Features in the curvature power spectrum after a turn of the inflationary trajectory

Xian Gao (高顯)

Department of Physics,

Tokyo Institute of Technology

18 October, 2013

Kavli Institute for the Physics and Mathematics of the Universe

Based on works with ***David Langlois*** and ***Shuntaro Mizuno***,

JCAP 10 (2012) 040 (arXiv:1205.5275)

JCAP 10 (2013) 023 (arXiv:1306.5680)

Single field inflation

- The latest observations on CMB are compatible with statistically Gaussian primordial perturbation, which has a nearly flat spectrum with negligible running spectral tilt.

- In particular, the data are **compatible** with the adiabaticity at 95% CL, which implies there is no evidence for the isocurvature modes and there is only one relevant degree of freedom responsible to the primordial perturbations.

Beyond the single field

- Theoretical motivations
- Observational hints (1)
  - **Asymmetries in the CMB**

$$\mathcal{P}_{\zeta}^{1/2}(k, \boldsymbol{x}) = \left[1 + A(k)\, \hat{\boldsymbol{p}} \cdot \boldsymbol{x}/x_{\mathrm{ls}} + \cdots\right] \mathcal{P}_{\zeta}^{1/2}(k)$$

Planck results: $|A| = 0.07 \pm 0.02$ for $l<64$

  - Modulation of a super-horizon long wavelength perturbation mode *[Erickcek, Kamionkowski & Carroll, 08']*

A consistency relation between factor *A* and local non-Gaussianity *[Lyth 13', Firouzjahi et al 13']*:

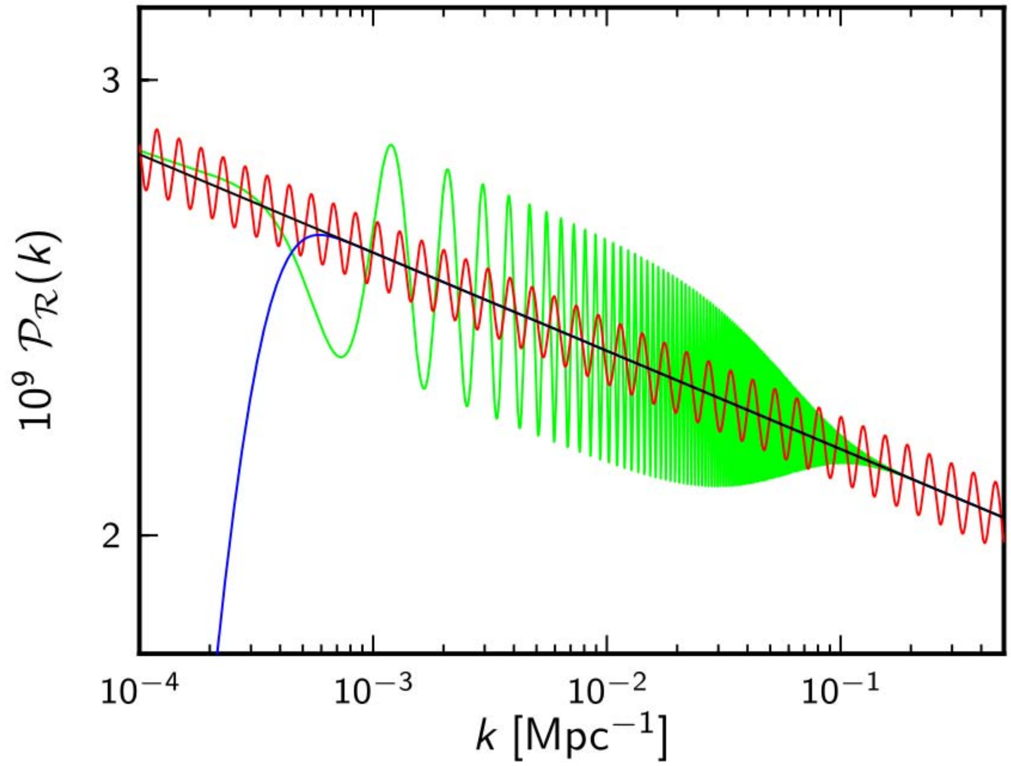
$$|A| \lesssim 10^{-1} \left| f_{\mathrm{NL}}^{(\mathrm{local})} \right| \sim 10^{-1} (n_s - 1) \sim 10^{-3}$$

Such anomaly cannot be generated in any single-field inflation model with attractor behavior.

(Curvaton *[Lyth 13']*, vector fields *[Chen & Wang 13']*, etc)

Beyond the single field

- Observational hints (2)
  - **Oscillatory features in the CMB power spectrum**



[Planck 2013]

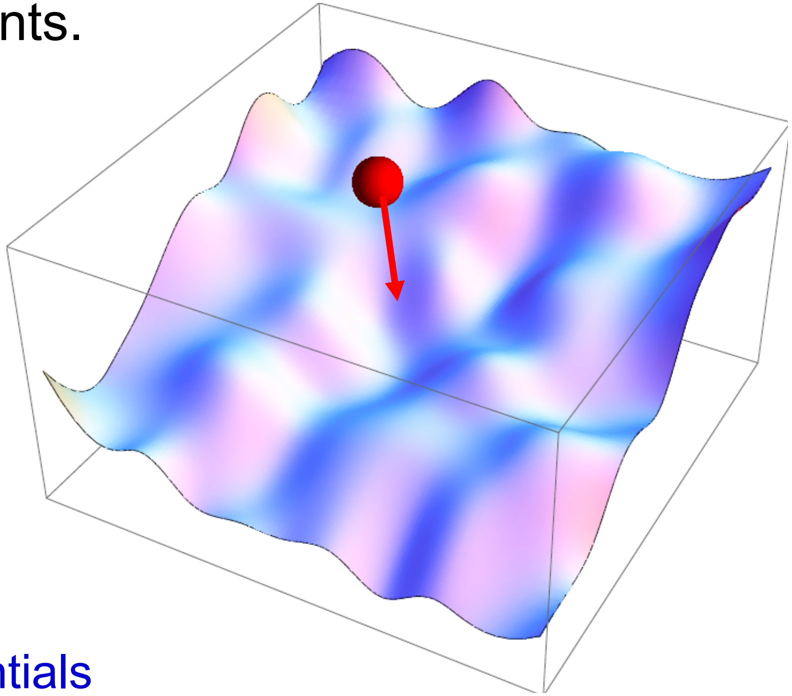
Oscillation periodic in cosmic time *t*:

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_0(k)\ \left\{1 + \alpha_{\mathrm{w}}\ \sin\left[\omega \ln\left(\frac{k}{k_*}\right) + \varphi\right]\right\}$$

Massive fields

Can massive ($M >= H$) fields be allowed and play some role in multi-field models?

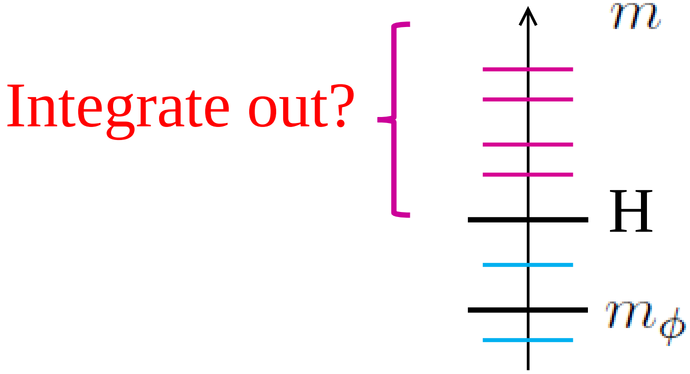
• As long as there is a **light (flat) direction** in the multi-field potential, inflation occurs, while other directions may be heavy.
• Perturbations probe the whole potential landscape, not only the light direction.
• Massive modes may have some imprints.



A landscape of potentials

Heavy modes?

- Naively, an effective theory for the light mode(s) is expected.



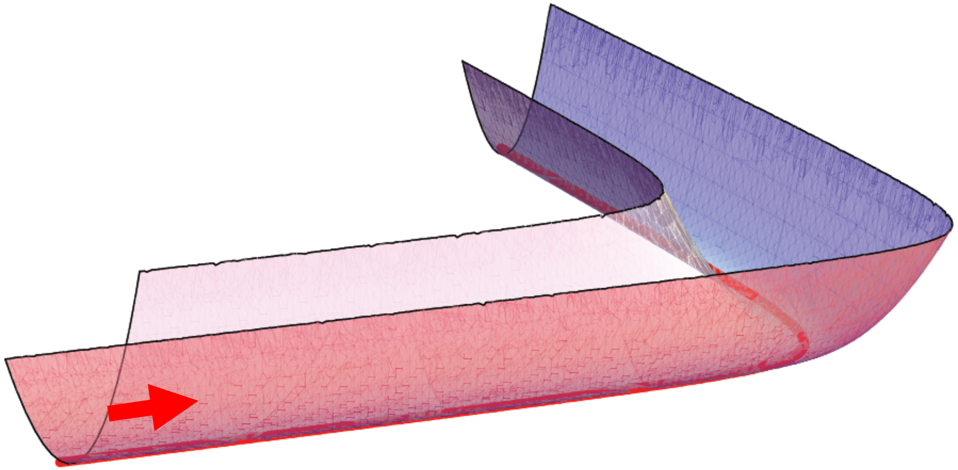
- If there is a bending trajectory: The trajectory generally deviates from the light direction. The **adiabatic** mode can become temporarily **heavy**. The effective single-field description may break down.
- Recent progress: *Tolley & Wyman `09. Cremonini, Lalak & Turzynski '10, Achucarro, Gong, Hardeman, Palma, Patil `1., Shiu & Xu `11, Watson et al '12. Chen & Wang `12, Gong, Pi & Sasaki '13, ...*

Background evolution

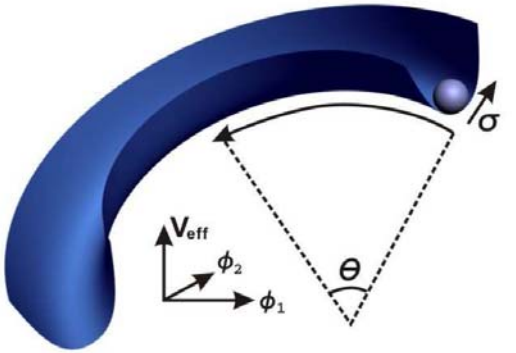
Heavy modes at work: Turning trajectory

Multi-field effects manifest themselves only when the background trajectory is **bending**.

We will concentrate on a **single turning process**, by requiring (the minimal deviation from the standard scenario):
1) the turning process occurs in a **finite** time interval
2) the potential trough is asymptotically **straight** before and after the turn.

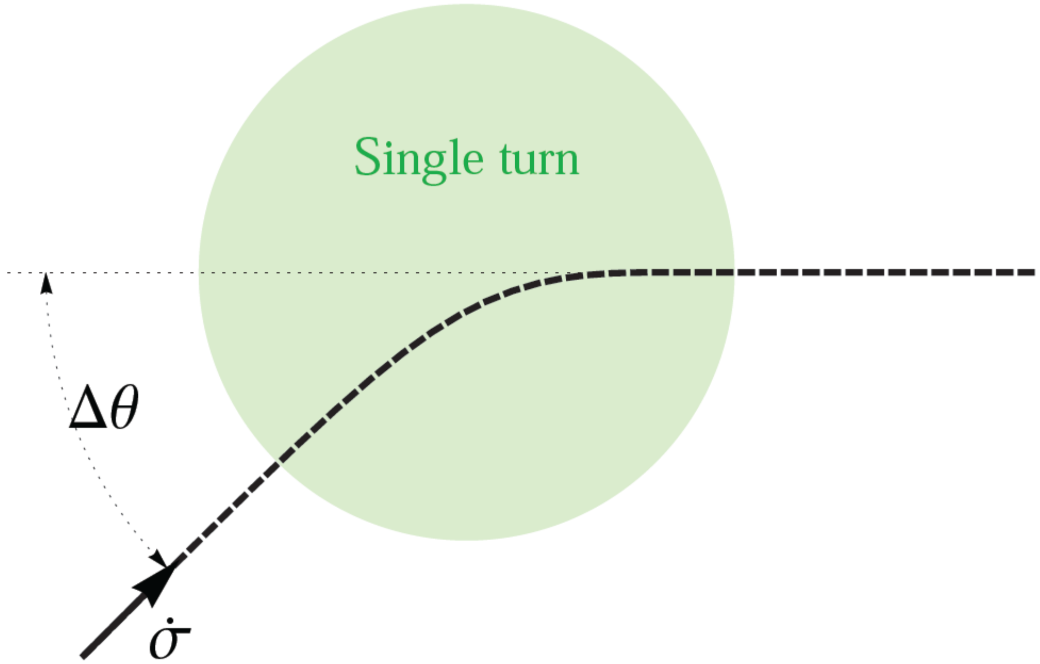


Different from "**constant** turn"



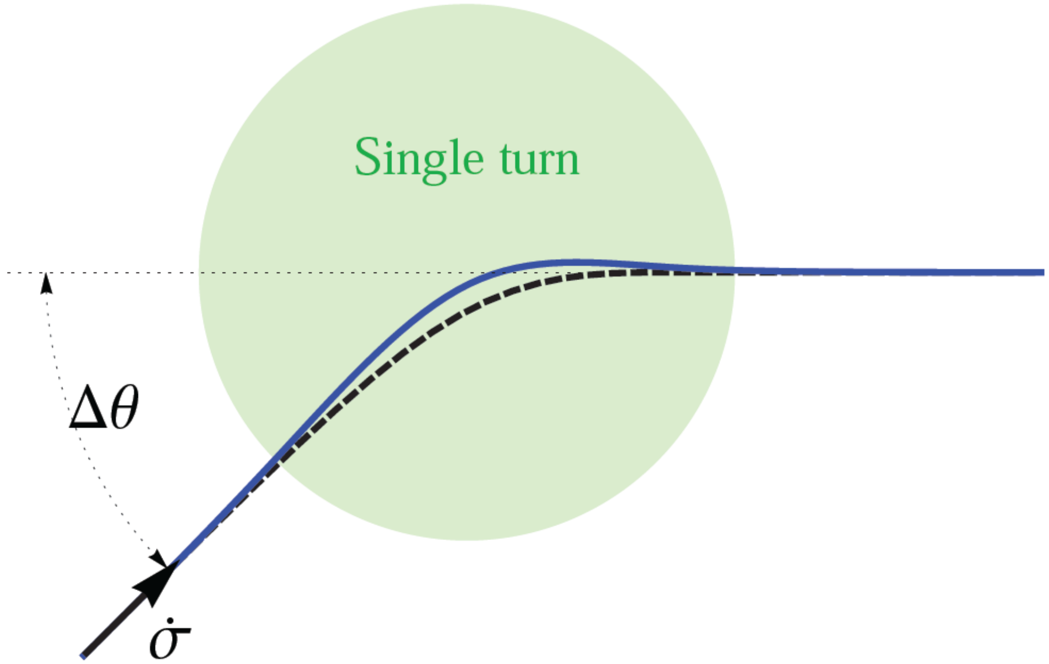
[*Chen & Wang '09, '12, Gong, Pi, Sasaki, '13, etc*]

Single turn: basic picture



Single turn: basic picture

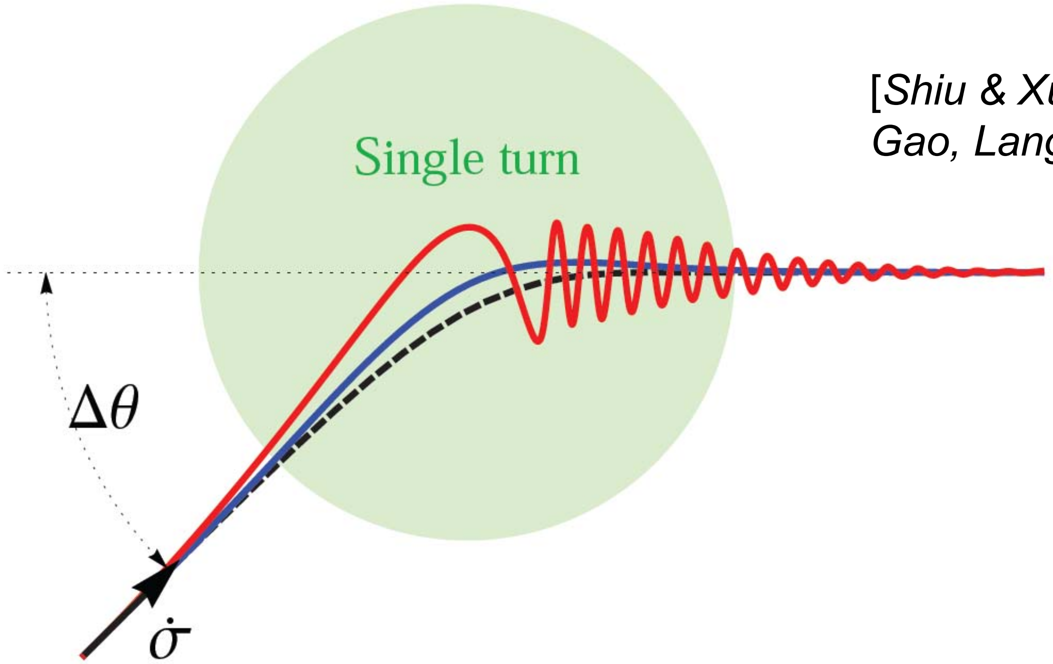
Intuitively, the **trajectory** **deviates** from the **light direction** of the valley due to the centrifugal force.



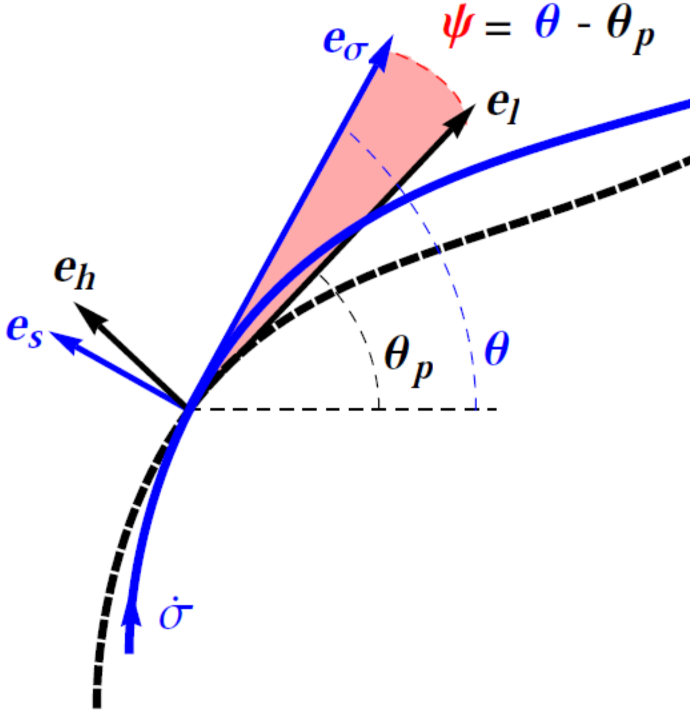
Single turn: basic picture

Intuitively, the **trajectory** **deviates** from the **light direction** of the valley due to the centrifugal force, and then **starts to oscillate**.

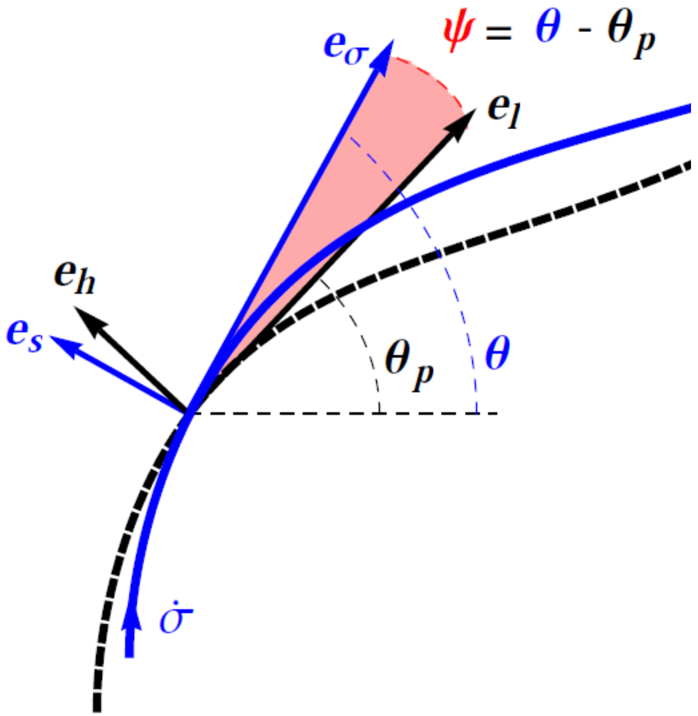
[*Shiu & Xu '11, Chen, '11, '12, Gao, Langlois, Mizuno, '12, '13*]



Turning trajectory: a two-field example



Turning trajectory: a two-field example



The background trajectory is characterized by: $\{\dot{\sigma}, \psi\}$

- **Velocity**: $\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0$

- **Direction**: **A simple approximate equation of motion for ψ ($|\psi| \ll 1$):**

$$\boxed{\ddot{\psi} + 3H\dot{\psi} + m_h^2\psi \simeq -\ddot{\theta}_p - 3H\dot{\theta}_p}$$

$$\boldsymbol{M} = \mathrm{diag}\{m_l^2, m_h^2\}, \qquad m_h \gtrsim H \gg M_l$$

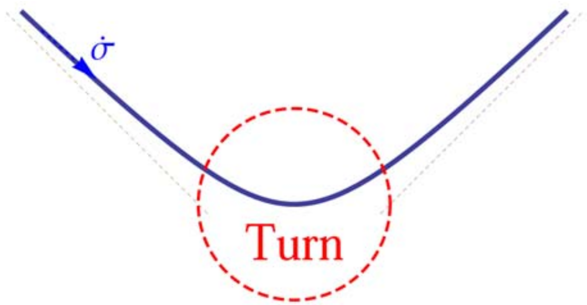
- In general, the trajectory (adiabatic direction) tends to deviate from the light direction, with turning light direction $\boldsymbol{\theta_p}$ serves as a driving force;

- ψ behaves as a damped oscillator with frequency controlled by $\boldsymbol{m_h}$;

A Gaussian toy model

A toy Gaussian ansatz:

$$\dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2}$$



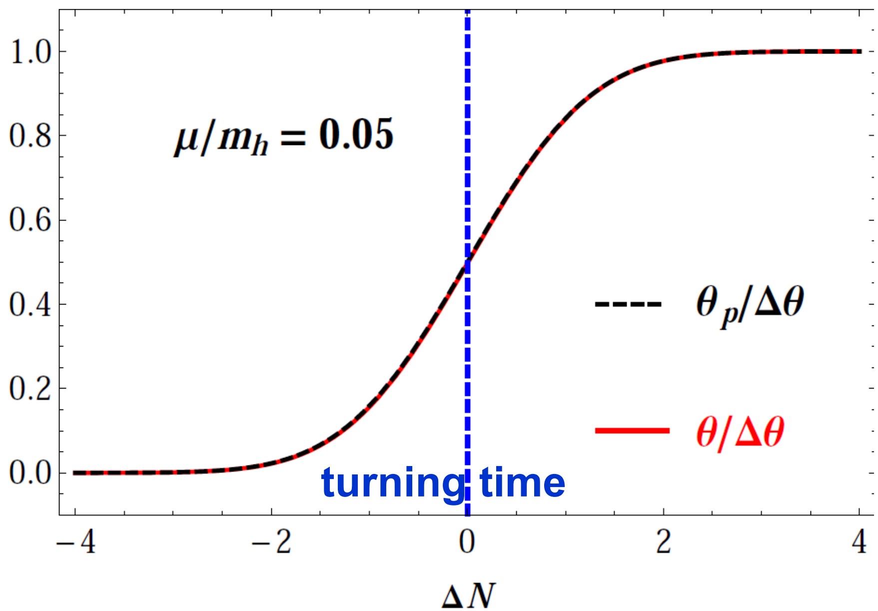
"Energy scale" of the turn: ***μ* = 1/Δt >> *H***

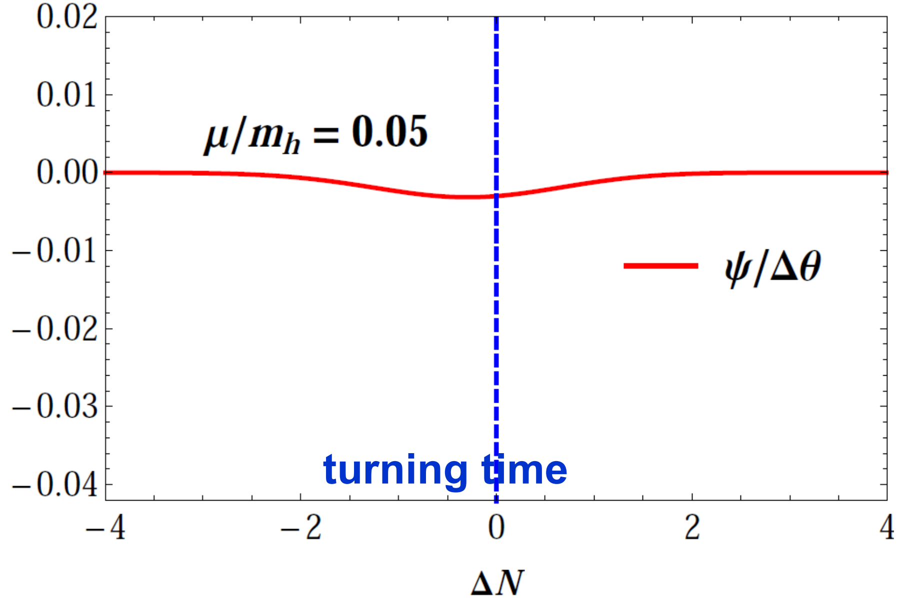
The qualitative behaviors of the trajectory and the perturbations are sensitive to the **ratio: μ/m_h**.

Soft turn ($\mu \ll m_h$)

Before ($-\mu t \gg 1$) and ***during*** ($|\mu t| \lesssim 1$) the soft turn:

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu}\right)$$





Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

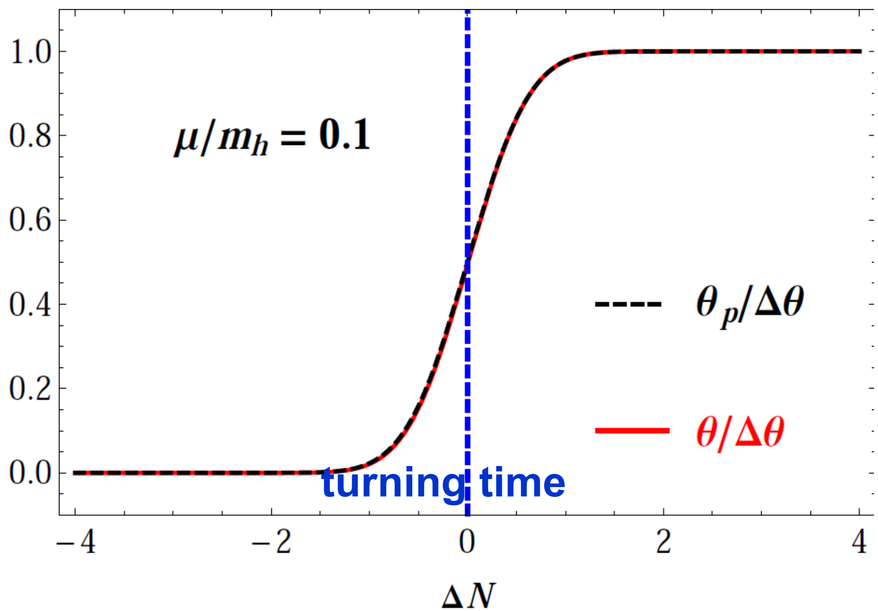
Evolution of $\psi = \theta - \theta_p$
(angle between trajectory & light direction)

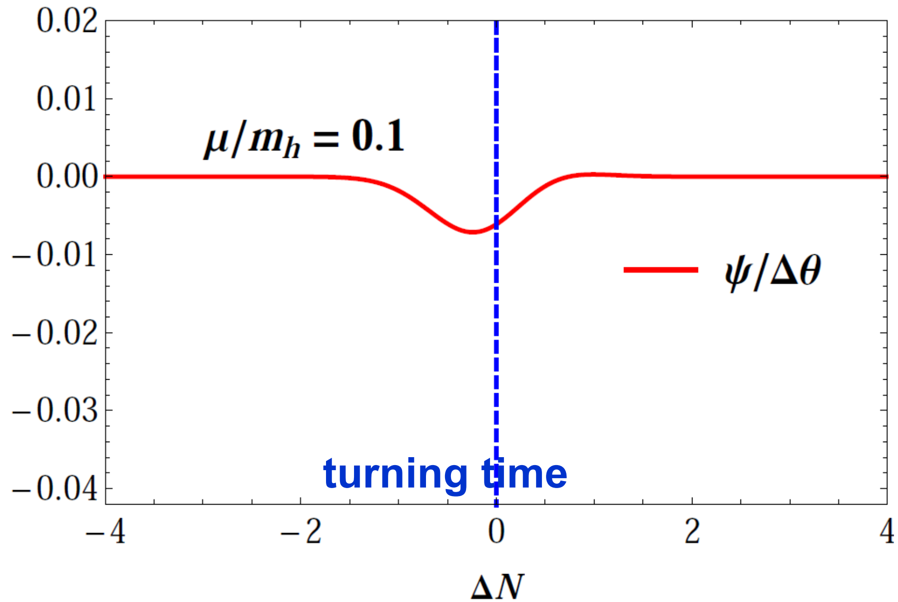
→ tiny deviation

Soft turn ($\mu << m_h$)

Before ($-\mu t >> 1$) and ***during*** ($|\mu t| <\approx 1$) the soft turn:

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu}\right)$$





Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

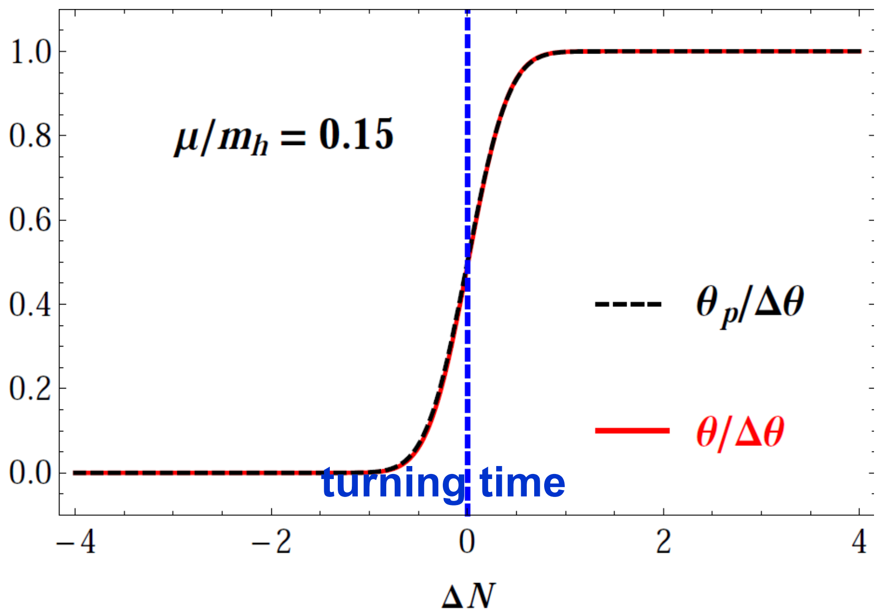
Evolution of $\psi = \theta - \theta_p$
(angle between trajectory & light direction)

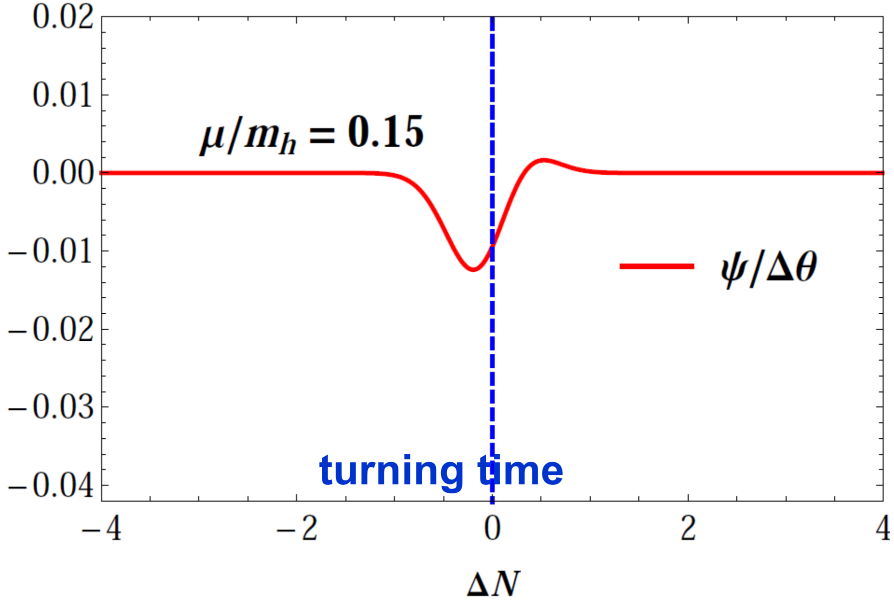
→ tiny deviation

Soft turn ($\mu << m_h$)

Before ($-\mu t >> 1$) and ***during*** ($|\mu t| <\approx 1$) the soft turn:

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}}\left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2}\left(\mu t - 3\frac{H}{\mu}\right)$$





Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

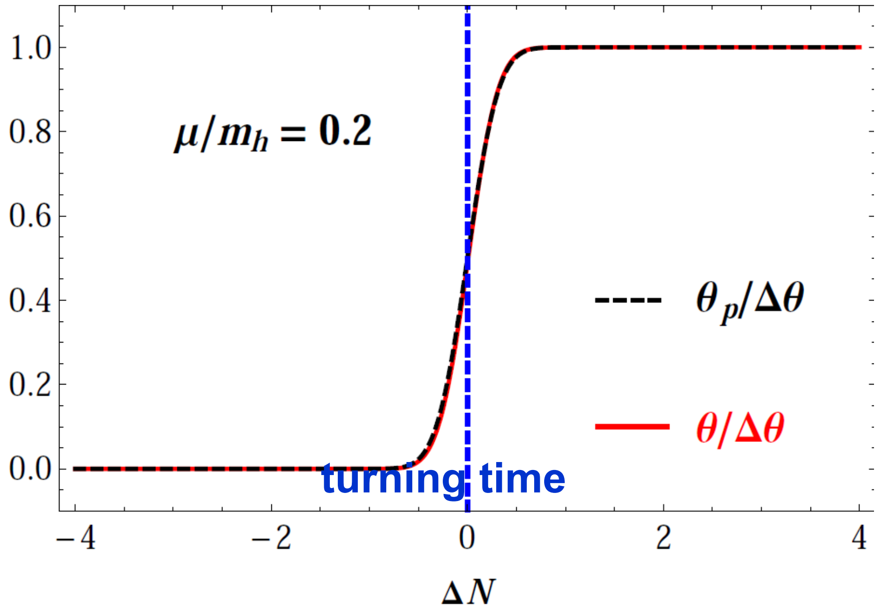
Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

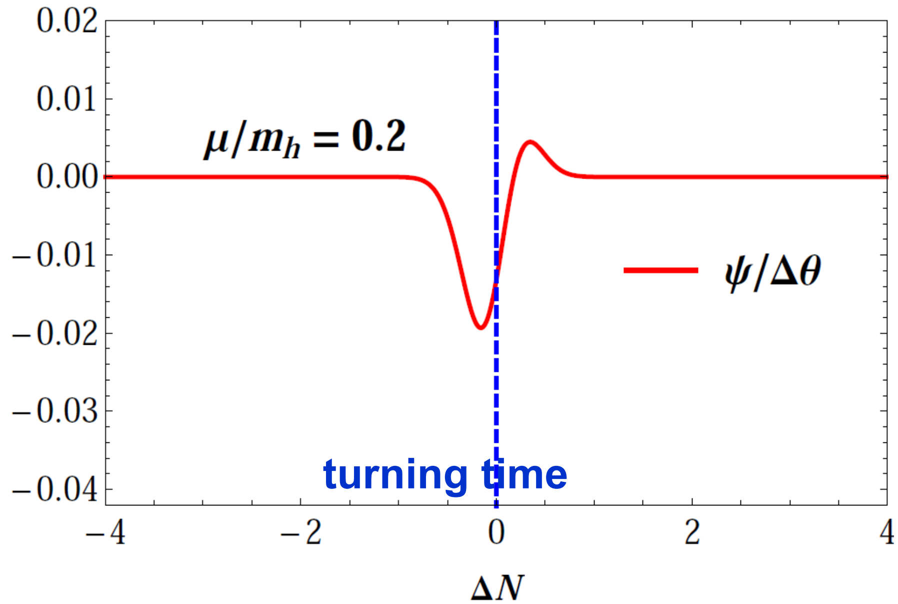
→ tiny deviation

Soft turn ($\mu \ll m_h$)

Before ($-\mu t \gg 1$) and ***during*** ($|\mu t| \lesssim 1$) the soft turn:

$$\psi(t) \approx \frac{\Delta\theta}{\sqrt{2\pi}} \left(\frac{\mu}{m_h}\right)^2 e^{-\frac{1}{2}\mu^2 t^2} \left(\mu t - 3\frac{H}{\mu}\right)$$





Evolution of:

θ (angle of the trajectory)

θ_p (angle of the light direction)

Evolution of $\psi = \theta - \theta_p$

(angle between trajectory & light direction)

→ tiny deviation

Soft turn ($\mu << m_h$)

Soft turn ($\mu << m_h$):

- If the turn is soft, the trajectory tightly follows the light direction, with tiny deviation just around the turning point.

- The "softer" the turn is, the closer the background trajectory is to the light direction.

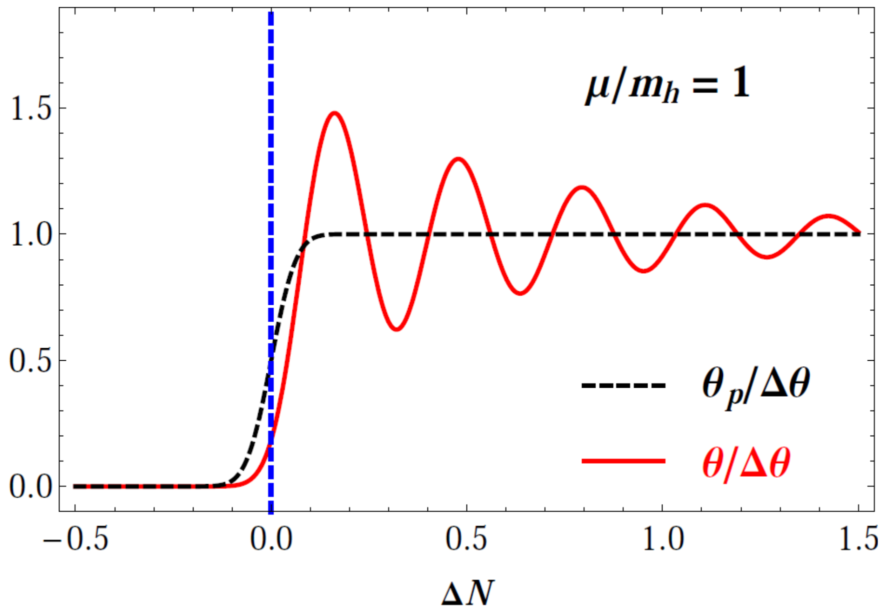
- After the turn, the trajectory soon relaxes and re-coincides with the light direction.

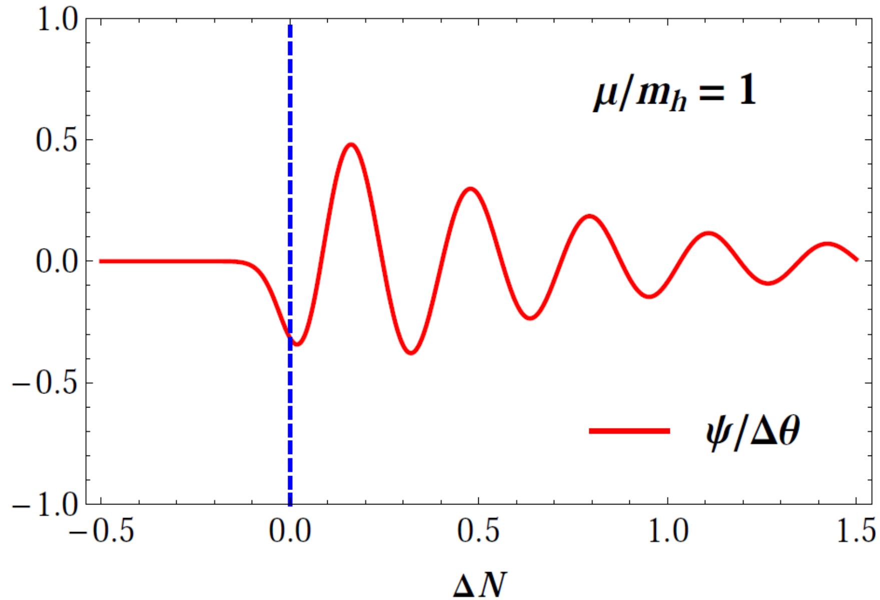
- There is no explicit oscillation of the trajectory.

- The adiabatic/entropic modes are approximately the light/heavy modes.

Sharp turn ($\mu >\approx m_h$)

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \mathrm{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos\left(m_h t - \text{phase}\right)$$





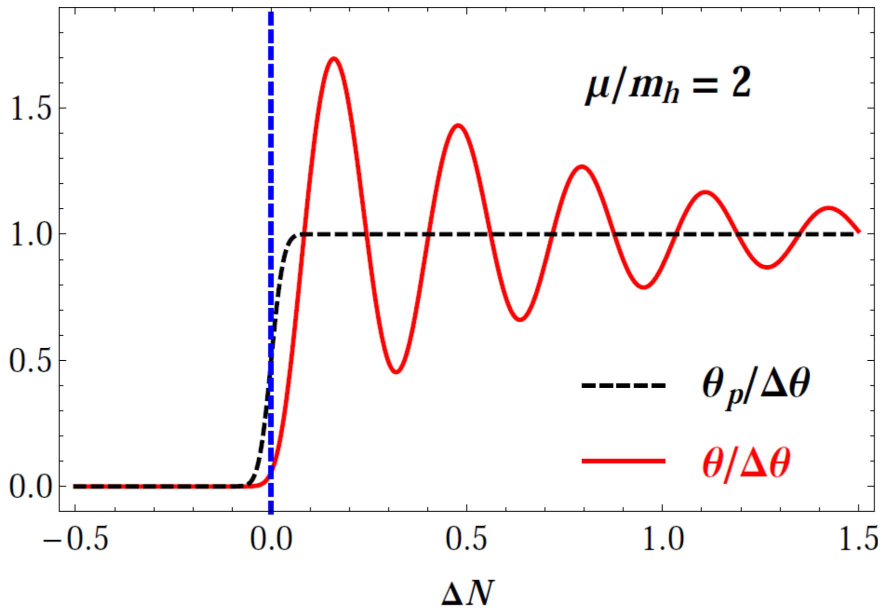
Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

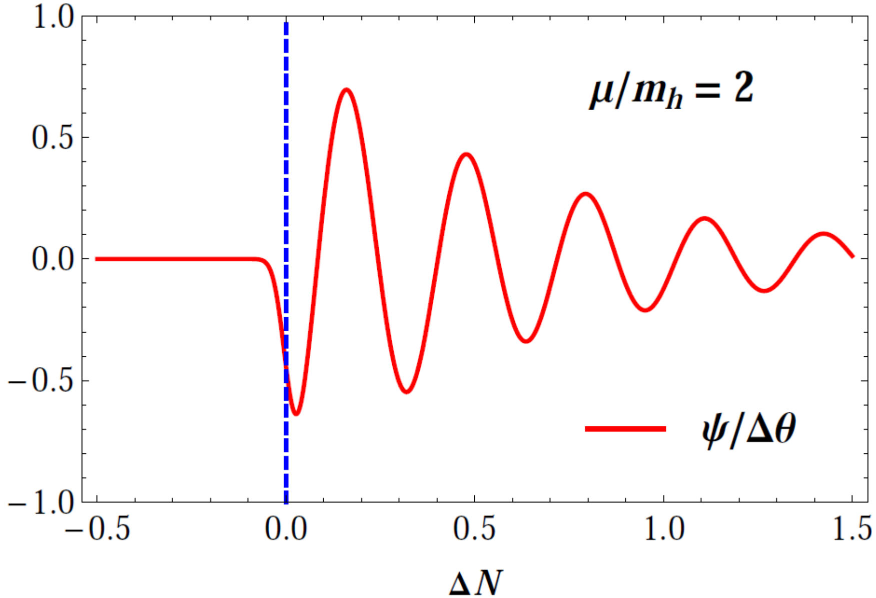
Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

→ large deviation with oscillation

Sharp turn ($\mu >\approx m_h$)

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \mathrm{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos\left(m_h t - \mathrm{phase}\right)$$





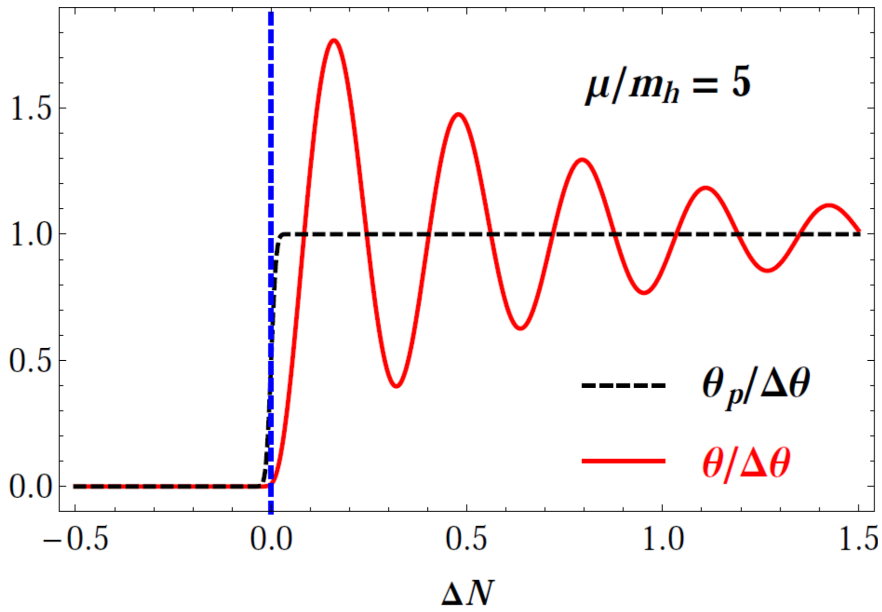
Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

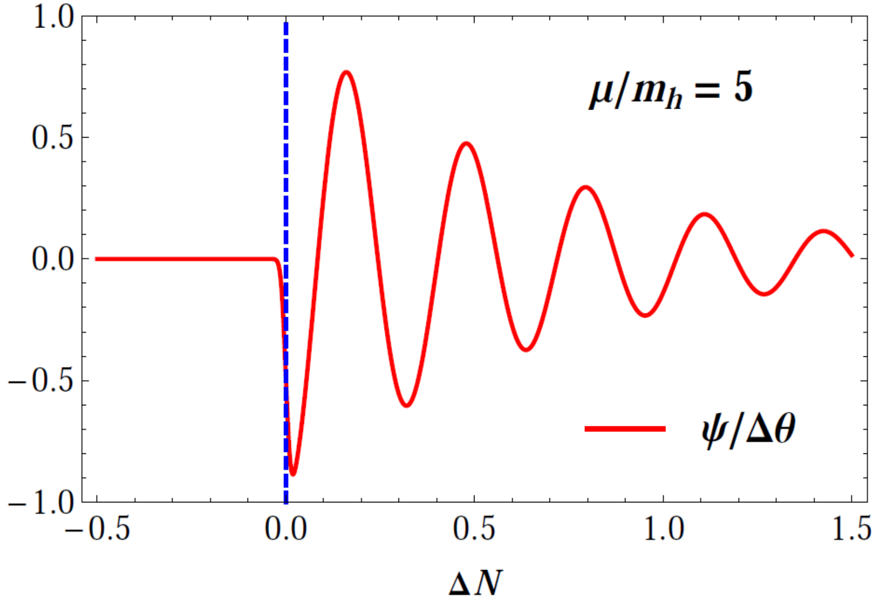
Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

→ **large deviation with oscillation**

Sharp turn (μ>≈m_h)

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \mathrm{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos\left(m_h t - \text{phase}\right)$$





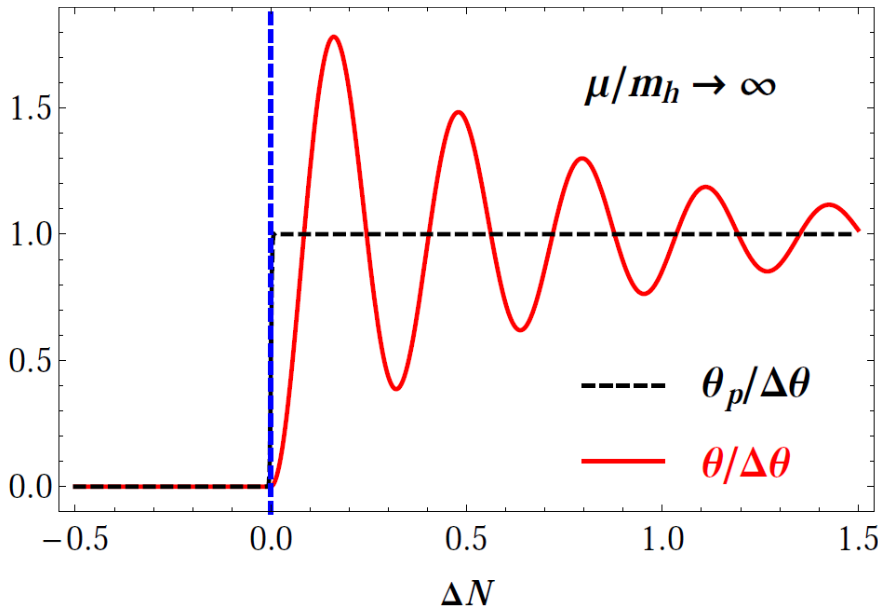
Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

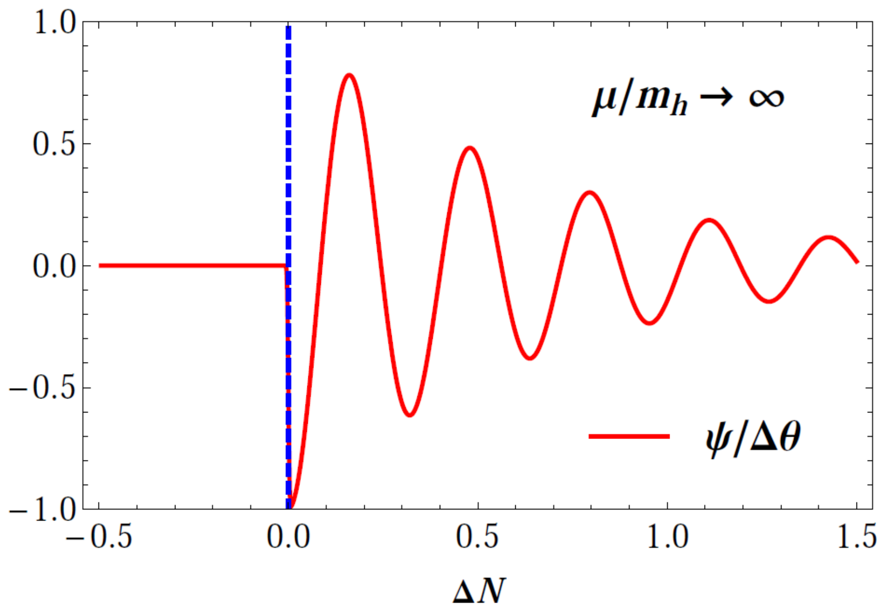
Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

→ **large deviation with oscillation**

Sharp turn (μ>≈m_h)

$$\psi(t) \approx -\frac{\Delta\theta}{2} e^{-\frac{m_h^2}{2\mu^2}} \mathrm{erfc}\left(-\frac{\mu t}{\sqrt{2}}\right) e^{-\frac{3}{2}Ht} \cos\left(m_h t - \text{phase}\right)$$





Evolution of:
θ (angle of the trajectory)
θ_p (angle of the light direction)

Evolution of $\psi=\theta-\theta_p$
(angle between trajectory & light direction)

→ large deviation with oscillation

Sharp turn ($\mu>\approx m_h$)

Sharp turn ($\mu>\approx m_h$):

• Soon *after* the sharp turn, the trajectory starts to oscillate, with considerable amplitude.

• The adiabatic/entropic-basis rapidly rotates.

• The adiabatic/entropic modes get rapidly mixed with light/heavy modes.

• The adibatic (curvature) mode has not necessarily to be light, which can be temporarily heavy around the turn.
[*Achucarro, Gong, Hardeman, Palma, Patil, '10. Shiu & Xu, '11*]

Oscillatory background during a sharp turn

When the turn is sharp, the oscillating trajectory will induce oscillatory parts in background quantities (*a*, *H* etc).

Deviation from the smooth value: $a = \bar{a} + \Delta a, \quad H = \bar{H} + \Delta H, \quad \epsilon = \bar{\epsilon} + \Delta\epsilon$

An equation of motion for $\Delta\epsilon$

$$\frac{d^2 \Delta\epsilon}{dt^2} + 3\bar{H}\frac{d\Delta\epsilon}{dt} - 12\bar{\epsilon}\bar{H}^2\Delta\epsilon = 2\bar{\epsilon}\left[\left(\dot{\theta}_p + \dot{\psi}\right)^2 - \hat{m}_h^2 \sin^2\psi\right]$$

Infinitely sharp turn limit (μ→∞):

$$\dot{\theta}_p = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 t^2} \xrightarrow{\mu\to\infty} \dot{\theta}_p = \Delta\theta\delta\left(t\right)$$

$$\psi\left(t\right) \approx -\Theta\left(t\right)\Delta\theta e^{-\frac{3}{2}\bar{H}t}\cos\left(\hat{m}_h t\right)$$

$$\boxed{\Delta\epsilon \approx \frac{\Theta\left(t\right)}{2}\bar{\epsilon}(\Delta\theta)^2 e^{-3\bar{H}t}\cos\left(2\hat{m}_h t\right) + \text{non-osci}}$$

Perturbations

Adiabatic / entropic v.s. light / heavy

Two possible decompositions:

Adiabatic / entropic decomposition → **kinematic** features
[Gordon, Wands, Bassett & Maartens '00, Groot Nibbelink & van Tent '01]

Light / heavy decomposition → **potential** features
[Gao, Langlois, Mizuno, '12, '13]

- Adiabatic/entropic decomposition has special advantage, since the adiabatic mode is directly related to the curvature perturbation.

- Light/heavy decomposition is directly related with the shape of the inflationary potential, which is (sometimes) more robust and simpler.

- **The final spectra for the curvature perturbation:**

  $u_\sigma = \cos\psi u_l + \sin\psi u_h$ after the turn $\psi \to 0, \quad u_\sigma \simeq u_l$

Two effects

Deviation from the single-field slow-roll (SFSL):

$$\begin{aligned}\mathcal{L} &= \mathcal{L}\left(\theta_m, a\right) \\ &= \mathcal{L}\left(\theta_m, \bar{a} + \Delta a\right) \\ &= \mathcal{L}_0\left(0, \bar{a}\right) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})}\left(\theta_m, \bar{a}\right) + \mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})}\left(0, \Delta a\right)\end{aligned}$$

"Free" part (SFSL limit):

$$\mathcal{L}_0^{\mathrm{l,h}} = \frac{1}{2}\left[u_{l,h}'^2 - \left(\partial u_{l,h}\right)^2 - \left(\bar{a}^2 m_{l,h}^2 - \bar{a}^2 \bar{H}^2\left(2-\bar{\epsilon}\right)\right) u_{l,h}^2\right]$$

$\bar{H}$ and $\bar{\epsilon}$ are evaluated by $\bar{a}$.

"Interaction" part (deviation from SFSL):

Effects 1: bending light direction (potential trough)

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{turn})} = \frac{1}{2}{\theta_m}'^2 u_l^2 + \frac{1}{2}{\theta_m}'^2 u_h^2 + 2{\theta_m}' u_l u_h' + {\theta_m}'' u_l u_h$$

Effects 2: oscillatory background

$$\mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})} = -\frac{1}{2}\left[\left(\Delta a\right)^2 m_{l,h}^2 - \Delta\left(a^2 H^2\left(2-\epsilon\right)\right)\right] u_{l,h}^2$$

Effects (I):
Contributions from the turning light direction

Perturbation equations

$$u_l'' + \left(k^2 + \bar{a}^2 m_l^2 - {\theta_m}'^2 - \frac{\bar{a}''}{a}\right) u_l = {\theta_m}'' u_h + 2{\theta_m}' u_h',$$

$$u_h'' + \left(k^2 + \bar{a}^2 m_h^2 - {\theta_m}'^2 - \frac{\bar{a}''}{a}\right) u_h = -{\theta_m}'' u_l - 2{\theta_m}' u_l'.$$

$\bar{a}$ is the smooth part of the scale factor,

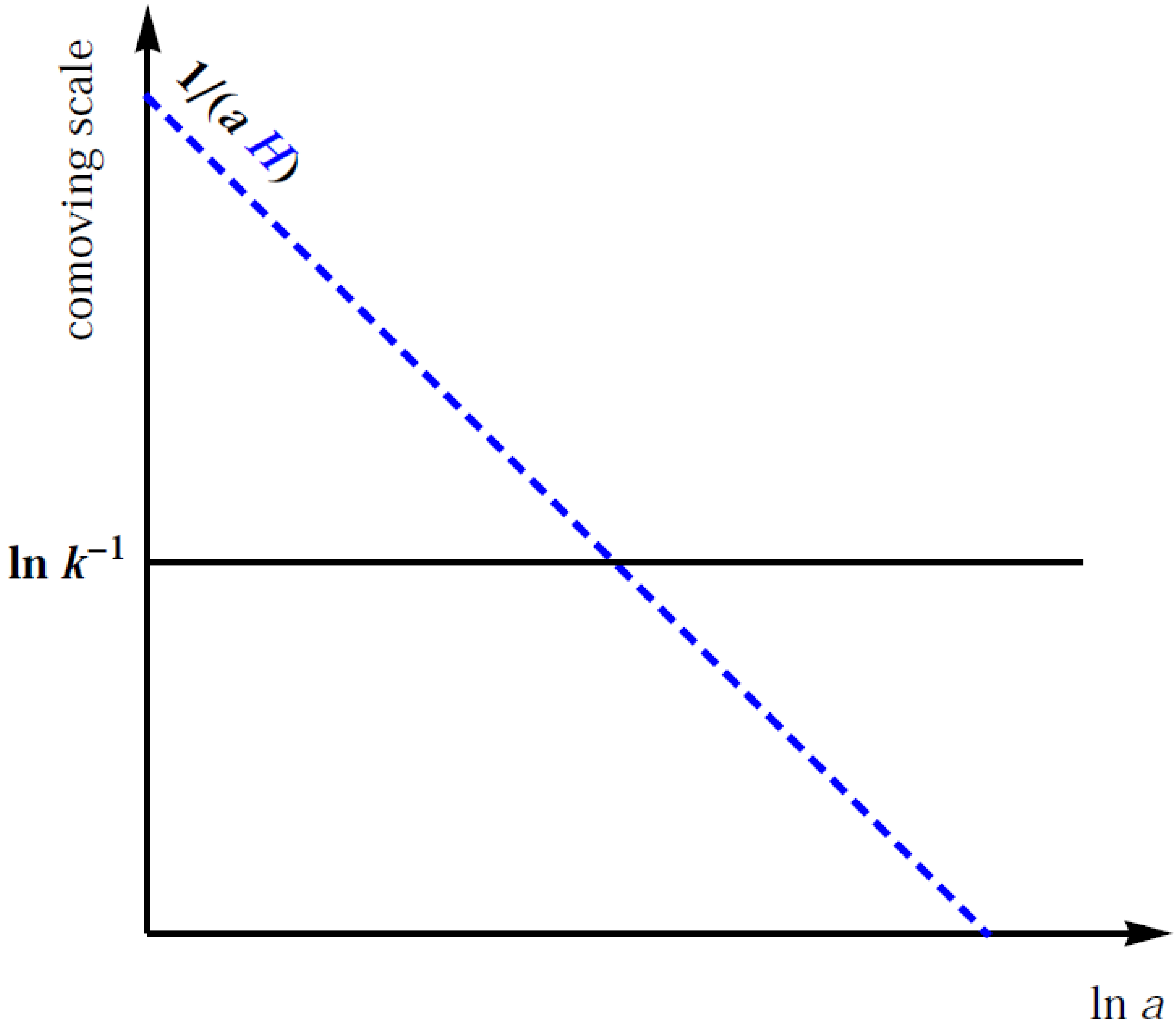
θ_m' is the turning rate of the light direction, which is roughly the turning rate of the potential trough.

Gaussian ansatz:

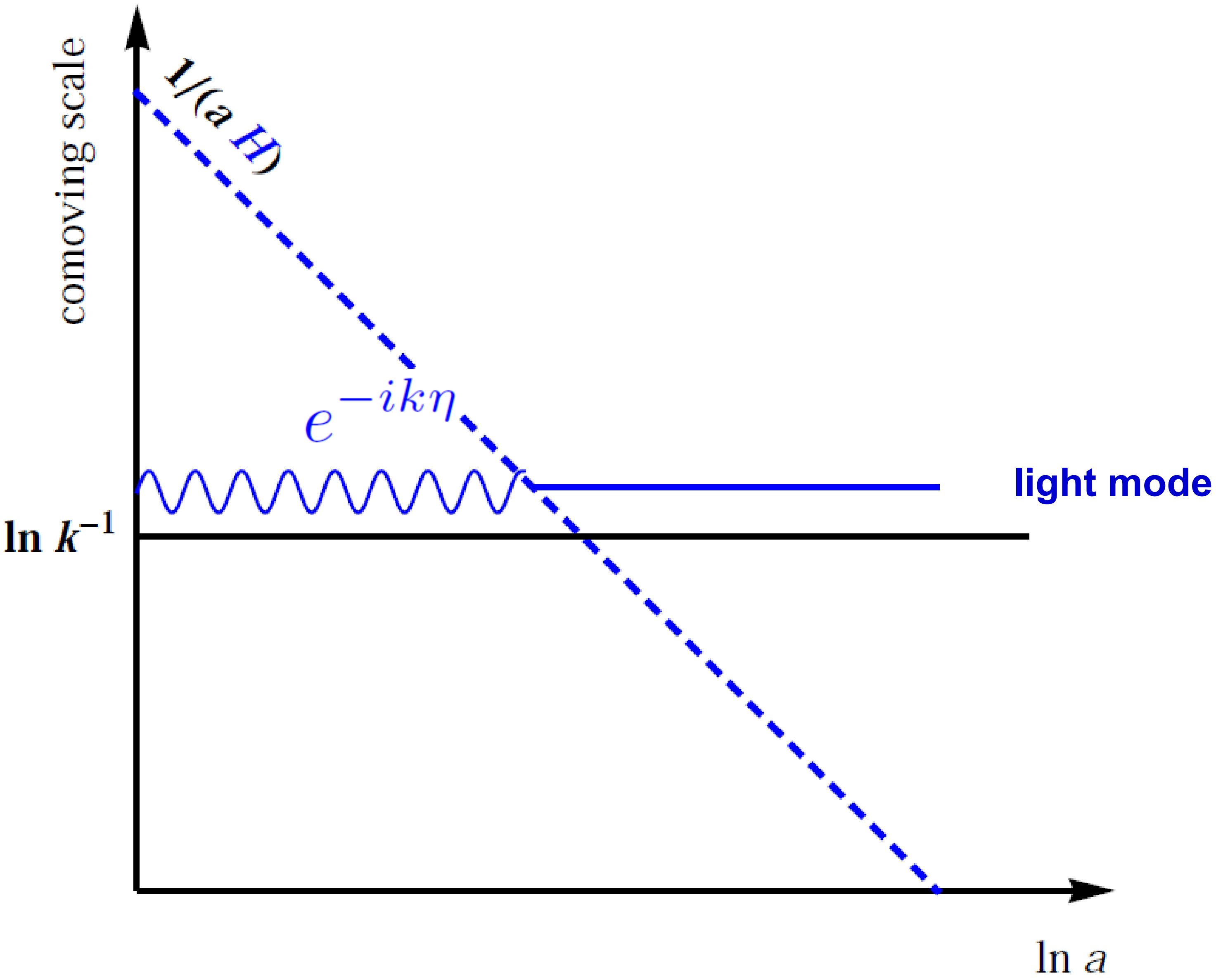
$$\dot{\theta}_m(t) \approx \dot{\theta}_p(t) = \Delta\theta \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2 (t-t_*)^2}$$

We will solve both the **full two-field system** as well as the **effective single light-field theory**.

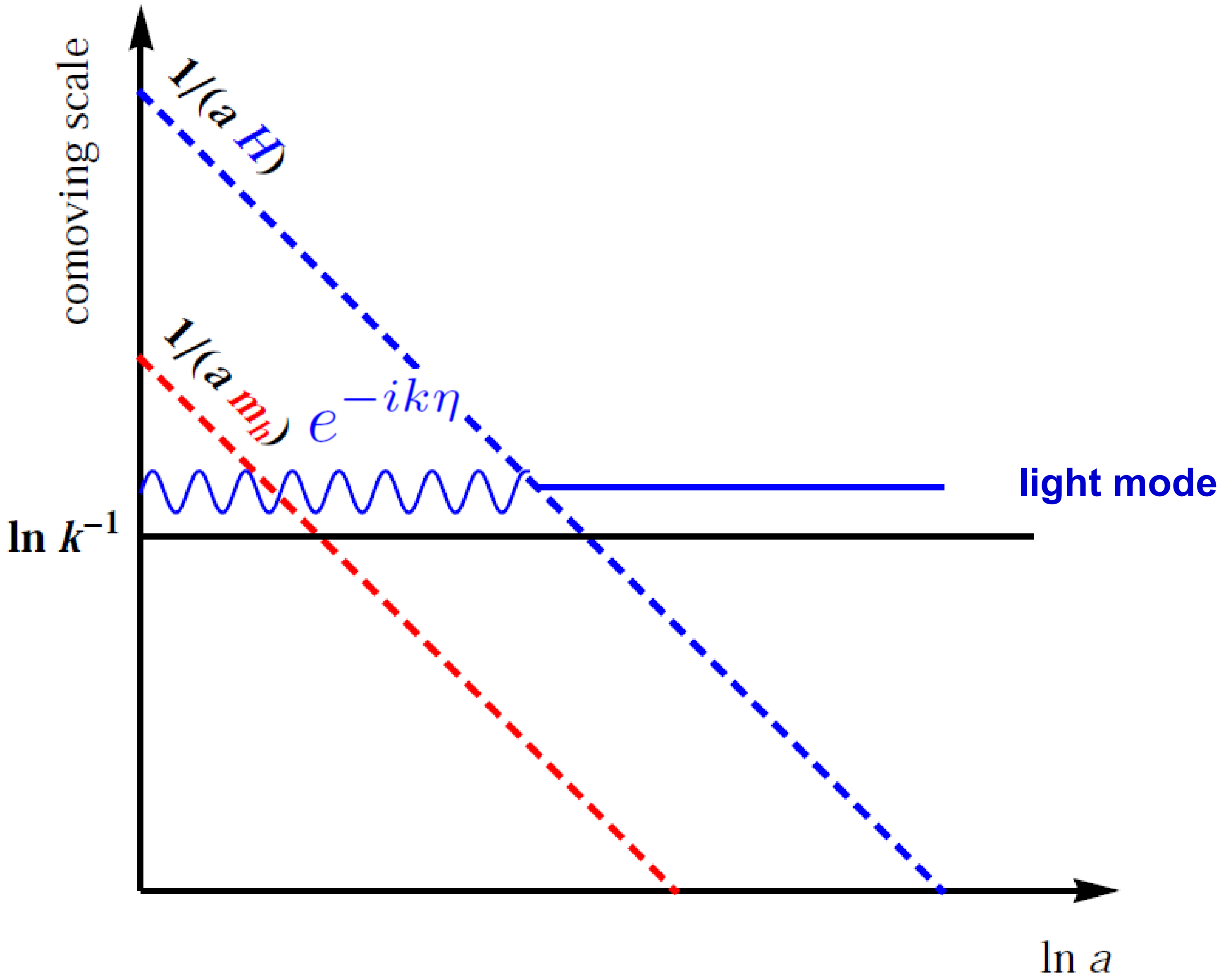
Characteristic scales



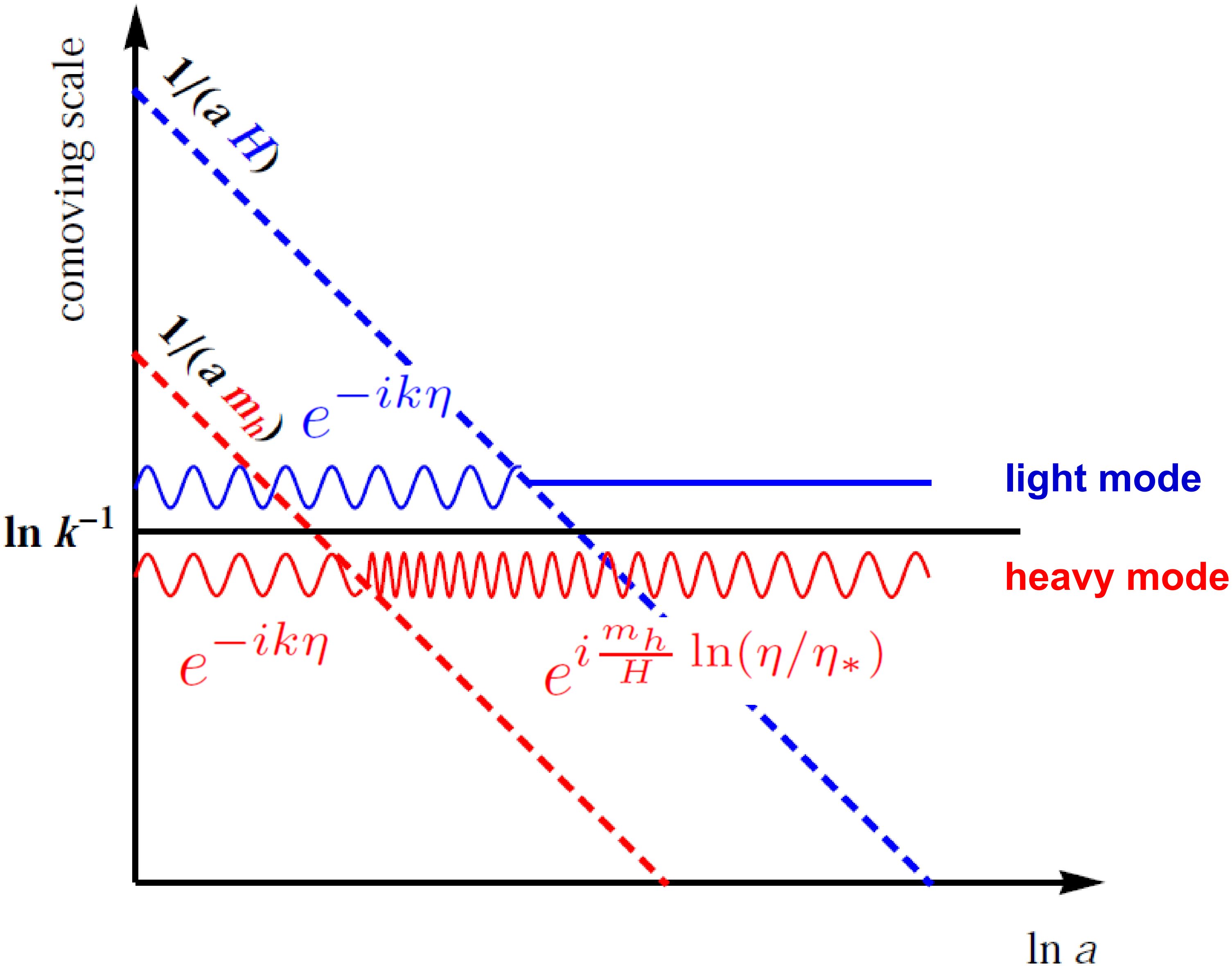
Characteristic scales



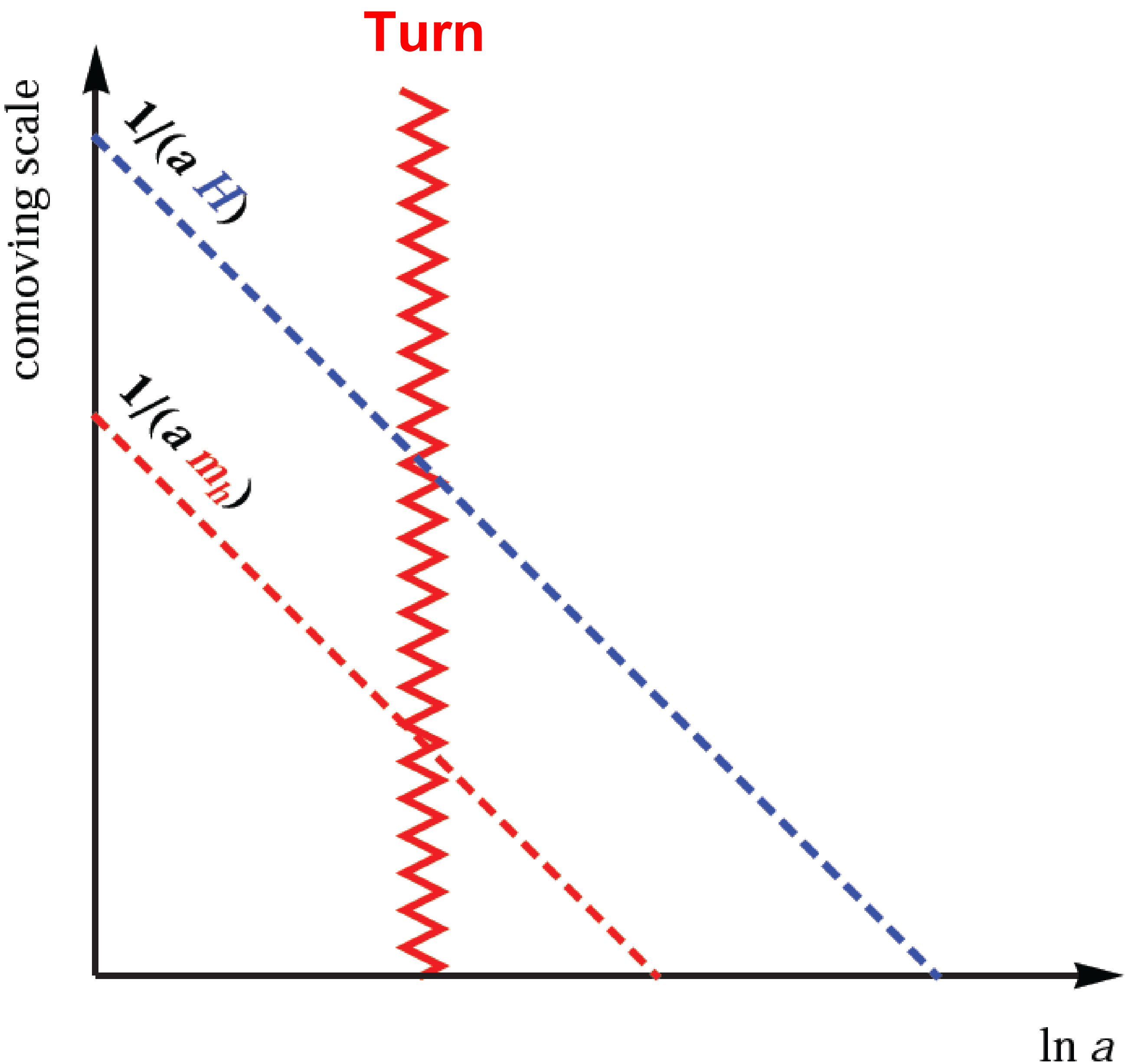
Characteristic scales



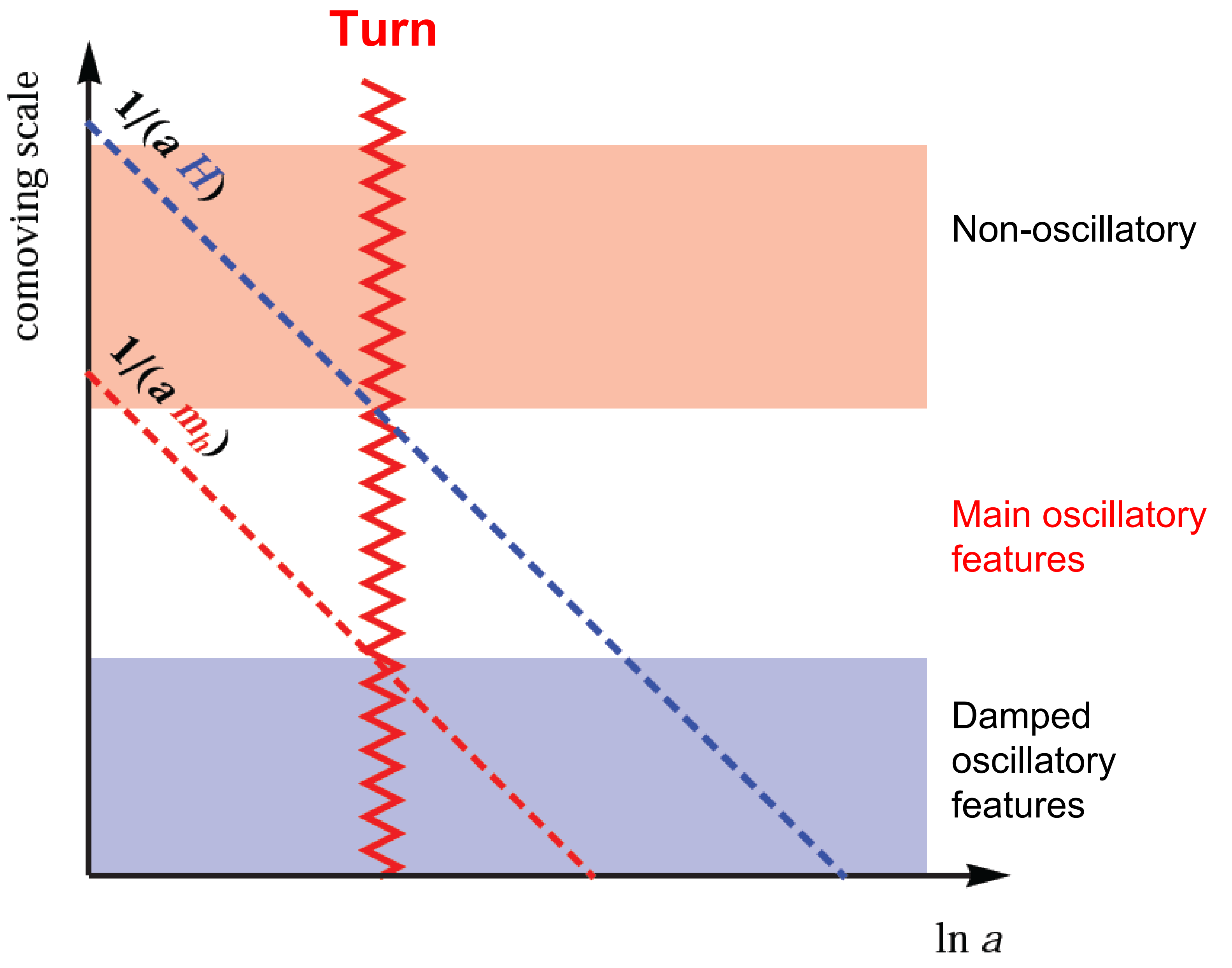
Characteristic scales



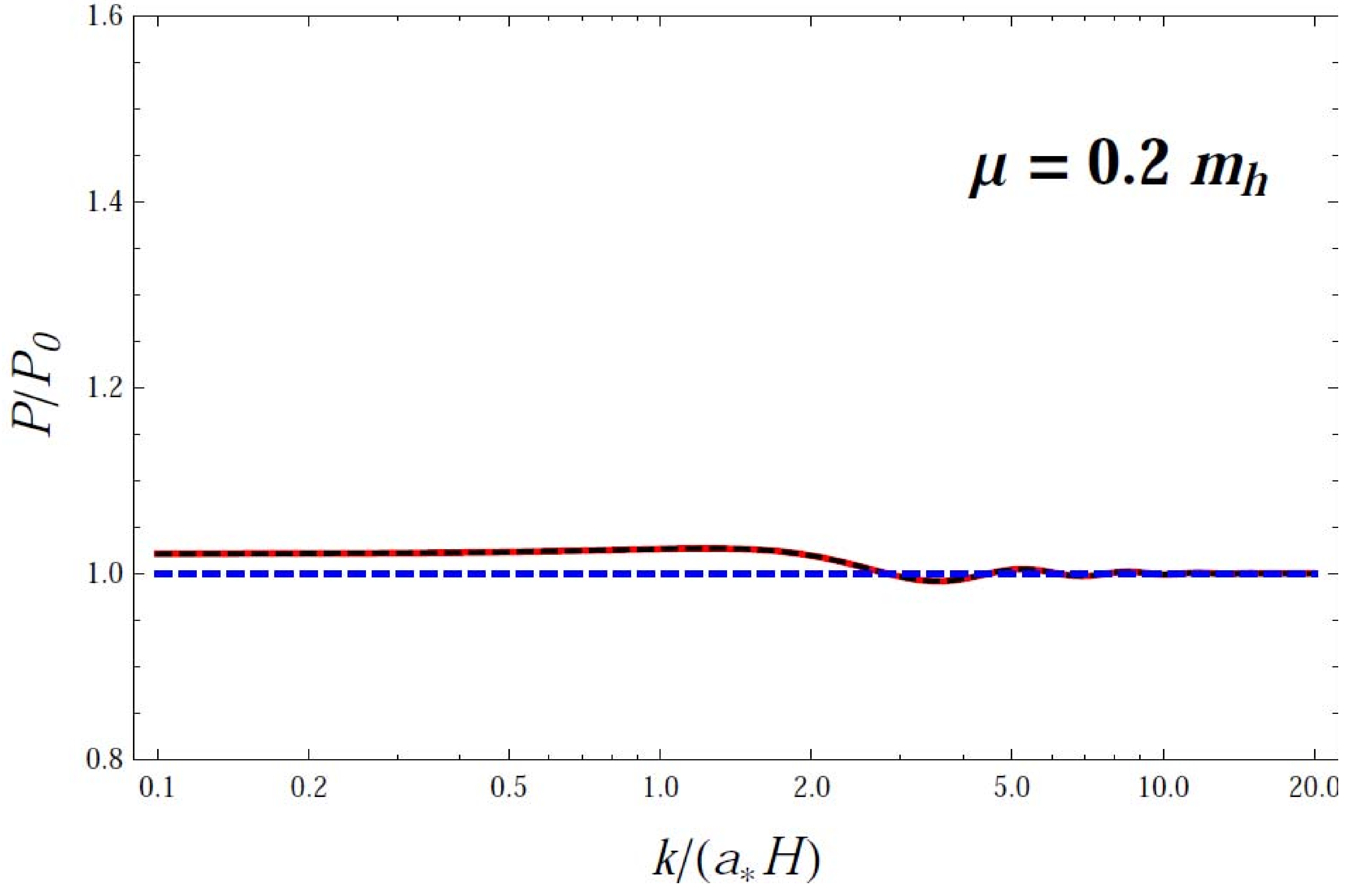
Characteristic scales



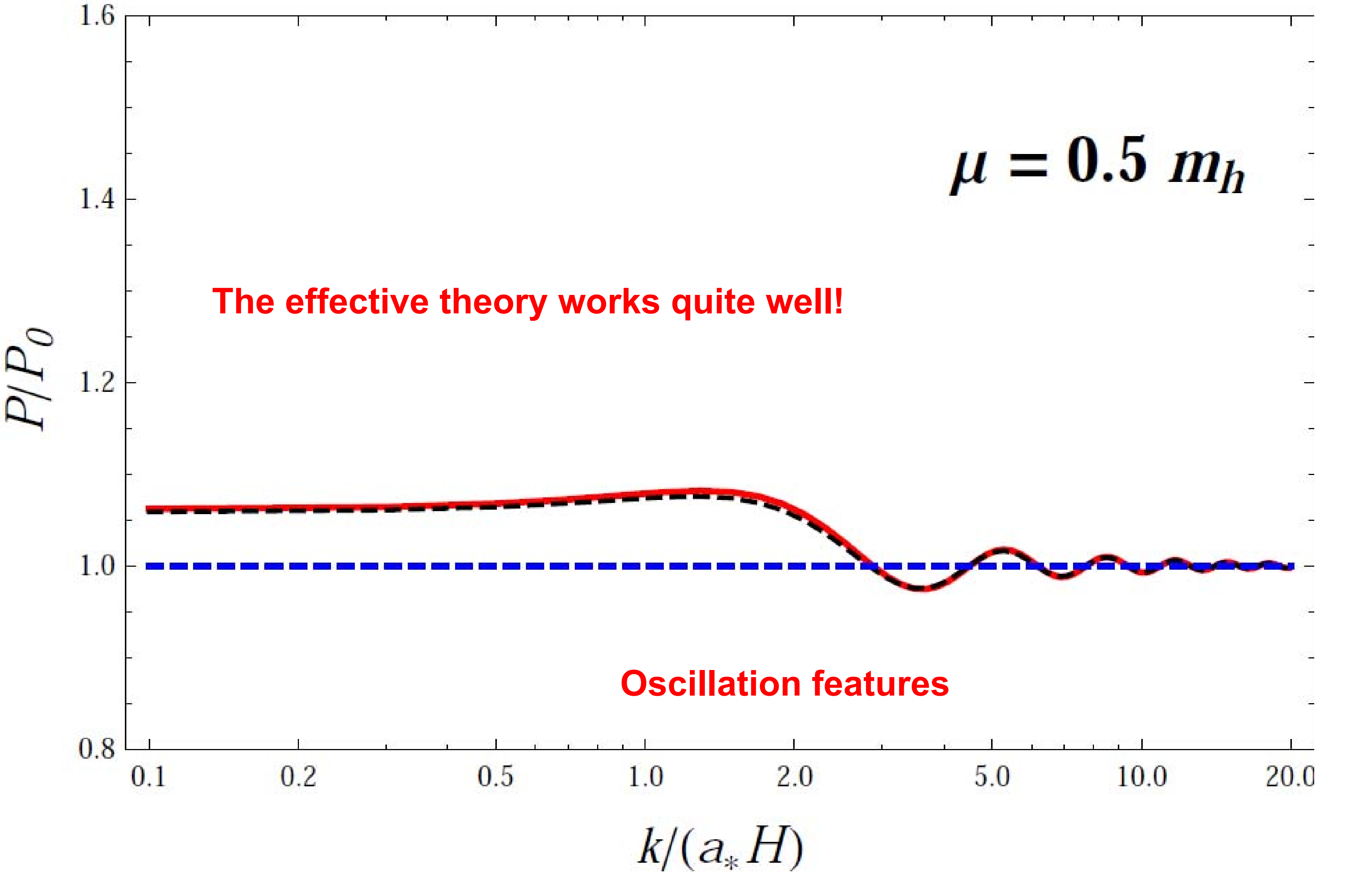
Characteristic scales



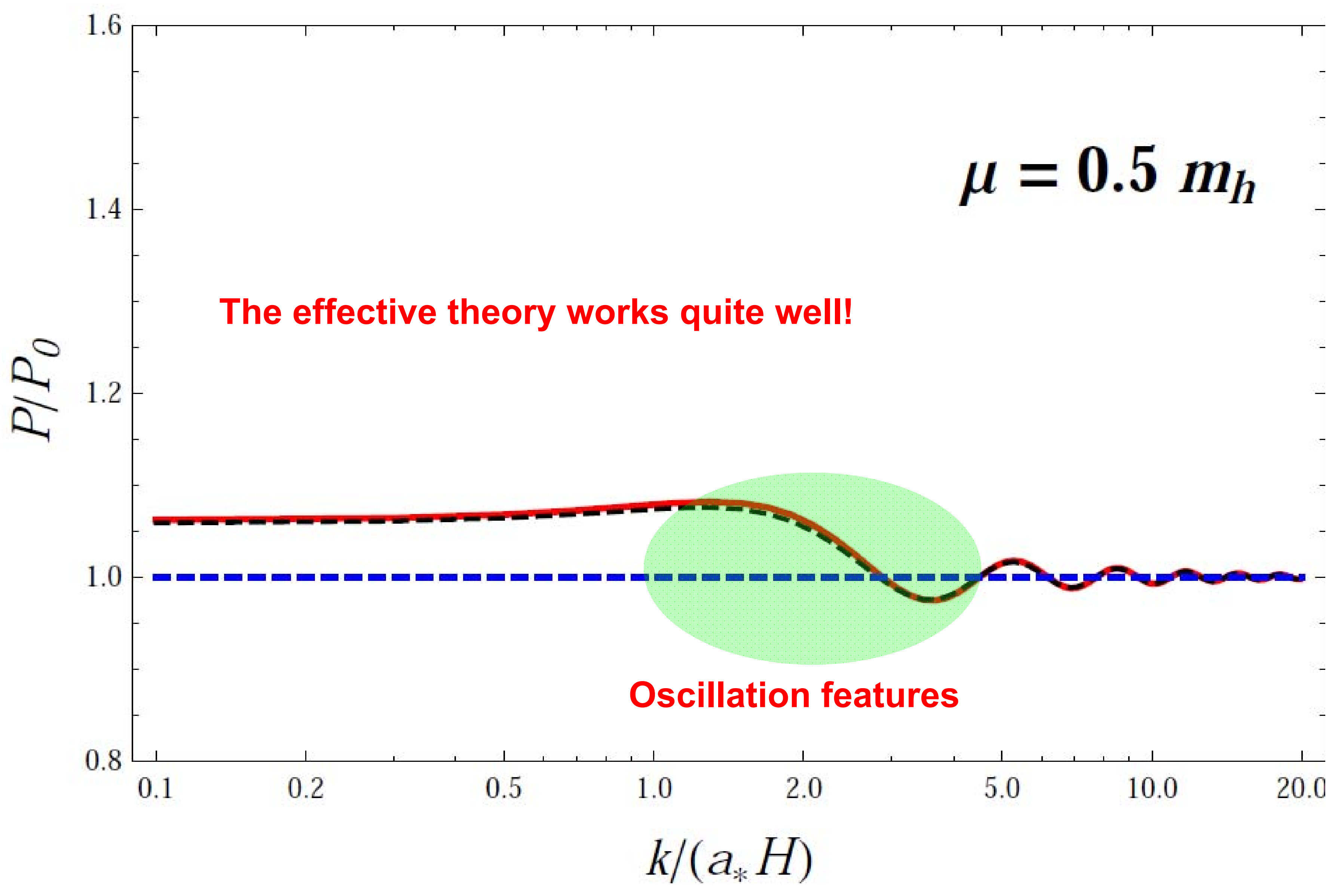
Numerics: fixed m_h



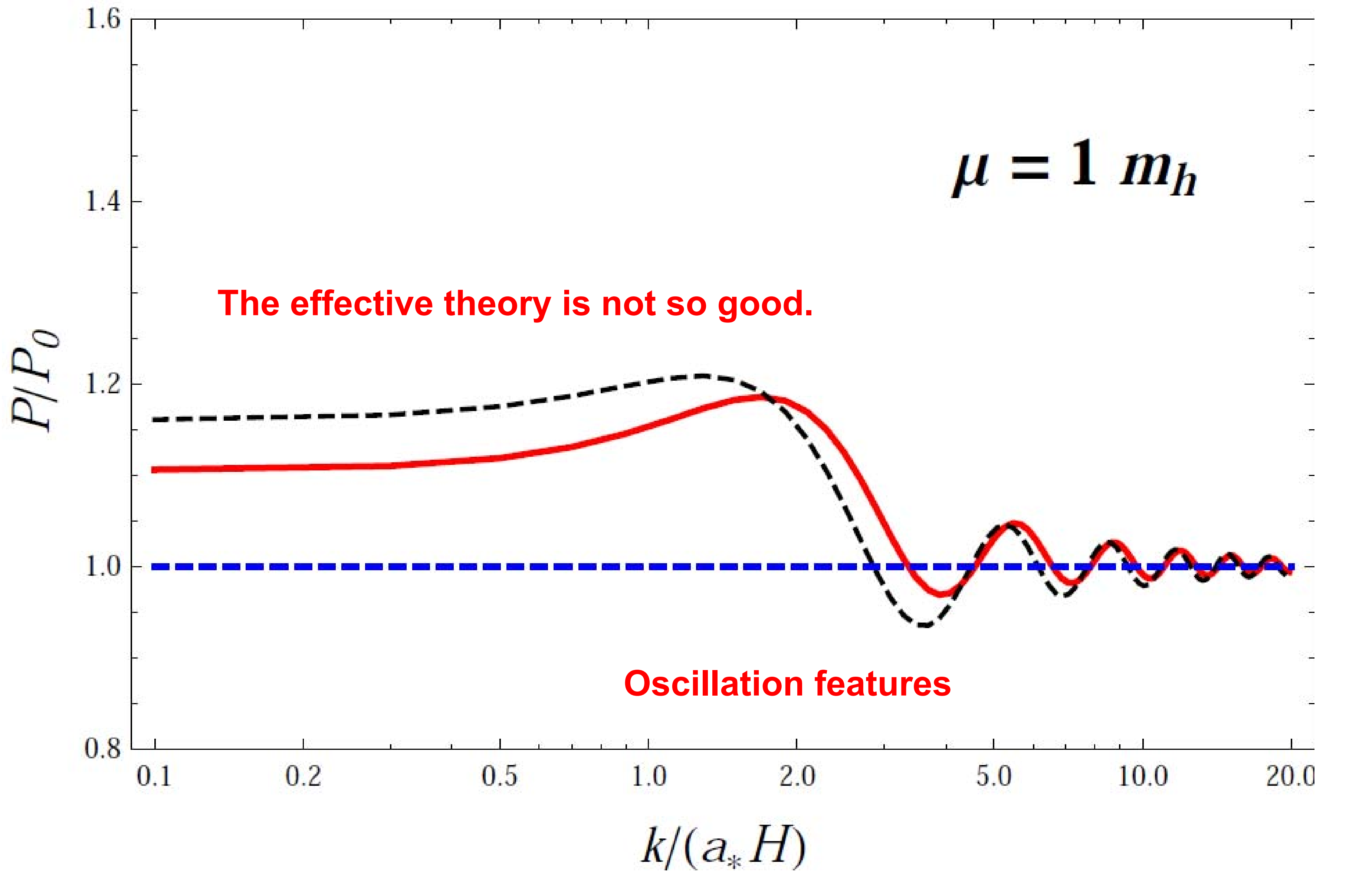
Numerics: fixed m_h



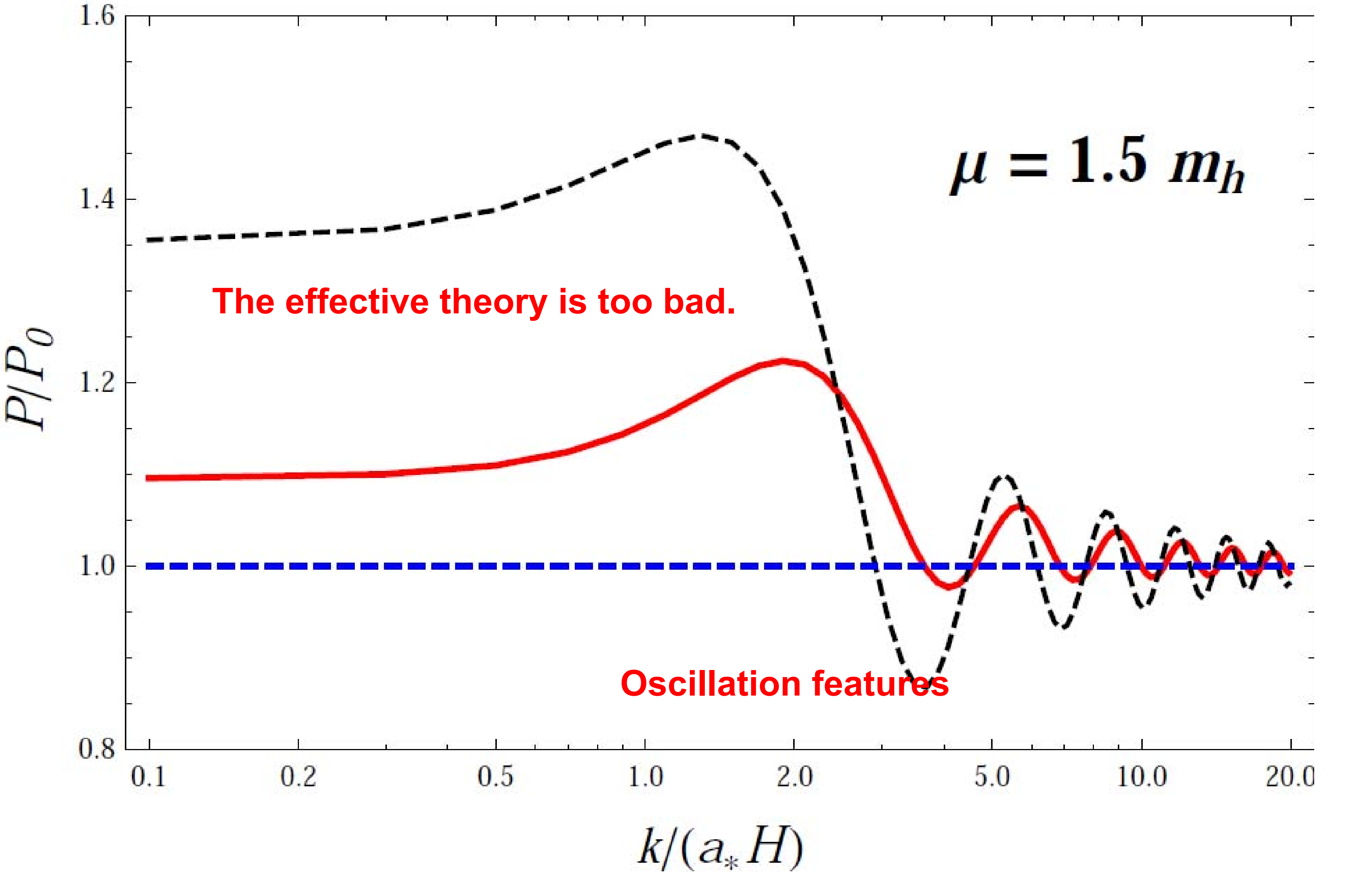
Numerics: fixed m_h



Numerics: fixed m_h



Numerics: fixed m_h



Effects (II):
Resonance from the oscillatory background

Resonance

For the light mode:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{I}}^{(\mathrm{resonance})} &= -\frac{1}{2}\left[(\Delta a)^2 m_l^2 - \Delta\left(a^2 H^2 (2-\epsilon)\right)\right] u_l^2 \\
&\simeq \frac{1}{2}\Delta\left(a^2 H^2 (2-\epsilon)\right) u_l^2 \\
&\simeq -\frac{1}{2}\bar{a}^2 \bar{H}^2 (\Delta\epsilon)_{\mathrm{osci}} u_l^2
\end{aligned}
$$

In the infinitely sharp turn limit, we have solved:

$$
\Delta\epsilon \approx \frac{\Theta(t)}{2}\bar{\epsilon}(\Delta\theta)^2 e^{-3\bar{H}t}\cos\left(2\hat{m}_h t\right) + \text{non-osci}
$$

An oscillation in background periodic in cosmic time *t* will induce resonance effect, which is period in (ln *k*), in the spectrum of perturbation.
[Chen '11, '12]

Resonance

Contribution to the spectrum of the light mode:

$$\left(\frac{\Delta P}{P}\right)_{\mathrm{res}} \approx \Theta\left(\frac{k}{a_* m_h} - 1\right) \frac{\sqrt{\pi}}{4} \bar{\epsilon}\,(\Delta\theta)^2 \left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}}$$
$$\times \left(\frac{a_* m_h}{k}\right)^3 \cos\left[2\frac{m_h}{\bar{H}} \ln\left(\frac{k}{a_* m_h}\right) + 2\frac{m_h}{\bar{H}} - \frac{\pi}{4}\right].$$

- The oscillation is periodic in **ln *k***, with frequency $2m_h/\bar{H} \gg 1$.
- The resonance features manifest themselves only on very small length scales: $k > a_* m_h \gg a_* \bar{H}$
- The amplitude is rather small: $\bar{\epsilon}\,(\Delta\theta)^2 \left(\frac{\bar{H}}{m_h}\right)^{\frac{3}{2}} \ll 1$
- The amplitude is even suppressed on small scales: $\sim 1/k^3$

The resonance feature is subdominant with respect to the oscillatory feature caused by the bending trajectory.

Conclusion: main message from this talk

- Heavy field(s) may play a role in the early Universe.
- Light/heavy decomposition may be more convenient.
- Effective single-field description may not be valid.
- Sharp turn may produce oscillatory features in the spectra of light mode(s).

Thank you for your attention!