Deformed supersymmetric gauge theories from String and M-Theory:

Introduction to the fluxtrap background

Susanne Reffert

based on work with with D. Orlando, S. Hellerman, N. Lambert
arXiv:1106.2097, 1108.0644, 1111.4811, 1204.4192, 1210.7805, 1304.3488, 1309.7350, work in progress

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AGT correspondence (Alday, Gaiotto, Tachikawa): relates Omega-deformed super-Yang-Mills theory to Liouville theory.

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    The string theory construction provides a unifying framework and a different point of view on the gauge theory problems.

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Use the fluxtrap construction to unify and meaningfully relate and reinterpret a large variety of existing results.

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Fluxtrap background as toolbox to generate deformed gauge theories and analyze them via string theoretic methods.

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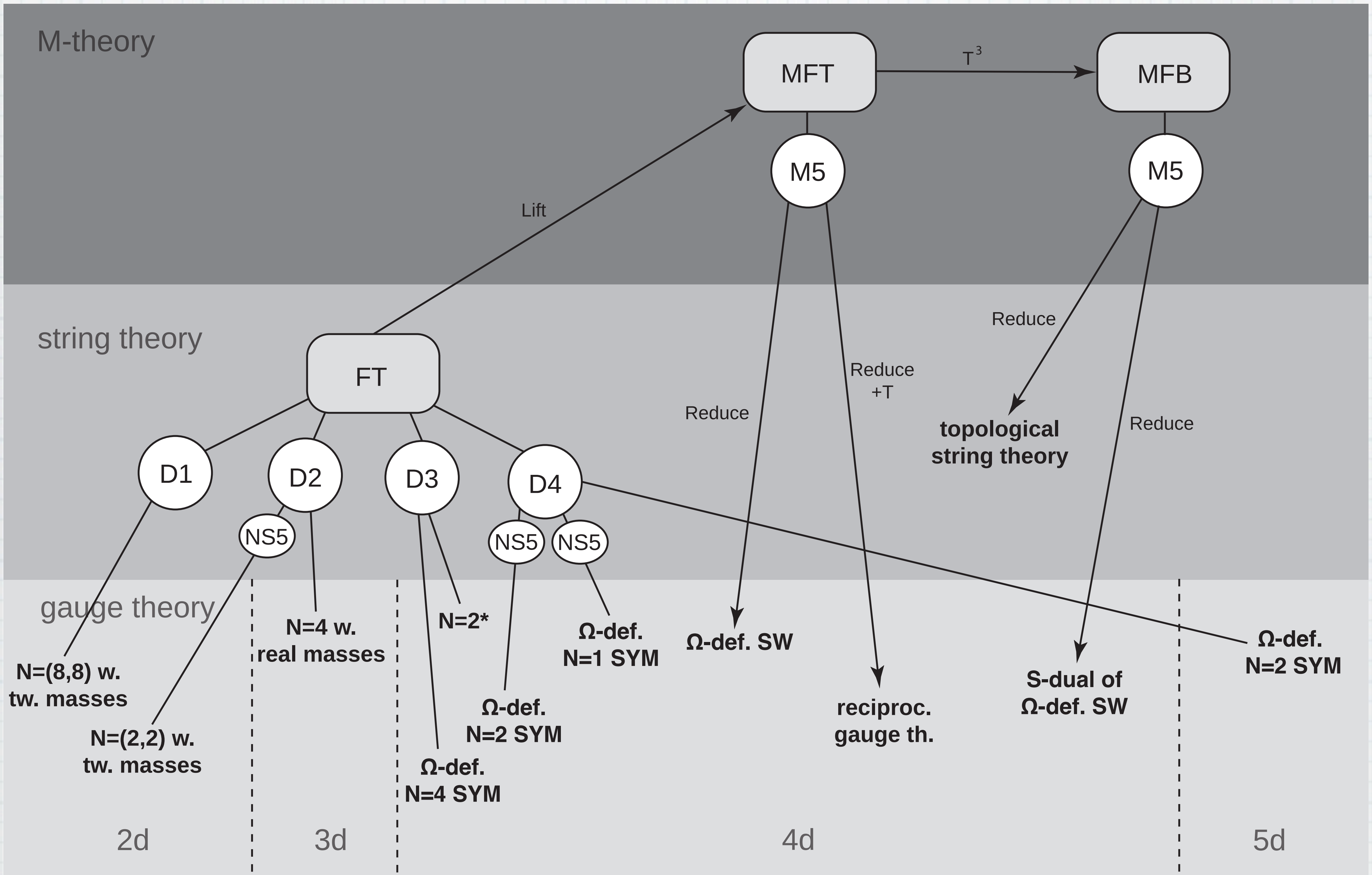

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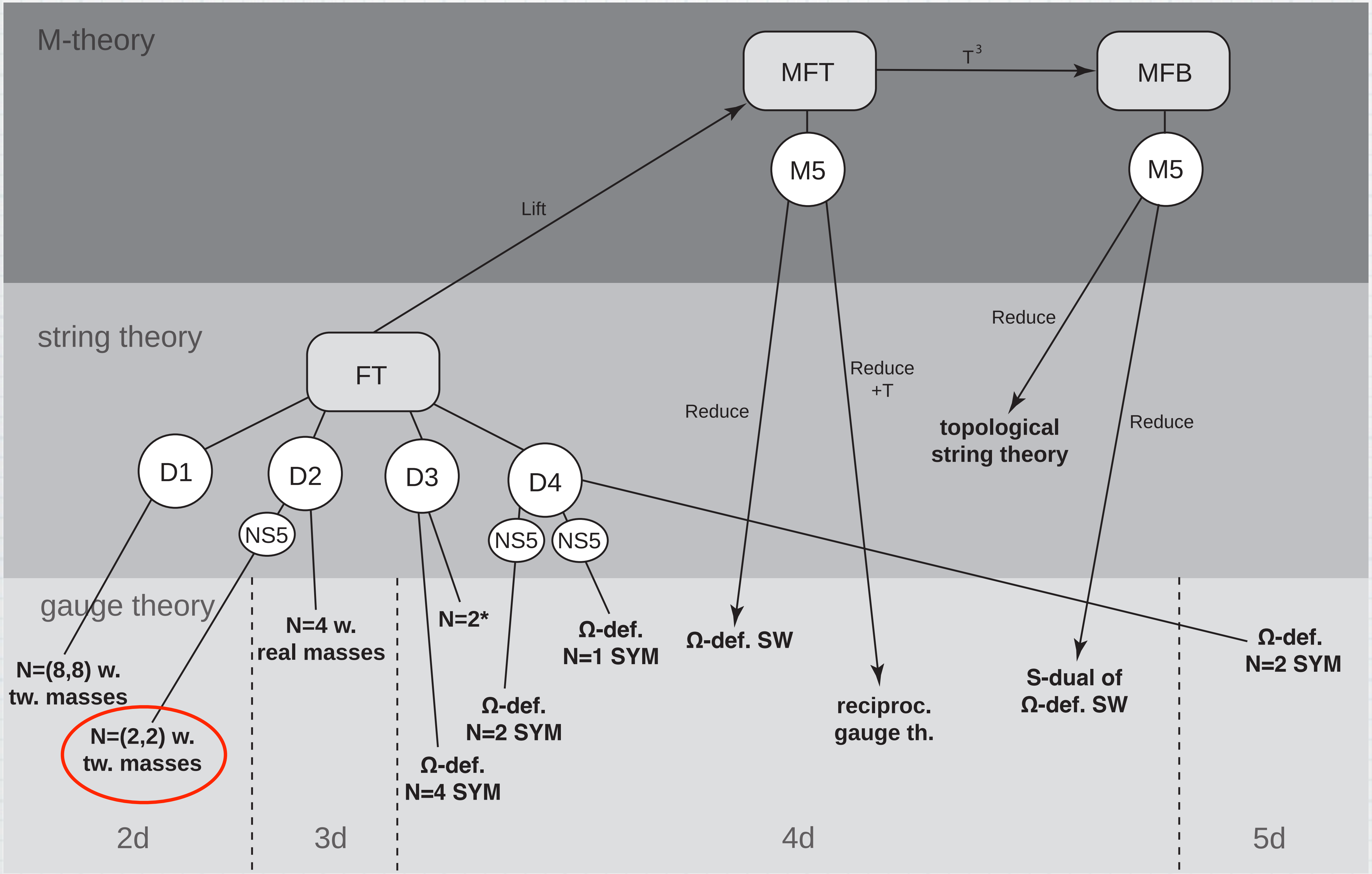

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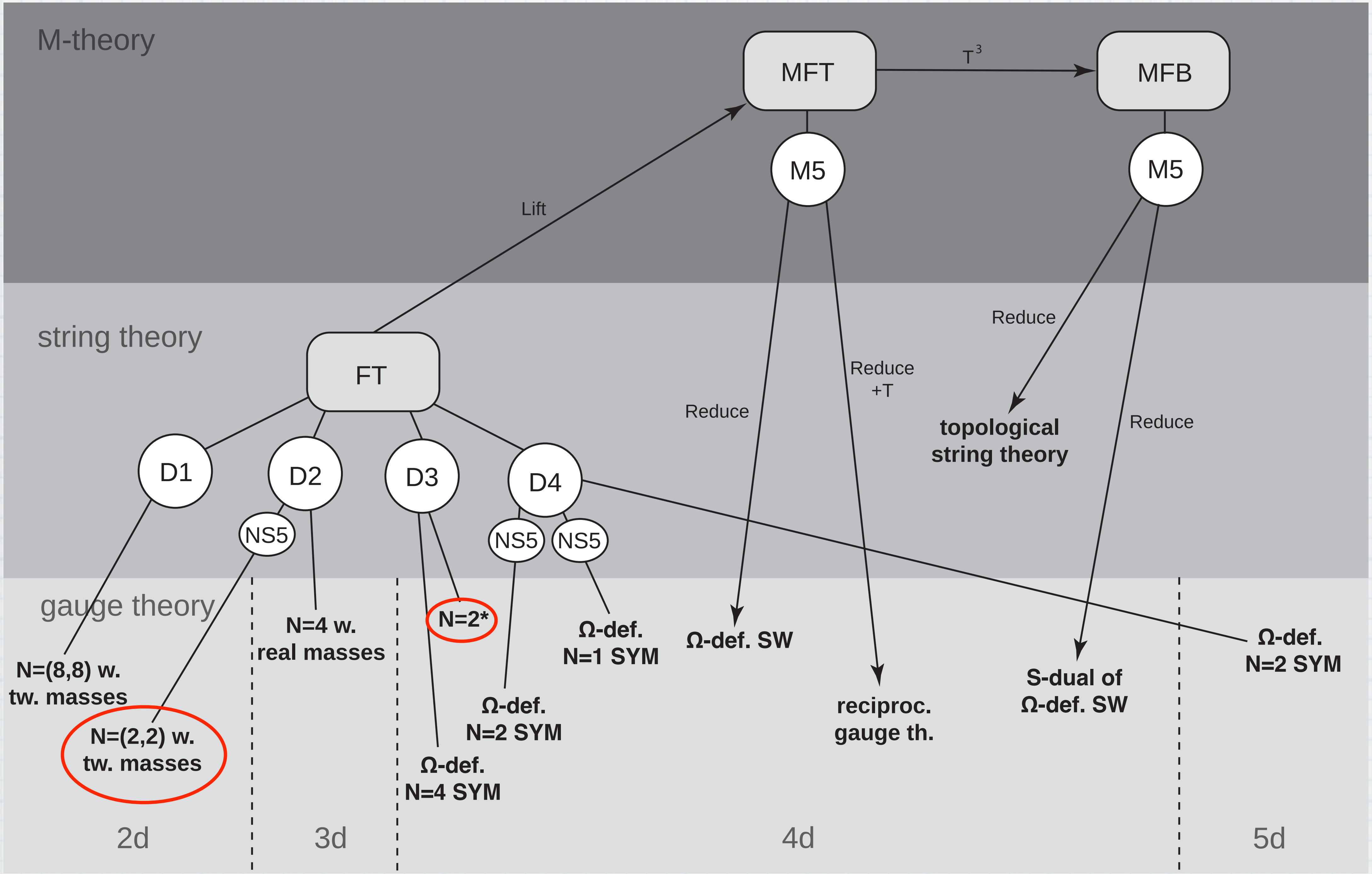

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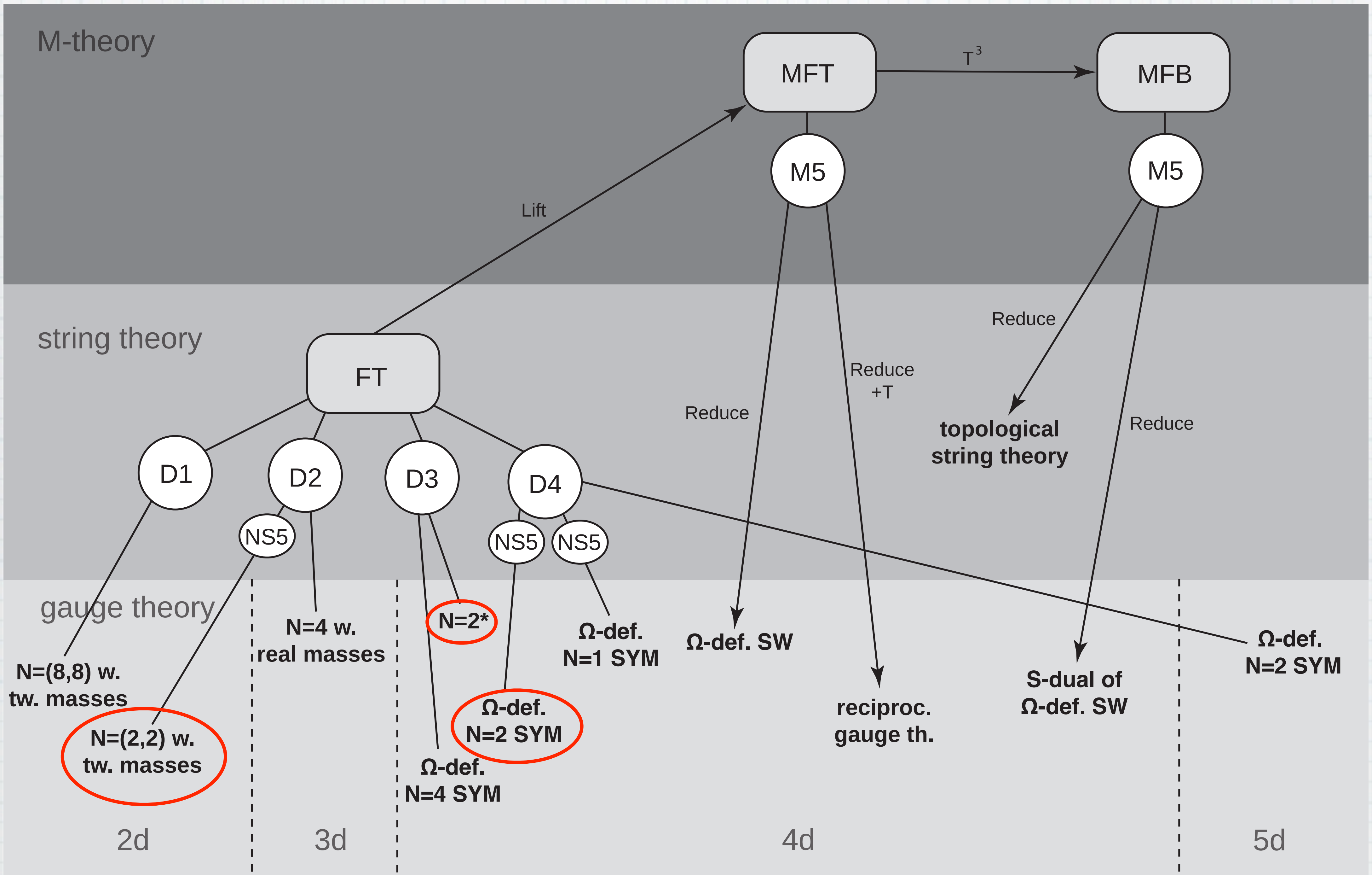

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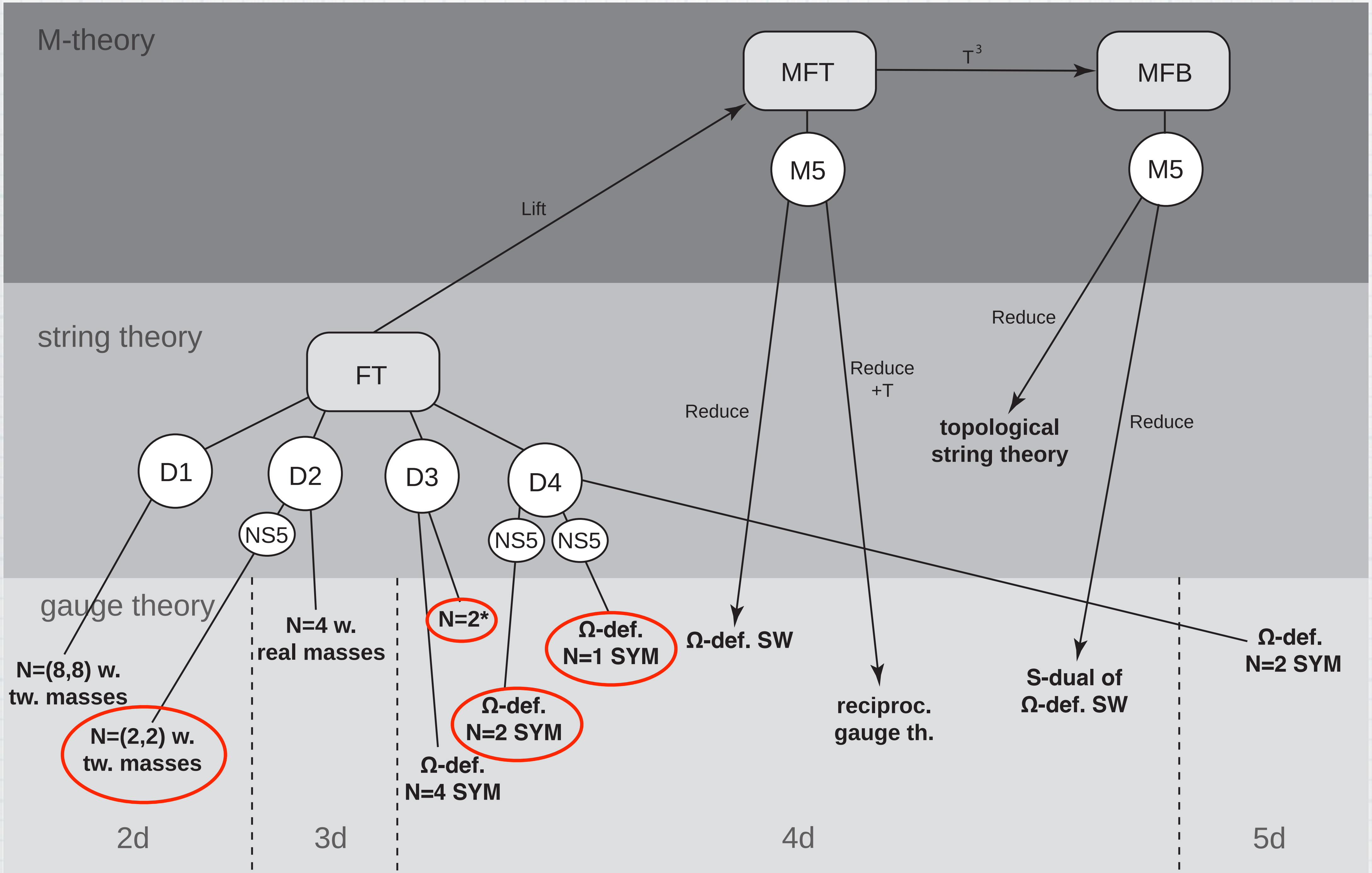

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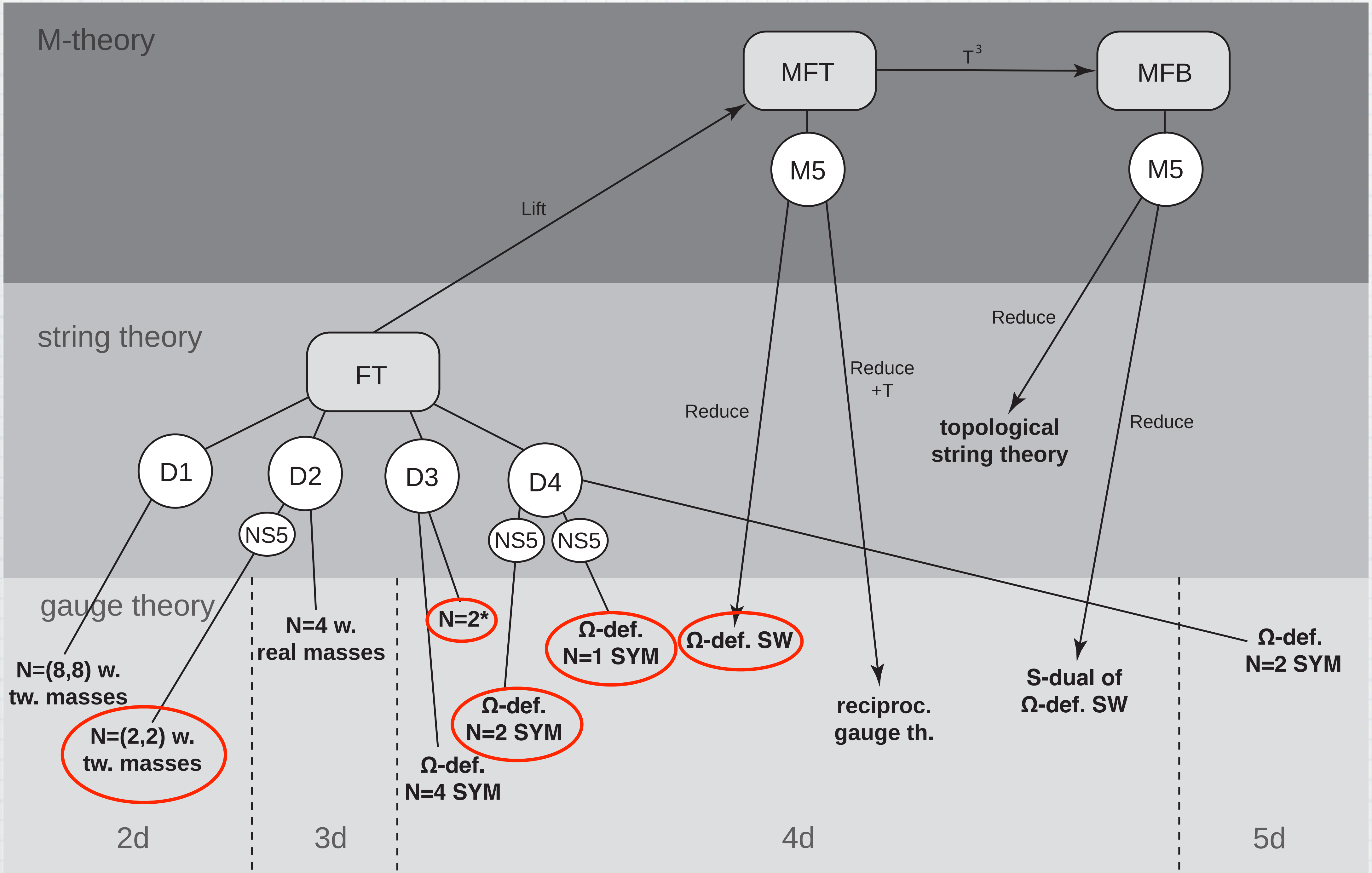

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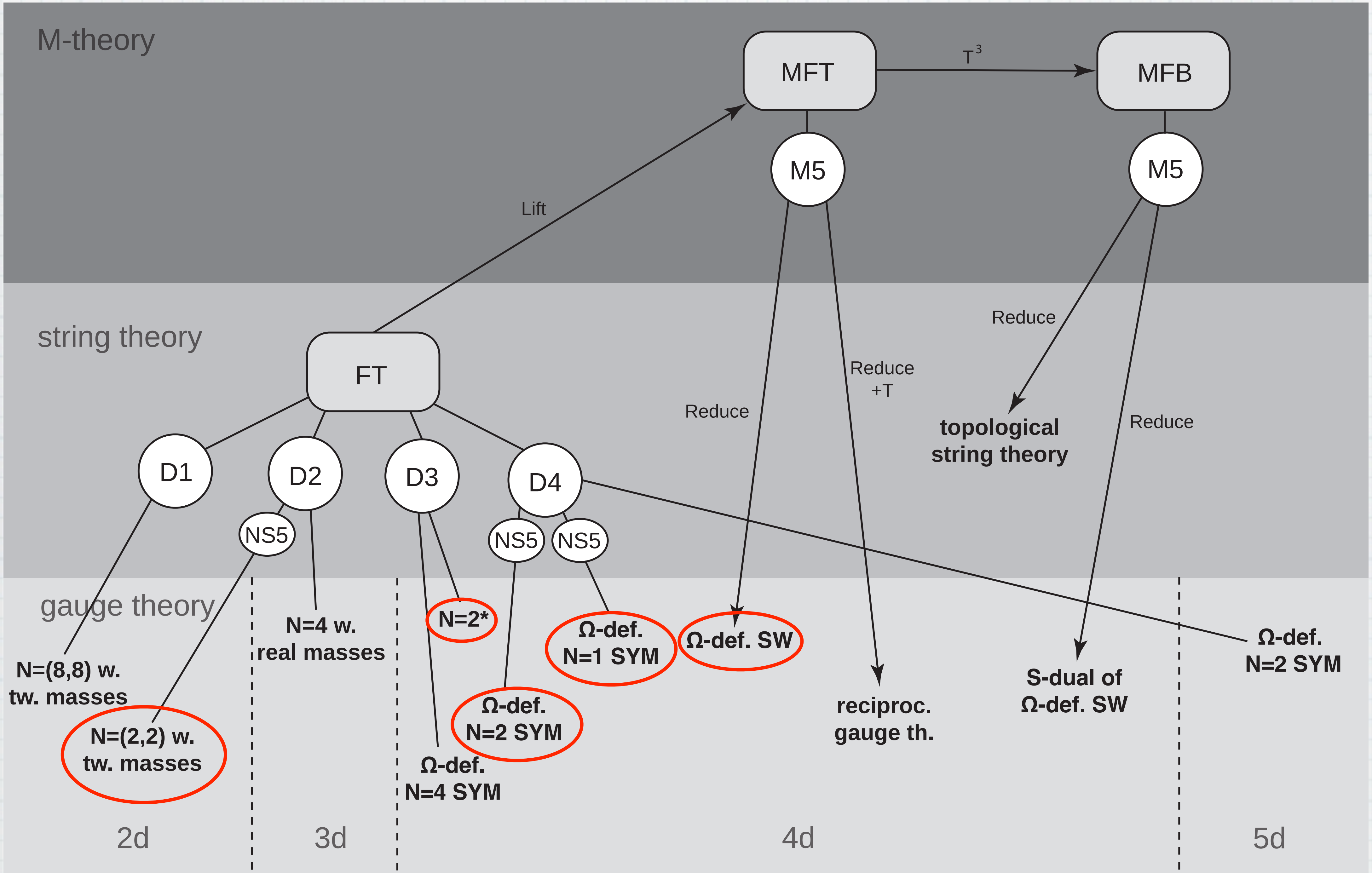

The same string theory background can give rise to many different deformations depending on how we place branes in it!

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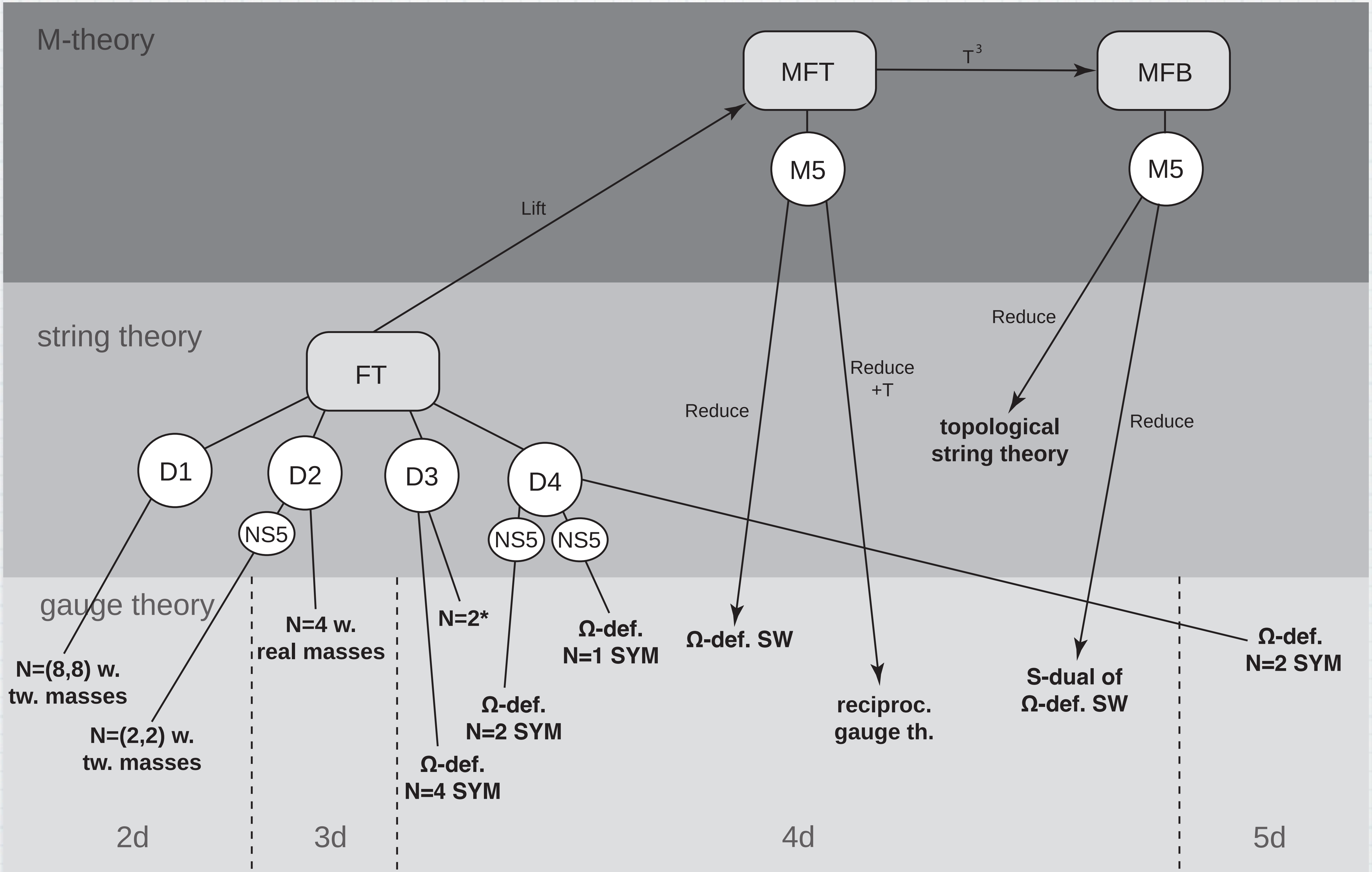

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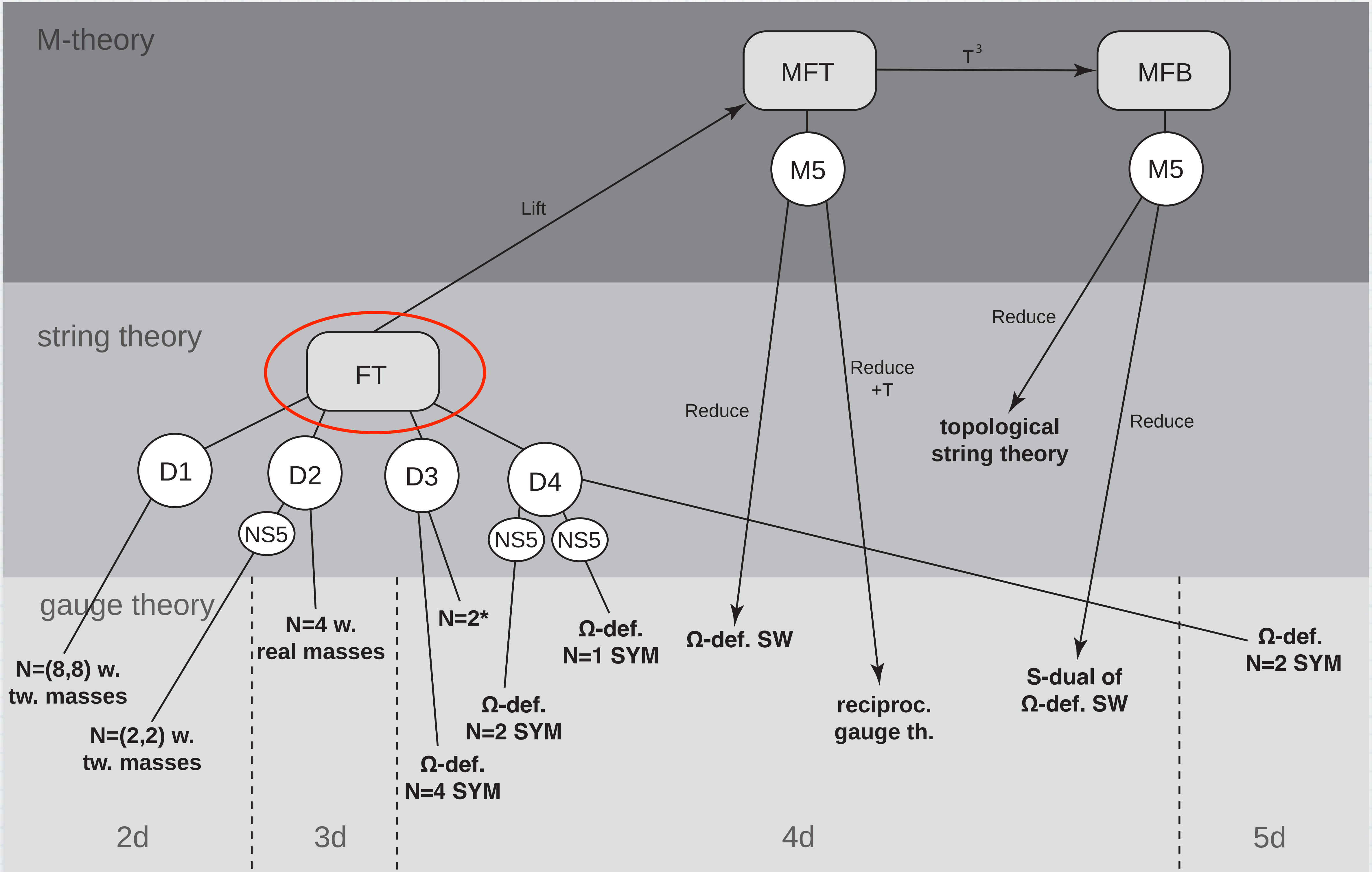

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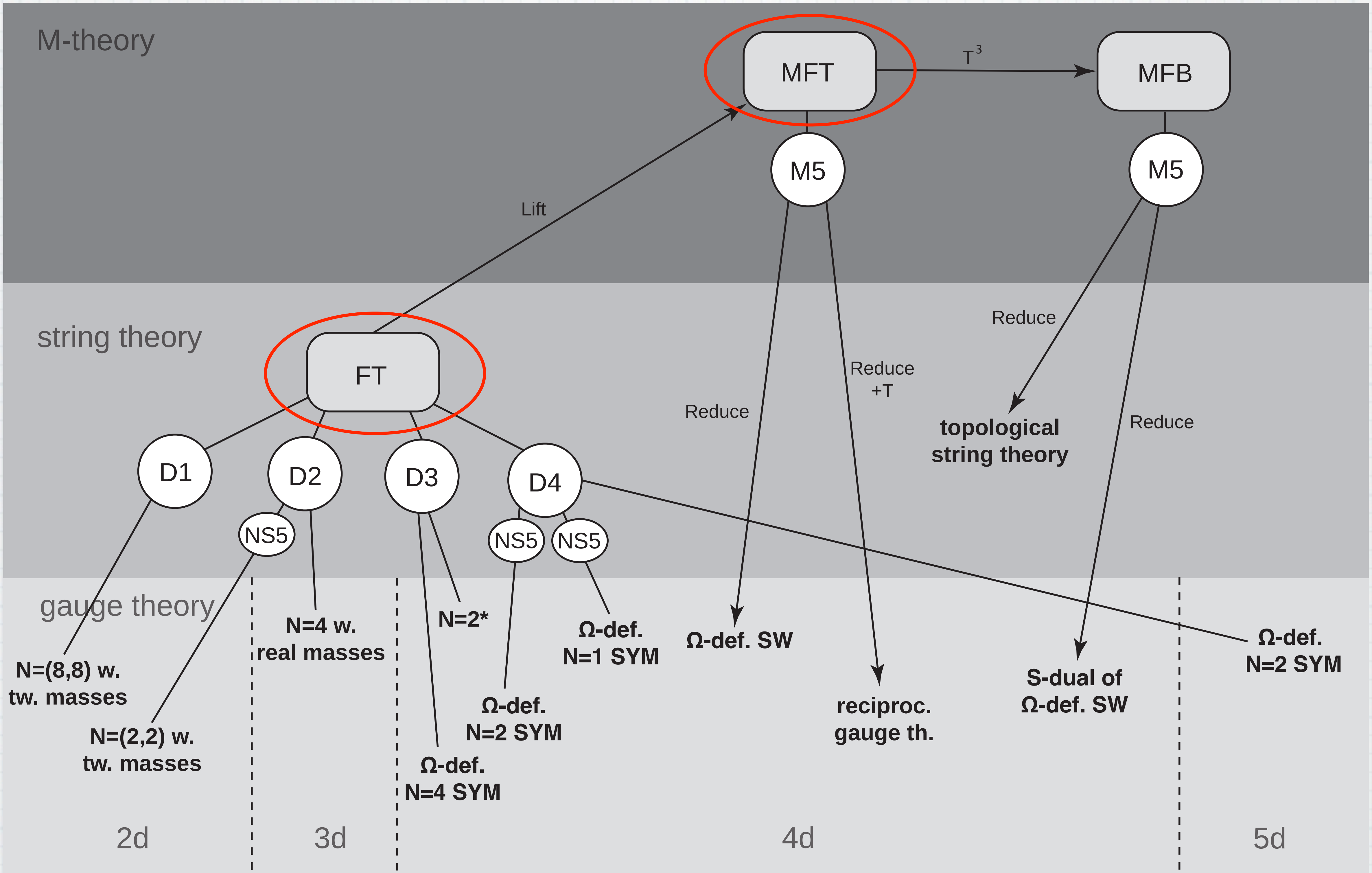

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This corresponds to the well-known Melvin or fluxbrane background.

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$$\eta_{\mathrm{IIB}} = \prod_{k=1}^{N} \exp[\frac{\phi_k}{2}\Gamma_{\rho_k\theta_k}] \exp[\tfrac{1}{2}\,\mathrm{Re}[\epsilon_k \bar{\widetilde{v}}]\Gamma_{\rho_k\theta_k}]\widetilde{\eta_w}$$

not inv. under identifications

$$\widetilde{\eta_w} = (\mathbf{1} + \Gamma_{11})\,(\widetilde{\eta}_0 + \mathrm{i}\,\widetilde{\eta}_1)$$

Impose condition $$\sum_{k=1}^{N} \epsilon_k = 0$$

Exponential can be re-written as

$$\prod_{k=1}^{N-1} \exp[\tfrac{1}{2}\,\mathrm{Re}[\epsilon_k \bar{\widetilde{v}}](\Gamma_{\rho_k\theta_k} - \Gamma_{\rho_N\theta_N})]$$

annihilated by projector

$$\Pi_k^{\mathrm{flux}} = \tfrac{1}{2}\,(\mathbf{1} - \gamma_{\rho_k\theta_k\rho_N\theta_N})$$

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number of deformation parameters

The Fluxtrap Background

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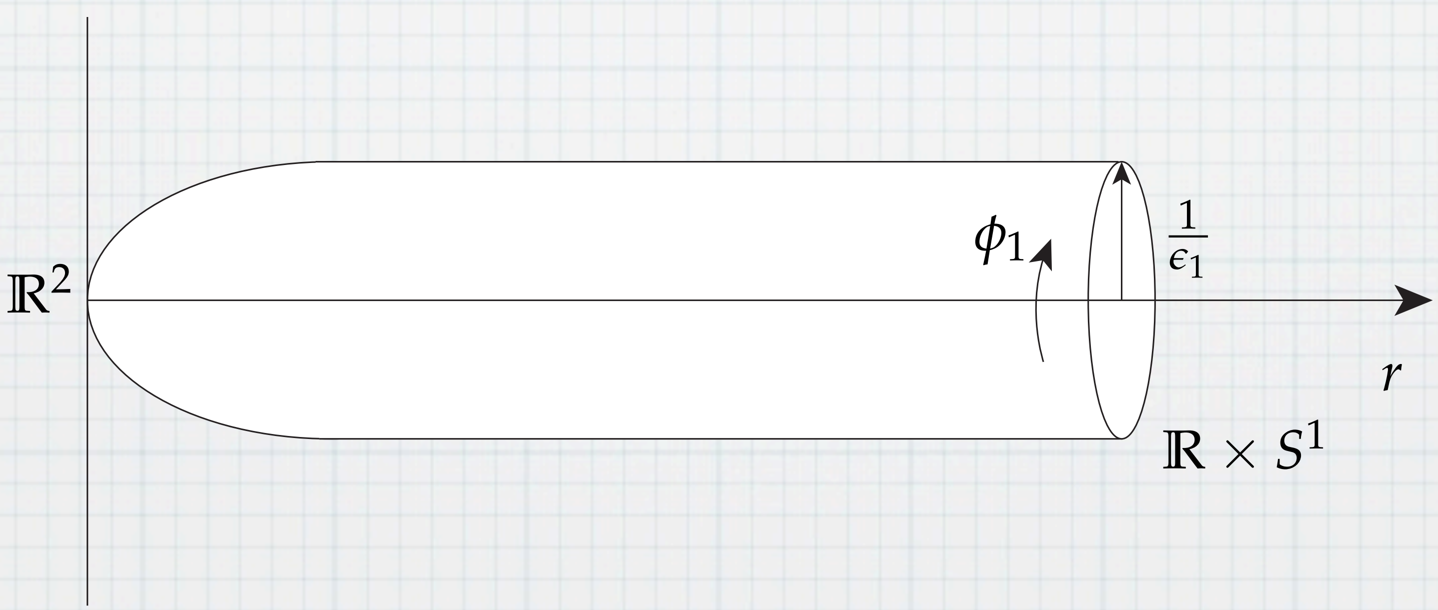

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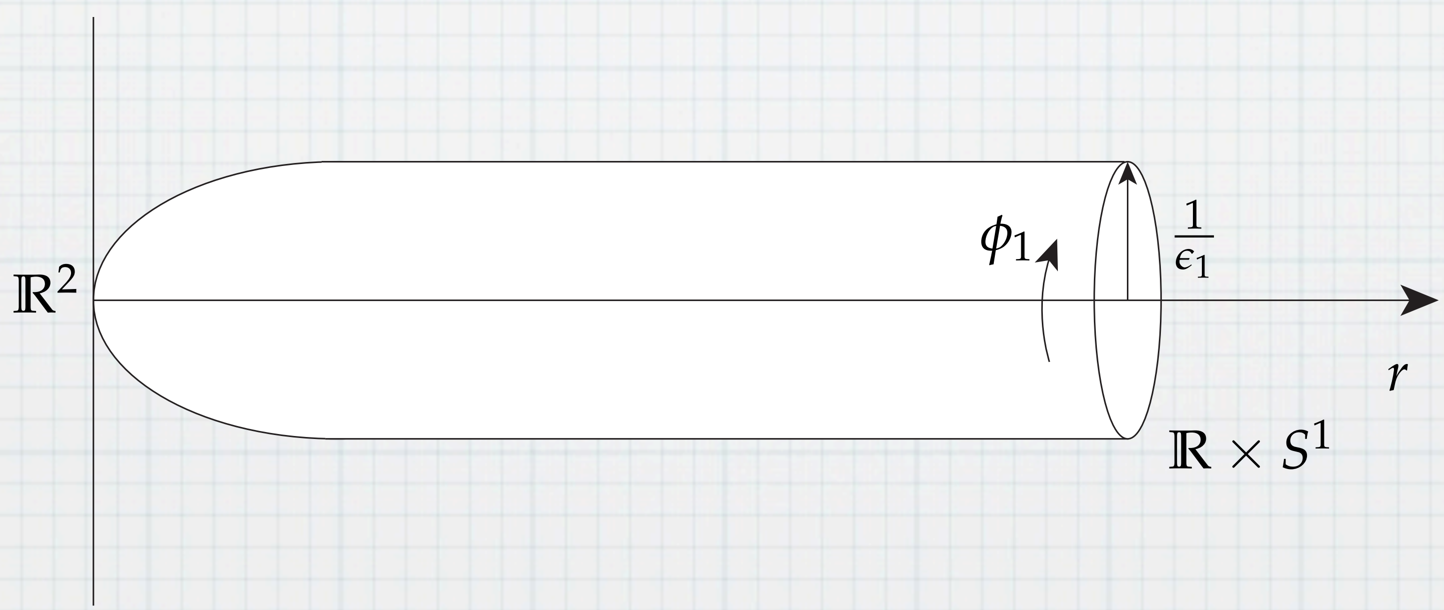

The generator of rotations is bounded (by asymptotic radius).

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$$\Gamma_8 = g_{8\mu}{e_a}^{\mu}\gamma^a$$

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Bulk Relations

Overview over duality web of FT BG in the bulk:

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$$
\begin{tikzcd}
\text{fluxbrane} \arrow[r, leftrightarrow, "\text{T–duality}"] \arrow[d, leftrightarrow, "\text{redox}"] & \text{NS fluxtrap} \arrow[r, leftrightarrow, "\text{S–duality}"] \arrow[dr, leftrightarrow, "\text{redox}"] & \text{RR fluxtrap} \\
\text{M–theory fluxbrane} \arrow[rr, leftrightarrow, "SL_2 \times SL_3"] \arrow[urr, leftrightarrow, "\text{redox and T}"] & & \text{M–theory fluxtrap}
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$$

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Example: Omega-deformed SW action

Omega-deformed SW action

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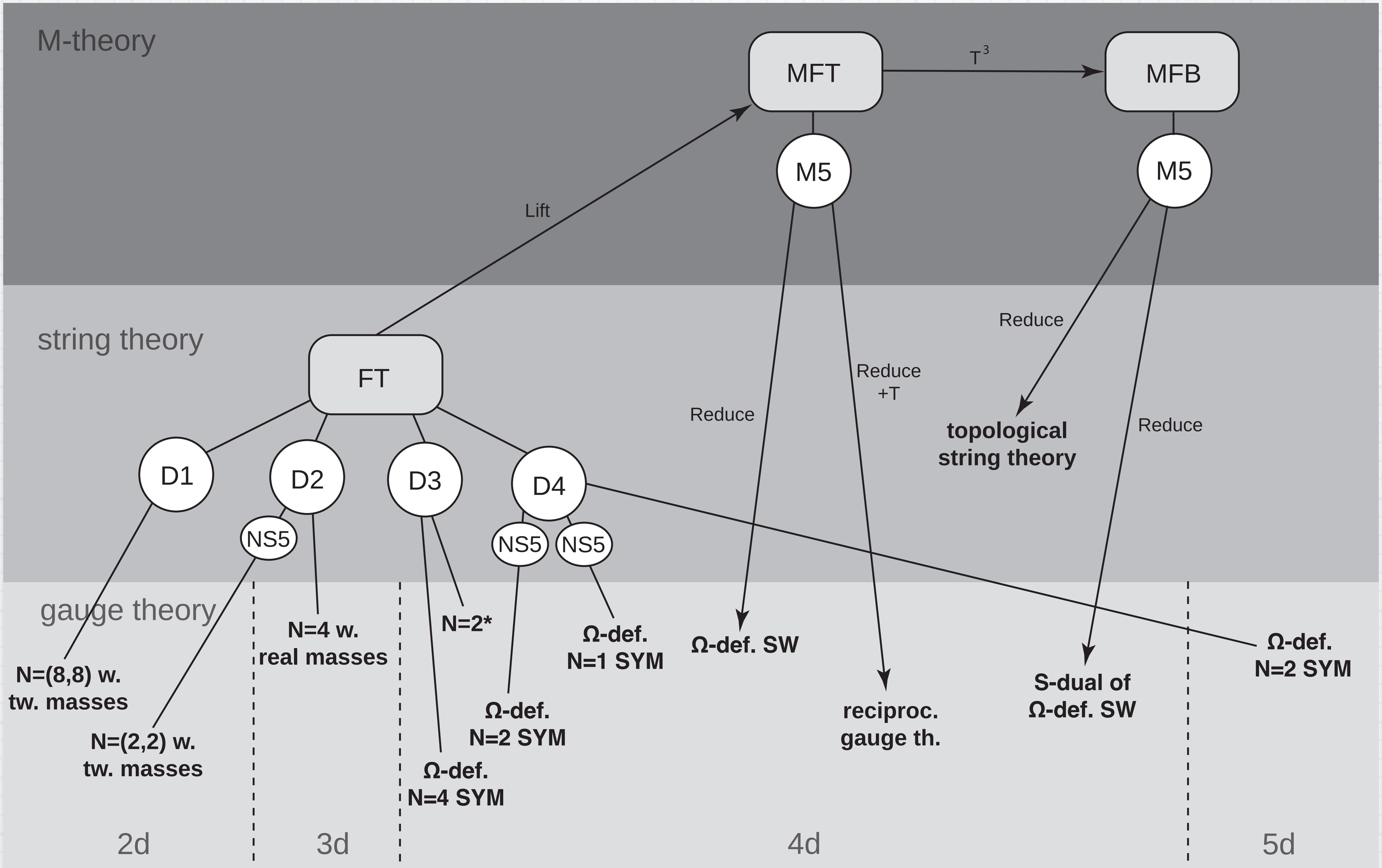

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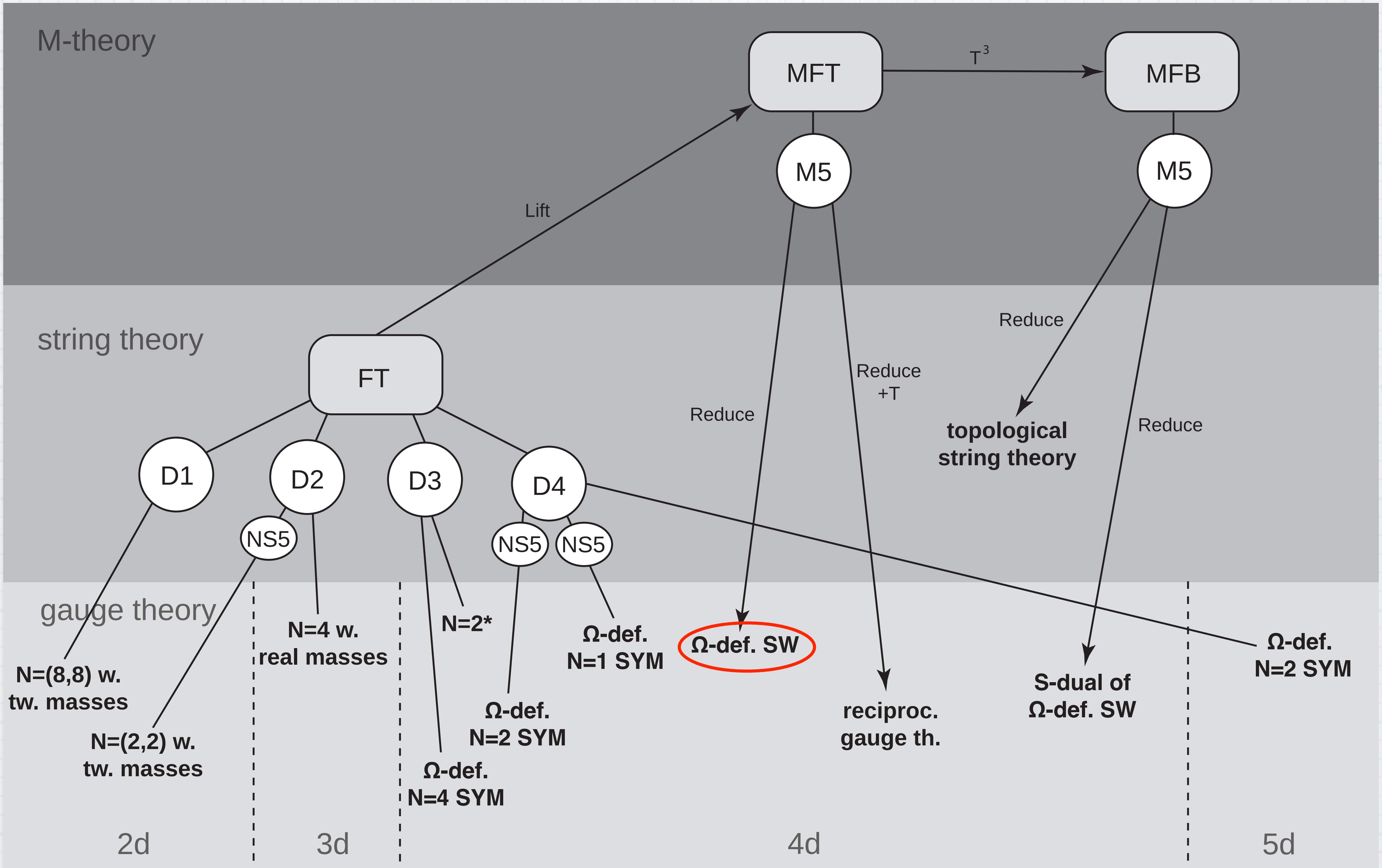

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Application: derive Omega-deformed Seiberg-Witten Lagrangian (eff. low energy action)

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Self-dual three-form on the brane.

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Captures all orders of the 4D gauge theory.

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Start with type IIA set-up of D4- and NS5-branes:

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	0	1	2	3	4	5	6	7	8	9
fluxtrap	ϵ_1		ϵ_2		ϵ_3		$\times$	$\times$	$\circ$	$\times$
NS5	$\times$	$\times$	$\times$	$\times$					$\times$	$\times$
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$$\mathscr{L}_{\mathrm{D4}} = \frac{1}{g_4^2}\,\mathrm{Tr}\Big[\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu} + \frac{1}{2}(\mathbf{D}_\mu\,\boldsymbol{\varphi} + \frac{1}{2}\mathbf{F}_{\mu\lambda}\hat{U}^\lambda)(\mathbf{D}_\mu\,\bar{\boldsymbol{\varphi}} + \frac{1}{2}\mathbf{F}_{\mu\rho}\hat{U}^\rho) \\ - \frac{1}{4}[\boldsymbol{\varphi},\bar{\boldsymbol{\varphi}}]^2 + \frac{1}{8}(\hat{U}^\mu\mathbf{D}_\mu(\boldsymbol{\varphi}-\bar{\boldsymbol{\varphi}}))^2\Big]$$

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Lifts to single M5 extended in $x^0,\dots,x^3$ and wrapping a 2-cycle in x^6, x^8, x^9, x^{10}.

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Choose embedding preserving same susy as in type IIA.

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Selfdual three-form on WV of the M5-brane:

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M5 still embedded holomorphically, implicit form for SU(2): $t^2 - 2B(z|u)t + \Lambda^4 = 0$,

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$$B(z|u) = \Lambda^4 z^2 - u$$

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Riemann surface with modulus u:

$$\Sigma = \{(z,s)|s = s(z|u)\}$$

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$$s = s(z|u(x^\mu)) \qquad \partial_\mu s(z|u(x^\mu)) = \partial_\mu u \frac{\partial s}{\partial u}$$

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Howe, Sezgin, West

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$$(\hat{g}^{mn} - 16h^{mpq}h^{n}{}_{pq})\,\nabla_m\nabla_n X^I = -\frac{2}{3}\hat{G}^I{}_{mnp}h^{mnp}\,,$$

vector equation

$$\mathrm{d}h_3 = -\tfrac{1}{4}\hat{G}_4\,,$$

Howe, Sezgin, West

These equations do not stem from an action in 6D.

Omega-deformed SW

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selfdual 3-form, encodes fluctuations of 4d gauge field

Omega-deformed SW

Omega-deformed SW

Want to relate Φ to 4d gauge field: only components

$$(\mu,\nu,z),\ \ (\mu,\nu,\bar{z})$$

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antiselfdual 2-form

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to ensure

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holomorphic 1-form on Riemann surface

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unique 1-form on Riemann surface

Omega-deformed SW

Integrals result in

Omega-deformed SW

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Omega-deformed SW

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The difference of these equations is solved by

$$\begin{cases} \mathcal{F} = (1 - *)\, F - (a - \bar a)\, \hat\omega^- \,, \\ \widetilde{\mathcal{F}} = (1 + *)\, F + (a - \bar a)\, \hat\omega^+ \,, \end{cases}$$

Omega-deformed SW

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where F satisfies the Bianchi identity: $\mathrm{d}F=0$

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F can now be identified with the gauge field in 4D!

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Substituting this back, the vector equation becomes

Omega-deformed SW

Integrals result in

$$(\tau-\bar\tau)\,(\partial_\mu\mathcal{F}_{\mu\nu}+\partial_\mu a\,\hat\omega_{\mu\nu})+\partial_\mu\tau\mathcal{F}_{\mu\nu}-\partial_\mu\bar\tau\widetilde{\mathcal{F}}_{\mu\nu}=0\,,$$

$$(\tau-\bar\tau)\left(\partial_\mu\widetilde{\mathcal{F}}_{\mu\nu}+\partial_\mu\bar a\,\hat\omega_{\mu\nu}\right)+\partial_\mu\tau\mathcal{F}_{\mu\nu}-\partial_\mu\bar\tau\widetilde{\mathcal{F}}_{\mu\nu}=0$$

The difference of these equations is solved by

$$\begin{cases}\mathcal{F}=(1-*)\,F-(a-\bar a)\,\hat\omega^-\,,\\ \widetilde{\mathcal{F}}=(1+*)\,F+(a-\bar a)\,\hat\omega^+\,,\end{cases}$$

where F satisfies the Bianchi identity: $\mathrm{d}F=0$

F can now be identified with the gauge field in 4D!

Substituting this back, the vector equation becomes

$$(\tau-\bar\tau)\left[\partial_\mu F_{\mu\nu}+\tfrac{1}{2}\,\partial_\mu(a+\bar a)\hat\omega_{\mu\nu}+\tfrac{1}{2}\,\partial_\mu(a-\bar a)^*\hat\omega_{\mu\nu}\right]$$
$$+\,\partial_\mu(\tau-\bar\tau)\left[F_{\mu\nu}+\tfrac{1}{2}\,(a-\bar a)\,{}^*\hat\omega_{\mu\nu}\right]-\partial_\mu(\tau+\bar\tau)\left[{}^*F_{\mu\nu}+\tfrac{1}{2}\,(a-\bar a)\,\hat\omega_{\mu\nu}\right]=0$$

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Scalar equation:

$$\left(\hat{g}^{mn} - 16 h^{mpq} h^{n}{}_{pq}\right) \nabla_m \nabla_n X^I = -\frac{2}{3} \hat{G}^I{}_{mnp} h^{mnp}$$

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Of the 11 equations, only the ones of the components $I = s, \bar{s}$ are nontrivial:

$$E \; = \partial_\mu \partial_\mu s - \partial \left[\frac{\partial_\rho s\, \partial_\rho s\, \bar{\partial} \bar{s}}{1 + |\partial s|^2} \right] - \frac{16\, \partial^2 s}{\left(1 + |\partial s|^2\right)^2} h_{\mu\nu\bar{z}} h_{\mu\nu\bar{z}}$$

$$- 2 \hat{\omega}^-_{\mu\nu} \mathcal{F}_{\mu\nu} \left(\frac{\mathrm{d}a}{\mathrm{d}u} \right)^{-1} \lambda_z + 2 \hat{\omega}^+_{\mu\nu} \widetilde{\mathcal{F}}_{\mu\nu} \left(\frac{\mathrm{d}\bar{a}}{\mathrm{d}\bar{u}} \right)^{-1} \bar{\lambda}_{\bar{z}} = 0 \,,$$

$$\bar{E} \; = \partial_\mu \partial_\mu \bar{s} - \bar{\partial} \left[\frac{\partial_\rho \bar{s}\, \partial_\rho \bar{s}\, \partial s}{1 + |\partial s|^2} \right] - \frac{16\, \bar{\partial}^2 \bar{s}}{\left(1 + |\partial s|^2\right)^2} h_{\mu\nu z} h_{\mu\nu z}$$

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$$\begin{aligned}
(\tau - \bar{\tau})\,\partial_\mu\partial_\mu a + \partial_\mu a\,\partial_\mu \tau + 2\frac{\mathrm{d}\bar{\tau}}{\mathrm{d}\bar{a}}\,(F_{\mu\nu}F_{\mu\nu} + F_{\mu\nu}{}^*F_{\mu\nu})& \\
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(\tau - \bar{\tau})\,\partial_\mu\partial_\mu \bar{a} - \partial_\mu \bar{a}\,\partial_\mu \bar{\tau} - 2\frac{\mathrm{d}\tau}{\mathrm{d}a}\,(F_{\mu\nu}F_{\mu\nu} - F_{\mu\nu}{}^*F_{\mu\nu})& \\
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Consistent result justifies earlier assumptions about foliation structure, form of fluctuations and integration measure.

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Why does this result of a classical M-theory calculation capture the quantum effects of gauge theory?

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Independent of compactification radius to IIA, which is related to gauge coupling in 4d.

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generalized covariant derivative for the scalar a,
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shift in the gauge field strength

Summary

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Constructed the fluxtrap background in string theory.

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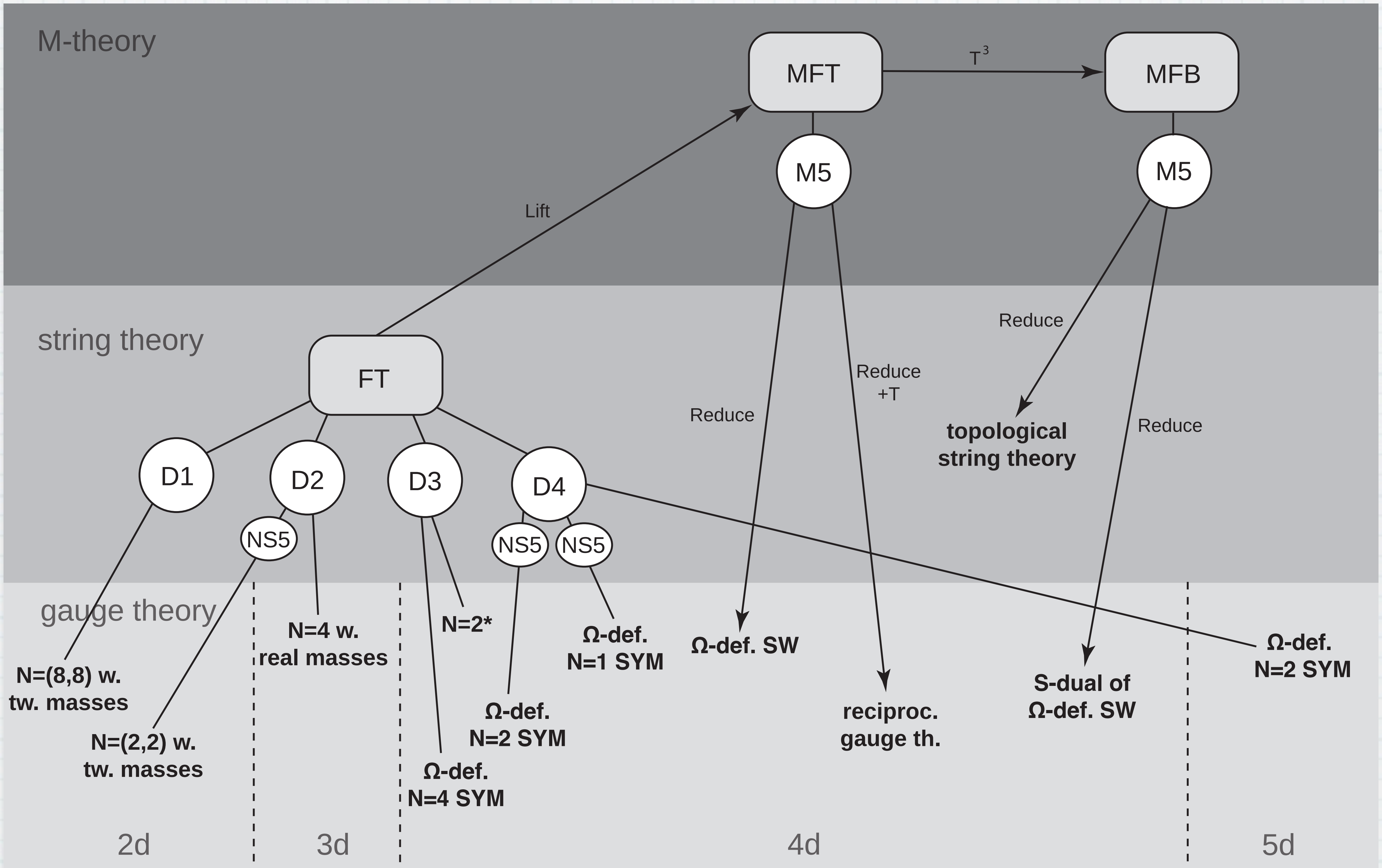

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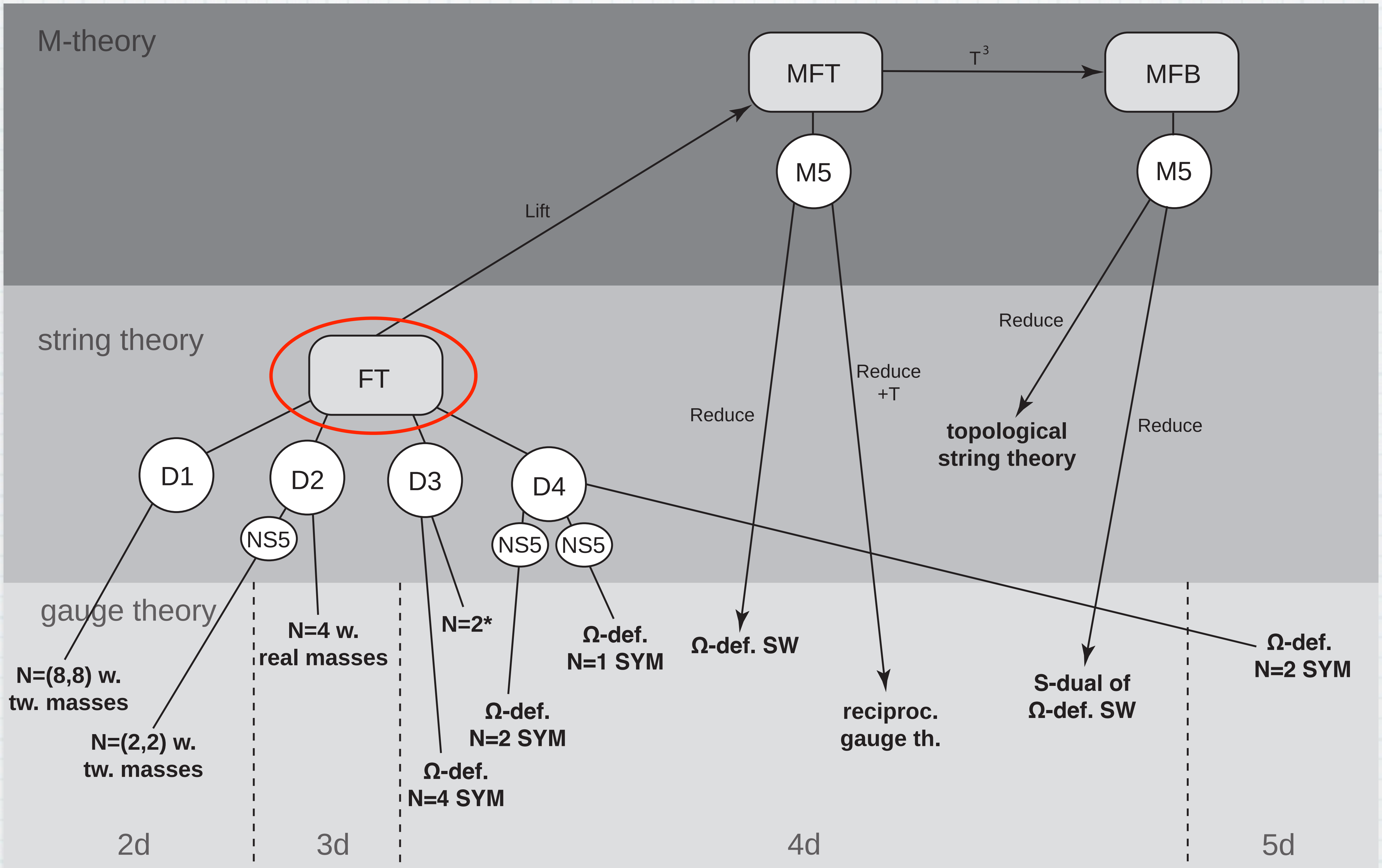

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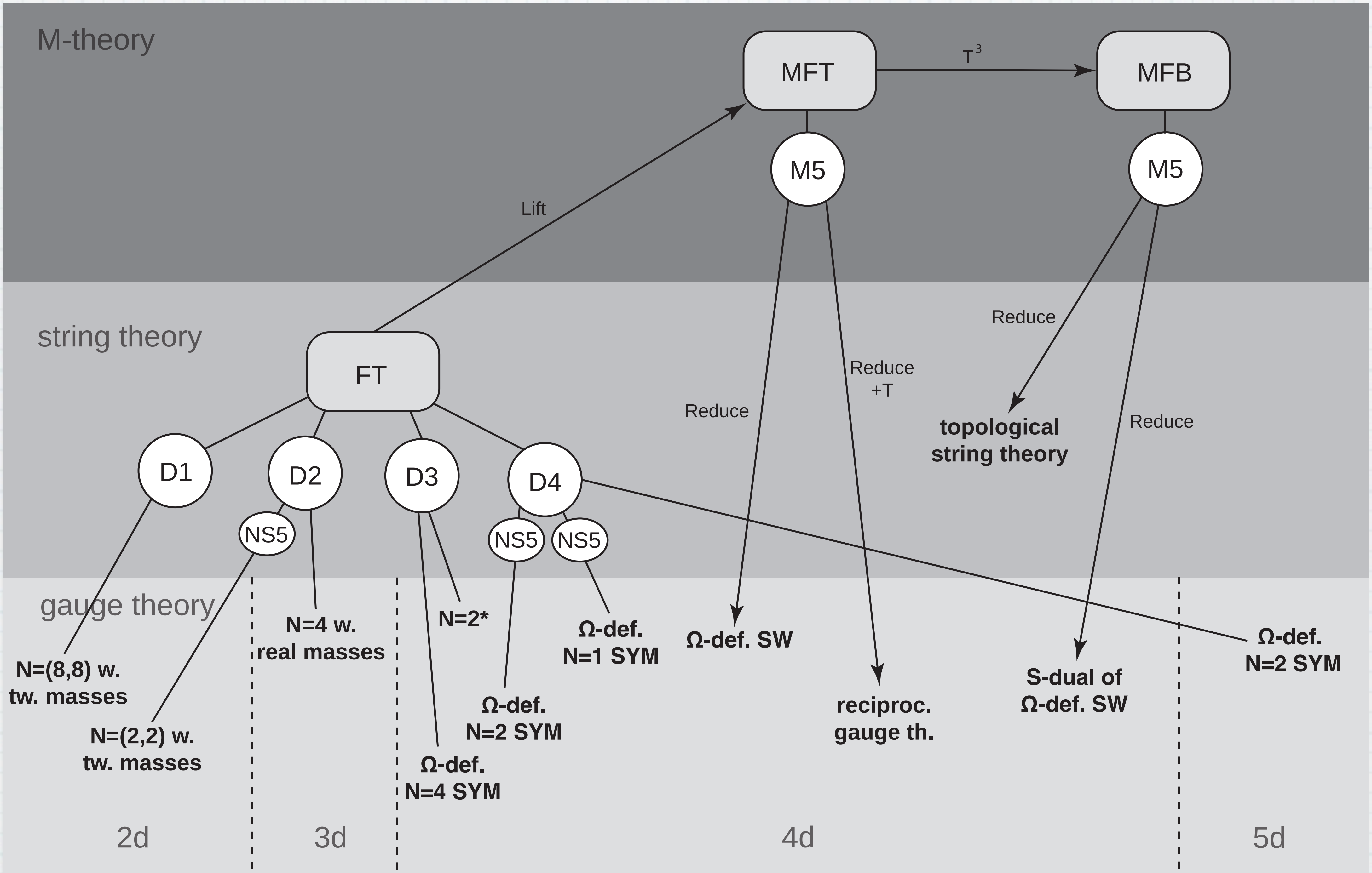
Can be lifted to M-theory: M-theory Fluxtrap



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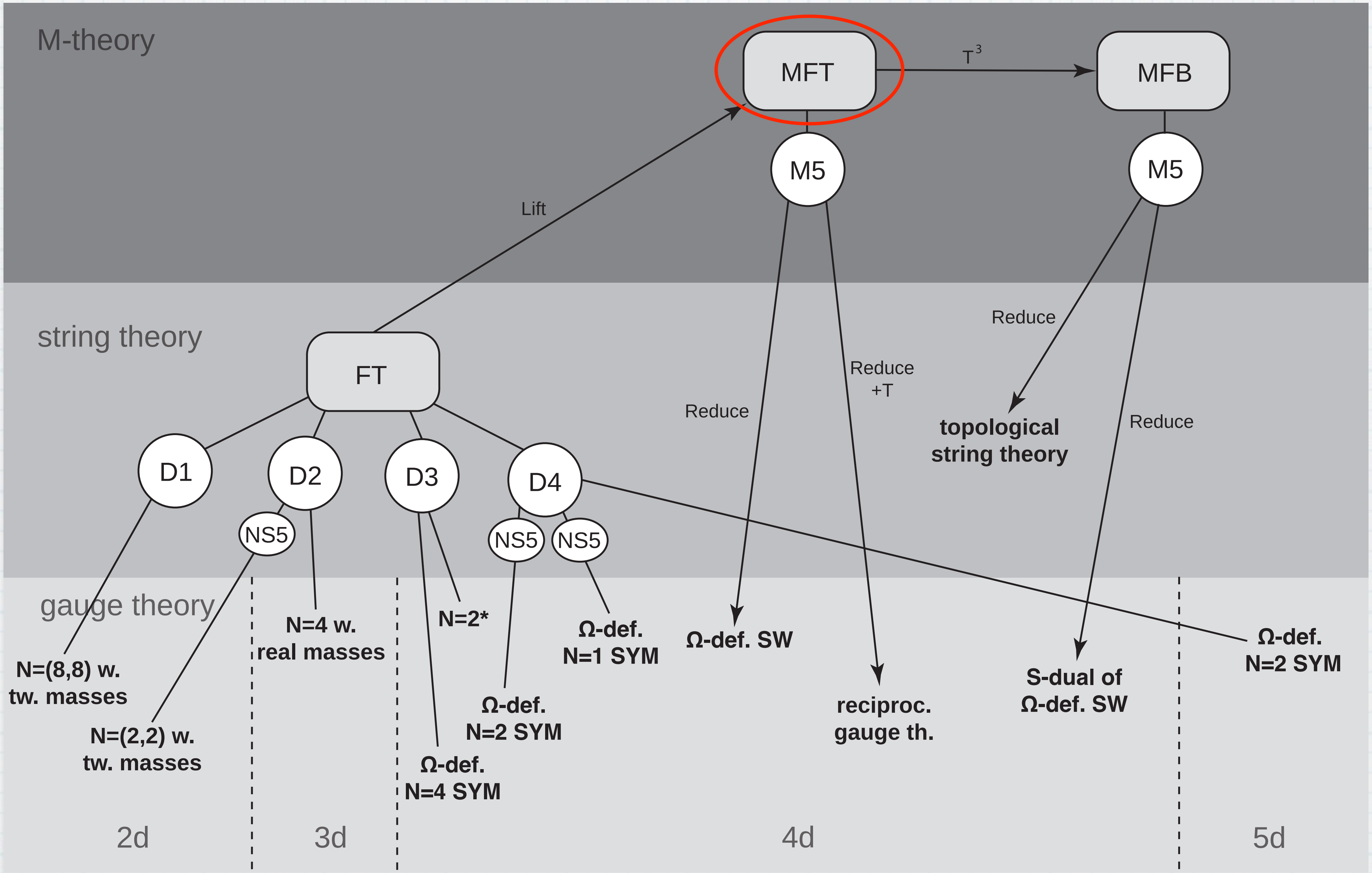

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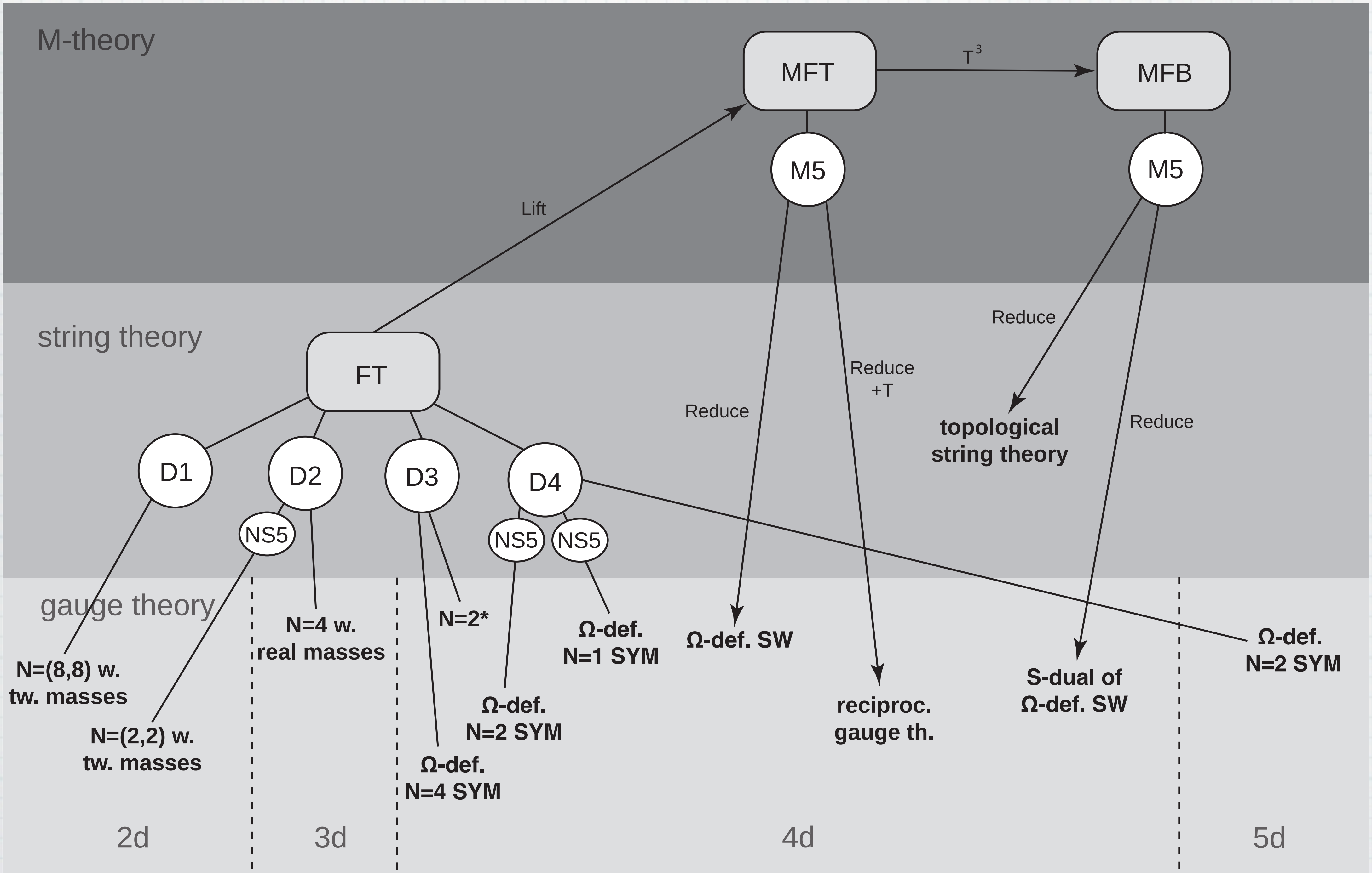
Derive Omega-deformed Seiberg-Witten Lagrangian and its S-dual

arXiv:1304.3488



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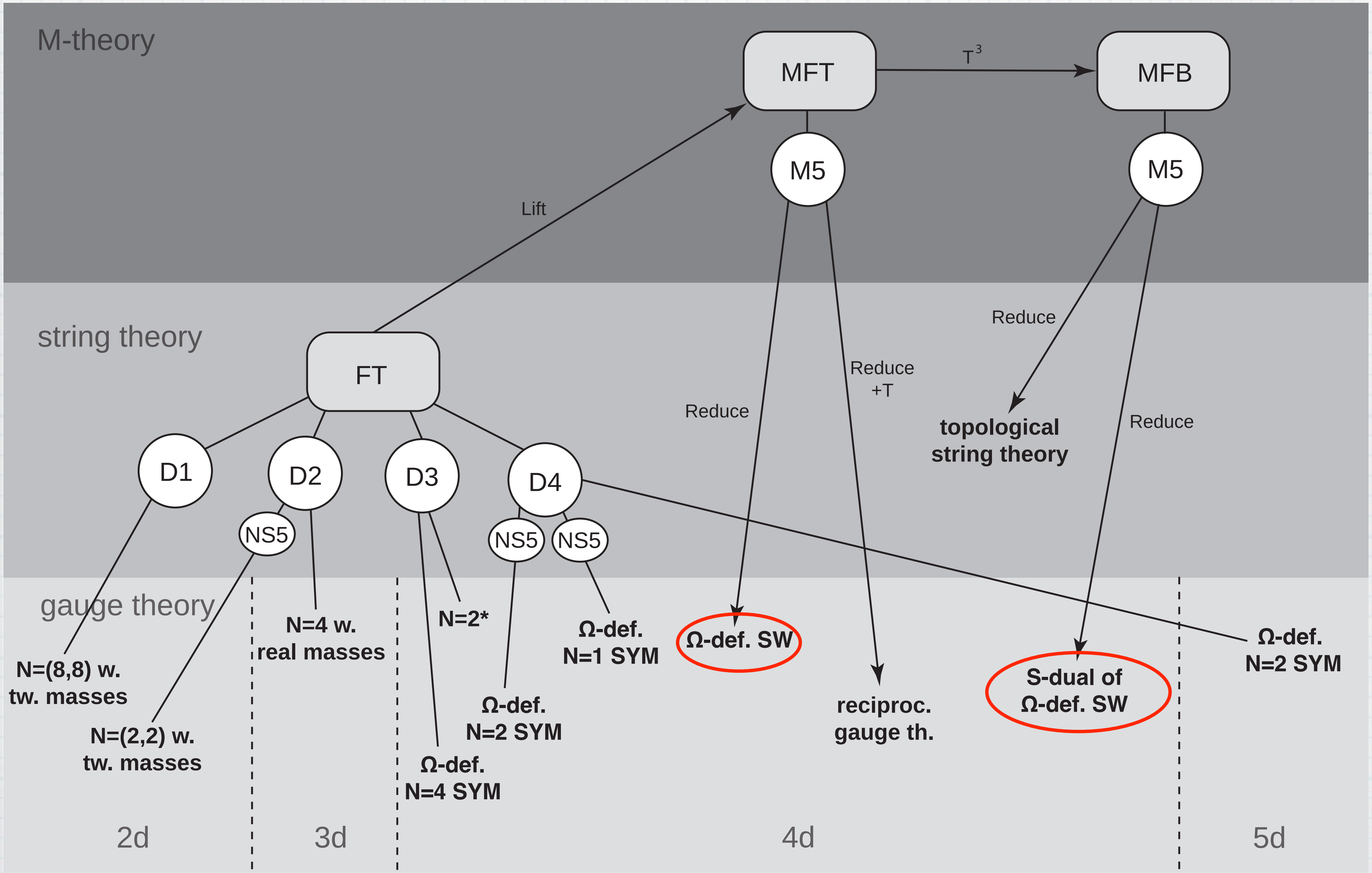

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The resulting 4d e.o.m. for the scalar and vector fields are Euler-Lagrange equations for a 4d action: Omega-deformed Seiberg-Witten Lagrangian!

Classical M-theory calculation yields quantum result, captures all orders of 4d gauge theory.

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Many interesting applications! Domenico's talk after the break.

Thank you for your attention!