Supersymmetric Spectroscopy

BPS spectrum of 4d $\mathcal{N} = 2$ SCFT via spectral network

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K. Maruyoshi, CYP, W. Yan, to appear

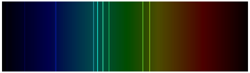
Spectroscopy we learned at high school

- A flame test is a qualitative method to identify an element.

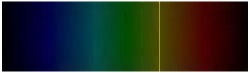
Cu

Na

- An emission spectrum is used to quantitatively distinguish different elements.



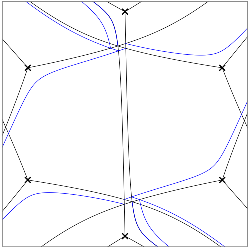
Cu



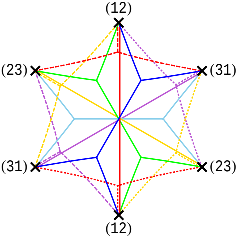
Na

Supersymmetric spectroscopy via spectral network

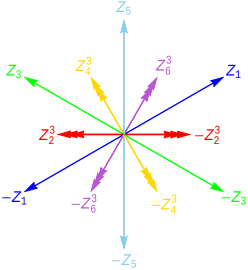
- Using spectral network, we can get the BPS spectrum of a 4d $\mathcal{N} = 2$ theory of class $\mathcal{S}$ on the Coulomb branch.
- We use the BPS spectrum to identify such a theory, which is useful when the theory is strongly coupled and we lack any perturbative approach to understand it.



spectral network



finite $\mathcal{S}$-walls



central charges

4d $\mathcal{N} = 2$ theory of class $\mathcal{S}$ and BPS states

- The low-energy effective theory of a 4d $\mathcal{N} = 2$ gauge theory can be understood as being from an M5-brane wrapping a complex 1-dimensional curve, called Seiberg-Witten curve.

  [Seiberg-Witten, 1994]
- A theory of class $\mathcal{S}$ is obtained from multiple M5-branes wrapping a punctured Riemann surface, which in the Coulomb branch merge into a single M5-brane wrapping a Seiberg-Witten curve. [Gaiotto, 2009]
- A 4d state that has its mass tied to the central charge of the SUSY algebra, called a BPS state, is identified with an M2-brane that ends along a 1-cycle of the Seiberg-Witten curve. Its mass is given by integrating a 1-form, called Seiberg-Witten differential, along the 1-cycle.

$\mathcal{S}$-walls and spectral network

- When we have a Seiberg-Witten curve $f(t,x)=0$ as a multi-sheeted cover over the t-plane and the corresponding Seiberg-Witten differential $\lambda = \lambda(t,x)\,dt$, we obtain an $\mathcal{S}_{jk}$-wall of a spectral network by solving

$$\frac{\partial \lambda_{jk}}{\partial \tau} = \left(\lambda_j(t,x) - \lambda_k(t,x)\right)\frac{\partial t}{\partial \tau} = e^{i\theta},$$

  where λ_i is the value of λ on the i-th sheet, and τ is a real parameter along the $\mathcal{S}_{jk}$-wall. [Klemm-Lerche-Mayr-Vafa-Warner, 1996] [Gaiotto-Moore-Neitzke, 2009,2010,2011,2012]

- The collection of the $\mathcal{S}$-walls at a value of θ is called a spectral network. [Gaiotto-Moore-Neitzke, 2012]

Punctures of the theory of class $\mathcal{S}$

- When we represent a Seiberg-Witten curve in $(x, t) \in T^*\mathcal{C}$ as

$$x^N + \sum_{k=2}^{N} \phi_k(t) x^{N-k} = 0,$$

where $\mathcal{C}$ is a Riemann sphere with punctures, each puncture is labeled by $D = (d_2, d_3, \dots, d_{N-1}, d_N)$, where

$$\phi_k = (dt)^k / t^{d_k} + \cdots .$$

- When a Seiberg-Witten differential $\lambda = x\, dt$ has a singularity at $t = 0$ such that the singular part of λ is described by

$$\sum_{i=0} \frac{c_i}{t^{a_i}} dt, \;\; a_i > a_{i+1} > 0,$$

we call the point a regular puncture if $a_0 \leq 1$, and an irregular puncture if $a_0 > 1$.

4d $\mathcal{N} = 2$ SCFT at AD point from 6d $\mathcal{N} = (2, 0)$ theory

- When a 4d $\mathcal{N} = 2$ theory has mutually nonlocal massless states in the IR limit, it flows to an interacting SCFT.

  [Argyres-Douglas, 1995][Argyres-Plesser-Seiberg-Witten, 1995]

- Compactifying the 6d $\mathcal{N} = (2, 0)$ A_N theory on a Riemann sphere with an irregular puncture leads to a 4d $\mathcal{N} = 2$ SCFT of Argyres-Douglas type. [Gaiotto-Moore-Neitzke, 2009][Cecotti-Neitzke-Vafa, 2010]

  [Alim-Cecotti-Cordova-Espahbodi-Rastogi-Vafa, 2011][Bonelli-Maruyoshi-Tanzini, 2012][Xie, 2013]

- The theory on the Coulomb branch of an SCFT from an Argyres-Douglas fixed point has massive BPS states that become mutually nonlocal and massless when we flow the theory to the fixed point.

- By studying the spectral network of the theory, we can find such BPS states.

4d $\mathcal{N} = 2$ SCFTs from 6d A_1

$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{n+5}]$

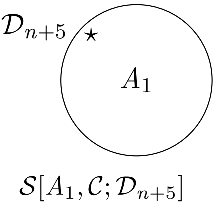
From compactifying 6d A_1 on a Riemann sphere $\mathcal{C}$ with

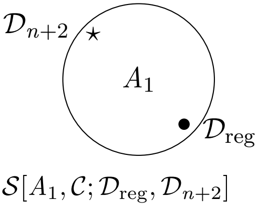
- an irregular puncture $\mathcal{D}_{n+5} = (d_2) = (n+5)$ at $t = \infty$

$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_{n+2}]$

From 6d A_1 on $\mathcal{C}$ with a regular puncture $\mathcal{D}_{\text{reg}}$ at $t = 0$ and

- an irregular puncture $\mathcal{D}_{n+2} = (d_2) = (n+2)$ at $t = \infty$





4d $\mathcal{N} = 2$ SCFTs from 6d A_{N-1}

$\mathcal{S}[A_{N-1}, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$, $\mathcal{S}[A_{N-1}, \mathcal{C}; \mathcal{D}_{\mathrm{II}}]$

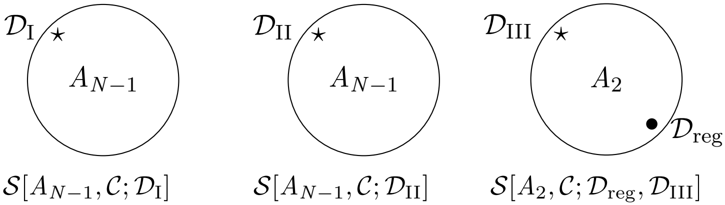
From 6d A_{N-1} on $\mathcal{C}$ with an irregular puncture at $t = \infty$,

- $\mathcal{D}_{\mathrm{I}} = (d_2, \ldots, d_N) = (4, 6, \ldots, 2N-2, 2N+2)$
- $\mathcal{D}_{\mathrm{II}} = (d_2, \ldots, d_N) = (4, 6, \ldots, 2N-4, 2N, 2N+2)$

$\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\mathrm{reg}}, \mathcal{D}_{\mathrm{III}}]$

From 6d A_2 on $\mathcal{C}$ with a regular puncture $\mathcal{D}_{\mathrm{reg}}$ at $t = 0$ and

- an irregular puncture $\mathcal{D}_{\mathrm{III}} = (d_2, d_3) = (3, 5)$ at $t = \infty$



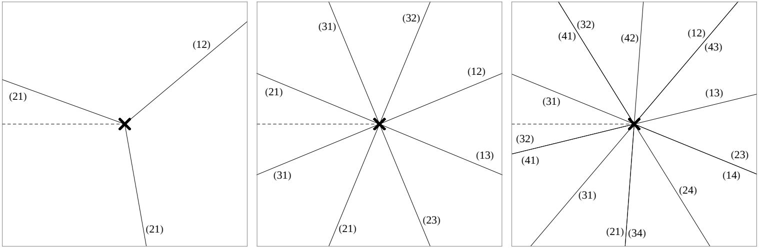
- We claim equivalences between 4d $\mathcal{N}=2$ SCFTs of different descriptions by analyzing the spectral network of the theory from the IR Coulomb branch of each SCFT.
- A minimal BPS spectrum of each class is represented by a quiver based on a Dynkin diagram Γ.
- Theories in the same Γ-class have the same BPS spectrum and exhibit the same wall-crossings.

Γ	6d A_1	6d A_{N-1}
$A_{n=N-1}$, $N \geq 3$	$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{n+5}]$	$\mathcal{S}[A_{N-1}, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$
$D_3 = A_3$	$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_8]$ $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{reg}, \mathcal{D}_5]$	$\mathcal{S}[A_3, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$
D_4	$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{reg}, \mathcal{D}_6]$	$\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\mathrm{II}}]$ $\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{reg}, \mathcal{D}_{\mathrm{III}}]$
$D_{n=N+1}$, $N \geq 4$	$\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{reg}, \mathcal{D}_{n+2}]$	$\mathcal{S}[A_{N-1}, \mathcal{C}; \mathcal{D}_{\mathrm{II}}]$

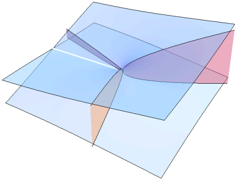
Outline

Outline

$\mathcal{S}$-walls around a branch point of ramification index N



$N = 2$ $\quad$ $N = 3$ $\quad$ $N = 4$

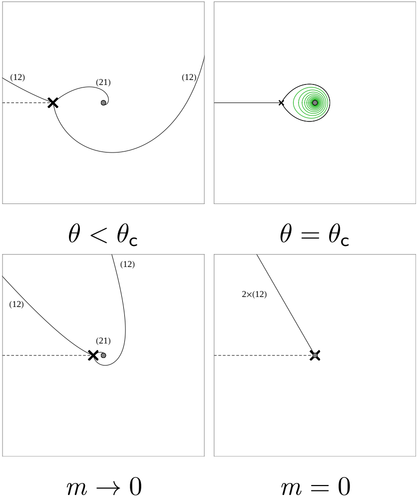


For $f(t, x) = t - x^N$ and $\lambda = x\, dt$, there are $N^2 - 1$ $\mathcal{S}_{jk}$-walls described by

$$t_{jk}(\tau) = (\exp(i\theta)/\omega_{jk})^{\frac{N}{N+1}}\, \tau,$$

where $\omega_{jk} = \omega_j - \omega_k$ and $\omega_k = \exp\left(\frac{2\pi i}{N} k\right)$.

$\mathcal{S}$-walls around an $\mathrm{SU}(N)$ regular puncture

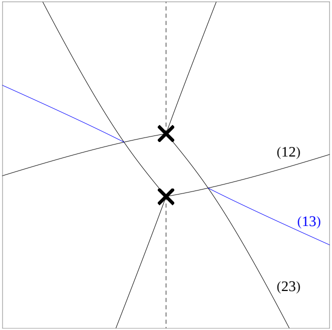


- There are $N-1$ branch points corresponding to the mass parameters.
- A closed $\mathcal{S}$-wall appears around a puncture when $\theta = \theta_c = \arg\left[i(m_j - m_k)\right]$
- When all $m_j = 0$, we have $f(t,v) = t - v^N$ and $\lambda = \frac{v}{t}dt$ that gives $N(N-1)$ $\mathcal{S}$-walls described by

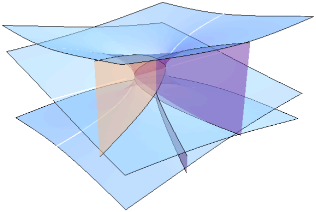
$$t(\tau) = \left(e^{i\theta}/\omega_{jk}\right)^N \tau$$

BPS joint of $\mathcal{S}$-walls

$\mathcal{S}$-walls $\mathcal{S}_{i_1 i_2}$, $\mathcal{S}_{i_2 i_3}$, ..., $\mathcal{S}_{i_n i_1}$ of the spectral network from $A_{N>1}$ can form a joint, where $\lambda_{i_1 i_2} + \lambda_{i_2 i_3} + \cdots + \lambda_{i_n i_1} = 0$ is satisfied.

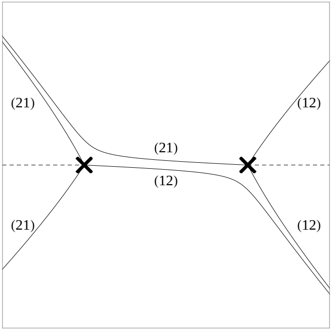


spectral network



Seiberg-Witten curve and $\mathcal{S}$-walls

4d BPS states from finite $\mathcal{S}$-walls and their central charge



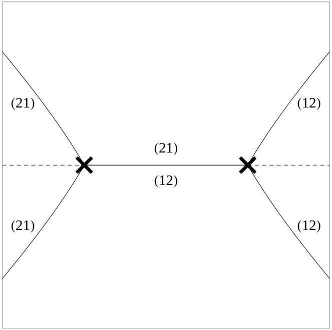
$\theta < \theta_c$

- We obtain the 4d BPS states from the cycles of the Seiberg-Witten curve.
- Spectral networks enable us to identify the cycles with finite $\mathcal{S}$-walls that have finite values of the integration of λ along the $\mathcal{S}$-wall,

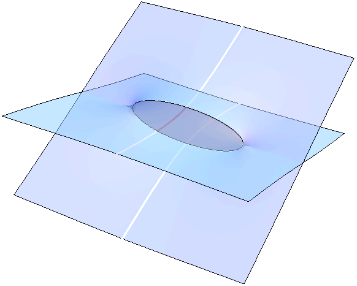
$$Z = \int_{\tau_i}^{\tau_f} \lambda_{jk}(t) \frac{\partial t}{\partial \tau} d\tau = \int_{\tau_i}^{\tau_f} e^{i\theta} d\tau,$$

where Z is the central charge of the BPS state.

4d BPS states from finite $\mathcal{S}$-walls and their central charge

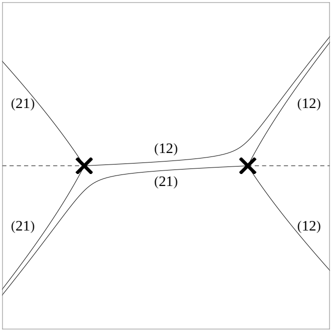


$\theta = \theta_c$



Seiberg-Witten curve from A_1 and a finite $\mathcal{S}$-wall

4d BPS states from finite $\mathcal{S}$-walls and their central charge



$\theta > \theta_c$

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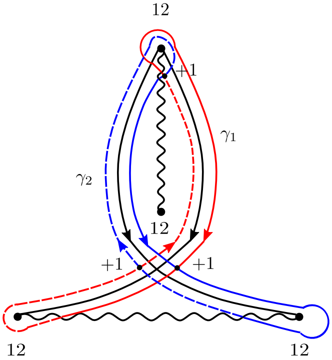
where Z is the central charge of the BPS state.

IR charges from finite $\mathcal{S}$-walls

- The intersection of two 1-cycles corresponding to two finite $\mathcal{S}$-walls give the skew-symmetric electric-magnetic inner product of the corresponding BPS states.

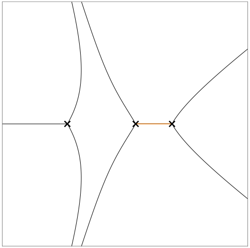
$$\langle \gamma_1, \gamma_2 \rangle = \langle \mathcal{S}_{ij}, \mathcal{S}_{jk} \rangle = \sum_r q_1^{(r)} p_2^{(r)} - p_1^{(r)} q_2^{(r)}$$



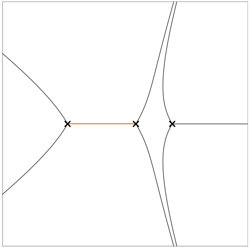
Outline

A_2-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_7]$ with minimal BPS spectrum

- Click here to see the animation of the spectral network.

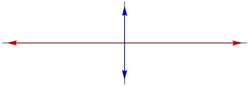


$\theta = \arg(Z_1)$



$\theta = \arg(Z_2)$

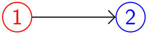
Finite $\mathcal{S}$-walls



central charges

state	(e, m)
1	$(1, 0)$
2	$(0, 1)$

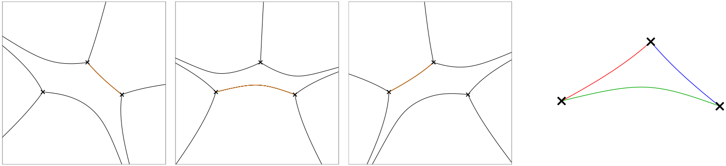
IR charges



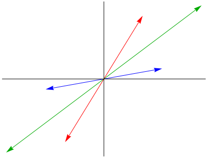
BPS quiver

A_2-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_7]$ with maximal BPS spectrum

- After a wall-crossing, $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_7]$ has a maximal BPS spectrum with three BPS states.
- Click here to see the animation of the spectral network.



$\theta = \arg(Z_2)$ $\theta = \arg(Z_3)$ $\theta = \arg(Z_1)$ finite $\mathcal{S}$-walls



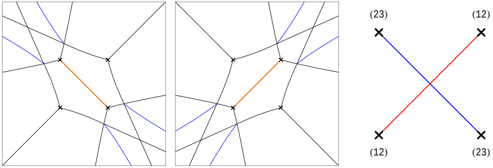
central charges

state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 1)$

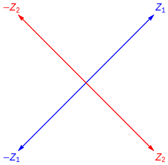
IR charges

A_2-class: $\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$ with minimal BPS spectrum

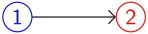
- Another SCFT in the same A_2-class as $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_7]$ can be obtained from 6d A_2 theory. [Gaiotto-Moore-Neitzke, 2012]
- Click here to see the animation of the spectral network.



$\theta = \arg(Z_1)$ $\theta = \arg(Z_2)$ finite $\mathcal{S}$-walls

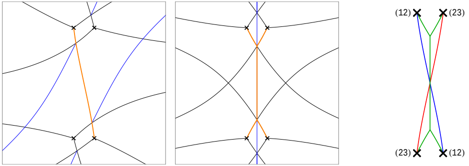


central charges

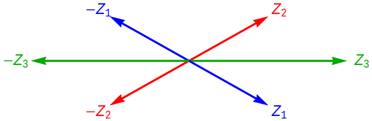


BPS quiver

- After a wall-crossing, there appears a joint of three $\mathcal{S}$-walls, giving the third BPS state. [Gaiotto-Moore-Neitzke, 2012]
- Click here to see the animation of the spectral network.



$\theta = \arg(Z_1)$ $\quad$ $\theta = \arg(Z_3)$ $\quad$ finite $\mathcal{S}$-walls



central charges

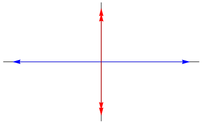
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 1)$

IR charges

A_3-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_8]$ with $\mathrm{SU}(2)_{\mathrm{f}}$

- A minimal BPS spectrum is represented with an A_3 quiver.
- There is a doublet of $\mathrm{SU}(2)_f$.

finite $\mathcal{S}$-walls

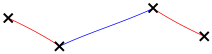


central charges

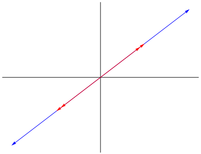
BPS quiver

A_3-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_8]$ with $\mathrm{SU}(2)_{\mathrm{f}}$

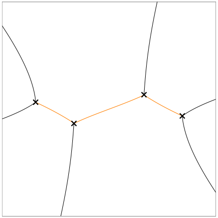
- BPS spectrum when the theory is on the BPS wall.
- With $\mathrm{SU}(2)_{\mathrm{f}}$ maintained, the central charges of three BPS states align at the same time.



finite $\mathcal{S}$-walls



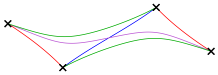
central charges



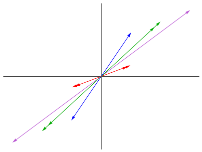
$\theta = \theta_{\mathrm{c}}$

A_3-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_8]$ with $\mathrm{SU}(2)_\mathrm{f}$

- After the wall-crossing with $\mathrm{SU}(2)_\mathrm{f}$, Maximal BPS spectrum has three additional BPS states, one doublet and one singlet.



finite $\mathcal{S}$-walls

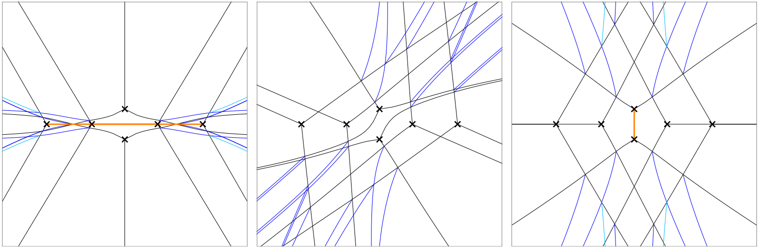


central charges

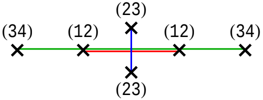
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 1)$
4	$(2, 1)$

IR charges

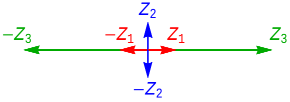
A_3-class: $\mathcal{S}[A_3, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$ with minimal BPS spectrum



$\theta = \arg(Z_1)$ | $\arg(Z_1) < \theta < \arg(Z_2)$ | $\theta = \arg(Z_2)$



finite $\mathcal{S}$-walls

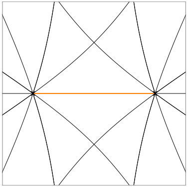


on the Z-plane

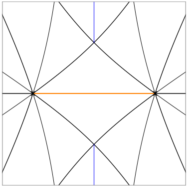
BPS quiver

A_3-class: $\mathcal{S}[A_3, \mathcal{C}; \mathcal{D}_{\mathrm{I}}]$ with maximal BPS spectrum

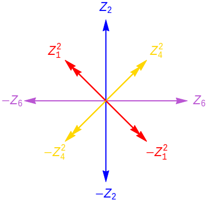
- When the spectral network is from two branch points of index 4, it gives the maximal, symmetric BPS spectrum.
- This spectrum contains two doublets and two singlets of $\mathrm{SU}(2)_f$, and each state has the same IR charge as the corresponding BPS state of $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_8]$.



$\theta = \arg(Z_1^{\mathbf{2}})$



$\theta = \arg(Z_2)$



central charges

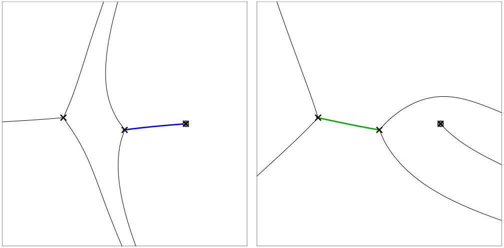
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
4	$(1, 1)$
6	$(2, 1)$

IR charges

Outline

$D_3(=A_3)$-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\rm reg}, \mathcal{D}_5]$ with $\mathrm{SU}(2)_{\rm f}$

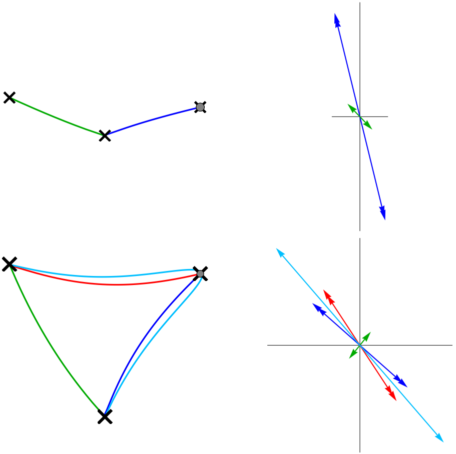
- From a regular puncture with $SU(2)_{\rm f}$, a doublet of $\mathcal{S}$-walls flow out of it.
- This provides a doublet of $SU(2)_{\rm f}$, which is also contained in the BPS spectrum of other theories of A_3-class, but how it realized in a spectral network is different.



$\theta = \arg(Z_1^{\mathbf{2}})$ $\qquad$ $\theta = \arg(Z_2)$

$D_3(= A_3)$-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_5]$ with $\mathrm{SU}(2)_{\mathrm{f}}$

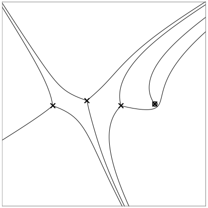
- Wall-crossing with $\mathrm{SU}(2)_{\mathrm{f}}$ maintained results in three additional BPS states, giving the maximal BPS spectrum of D_3-class ($= A_3$-class).



$$1 \longrightarrow 2 \longleftarrow 1$$

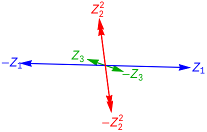
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 1)$
4	$(2, 1)$

D_4-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_6]$, minimal BPS spectrum



at general θ

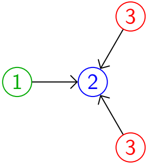
finite $\mathcal{S}$-walls



central charges

state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 0)$

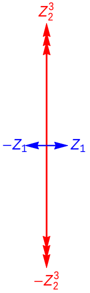
IR charges



BPS quiver

D_4-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\mathrm{reg}}, \mathcal{D}_6]$, wall-crossing with $\mathrm{SU}(3)_{\mathrm{f}}$

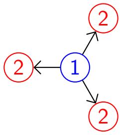
finite $\mathcal{S}$-walls



central charges

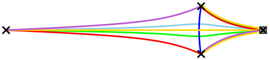
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$

IR charges

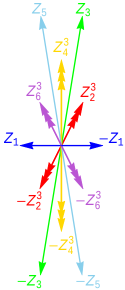


BPS quiver

D_4-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_6]$, wall-crossing with $\mathrm{SU}(3)_{\mathrm{f}}$



finite $\mathcal{S}$-walls

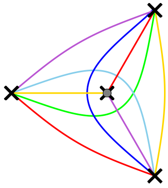


central charges

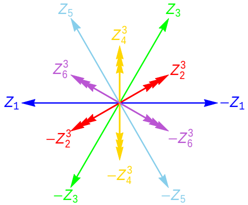
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 3)$
4	$(1, 2)$
5	$(2, 3)$
6	$(1, 1)$

IR charges

D_4-class: $\mathcal{S}[A_1, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_6]$, wall-crossing with $\text{SU}(3)_{\text{f}}$



finite $\mathcal{S}$-walls



central charges

state	(e, m)
1	$(1, 0)$
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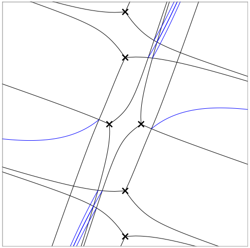
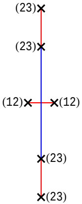
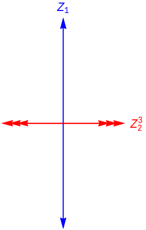
3	$(1, 3)$
4	$(1, 2)$
5	$(2, 3)$
6	$(1, 1)$

IR charges





$\arg(Z_2^{\mathbf{3}}) < \theta < \arg(Z_1)$ finite $\mathcal{S}$-walls



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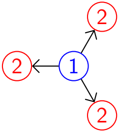
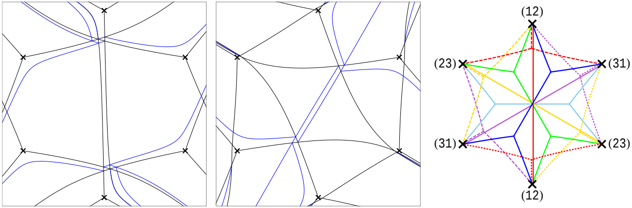
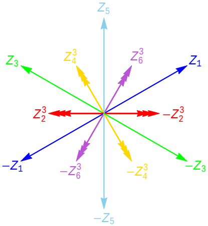


central charges IR charges BPS quiver

D_4-class: $\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\mathrm{II}}]$, maximal symmetric BPS spectrum



$\theta \approx \arg(Z_1^{\mathbf{3}})$ $\theta \approx \arg(Z_2)$ finite $\mathcal{S}$-walls

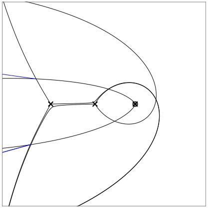


central charges

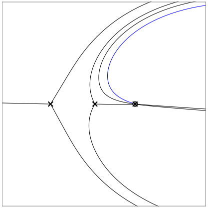
state	(e, m)
1	$(1, 0)$
2	$(0, 1)$
3	$(1, 3)$
4	$(1, 2)$
5	$(2, 3)$
6	$(1, 1)$

IR charges

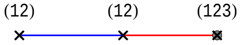
D_4-class: $\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\mathrm{reg}}, \mathcal{D}_{\mathrm{III}}]$, minimal BPS spectrum



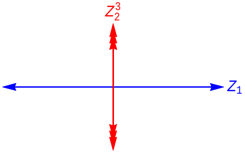
$\theta \approx \arg(Z_1)$



$\theta \approx \arg(Z_2^{\mathbf{3}})$



finite $\mathcal{S}$-walls



central charges

state	(e, m)
1	$(1, 0)$
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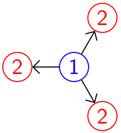
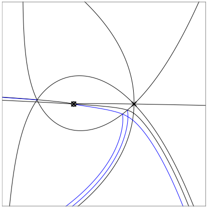
IR charges

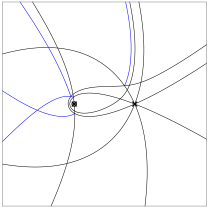


BPS quiver

D_4-class: $\mathcal{S}[A_2, \mathcal{C}; \mathcal{D}_{\text{reg}}, \mathcal{D}_{\text{III}}]$, maximal BPS spectrum

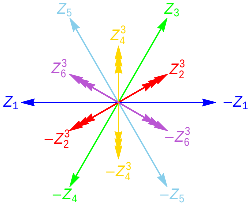


$\theta \approx \arg(Z_2^{\mathbf{3}})$



$\theta \approx \arg(Z_1)$

finite $\mathcal{S}$-walls



central charges

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1	$(1, 0)$
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6	$(1, 1)$

IR charges

Outline

Summary

- Using spectral network we can find out the BPS spectra of 4d $\mathcal{N} = 2$ theories of class $\mathcal{S}$, including SCFTs at Argyres-Douglas fixed points.
- BPS spectrum obtained via spectral network illustrates how massless nonlocal states of an SCFT at an Argyres-Douglas fixed point arise and how flavor symmetry enhancement occurs.
- Matching the BPS spectra of theories with different descriptions show there are equivalent classes of such theories, each class being represented by a BPS quiver.

Outlook

- Study SCFTs at AD points from other types of irregular punctures [Xie,2012][Kanno-Maruyoshi-Shiba-Taki, 2013] and regular punctures [Cecotti-Del Zotto, 2012][Cecotti-Del Zotto-Giacomelli, 2013]
- Understand how to use spectral network and BPS quiver in a complementary way. [Cecotti-Neitzke-Vafa, 2010][Cecotti-Vafa, 2011] [Alim-Cecotti-Cordova-Espahbodi-Rastogi-Vafa, 2011]
- Study more general SCFTs of class $\mathcal{S}$ via spectral network. [Gaiotto-Tachikawa-Seiberg, 2011][Chacaltana-Distler, 2010]
- Use spectral network to understand 6d $(2,0)$ theory in the Coulomb branch.