Naturalness, Conformal Symmetry and Duality

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Y.K., “Naturalness, conformal symmetry and duality”, to appear in *Prog. Theor. Exp. Phys.* arXiv:1308.5069 [hep-ph]

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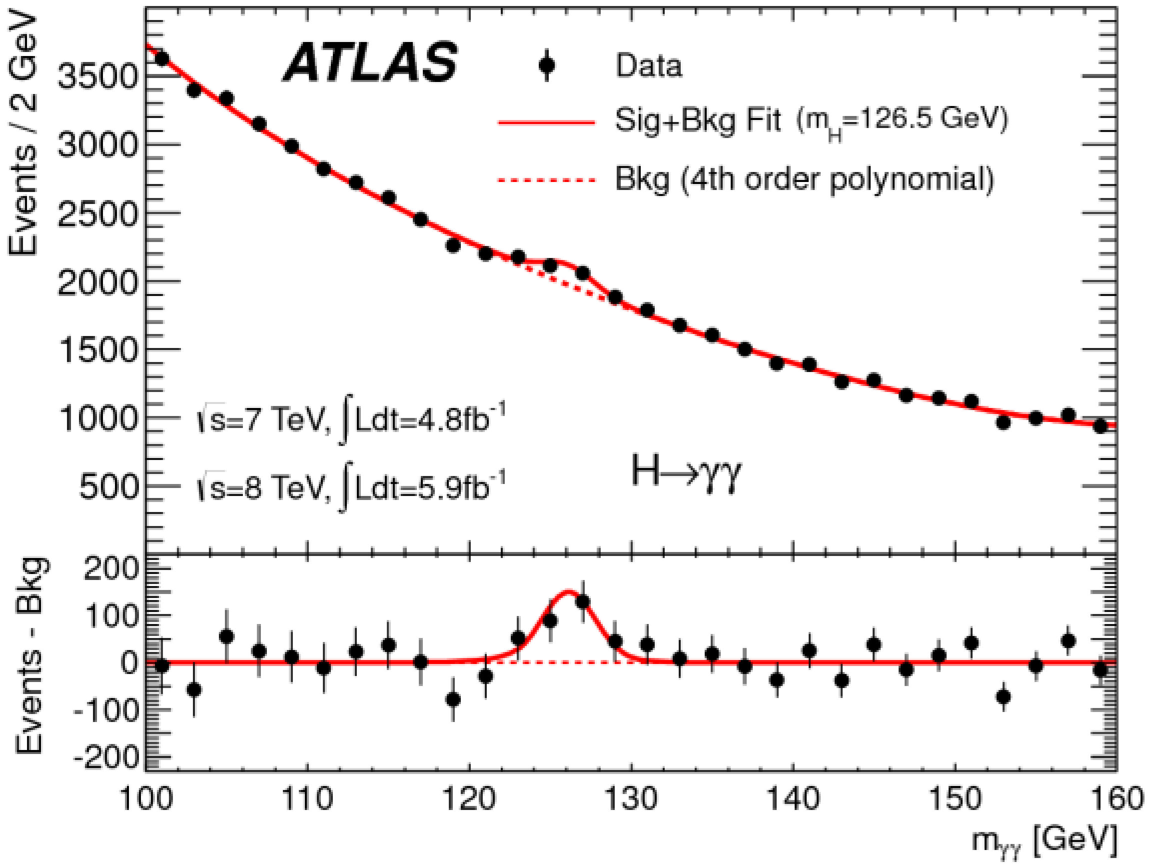


$m_h \simeq 126\,\mathrm{GeV}$

Why is m_h much smaller than M_{Pl} ?

What does $m_h \cong 126\,\mathrm{GeV}$ mean?

Nobody knows a definite answer.

According to the standard model,

$m_h = \sqrt{2\lambda}\ \mathrm{v}$ $\mathrm{v} \cong 246\,\mathrm{GeV}$

→ $\lambda \cong 0.131$

at the weak scale.

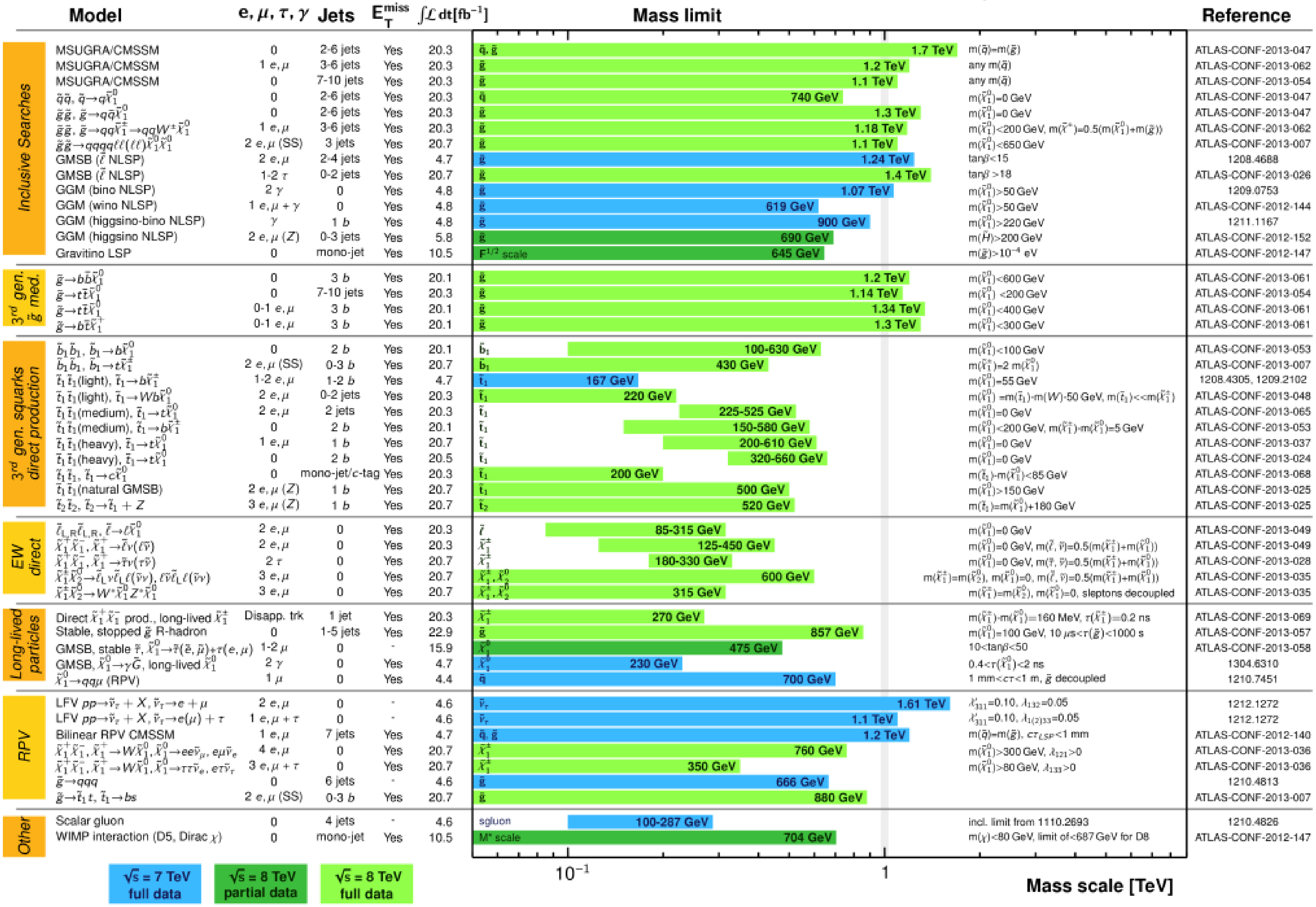
No evidences from Supersymmetry, @ LHC

ATLAS SUSY Searches* - 95% CL Lower Limits

Status: EPS 2013

ATLAS Preliminary

$\int\mathcal{L}\,dt = (4.4 - 22.9)\ \mathrm{fb}^{-1}$ $\sqrt{s} = 7, 8$ TeV



	Model	e, μ, τ, γ	Jets	E_T^{miss}	$\int\mathcal{L}\,dt[\mathrm{fb}^{-1}]$	Mass limit		Reference
Inclusive Searches	MSUGRA/CMSSM	0	2-6 jets	Yes	20.3	$\tilde{q}, \tilde{g}$ 1.7 TeV	$m(\tilde{q})=m(\tilde{g})$	ATLAS-CONF-2013-047
	MSUGRA/CMSSM	1 e, μ	3-6 jets	Yes	20.3	$\tilde{g}$ 1.2 TeV	any $m(\tilde{q})$	ATLAS-CONF-2013-062
	MSUGRA/CMSSM	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.1 TeV	any $m(\tilde{q})$	ATLAS-CONF-2013-054
	$\tilde{q}\tilde{q}, \tilde{q}\to q\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{q}$ 740 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-047
	$\tilde{g}\tilde{g}, \tilde{g}\to q\bar{q}\tilde{\chi}_1^0$	0	2-6 jets	Yes	20.3	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-047
	$\tilde{g}\tilde{g}, \tilde{g}\to qq\tilde{\chi}_1^\pm\to qqW^\pm\tilde{\chi}_1^0$	1 e, μ	3-6 jets	Yes	20.3	$\tilde{g}$ 1.18 TeV	$m(\tilde{\chi}_1^0)$<200 GeV, $m(\tilde{\chi}^\pm)=0.5(m(\tilde{\chi}_1^0)+m(\tilde{g}))$	ATLAS-CONF-2013-062
	$\tilde{g}\tilde{g}\to qqqq\ell\ell(\ell\ell)\tilde{\chi}_1^0\tilde{\chi}_1^0$	2 e, μ (SS)	3 jets	Yes	20.7	$\tilde{g}$ 1.1 TeV	$m(\tilde{\chi}_1^0)$<650 GeV	ATLAS-CONF-2013-007
	GMSB ($\tilde{\ell}$ NLSP)	2 e, μ	2-4 jets	Yes	4.7	$\tilde{g}$ 1.24 TeV	$\tan\beta<15$	1208.4688
	GMSB ($\tilde{\ell}$ NLSP)	1-2 τ	0-2 jets	Yes	20.7	$\tilde{g}$ 1.4 TeV	$\tan\beta$ >18	ATLAS-CONF-2013-026
	GGM (bino NLSP)	2 γ	0	Yes	4.8	$\tilde{g}$ 1.07 TeV	$m(\tilde{\chi}_1^0)$>50 GeV	1209.0753
	GGM (wino NLSP)	1 $e, \mu + \gamma$	0	Yes	4.8	$\tilde{g}$ 619 GeV	$m(\tilde{\chi}_1^0)$>50 GeV	ATLAS-CONF-2012-144
	GGM (higgsino-bino NLSP)	γ	1 b	Yes	4.8	$\tilde{g}$ 900 GeV	$m(\tilde{\chi}_1^0)$>220 GeV	1211.1167
	GGM (higgsino NLSP)	2 e, μ (Z)	0-3 jets	Yes	5.8	$\tilde{g}$ 690 GeV	$m(\tilde{H})$>200 GeV	ATLAS-CONF-2012-152
	Gravitino LSP	0	mono-jet	Yes	10.5	$F^{1/2}$ scale 645 GeV	$m(\tilde{g})>10^{-4}$ eV	ATLAS-CONF-2012-147
3^{rd} gen. $\tilde{g}$ med.	$\tilde{g}\to b\bar{b}\tilde{\chi}_1^0$	0	3 b	Yes	20.1	$\tilde{g}$ 1.2 TeV	$m(\tilde{\chi}_1^0)$<600 GeV	ATLAS-CONF-2013-061
	$\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0	7-10 jets	Yes	20.3	$\tilde{g}$ 1.14 TeV	$m(\tilde{\chi}_1^0)$ <200 GeV	ATLAS-CONF-2013-054
	$\tilde{g}\to t\bar{t}\tilde{\chi}_1^0$	0-1 e, μ	3 b	Yes	20.1	$\tilde{g}$ 1.34 TeV	$m(\tilde{\chi}_1^0)$<400 GeV	ATLAS-CONF-2013-061
	$\tilde{g}\to b\bar{t}\tilde{\chi}_1^+$	0-1 e, μ	3 b	Yes	20.1	$\tilde{g}$ 1.3 TeV	$m(\tilde{\chi}_1^0)$<300 GeV	ATLAS-CONF-2013-061
3^{rd} gen. squarks direct production	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to b\tilde{\chi}_1^0$	0	2 b	Yes	20.1	$\tilde{b}_1$ 100-630 GeV	$m(\tilde{\chi}_1^0)$<100 GeV	ATLAS-CONF-2013-053
	$\tilde{b}_1\tilde{b}_1, \tilde{b}_1\to t\tilde{\chi}_1^\pm$	2 e, μ (SS)	0-3 b	Yes	20.7	$\tilde{b}_1$ 430 GeV	$m(\tilde{\chi}_1^\pm)=2\,m(\tilde{\chi}_1^0)$	ATLAS-CONF-2013-007
	$\tilde{t}_1\tilde{t}_1$(light), $\tilde{t}_1\to b\tilde{\chi}_1^\pm$	1-2 e, μ	1-2 b	Yes	4.7	$\tilde{t}_1$ 167 GeV	$m(\tilde{\chi}_1^0)$=55 GeV	1208.4305, 1209.2102
	$\tilde{t}_1\tilde{t}_1$(light), $\tilde{t}_1\to Wb\tilde{\chi}_1^0$	2 e, μ	0-2 jets	Yes	20.3	$\tilde{t}_1$ 220 GeV	$m(\tilde{\chi}_1^0)$ $=m(\tilde{t}_1)-m(W)$-50 GeV, $m(\tilde{t}_1)<<m(\tilde{\chi}_1^\pm)$	ATLAS-CONF-2013-048
	$\tilde{t}_1\tilde{t}_1$(medium), $\tilde{t}_1\to t\tilde{\chi}_1^0$	2 e, μ	2 jets	Yes	20.3	$\tilde{t}_1$ 225-525 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-065
	$\tilde{t}_1\tilde{t}_1$(medium), $\tilde{t}_1\to b\tilde{\chi}_1^+$	0	2 b	Yes	20.1	$\tilde{t}_1$ 150-580 GeV	$m(\tilde{\chi}_1^0)$<200 GeV, $m(\tilde{\chi}_1^\pm)-m(\tilde{\chi}_1^0)$=5 GeV	ATLAS-CONF-2013-053
	$\tilde{t}_1\tilde{t}_1$(heavy), $\tilde{t}_1\to t\tilde{\chi}_1^0$	1 e, μ	1 b	Yes	20.7	$\tilde{t}_1$ 200-610 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-037
	$\tilde{t}_1\tilde{t}_1$(heavy), $\tilde{t}_1\to t\tilde{\chi}_1^0$	0	2 b	Yes	20.5	$\tilde{t}_1$ 320-660 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-024
	$\tilde{t}_1\tilde{t}_1, \tilde{t}_1\to c\tilde{\chi}_1^0$	0	mono-jet/c-tag	Yes	20.3	$\tilde{t}_1$ 200 GeV	$m(\tilde{t}_1)-m(\tilde{\chi}_1^0)$<85 GeV	ATLAS-CONF-2013-068
	$\tilde{t}_1\tilde{t}_1$(natural GMSB)	2 e, μ (Z)	1 b	Yes	20.7	$\tilde{t}_1$ 500 GeV	$m(\tilde{\chi}_1^0)$>150 GeV	ATLAS-CONF-2013-025
	$\tilde{t}_2\tilde{t}_2, \tilde{t}_2\to\tilde{t}_1 + Z$	3 e, μ (Z)	1 b	Yes	20.7	$\tilde{t}_2$ 520 GeV	$m(\tilde{t}_1)=m(\tilde{\chi}_1^0)$+180 GeV	ATLAS-CONF-2013-025
EW direct	$\tilde{\ell}_{L,R}\tilde{\ell}_{L,R}, \tilde{\ell}\to\ell\tilde{\chi}_1^0$	2 e, μ	0	Yes	20.3	$\tilde{\ell}$ 85-315 GeV	$m(\tilde{\chi}_1^0)$=0 GeV	ATLAS-CONF-2013-049
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to\tilde{\ell}\nu(\ell\tilde{\nu})$	2 e, μ	0	Yes	20.3	$\tilde{\chi}_1^\pm$ 125-450 GeV	$m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\ell}, \tilde{\nu})=0.5(m(\tilde{\chi}_1^\pm)+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-049
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to\tilde{\tau}\nu(\tau\tilde{\nu})$	2 τ	0	Yes	20.7	$\tilde{\chi}_1^\pm$ 180-330 GeV	$m(\tilde{\chi}_1^0)$=0 GeV, $m(\tilde{\tau}, \tilde{\nu})=0.5(m(\tilde{\chi}_1^\pm)+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-028
	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to\tilde{\ell}_L\nu\tilde{\ell}_L\ell(\tilde{\nu}\nu), \ell\tilde{\nu}\tilde{\ell}_L\ell(\tilde{\nu}\nu)$	3 e, μ	0	Yes	20.7	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ 600 GeV	$m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, $m(\tilde{\ell}, \tilde{\nu})=0.5(m(\tilde{\chi}_1^\pm)+m(\tilde{\chi}_1^0))$	ATLAS-CONF-2013-035
	$\tilde{\chi}_1^\pm\tilde{\chi}_2^0\to W^*\tilde{\chi}_1^0 Z^*\tilde{\chi}_1^0$	3 e, μ	0	Yes	20.7	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^0$ 315 GeV	$m(\tilde{\chi}_1^\pm)=m(\tilde{\chi}_2^0)$, $m(\tilde{\chi}_1^0)$=0, sleptons decoupled	ATLAS-CONF-2013-035
Long-lived particles	Direct $\tilde{\chi}_1^+\tilde{\chi}_1^-$ prod., long-lived $\tilde{\chi}_1^\pm$	Disapp. trk	1 jet	Yes	20.3	$\tilde{\chi}_1^\pm$ 270 GeV	$m(\tilde{\chi}_1^\pm)-m(\tilde{\chi}_1^0)$=160 MeV, $\tau(\tilde{\chi}_1^\pm)$=0.2 ns	ATLAS-CONF-2013-069
	Stable, stopped $\tilde{g}$ R-hadron	0	1-5 jets	Yes	22.9	$\tilde{g}$ 857 GeV	$m(\tilde{\chi}_1^0)$=100 GeV, 10 μs<$\tau(\tilde{g})$<1000 s	ATLAS-CONF-2013-057
	GMSB, stable $\tilde{\tau}$, $\tilde{\chi}_1^0\to\tilde{\tau}(\tilde{e}, \tilde{\mu})+\tau(e, \mu)$	1-2 μ	0	-	15.9	$\tilde{\chi}_1^0$ 475 GeV	10<$\tan\beta$<50	ATLAS-CONF-2013-058
	GMSB, $\tilde{\chi}_1^0\to\gamma\tilde{G}$, long-lived $\tilde{\chi}_1^0$	2 γ	0	Yes	4.7	$\tilde{\chi}_1^0$ 230 GeV	0.4<$\tau(\tilde{\chi}_1^0)$<2 ns	1304.6310
	$\tilde{\chi}_1^0\to qq\mu$ (RPV)	1 μ	0	Yes	4.4	$\tilde{q}$ 700 GeV	1 mm<$c\tau$<1 m, $\tilde{g}$ decoupled	1210.7451
RPV	LFV $pp\to\tilde{\nu}_\tau + X, \tilde{\nu}_\tau\to e + \mu$	2 e, μ	0	-	4.6	$\tilde{\nu}_\tau$ 1.61 TeV	λ'_{311}=0.10, λ_{132}=0.05	1212.1272
	LFV $pp\to\tilde{\nu}_\tau + X, \tilde{\nu}_\tau\to e(\mu) + \tau$	1 $e, \mu + \tau$	0	-	4.6	$\tilde{\nu}_\tau$ 1.1 TeV	λ'_{311}=0.10, $\lambda_{1(2)33}$=0.05	1212.1272
	Bilinear RPV CMSSM	1 e, μ	7 jets	Yes	4.7	$\tilde{q}, \tilde{g}$ 1.2 TeV	$m(\tilde{q})=m(\tilde{g})$, $c\tau_{LSP}$<1 mm	ATLAS-CONF-2012-140
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to ee\tilde{\nu}_\mu, e\mu\tilde{\nu}_e$	4 e, μ	0	Yes	20.7	$\tilde{\chi}_1^\pm$ 760 GeV	$m(\tilde{\chi}_1^0)$>300 GeV, λ_{121}>0	ATLAS-CONF-2013-036
	$\tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{\chi}_1^+\to W\tilde{\chi}_1^0, \tilde{\chi}_1^0\to\tau\tau\tilde{\nu}_e, e\tau\tilde{\nu}_\tau$	3 $e, \mu + \tau$	0	Yes	20.7	$\tilde{\chi}_1^\pm$ 350 GeV	$m(\tilde{\chi}_1^0)$>80 GeV, λ_{133}>0	ATLAS-CONF-2013-036
	$\tilde{g}\to qqq$	0	6 jets	-	4.6	$\tilde{g}$ 666 GeV		1210.4813
	$\tilde{g}\to\tilde{t}_1 t, \tilde{t}_1\to bs$	2 e, μ (SS)	0-3 b	Yes	20.7	$\tilde{g}$ 880 GeV		ATLAS-CONF-2013-007
Other	Scalar gluon	0	4 jets	-	4.6	sgluon 100-287 GeV	incl. limit from 1110.2693	1210.4826
	WIMP interaction (D5, Dirac χ)	0	mono-jet	Yes	10.5	M* scale 704 GeV	$m(\chi)$<80 GeV, limit of<687 GeV for D8	ATLAS-CONF-2012-147

*Only a selection of the available mass limits on new states or phenomena is shown. All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.

No evidences from Supersymmetry @ LHC

No evidences from Compositeness and Extra dimensions @ LHC

➔ No evidences from Physics beyond the Standard Model

➔ Naturalness problem and Gauge hierarchy problem would be revisited !?

In this talk,

Naturalness problem

A narrow definition

It is the fine tuning problem relating radiative corrections on the Higgs boson mass in the framework of low-energy effective theory such as the SM.

【Naturalness problem】

$$\delta m_h^2 = \boxed{C_h \Lambda^2} + C'_h m_h^2 \ln \Lambda^2 / m_h^2 + \cdots$$

If $\sqrt{C_h}\Lambda >> m_h$

Λ : a cutoff scale

unnatural because we need a fine tuning ??

Way outs to escape a fine tuning,

$C_h \approx 0$ and/or $\Lambda \le O(1)\mathrm{TeV}$

【Naturalness problem】

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \cdots$$

$$C_h = \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

at the one-loop level

$$C_h = 0 \Rightarrow m_h^2 = 4m_t^2 - 2M_W^2 - M_Z^2$$

Veltman condition

【Naturalness problem】

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 / m_h^2 + \cdots$$

$$C_h = \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$$C_h = 0\big|_{M_Z} \Rightarrow m_h \cong 320\mathrm{GeV}$$

$$C_h \approx 0\big|_{M_{\mathrm{Pl}}}$$

Y. Hamada, H. Kawai, & K. Oda,
Phys. Rev. **D87, 053009 (2013)**

【Naturalness problem】

$$\delta m_h^2 = C_h \Lambda^2 + C_h' m_h^2 \ln \Lambda^2 \big/ m_h^2 + \cdots$$

$$C_h = \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$\Lambda \le O(1)\mathrm{TeV}$

➔New Physics @ Terascale

Candidates：SUSY, Compositeness, Extra Dimensions, ･･･

【Naturalness problem】

$$\delta m_h^2 = C_h \Lambda^2 + C'_h m_h^2 \ln \Lambda^2 \big/ m_h^2 + \cdots$$

$$C_h = \frac{1}{16\pi^2}\left(6\lambda + \frac{9}{4}g^2 + \frac{3}{4}g'^2 - 6y_t^2\right)$$

$$= \frac{3}{16\pi^2 \mathrm{v}^2}\left(m_h^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right)$$

$\Lambda \leq O(1)\mathrm{TeV}$ ➔New Physics @ Terascale

Problem revisited because of no evidences

For a sake of completeness,

Gauge hierarchy problem

(A narrow definition)

It is the fine tuning problem relating radiative corrections on the Higgs boson mass in the framework of effective theory **including heavy particles** beyond the SM.

【Gauge hierarchy problem】

Even if quadratic div. are removed,

$$\delta m_h^2 = C_h' m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \sum_k \boxed{C_h'' M_k^2} \ln \Lambda^2 \Big/ M_k^2 + \cdots$$

If $M_k >> m_h$ unnatural because we need a fine tuning ??

Serious problem for Grand Unified Theory ➔ Supersymmetry ?

【Gauge hierarchy problem】

Even if quadratic div. are removed,

$$\delta m_h^2 = C'_h m_h^2 \ln \Lambda^2 \big/ m_h^2 + \sum_k \boxed{C''_h M_k^2} \ln \Lambda^2 \big/ M_k^2 + \cdots$$

If $M_k >> m_h$ **unnatural because we need a fine tuning ??**

Way outs to escape a fine tuning,

$C''_h \approx 0$ and/or $M_k \leq O(1)\mathrm{TeV}$

No high-energy physics relevant to the SM?

【Gauge hierarchy problem】

Even if quadratic div. are removed,

$$\delta m_h^2 = C'_h m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \sum_k \boxed{C''_h M_k^2} \ln \Lambda^2 \Big/ M_k^2 + \cdots$$

If $M_k >> m_h$ unnatural because we need a fine tuning ??

Way outs to escape a fine tuning,

or Miracle such as $\sum_k C''_h M_k^2 \ln \Lambda^2 \Big/ M_k^2 = 0$

【Gauge hierarchy problem】

Even if quadratic div. are removed,

$$\delta m_h^2 = C_h' m_h^2 \ln \Lambda^2 / m_h^2 + \sum_k C_h'' M_k^2 \ln \Lambda^2 / M_k^2 + \cdots$$

Way outs to escape a fine tuning,

with a miracle such as $\sum_k C_h'' M_k^2 \ln \Lambda^2 / M_k^2 = 0$ at M_{Pl}

If new particles exist below M_{Pl},
they would be around the terascale.

【An unusual scenario】

The SM＋New particles around the terascale (without SUSY)

↓ Big desert

An ultimate theory
@ the Planck scale

Gauge hierarchy problem at $M_{\rm Pl}$ is solved by miracle ?

The naturalness problem➔Theme of Sect. 2 & 3

【Naturalness problem】

$$\delta m_h^2 = \boxed{C_h \Lambda^2} + C_h' m_h^2 \ln {}^{\Lambda^2}\!\big/_{m_h^2} + \cdots$$

If $\sqrt{C_h}\,\Lambda >> m_h$ unnatural because we need a fine tuning ??

2． Naturalness and Conformal Symmetry

2.1 What is naturalness?

(Naturalness, Chiral symmetry, Conformal symmetry)

2.2 Is a scalar mass natural?

2.1 What is naturalness?

G.’t Hooft,（1979）

The concept based on the dogma,

“at any energy scale μ, a physical parameter or set of physical parameters $a_i(\mu)$ is allowed to be very small, only if the replacement $a_i(\mu)=0$ would increase the symmetry of the system.”

$$\delta a = a\, h(\Lambda^2) + \cancel{k(\Lambda^2)}$$

by some symmetry

Hereafter,
we refer to a parameter with the feature that the symmetry of the system enhances when its value approaches zero as a natural parameter.

【Example】 Electron mass m_e

$m_e \to 0$ ➔ chiral symmetry

$$\psi_L \to e^{i\theta_L}\psi_L, \quad \psi_R \to e^{i\theta_R}\psi_R$$
$$\left(\theta_L, \theta_R : \text{real parameters}\right)$$

For $\theta_L = -\theta_R$,

$$\langle \partial_\mu j_A^\mu \rangle = 2i(m_e + \delta m_e)(\psi_L^\dagger \psi_R - \psi_R^\dagger \psi_L) + \frac{e^2}{16\pi^2}\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$$

$$\delta m_e = \frac{3\alpha}{4\pi} m_e \left(\ln \frac{\Lambda^2}{m_e^2} + \frac{1}{2} \right)$$

For $m_e \to 0, \ \delta m_e \to 0$

Chiral symmetry controls quantum corrections.

【Supplement】

$m_e \to 0$ ➔ **Scale invariance**

$$\psi_L \to e^{\rho/2}\psi_L, \quad \psi_R \to e^{\rho/2}\psi_R$$

$(\rho : \text{real parameter})$

$$\langle T_\mu^\mu \rangle = 2(m_e + \delta m_e)(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) + \frac{\beta_\alpha}{\alpha} F^{\mu\nu} F_{\mu\nu}$$

$$\delta m_e = \frac{3\alpha}{4\pi} m_e \left(\ln \frac{\Lambda^2}{m_e^2} + \frac{1}{2} \right)$$

For $m_e \to 0, \ \delta m_e \to 0$

Conformal symmetry plays the same role as chiral symmetry does.

In the Standard Model, chiral symmetry has a superior quality to conformal symmetry.

The chiral symmetry such as $SU(2)_L \times U(1)_Y$ is a local one and unbroken perturbatively and anomalously.

The conformal symmetry is a global one and broken down explicitely and anomalously.

The chiral gauge symmetry is broken down spontaneously by the VEV of Higgs boson v=246GeV, and fermions acquire masses

$$m_f = y_f \mathrm{v} \big/ \sqrt{2}\,.$$

The smallness of $m_f = y_f \mathrm{v} \big/ \sqrt{2} << M_{\mathrm{Pl}}$ stems from the smallness of $\mathrm{v}(<< M_{\mathrm{Pl}})$.

The (chiral) gauge symmetry enhances in the limit of $\mathrm{v} \to 0$.

2. 2 Is a scalar mass natural?

A scalar mass m_ϕ is a natural parameter or not ?

$m_\phi \to 0$ ➔ Scale invariance ?

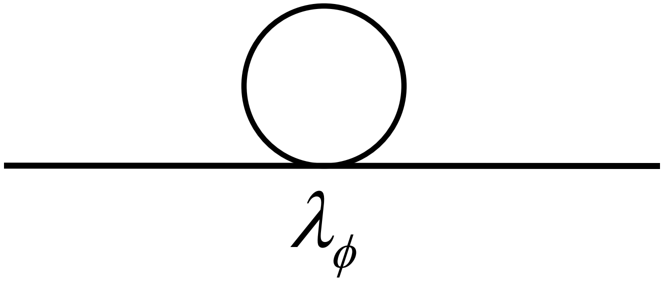
$$\langle T_\mu^\mu \rangle = 2\left(m_\phi^2 + \delta m_\phi^2\right)\phi^2 + \sum_k \beta_k O_k$$

O_k : Operators with the mass dimension 4

For $m_\phi^2 \to 0,\quad \delta m_\phi^2 \to 0$?

$\delta m_\phi^2 \propto m_\phi^2$?

In ϕ^4 theory, $\delta m_\phi^2 =$



$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m_\phi^2} = \frac{\lambda_\phi}{2}\frac{\pi^2}{(2\pi)^4}\int_0^{\infty}\frac{p^2dp^2}{p^2+m_\phi^2}$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\infty}dp^2 - m_\phi^2\int_0^{\infty}\frac{dp^2}{p^2+m_\phi^2}\right)$$

Regularization

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2}dp^2 - m_\phi^2\int_0^{\Lambda^2-m_\phi^2}\frac{dp^2}{p^2+m_\phi^2}\right)$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\Lambda^2 - m_\phi^2 - m_\phi^2\ln\frac{\Lambda^2}{m_\phi^2}\right)$$

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\left(\Lambda^2 - m_\phi^2 - m_\phi^2 \ln\frac{\Lambda^2}{m_\phi^2}\right)$$

$$\text{For } m_\phi^2 \to 0, \quad \delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\Lambda^2 \neq 0$$

➔ The scale invariance is not recovered, and hence it is widely thought that m_ϕ is not a natural parameter. This is the root of naturalness problem.

Is it true ?

【Bardeen's argument】

Anomalous relation

$$\left\langle T_\mu^\mu \right\rangle = m_h^2 + \delta m_h^2 + \sum_k \beta_k O_k$$

For $m_h^2 \to 0$ and $\beta_k \to 0$, **the classical scale invariance should be restored.**

$$\delta m_h^2 = \cancel{C\Lambda^2} + C' m_h^2 \ln \Lambda^2 \Big/ m_h^2 + \cdots$$

Ambiguities can exist in the regularization procedure.

Such ambiguities, in most case, are resolved by considering symmetries realized manifestly.

Quantities depending on the regularization method should be subtracted, unless the subtraction induces any physical effects.

Ambiguities can exist in the regularization procedure.

- In the dimensional regularization

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2} m_\phi^2 \left(-\frac{2}{\varepsilon} + \gamma - 1 + \cdots \right)$$

- Proposal for subtractive renormalization

K. Fujikawa,
Phys. Rev. D**83**, 105012（2011)

- From the viewpoint of the Wilsonian renormalization group

H. Aoki & S. Iso,
Phys. Rev. D**86**, 013001（2012)

3． Naturalness and Duality

3. 1 Basic idea

3. 2 Radiative corrections on scalar mass

3.1 Basic idea

【Expectation】

- Quadratic div. might be artifact of regularization procedure.
- The calculation scheme can be selected by the physics.

【Conjecture】

An ultimate theory does not induce any large radiative corrections for low-energy fields owing to a symmetry, and such a symmetry is hidden in the Standard Model.

Cf. K. Dienes,
Nucl. Phys. B611, 146（2001）

【Assumptions】

(a) There is an ultimate theory with a fundamental scale Λ.

(b) It has a following duality.

$$\text{The physics}@E(\geq \Lambda) \ \sim \ \text{The physics}@E(\leq \Lambda)$$

(b1) The physics is invariant under the duality.

(b2) The physics is only described by one of the two regions.

(c) A remnant of the duality is hidden in quantities of the low-energy physics involved with Λ.

Quantum corrections on a

$$\delta a = \int_0^\infty f(p^2)dp^2$$

p^2 : Euclidean momentum squared for a massless virtual particles running in the loop

When δa diverges at $p^2 = \infty$ and $p^2 = 0$, it is ordinarily regularized as

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$$

μ_0 : a fictitious mass parameter

$$\delta a = \int_{0}^{\infty} f(p^2)dp^2 \rightarrow \delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$$

In our method, this procedure is not a mere regularization, but has a deep physical meaning.

Let us show $\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$ is necessarily obtained and the integral is restricted by the above-mentioned assumptions.

$$\delta a=\int_0^{\infty} f(p^2)dp^2 \Rightarrow \int_{\mu_0^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

$$\left(\xrightarrow[\mu_0^2\to 0]{}\delta a=\int_0^{\infty} f(p^2)dp^2\right)$$

$$\delta a=\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2+\int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

If a remnant of duality holds with $\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2=\int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$,

we obtain $\delta a=\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$.

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 = \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(p^2)dp^2$$

Furthermore, for $\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2$ **,**

we take $p^2 \to p'^2 = \Lambda^4 / p^2$ **as the remnant of duality transf.**

$$\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 = \int_{\Lambda^4/\mu_0^2}^{\Lambda^2} f(p'^2)dp'^2 = \int_{\Lambda^2}^{\Lambda^4/\mu_0^2} f(\Lambda^4/p^2)\frac{\Lambda^4}{p^4}dp^2$$

Unless $f(p^2)$ contains Λ, $f(p^2) = \frac{c_{-1}}{p^2}$

Then, $\delta a = \int_{\mu_0^2}^{\Lambda^2} \frac{c_{-1}}{p^2}dp^2 = c_{-1} \ln \frac{\Lambda^2}{\mu_0^2}$

Our procedure can be not a mere regularization, but a recipe to obtain finite physical values, because Λ is (large but) finite and infinities are taken away by the symmetry relating integration variables, like world-sheet modular invariance in string theory.

From world-sheet modular invariance for the closed string,

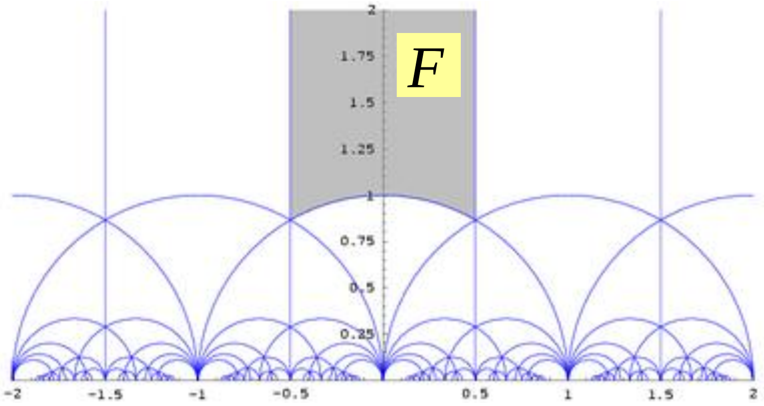
$$\delta a = \int_F \frac{d^2\tau}{\tau_2^2} G(\tau)$$

$$\tau = \tau_1 + i\tau_2$$

$$F = \left\{ \tau : \left| \mathrm{Re}\, \tau \right| \le 1/2,\ \ 1 \le \left| \tau \right| \right\}$$

$G(\tau)$: a world-sheet modular invariant function

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \qquad (a, b, c, d \in Z,\ \ ad - bc = 1)$$



$$\left(\tau \rightarrow -\frac{1}{\tau}, \quad \tau \rightarrow \tau + 1 \right)$$

From Wikipedia

In string theory, the world-sheet modular invariance is deeply connected to the consistency of the theory, and radiative corrections should be given in the world-sheet modular invariance form.

In an ultimate theory, the duality is connected to the consistency of the theory.

$$\delta a = \left(\text{Duality invariant terms}\right)$$

In the effective field theory, a remnant of duality is hidden, and it is not connected to the consistency of the theory.

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)\,dp^2 \Rightarrow \left(\text{Duality invariant terms}\right)$$

Projection is needed.

$$\delta a = \int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2 \Rightarrow (\text{Duality invariant terms})$$

Projection is needed.

We denote the operation (projection) as $\mathrm{Du}[*]$.

In the case that $f(p^2)$ does not contain Λ,

we expand in a series such as $f(p^2) = \sum_n c_n (p^2)^n$.

$$\delta a = \mathrm{Du}\left[\int_{\mu_0^2}^{\Lambda^2} f(p^2)dp^2\right] = \mathrm{Du}\left[\int_{\mu_0^2}^{\Lambda^2} \sum_n c_n (p^2)^n dp^2\right]$$

$$= \int_{\mu_0^2}^{\Lambda^2} \frac{c_{-1}}{p^2} dp^2 = c_{-1} \ln \frac{\Lambda^2}{\mu_0^2}$$

3. 2 Radiative corrections on scalar mass

In the massless case,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2} = \frac{\lambda_\phi}{32\pi^2}\int_0^{\infty} dp^2$$

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\int_{\mu_0^2}^{\Lambda^4/\mu_0^2} dp^2 = \frac{\lambda_\phi}{32\pi^2}\int_{\mu_0^2}^{\Lambda^2} dp^2 + \frac{\lambda_\phi}{32\pi^2}\int_{\Lambda^2}^{\Lambda^4/\mu_0^2} dp^2$$

$\Downarrow$

$$p^2 \to p'^2 = \Lambda^4/p^2$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^{\infty} dp^2\right] = 0$$

For the massive case,

- Momentum cutoff method
- Proper time method

Using momentum cutoff method,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\left(\int_0^\infty dp^2 - m_\phi^2 \int_0^\infty \frac{dp^2}{p^2+m_\phi^2}\right)$$

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda_\phi^2} dp^2 - m_\phi^2 \int_0^{\Lambda_\phi^2} \frac{dp^2}{p^2+m_\phi^2}\right)$$

$$\Lambda_\phi^2 \equiv \left(\Lambda^4 / m_\phi^2\right) - m_\phi^2$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2} dp^2 + \int_{\Lambda^2-m_\phi^2}^{\Lambda_\phi^2} dp^2\right) - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\left(\int_0^{\Lambda^2-m_\phi^2} \frac{dp^2}{p^2+m_\phi^2} + \int_{\Lambda^2-m_\phi^2}^{\Lambda_\phi^2} \frac{dp^2}{p^2+m_\phi^2}\right)$$

⬇

$$p^2 + m_\phi^2 \rightarrow \Lambda^4 / \left(p^2 + m_\phi^2\right)$$

$$\delta m_\phi^2 = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_0^{\Lambda^2-m_\phi^2} \frac{dp^2}{p^2+m_\phi^2} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\ln\frac{\Lambda^2}{m_\phi^2}$$

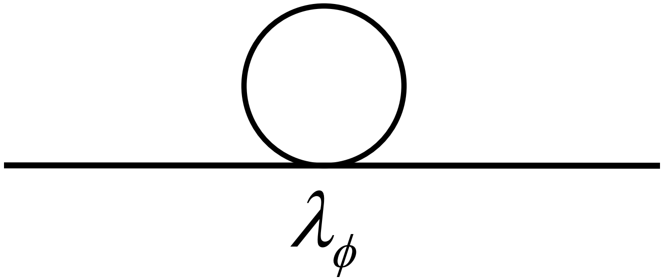
Using the proper time method,

$$\delta m_\phi^2 = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m_\phi^2} = \frac{\lambda_\phi}{2}\int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\int_0^\infty e^{-(p^2+m_\phi^2)t}dt$$

$$= \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt$$

t : proper time

$$\Rightarrow \frac{\lambda_\phi}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t^2} - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t} + \frac{\lambda_\phi m_\phi^4}{64\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}dt + \cdots$$



$$\tilde{\Lambda}_\phi^2 \equiv \Lambda^4 / m_\phi^2$$

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t^2} - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2}\frac{dt}{t} + \frac{\lambda_\phi m_\phi^4}{32\pi^2}\int_{1/\tilde{\Lambda}_\phi^2}^{1/m_\phi^2} dt + \cdots$$

$$= \frac{\lambda_\phi}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t^2} + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2}\frac{dt}{t^2}\right) - \frac{\lambda_\phi m_\phi^2}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t} + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2}\frac{dt}{t}\right)$$

$$+ \frac{\lambda_\phi m_\phi^4}{32\pi^2}\left(\int_{1/\Lambda^2}^{1/m_\phi^2} dt + \int_{1/\tilde{\Lambda}_\phi^2}^{1/\Lambda^2} dt\right) + \cdots$$

$$\tilde{\Lambda}_\phi^2 \equiv \Lambda^4 / m_\phi^2$$

$$\Downarrow$$

$$t \rightarrow 1/(\Lambda^4 t)$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt\right] = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\Lambda^2}^{1/m_\phi^2}\frac{dt}{t} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\ln\frac{\Lambda^2}{m_\phi^2}$$

It is important to examine the applicable scope of our method.

Here, we point out that the result depends on the choice of duality transformation.

Different choice

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2} dt$$

$$\tau_2 \equiv \Lambda^2 t$$

$$\tau = \tau_1 + i\tau_2$$

⬇

$$\tau \to -\frac{1}{\tau}, \quad \tau \to \tau + 1$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2} dt\right]$$

$$= \frac{\lambda_\phi \Lambda^2}{32\pi^2}\int_F \frac{d^2\tau}{\tau_2^2} = \frac{\lambda_\phi}{32\pi^2}\frac{\pi}{2}\Lambda^2$$

$$F = \left\{\tau : \left|\mathrm{Re}\,\tau\right| \le 1/2,\ \ 1 \le \left|\tau\right|\right\}$$

$$\tau \to -\frac{1}{\tau}, \quad \tau \to \tau + 1$$

$$F = \left\{\tau : |\mathrm{Re}\,\tau| \le 1/2,\ \ 1 \le |\tau|\right\}$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2} dt\right] = \frac{\lambda_\phi \Lambda^2}{32\pi^2}\int_F \boxed{\frac{d^2\tau}{\tau_2^2}} = \frac{\lambda_\phi}{32\pi^2}\frac{\pi}{2}\Lambda^2$$

↕ **Difference of invariant measures**

$$t \to 1/(\Lambda^4 t)$$

$$\delta m_\phi^2 = \mathrm{Du}\left[\frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt\right] = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\int_{1/\Lambda^2}^{1/m_\phi^2}\boxed{\frac{dt}{t}} = -\frac{\lambda_\phi m_\phi^2}{32\pi^2}\ln\frac{\Lambda^2}{m_\phi^2}$$

We need to specify the duality in order to obtain a phys. result.

Different choice

➔ The form of duality could be determined by matching the counterpart in the ultimate theory.

$$\delta m_\phi^2 = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty \frac{e^{-m_\phi^2 t}}{t^2}dt = \frac{\lambda_\phi}{32\pi^2}\int_0^\infty d\tau_2 \int_{-1/2}^{1/2} d\tau_1 \frac{\Lambda^2}{\tau_2^2} e^{-\frac{m_\phi^2}{\Lambda^2}\tau_2} dt$$

$$\tau \to \frac{a\tau+b}{c\tau+d} \quad \leftarrow \quad \tau \to -\frac{1}{\tau} \quad \& \quad \tau \to \tau+1$$

$$\tau = \tau_1 + i\tau_2$$

Field th. limit

$$\tau_1 = 0$$

$$\downarrow$$

$$\tau_2 \equiv \Lambda^2 t$$

$$\tau_2 \to \frac{1}{\tau_2} \quad \rightarrow \quad t \to 1/(\Lambda^4 t)$$

4．Conclusions

We have reconsidered naturalness, from the viewpoint of effective field theories including the SM.

【Expectation】

- Quadratic div. might be artifact of regularization procedure.
- The calculation scheme can be selected by the physics.

We have found that the quadratic divergences can be subtracted by imposing a hidden duality on radiative corrections of parameters.

【Conjecture】

An ultimate theory does not induce any large corrections for low-energy fields owing to a symmetry, and such a symmetry is hidden in the Standard Model.

Future Subjects

Justification of our method and/or Understanding of applicable scope

- Beyond the one-loop analysis

  Multi parameters

- Several scalar fields with different masses

Message

Even if our scheme has a limit of application and/or the hidden duality is a product of fantasy, I hope the expectation would survive.

- The calculation scheme can be selected by the physics.
- Radiative corrections can be constrained by a remnant of symmetries in an ultimate theory.

Thank you for your attention.