Deformed supersymmetric gauge theories from String and M-Theory: Introduction to the fluxtrap background

Susanne Reffert

based on work with D. Orlando, S. Hellerman, N. Lambert
arXiv:1106.2097, 1108.0644, 1111.4811, 1204.4192, 1210.7805, 1304.3488, 1309.7350, work in progress
Introduction

In recent years, \textit{N}=2 supersymmetric gauge theories and their deformations have played an important role in theoretical physics - \textit{very active research topic}.
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2. The deformed gauge theories in question can be realized in string theory via the **fluxtrap background**!
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2. The deformed gauge theories in question can be realized in string theory via the fluxtrap background!

   The string theory construction provides a unifying framework and a different point of view on the gauge theory problems.
Realize **deformed** supersymmetric gauge theories via **string theory**. Gauge theories encode fluctuations on the world-volume of D-branes. Many parameters can be tuned by varying brane geometry.
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⇒ different brane set-ups give rise to different gauge theories with seemingly unrelated deformations!

Use the fluxtrap construction to **unify** and meaningfully **relate** and **reinterpret** a large variety of existing results.
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Our string theoretic approach enables us moreover to generate new deformed gauge theories in a simple and algorithmic way.
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- 2d effective gauge theories with deformations
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Fluxtrap background as toolbox to generate deformed gauge theories and analyze them via string theoretic methods.
Introduction
Introduction
Introduction
Introduction

M-theory

string theory

FT

MFT

MFB

M5

D1

D2

D4

NS5

D3

NS5

Lift

Reduce

N=(2,2) w.

tw. masses

N=(8,8) w.

tw. masses

N=4 w.

real masses

N=2+

Ω-def.

N=2 SYM

Ω-def.

N=1 SYM

Ω-def. SW

Ω-def.

N=4 SYM

reciproc. gauge th.

S-dual of

Ω-def. SW

Ω-def. N=2 SYM

2d

3d

4d

5d
Introduction
Introduction
Introduction
The same string theory background can give rise to many different deformations depending on how we place branes in it!
Outline
Outline

• Introduction, Motivation
Outline

• Introduction, Motivation
• The Fluxbrane Background
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• Supersymmetry
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  • $\Omega$–deformed SW
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- Introduction, Motivation
- The Fluxbrane Background
- Supersymmetry
- The Fluxtrap Background
- M-theory Lift
- Deformed gauge theories
  - $\Omega$–deformed SW
- Summary
The Fluxtrap
Background
The Fluxtrap Background

M-theory

string theory

gauge theory

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M5

MFB

T^3

Lift

Reduce

Reduce +T

N=(8,8) w. tw. masses

N=(2,2) w. tw. masses

N=4 w. real masses

N=2^*

Ω-def. N=2 SYM

Ω-def. N=1 SYM

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reciproc. gauge th.

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N=2 SYM

S-dual of Ω-def. SW

Ω-def. N=2 SYM
The Fluxtrap Background

M-theory

string theory

FT

MFT

MFB

M5

Lift

T³

Reduce

M5

topological string theory

gauge theory

FT

D1

D2

D3

D4

NS5

N=(2,2) w. tw. masses

N=(8,8) w. tw. masses

N=4 w. real masses

N=2* w. real masses

Ω-def. N=4 SYM

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N=2 SYM

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reciproc. gauge th.

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5d

4d

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2d

N=(8,8) w.

tw. masses

N=(2,2) w.

tw. masses

N=4 w.

real masses

N=2* w.

real masses

T³

Reduce

Topological string theory

Gauge theory

String theory

M-theory
The Fluxxtrap Background

- M-theory
- String theory
- Gauge theory

Fluxtrap Background:
- FT
- D1
- D2
- D3
- D4
- NS5

Reductions:
- Lift
- Reduce
- Reduce +T

Topological String Theory:
- N=(2,2) w. tw. masses
- N=(8,8) w. tw. masses
- N=4 w. real masses
- N=2 w. real masses
- N=2 w. complex masses

symplectic
- \(\Omega\)-def. SW
- \(\Omega\)-def. N=1 SYM
- \(\Omega\)-def. N=2 SYM
- \(\Omega\)-def. N=4 SYM

S-dual of
- \(\Omega\)-def. SW
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Lift
- M-theory
- String theory

Real Masses:
- N=4 w. real masses

Complex Masses:
- N=2 w. complex masses

Gauge Theory:
- \(\Omega\)-def. gauge th.

Dimensions:
- 2d
- 3d
- 4d
- 5d
The Fluxbrane Background

Geometrical realization of Nekrasov's construction of the equivariant gauge theory.
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Geometrical realization of Nekrasov's construction of the equivariant gauge theory.

Start with metric with 2 periodic directions and at least a $U(1) \times U(1)$ symmetry, no B-field, constant dilaton.
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Fluxbrane background with 3 independent deformation parameters:
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Fluxbrane background with 3 independent deformation parameters:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ρ₁, θ₁)</td>
<td>(ρ₂, θ₂)</td>
<td>(ρ₃, θ₃)</td>
<td>(ρ₄, θ₄)</td>
<td>v</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluxbrane</td>
<td>ε₁</td>
<td>ε₂</td>
<td>ε₃</td>
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<tbody>
<tr>
<td>$(\rho_1, \theta_1)$</td>
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<td>$\epsilon_1$</td>
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<td>$\circ$</td>
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$\tilde{x}^8 \simeq \tilde{x}^8 + 2\pi \tilde{R}_8$

$\tilde{x}^9 \simeq \tilde{x}^9 + 2\pi \tilde{R}_9$
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<tr>
<td></td>
<td>$\rho_1, \theta_1$</td>
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<td>$\rho_3, \theta_3$</td>
<td>$\rho_4, \theta_4$</td>
<td>$\nu$</td>
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Impose identifications:

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<td></td>
<td></td>
<td>$T^2$</td>
<td></td>
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Impose identifications:

\[
\begin{align*}
\tilde{x}^8 & \simeq \tilde{x}^8 + 2\pi \tilde{R}_8 n_8 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^{R} \tilde{R}_8 n_8 \\
\tilde{x}^9 & \simeq \tilde{x}^9 + 2\pi \tilde{R}_9 n_9 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^{I} \tilde{R}_9 n_9
\end{align*}
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| fluxbrane | $\epsilon_1$ | $\epsilon_2$ | $\epsilon_3$ | $\epsilon_4$ | $\circ$ | $\circ$

Impose identifications:

- Fluxbrane parameters

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\begin{align*}
\tilde{x}^8 & \simeq \tilde{x}^8 + 2\pi \tilde{R}_8 n_8 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^R \tilde{R}_8 n_8
\end{align*}
\]

\[
\begin{align*}
\tilde{x}^9 & \simeq \tilde{x}^9 + 2\pi \tilde{R}_9 n_9 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^R \tilde{R}_9 n_9
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\[
\begin{array}{cccccccc}
\times & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
(\rho_1, \theta_1) & (\rho_2, \theta_2) & (\rho_3, \theta_3) & (\rho_4, \theta_4) & & & & & & \\
\hline
\text{fluxbrane} & \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 & & & & & & \\
\end{array}
\]

\[
\begin{array}{c}
\tilde{x}^8 \simeq \tilde{x}^8 + 2\pi \tilde{R}_8 \\
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\end{array}
\]

Impose identifications: fluxbrane parameters

\[
\begin{align*}
\tilde{x}^8 & \simeq \tilde{x}^8 + 2\pi \tilde{R}_8 n_8 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^R \tilde{R}_8 n_8 \\
\tilde{x}^9 & \simeq \tilde{x}^9 + 2\pi \tilde{R}_9 n_9 \\
\theta_k & \simeq \theta_k + 2\pi \epsilon_k^I \tilde{R}_9 n_9 \\
\end{align*}
\]

This corresponds to the well-known Melvin or fluxbrane background.
Introduce new angular variables with disentangled periodicities:
The Fluxbrane Background

Introduce new angular variables with disentangled periodicities:

\[ \phi_k = \theta_k - \epsilon_k^R \tilde{x}^8 - \epsilon_k^I \tilde{x}^9 = \theta_k - \Re(\epsilon_k \tilde{v}) \]

\[ \epsilon_k = \epsilon_k^R + i \epsilon_k^I \]

\[ \tilde{v} = \tilde{x}^8 + i \tilde{x}^9 \]
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\[ \epsilon_k = \epsilon_k^R + i \epsilon_k^I \quad \tilde{v} = \tilde{x}^8 + i \tilde{x}^9 \]

Fluxbrane metric (\(T^2\)-fibration over \(\Omega\)-deformed \(\mathbb{R}^8\)): 

\[ \text{Fluxbrane metric (}T^2\text{-fibration over }\Omega\text{-deformed }\mathbb{R}^8\text{):} \]
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Introduce new angular variables with disentangled periodicities:

\[
\phi_k = \theta_k - \epsilon_k^R x^8 - \epsilon_k^I x^9 = \theta_k - \text{Re}(\epsilon_k \tilde{v})
\]

\[
\epsilon_k = \epsilon_k^R + i \epsilon_k^I \quad \tilde{v} = x^8 + i x^9
\]

Fluxbrane metric \((T^2\text{-fibration over } \Omega\text{-deformed } \mathbb{R}^8)\):

\[
ds^2 = d\tilde{x}_0^2 \cdots 7 - \frac{V^R_i V^R_j dx^i dx^j}{1 + V^R \cdot V^R} - \frac{V^R_i V^R_j dx^i dx^j}{1 + V^R \cdot V^R} \\
+ (1 + V^R \cdot V^R) \left[ (dx^8)^2 - \frac{V^R_i dx^i}{1 + V^R \cdot V^R} \right]^2 \\
+ (1 + V^I \cdot V^I) \left[ (dx^9)^2 - \frac{V^I_i dx^i}{1 + V^I \cdot V^I} \right]^2 + 2V^R \cdot V^I dx^8 dx^9
\]
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Introduce new angular variables with disentangled periodicities:

\[ \phi_k = \theta_k - \epsilon_k^R \tilde{x}^8 - \epsilon_k^I \tilde{x}^9 = \theta_k - \text{Re}(\epsilon_k \tilde{v}) \]

\[ \epsilon_k = \epsilon_k^R + i \epsilon_k^I \]

\[ \tilde{v} = \tilde{x}^8 + i \tilde{x}^9 \]

Fluxbrane metric \((T^2\text{-fibration over } \Omega\text{-deformed } \mathbb{R}^8)\):

\[
\begin{align*}
    ds^2 &= d\tilde{x}_0^2 \ldots 7 - \frac{V_i^R V_j^R \, dx^i \, dx^j}{1 + V^R \cdot V^R} - \frac{V_i^R V_j^R \, dx^i \, dx^j}{1 + V^R \cdot V^R} \\
    &+ (1 + V^R \cdot V^R) \left[ (dx^8)^2 - \frac{V_i^R \, dx^i}{1 + V^R \cdot V^R} \right]^2 \\
    &+ (1 + V^I \cdot V^I) \left[ (dx^9)^2 - \frac{V_i^I \, dx^i}{1 + V^I \cdot V^I} \right]^2 + 2V^R \cdot V^I \, dx^8 \, dx^9
\end{align*}
\]

Generator of rotations:
The Fluxbrane Background

Introduce new angular variables with disentangled periodicities:

\[ \phi_k = \theta_k - \epsilon_k^R \varepsilon^8 - \epsilon_k^I \varepsilon^9 = \theta_k - \text{Re}(\epsilon_k \tilde{v}) \]

\[ \epsilon_k = \epsilon_k^R + i \epsilon_k^I \quad \tilde{v} = \varepsilon^8 + i \varepsilon^9 \]

Fluxbrane metric \((T^2\text{-fibration over } \Omega\text{-deformed } \mathbb{R}^8)\):

\[
\begin{align*}
  ds^2 &= d\tilde{x}_0^2 - \frac{V_i^R V_j^R \, dx^i \, dx^j}{1 + V_R \cdot V_R} - \frac{V_i^R V_j^R \, dx^i \, dx^j}{1 + V_R \cdot V_R} \\
       &\quad + (1 + V_R \cdot V_R) \left[ (dx^8)^2 - \frac{V_i^R \, dx^i}{1 + V_R \cdot V_R} \right]^2 \\
       &\quad + (1 + V_I \cdot V_I) \left[ (dx^9)^2 - \frac{V_i^I \, dx^i}{1 + V_I \cdot V_I} \right]^2 + 2V_R \cdot V_I \, dx^8 \, dx^9
\end{align*}
\]

Generator of rotations:

\[ V = V^R + i V^I = \epsilon_1 \left( x^1 \, \partial_0 - x^0 \, \partial_1 \right) + \epsilon_2 \left( x^3 \, \partial_2 - x^2 \, \partial_3 \right) + \epsilon_3 \left( x^5 \, \partial_4 - x^4 \, \partial_5 \right) + \epsilon_4 \left( x^7 \, \partial_6 - x^6 \, \partial_7 \right) \]
Supersymmetry

To find preserved supersymmetries, start with the 32 (IIB)constant Killing spinors of flat space and project out those that are not compatible with the fluxbrane identifications. General case breaks all supersymmetries.
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To find preserved supersymmetries, start with the 32 (IIB) constant Killing spinors of flat space and project out those that are not compatible with the fluxbrane identifications. General case breaks all supersymmetries.

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annihilated by projector

\[ \Pi_{k}^{\text{flux}} = \frac{1}{2} (1 - \gamma_{\rho_k \theta_k} \gamma_{\rho_N \theta_N}) \]
Supersymmetry

Find preserved Killing spinor in fluxbrane BG which respects the boundary conditions:
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number of deformation parameters
The Fluxtrap Background

T-dualize along torus directions and take decompactification limit to discard torus momenta:
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*Fluxtrap background*
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Before T-duality, locally, the metric was still flat, but some of the rotation symmetries were broken globally.
The Fluxtrat Background

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Bulk fields after T-duality (case $V^R \cdot V^I = 0$, $\epsilon_1 \in \mathbb{R}$, $\epsilon_2 \in i \mathbb{R}$, $\epsilon_3 = \epsilon_4 = 0$):
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\[
\begin{align*}
    ds^2 &= d\rho_1^2 + \frac{\rho_1^2 d\phi_1^2 + dx_8^2}{1 + \epsilon_1^2 \rho_1^2} + d\rho_2^2 + \frac{\rho_2^2 d\phi_2^2 + dx_9^2}{1 + \epsilon_2^2 \rho_2^2} + \sum_{k=4}^{7} (dx^k)^2, \\
    B &= \epsilon_1 \frac{\rho_1^2}{1 + \epsilon_1^2 \rho_1^2} d\phi_1 \wedge dx_8 + \epsilon_2 \frac{\rho_2^2}{1 + \epsilon_2^2 \rho_2^2} d\phi_2 \wedge dx_9, \\
    e^{-\Phi} &= \frac{\sqrt{\alpha'} e^{-\Phi_0}}{R} \sqrt{(1 + \epsilon_1^2 \rho_1^2)(1 + \epsilon_2^2 \rho_2^2)}
\end{align*}
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The Fluxtrap Background

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B-field has appeared

creates a potential that localizes instantons
The Fluxtrap Background

Study resulting geometry.
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The Fluxtrap Background

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\[ \mathbb{R}^2 \]

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The Fluxtrap Background

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R-foliation over the cigar

cigar \( \langle \rho_1, \phi_1 \rangle \)

The generator of rotations is bounded (by asymptotic radius).
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Reuse result of fluxbrane BG.
The effect of T–duality is to multiply half of the preserved spinors by the gamma matrices in the directions of the T–dualities:

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\[
\Gamma_8 = g_8 \mu e_\mu a \gamma^a
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M-Theory Lift

Now we want to lift to \textit{M-theory}:
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\[ ds^2 = \left( \Delta_1 \Delta_2 \right)^{2/3} \left[ d\rho_1^2 + \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} d\sigma_1^2 + \frac{dx_8^2}{1 + \epsilon_1^2 \rho_1^2} + d\rho_2^2 + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} d\sigma_2^2 + \frac{dx_9^2}{1 + \epsilon_2^2 \rho_2^2} + d\rho_3^2 + \rho_3^2 d\psi^2 + dx_6^2 + dx_7^2 \right] + \left( \Delta_1 \Delta_2 \right)^{-4/3} dx_{10}^2, \]

\[ A_3 = \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} d\sigma_1 \wedge dx_8 \wedge dx_{10} + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} d\sigma_2 \wedge dx_9 \wedge dx_{10} \]
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    &\quad + d\rho_3^2 + \rho_3^2 d\psi^2 + dx_6^2 + dx_7^2 \right] + (\Delta_1 \Delta_2)^{-4/3} dx_{10}^2,
\end{align*}
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\]

\[
\sigma_i = \frac{\phi_i}{\epsilon_i} \quad \Delta_i^2 = 1 + \epsilon_i^2 \rho_i^2 \quad x_{10} = x_{10} + 2\pi R_{10}
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Consider only linear order in \( \epsilon \):
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\mathrm{d}s^2 &= (\Delta_1 \Delta_2)^{2/3} \left[ \mathrm{d}\rho_1^2 + \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \, \mathrm{d}\sigma_1^2 + \frac{\mathrm{d}x_8^2}{1 + \epsilon_1^2 \rho_1^2} + \mathrm{d}\rho_2^2 + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \, \mathrm{d}\sigma_2^2 + \frac{\mathrm{d}x_9^2}{1 + \epsilon_2^2 \rho_2^2} \\
&\quad + \mathrm{d}\rho_3^2 + \rho_3^2 \, \mathrm{d}\psi^2 + \mathrm{d}x_6^2 + \mathrm{d}x_7^3 \right] + (\Delta_1 \Delta_2)^{-4/3} \, \mathrm{d}x_{10}^2, \\
A_3 &= \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \, \mathrm{d}\sigma_1 \wedge \mathrm{d}x_8 \wedge \mathrm{d}x_{10} + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \, \mathrm{d}\sigma_2 \wedge \mathrm{d}x_9 \wedge \mathrm{d}x_{10} \\
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\end{align*}
\]

Consider only linear order in \( \epsilon \):

\[
\begin{align*}
g_{MN} &= \delta_{MN} + \mathcal{O}(\epsilon^2), \\
G_4 &= (\mathrm{d}\zeta + \mathrm{d}\bar{\zeta}) \wedge (\mathrm{d}s + \mathrm{d}\bar{s}) \wedge \omega
\end{align*}
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\[ ds^2 = (\Delta_1 \Delta_2)^{2/3} \left[ d\rho_1^2 + \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} d\sigma_1^2 + \frac{dx_8^2}{1 + \epsilon_1^2 \rho_1^2} + dp_2^2 + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} d\sigma_2^2 + \frac{dx_9^2}{1 + \epsilon_2^2 \rho_2^2} + d\rho_3^2 + \rho_3^2 d\psi^2 + dx_6^2 + dx_7^2 \right] + (\Delta_1 \Delta_2)^{-4/3} dx_{10}^2 , \]

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Consider only linear order in \( \epsilon \):

\[ g_{MN} = \delta_{MN} + \mathcal{O}(\epsilon^2) , \]
\[ G_4 = (dz + d\bar{z}) \wedge (ds + d\bar{s}) \wedge \omega \]
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Now we want to lift to **M-theory**:

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\begin{align*}
\text{ds}^2 &= (\Delta_1 \Delta_2)^{2/3} \left[ \text{d}\rho_1^2 \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \text{d}\sigma_1^2 + \frac{\text{d}x_8^2}{1 + \epsilon_1^2 \rho_1^2} + \text{d}\rho_2^2 \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \text{d}\sigma_2^2 + \frac{\text{d}x_9^2}{1 + \epsilon_2^2 \rho_2^2} \\
&\quad + \text{d}\rho_3^2 + \rho_3^2 \text{d}\psi^2 + \text{d}x_6^2 + \text{d}x_7^2 \right] + (\Delta_1 \Delta_2)^{-4/3} \text{d}x_{10}^2 ,
\end{align*}
\]

\[
A_3 = \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \text{d}\sigma_1 \wedge \text{d}x_8 \wedge \text{d}x_{10} + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \text{d}\sigma_2 \wedge \text{d}x_9 \wedge \text{d}x_{10}
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\[
z = x^8 + i x^9 \quad s = x^6 + i x^{10}
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Now we want to lift to M-theory:

\[ ds^2 = (\Delta_1 \Delta_2)^{2/3} \left[ d\rho_1^2 + \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \, d\sigma_1^2 + \frac{dx_8^2}{1 + \epsilon_1^2 \rho_1^2} + d\rho_2^2 + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \, d\sigma_2^2 + \frac{dx_9^2}{1 + \epsilon_2^2 \rho_2^2} + d\rho_3^2 + \rho_3^2 \, d\psi^2 + dx_6^2 + dx_7^3 \right] + (\Delta_1 \Delta_2)^{-4/3} \, dx_{10}^2, \]

\[ A_3 = \frac{\epsilon_1^2 \rho_1^2}{1 + \epsilon_1^2 \rho_1^2} \, d\sigma_1 \wedge dx_8 \wedge dx_{10} + \frac{\epsilon_2^2 \rho_2^2}{1 + \epsilon_2^2 \rho_2^2} \, d\sigma_2 \wedge dx_9 \wedge dx_{10} \]

\[ \sigma_i = \frac{\phi_i}{\epsilon_i}, \quad \Delta_i^2 = 1 + \epsilon_i^2 \rho_i^2, \quad x_{10} = x_{10} + 2\pi R_{10} \]

Consider only linear order in \( \epsilon \):

\[ g_{MN} = \delta_{MN} + \mathcal{O}(\epsilon^2), \]

\[ G_4 = (dz + d\bar{z}) \wedge (ds + d\bar{s}) \wedge \omega \]

\[ z = x^8 + i \, x^9 \quad s = x^6 + i \, x^{10} \]

\[ \omega = \epsilon_1 \, dx^0 \wedge dx^1 + \epsilon_2 \, dx^2 \wedge dx^3 + \epsilon_3 \, dx^4 \wedge dx^5 \]
Bulk Relations

Overview over duality web of FT BG in the bulk:
Overview over duality web of FT BG in the bulk:

Bulk Relations

fluxbrane $\overset{T\text{-duality}}{\longrightarrow}$ NS fluxtrap $\overset{S\text{-duality}}{\longrightarrow}$ RR fluxtrap

redox $\downarrow$

redox and T

redox

M–theory fluxbrane $\overset{SL_2 \times SL_3}{\longrightarrow}$ M–theory fluxtrap
Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap with respect to the monodromies:
Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap with respect to the monodromies:

Deformation not on brane world-volume:
  mass deformation
Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap \textit{with respect to the monodromies}:

Deformation \textbf{not} on brane world-volume: \textit{mass deformation}

\[
\begin{array}{ccc}
\text{fluxtrap} & \times & \times \\
\text{D-brane} & \times & \times \\
\end{array}
\]

\[
\phi_i \quad \epsilon_i \quad \epsilon_j
\]
Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap with respect to the monodromies:

Deformation not on brane world-volume: mass deformation

Deformation on brane world-volume: $\Omega$-type deformation, Lorentz invariance broken
Deformed gauge theories

The type of deformation resulting from the fluxbrane background depends on how D-branes are placed into the fluxtrap with respect to the monodromies:

Deformation not on brane world-volume: mass deformation

Deformation on brane world-volume: $\Omega$-type deformation, Lorentz invariance broken
Example: Omega-deformed SW action
Omega-deformed SW action
Omega-deformed SW action
Omega-deformed SW action

M-theory

string theory

FT

MFT

MFB

M5

D1

D2

D3

D4

NS5

NS5

NS5

Lift

M-theory

string theory

2d

3d

4d

5d

N=(2,2) w. tw. masses
N=(8,8) w. tw. masses
N=2* w. real masses
N=4 w. real masses
N=2 SYM
N=4 SYM
Ω-def. N=1 SYM
Ω-def. N=2 SYM
Ω-def. N=4 SYM
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reciproc. gauge th.
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S-dual of Ω-def. SW
N=(8,8) w. tw. masses
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Omega-deformed SW

**Application:** derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)
Omega-deformed SW

**Application:** derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)
Use **M-theory lift** of fluxtrap BG.
Omega-deformed SW

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Classical computation yields *quantum* result.
Omega-deformed SW

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Embed M5-brane into fluxtrap BG.
Omega-deformed SW

**Application**: derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)
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Classical computation yields **quantum** result.
Embed M5-brane into fluxtrap BG.
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Omega-deformed SW

**Application**: derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)

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Embed M5-brane into fluxtrap BG.

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- Still **wrapped on a Riemann** surface at linear order.
Omega-deformed SW

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Embed M5-brane into fluxtrap BG.

- **Self-dual three-form** on the brane.
- Still wrapped on a **Riemann** surface at linear order.

Take **vector** and **scalar** equations of motion in 6d (not from an action!).
**Omega-deformed SW**

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Integrate equations over Riemann surface.
Omega-deformed SW

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4d equations of motion are **Euler-Lagrange** equations of an action.
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**Integrate** equations over Riemann surface.  
4d equations of motion are **Euler-Lagrange** equations of an action.  
This action reduces to the **Seiberg-Witten** action in the undeformed case.
Omega-deformed SW

**Application:** derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)

Use **M-theory lift** of fluxtrap BG.

Classical computation yields **quantum** result.

Embed M5-brane into fluxtrap BG.

Self-dual three-form on the brane.

Still wrapped on a Riemann surface at linear order.

Take vector and scalar equations of motion in 6d (not from an action!).

Integrate equations over Riemann surface.

4d equations of motion are **Euler-Lagrange** equations of an action.

This action reduces to the **Seiberg-Witten** action in the undeformed case.

Captures all orders of the 4D gauge theory.
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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Notes:

- The table represents a flux trap background with a D4-brane suspended between two parallel NS5-branes.
- The table indicates the presence or absence of branes in various dimensions.
- The crosses (×) indicate the presence of branes, while the circles (○) indicate a vacuum.
- The T-duality direction is indicated by the presence of the circle.
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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Non-abelian generalization of bosonic world-volume action for D4-branes suspended between NS5-branes in fluxtrap BG:
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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Non-abelian generalization of bosonic world-volume action for D4-branes suspended between NS5-branes in fluxtrap BG:

$$L_{D4} = \frac{1}{g_4^2} \text{Tr} \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \varphi + \frac{1}{2} F_{\mu\lambda} \hat{U}^\lambda)(D_\mu \bar{\varphi} + \frac{1}{2} F_{\mu\rho} \hat{U}^\rho) 
- \frac{1}{4} [\varphi, \bar{\varphi}]^2 + \frac{1}{8} (\hat{U}_\mu D_\mu (\varphi - \bar{\varphi}))^2 \right]$$
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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</table>

Non-abelian generalization of bosonic world-volume action for D4-branes suspended between NS5-branes in fluxtrap BG:

$$\mathcal{L}_{D_4} = \frac{1}{g_4^2} \text{Tr} \left[ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \varphi + \frac{1}{2} F_{\mu\lambda} \hat{U}^\lambda) (D_\mu \bar{\varphi} + \frac{1}{2} F_{\mu\rho} \hat{U}^\rho) ight]$$

$$- \frac{1}{4} [\varphi, \bar{\varphi}]^2 + \frac{1}{8} (\hat{U}^\mu D_\mu (\varphi - \bar{\varphi}))^2$$

Lifts to single M5 extended in $x^0, \ldots, x^3$ and wrapping a 2-cycle in $x^6, x^8, x^9, x^{10}$. 
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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<th>0 1 2 3 4 5 6 7 8 9</th>
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<tr>
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Non-abelian generalization of bosonic world-volume action for D4-branes suspended between NS5-branes in fluxtrap BG:

\[
\mathcal{L}_{D₄} = \frac{1}{g₄^2} \text{Tr} \left[ \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (D_\mu \varphi + \frac{1}{2} F_{\mu \lambda} \hat{U}^\lambda) (D_\mu \bar{\varphi} + \frac{1}{2} F_{\mu \rho} \hat{U}^\rho) \right. \\
\left.\quad - \frac{1}{4} [\varphi, \bar{\varphi}]^2 + \frac{1}{8} (\hat{U}^\mu D_\mu (\varphi - \bar{\varphi}))^2 \right]
\]

Lifts to single M5 extended in \(x^0, \ldots, x^3\) and wrapping a 2-cycle in \(x^6, x^8, x^9, x^{10}\).

\[
z = x^8 + i x^9 \\
s = x^6 + i x^{10}
\]
Omega-deformed SW

Start with type IIA set-up of D4- and NS5-branes:

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$$

Lifts to single M5 extended in $x^0, \ldots, x^3$ and wrapping a 2-cycle in $x^6, x^8, x^9, x^{10}$.

$$
z = x^8 + i x^9 \quad \quad s = x^6 + i x^{10}
$$

Choose embedding preserving same susy as in type IIA.
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:
**Omega-deformed SW**

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \]

\[ H_3 = h_3 + O(h_3^3) \]
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

$$dH_3 = -\frac{1}{4} \hat{G}_4$$

$$H_3 = h_3 + O(h_3^3)$$
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \quad \Rightarrow \quad H_3 = h_3 + O(h_3^3) \]

For \( \epsilon = 0 \), we have \( h_3 = 0 \) and the M5-brane wraps a Riemann surface \( \bar{\partial}s = 0 \).
For $\epsilon = 0$, we have $h_3 = 0$ and the M5-brane wraps a Riemann surface $\bar{\partial}s = 0$.
At linear order, pullback only depends holomorphically on $s(z)$:
Omega-deformed SW

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At linear order, pullback only depends holomorphically on \( s(z) \):

\[ \hat{G}_4 = - (\partial s - \bar{\partial}s) \, dz \wedge d\bar{z} \wedge \hat{\omega} + O(\epsilon^2) \].
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \quad \text{selfdual} \]
\[ H_3 = h_3 + O(h_3^3) \]

For \( \epsilon = 0 \), we have \( h_3 = 0 \) and the M5-brane wraps a Riemann surface \( \bar{\partial}s = 0 \).

At linear order, pullback only depends holomorphically on \( s(z) \):

\[ \hat{G}_4 = - (\partial s - \bar{\partial}\bar{s}) \, dz \wedge d\bar{z} \wedge \hat{\omega} + O(\epsilon^2) . \]

From the susy condition, we find
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \]

\[ H_3 = h_3 + \mathcal{O}(h_3^3) \]

For \( \epsilon = 0 \), we have \( h_3 = 0 \) and the M5-brane wraps a Riemann surface \( \bar{\partial} s = 0 \).

At linear order, pullback only depends holomorphically on \( s(z) \):

\[ \hat{G}_4 = - (\partial s - \bar{\partial} \bar{s}) \, dz \wedge d\bar{z} \wedge \hat{\omega} + \mathcal{O}(\epsilon^2) \]

From the susy condition, we find

\[ h_3 = \frac{1}{4} (\bar{s} - \bar{z} \partial s) \, dz \wedge \hat{\omega}^- + \frac{1}{4} (s - z \bar{\partial} \bar{s}) \, d\bar{z} \wedge \hat{\omega}^+ \]
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \quad \text{selfdual} \]
\[ H_3 = h_3 + O(h_3^3) \]

For \( \epsilon = 0 \), we have \( h_3 = 0 \) and the M5-brane wraps a Riemann surface \( \bar{\partial}s = 0 \).

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\[ \hat{G}_4 = -(\partial s - \bar{\partial}\bar{s}) \, dz \wedge d\bar{z} \wedge \hat{\omega} + O(\epsilon^2). \]

From the susy condition, we find

\[ \hat{\omega}^- = \frac{\epsilon_1 - \epsilon_2}{2} \left( dx^0 \wedge dx^1 - dx^2 \wedge dx^3 \right) \]
\[ h_3 = \frac{1}{4} (\bar{s} - \bar{z} \, \partial s) \, dz \wedge \hat{\omega}^- + \frac{1}{4} (s - z \, \bar{\partial}\bar{s}) \, d\bar{z} \wedge \hat{\omega}^+ \]
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

\[ dH_3 = -\frac{1}{4} \hat{G}_4 \quad \text{selfdual} \]

\[ H_3 = h_3 + O(h_3^3) \]

For \( \epsilon = 0 \), we have \( h_3 = 0 \) and the M5-brane wraps a Riemann surface \( \bar{s} = 0 \).

At linear order, pullback only depends holomorphically on \( s(z) \):

\[ \hat{G}_4 = - (\partial s - \bar{\partial} \bar{s}) \, dz \wedge d\bar{z} \wedge \hat{\omega} + O(\epsilon^2). \]

From the susy condition, we find

\[ \hat{\omega}^- = \frac{\epsilon_1 - \epsilon_2}{2} (dx^0 \wedge dx^1 - dx^2 \wedge dx^3) \quad \hat{\omega}^+ = \frac{\epsilon_1 + \epsilon_2}{2} (dx^0 \wedge dx^1 + dx^2 \wedge dx^3) \]

\[ h_3 = \frac{1}{4} (\bar{s} - \bar{z} \, \partial s) \, dz \wedge \hat{\omega}^- + \frac{1}{4} (s - z \, \bar{\partial} \bar{s}) \, d\bar{z} \wedge \hat{\omega}^+ \]
Omega-deformed SW

Selfdual three-form on WV of the M5-brane:

$$dH_3 = -\frac{1}{4} \hat{G}_4$$

$$h_3 = h_3 + O(h_3^3)$$

For $\epsilon = 0$, we have $h_3 = 0$ and the M5-brane wraps a Riemann surface $\bar{s}s = 0$.

At linear order, pullback only depends holomorphically on $s(z)$:

$$\hat{G}_4 = -(\partial s - \bar{\partial} \bar{s}) dz \wedge d\bar{z} \wedge \hat{\omega} + O(\epsilon^2).$$

From the susy condition, we find

$$\hat{\omega}^- = \frac{\epsilon_1 - \epsilon_2}{2} (dx^0 \wedge dx^1 - dx^2 \wedge dx^3)$$

$$\hat{\omega}^+ = \frac{\epsilon_1 + \epsilon_2}{2} (dx^0 \wedge dx^1 + dx^2 \wedge dx^3)$$

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M5 still embedded holomorphically, implicit form for SU(2):
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\[ t^2 - 2B(z|u)t + \Lambda^4 = 0 \],
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Riemann surface with modulus \( u \):

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Omega-deformed SW

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\[ \Sigma = \{ (z, s) | s = s(z|u) \} \]
Omega-deformed SW

Want to describe the low energy dynamics of the fluctuations around the equilibrium.
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The modulus $u$ of the Riemann surface is a function of the worldvolume coordinates and the embedding is still formally defined by the same equation:
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The modulus $u$ of the Riemann surface is a function of the worldvolume coordinates and the embedding is still formally defined by the same equation:

$$s = s(z|u(x^\mu)) \quad \partial_\mu s(z|u(x^\mu)) = \partial_\mu u \frac{\partial s}{\partial u}$$

$$z = x^8 + i x^9 \quad s = x^6 + i x^{10}$$
Omega-deformed SW

6D Equations of Motion:
Omega-deformed SW

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Dynamics can be obtained by evaluating the M5-brane equations of motion (bosonic fields).
Omega-deformed SW

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\[(\hat{g}^{mn} - 16h^{mpq}h^{n}_{\quad pq}) \nabla_m \nabla_n X^I = -\frac{2}{3}\hat{G}^I_{\quad mnp}h^{mnp},\]

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Omega-deformed SW
Omega-deformed SW

Want to relate $\Phi$ to 4d gauge field: only components

$(\mu, \nu, z)$, $(\mu, \nu, \bar{z})$
Omega-deformed SW

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Omega-deformed SW

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$$+ \frac{1}{1 + |\partial s|^2} \frac{1}{3!} \varepsilon_{\mu \nu \rho \sigma} \left( \partial^\tau s \, \bar{\partial} \bar{s} \, \kappa F_{\sigma \tau} - \partial^\tau \bar{s} \, \partial s \, \bar{\kappa} \bar{F}_{\sigma \tau} \right) \, dx^\mu \wedge dx^\nu \wedge dx^\rho .$$

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\]

antiselfdual 2-form

\[^*4 F = -F, \quad \tilde{F} = \star F\]
Omega-deformed SW

Want to relate $\Phi$ to 4d gauge field: only components

$$\begin{align*}
(\mu, \nu, z), & \quad (\mu, \nu, \bar{z}) \\
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& \quad + \frac{1}{1 + |s|^2} \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} \left( \partial^\tau s \bar{\partial} \bar{s} \kappa F_{\sigma\tau} - \partial^\tau \bar{s} \partial s \bar{\kappa} \bar{F}_{\sigma\tau} \right) \, dx^\mu \wedge dx^\nu \wedge dx^\rho.
\end{align*}$$

to ensure self-duality \hspace{1cm} \ast_4 F = - F, \hspace{1cm} \ast_4 \bar{F} = \bar{F}
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$\ast_4 F = - F,$

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SW notation:
Omega-deformed SW

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SW notation:

$$a = \int_A \lambda_{SW}, \quad a_D = \int_B \lambda_{SW}, \quad \tau = \frac{da_D}{da}, \quad \lambda = \frac{\partial \lambda_{SW}}{\partial u}$$
Omega-deformed SW

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SW notation:

scalar field

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Omega-deformed SW

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Holomorphic 1-form on Riemann surface
Want to relate $\Phi$ to 4d gauge field: only components $(\mu, \nu, z), (\mu, \nu, \bar{z})$

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To ensure self-duality

$$*_4 F = -F, \quad *_4 \tilde{F} = \tilde{F}$$

**SW notation:**

Scalar field

$$a = \oint_A \lambda_{SW}, \quad a_D = \oint_B \lambda_{SW}, \quad \tau = \frac{da_D}{da}, \quad \lambda = \frac{\partial \lambda_{SW}}{\partial u}$$

Holomorphic 1-form on Riemann surface

$$\kappa = \frac{ds}{da} = \left( \frac{da}{du} \right)^{-1} \lambda_z \quad \lambda = \lambda_z dz \quad \frac{da}{du} = \oint_A \lambda$$
Omega-deformed SW

Want to relate \( \Phi \) to 4d gauge field: only components

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\Phi = \frac{\kappa}{2} \mathcal{F}_{\mu \nu} \, dx^\mu \wedge dx^\nu \wedge dz + \frac{\bar{\kappa}}{2} \tilde{\mathcal{F}}_{\mu \nu} \, dx^\mu \wedge dx^\nu \wedge d\bar{z} + \frac{1}{1 + |\partial s|^2} \frac{1}{3!} \epsilon_{\mu \nu \rho \sigma} \left( \partial^\tau s \, \bar{\partial} \bar{s} \, \kappa \mathcal{F}_{\sigma \tau} - \partial^\tau \bar{s} \, \partial s \, \bar{\kappa} \tilde{\mathcal{F}}_{\sigma \tau} \right) \, dx^\mu \wedge dx^\nu \wedge dx^\rho.
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to ensure self-duality 

\[ *_4 \mathcal{F} = -\mathcal{F}, \quad *_4 \tilde{\mathcal{F}} = \tilde{\mathcal{F}} \]

SW notation:

- scalar field
  \[ a = \oint_A \lambda_{SW}, \quad a_D = \oint_B \lambda_{SW}, \quad \tau = \frac{da_D}{da}, \quad \lambda = \frac{\partial \lambda_{SW}}{\partial u} \]
- holomorphic fn
  \[ \kappa = \frac{ds}{da} = \left( \frac{da}{du} \right)^{-1} \lambda_z \]
- holomorphic 1-form on Riemann surface
  \[ \lambda = \lambda_z \, dz, \quad \frac{da}{du} = \oint_A \lambda \]
Omega-deformed SW

Vector equation:

\[ dh_3 = -\frac{1}{4} \hat{H}_4 \]
Omega-deformed SW

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”The threeform on the brane is the (generalized) pullback of the threeform in the bulk.”
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source for fluctuations

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With our ansatz, the only nonvanishing components result in
Omega-deformed SW

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\[ \ast_6 d(\Phi - i \ast_6 \hat{C}_3) = \frac{1}{2} E_{\mu z} dx^\mu \wedge dz + \frac{1}{2} E_{\mu \bar{z}} dx^\mu \wedge d\bar{z} = 0 \]
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To obtain the e.o.m. of the vector fields in four dimensions we need to reduce these equations on the Riemann surface:
Omega-deformed SW

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\[ \star_6 \text{d}(\Phi - i \star_6 \hat{C}_3) = \frac{1}{2} E_{\mu z} \text{d}x^\mu \wedge \text{d}z + \frac{1}{2} E_{\mu \bar{z}} \text{d}x^\mu \wedge \text{d}\bar{z} = 0 \]

To obtain the e.o.m. of the vector fields in four dimensions we need to reduce these equations on the Riemann surface:

\[ \int \star_6 \text{d}(\Phi - i \text{d}\star \hat{C}_3) \wedge \check{\lambda} = \text{d}x^\mu \wedge \int \Sigma E_{\mu z} \text{d}z \wedge \check{\lambda} = 0, \]

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Omega-deformed SW

Vector equation:

\[ h_3 = -\frac{1}{4} \left( \hat{C}_3 + i \ast_6 \hat{C}_3 - \Phi \right) \]
\[ dh_3 = -\frac{1}{4} \hat{H}_4 \]

"The threeform on the brane is the (generalized) pullback of the threeform in the bulk."

\[ d\Phi = i \, d\ast_6 \hat{C}_3 \]

With our ansatz, the only nonvanishing components result in

\[ \ast_6 d(\Phi - i \ast_6 \hat{C}_3) = \frac{1}{2} E_{\mu z} \, dx^\mu \wedge dz + \frac{1}{2} E_{\mu \bar{z}} \, dx^\mu \wedge d\bar{z} = 0 \]

To obtain the e.o.m. of the vector fields in four dimensions we need to reduce these equations on the Riemann surface:

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Omega-deformed SW

Integrals result in
Omega-deformed SW

Integrals result in

\[ (\tau - \bar{\tau}) (\partial_\mu F_{\mu\nu} + \partial_\mu a \hat{\omega}_{\mu\nu}) + \partial_\mu \tau F_{\mu\nu} - \partial_\mu \bar{\tau} \tilde{F}_{\mu\nu} = 0, \]

\[ (\tau - \bar{\tau}) \left( \partial_\mu \tilde{F}_{\mu\nu} + \partial_\mu \bar{a} \hat{\omega}_{\mu\nu} \right) + \partial_\mu \tau F_{\mu\nu} - \partial_\mu \bar{\tau} \tilde{F}_{\mu\nu} = 0 \]
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The difference of these equations is solved by
Omega-deformed SW

Integrals result in

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(\tau - \bar{\tau})(\partial_\mu \mathcal{F}_{\mu\nu} + \partial_\mu a \hat{\omega}_{\mu\nu}) + \partial_\mu \tau F_{\mu\nu} - \partial_\mu \bar{\tau} \tilde{F}_{\mu\nu} = 0,
\]

\[
(\tau - \bar{\tau}) \left( \partial_\mu \tilde{\mathcal{F}}_{\mu\nu} + \partial_\mu \bar{a} \hat{\omega}_{\mu\nu} \right) + \partial_\mu \tau F_{\mu\nu} - \partial_\mu \bar{\tau} \tilde{F}_{\mu\nu} = 0
\]

The difference of these equations is solved by

\[
\begin{align*}
\mathcal{F} &= (1 - \ast) F - (a - \bar{a}) \hat{\omega}^-,
\tilde{\mathcal{F}} &= (1 + \ast) F + (a - \bar{a}) \hat{\omega}^+,
\end{align*}
\]
Omega-deformed SW

Integrals result in
\[
(\tau - \bar{\tau}) \left( \partial_\mu F_{\mu \nu} + \partial_\mu a \dot{\omega}_{\mu \nu} \right) + \partial_\mu \tau F_{\mu \nu} - \partial_\mu \bar{\tau} \tilde{F}_{\mu \nu} = 0, \\
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\]

The difference of these equations is solved by
\[
\begin{cases}
F = (1 - *) F - (a - \bar{a}) \dot{\omega}^- , \\
\tilde{F} = (1 + *) F + (a - \bar{a}) \dot{\omega}^+ ,
\end{cases}
\]

where $F$ satisfies the Bianchi identity: $dF = 0$
Omega-deformed SW

Integrals result in

\[(\tau - \bar{\tau}) (\partial_\mu F_{\mu \nu} + \partial_\mu a \hat{\omega}_{\mu \nu}) + \partial_\mu \tau F_{\mu \nu} - \partial_\mu \bar{\tau} \bar{F}_{\mu \nu} = 0,\]

\[(\tau - \bar{\tau}) \left( \partial_\mu \bar{F}_{\mu \nu} + \partial_\mu \bar{a} \hat{\omega}_{\mu \nu} \right) + \partial_\mu \tau F_{\mu \nu} - \partial_\mu \bar{\tau} \bar{F}_{\mu \nu} = 0\]

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\end{cases}\]

where \( F \) satisfies the Bianchi identity: \( dF = 0 \)

F can now be identified with the gauge field in 4D!
Omega-deformed SW

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Substituting this back, the vector equation becomes
Omega-deformed SW

Integrals result in

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Substituting this back, the vector equation becomes

\[
(\tau - \bar{\tau}) \left[ \partial_\mu F_{\mu \nu} + \frac{1}{2} \partial_\mu (a + \bar{a}) \hat{\omega}_{\mu \nu} + \frac{1}{2} \partial_\mu (a - \bar{a})^* \hat{\omega}_{\mu \nu} \right]
\]

\[
+ \partial_\mu (\tau - \bar{\tau}) \left[ F_{\mu \nu} + \frac{1}{2} (a - \bar{a})^* \hat{\omega}_{\mu \nu} \right] - \partial_\mu (\tau + \bar{\tau}) \left[ \* F_{\mu \nu} + \frac{1}{2} (a - \bar{a}) \hat{\omega}_{\mu \nu} \right] = 0
\]
Omega-deformed SW

Scalar equation:

\[
(\hat{g}^{mn} - 16h^{mpq}h^{n}_{pq}) \nabla_m \nabla_n X^I = -\frac{2}{3} \hat{G}^I_{mnp} h^{mnp}
\]
Omega-deformed SW

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\[(\hat{g}^{mn} - 16h^{mpq}h_{pq}^n) \nabla_m \nabla_n X^I = -\frac{2}{3} \hat{G}^I_{mnp} h^{mnp}\]

"The M5 brane is a (generalized) minimal surface."
Omega-deformed SW

Scalar equation:

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"The M5 brane is a (generalized) minimal surface."

Of the 11 equations, only the ones of the components $I = s, \bar{s}$ are nontrivial:
The M5 brane is a (generalized) minimal surface.

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(\hat{g}^{mn} - 16h^{mpq}h^{n}_{\quad pq}) \nabla_m \nabla_n X^I = -\frac{2}{3} \hat{G}^I_{\quad mnp} h^{mnp}
\]

\[
E = \partial_\mu \partial_{\bar{\mu}} s - \partial \left[ \frac{\partial_\rho s \partial_\rho \bar{s} \bar{\partial} \bar{s}}{1 + |\partial s|^2} \right] - \frac{16 \partial^2 s}{(1 + |\partial s|^2)^2} h_{\mu \nu \bar{z}} h_{\mu \nu \bar{z}}
\]

\[
-2\hat{\omega}_{\mu \nu} F_{\mu \nu} \left( \frac{da}{du} \right)^{-1} \lambda_z + 2\hat{\omega}^+_{\mu \nu} \tilde{F}_{\mu \nu} \left( \frac{da}{d\bar{u}} \right)^{-1} \bar{\lambda}_{\bar{z}} = 0,
\]

\[
\bar{E} = \partial_\mu \partial_{\bar{\mu}} \bar{s} - \partial \left[ \frac{\partial_\rho \bar{s} \partial_\rho \bar{s} \partial s}{1 + |\partial s|^2} \right] - \frac{16 \partial^2 \bar{s}}{(1 + |\partial s|^2)^2} h_{\mu \nu \bar{z}} h_{\mu \nu \bar{z}}
\]

\[
-2\hat{\omega}_{\mu \nu} \bar{F}_{\mu \nu} \left( \frac{da}{du} \right)^{-1} \lambda_z + 2\hat{\omega}^+_{\mu \nu} \tilde{\bar{F}}_{\mu \nu} \left( \frac{da}{d\bar{u}} \right)^{-1} \bar{\lambda}_{\bar{z}} = 0.
\]
Omega-deformed SW

Integrate over the Riemann surface:
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\[ \int_{\Sigma} E \, dz \wedge \bar{\lambda} = \int_{\Sigma} \bar{E} \, d\bar{z} \wedge \lambda = 0 \]
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The final form of the 4d scalar equations is:
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\[
\left(\dot{\tau} - \dot{\bar{\tau}}\right) \partial_\mu \partial_\mu a + \partial_\mu a \partial_\mu \bar{\tau} + 2 \frac{d\bar{\tau}}{d\bar{a}} \left( F_{\mu\nu} F_{\mu\nu} + F_{\mu\nu}^* F_{\mu\nu} \right) + 4 \frac{d\bar{\tau}}{d\bar{a}} \left( a - \bar{a} \right) \hat{\omega}^+_{\mu\nu} F_{\mu\nu} - 4 \left( \tau - \bar{\tau} \right) \hat{\omega}^-_{\mu\nu} F_{\mu\nu} = 0 ,
\]

\[
\left(\tau - \bar{\tau}\right) \partial_\mu \partial_\mu \bar{a} - \partial_\mu \bar{a} \partial_\mu \bar{\tau} - 2 \frac{d\tau}{da} \left( F_{\mu\nu} F_{\mu\nu} - F_{\mu\nu}^* F_{\mu\nu} \right) + 4 \frac{d\tau}{da} \left( a - \bar{a} \right) \hat{\omega}^-_{\mu\nu} F_{\mu\nu} - 4 \left( \tau - \bar{\tau} \right) \hat{\omega}^+_{\mu\nu} F_{\mu\nu} = 0 .
\]

Integrate over the Riemann surface:

\[
\int_{\Sigma} E \, dz \wedge \bar{\lambda} = \int_{\Sigma} \bar{E} \, d\bar{z} \wedge \lambda = 0
\]
Omega-deformed SW

Our calculation results in the 4d e.o.m. for the Omega-deformation of Seiberg Witten theory:
Omega-deformed SW

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Vector equation:
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\[
(\tau - \bar{\tau}) \left[ \partial_\mu F_{\mu\nu} + \frac{1}{2} \partial_\mu (a + \bar{a}) \hat{\omega}_{\mu\nu} + \frac{1}{2} \partial_\mu (a - \bar{a})^* \hat{\omega}_{\mu\nu} \right] \\
+ \partial_\mu (\tau - \bar{\tau}) \left[ F_{\mu\nu} + \frac{1}{2} (a - \bar{a})^* \hat{\omega}_{\mu\nu} \right] - \partial_\mu (\tau + \bar{\tau}) \left[ *F_{\mu\nu} + \frac{1}{2} (a - \bar{a}) \hat{\omega}_{\mu\nu} \right] = 0
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Scalar equations:
Omega-deformed SW

Our calculation results in the 4d e.o.m. for the Omega-deformation of Seiberg Witten theory:

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\]

**Scalar equations:**

\[
(\tau - \bar{\tau}) \partial_\mu \partial_\nu a + \partial_\nu a \partial_\mu \tau + 2 \frac{d\tau}{da} \left( F_{\mu\nu} F_{\mu\nu} + F_{\mu\nu}^* F_{\mu\nu} \right) \\
+ 4 \frac{d\tau}{da} (a - \bar{a}) \hat{\omega}_{\mu\nu}^+ F_{\mu\nu} - 4 (\tau - \bar{\tau}) \hat{\omega}_{\mu\nu}^- F_{\mu\nu} = 0 ,
\]

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(\tau - \bar{\tau}) \partial_\mu \partial_\nu \bar{a} - \partial_\nu \bar{a} \partial_\mu \bar{\tau} - 2 \frac{d\tau}{da} \left( F_{\mu\nu} F_{\mu\nu} - F_{\mu\nu}^* F_{\mu\nu} \right) \\
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\[+ 4 \frac{d\bar{\tau}}{d\bar{a}} (a - \bar{a}) \hat{\omega}^+_{\mu\nu} F_{\mu\nu} - 4 (\tau - \bar{\tau}) \hat{\omega}^-_{\mu\nu} F_{\mu\nu} = 0,\]

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Consistent result justifies earlier assumptions about foliation structure, form of fluctuations and integration measure.
Omega-deformed SW

Expect to have a **Lagrangian description**!
Omega-deformed SW

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The vector and scalar e.o.m. are the Euler Lagrange equations of the following Lagrangian:
Omega-deformed SW

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The vector and scalar e.o.m. are the Euler Lagrange equations of the following Lagrangian:

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i \mathcal{L} = - (\tau - \bar{\tau}) \left[ \frac{1}{2} \partial_\mu a \partial_\mu \bar{a} + F_{\mu\nu} F^{\mu\nu} + (a - \bar{a}) \omega_{\mu\nu} F_{\mu\nu} - 2 \partial_\mu (a + \bar{a}) \omega_{\nu} F^\nu - \hat{\omega}_\mu F_{\mu\nu} \hat{U}^\nu \right] \\
\quad + (\tau + \bar{\tau}) \left[ F_{\mu\nu} \omega_{\mu\nu} F^{\mu\nu} + (a - \bar{a}) \omega_{\mu\nu} F_{\mu\nu} + 2 \partial_\mu (a - \bar{a}) \omega_{\nu} F^\nu - \hat{\omega}_\mu F_{\mu\nu} \hat{U}^\nu \right]
\]

\[
\omega = dU
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&\quad + (\tau + \bar{\tau}) \left[ F_{\mu\nu} \ast F_{\mu\nu} + (a - \bar{a}) \hat{\omega}_{\mu\nu} F_{\mu\nu} + 2 \partial_\mu (a - \bar{a}) \ast F_{\mu\nu} \ast \hat{U}_\nu \right] \\
\omega &= dU
\end{align*}
\]

For \( \epsilon = 0 \), this reproduces the Seiberg-Witten Lagrangian.
Omega-deformed SW

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Why does this result of a classical M-theory calculation capture the quantum effects of gauge theory?
Omega-deformed SW

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\]

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For \( \epsilon = 0 \), this **reproduces the Seiberg-Witten Lagrangian**.

Why does this result of a classical M-theory calculation capture the quantum effects of gauge theory?

Independent of compactification radius to IIA, which is related to gauge coupling in 4d.
Omega-deformed SW

Write the action in a more supersymmetric form as a sum of squares.
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Generalization to arbitrary gauge group and matter content, prediction for $\epsilon^2$ terms:
Omega-deformed SW

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Generalization to arbitrary gauge group and matter content, prediction for $\epsilon^2$ terms:

\[
i \mathcal{L} = - (\tau_{ij} - \bar{\tau}_{ij}) \left[ \frac{1}{2} \left( \partial_\mu a^i + 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right) i_k F^k_{\mu \nu} * U_\nu \right) \left( \partial_\mu \bar{a}^j - 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right) j_l F^l_{\mu \nu} * U_\nu \right) \right.
\]

\[
+ \left( F^i_{\mu \nu} + \frac{1}{2} (a^i - \bar{a}^i) * \hat{\omega}_{\mu \nu} \right) \left( F^j_{\mu \nu} + \frac{1}{2} (a^j - \bar{a}^j) * \hat{\omega}_{\mu \nu} \right) \right]
\]

\[
+ (\tau_{ij} + \bar{\tau}_{ij}) \left( F^i_{\mu \nu} + \frac{1}{2} (a^i - \bar{a}^i) * \hat{\omega}_{\mu \nu} \right) \left( * F^j_{\mu \nu} + \frac{1}{2} (a^j - \bar{a}^j) \hat{\omega}_{\mu \nu} \right)
\]
Write the action in a more supersymmetric form as a sum of squares.

Generalization to arbitrary gauge group and matter content, prediction for $\epsilon^2$ terms:

generalized covariant derivative for the scalar $a$, non-minimal coupling to the gauge field.

$$i\mathcal{L} = -(\tau_{ij} - \bar{\tau}_{ij}) \left[ \frac{1}{2} \left( \partial_\mu a^i + 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right)_{i\bar{k}} F^k_{\mu\nu} \ast \hat{U}_\nu \right) \left( \partial_\mu \bar{a}^j - 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right)_{j\bar{l}} F^l_{\mu\nu} \ast \hat{U}_\nu \right) \right]$$

$$+ \left( F^i_{\mu\nu} + \frac{1}{2} (a^i - \bar{a}^i) \ast \hat{\omega}_{\mu\nu} \right) \left( F^j_{\mu\nu} + \frac{1}{2} (a^j - \bar{a}^j) \ast \hat{\omega}_{\mu\nu} \right)$$

$$+ (\tau_{ij} + \bar{\tau}_{ij}) \left( F^i_{\mu\nu} + \frac{1}{2} (a^i - \bar{a}^i) \ast \hat{\omega}_{\mu\nu} \right) \left( F^j_{\mu\nu} + \frac{1}{2} (a^j - \bar{a}^j) \hat{\omega}_{\mu\nu} \right)$$
Omega-deformed SW

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i \mathcal{L} = - (\tau_{ij} - \bar{\tau}_{ij}) \left[ \frac{1}{2} \left( \partial_\mu a^i + 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right)_{ik} F^k_{\mu \nu} \ast \hat{U}_\nu \right) \left( \partial_\mu \bar{a}^j - 2 \left( \frac{\tau}{\tau - \bar{\tau}} \right)_{jl} F_{\mu \nu}^l \ast \hat{U}_\nu \right) \\
+ (F^i_{\mu \nu} + \frac{1}{2} (a^i - \bar{a}^i) \ast \hat{\omega}_{\mu \nu}) (F^j_{\mu \nu} + \frac{1}{2} (a^j - \bar{a}^j) \ast \hat{\omega}_{\mu \nu}) \left] \right. \\
+ (\tau_{ij} + \bar{\tau}_{ij}) (F^i_{\mu \nu} + \frac{1}{2} (a^i - \bar{a}^i) \ast \hat{\omega}_{\mu \nu}) (\ast F^j_{\mu \nu} + \frac{1}{2} (a^j - \bar{a}^j) \hat{\omega}_{\mu \nu}) \right]
\]

shift in the gauge field strength
Summary
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Constructed the **fluxtrap background** in string theory.
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Summary

Can be lifted to M-theory: M-theory Fluxtrap

M-theory

string theory

gauge theory

N=(8,8) w. tw. masses
N=(2,2) w. tw. masses
N=4 w. real masses
N=2* reciprocal gauge th.
Ω-def. N=2 SYM
Ω-def. SW
S-dual of Ω-def. SW
Ω-def. N=2 SYM

FT

MFT

M5

MFB

T^3

Lift

Reduce

Reduce +T

topological string theory

Ω-def. SW
Ω-def. N=2 SYM
Ω-def. N=1 SYM
Ω-def.
N=4 SYM

N=(2,2) SYM

2d

3d

4d

5d
Summary

Can be lifted to M-theory: M-theory Fluxtrap
Derive **Omega-deformed Seiberg-Witten Lagrangian** and its S-dual

arXiv:1304.3488
Summary

Derive **Omega-deformed Seiberg-Witten Lagrangian and its S-dual**

[Diagram showing M-theory, string theory, and gauge theory relationships, including FT, MFT, MFB, M5, NS5, D1, D2, D3, D4, Lift, T^3, topological string theory, and S-dual of Omega-def. SW.]

Derive Omega-deformed Seiberg-Witten Lagrangian and its S-dual. [arXiv:1304.3488]
Summary

Use M-theory lift of fluxtrap BG, embed M5-brane, reduce 6d e.o.m. on Riemann surface.
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The resulting 4d e.o.m. for the scalar and vector fields are Euler-Lagrange equations for a 4d action: Omega-deformed Seiberg-Witten Lagrangian!
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The resulting 4d e.o.m. for the scalar and vector fields are Euler-Lagrange equations for a 4d action: **Omega-deformed Seiberg-Witten Lagrangian**!

Classical M-theory calculation yields quantum result, captures all orders of 4d gauge theory.
Summary

The fluxtrap construction allows us to study different gauge theories of interest via string theoretic methods.
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Many interesting applications! Domenico’s talk after the break.
Thank you for your attention!