Re-interpreting kinematic variables @ the LHC searches

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Talk will be mostly based on
"Re-interpreting the Oxbridge stransverse mass variable MT2 in general cases" (arxiv:1212.1720)
"Cracking the dark matter code at the LHC." (arxiv:1206.1546)

- In this talk, I will focus on two very well-known variables in the new physics search at colliders.
- MT2
- Invariant mass

Understanding physics@LHC

- If we know the physics model (theory [=Lagrangian]) and want to determine parameters, we can directly compare Monte Carlo (MC) simulations to data

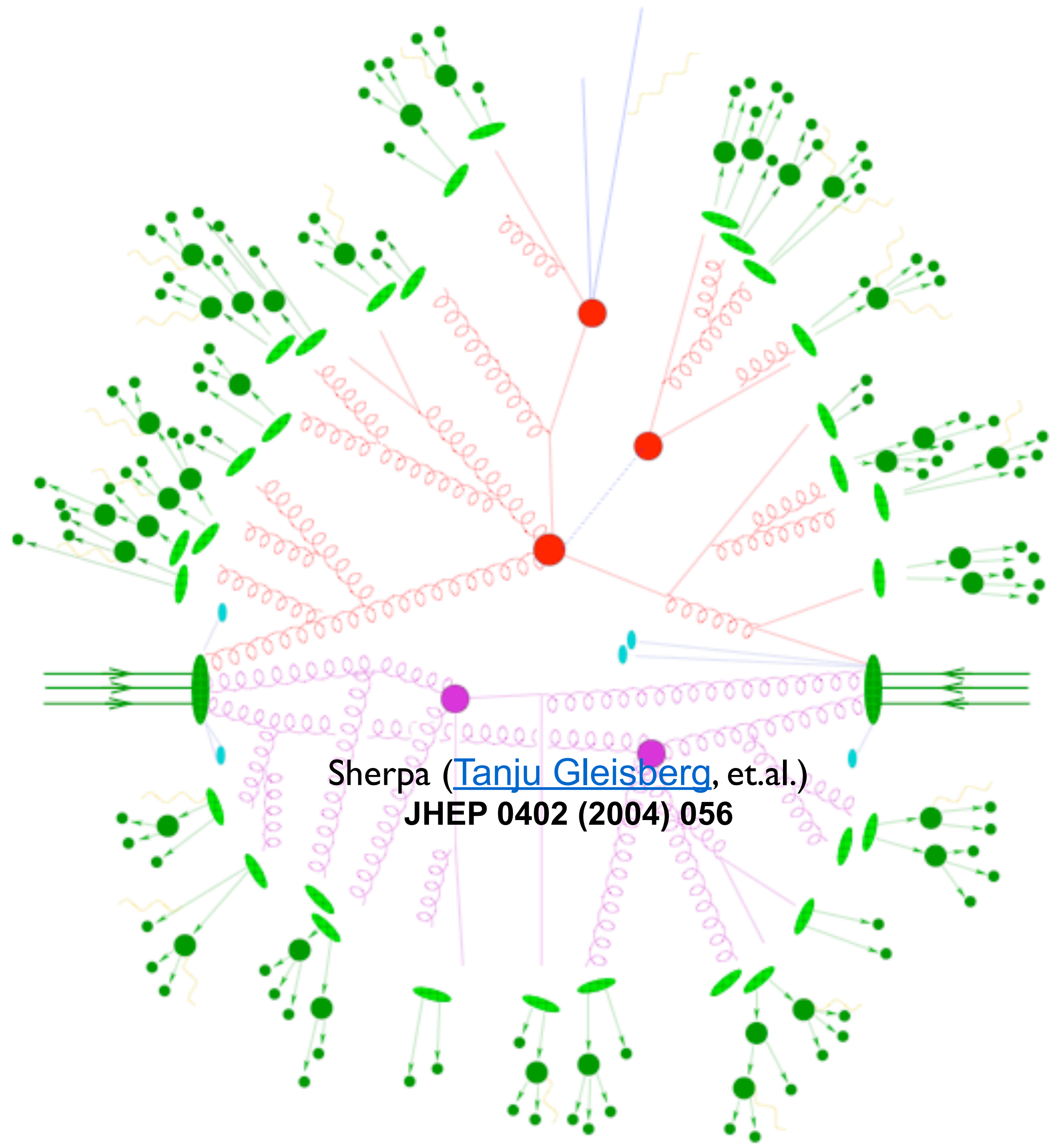
$$\mathcal{L} \rightarrow \mathrm{MC} \xleftrightarrow{\text{compare FITTING...}} \mathrm{DATA}\ \text{(LHC)}$$

tuning parameters

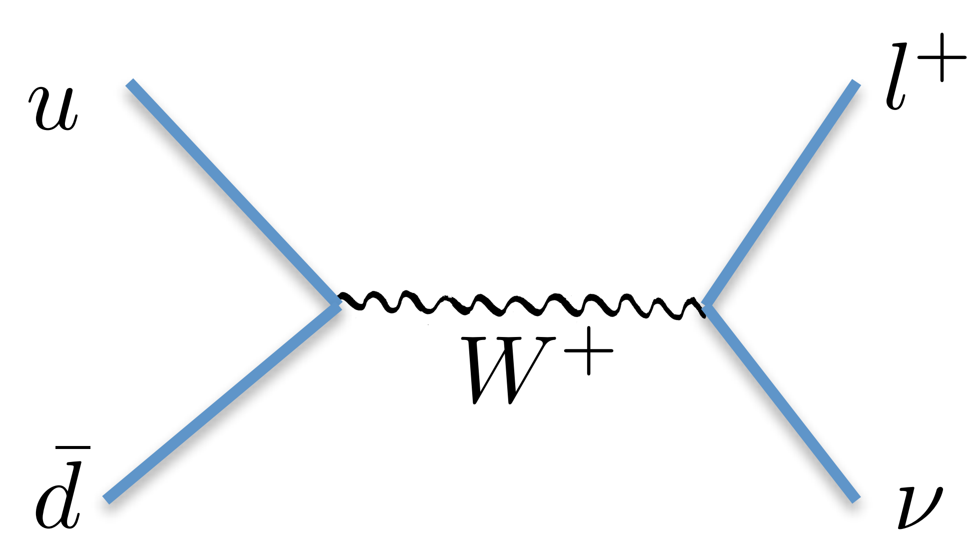
Parton level MC
(eg:MadGraph)

Hadronization
Radiation, ...
(eg: Pythia)

Detector effect
(eg: PGS)

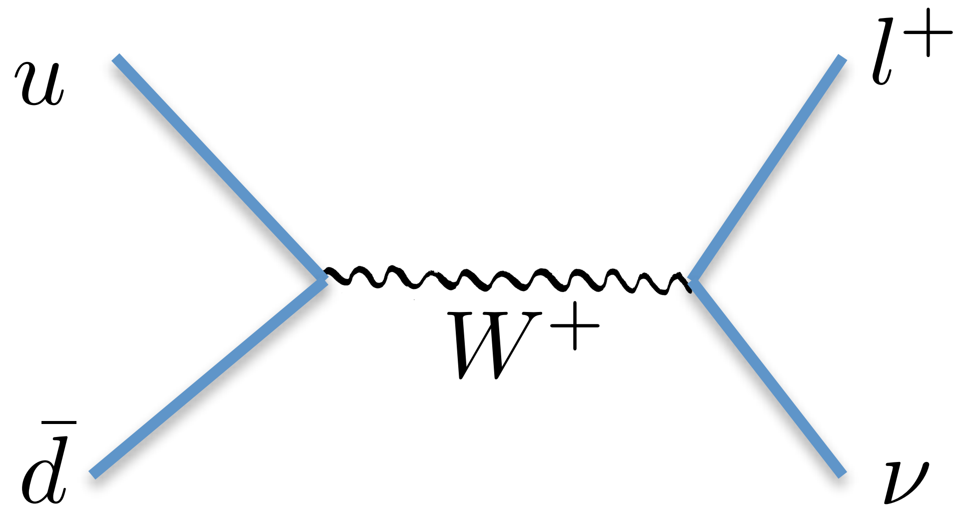


- Huge amount of community's efforts have been focused on MC to describe physics more precisely. (to remove systematic uncertainties from MC, DATA comparison)
- We need observables(histograms) to compare MC outputs with data.
- One example: W-boson mass measurement@Tevatron



At LO, W boson's transverse momentum (orthogonal to beam direction) ~ 0. Using the change of variable, we have a well-known Jacobian peak in leptons' PT distribution.

$$\frac{d\sigma}{dp_\perp} = \frac{d\cos\hat{\theta}}{dp_\perp}\frac{d\sigma}{d\cos\hat{\theta}} = \frac{p_\perp}{\sqrt{\left(\frac{M_W}{2}\right)^2 - p_\perp^2}}\frac{d\sigma}{d\cos\hat{\theta}}$$



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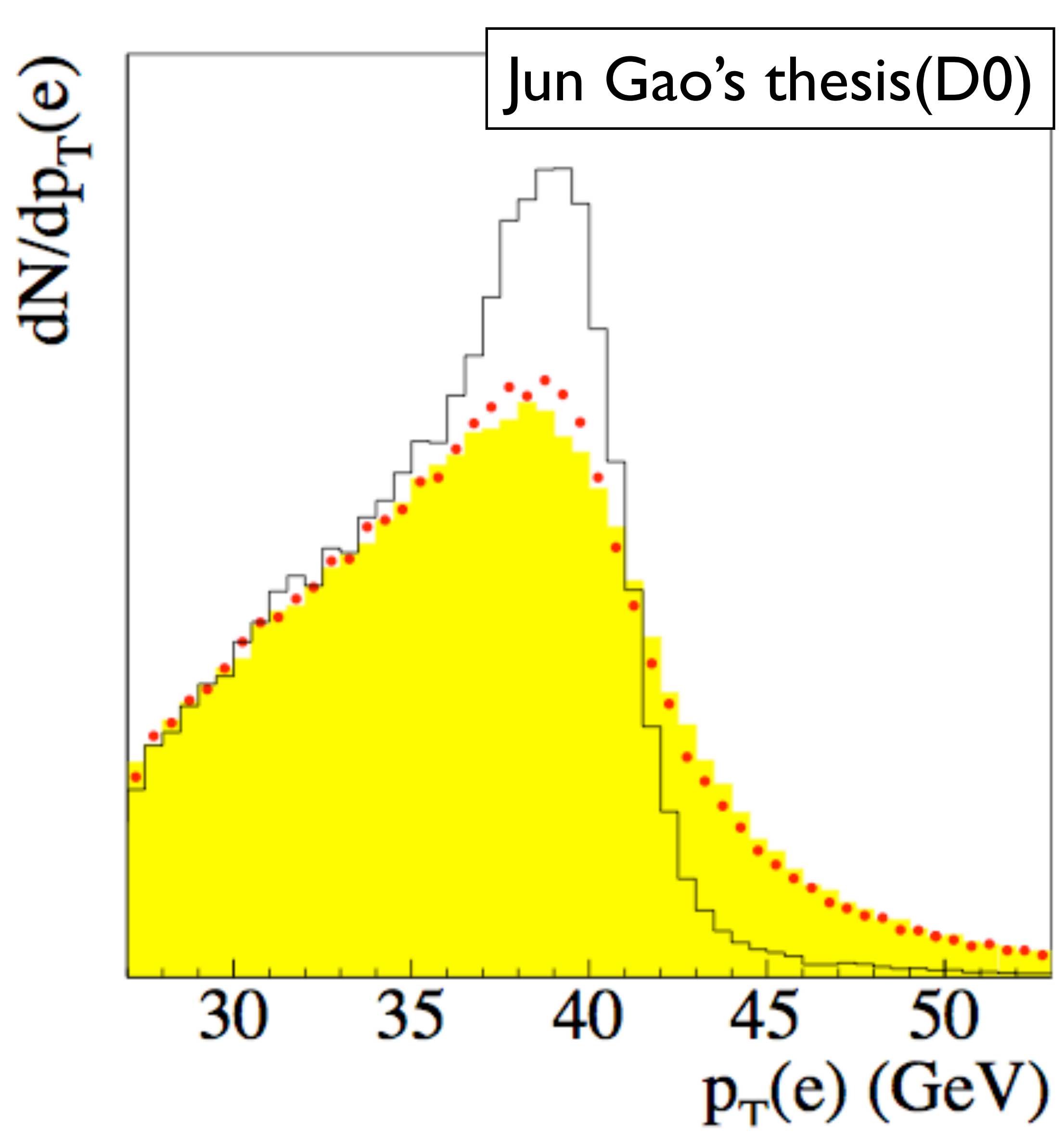
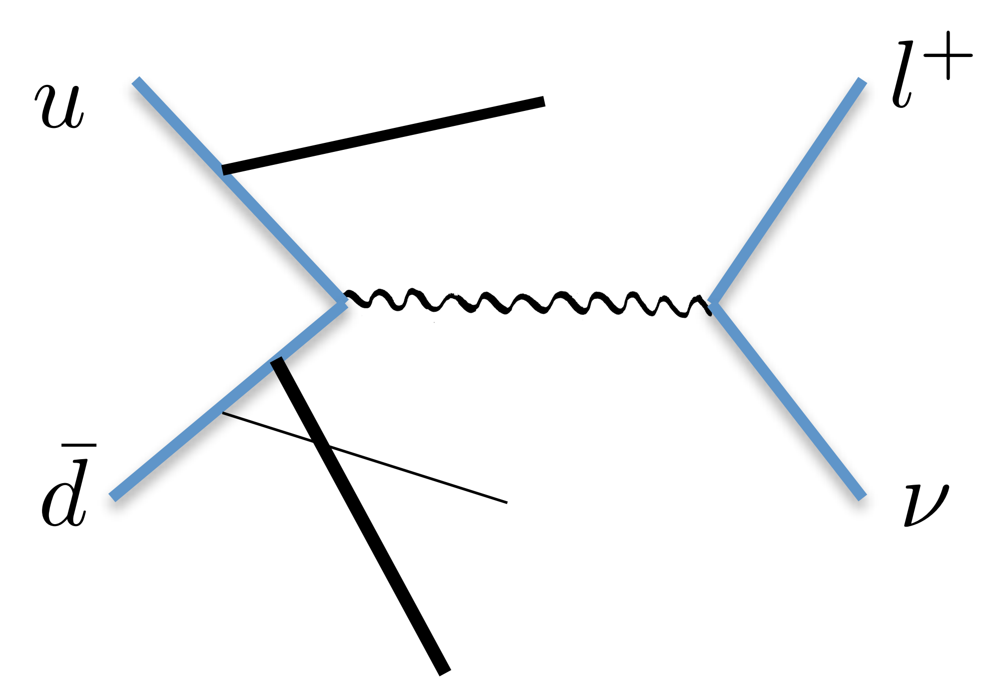
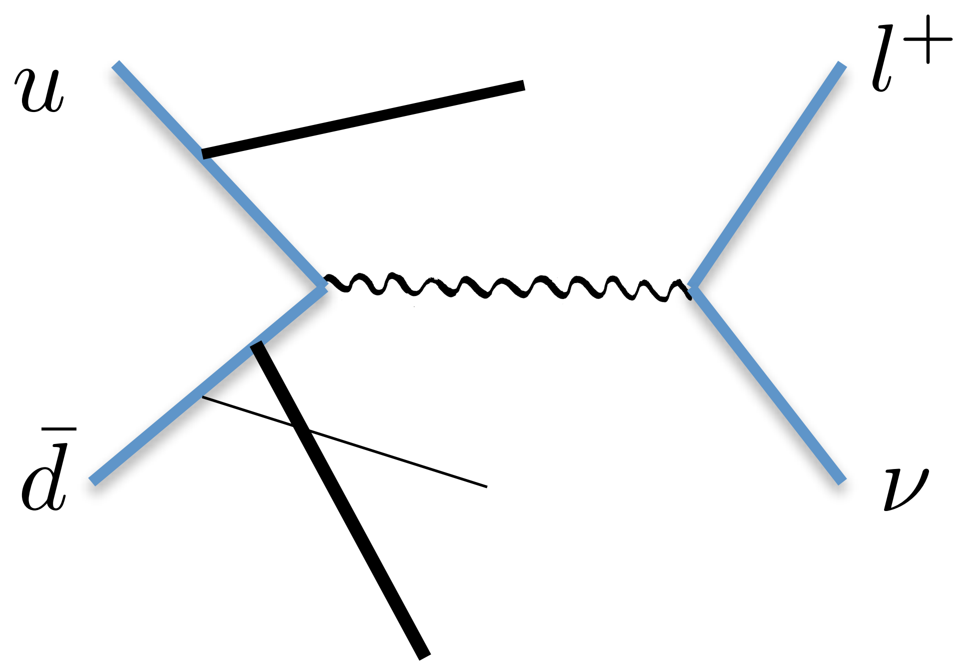


Black: Parton level [PT(W)=0]
RED: Parton level [PT(W) !=0]
Yellow: Detector level



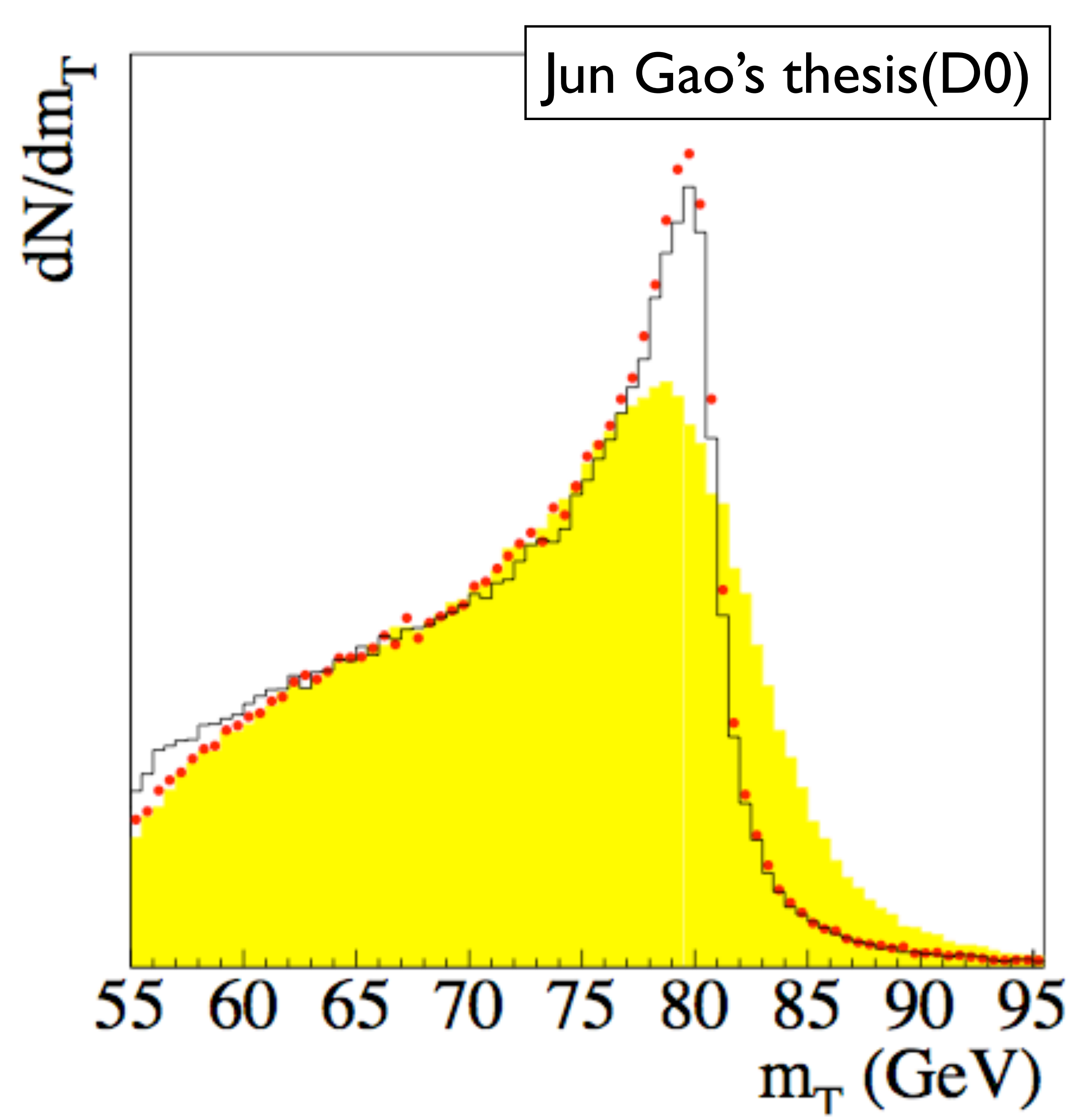
In reality, W boson will be kicked off by extra jets!
Thus, precise measurement will be highly dependent on the goodness of MC tools.
Option: We can make some special variable to remove effects from this effect.



Like as invariant mass is boost-invariant, a "transverse mass" will be invariant under the boost along transverse direction.

$$m_T^2 = (|\mathbf{p}_\perp^\nu| + |\mathbf{p}_\perp^\ell|)^2 - (\mathbf{p}_\perp^\nu + \mathbf{p}_\perp^\ell)^2$$

This variable is bounded by the mass of W boson, and have Jacobian peak just like lepton's PT distribution.



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What we learned from old days:

It is important to design observables that are strong under (complicated, uncontrolled) effects.

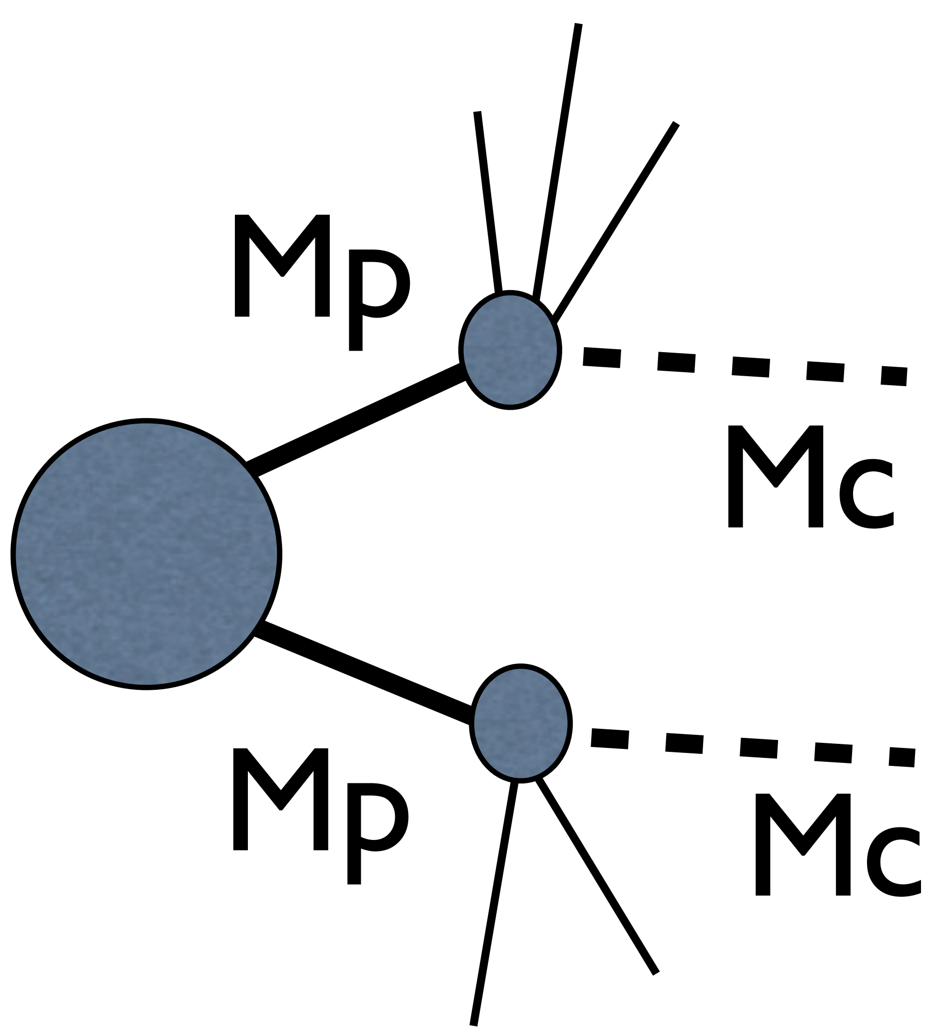
TABLE II: Systematic uncertainties of the M_W measurement.

	ΔM_W (MeV)		
Source	m_T	p_T^e	$\not{E}_T$
Electron energy calibration	34	34	34
Electron resolution model	2	2	3
Electron shower modeling	4	6	7
Electron energy loss model	4	4	4
Hadronic recoil model	6	12	20
Electron efficiencies	5	6	5
Backgrounds	2	5	4
Experimental Subtotal	35	37	41
PDF	10	11	11
QED	7	7	9
Boson p_T	2	5	2
Production Subtotal	12	14	14
Total	37	40	43

D0(arxiv:0908.0766)

Design variable for new physics search @LHC

- Example: Supersymmetry (RPC)

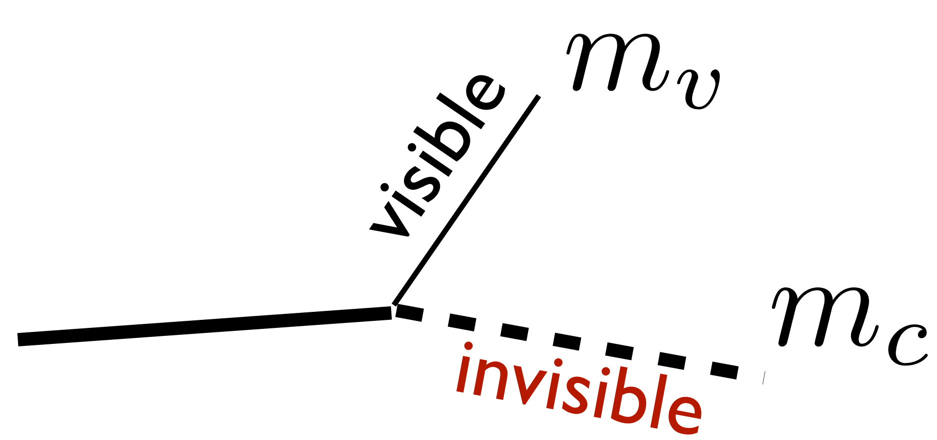


- Pair produced Heavy particle decays into visible(at the LHC) particles & invisible particle (LSP)

- We may get a hint to design a new variable from Tevatron example.

Design variable for new physics search @LHC

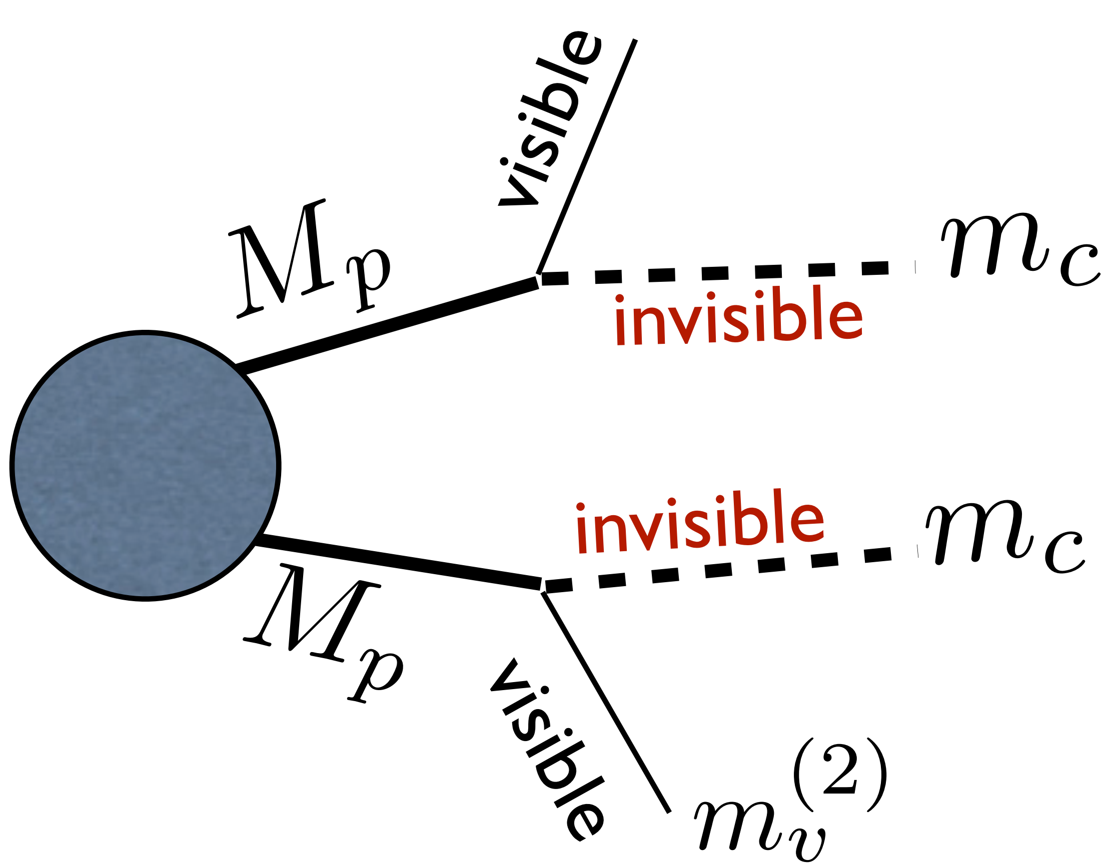
- A transverse mass M_T:



$$M_T\,(m_c) = \sqrt{m_v^2 + m_c^2 + 2\left(e_v e_c - \vec{p}_T^{\,v} \cdot \vec{p}_T^{\,(c)}\right)}$$

with a transverse energy, $e_c = \sqrt{\vec{p}_T^{\,(c)} \cdot \vec{p}_T^{\,(c)} + m_c^2}$

- For double decay chain event: Let's use M_T twice. C.Lester, D. Summers (hep-ph/9906349)

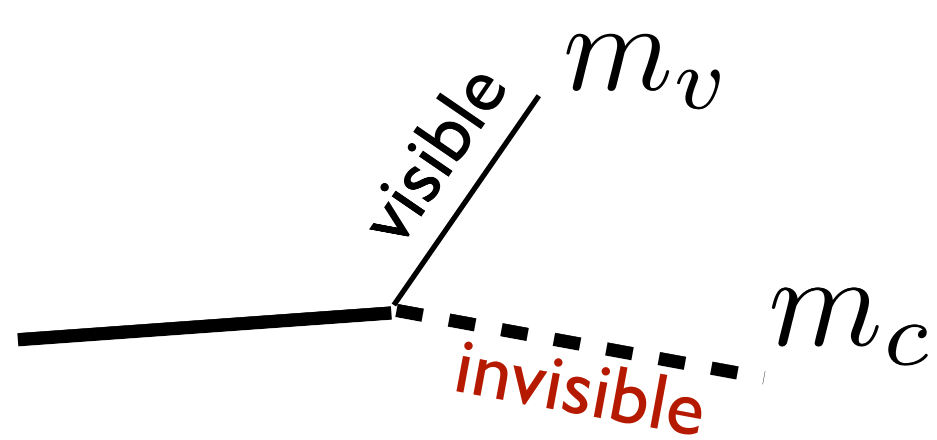


Assumptions:

I. Decaying particle in both chain has a common mass : M_p
II. Invisible particle in both chain has a common mass : m_c
III. No invisible particles except LSP

Design variable for new physics search @LHC

- A transverse mass M_T:



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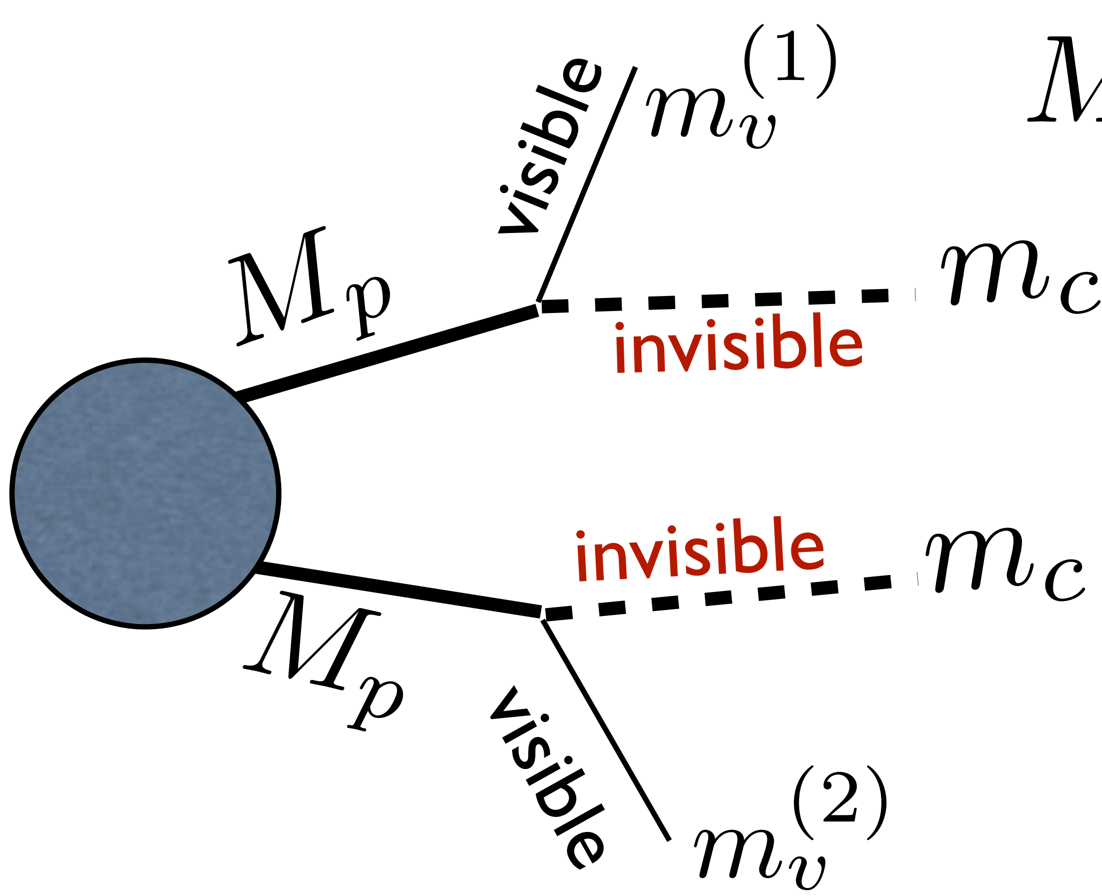
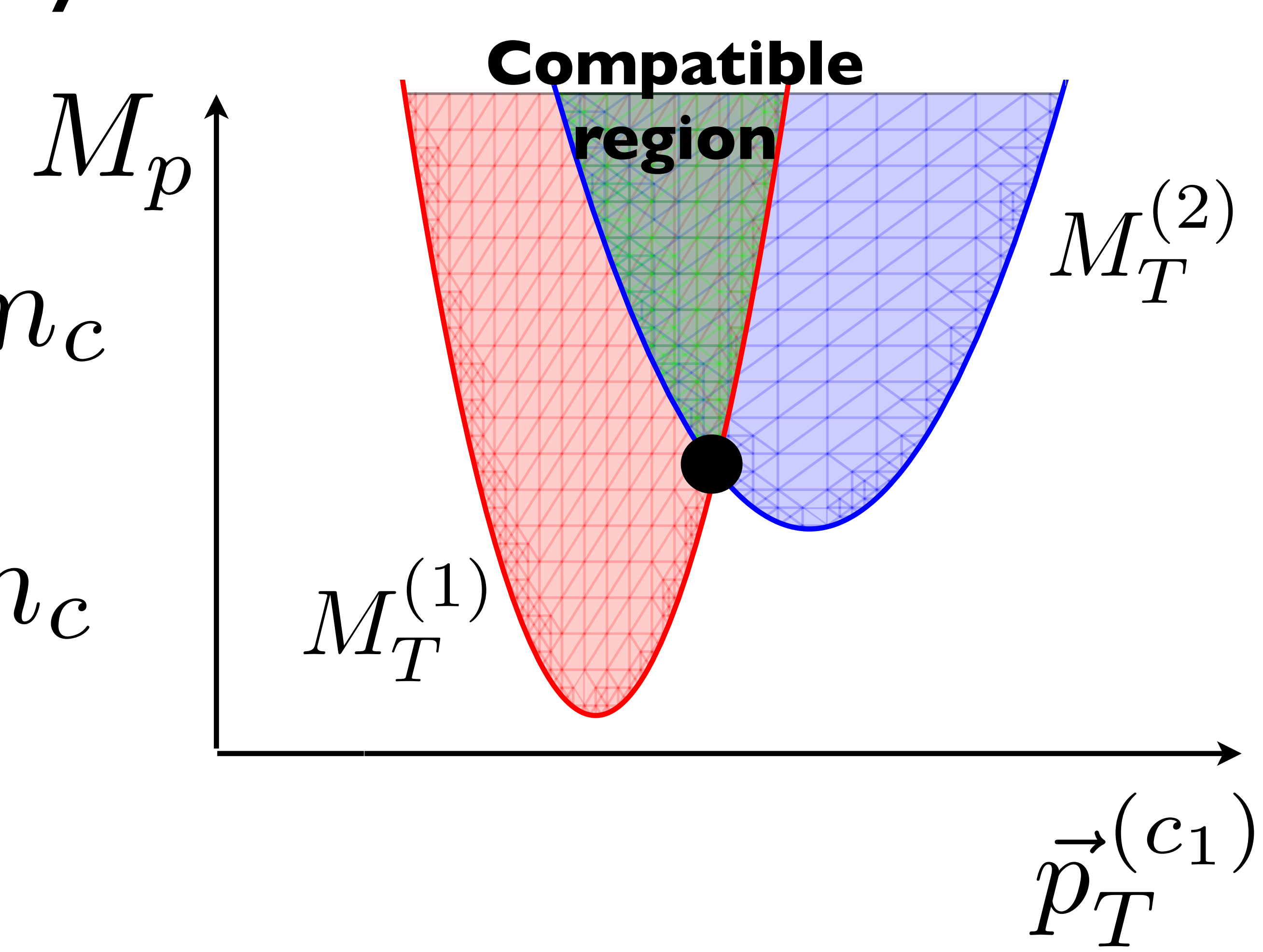




$$M_T^{(1)} \leq M_p \text{ \& } M_T^{(2)} \leq M_p$$

(Transverse mass is less than the actual mass.)

Get the minimum possible Mp with above kinematics constraints= M_{T2}

$$M_{T2}(m_c) = \min\left(\max[M_T^{(1)}(m_c), M_T^{(2)}(m_c)]\right)$$

this interpretation was provided by H.-C. Cheng and Z. Han (hep-ph:0810.5178)

New variable as a CUT

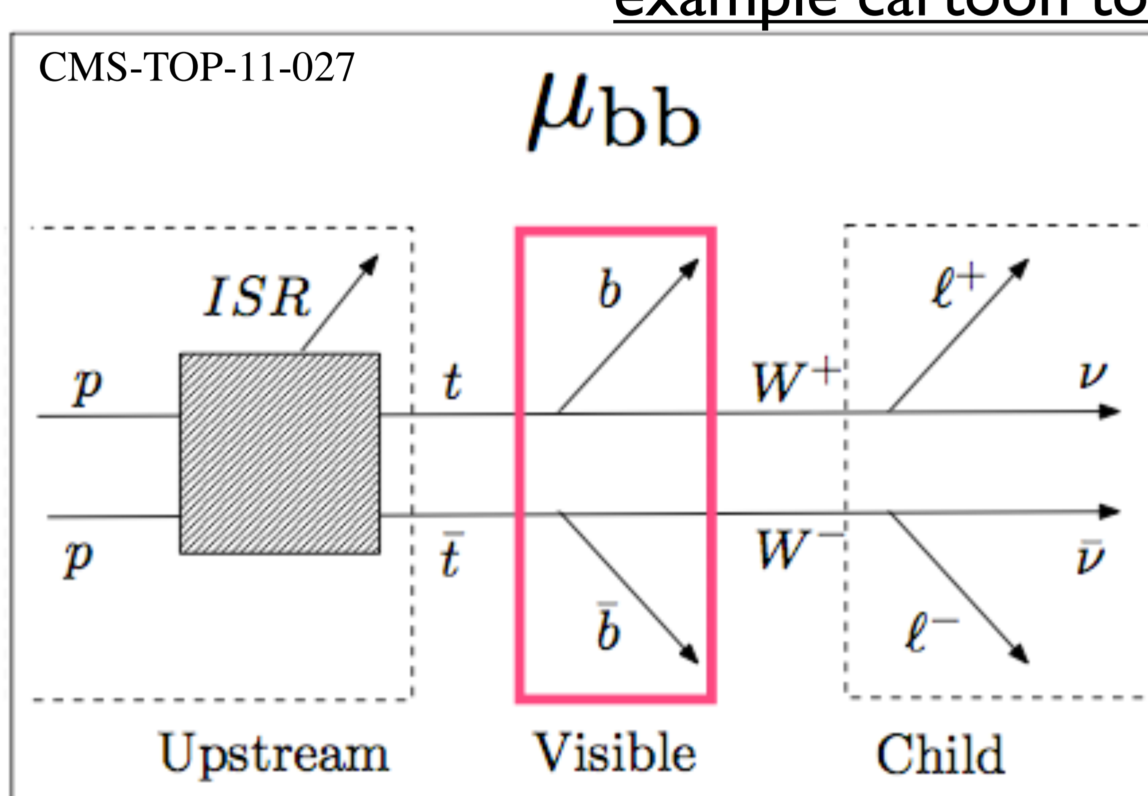
- M_{T2} inherits the good property of M_T: Transverse boost invariance!
(<u>But only when you put the the right value for m_c</u>)

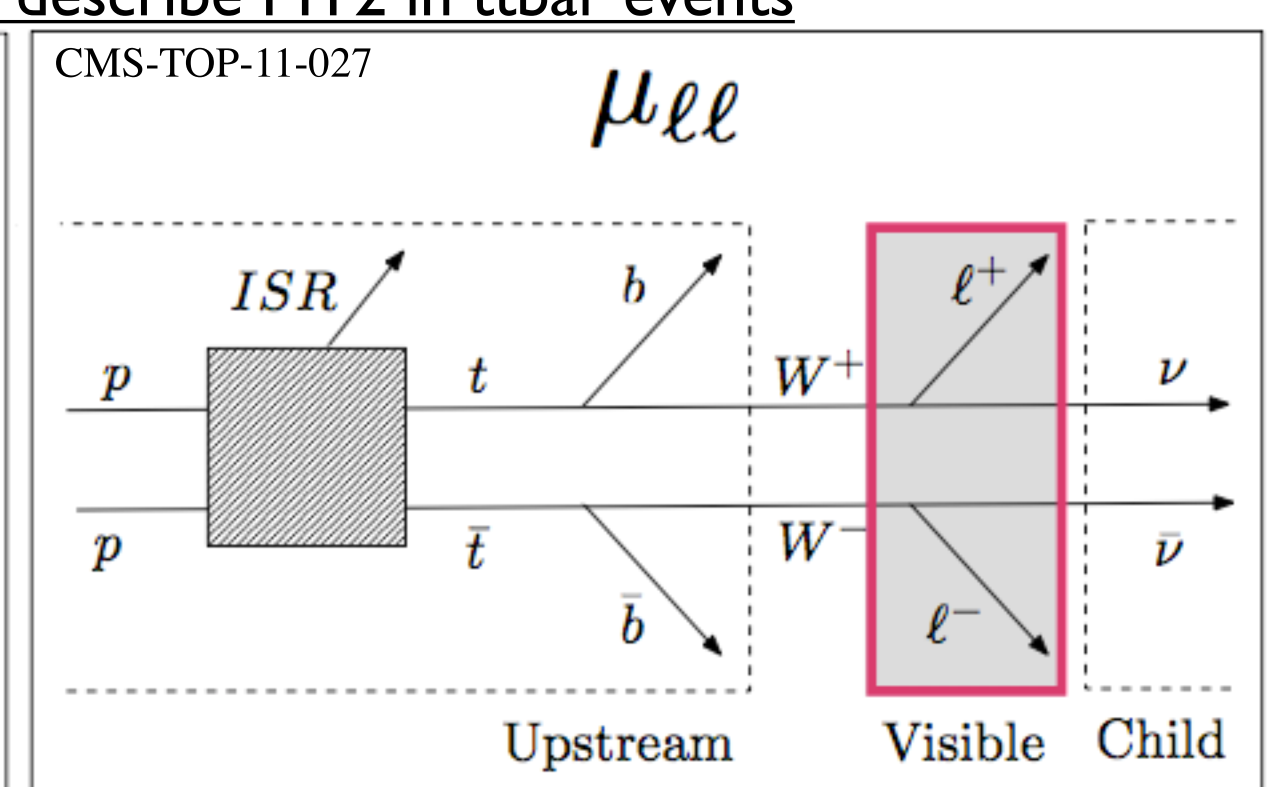
- Since we don't know the true mass of LSP, we only can get the constraint of decayed particle in terms of LSP mass.

- But we know what will be missing particles of background (Standard Model) : neutrino
- Thus, experimentalists started to use this variable as one of cuts.
(by Alan Barr, Claire Gwenlan : arxiv:0907.2713)

Process	$m_{T2}(v_1, v_2, \not{p}_T, 0, 0)$	Comments
QCD di-jet → hadrons	$= \max m_j$ by Lemmas 1,4	
QCD multi jets → hadrons	$= \max m_j$ by Lemma 4	
$t\bar{t}$ production	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\leq m_t$ by Lemmas 1,7	any leptonic decays
Single top / tW	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\leq m_t$ by Lemmas 2,7	any leptonic decays
Multi jets: "fake" $\not{p}_T$	$= \max m_j$ by Lemma 5	single mismeasured jet[a]
	$= \max m_j$ by Lemma 6	two mismeasured jets[a]
Multi jets: "real" $\not{p}_T$	$= \max m_j$ by Lemma 5	single jet with leptonic b decay[a]
	$= \max m_j$ by Lemma 6	two jets with leptonic b decays[a]
$Z \to \nu\bar{\nu}$	$= 0$ by Lemma 3	
$Z\,j \to \nu\bar{\nu}\,j$	$= m_j$ by Lemma 3	one ISR jet[a]
$W \to \ell\nu$ [b]	$= m_\ell$ by Lemma 3	
$W\,j \to \ell\nu\,j$ [b]	$\leq m_W$ by Lemma 2	one ISR jet[a]
$WW \to \ell\nu\ell\nu$ [b]	$\leq m_W$ by Lemma 1	
$ZZ \to \nu\bar{\nu}\nu\bar{\nu}$	$= 0$ by Lemma 3	also $= m_j$ for one ISR jet[a]
$LQ\,\overline{LQ} \to q\nu\bar{q}\bar{\nu}$	$\leq m_{LQ}$	} i.e. can take large values
$\tilde{q}\,\bar{\tilde{q}} \to q\tilde{\chi}_1^0\,\bar{q}\tilde{\chi}_1^0$	$\leq m_{\tilde{q}}$	
$q_1, \bar{q}_1 \to q\gamma_1, \bar{q}\gamma_1$	$\leq m_{q_1}$	

<u>example cartoon to describe MT2 in ttbar events</u>



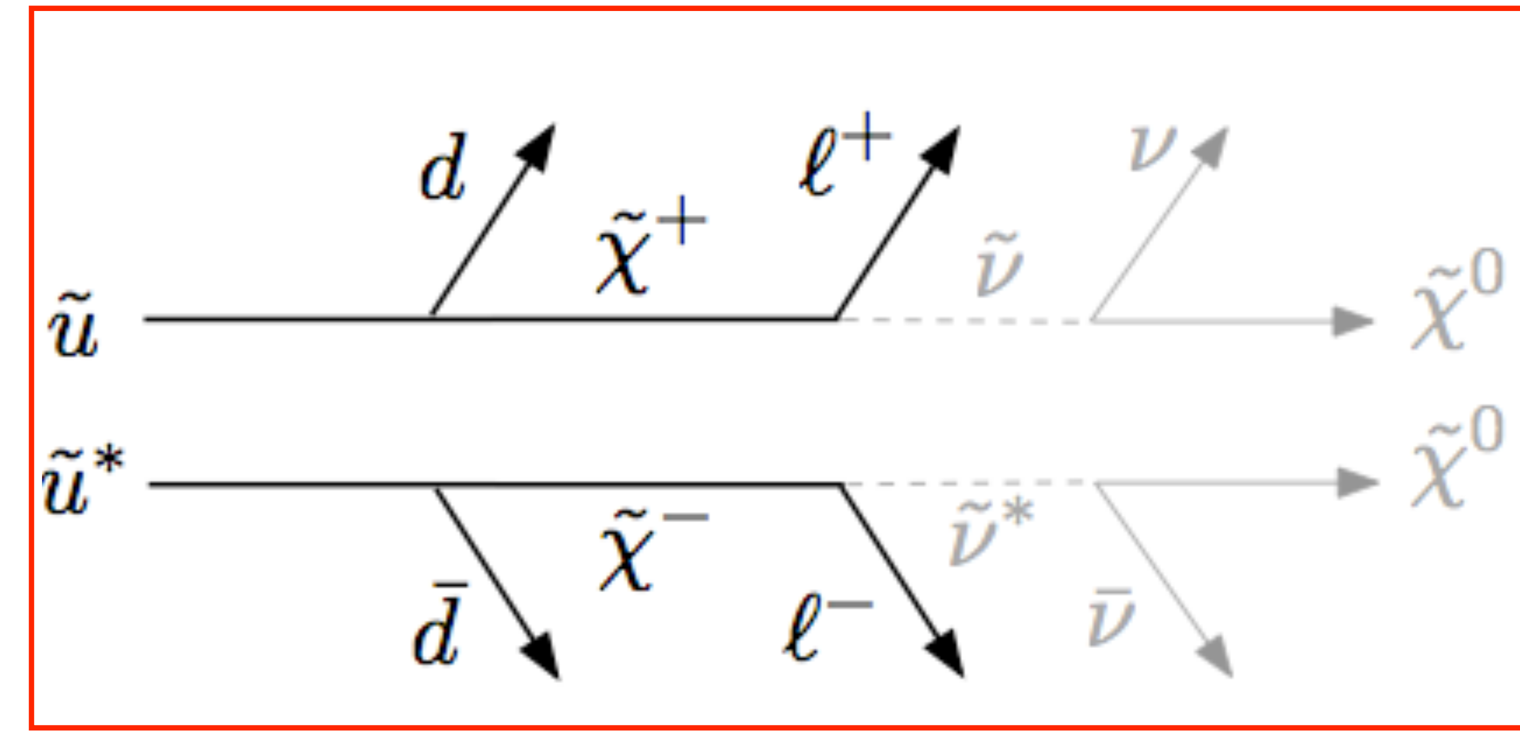


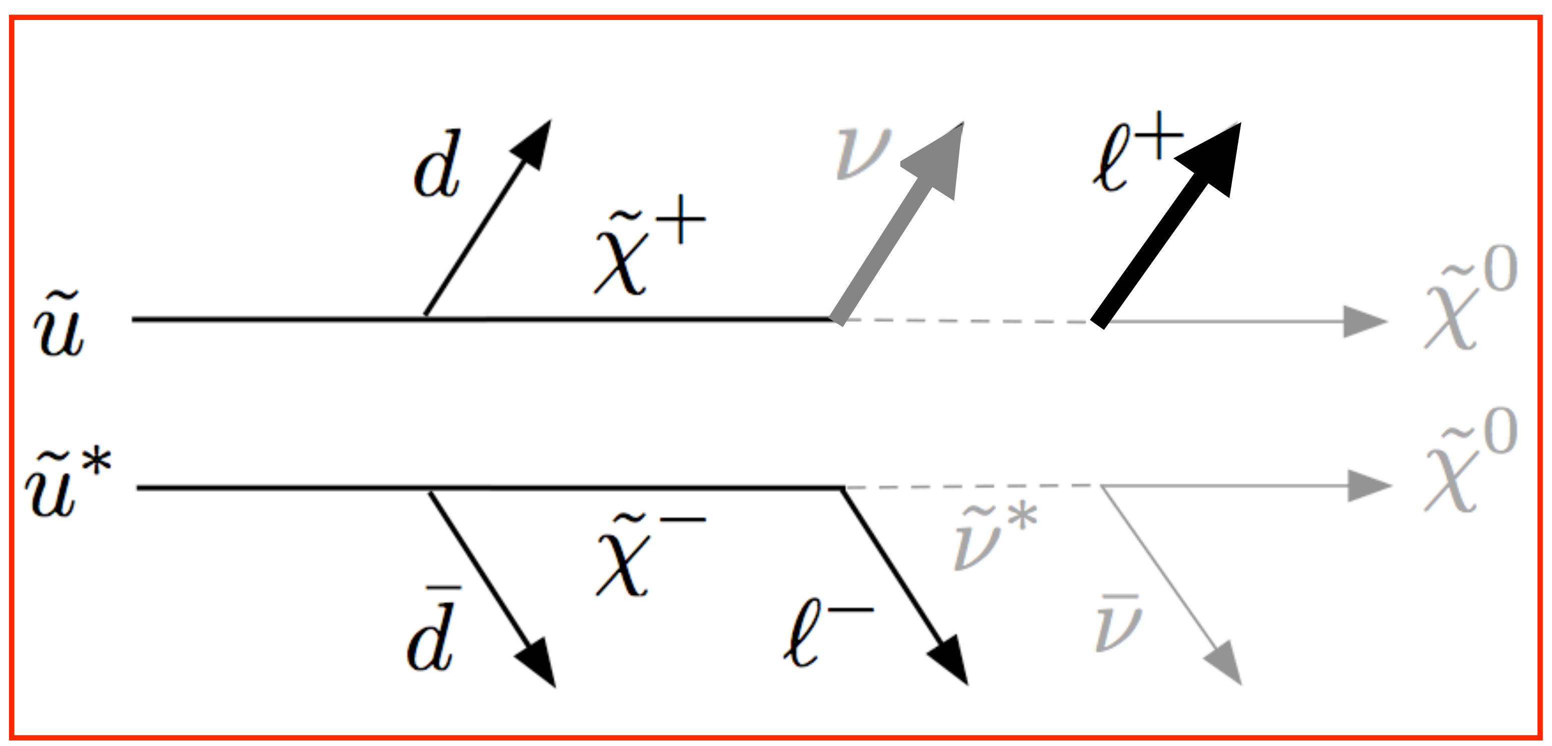
One concern

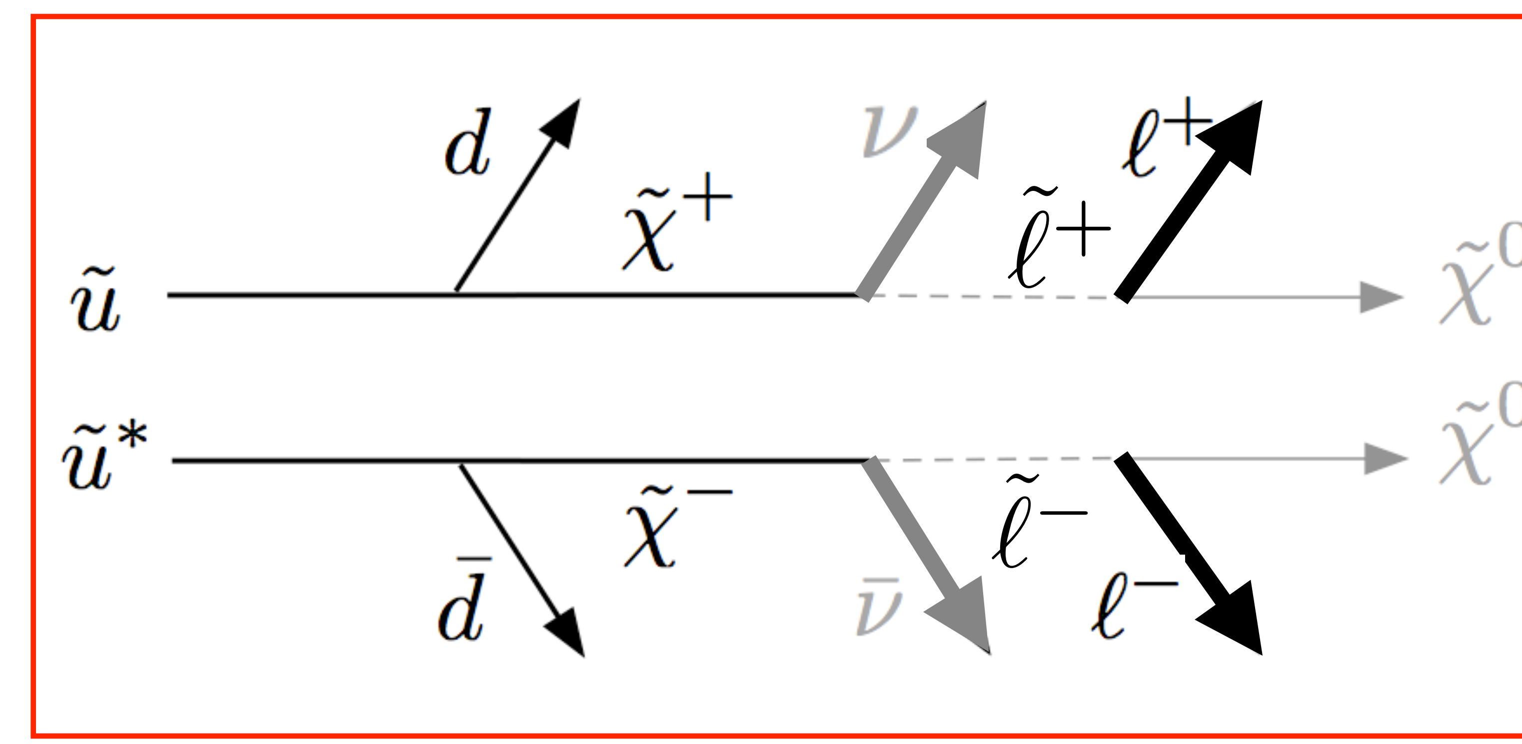
- Now M_{T2} is one of the standard variables on the market for the new physics search.
- I would like to remind you that M_{T2} was based on three big assumptions.
- Thus if most of signals (the new physics) violate at least one of these assumptions, is there any chance for signals can hide behind Backgrounds?
- I would like to study the behavior of M_{T2} when signals break some (all) of these assumptions.

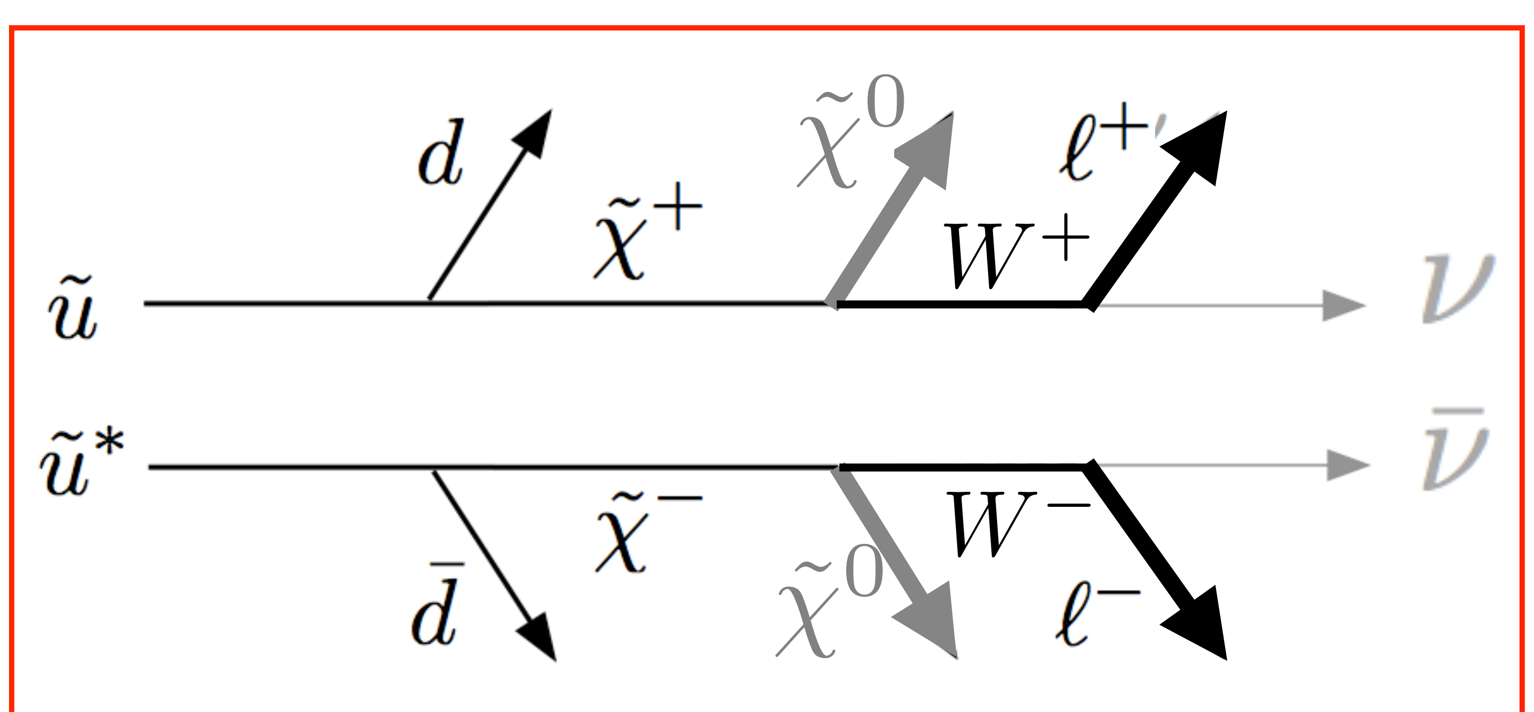
Various possibilities

- There may be more than one diagram in the BSM with the same signature. Some can violate assumptions of MT2







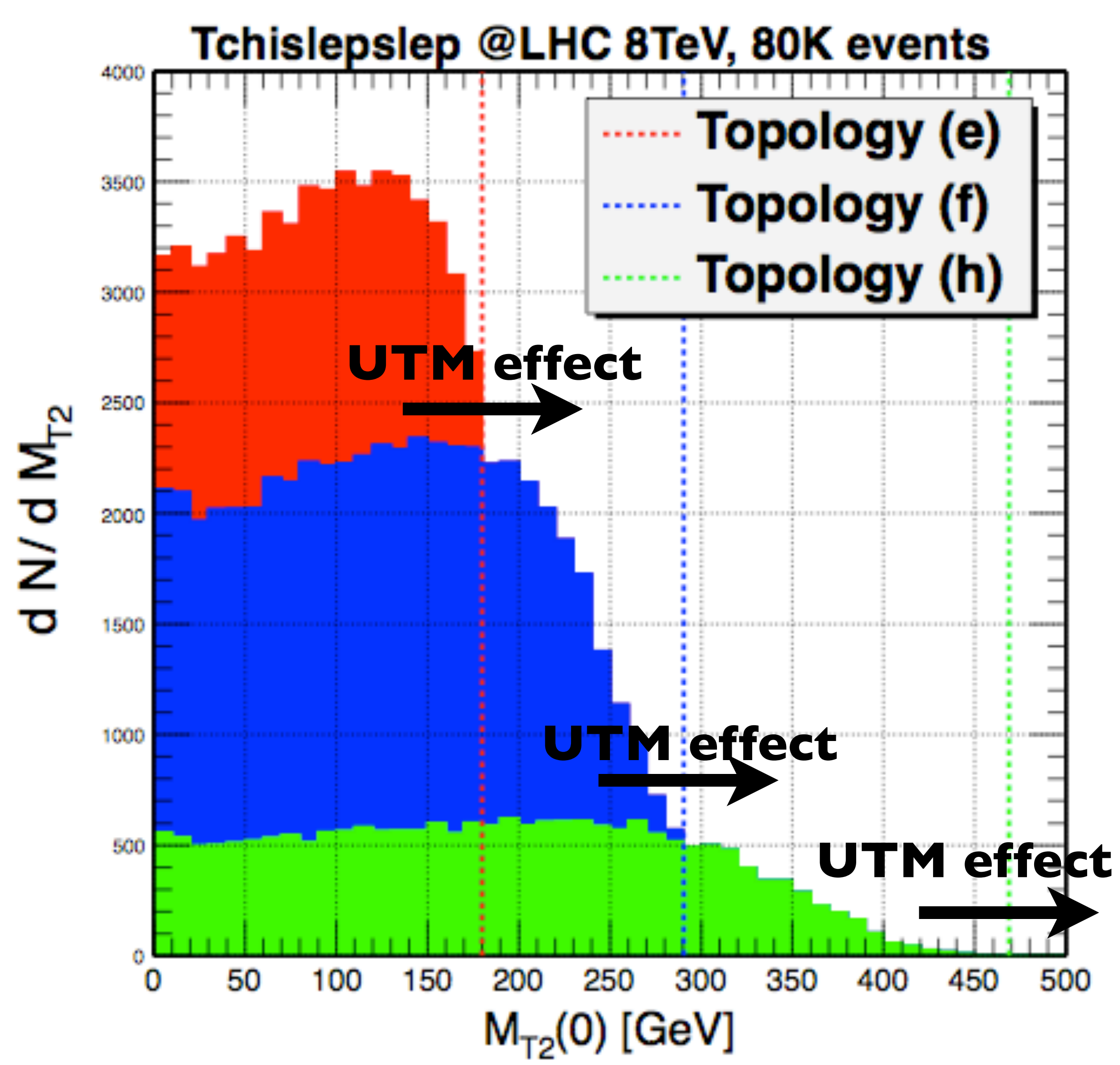


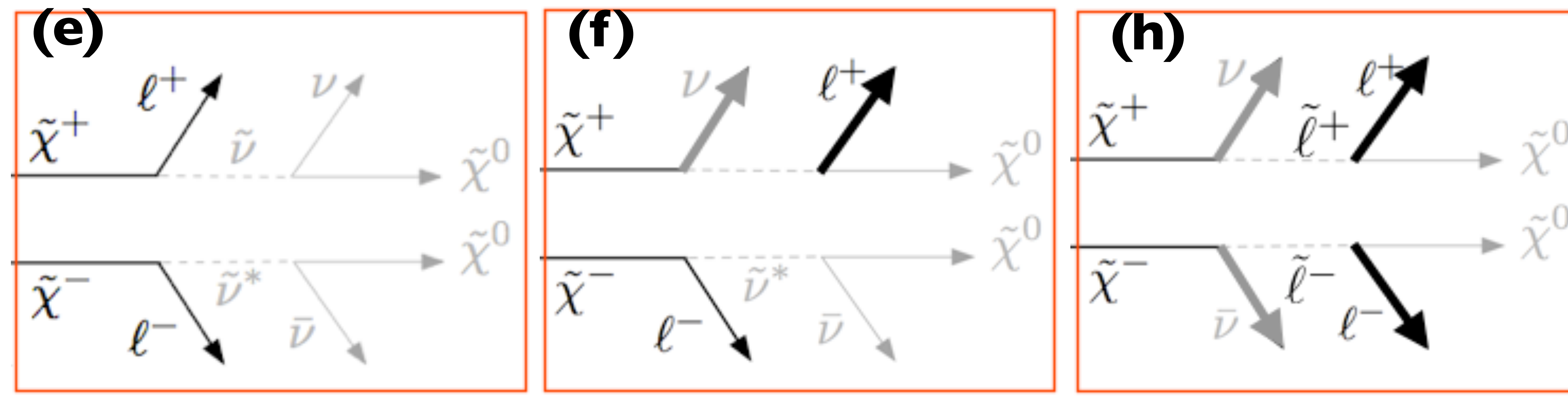
+ more if we consider different signals: squarks decays through long cascade (four leptons signals)

Various possibilities

- As an example, we generated CMS Tchislepslep simplified model with

$$M_{\tilde{\chi}^+} > M_{\tilde{\nu}_L} \simeq M_{\tilde{\ell}_L^+} > M_{\tilde{\chi}^0}$$





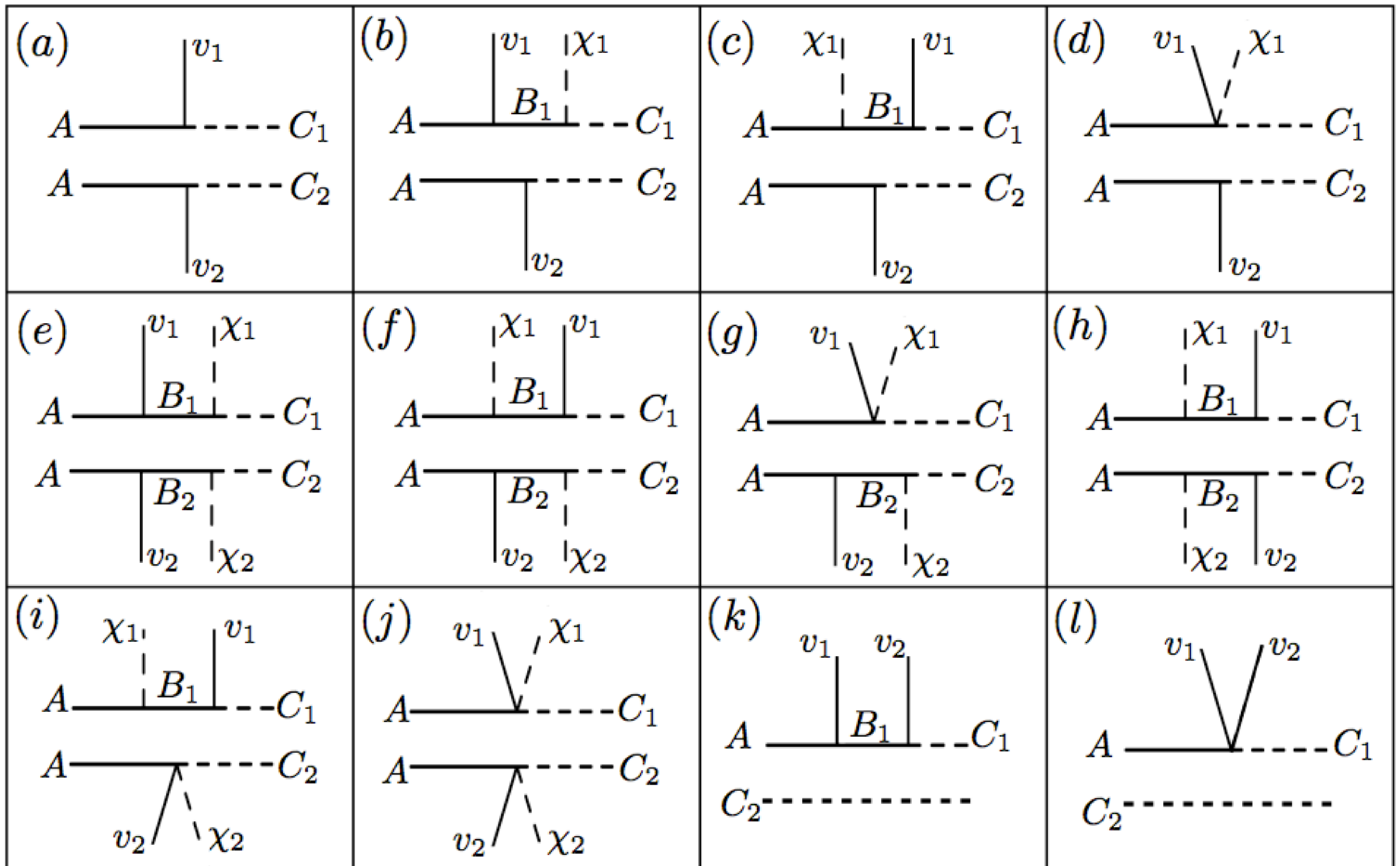
- Simulated[parton level] with masses:
  chargino 500GeV
  slepton(sneutrino) 400GeV
  LSP: 100GeV

This is chargino direct productions. When mostly charginos are produced via squark/gluino decays or with ISR, endpoints will be smeared from UTM boost effect. (since we don't know the right value for the mass of missing particle[s].)

As we can see the distribution of MT2
for (h) is not well saturated near the endpoint.
[I will explain how we can get the theoretical expectation of the endpoint of MT2 histogram]

Let's count how many possible diagrams that we have for this signature (two leptons, MET [+jets])

Various possibilities

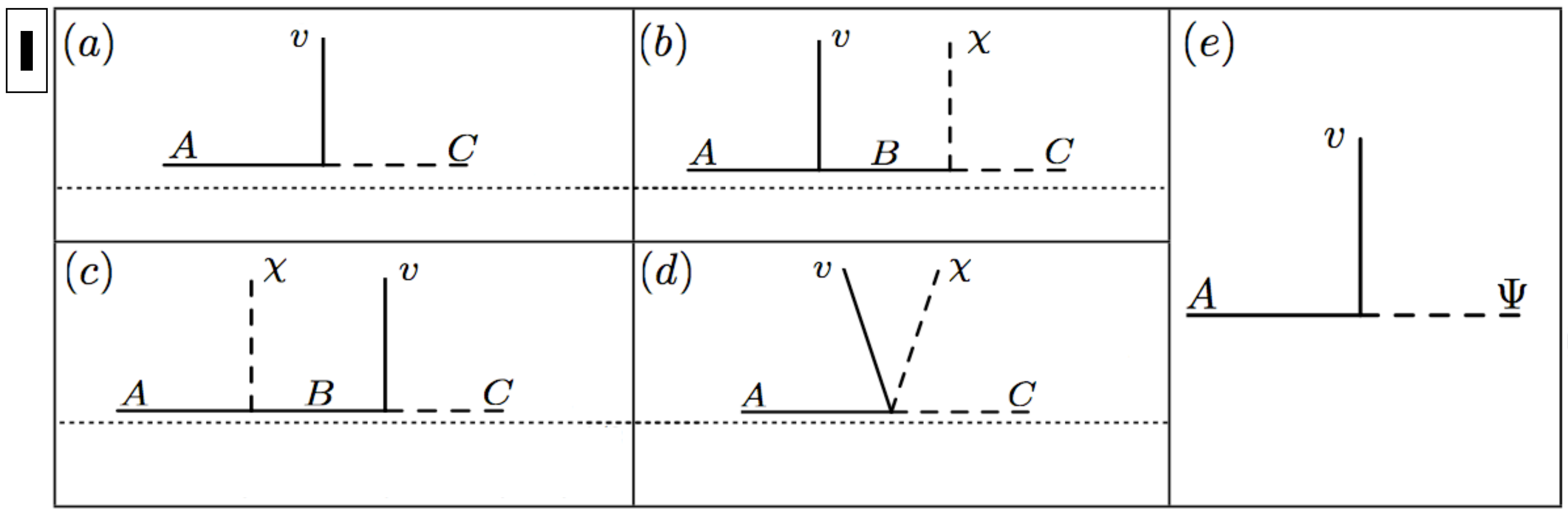


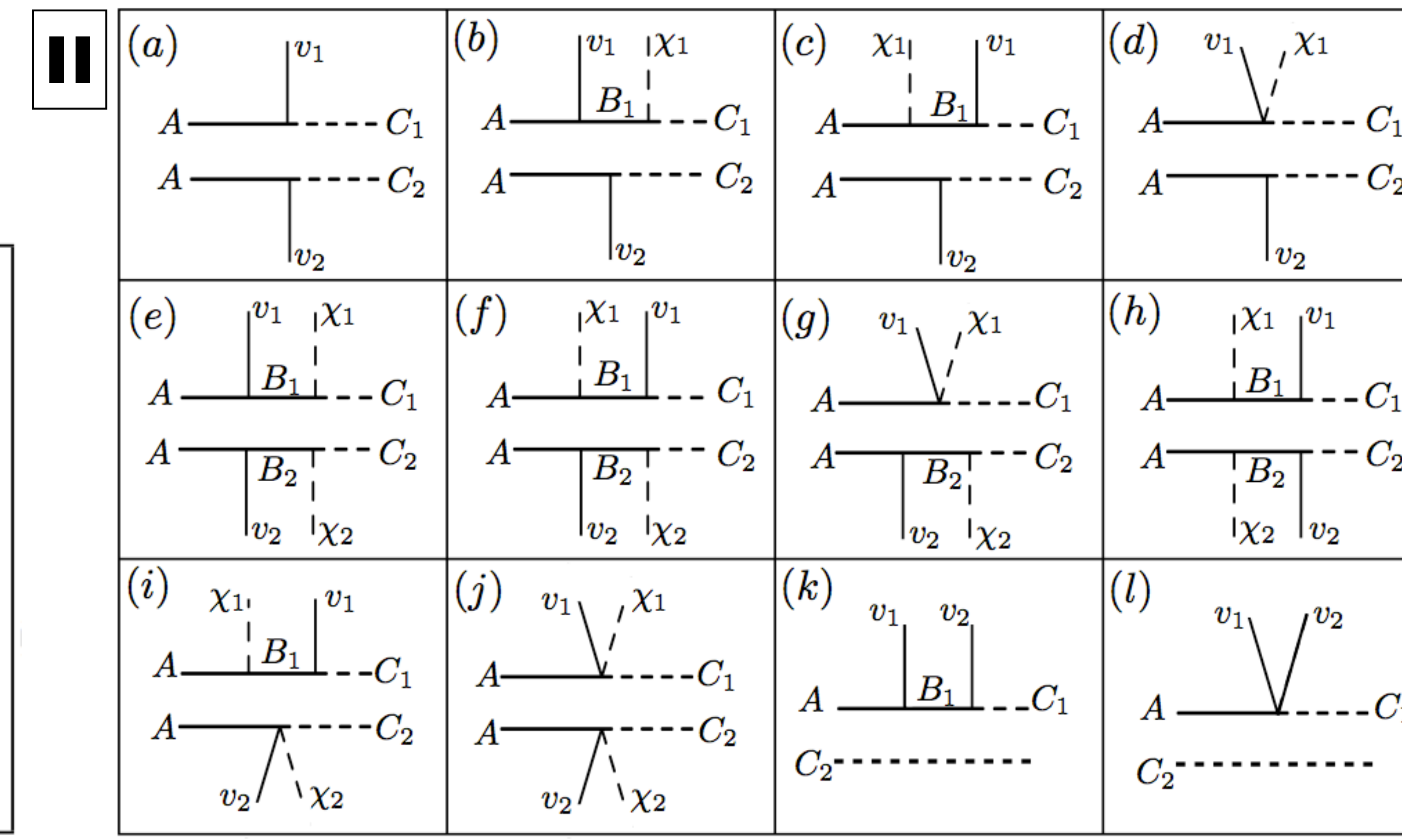
- There are 12 (sub) diagrams that have two visible particles and up to four invisible particles.
- We have options:
  1. we need to invent new observables based on each event-topology.

  2. And/Or we need to <u>understand how to interpret a result of existing observables (e.g. MT2) for each event-topology case.</u>

Effective event-topology

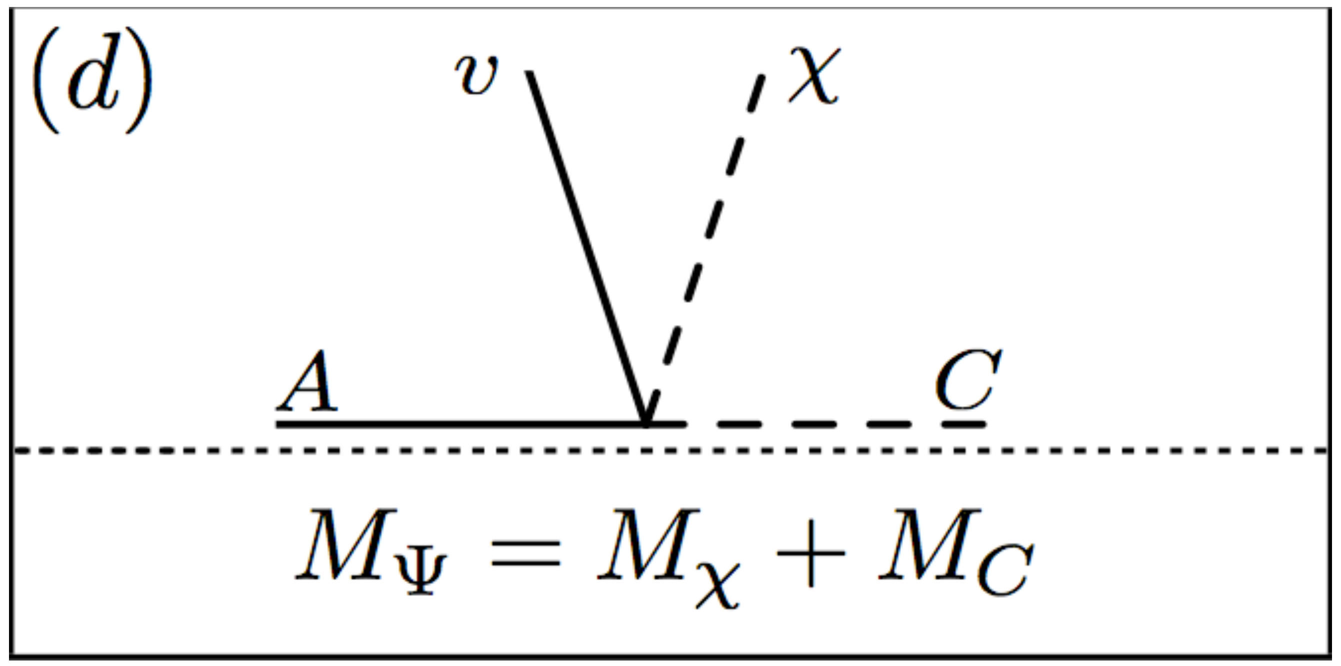
1. Number of invisible particle:
Introduce Equivalent event-topology method





- We apply an observable that was motivated initially for the II (a) assumptions, and want to interpret results (endpoint of distributions) in various cases.
- Diagrams in II (except k,l) are combinations of a basic decaying leg I (a), (b), (c), and (d).
- For example, in I (b), we can treat B that decays invisibly as invisible particle.
- The only non-trivial case will be I (c).

- We are interested in the endpoint of distributions. Thus we need to focus on the range of a (transverse) momentum of visible particle v (at the rest frame of A.)

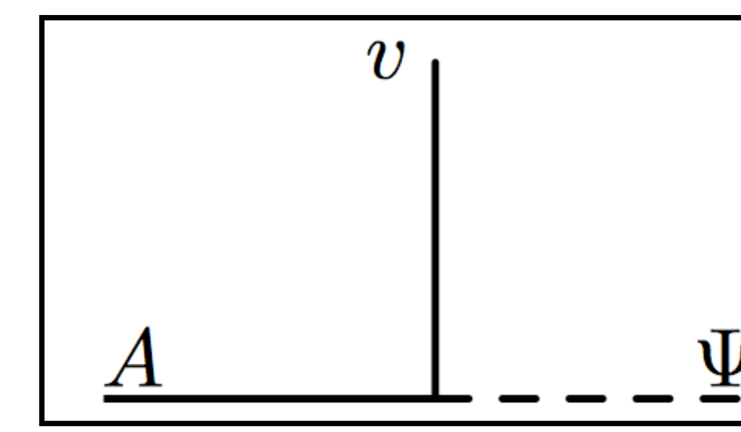


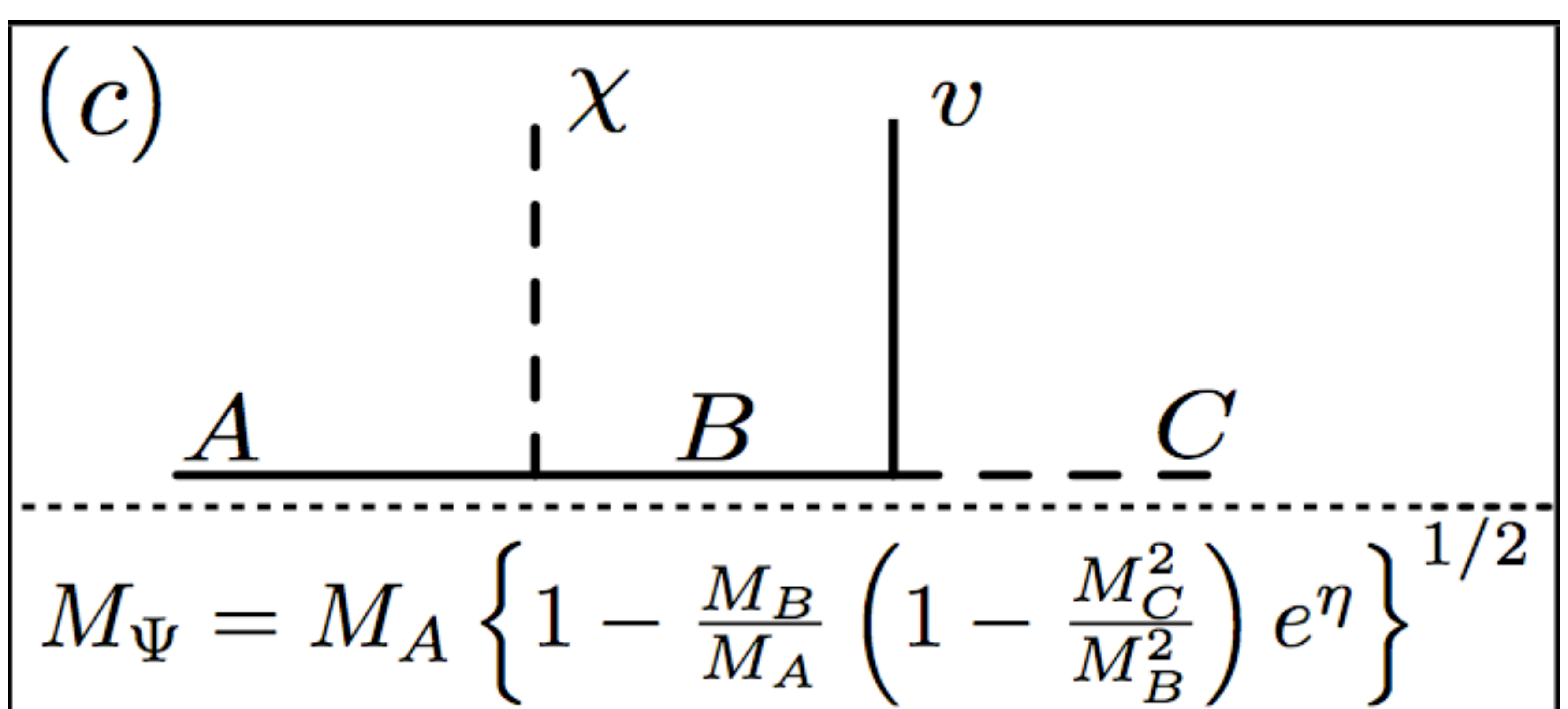
At A's rest frame, a range of transverse momentum of v

$$0 \leq P_T \leq \frac{M_A}{2}\left(1 - \frac{M_{C\chi}^2}{M_A^2}\right)$$

Thus, P_T will have a maximum when the invariant mass $M_{C\chi}$ (of C and chi) has a minimum value $= M_C + M_\chi$

This range of P_T also come from the right diagram where a particle Ψ with a mass of $M_\Psi = M_C + M_\chi$. Thus we can replace (d) with a right diagram for the endpoint of transverse observables.





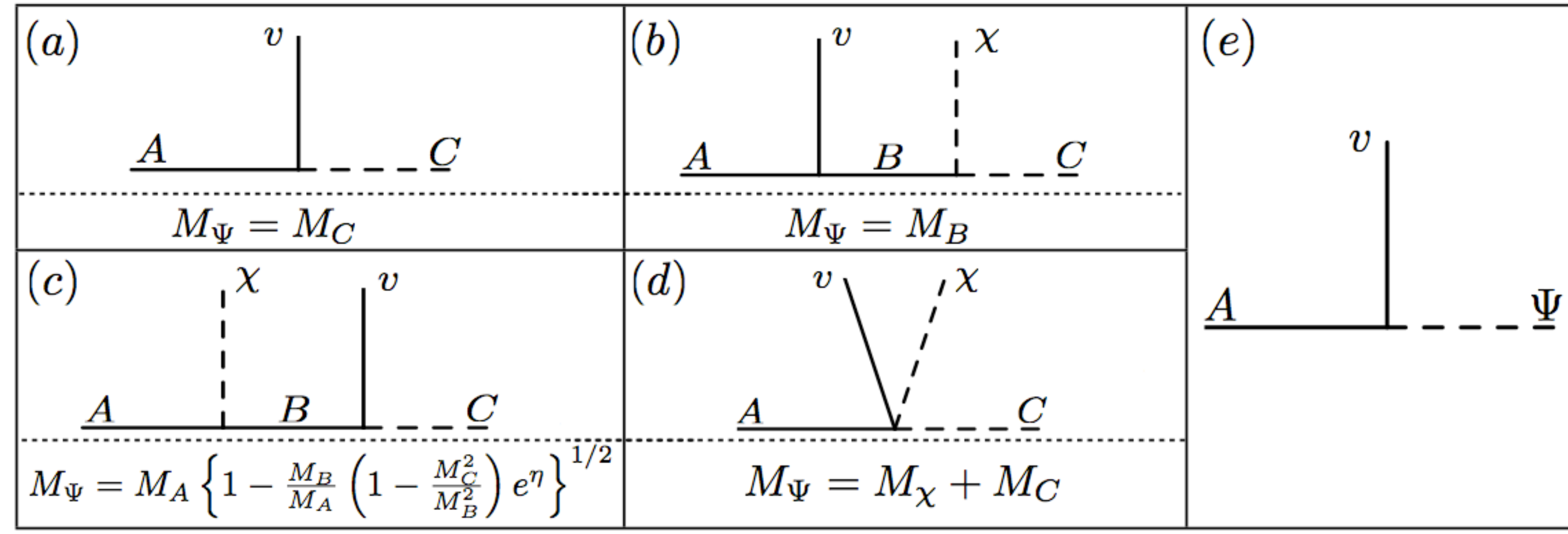
Similarly, in this case at the A's rest frame

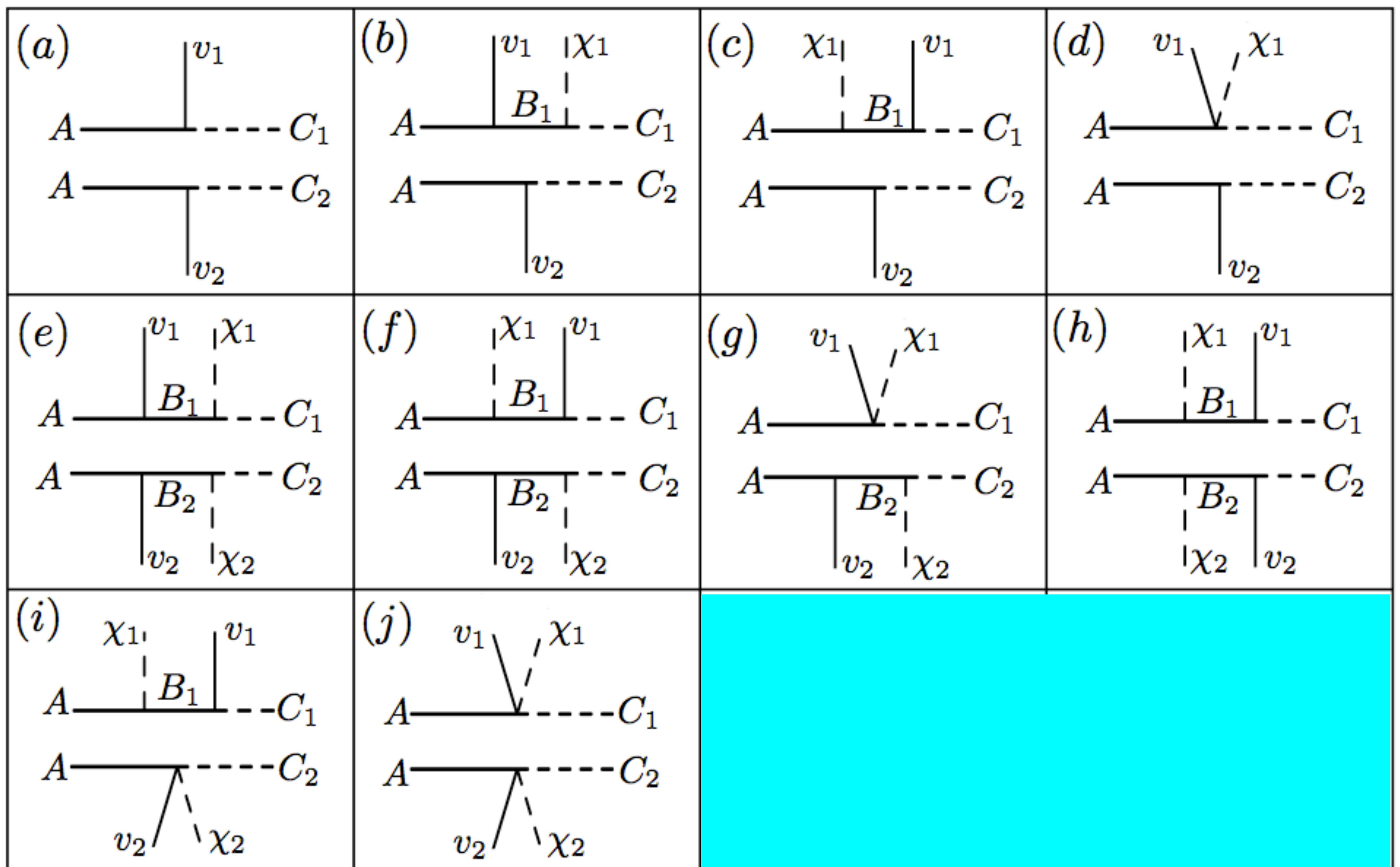
$$0 \leq P_{vT}^{(A)} \leq \frac{M_B}{2}\left(1 - \frac{M_C^2}{M_B^2}\right)(\cosh\eta + \sinh\eta) = \frac{M_B}{2}\left(1 - \frac{M_C^2}{M_B^2}\right) e^\eta$$

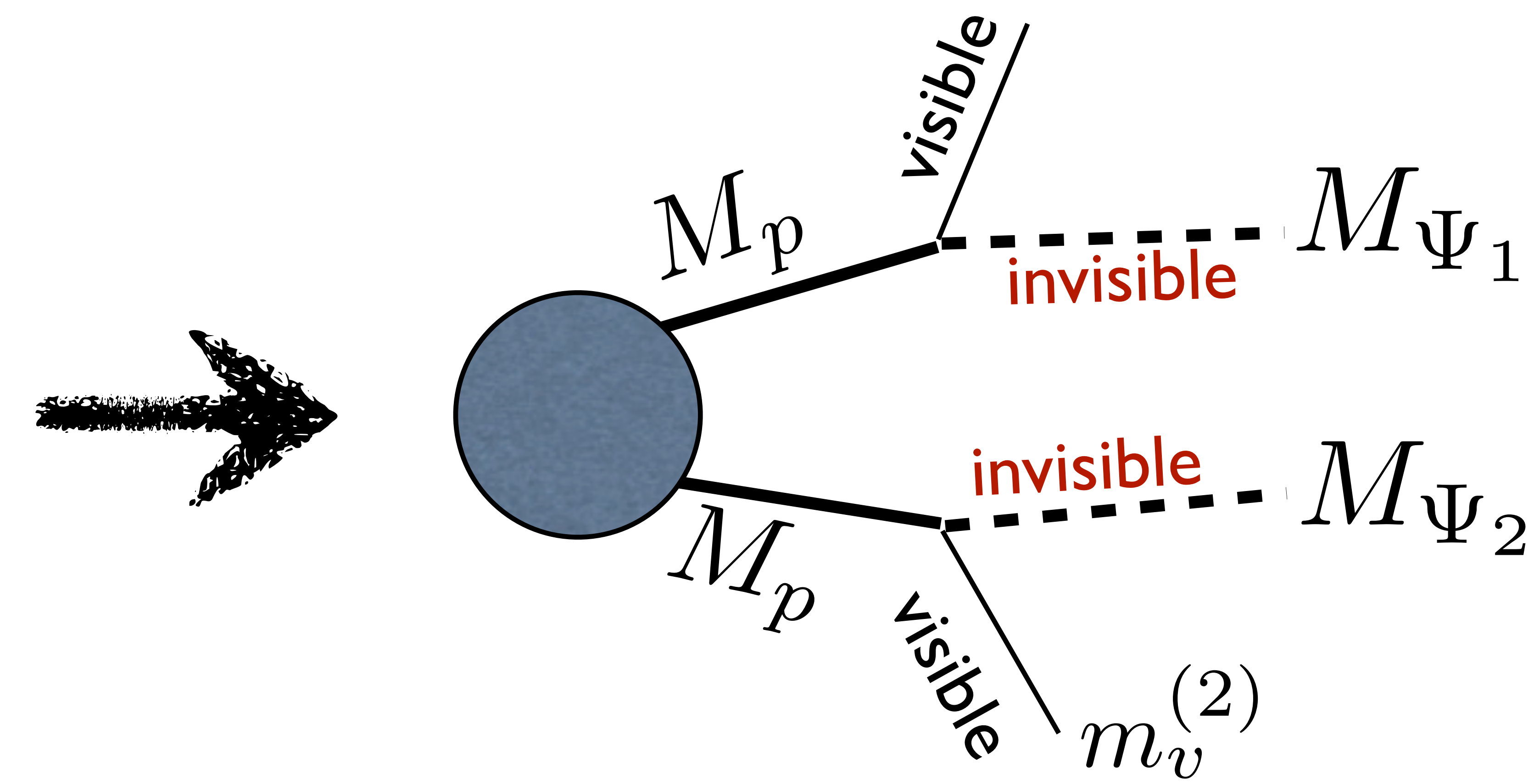
Identify as $= \frac{M_A}{2}\left(1 - \frac{M_\Psi^2}{M_A^2}\right)$

where η is a Lorentz boost factor from a rest frame of B to the rest frame of A. $\begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}\begin{pmatrix} M_B \\ 0 \end{pmatrix} = \begin{pmatrix} E_B^{(A)} \\ P_B^{(A)} \end{pmatrix}$

Using "Equivalent event-topology method", we can change event-topologies with multi-invisible particles into an event-topology with two invisible particle.

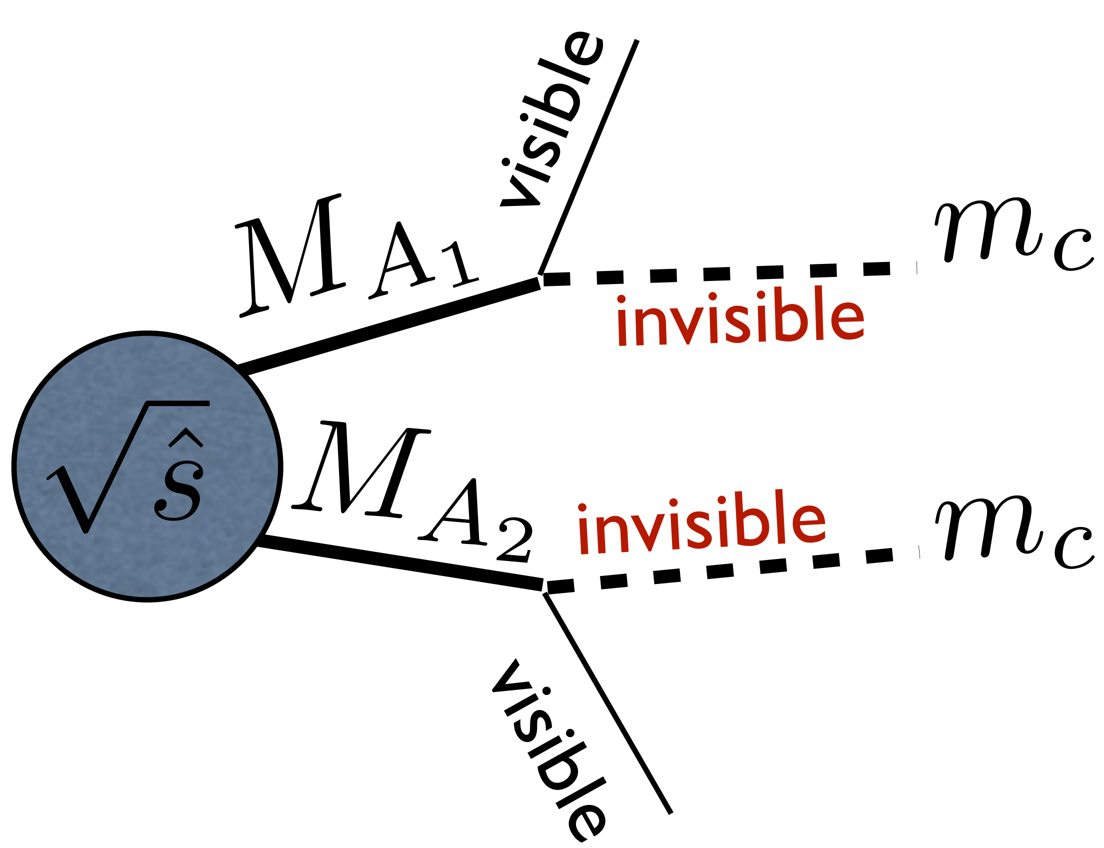






- But, now we need to deal with the case with different types of invisible particle ($M_{\Psi_1} \neq M_{\Psi_2}$): Studied by P. Konar, K.Matchev, K.Kong. MP [arxiv:0911.4126]

When decaying particles are different



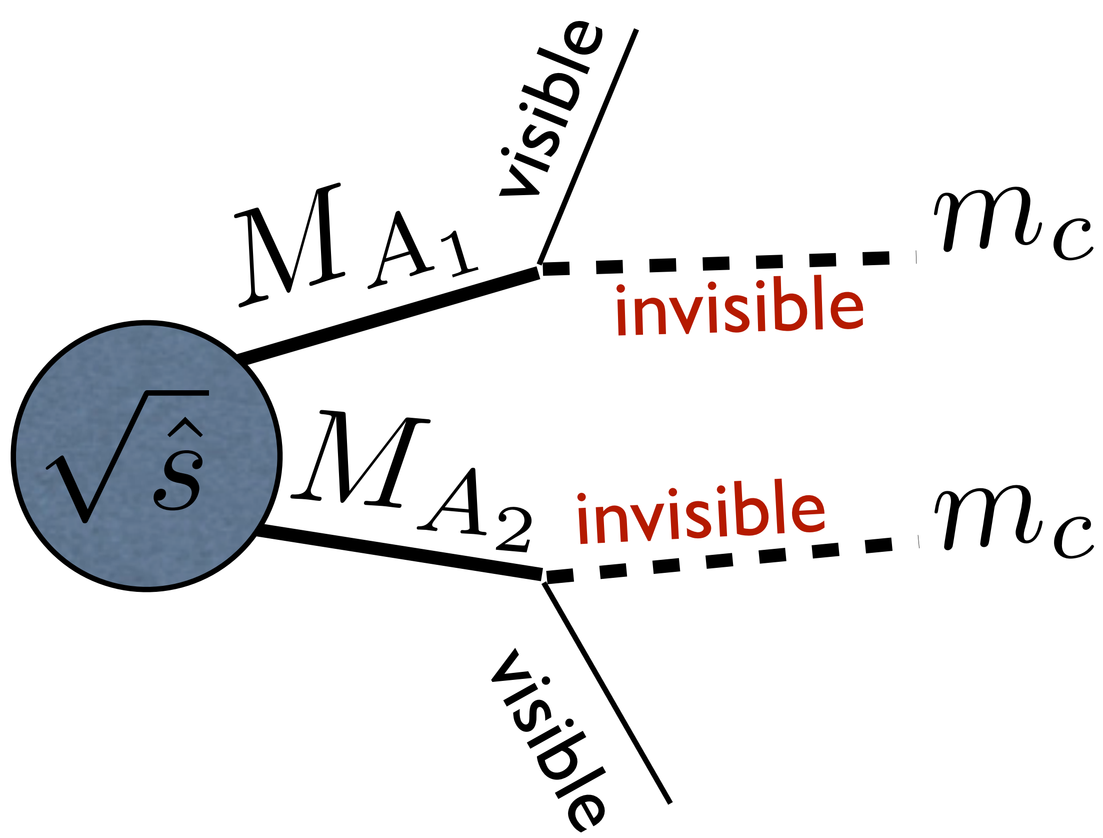
If $M_{A2} > M_{A1}$, then A_1 get the additional boost $\delta\eta\left(\sqrt{\hat{s}}\right)$ from E_{CM} ($\sqrt{\hat{s}}$) compared to A_2.

$$\eta_1(\sqrt{\hat{s}}) = \cosh^{-1}\left(\frac{\hat{s}+M_{A_1}^2-M_{A_2}^2}{2\hat{s}\,M_{A_1}}\right) \qquad \eta_2(\sqrt{\hat{s}}) = \cosh^{-1}\left(\frac{\hat{s}+M_{A_2}^2-M_{A_1}^2}{2\hat{s}\,M_{A_2}}\right)$$

$$\delta\eta(\sqrt{\hat{s}}) \equiv \eta_1(\sqrt{\hat{s}}) - \eta_2(\sqrt{\hat{s}}) = \cosh^{-1}\left[\frac{M_{A_2}^2+M_{A_1}^2-\left(\frac{M_{A_2}^2-M_{A_1}^2}{\sqrt{\hat{s}}}\right)^2}{2M_{A_2}M_{A_1}}\right]$$

This additional boost will give effect on the visible part on A_1, and we can mimic this situation by inserting invisible particle in front of A_1

When decaying particles are different



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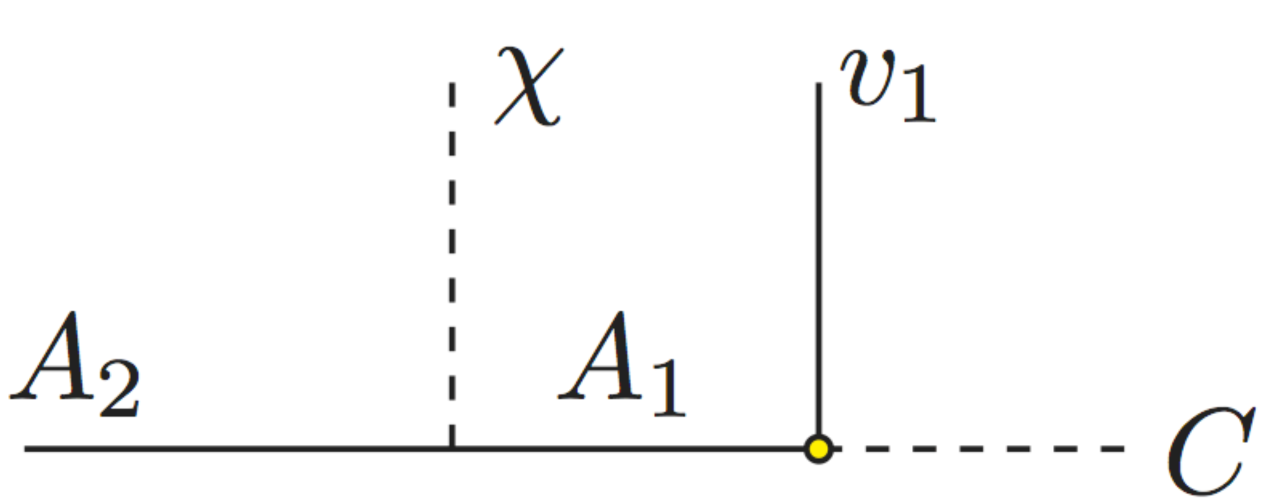


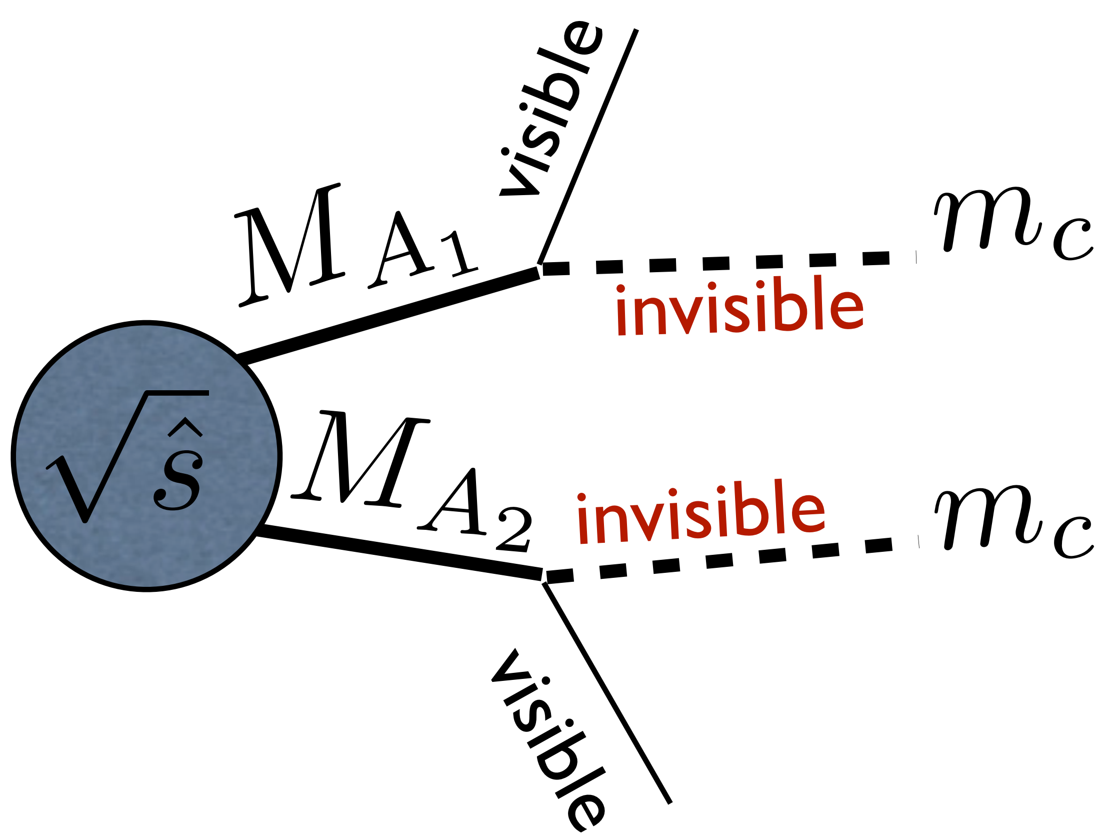
$$M_\Psi = M_{A_2}\left\{1 - \frac{M_{A_1}}{M_{A_2}}\left(1 - \frac{m_c^2}{M_{A_1}^2}\right)e^\eta\right\}^{1/2}$$

$$\eta = \cosh^{-1}\left(\frac{M_{A_2}^2 + M_{A_1}^2 - M_\chi^2}{2M_{A_2}M_{A_1}}\right)$$

Thus we can re-interpret this situation by putting invisible particle with mass

$$M_\chi(\sqrt{\hat{s}}) \equiv \frac{M_{A_2}^2 - M_{A_1}^2}{\sqrt{\hat{s}}}$$

When decaying particles are different



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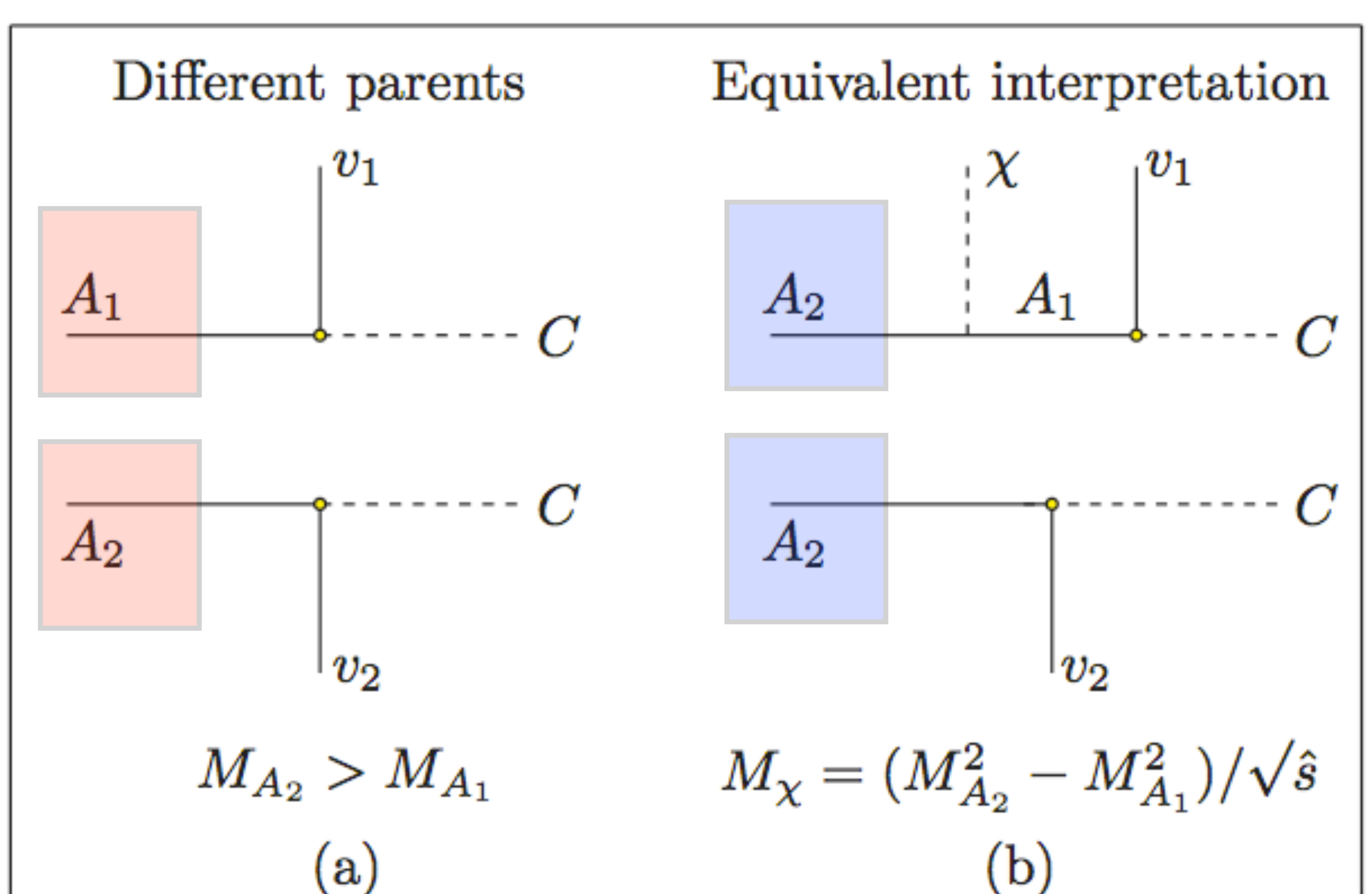
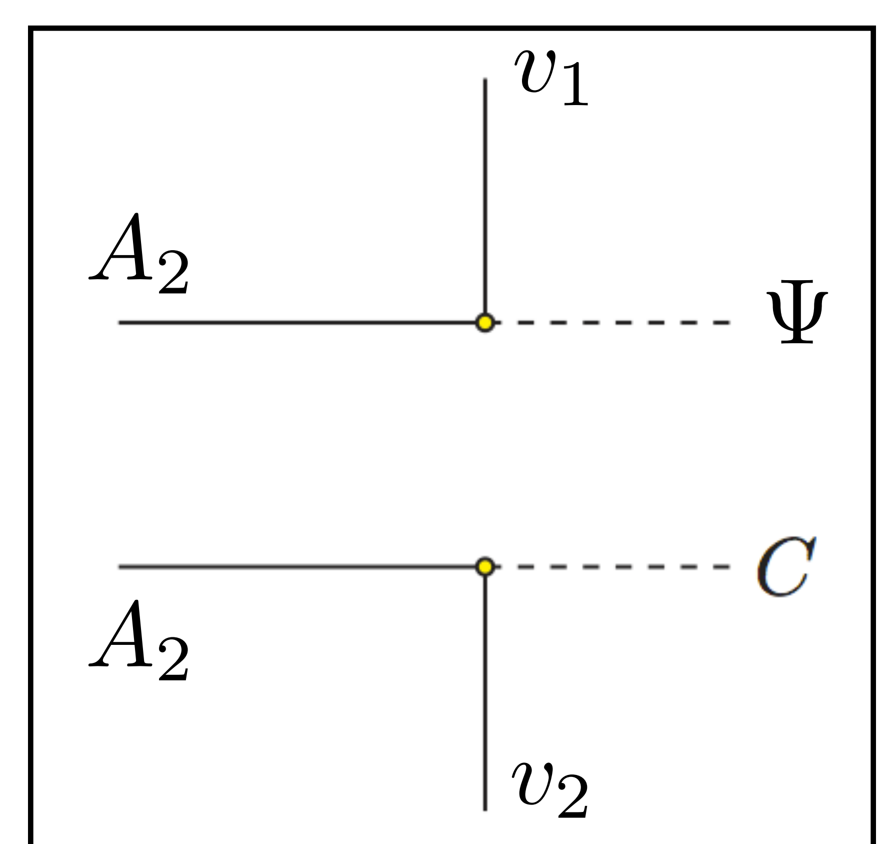
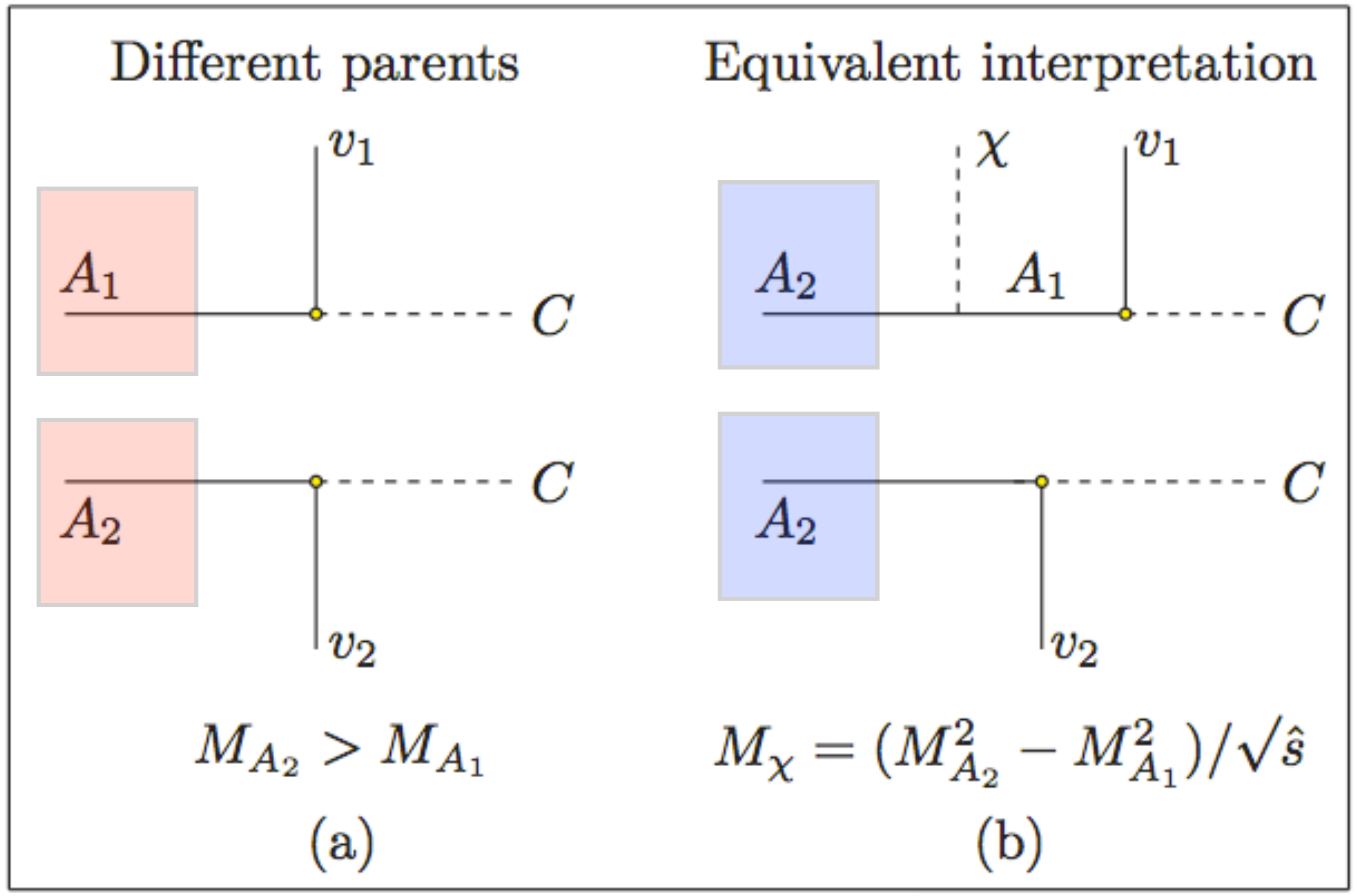
This additional boost will give effect on the visible part from A1, and we can mimic this situation by inserting invisible particle in front of A1

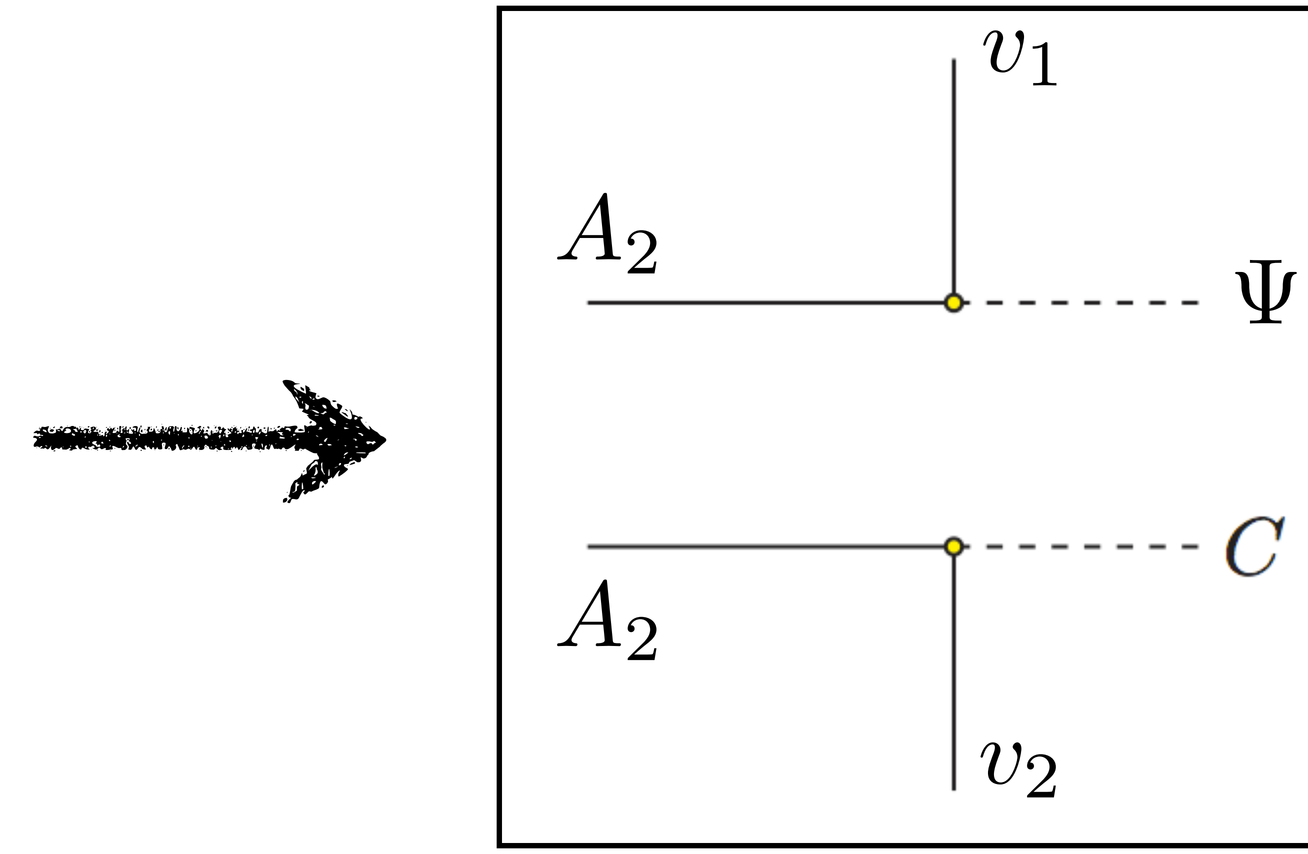




$$\eta = \cosh^{-1}\left(\frac{M_{A_2}^2 + M_{A_1}^2 - M_\chi^2}{2M_{A_2}M_{A_1}}\right)$$

$$M_\Psi = M_{A_2}\left\{1 - \frac{M_{A_1}}{M_{A_2}}\left(1 - \frac{m_c^2}{M_{A_1}^2}\right)e^\eta\right\}^{1/2}$$

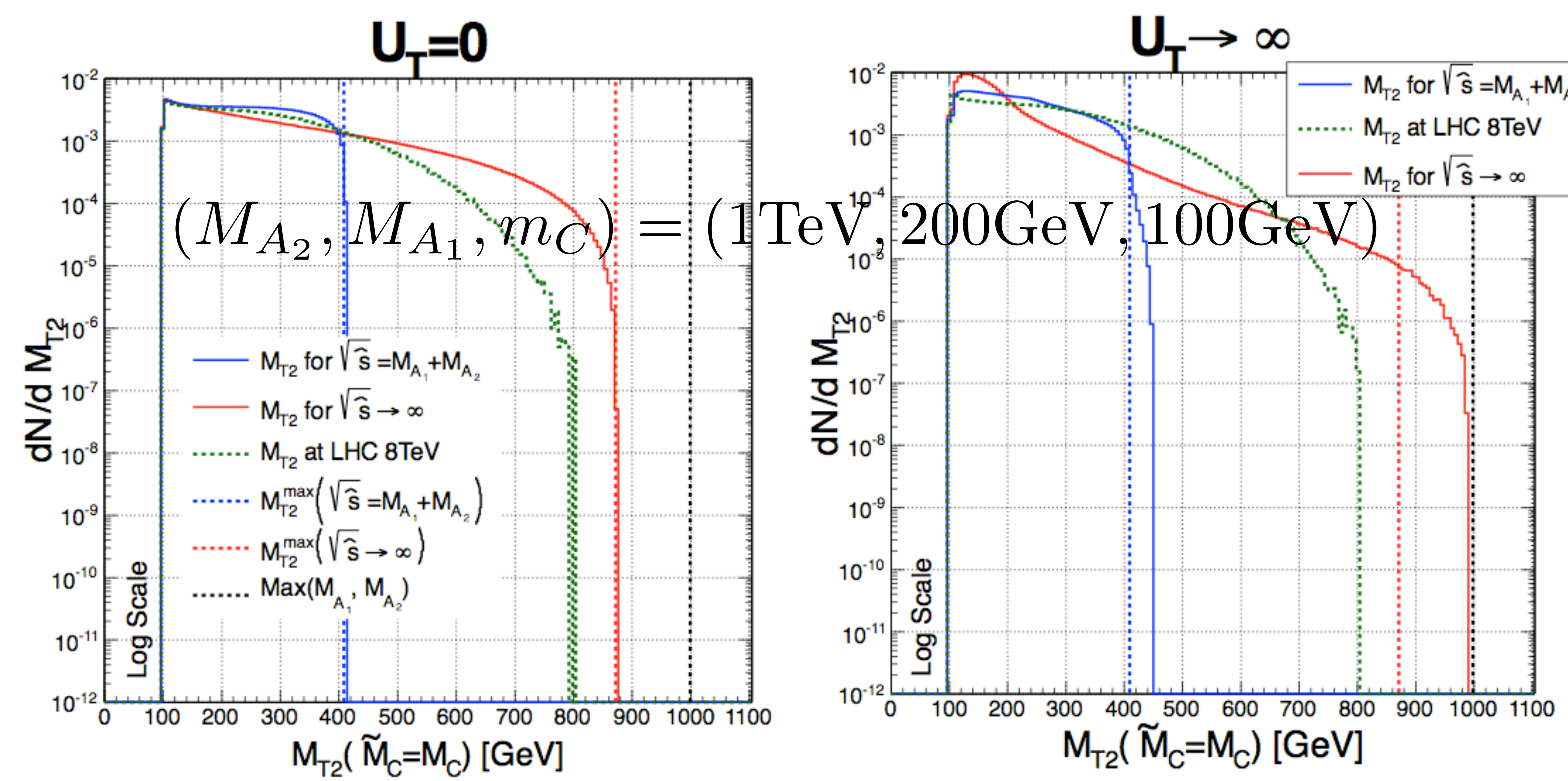




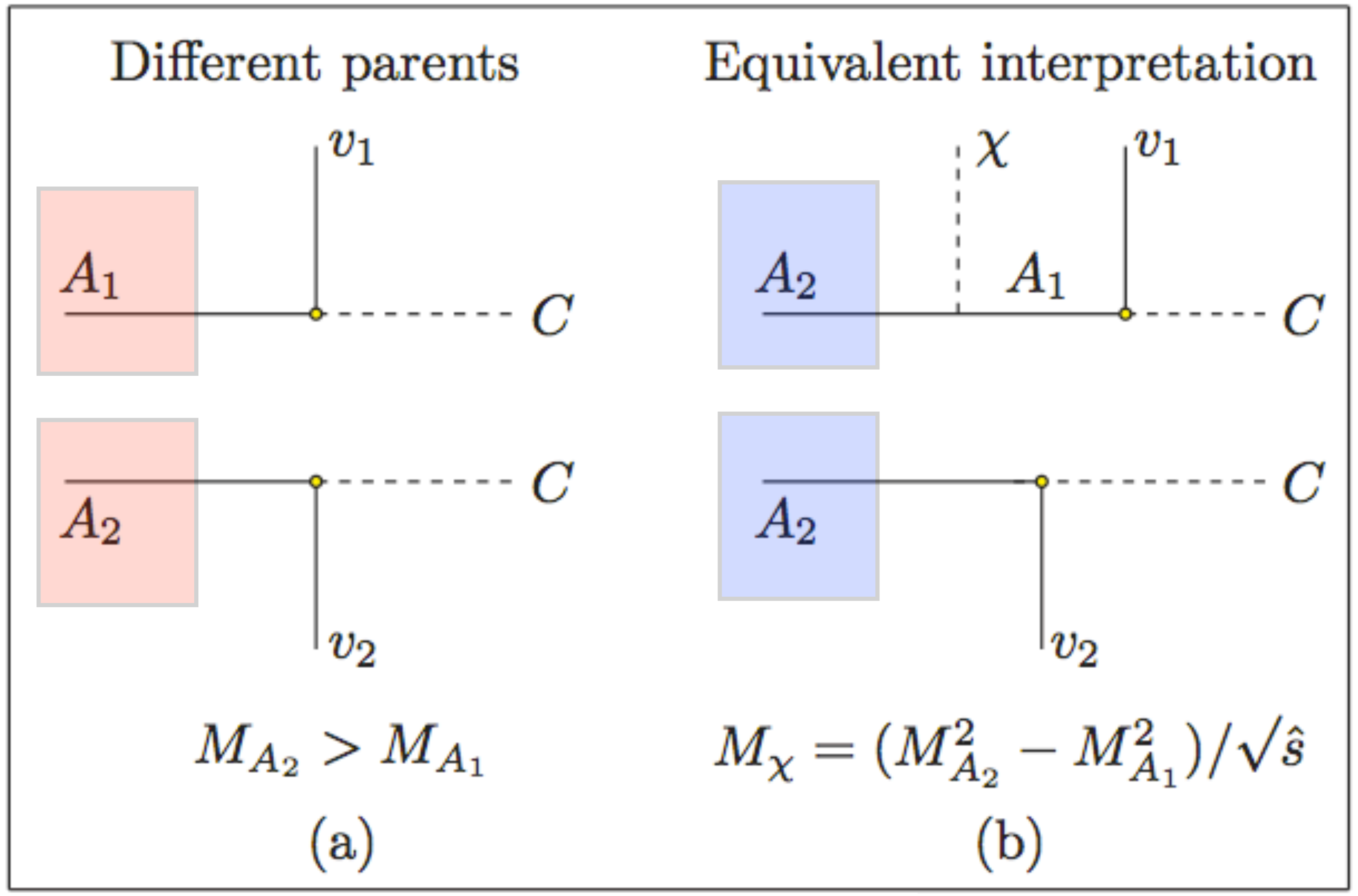
$0 \leq M_\chi(\sqrt{\hat{s}}) \leq M_{A_2} - M_{A_1}$ **with** $M_{A_1} + M_{A_2} \leq \sqrt{\hat{s}} < \infty$

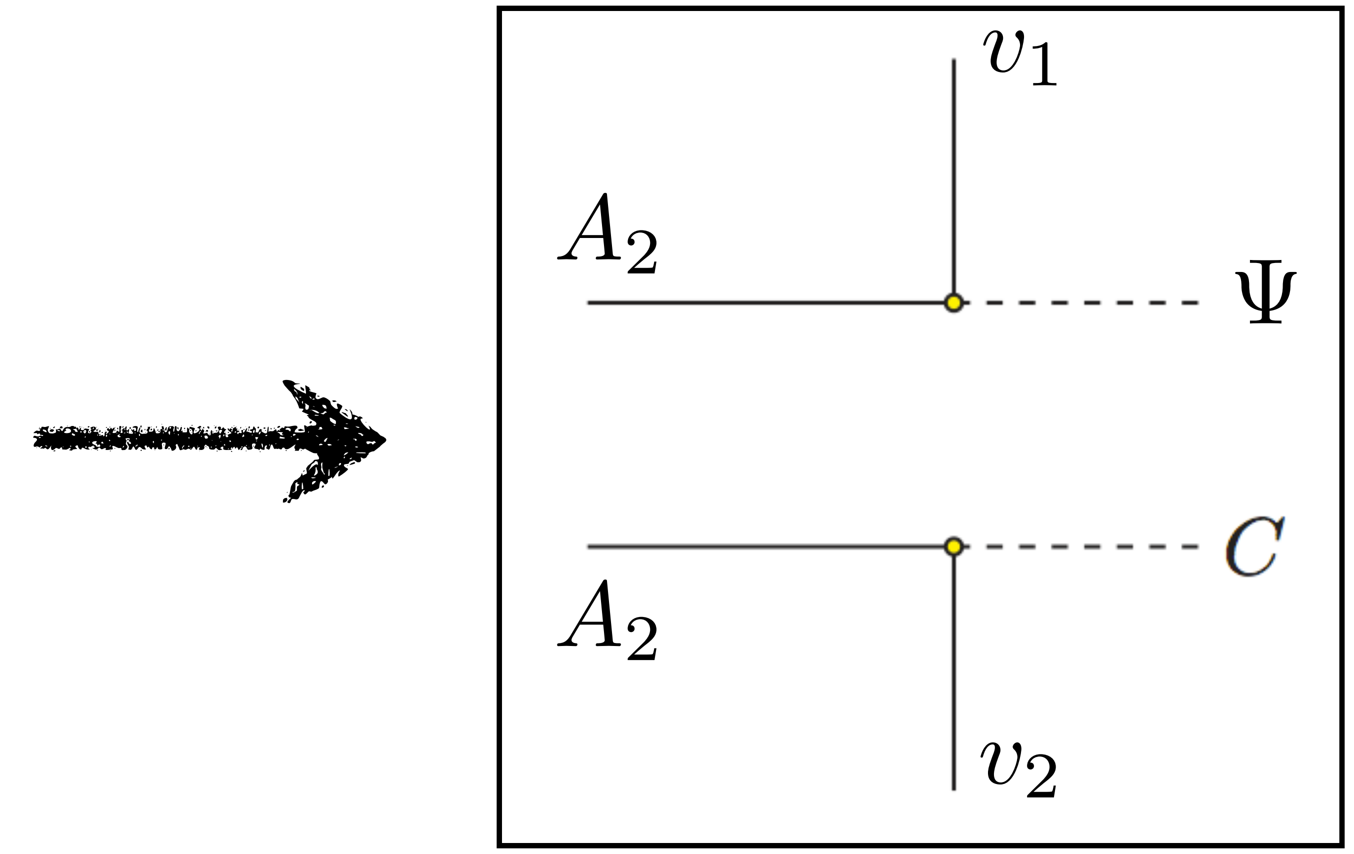
resulting in the effective particle Ψ's mass dependency on the $\sqrt{\hat{s}}$

In this example, we put the correct LSP mass for the input mass of MT2. But still MT2 distribution will be shifted by UT (Upstream Transverse momentum).
This is because the actual kinematics at the endpoint is described by Psi and C, instead of single C.



$(M_{A_2}, M_{A_1}, m_C) = (1\mathrm{TeV}, 200\mathrm{GeV}, 100\mathrm{GeV})$



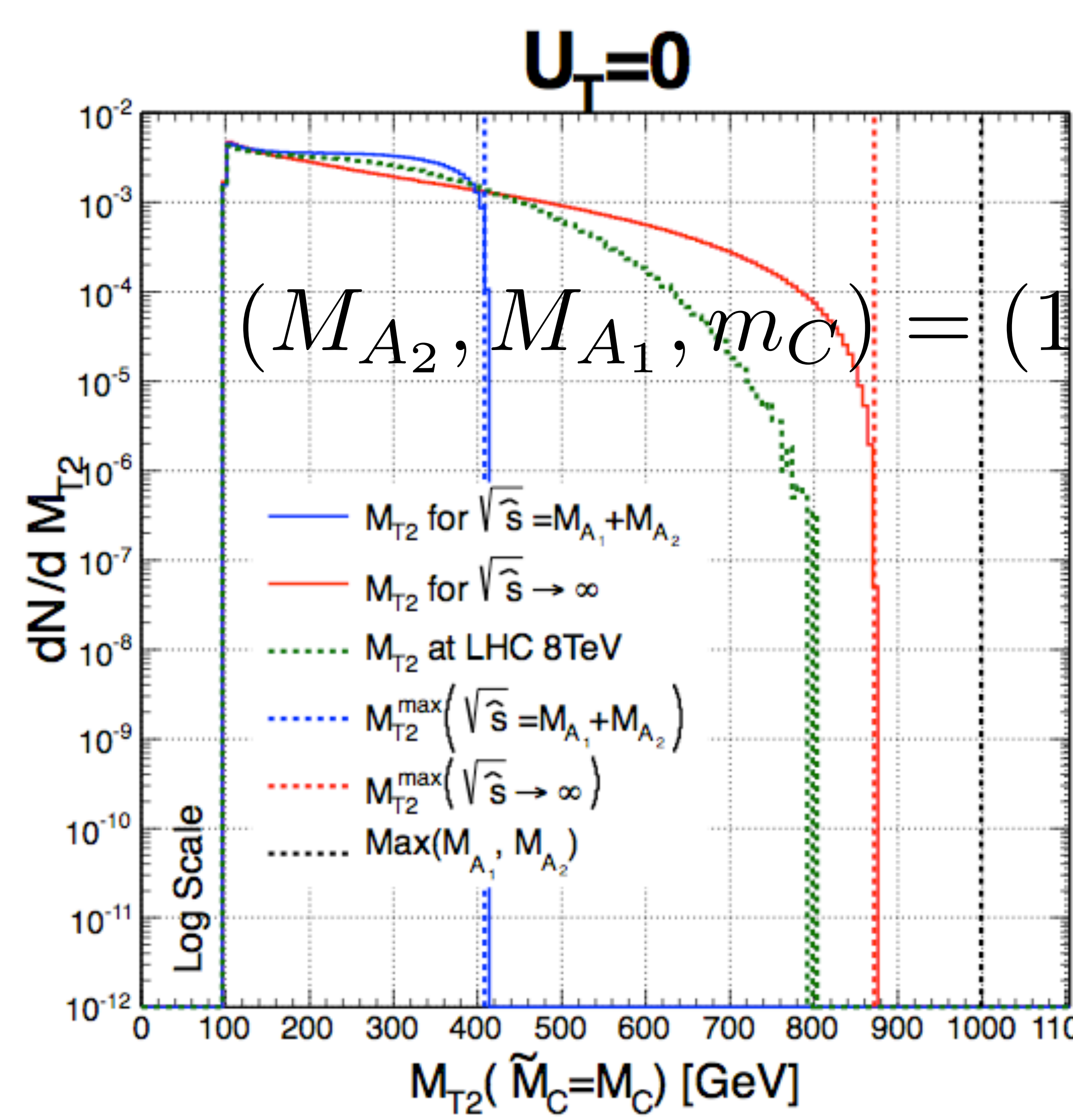


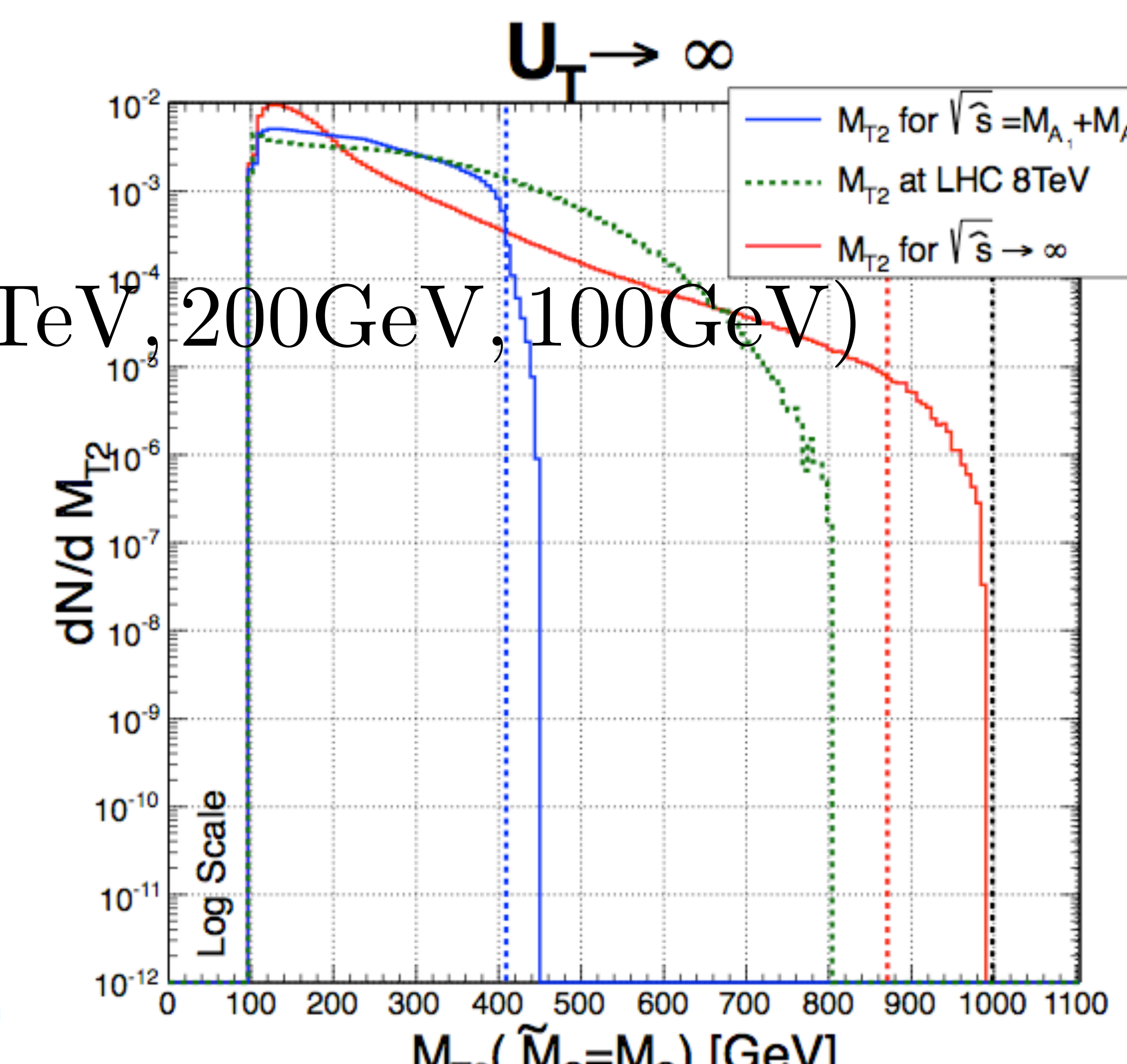
$$0 \leq M_\chi(\sqrt{\hat{s}}) \leq M_{A_2} - M_{A_1}$$ **with** $$M_{A_1} + M_{A_2} \leq \sqrt{\hat{s}} < \infty$$

resulting in the effective particle Ψ's mass dependency on the $\sqrt{\hat{s}}$

Previously it was believed that M_{T2} with different parent mass is bounded by the largest mass (M_{A2}).
We showed that only when UT and $\sqrt{\hat{s}}$ goes to infinity, M_{T2} is getting close to the M_{A2}

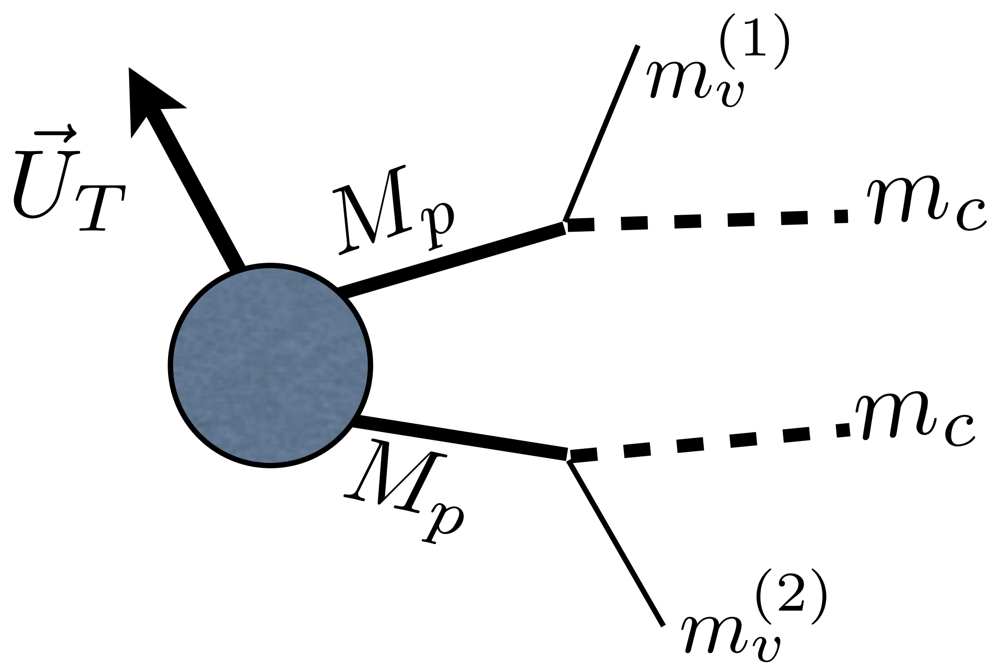
$$(M_{A_2}, M_{A_1}, m_C) = (1\text{TeV}, 200\text{GeV}, 100\text{GeV})$$

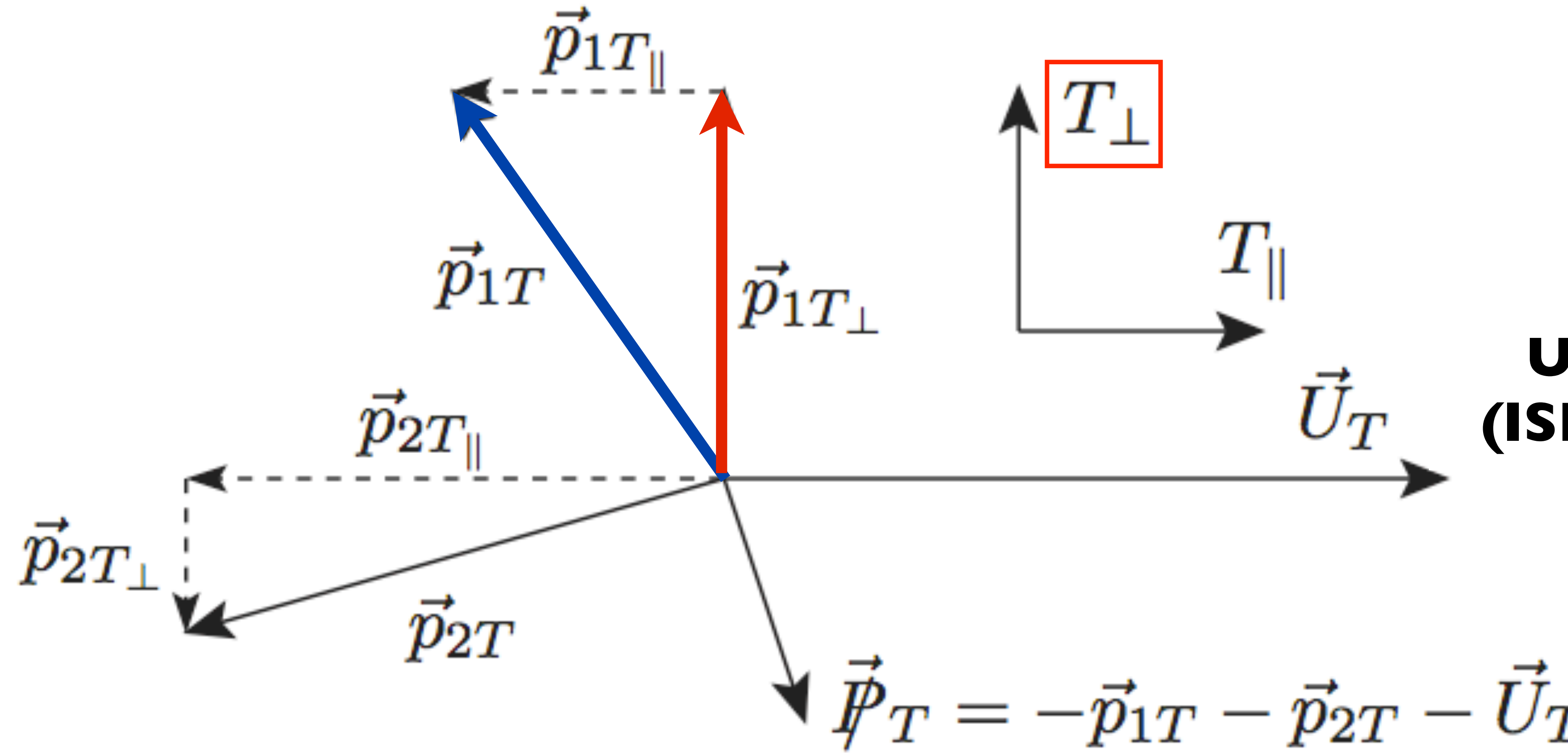




Shape of MT2 (near endpoint)

- By now, we focused on the endpoint.
- Studying the shape near the endpoint is rather difficult due to the dependency of MT2 on UTM (upstream transverse momentum) from ISR or visible particles in the upper levels.
- Thus we will project visible particles' transverse momentum to the direction (orthogonal direction to UTM)



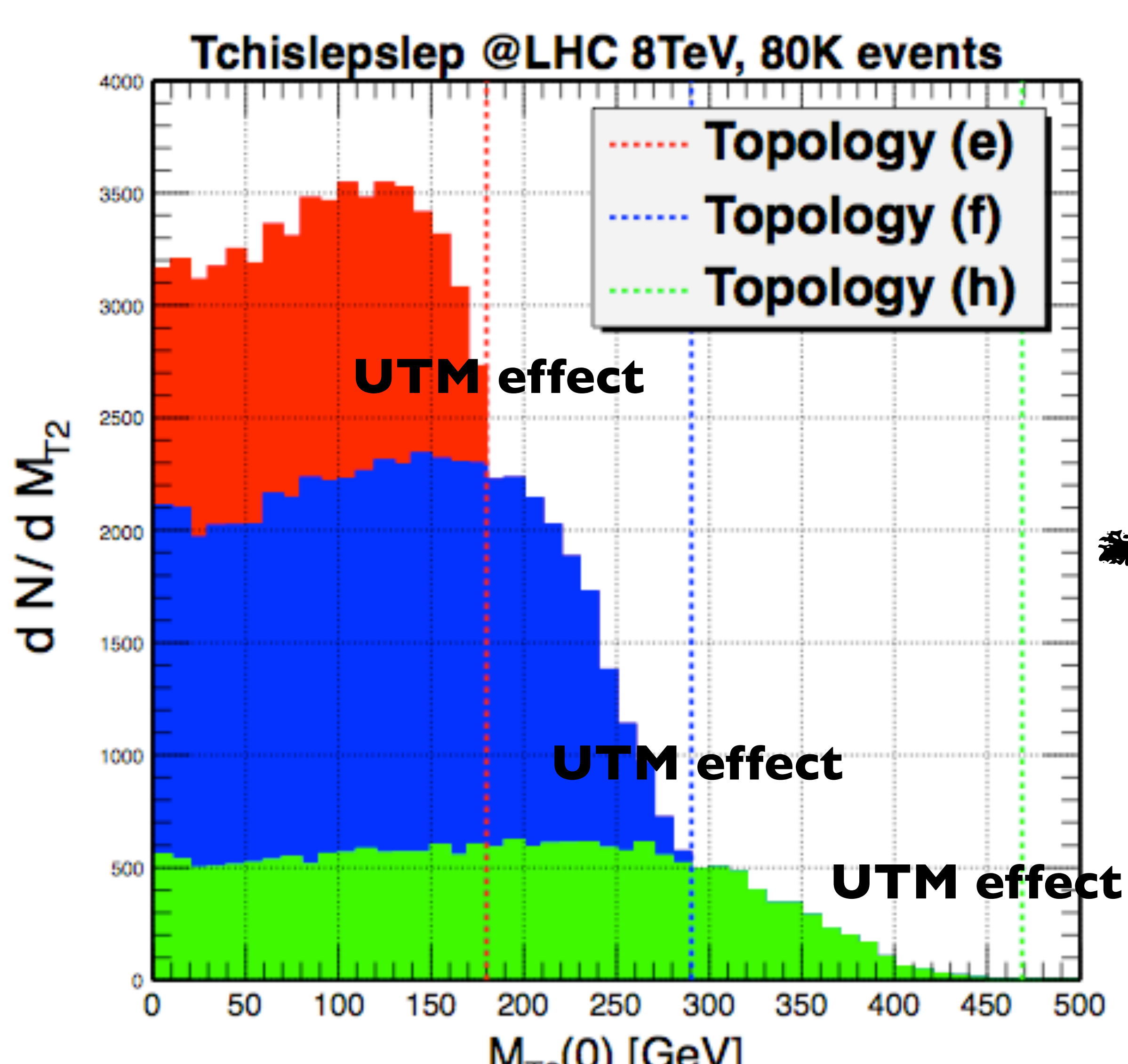


Transverse plane w.r.t the beam direction

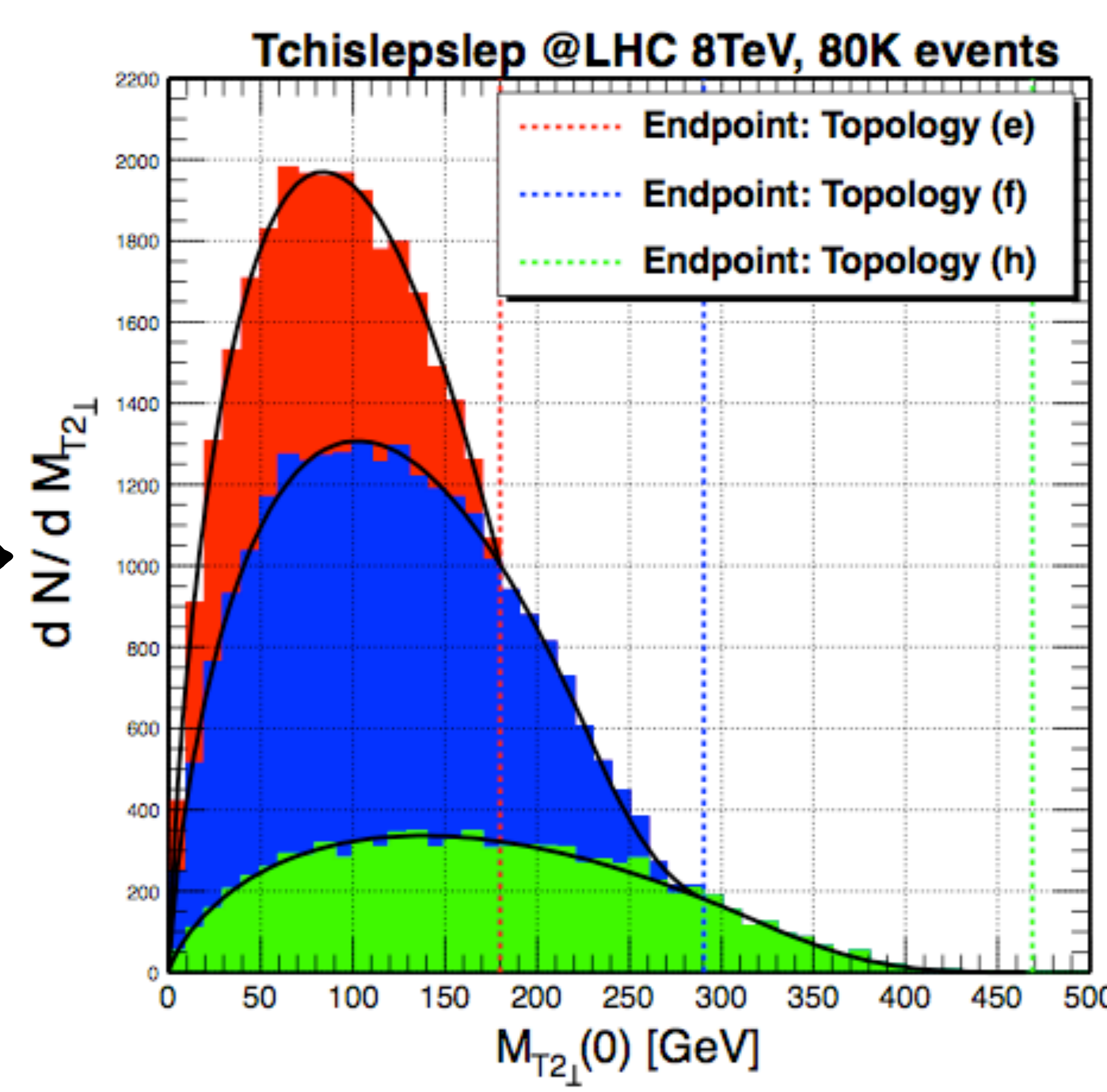
Upstream transverse momentum (ISR or bjets for leptonic subsystem)

K. Matchev, MP (arxiv:0910.1584)

Use only these one dimensional momentum of visible particles for MT2



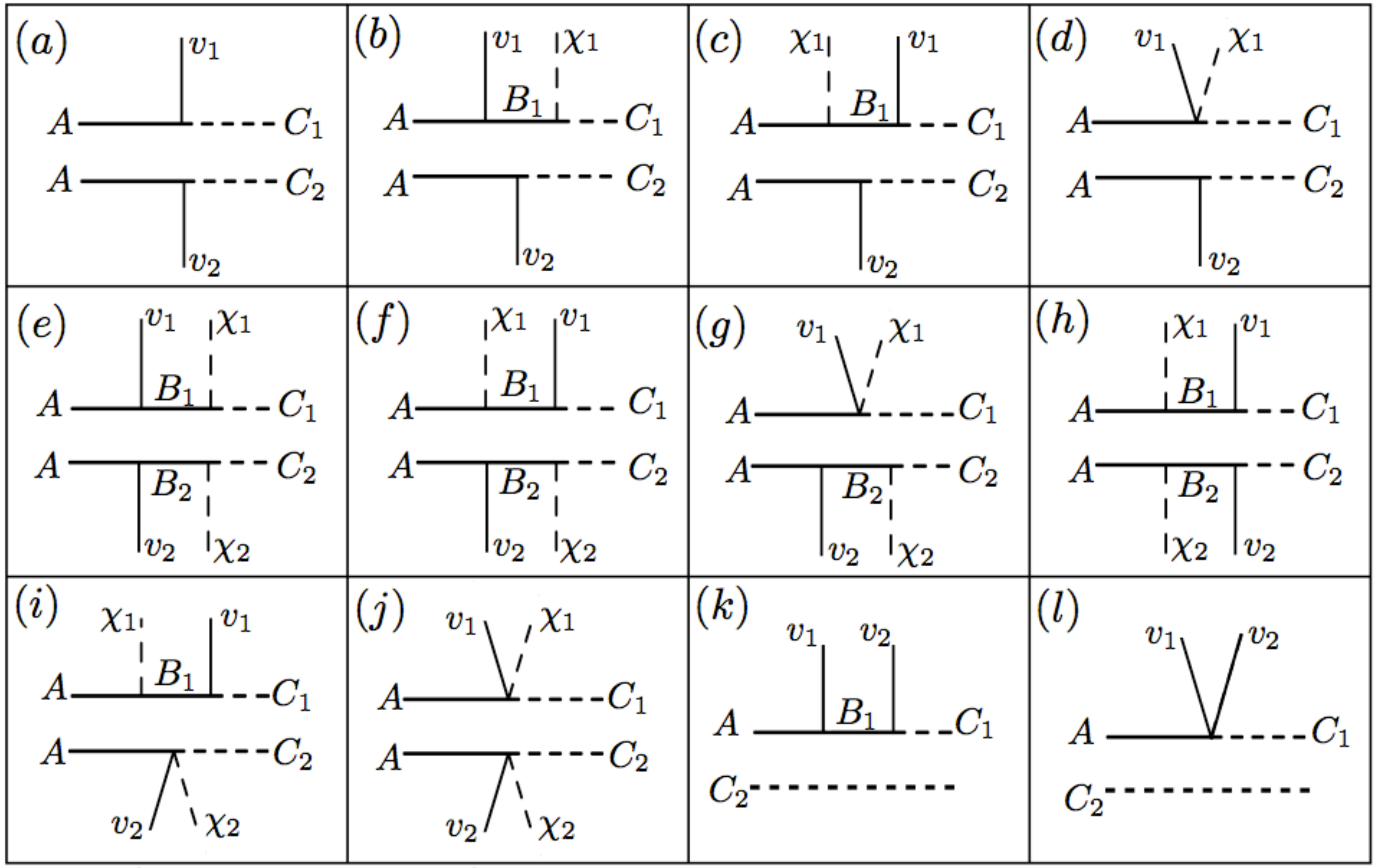
With full transverse information

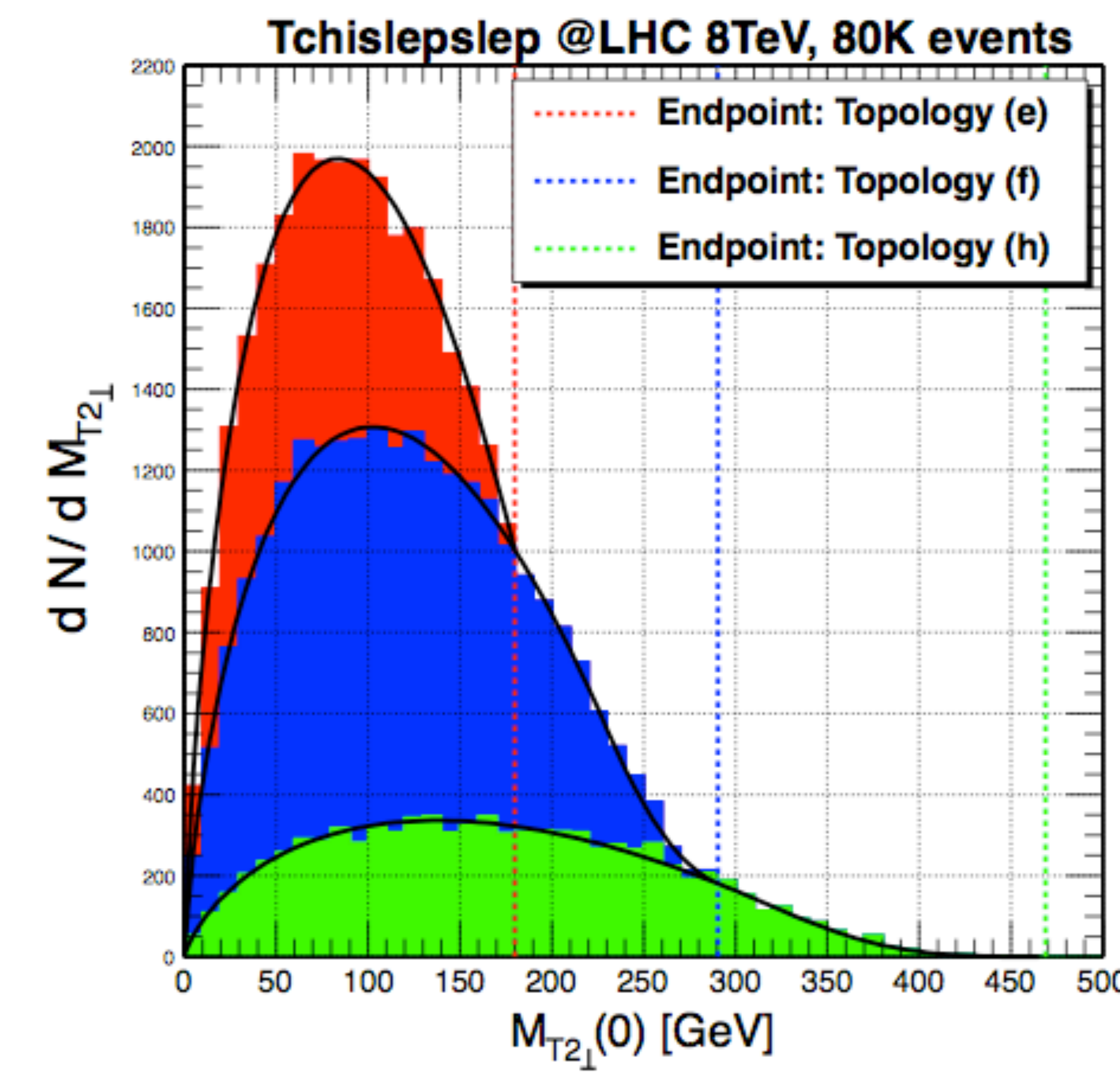


With only 1D information

- With this projection, we quantify the slope near the endpoint.

As we can see when there are invisible upstream momentum in front of visible particle, the distribution get more tail-like structure and will **not** be saturated at the endpoint.





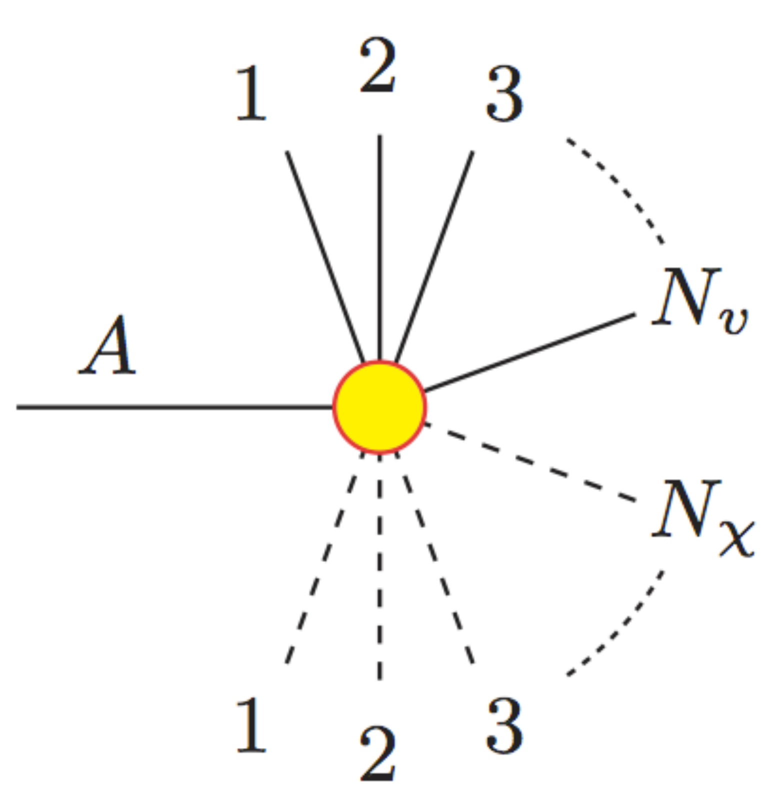
N_{inv}	On-shell topologies: topology	On-shell topologies: near-endpoint behavior	Off-shell topologies: topology	Off-shell topologies: near-endpoint behavior
2	(a)	ϵ		
3	(c),(f)	ϵ^2	(d),(g)	ϵ^3
4	(h)	ϵ^3	(i)	ϵ^4
			(j)	ϵ^5

ϵ : Distance near the endpoint

Though this behavior near the endpoint is with MT2(one-dim), we can get hint how MT2 shape will depend on the event-topology.

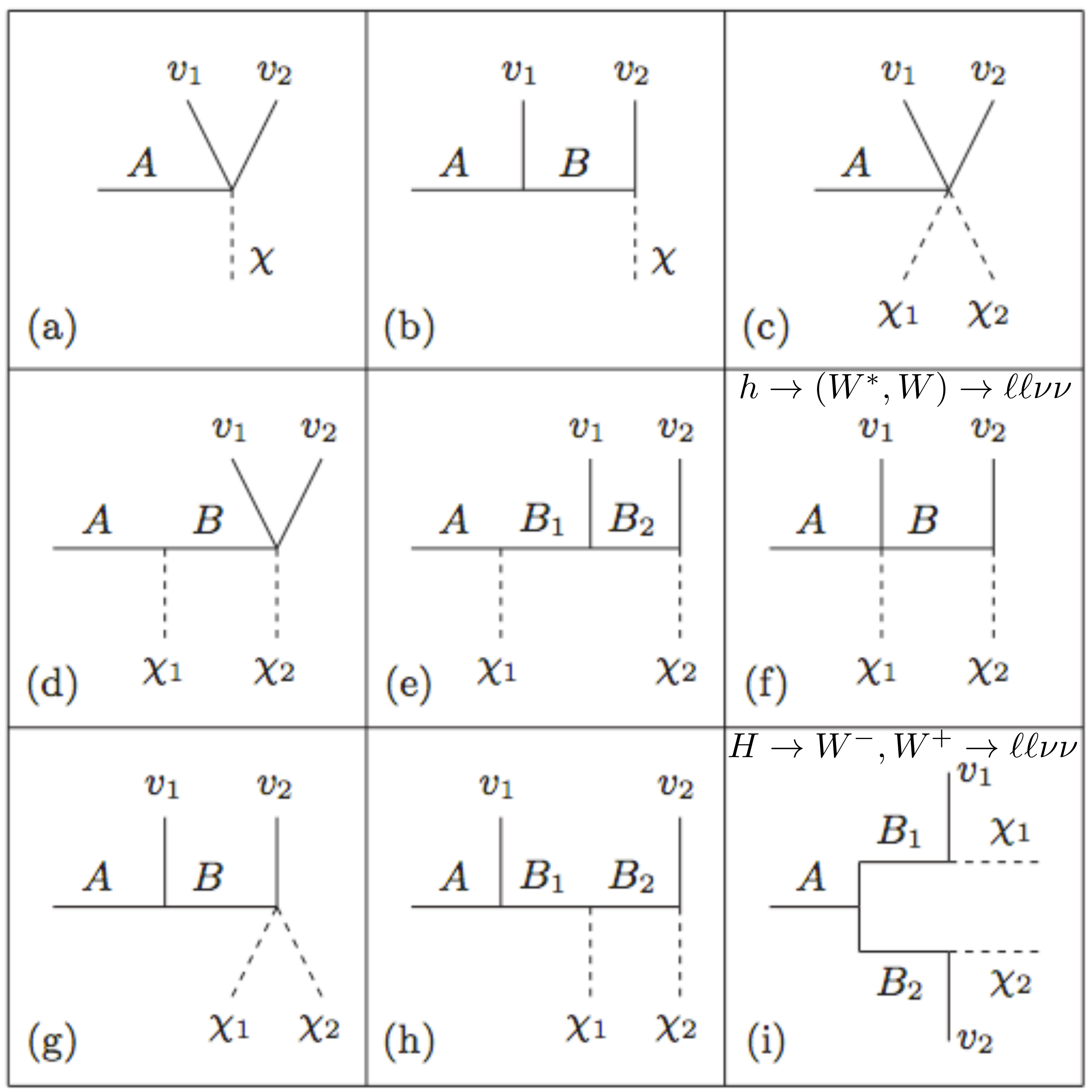
Invariant mass

- Similar concept can be applied for the invariant mass variable.
- There will be many diagrams with given number of visible particles.

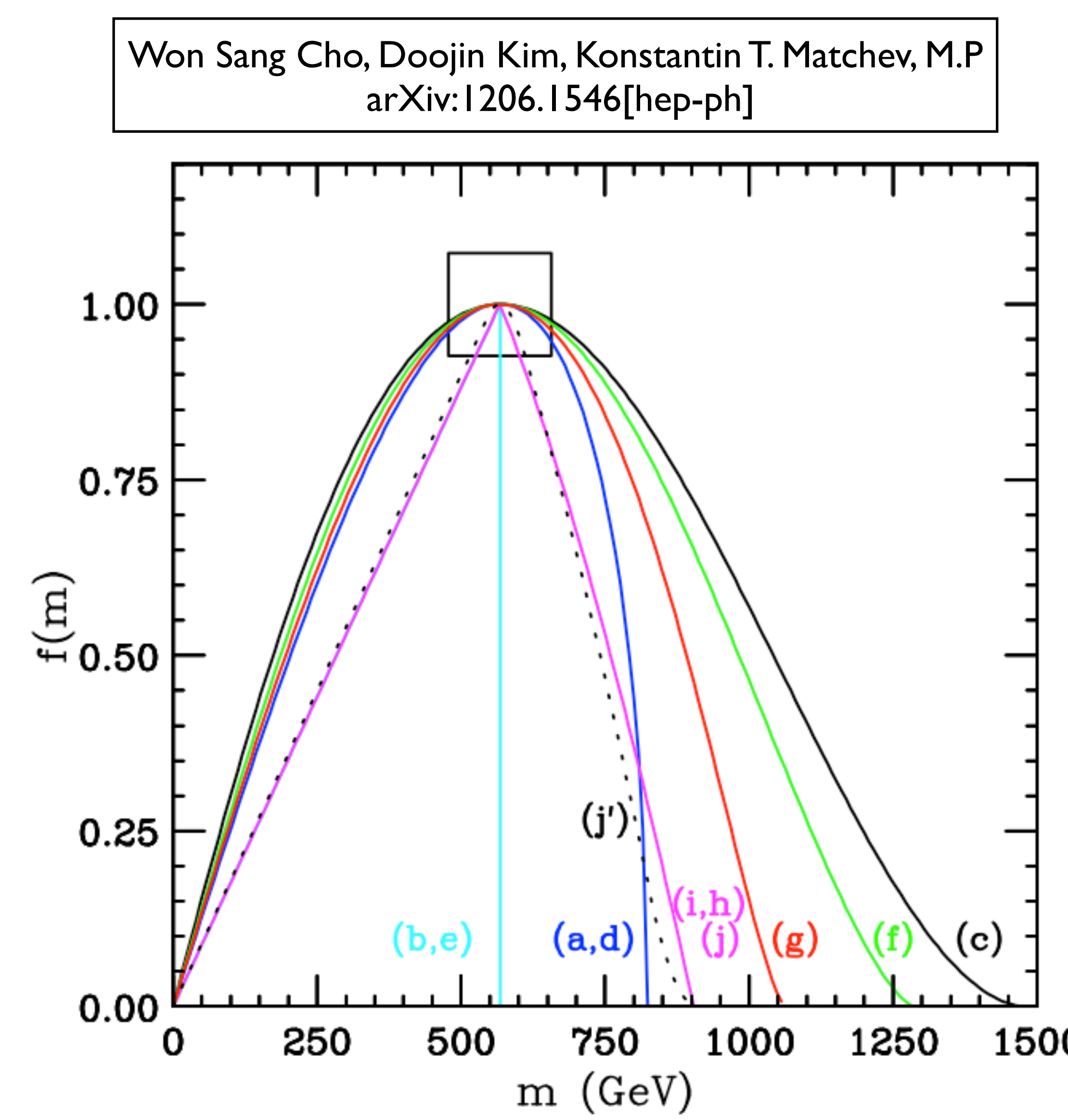


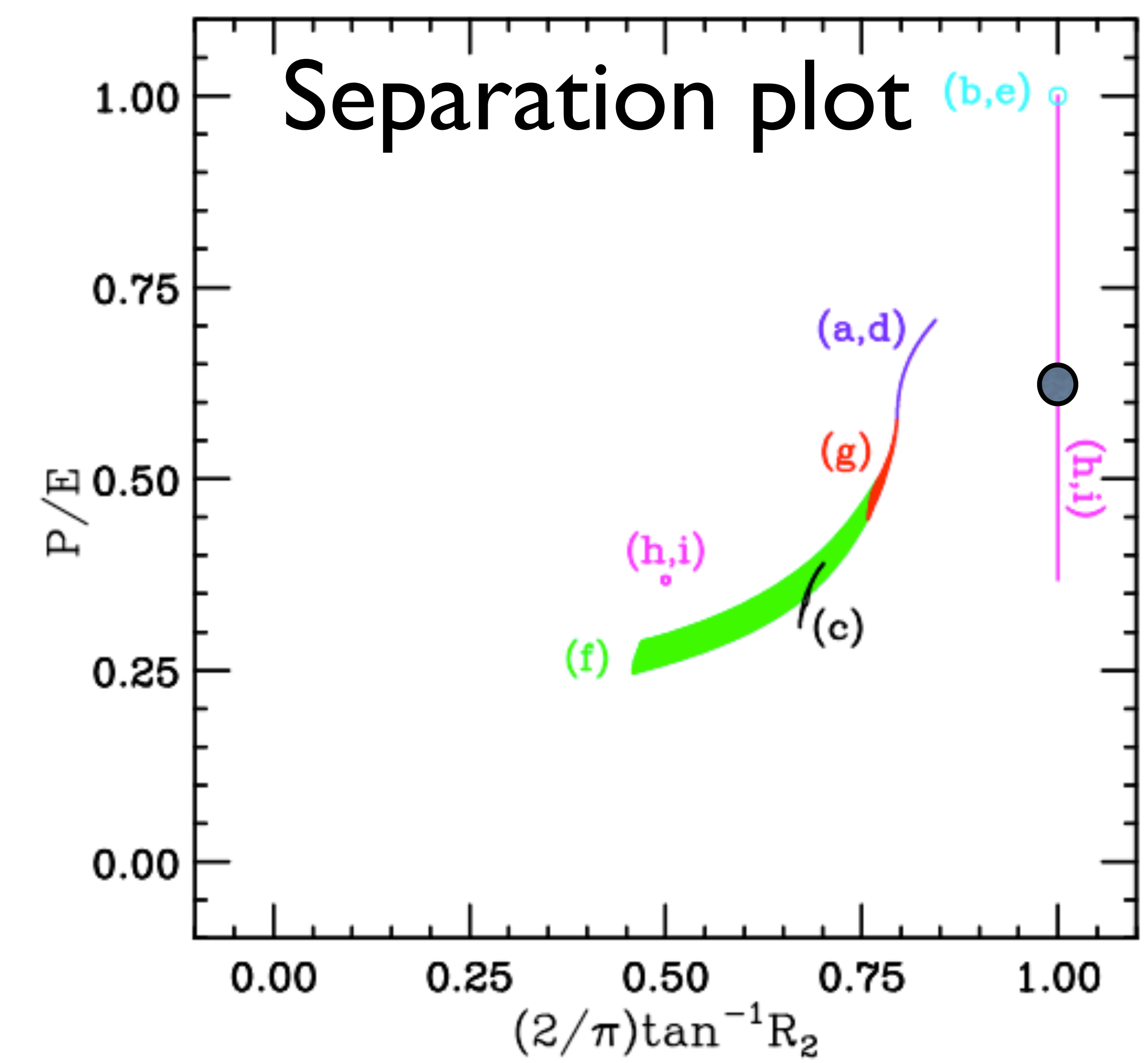
N_v \ N_χ	1	2	3	4	5
1	1	2	4	8	16
2	2	7	20	55	142
3	4	20	78	270	860
4	8	55	270	1138	4294

Example:two visible particles



We focus on the PEAK location and the curvature (R2) at the PEAK to characterize the shape of invariant mass distributions.





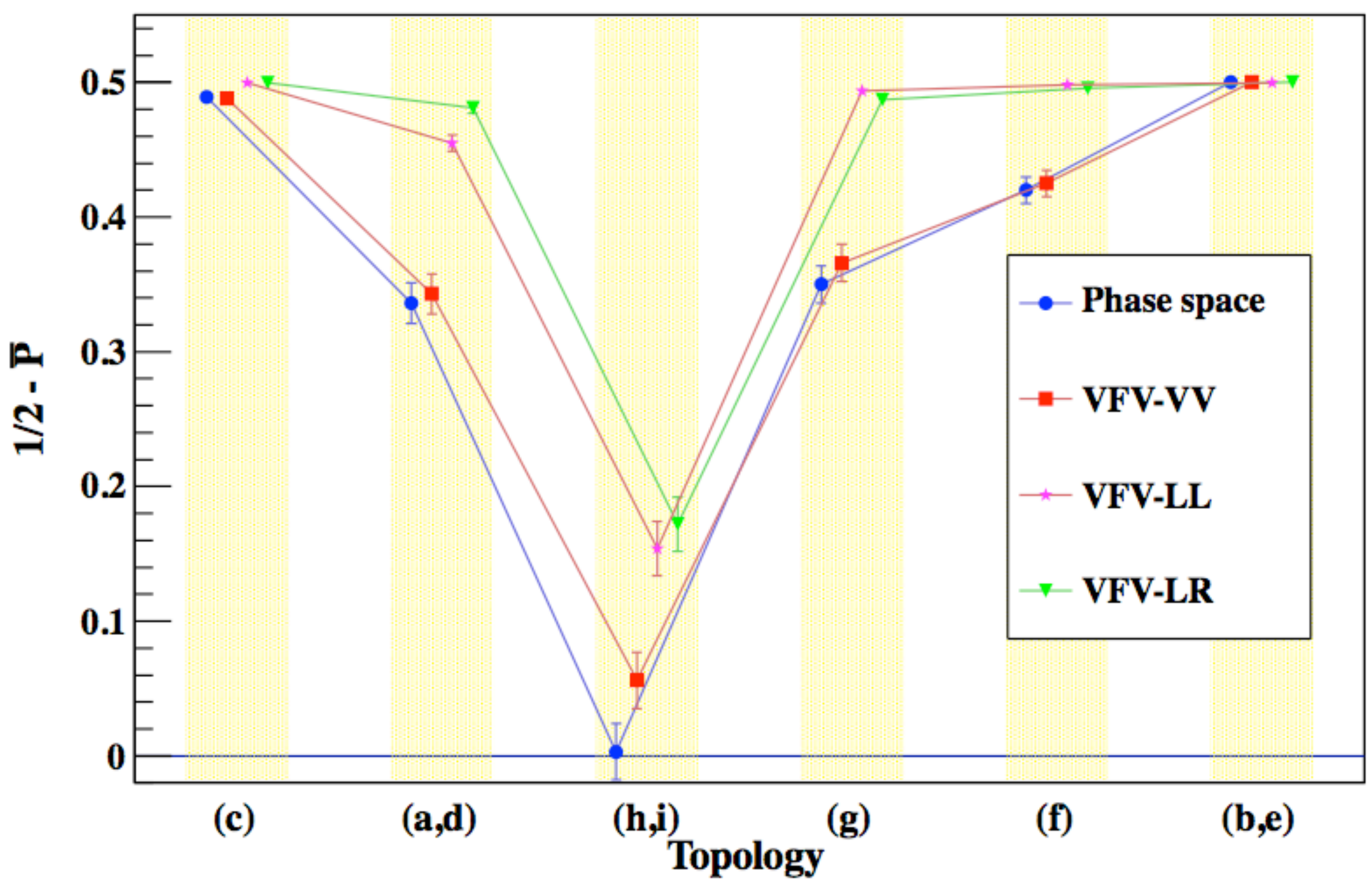
Data	(a,d)	(b,e)	(c)	(f)	(g)	(h,i)
(i)	698	37	96	275	698	∞

Number of events that we need to have to discriminate topology with 97% confidence level.

Conclusion

- To have more precise measurements on new physics or to be less dependent on systematic uncertainties, we need to develop variables(observables).
- But as I showed, there will be no SUPER variables and we need to understand their limits before applying these variables to the real data.
- We provide understanding of limits for MT2 and invariant mass variables in the analytical level.
- To overcome limits of MT2, we are developing more constrained variables for each event-topology (Tailor's variable)

Back up



Results from a quantitative topology disambiguation exercise using χ^2 as a test statistics.

- Data from (i) [antler topology]
To see the effect of spins, we simulated (i) topology with Vector > Fermion+Fermion, Fermion> visible+V

- Average p-value obtained in 200 pseudo experiments, each pseudo experiments has 500 signal events.
[parton level, no Detector simulation
to single out the effect of spin correlation]