}^{-n}\right)$$

Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]

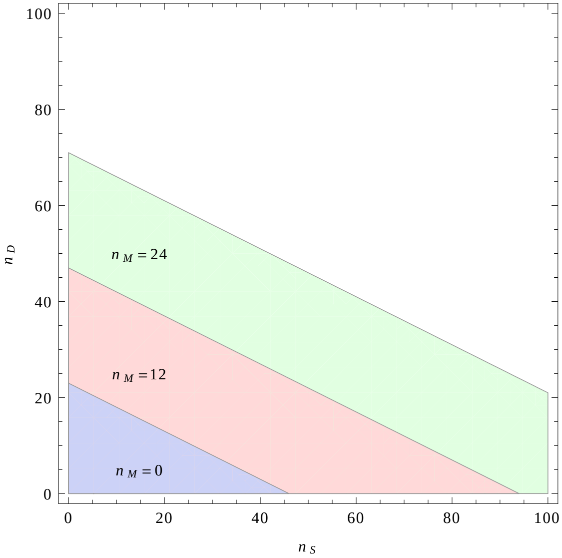
$f(R)$ gravity, functional treatment

Theorem 1: $\Gamma_*(g_{\mu\nu}) = A_* \int d^4x \sqrt{g} R^2$, $A_* \neq 0$

Theorem 2: if $\tilde{f}_*$ exists, the spectrum of perturbations is discrete, real, and there are at most finitely many relevant direction.

D. Benedetti, arXiv:1301.4422 [hep-th]

One loop, with matter



Gravity+scalar

$$\Gamma_k[g,\phi] = \int d^dx\,\sqrt{g}\left(V(\phi^2) - F(\phi^2)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

[R.P., D.Perini, Phys. Rev. D68, 044018 (2004)]
[G. Narain, R.P., Class. and Quantum Grav. 27, 075001 (2010)]

$$\Gamma[g,\phi] = \int d^dx\,\sqrt{g}\,\left(L(\phi^2,R) + \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi\right)$$

[G. Narain, C. Rahmede, Class. and Quantum Grav. 27, 075002 (2010)]

Functional flow of F, V

$$\partial_t V = \frac{k^4}{192\pi^2}\left\{6 + \frac{30\,V}{\Psi} + \frac{6(k^2\,\Psi + 24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,F\Sigma_1)}{\Delta} + \left(\frac{4}{F} + \frac{5\,k^2}{\Psi} + \frac{k^2\,\Sigma_1}{\Delta}\right)\partial_t F + \frac{24\,\phi^2\,k^2\,\Psi'}{\Delta}\,\partial_t F'\right\}\;,$$

$$\begin{aligned}
\partial_t F = \frac{k^2}{2304\pi^2}\Bigg\{&150 + \frac{120\,k^2\,F\,(3\,k^2 F - V)}{\Psi^2} - \frac{24}{\Delta}\left(24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,\Psi + k^2\,F\Sigma_1\right)\\
&-\frac{36}{\Delta^2}\Big[-4\,\phi^2\,(6\,k^4\,F'^2 + \Psi'^2)\,\Delta + 4\,\phi^2\,\Psi\,\Psi'\,(7\,k^2\,F' - V')\,(\Sigma_1 - k^2) + 4\,\phi^2\Sigma_1\,(7\,k^2\,F' - V')\,(2\,\Psi\,V' - V\,\Psi')\\
&+2\,k^4\,\Psi^2\,\Sigma_2 + 48\,k^4\,F'\,\phi^2\,\Psi\,\Psi'\,\Sigma_2 - 24\,k^4 F\,\phi^2\,\Psi'^2\,\Sigma_2\Big]\\
&-\frac{\partial_t F}{F}\Bigg[30 - \frac{10\,k^2 F\,(7\,\Psi + 4\,V)}{\Psi^2} + \frac{6}{\Delta^2}\Big(k^2\,F\,\Sigma_1\,\Delta + 4\,\phi^2\,V'\,\Psi'\,\Delta - 24\,k^4\,F\,\phi^2\,\Psi'^2\,\Sigma_2\\
&-4\,\phi^2\,k^2\,F\,\Psi'\,\Sigma_1(7\,k^2\,F' - V')\Big)\Bigg] + \partial_t F'\,\frac{24\,k^2\,\phi^2}{\Delta^2}\left[(k^2\,F' + 5\,V')\Delta - 12\,k^2\,\Psi\,\Psi'\,\Sigma_2 - 2\,(7\,k^2 F' - V')\,\Psi\,\Sigma_1\right]\Bigg\}
\end{aligned}$$

where we have defined the shorthands:

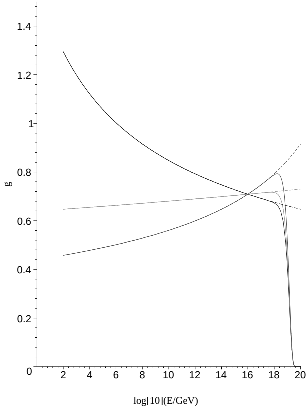
$$\Psi = k^2\,F - V\,;\quad \Sigma_1 = k^2 + 2\,V' + 4\,\phi^2\,V''\;;\quad \Sigma_2 = 2\,F' + 4\,\phi^2\,F''\;;\quad \Delta = \left(12\,\phi^2\,\Psi'^2 + \Psi\,\Sigma_1\right).$$

Polynomial truncations

$$\tilde{V}(\tilde{\phi}^2) = \tilde{\lambda}_0 + \tilde{\lambda}_2\tilde{\phi}^2 + \tilde{\lambda}_4\tilde{\phi}^4 + \dots$$
$$\tilde{F}(\tilde{\phi}^2) = \tilde{\xi}_0 + \tilde{\xi}_2\tilde{\phi}^2 + \dots$$

$$\partial_t\tilde{\lambda}_4 = \frac{9\lambda_4^2}{2\pi^2} + \frac{\tilde{G}\lambda_4}{\pi} + \dots$$

Gauge fields + gravity



[S.P. Robinson, F. Wilczek Phys. Rev. Lett. 96:231601, (2006)]

SM+gravity

$$\frac{dh}{dt} = \beta_h^{SM} + \frac{a_h}{8\pi}\tilde{G}(\mu)h$$

$$\beta_1^{SM} = \frac{1}{16\pi^2}\frac{41}{6}g_1^3 \; ; \qquad \beta_2^{SM} = -\frac{1}{16\pi^2}\frac{19}{6}g_2^3 \; ; \qquad \beta_3^{SM} = -\frac{1}{16\pi^2}7g_3^3 \; ;$$

$$\beta_y^{SM} = \frac{1}{16\pi^2}\left[\frac{9}{2}y^3 - 8g_3^2 y - \frac{9}{4}g_2^2 y - \frac{17}{21}g_1^2 y\right]$$

$$\beta_\lambda^{SM} = \frac{1}{16\pi^2}\left[24\lambda^2 + 12\lambda y^2 - 9\lambda\left(g_2^2 + \frac{1}{3}g_1^2\right) - 6y^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right]$$

If $a_1 = a_2 = a_3 < 0$, $a_y < 0$, $a_\lambda > 0$, predict m_h=126GeV

[M. Shaposhnikov and C. Wetterich, Phys.Lett. B683, 196 (2010)]

Summary

- use continuum covariant QFT
- bottom up approach
- guaranteed to give correct low energy limit
- growing evidence
- hints of agreement with CDT

Outlook

- FRGE with more complicated truncations
- other approximations, e.g. two loops
- matter coupled to gravity
- observable consequences
- describe as CFT

One Loop Corrections in Einstein's Theory

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = ck^{d-2}$$

$$k\frac{dG}{dk} = -16\pi c G^2 k^{d-2}$$

$$\tilde{G} = Gk^{d-2}$$

$$k\frac{d\tilde{G}}{dk} = (d-2)\tilde{G} - 16\pi c\tilde{G}^2$$

fixed point at $\tilde{G} = (d-2)/16\pi c$

Exorcizing the ghosts

- Julve-Tonin-Salam-Strathdee argument ($m^2_{phys} = m^2(p^2 = m^2_{phys})$)
- artifact of polynomial truncation ($g_2 p^2 + g_4 p^4 + \ldots = 0$)
- Bonanno and Reuter arXiv:1302.2928 [hep-th]: expanding around wrong vacuum
- Mukohyama arXiv:1303.1409 [hep-th]: $- + ++$ is emergent