# Equilibrating the Quark-Gluon Plasma

## Flow, Fluctuations, Thermalization

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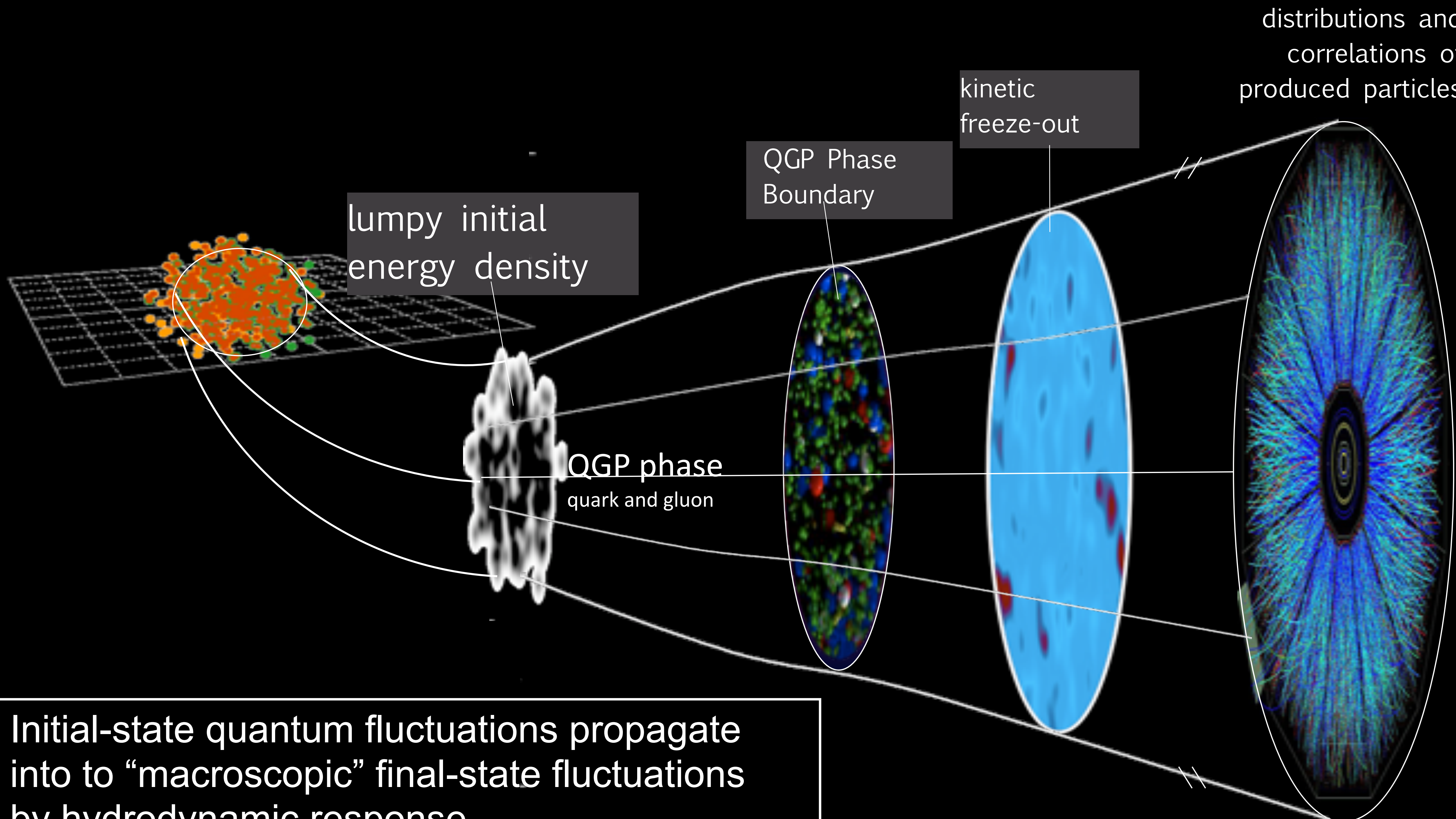
Some possibly useful reviews:
*General QGP (with B. Jacak):*
Science 337, 310 (2012)
*Entropy production (with A. Schäfer):*
arXiv:1110.2378 (IJMPE 20, 2235)

# Introduction

# Little Bang vs. Big Bang




Initial-state quantum fluctuations propagate into to “macroscopic” final-state fluctuations by hydrodynamic response.

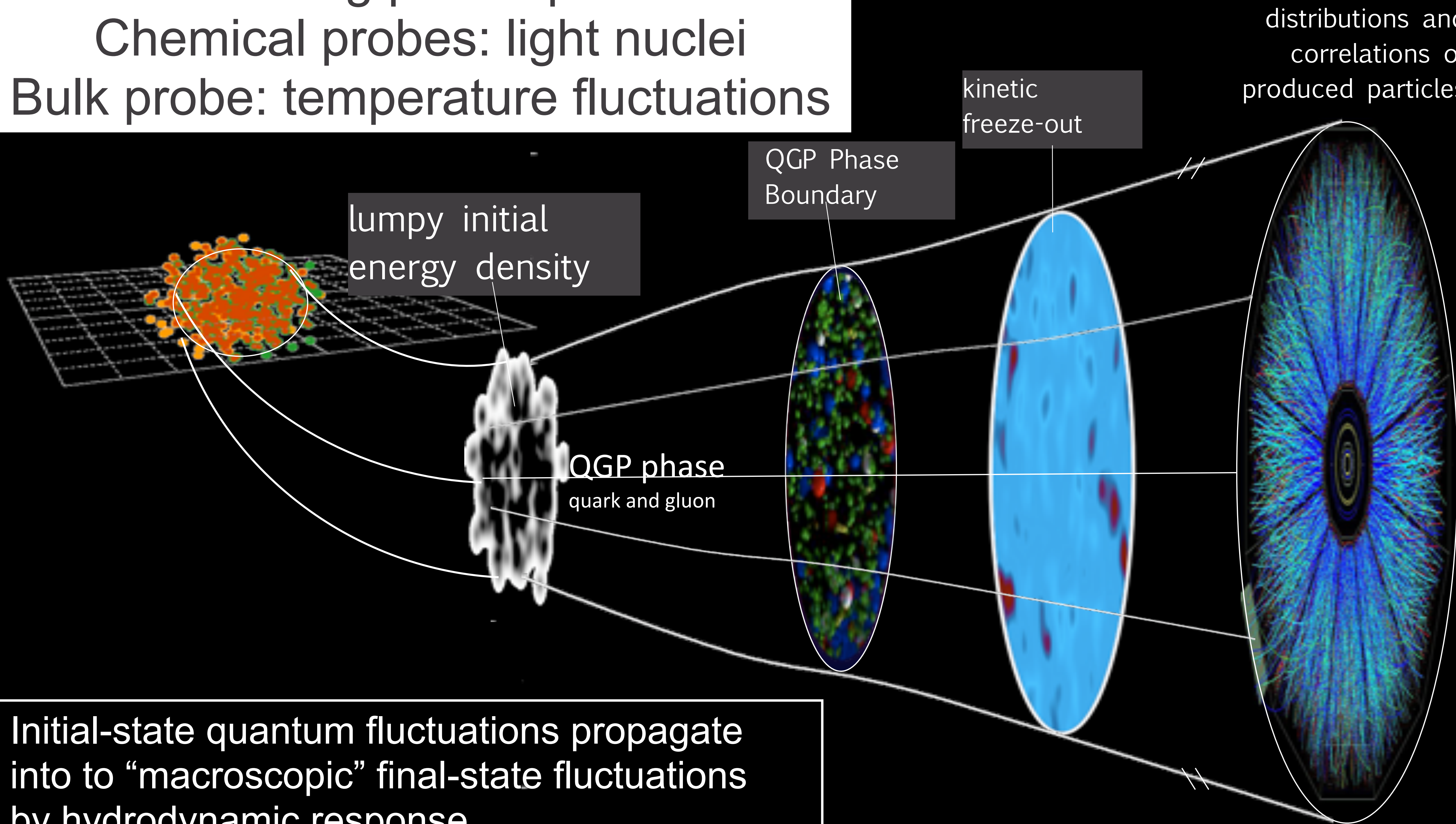
# Little Bang vs. Big Bang

"Big Bang"

Penetrating probe: photons

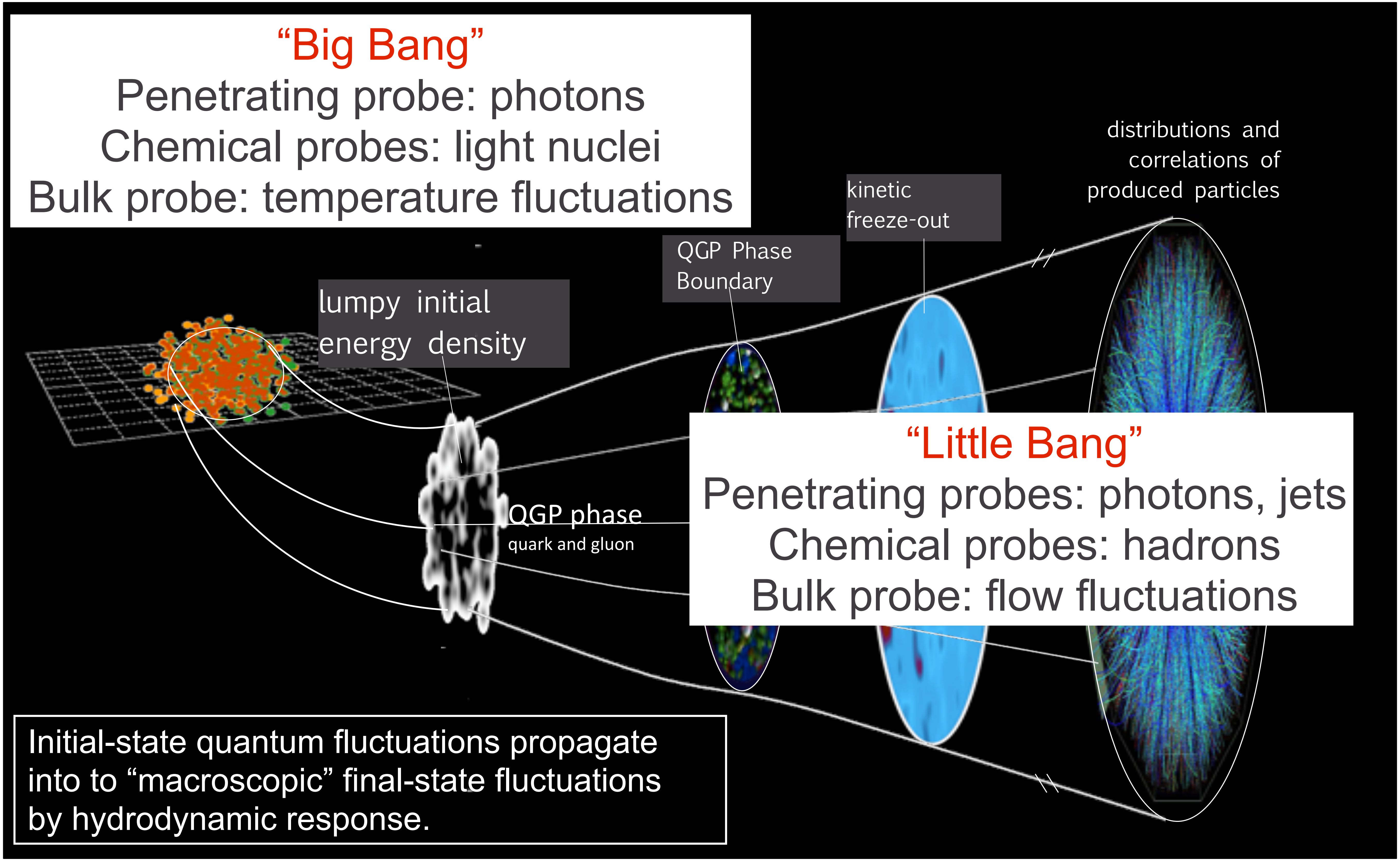
Chemical probes: light nuclei

Bulk probe: temperature fluctuations

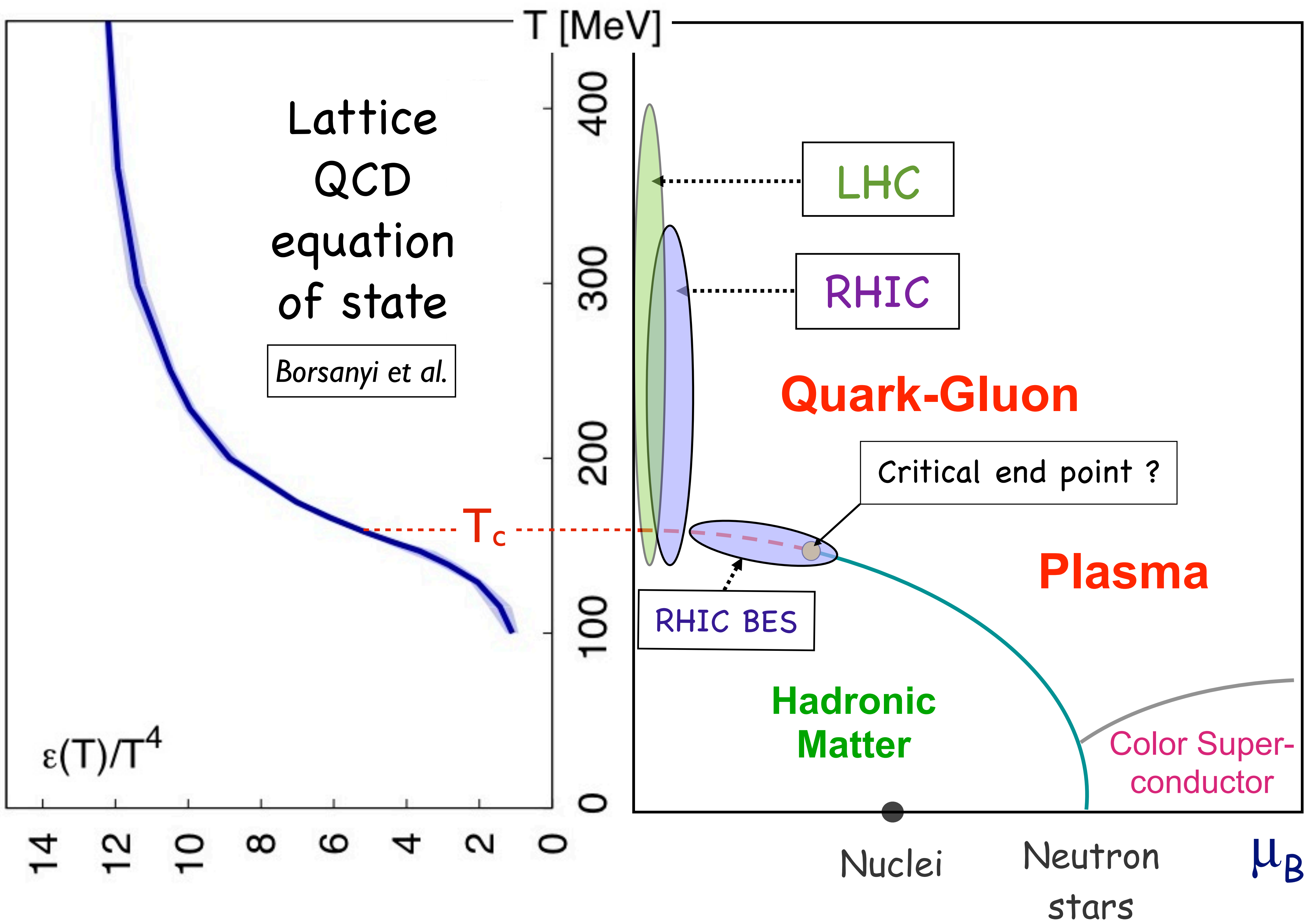



Initial-state quantum fluctuations propagate into to "macroscopic" final-state fluctuations by hydrodynamic response.

# Little Bang vs. Big Bang



# QCD Phase Diagram



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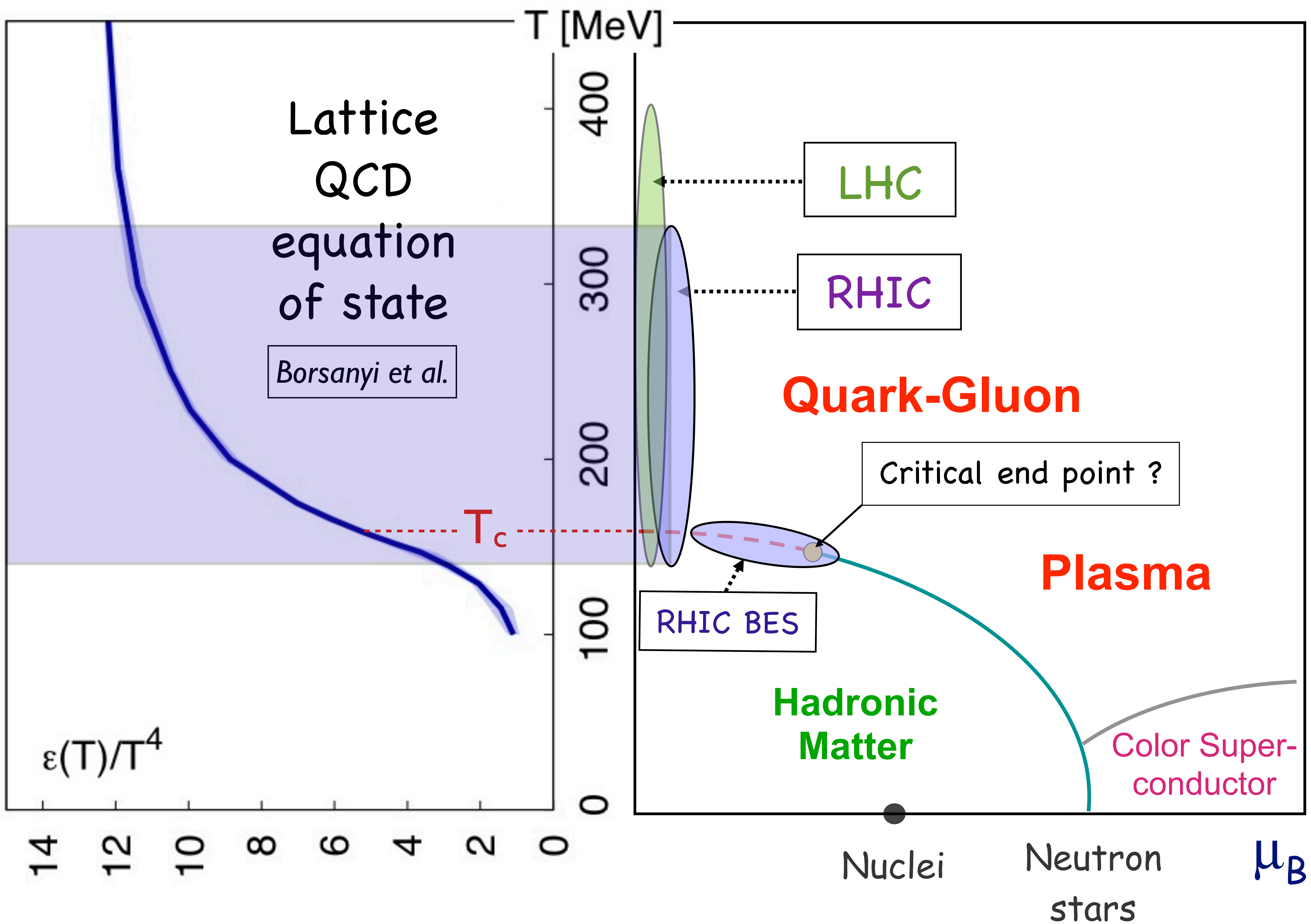

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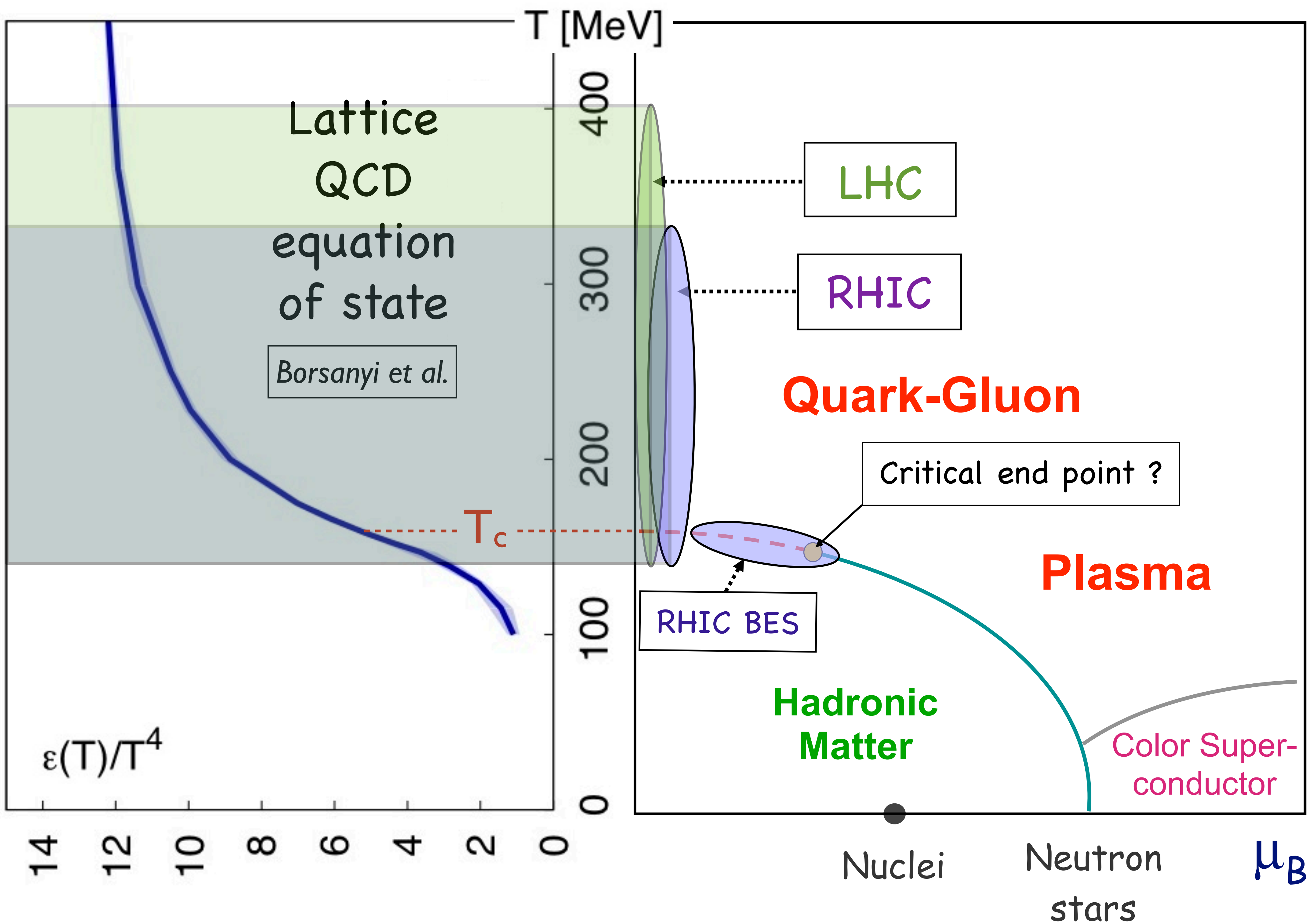
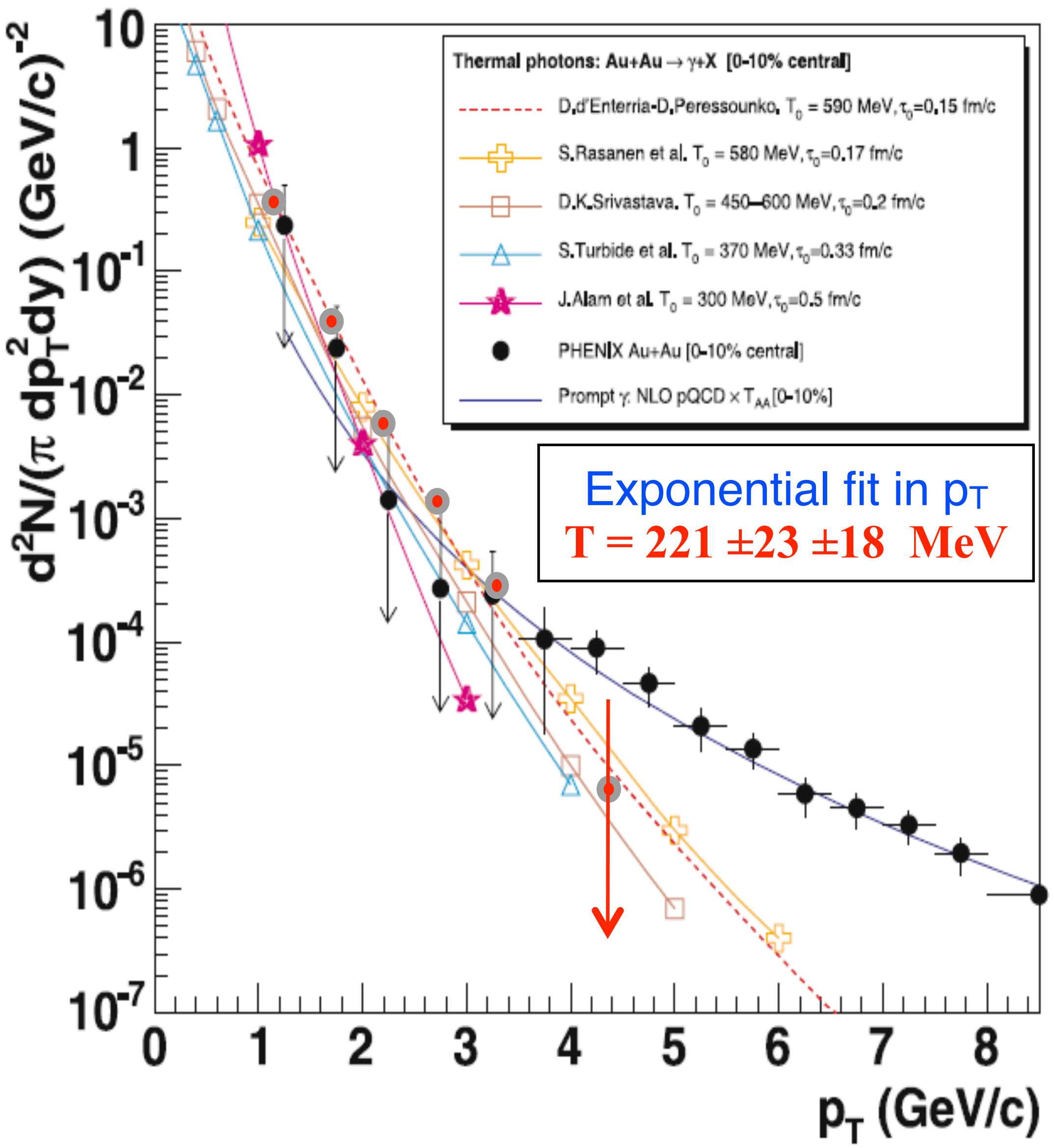
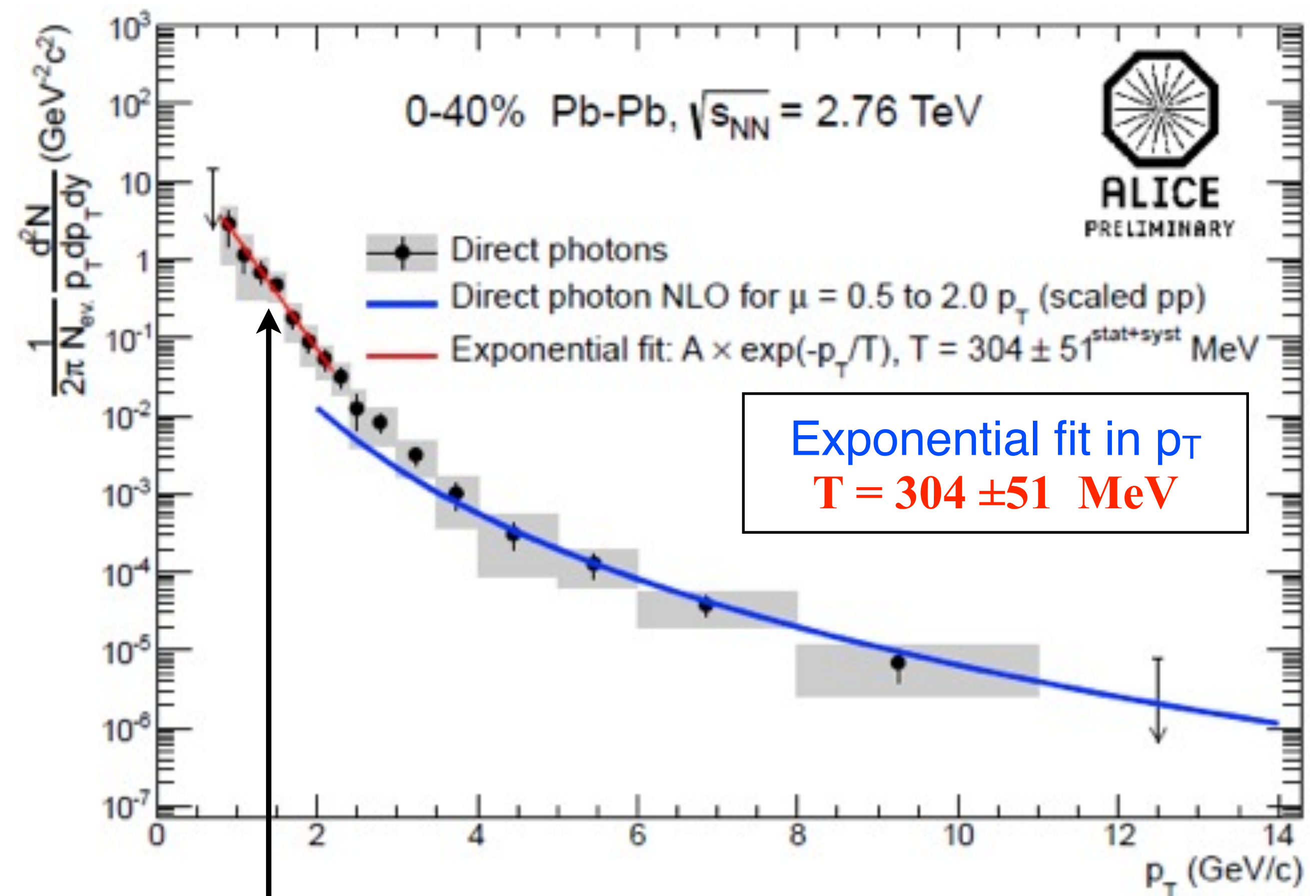


# Taking the heat




Hydro fits

$T_{init} \geq 300$ MeV

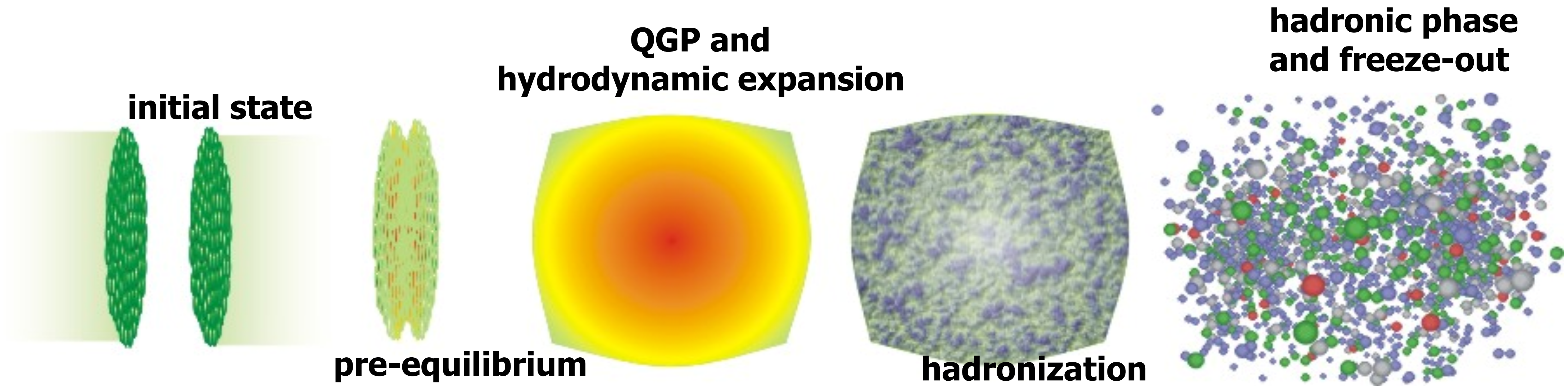



New **record "temperature"** measured in Pb+Pb at LHC:

$T_{LHC} = 1.37\ T_{RHIC}$.

Reflects larger initial temperature $T_{in}$, but not to be identified with $T_{in}$.

# Standard collision model




CGC | "Glasma" | Hydrodynamics | Hadronic gas

Not quantitatively established yet.

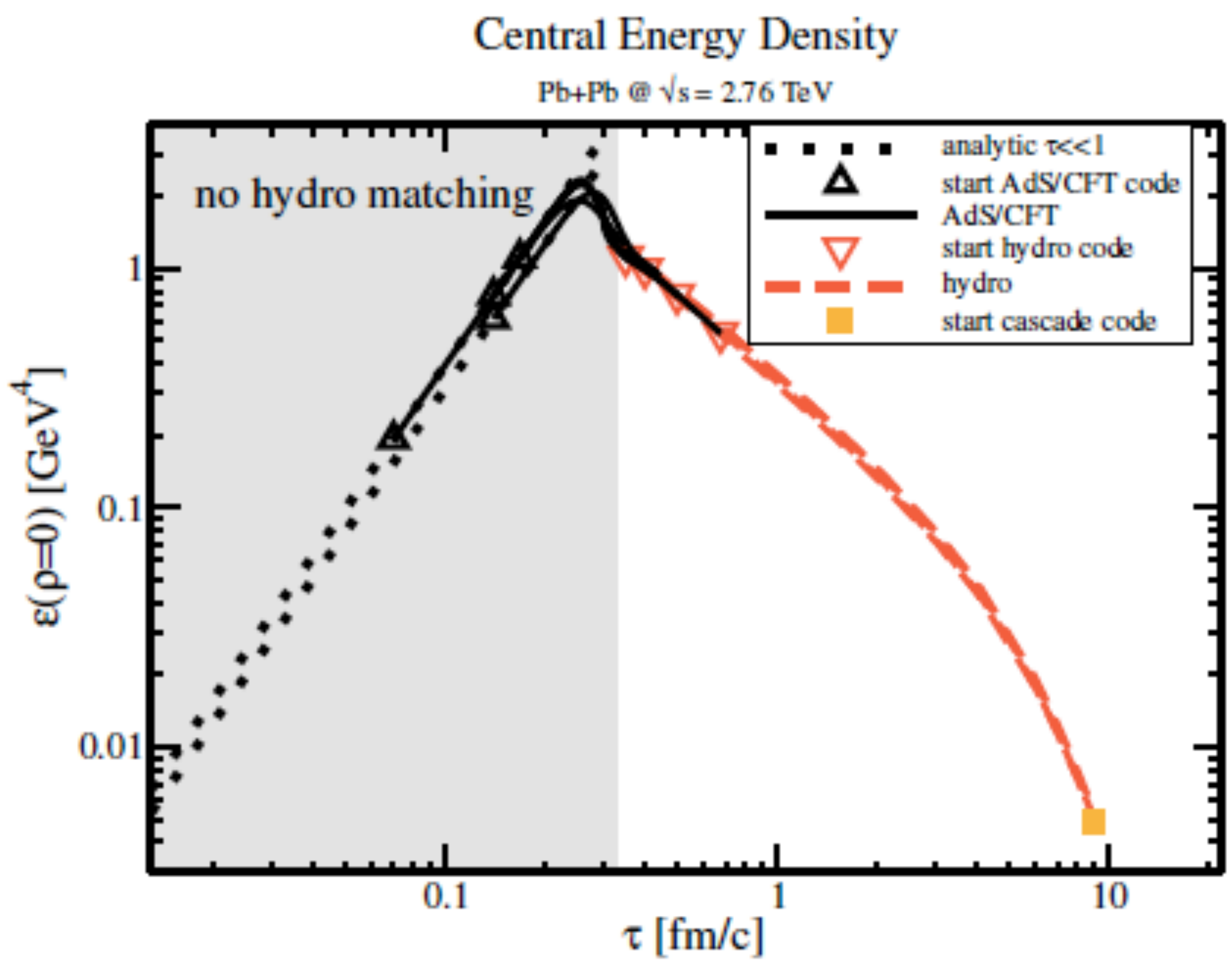
Unknown: Semiclassical "turbulent" gauge plasma or strongly coupled "muck" best described in AdS/CFT?

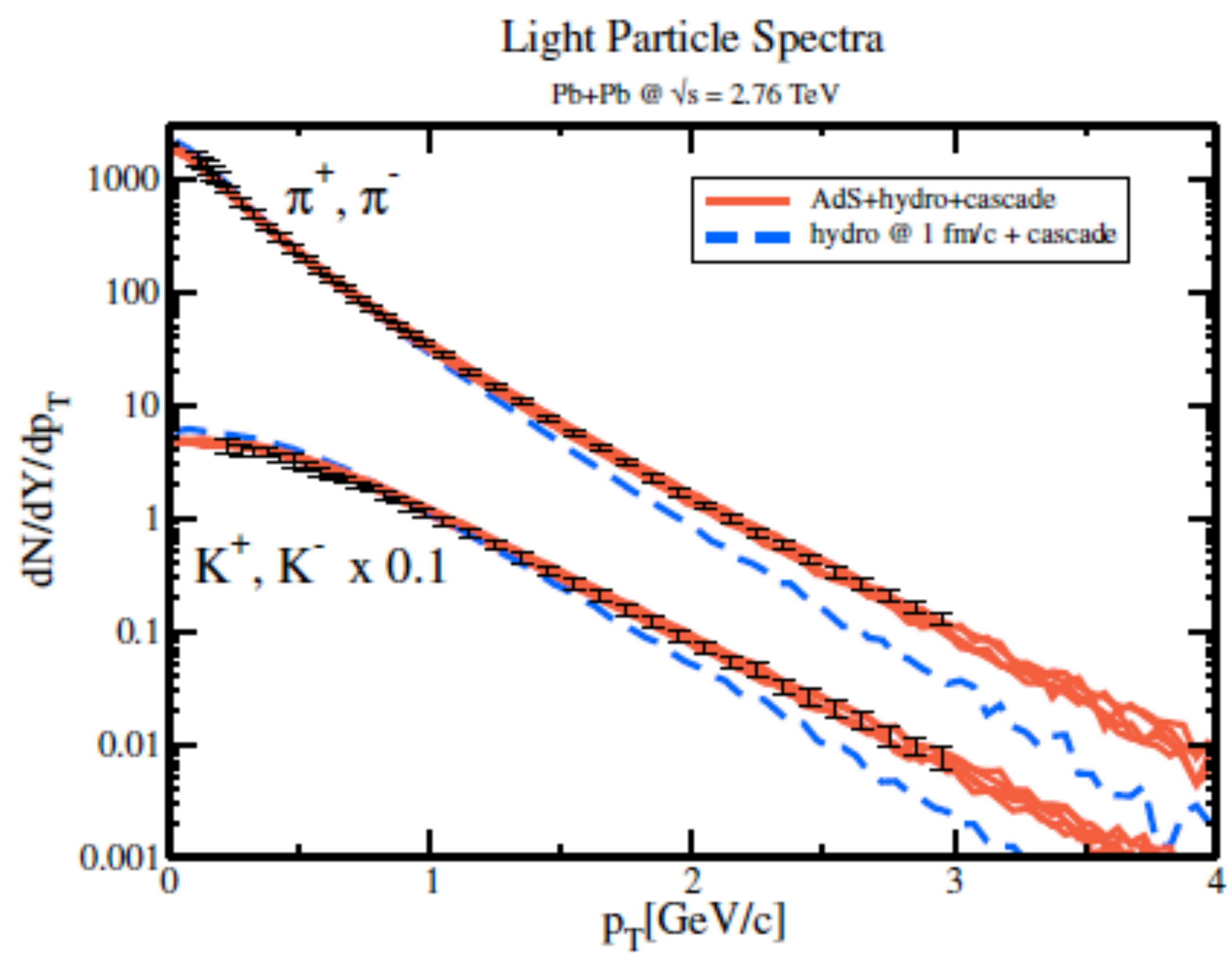
Well understood and flexible.
Requires initial conditions.
Nowadays typically second order relativistic viscous FD.

Relativistic Boltzmann for hadron resonance gas with empirical differential cross sections.
Diluteness assumption.

# AdS/CFT + Hydro works

*van der Schee, Romatschke, Pratt*: Collision of two nuclear "shock waves" in AdS/CFT, followed by hydrodynamical evolution and hadronic cascade final scattering. Hydro works after 0.35 fm/c; spectra fit LHC and RHIC data (w/o free parameters).



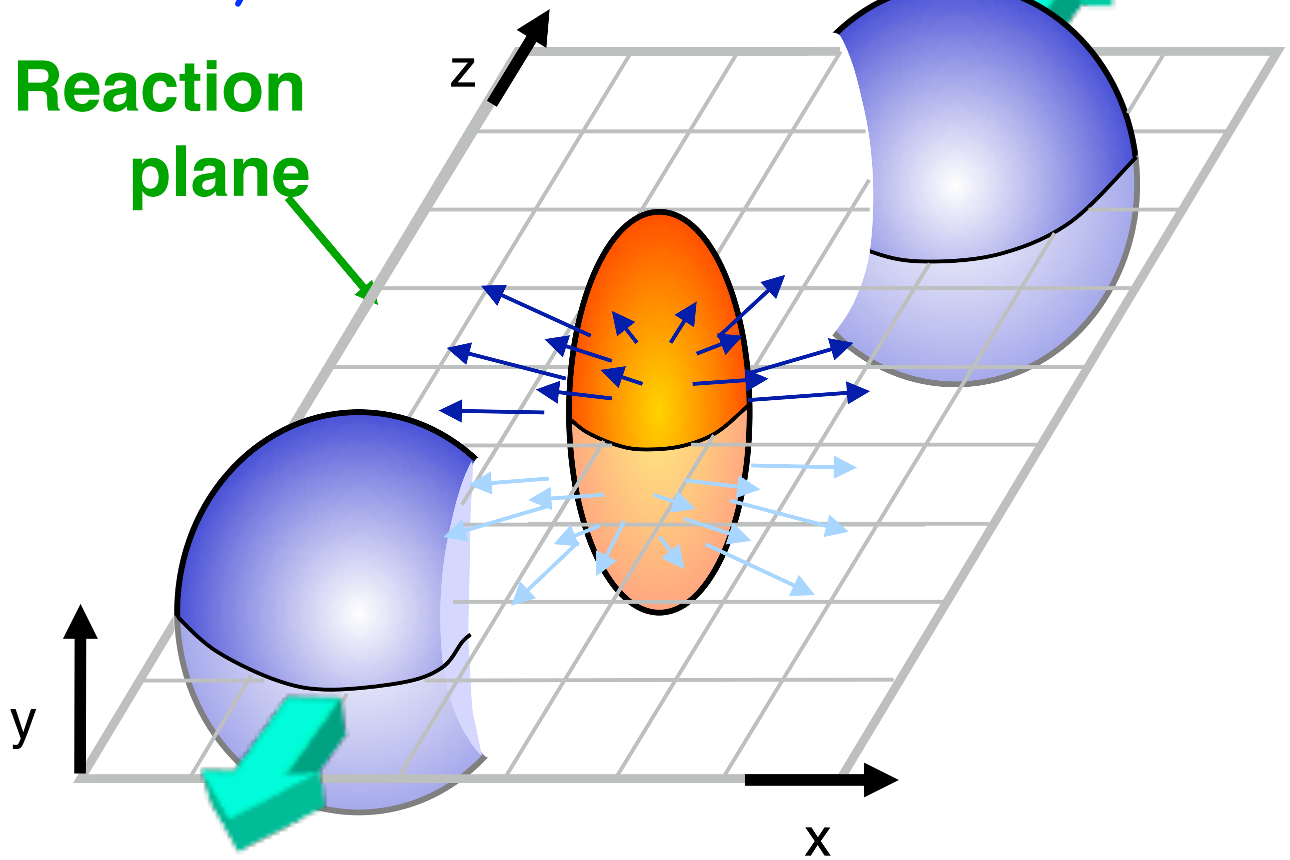



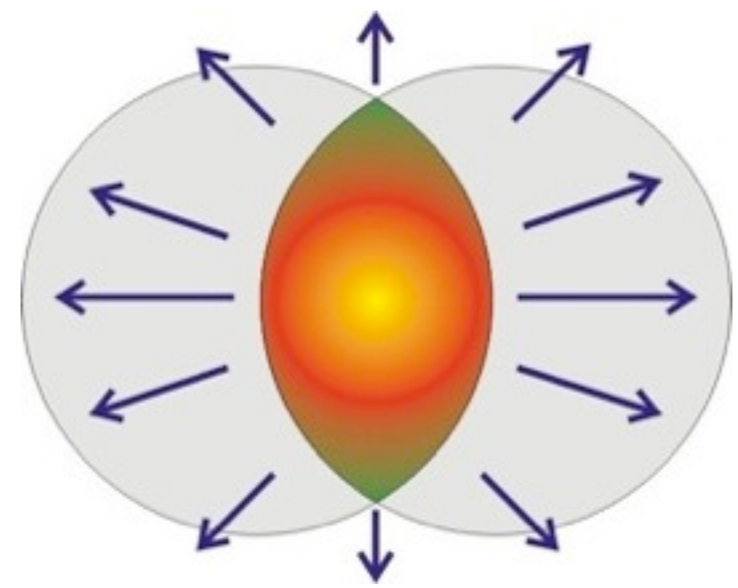

arXiv:1307.2539, to appear in PRL

# The “perfect” fluid

# Elliptic flow

- two nuclei collide rarely head-on, but mostly with an offset:



only matter in the overlap area gets compressed and heated:

Expansion is anisotropic

$$2\pi \frac{dN}{d\phi} = N_0 \left( 1 + 2 \sum_n \mathrm{v}_n(p_T, \eta) \cos n\left(\phi - \psi_n(p_T, \eta)\right) \right)$$

anisotropic flow coefficients

event plane angle

# Viscous hydrodynamics

Hydrodynamics = effective theory of energy and momentum conservation

**energy-momentum tensor** = **ideal fluid** + **dissipation**

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \Pi^{\mu\nu}$$

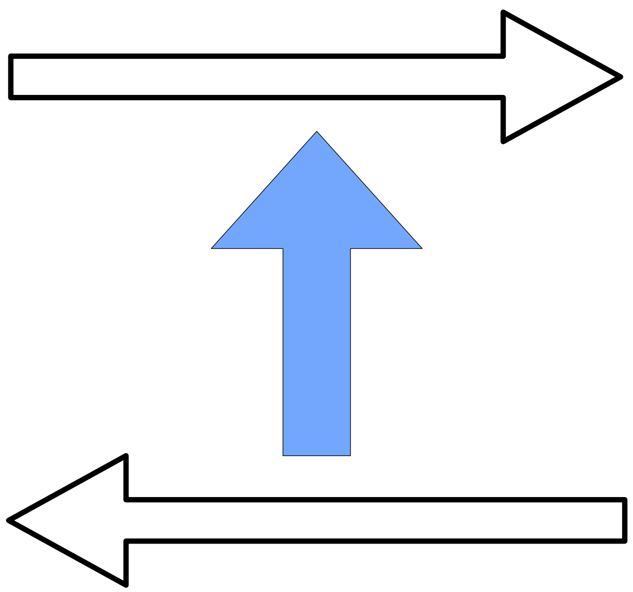
$$\tau_\Pi \left[ \frac{d\Pi^{\mu\nu}}{d\tau} + \left(u^\mu \Pi^{\nu\lambda} + u^\nu \Pi^{\mu\lambda}\right) \frac{du^\lambda}{d\tau} \right] = \eta\left(\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace}\right) - \Pi^{\mu\nu}$$

Input: Equation of state P(ε), shear viscosity, initial conditions ε(x,0), $u^\mu$(x,0)

Shear viscosity is normalized by density: kinematic viscosity η/ρ.

Relativistically, the appropriate normalization factor is the entropy density s = (ε+P)/T, because the particle density is not conserved: η/s.

# Shear viscosity



Shear viscosity describes ability to transport momentum across flow gradients! Kinetic theory:

$$\eta \approx \frac{1}{3} n \bar{p} \lambda_f \qquad \lambda_f = \frac{1}{n\sigma} \quad \rightarrow \quad \eta \approx \frac{\bar{p}}{3\sigma}$$

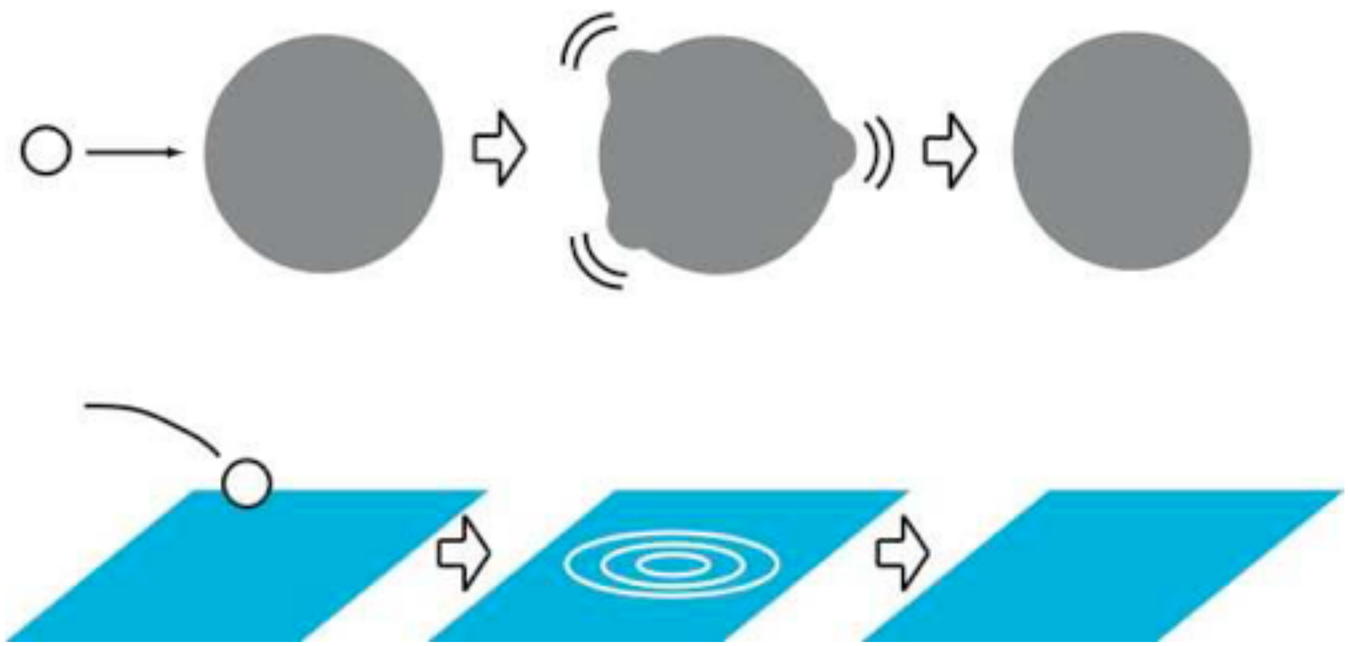
$$\sigma \leq \frac{4\pi}{\bar{p}^2} \quad \rightarrow \quad \eta \geq \frac{\bar{p}^3}{12\pi}$$

Relativistic system of massless particles: $\bar{p} \sim T \quad \rightarrow \quad \bar{p}^3 \sim T^3 \sim s$

$$\Rightarrow \quad \frac{\eta}{s} \geq \text{some lower bound} = \# \cdot \left[ \frac{\hbar}{k_B} \right]$$

# The Black Hole connection

Dynamics of hot QCD matter can be mathematically (holography) mapped onto black hole dynamics in 4+1 dimensions ($AdS_5$ space).



BH swallowing matter

⇩

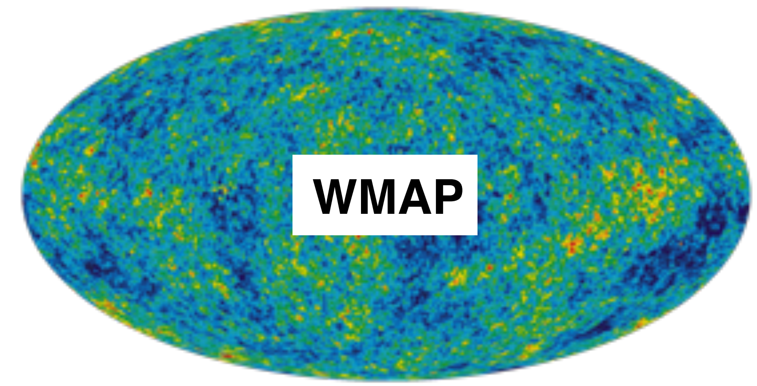
Perfect fluid hydrodynamics

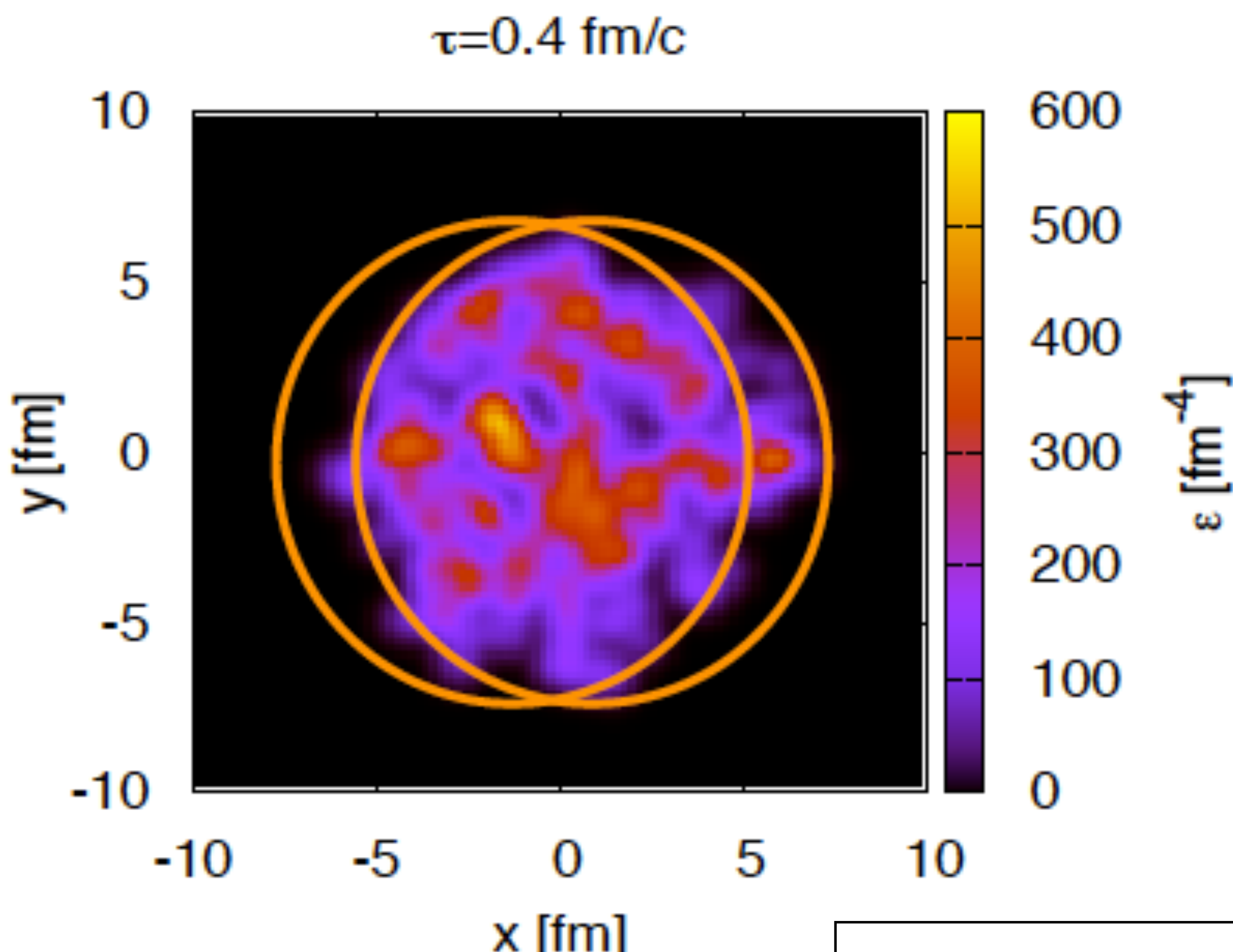
Formation of hot QCD matter at RHIC is similar to formation of a black hole, tied to information loss. Relies on the notion that ‘t Hooft coupling $g^2 N_c \sim 12$ is large enough to apply the classical limit of the dual theory:

$$\boxed{\eta / s \geq 1/4\pi}$$ (KSS bound)

# Event-by-event fluctuations

Initial state generated in A+A collision is grainy
event plane ≠ reaction plane
⇒ eccentricities $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\varepsilon_4$, etc. ≠ 0

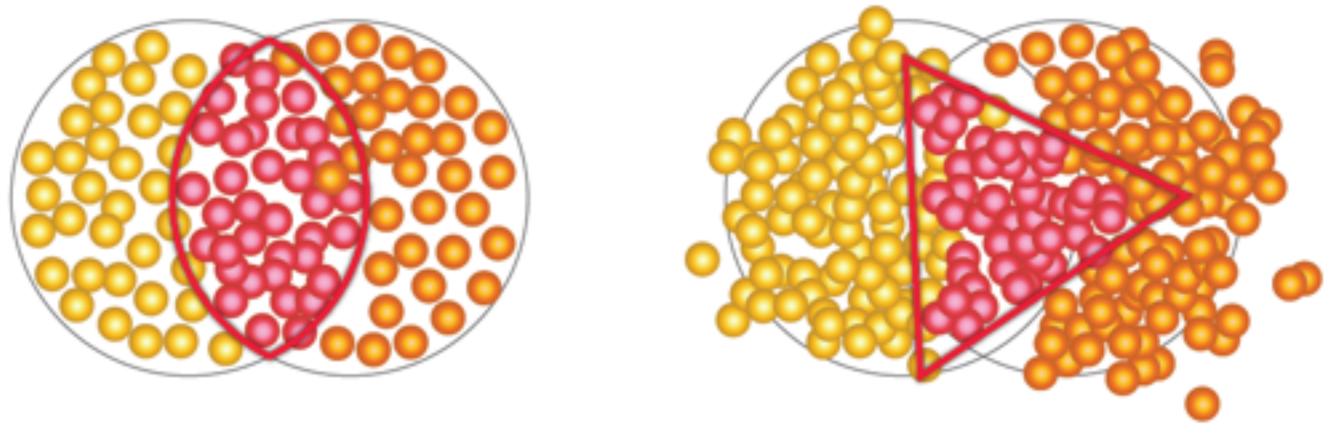



Idea: Energy density fluctuations in transverse plane from initial state quantum fluctuations. These thermalize to different temperatures locally and then propagate hydrodynamically to generate angular flow velocity fluctuations in the final state.

⇒ flows $v_1$, $v_2$, $v_3$, $v_4$,...

# Color charge fluctuations



Quantum fluctuations in the positions of the colliding nucleons give rise to a position dependent density of valence partons and other hard partons: $\mu^2(x)$.

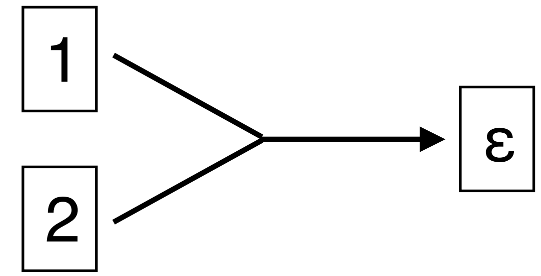
For given $\mu$, color charges of the partons combine in a random walk in SU(3). This generates an approximately Gaussian distribution of color charges $\rho^a(x)$.

$$P[\rho] \propto \exp\left( -\frac{1}{2g^2\mu^2} \int d^2x \, \rho^a(x)\rho^a(x) \right)$$

Neglected: transverse correlations among color charges, x-dependence of $\mu$, confinement related effects, etc.

# Energy density fluctuations

Quantity to calculate: $\langle \varepsilon(\mathbf{x})\varepsilon(\mathbf{y})\rangle - \langle \varepsilon(\mathbf{x})\rangle \langle \varepsilon(\mathbf{y})\rangle$




Energy density deposited by two colliding sheets of CGC:

$$\varepsilon(\mathbf{x}) = \frac{1}{4} F_{ij}^c(\mathbf{x}) F_{ij}^c(\mathbf{x}) + 2 A^{\eta c}(\mathbf{x}) A^{\eta c}(\mathbf{x}) \qquad F_{ij}^c(\mathbf{x}) = g f_{abc}\Big(A_i^a(1;\mathbf{x})A_j^b(2;\mathbf{x}) + A_i^a(2;\mathbf{x})A_j^b(1;\mathbf{x})\Big)$$

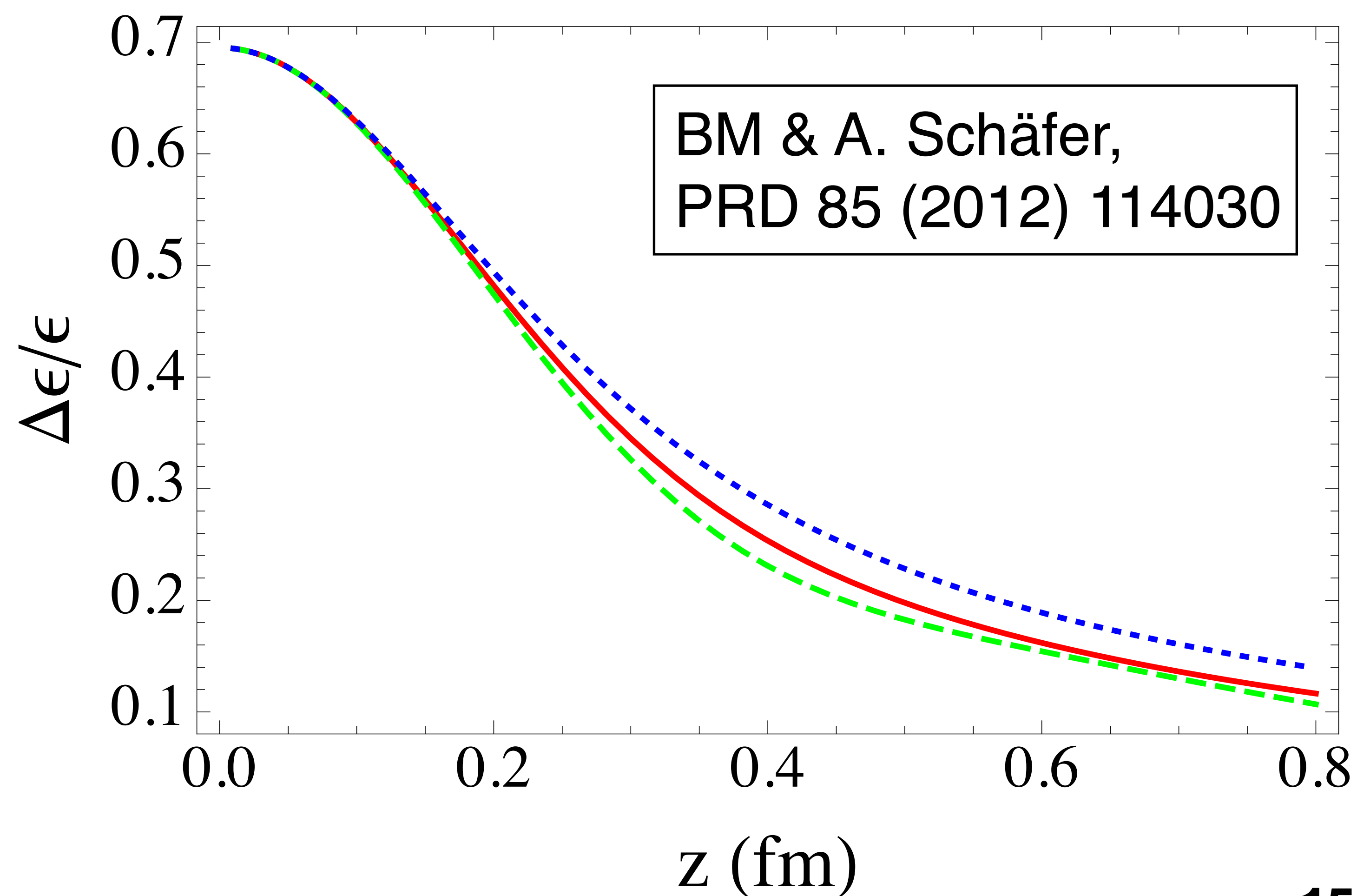
Scaling function for transverse color correlations:

$$G(|\mathrm{x}|) = G_0\, \phi(|\mathrm{x}|^2 / \xi^2)$$

with

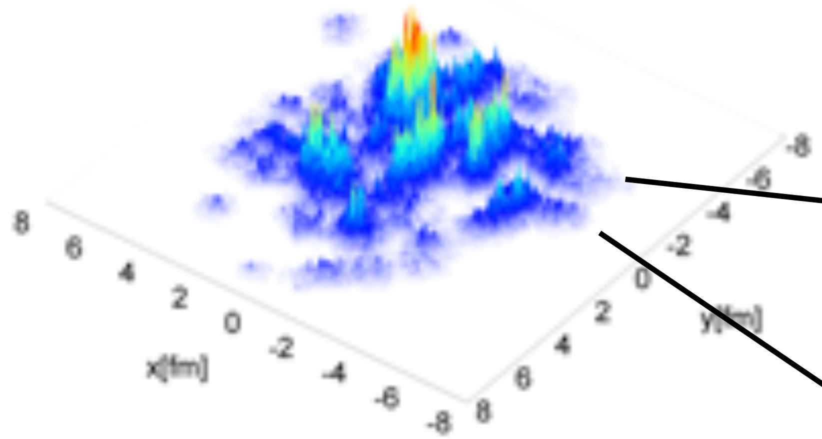
$$G_0 = \tfrac{4}{9}\pi\mu^2$$

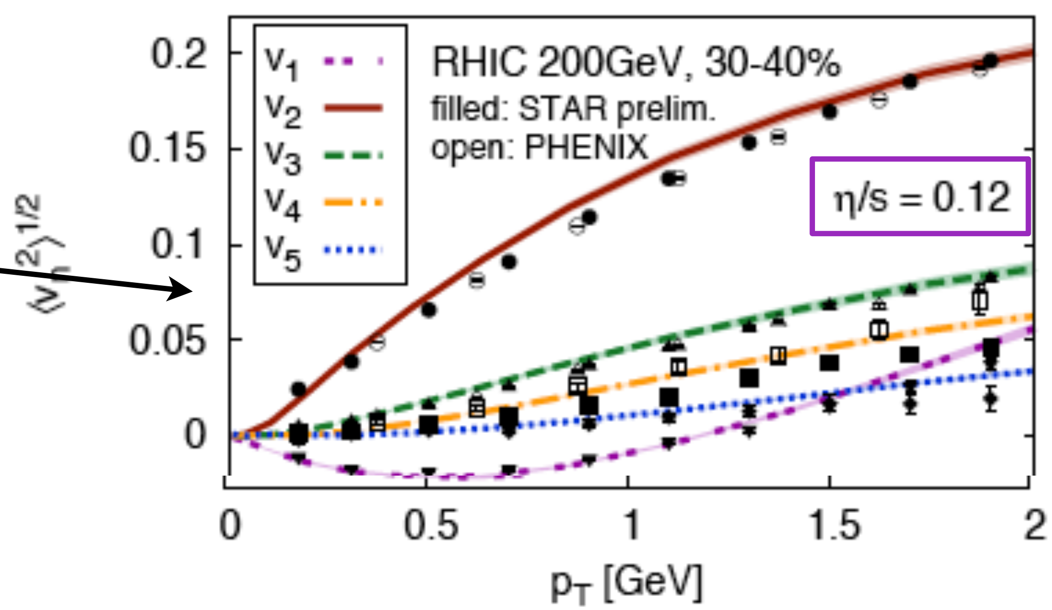
$$1/\xi^2 = \tfrac{1}{9} N\pi (g\mu)^2$$



# QCD Matter at RHIC is most “perfect”

Initial density distribution

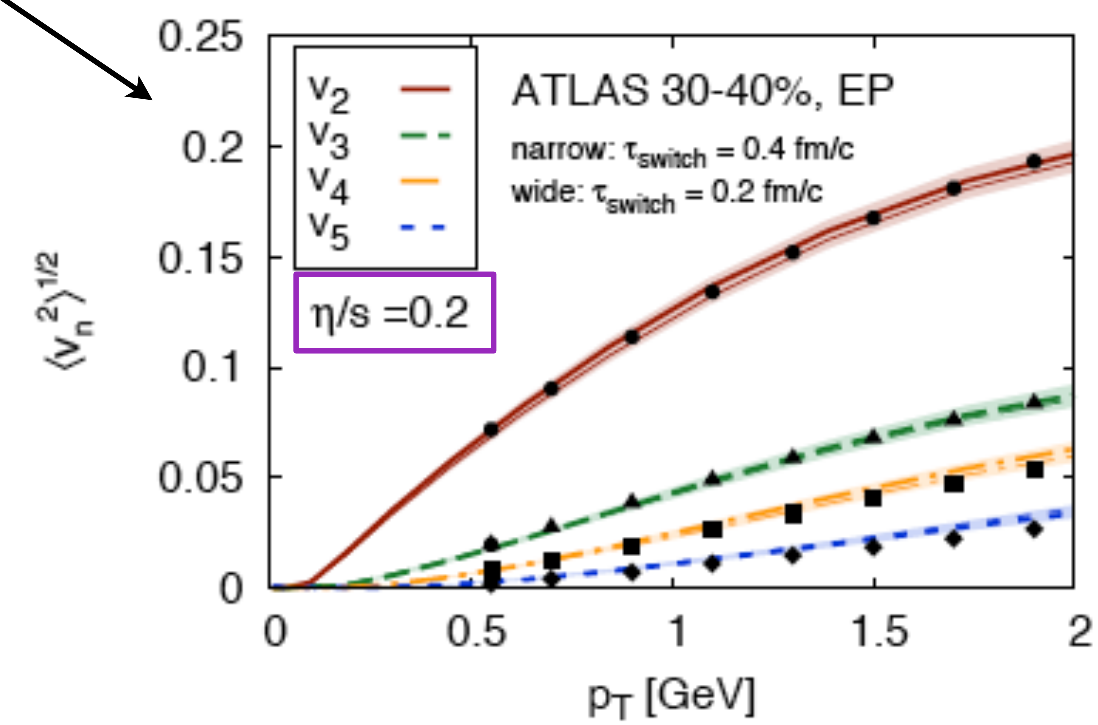



RHIC

Quantum limit: $\eta/s = 0.08$.

$(\eta/s)_{RHIC} \approx 0.6\ (\eta/s)_{LHC}$

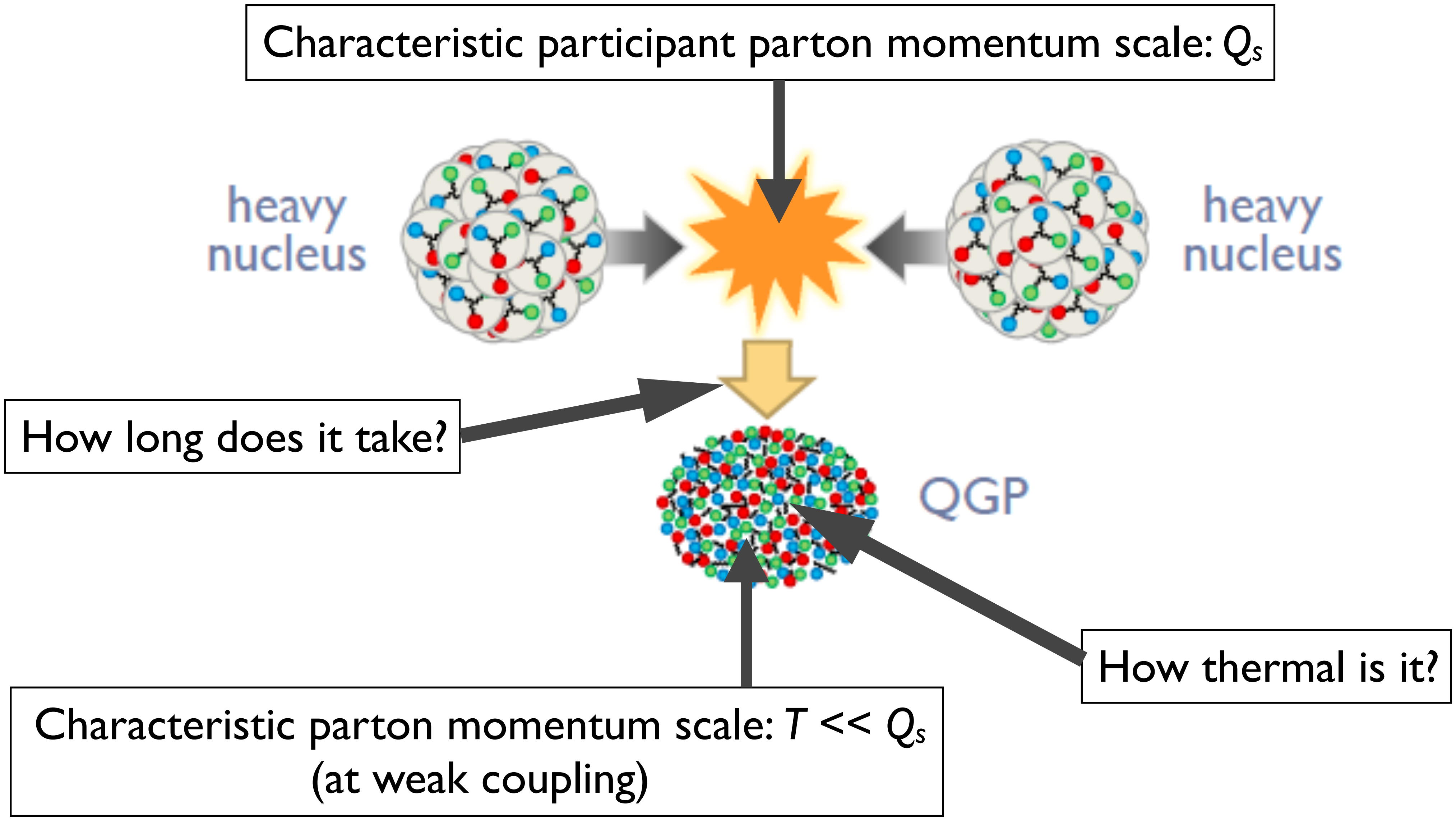



LHC

This is an average statement.
The temperature dependence
of $\eta/s$ may be much steeper.

# Thermalization

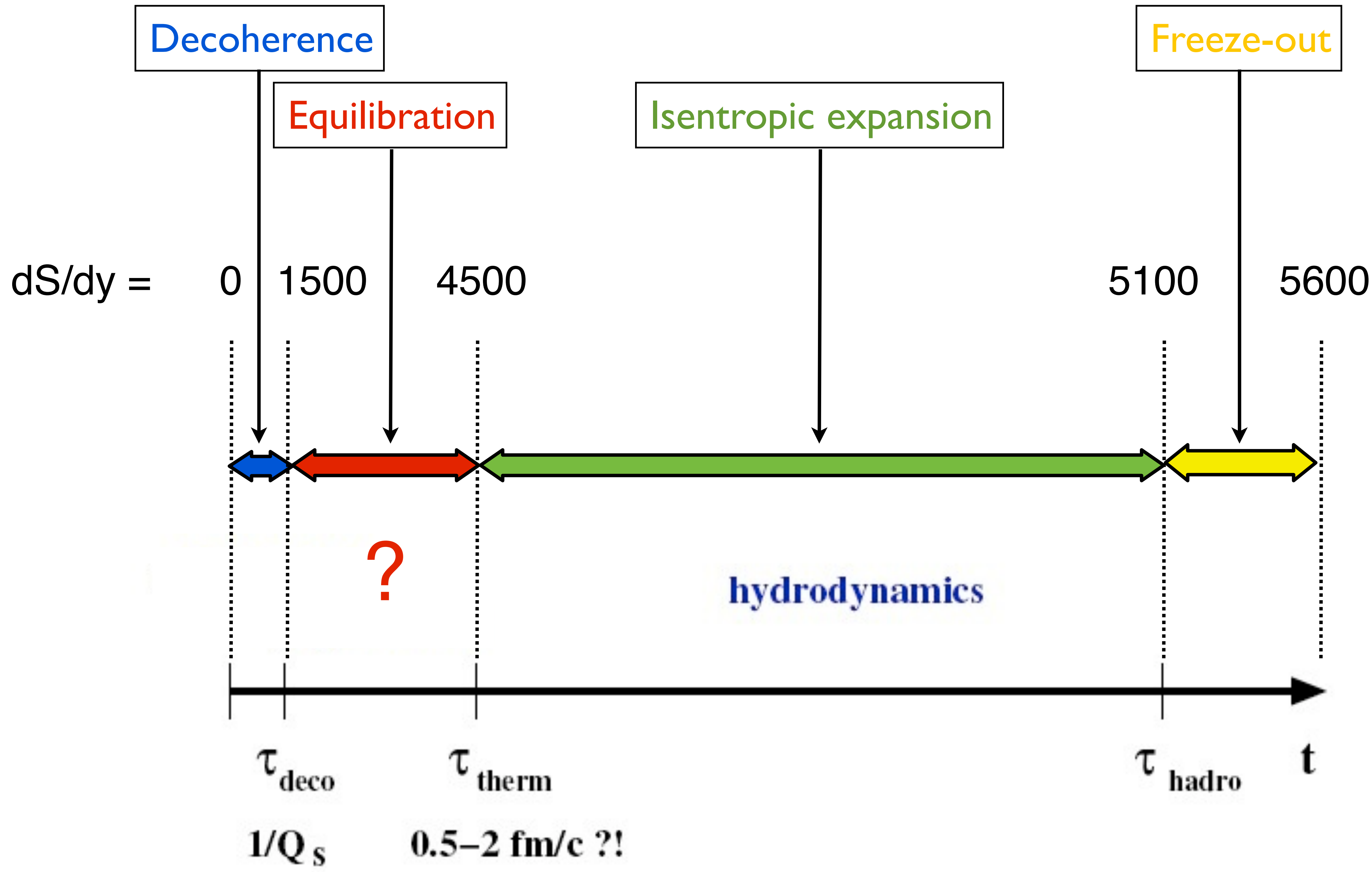
# Thermalization




**How does the thermalization process work at strong coupling?**

If not "bottom up", what else?

# Entropic history of a HI collision



# Thermalization

Thermalization means that a system loses all information about its history.

This can happen in two ways:

1. The system exchanges information with its environment (heat bath). This is true thermalization. The thermal state of the system is characterized by a density matrix, which only depends on the conserved quantum numbers (energy, particle number, charge, etc.). The entropy of the system is a measure of its information loss to the environment. In this case, the quantum state of the system becomes *entangled* with the quantum state of its environment.

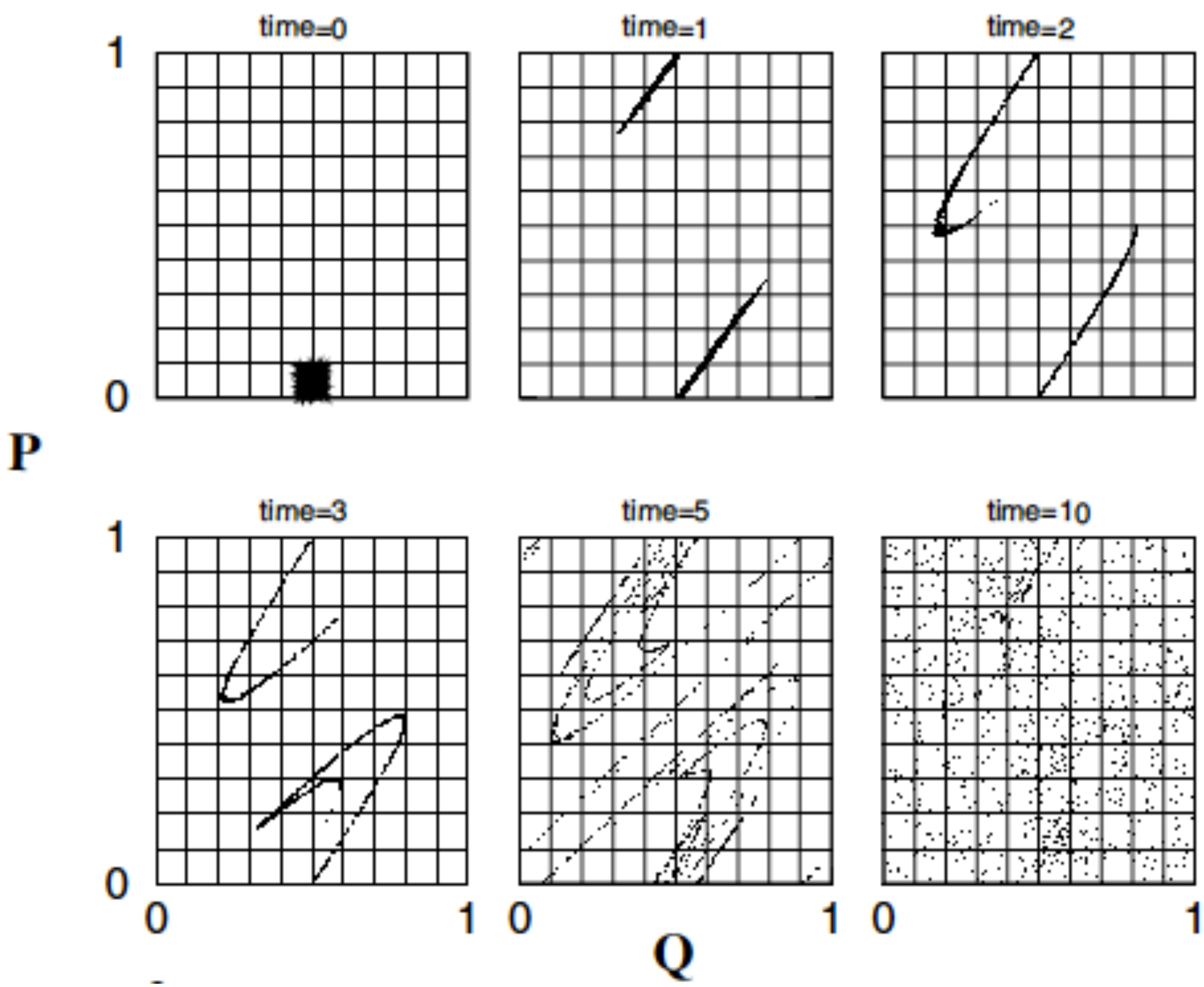
2. The state of the system evolves by itself into a complicated superposition of components that cannot be distinguished by any practical measurements. This is apparent thermalization, implied by the *coarse graining* inherent in physical observations. A single eigenstate of the system can <u>appear</u> thermal (*eigenstate thermalization*). The physical mechanism by which a system can evolve into such complex states under its own dynamics is called *quantum chaos*.

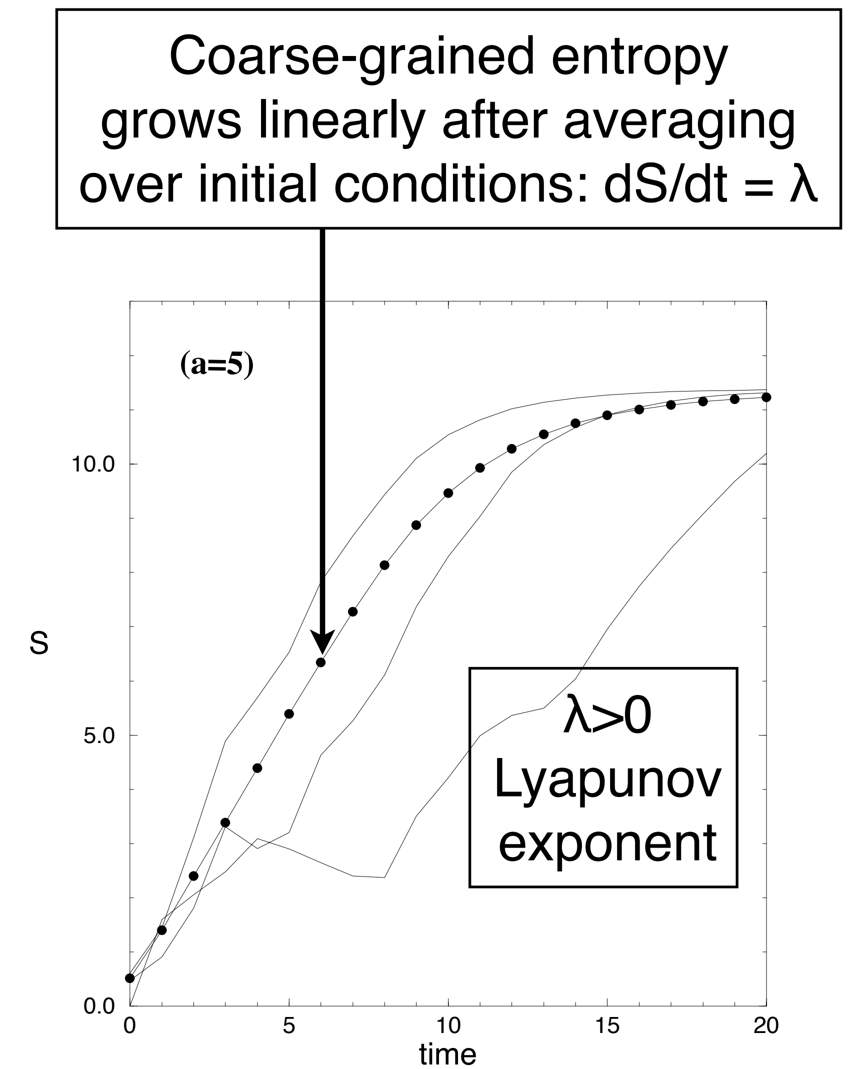
See e.g. entropy review: BM & A. Schäfer, arXiv:1110/2378

# Coarse-grained entropy

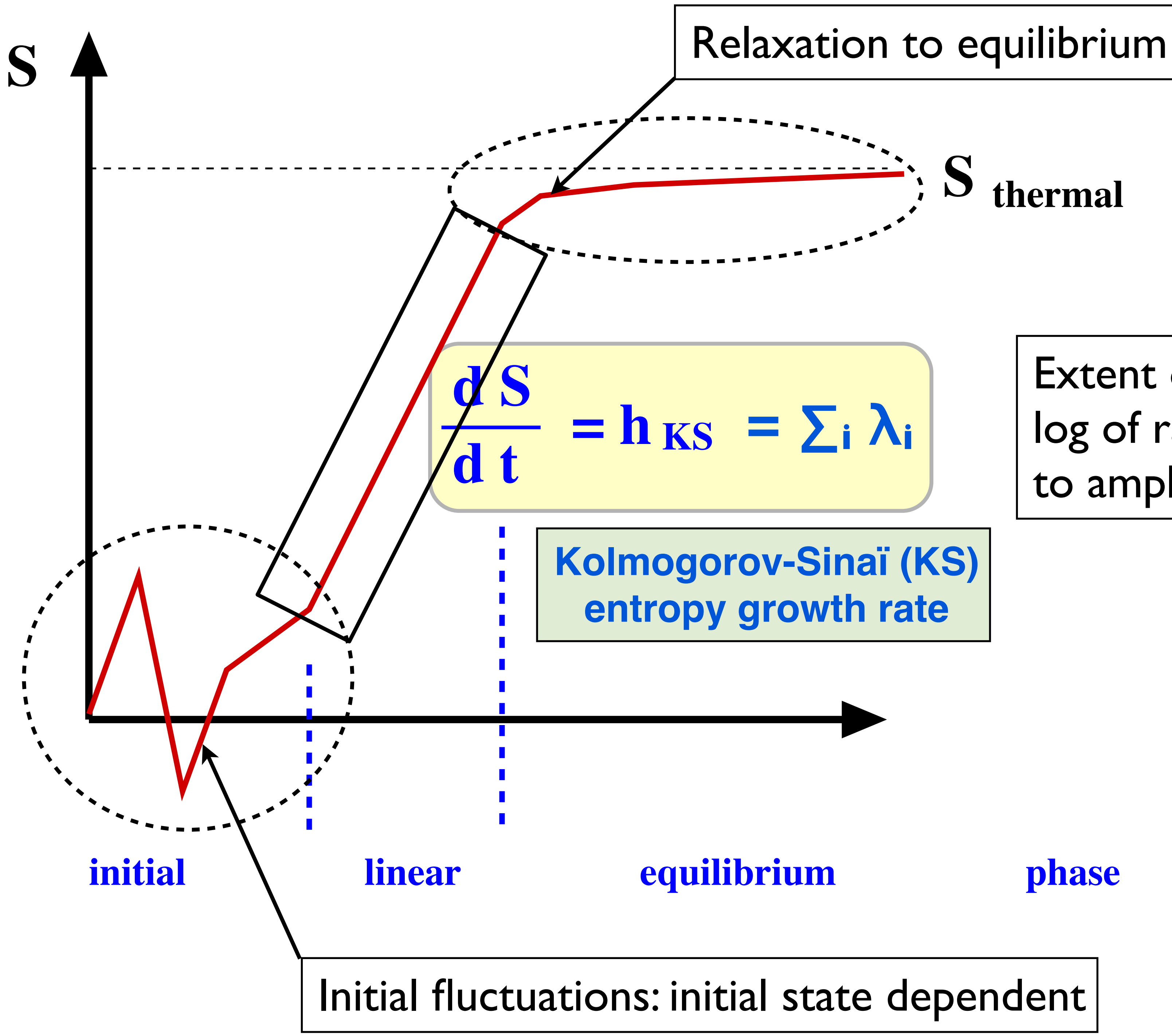
Evolution of the "standard map" system: $p_{n+1} = p_n + \frac{a}{2\pi}\sin(2\pi q_n); \quad q_{n+1} = q_n + p_n \quad (\text{mod } 1)$

[M. Baranger, V. Latora, A. Rapisarda, *Chaos, Solitons and Fractals* 13 (2002) 471]



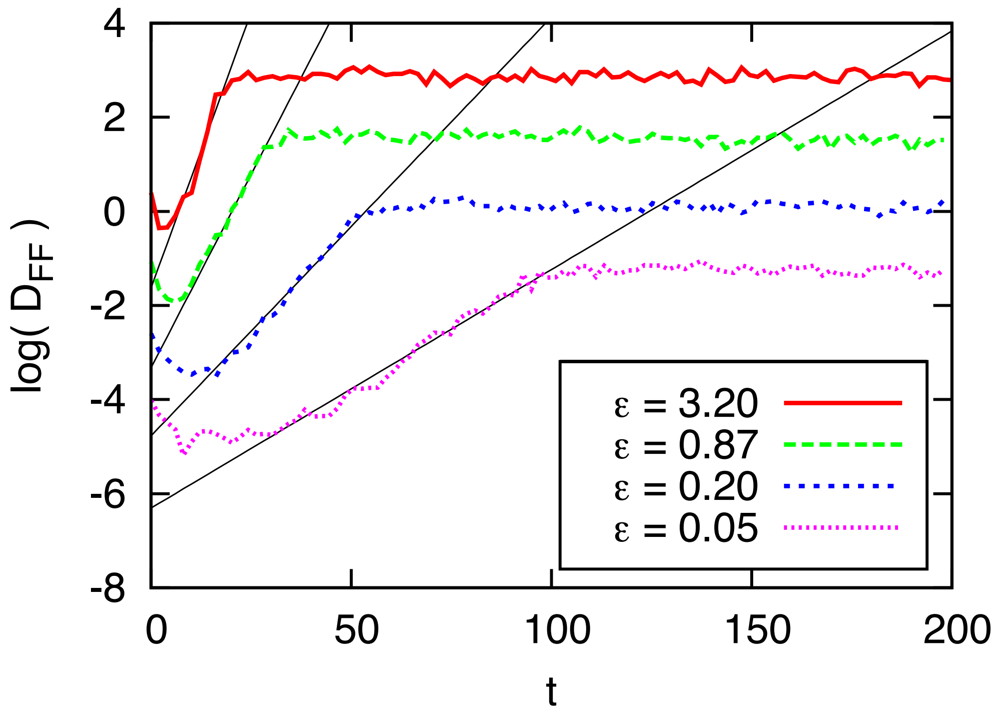



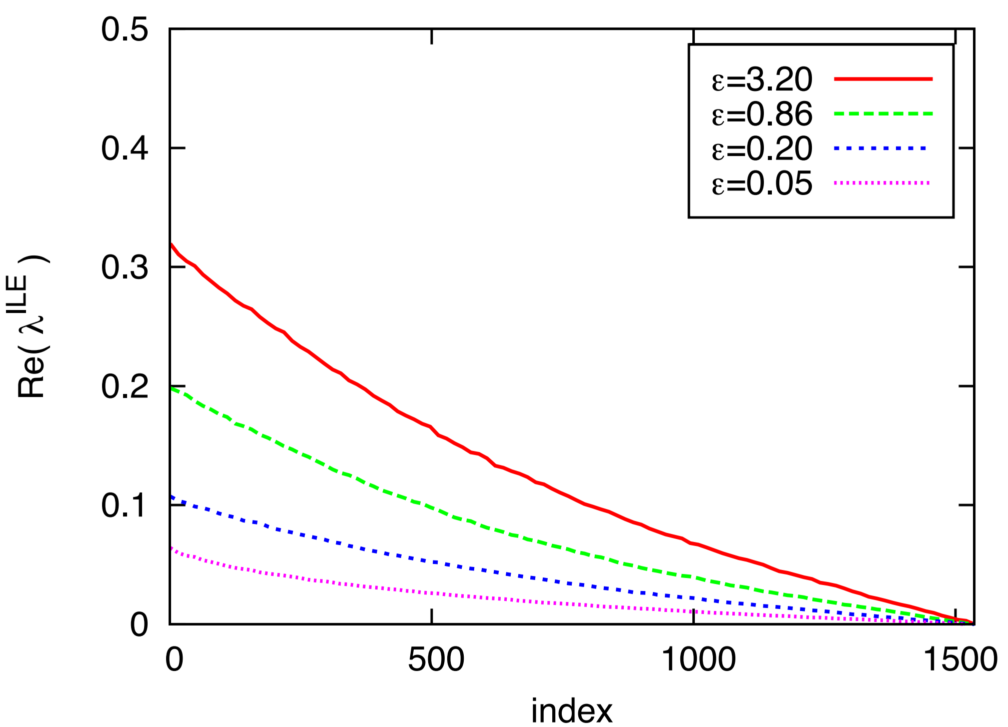
# General picture




Extent of linear region depends on log of ratio of thermal fluctuations to amplitude of initial fluctuations.

# Classical lattice YM theory




T. Kunihiro, BM, A. Ohnishi, A. Schäfer, T. Takahashi & A. Yamamoto, PRD 82 (2010) 114015
H. Iida, T. Kunihiro, BM, A. Ohnishi, A. Schäfer & T. Takahashi, PRD88 (2013) 094006

Lattice gauge fields exhibit extensive spectrum of positive (intermediate) Lyapunov exponents.

➥ Finite KS entropy density.

ILE's initially depend on time and initial cond's, but soon approach a universal spectrum.

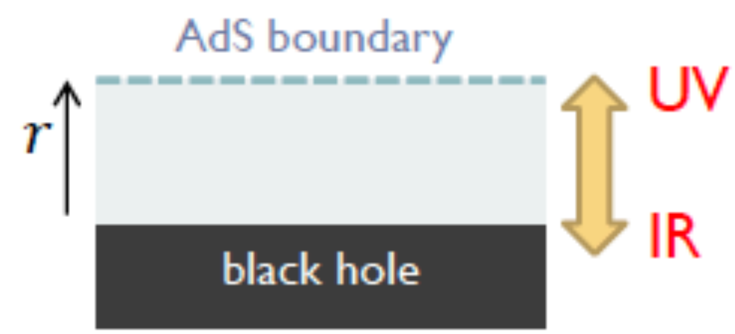
Classical approach only applies at weak coupling; there a universal scaling ("turbulent") domain is reached, which is not equilibrated (*Berges et al.*). Approach to equilibrium and time scale remains an open problem.

# AdS/CFT dictionary

- Want to study strongly coupled phenomena in QCD
- Toy model: $\mathcal{N} = 4\ SU(N_c)$ SYM

| $\mathcal{N} = 4$ SYM | holo. dual ⟷ AdS/CFT | String theory in $AdS_5$ |
|---|---|---|
| Strong coupling | ⟷ | Weak coupling |
| Vacuum | ⟷ | Empty $AdS_5$ |
| Thermal state (equilibrated plasma) | ⟷ | AdS BH |
| Thermalization | ⟷ | BH formation |
| HI collision | ⟷ | Energy injection |

$$\left(\frac{R_{AdS}}{l_s}\right)^4 = g_{YM}^2 N_c \equiv \lambda$$



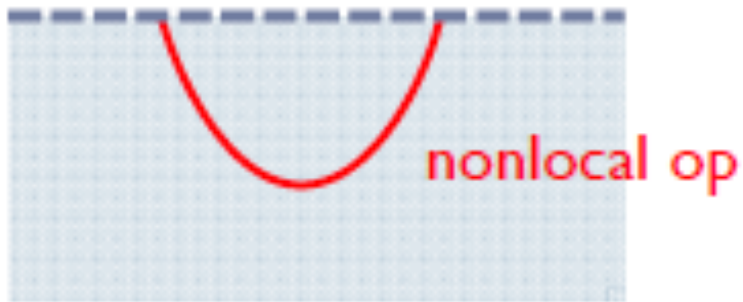
# Questions to answer

- What is the measure of thermalization on the boundary?
  - Local operators are not sufficient

$$\langle T_{\mu\nu} \rangle \text{ etc.}$$


  - Nonlocal operators are more sensitive

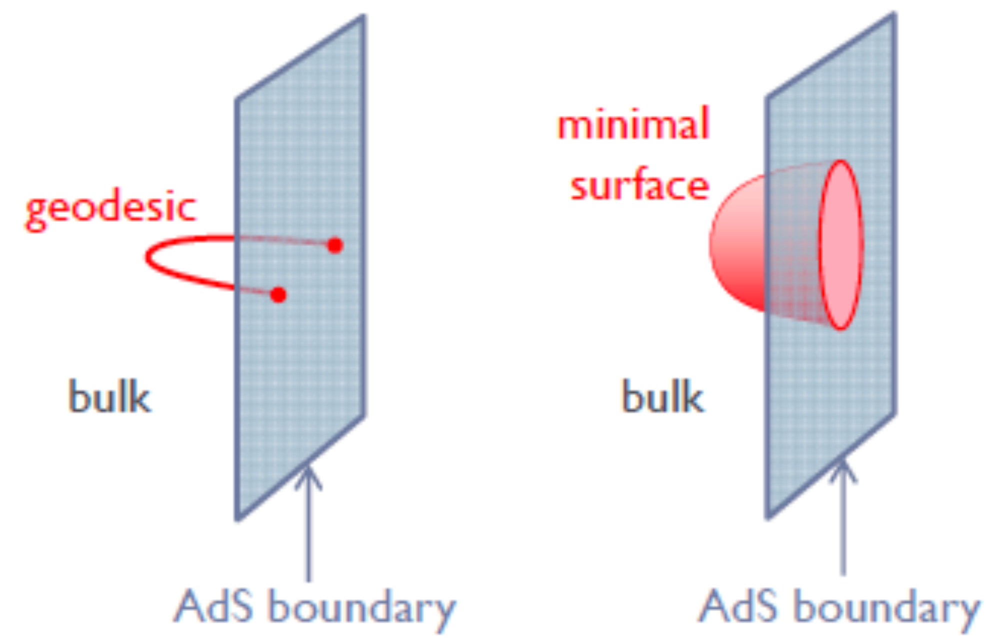
$$\langle O(x) O(x') \rangle \text{ etc.}$$




- What is the thermalization time?
  - When observables reach their thermal values

# Thermality probes

- 2-point function
  - $\langle \mathcal{O}(x)\mathcal{O}(x) \rangle$
  - Bulk: geodesic (1D)
- Wilson line
  - $W = P\{\exp[\int_C A_\mu(x)dx^\mu]\}$
  - Bulk: minimal surface (2D)
- Entanglement entropy
  - $S_A = -\mathrm{Tr}_A[\rho_A \log \rho_A], \quad \rho_A = \mathrm{Tr}_B[\rho_{tot}]$
  - Bulk: codim-2 hypersurface (same dimension as boundary space)




Use semiclassical approximation

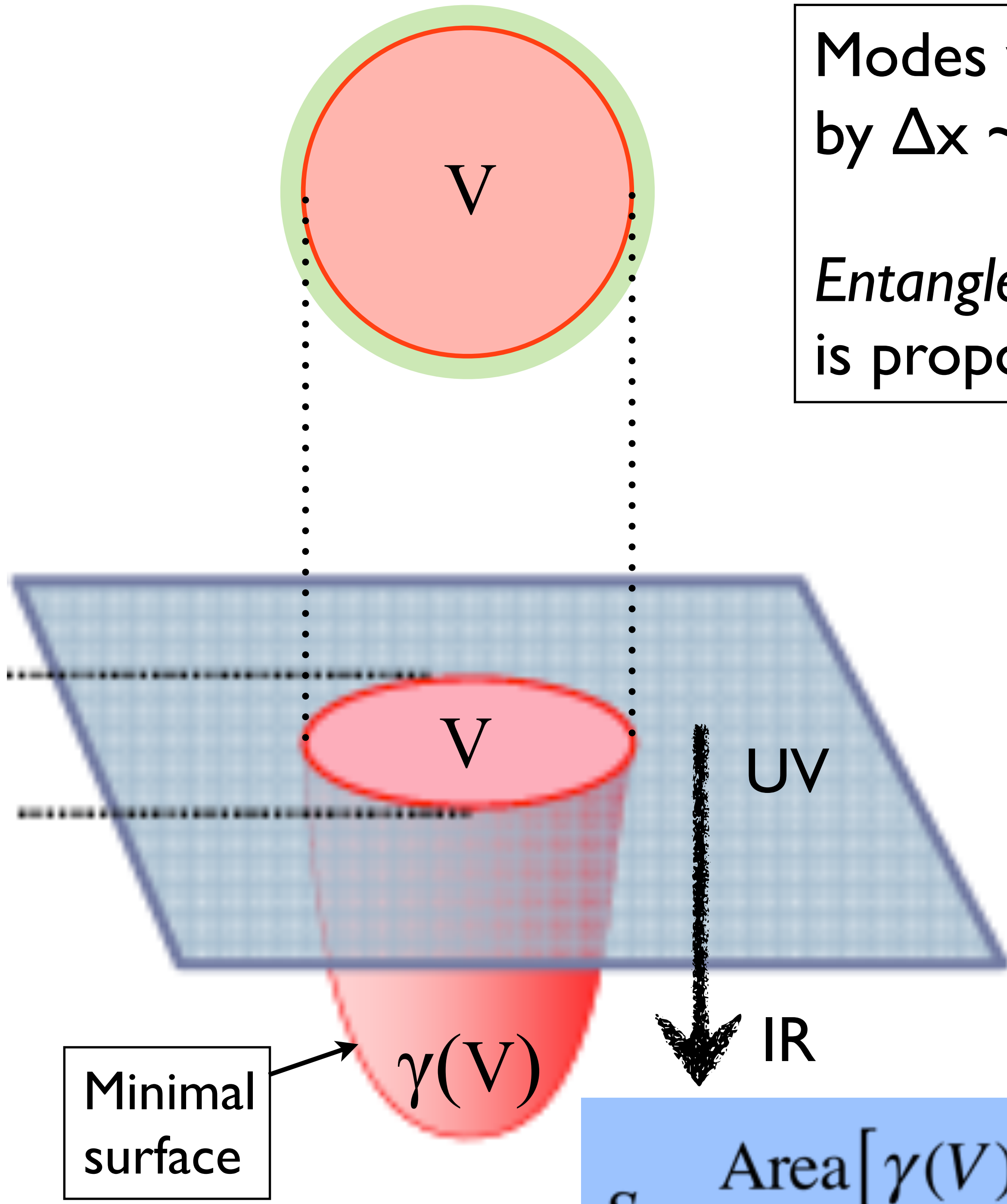
For details: *V. Balasubramanian, et al.*, PRL **106**, 191601 (2011); PRD 84, 026010

See also: S. Caron-Huot, P.M. Chesler & D. Teaney, arXiv:1102.1073

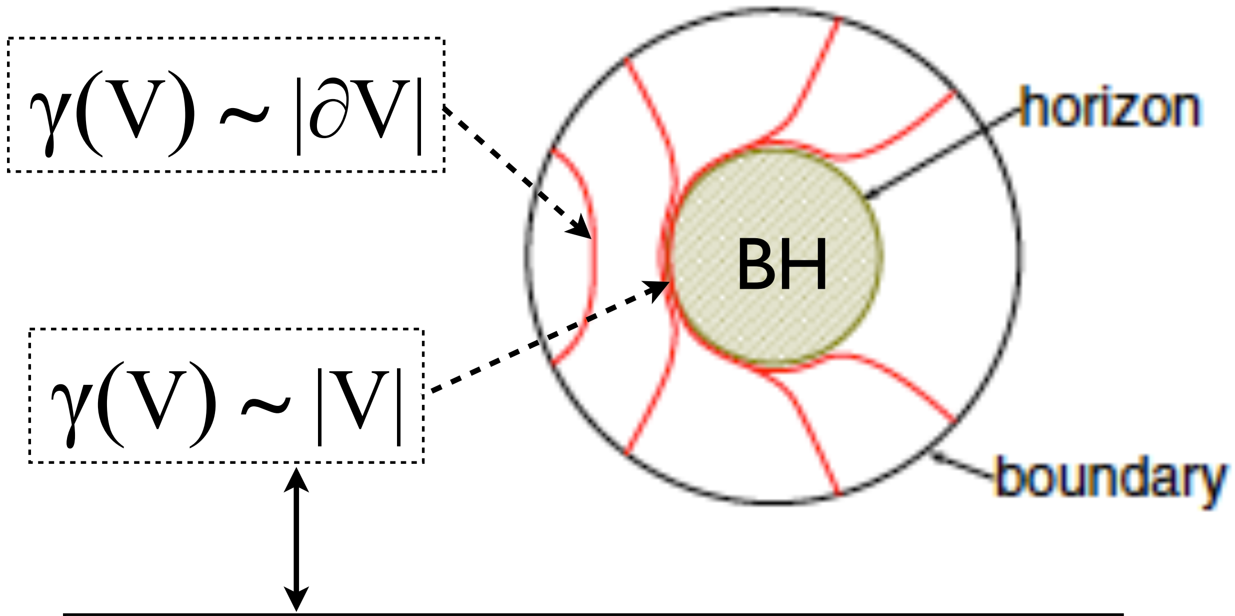
# Entanglement entropy

Modes with momentum *k* "leak" into surrounding by $\Delta x \sim 1/k$ ➠ *entanglement* with environment

*Entanglement entropy* of localized vacuum domain is proportional to surface area (*Srednicki* 1994).




$$S = \frac{\mathrm{Area}[\gamma(V)]}{4G_N}$$




$T \neq 0$: $S$ proportional to volume
⇔ area of horizon of dual BH

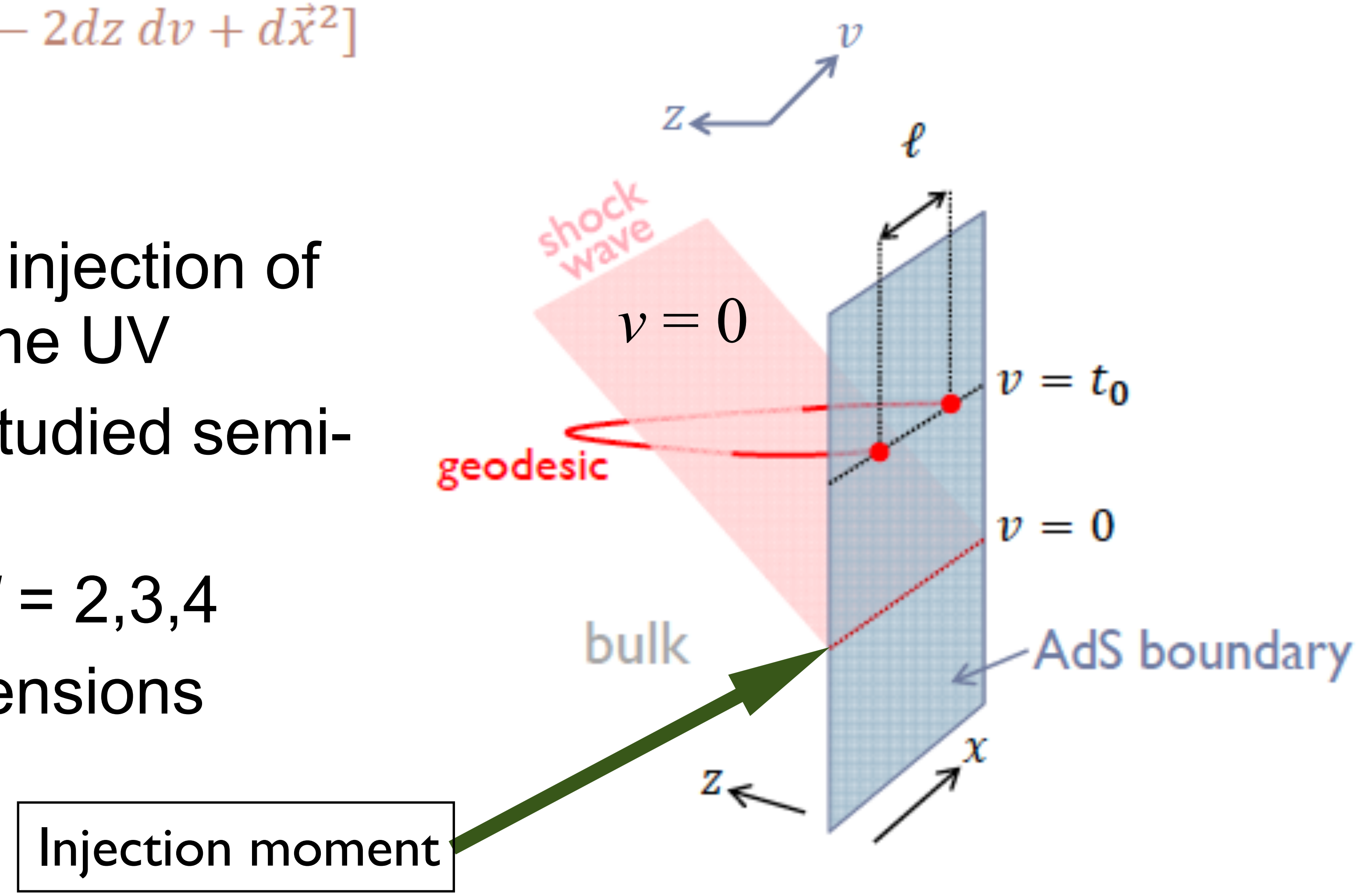
(*Ryu & Takayanagi* 2006)

# Vaidya-AdS geometry

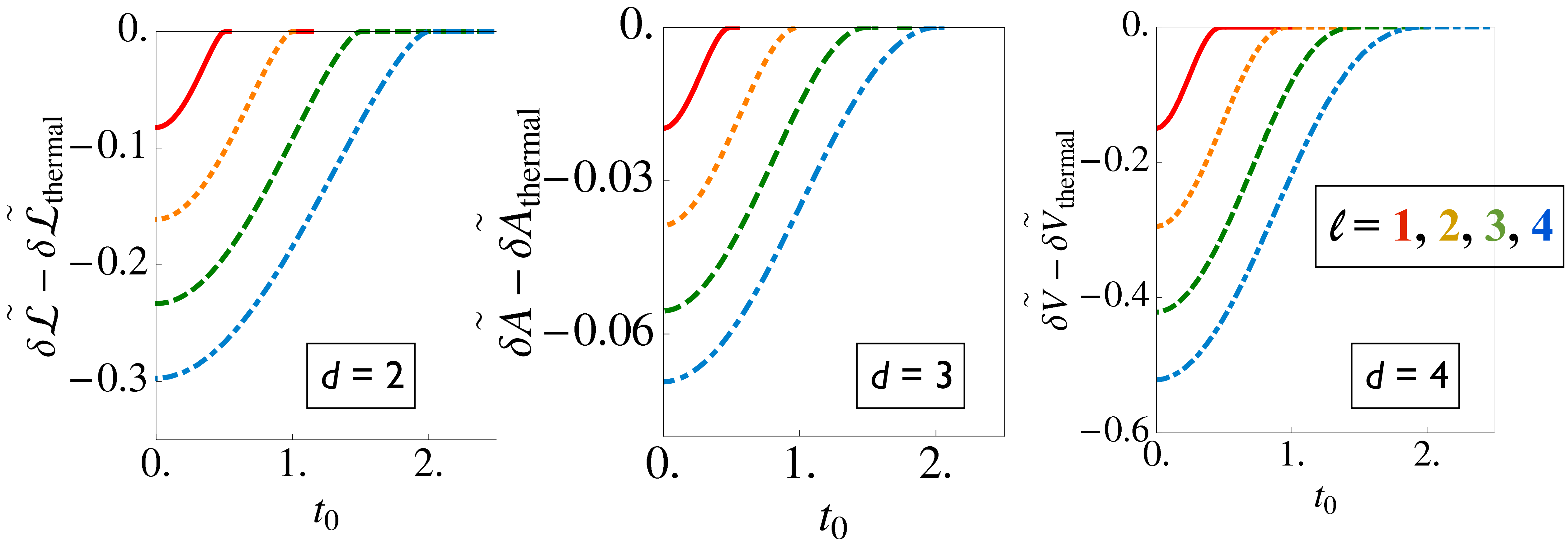
- Light-like (null) infalling energy shell in AdS (shock wave in bulk)
  - *Vaidya-AdS space-time* (analytical)

$$ds^2 = \frac{1}{z^2}\left[-\left(1 - m(v)z^d\right)dv^2 - 2dz\,dv + d\vec{x}^2\right]$$

  - $z = 0$: UV $\quad z = \infty$: IR
  - Homogeneous, sudden injection of entropy-free energy in the UV
  - Thin-shell limit can be studied semi-analytically
  - We studied AdS$_{d+1}$ for $d$ = 2,3,4
  - ⇔ Field theory in $d$ dimensions



# Entanglement entropy




**Thermalization time for entanglement entropy:** $\tau_{\rm th} = \ell/2$

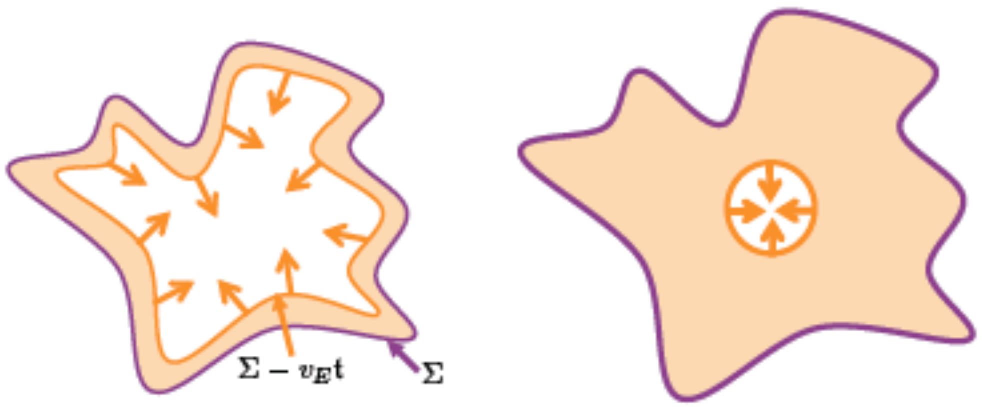
**= time for light to escape from the center of the volume to the surface.**

**Other observables thermalize faster.**

Crude estimate: $\tau_{\rm crit}$ **~ 0.5 ħ/*T* ≈ 0.3 fm/c for *T* = 300 − 400 MeV**

# Entanglement "tsunami"

H. Liu & S. J. Suh, arXiv:1205.7244 - Entanglement entropy growth of a large domain after a quantum quench modeled by thin collapsing shell in AdS. Find three regimes:




Local equilibration ($t < \ell_{eq} \sim 1/T$):

$$\Delta S_\Sigma(t) = \frac{\pi}{d-1}\mathcal{E} A_\Sigma t^2 + \cdots$$

Ingoing entanglement "tsunami" ($t > \ell_{eq}$):

$$\Delta S_\Sigma(t) = v_E s_{eq} A_\Sigma t + \cdots$$

Late time behavior:

$$S(R,t) - S^{(eq)}(R) \propto -(t_s - t)^\gamma$$

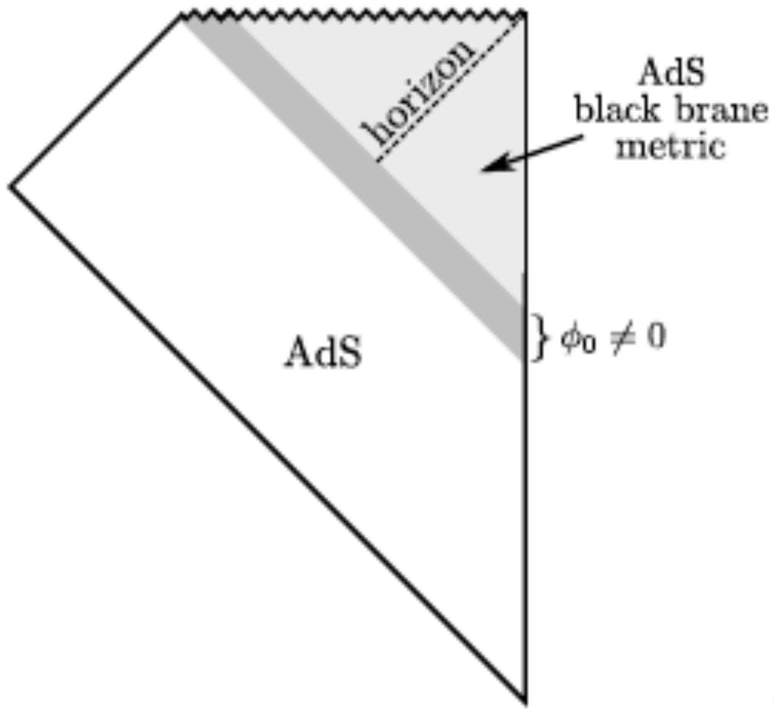
$$t_s(R) = \frac{1}{c_E} R - \frac{d-2}{4\pi T}\log R + O(R^0)$$

$d = 2: \quad v_E = c_E = 1$

$d \geq 3: \quad v_E < c_E < 1$

e.g.: $v_E = 0.62$ in $d = 4$

# Inhomogeneities




AdS$_4$ gravity plus scalar field, which is briefly turned on at the boundary to inject “cold” energy

$$ds^2 = -h(v,r,x)dv^2 + 2dv\,(dr + k(v,r,x)dx) + $$
$$+ f(v,r,x)^2 e^{B(v,r,x)} dx^2 + f(v,r,x)^2 e^{-B(v,r,x)} dy^2 \,,$$
$$\phi = \phi(v,r,x)\,,$$

$$\lim_{r\to\infty} \phi(v,r,x) = \varphi(v,x)$$

$$\begin{array}{|ll|} \varphi(v,x) = 0\,, & v \le 0 \\ \varphi(v,x) = \epsilon\varphi_0(v,x)\,, & 0 < v < \delta t \\ \varphi(v,x) = 0\,, & v \ge \delta t\,, \end{array}$$

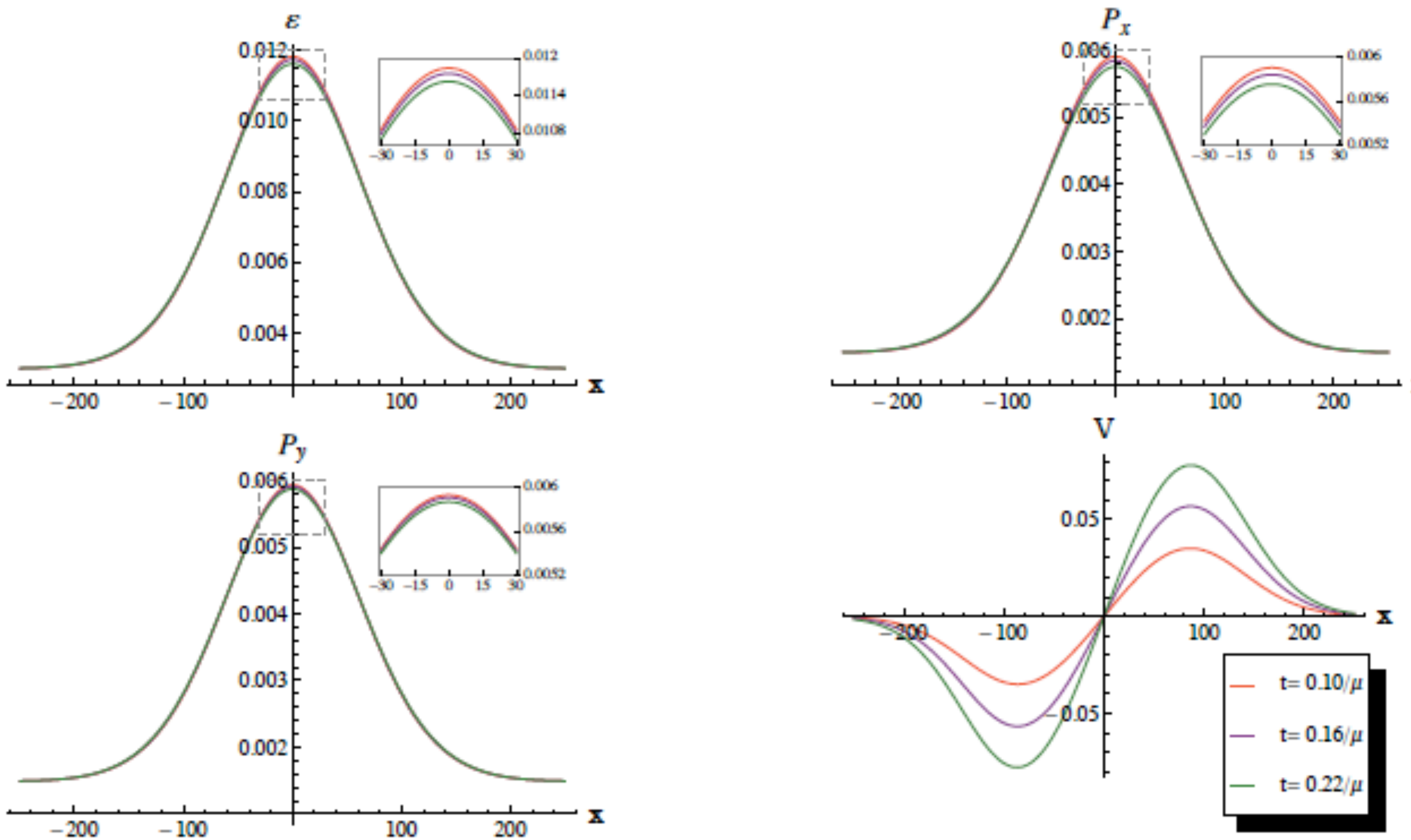
Perturbative solution following Bhattacharyya and Minwalla (arXiv:0904.0464).

Allows for analytical description, but fails after some time before formation of the event horizon - full thermalization requires numerical solution.

See: *V. Balasubramanian et al.*, arXiv:1307.1487 (PRL) and 1307.7086.

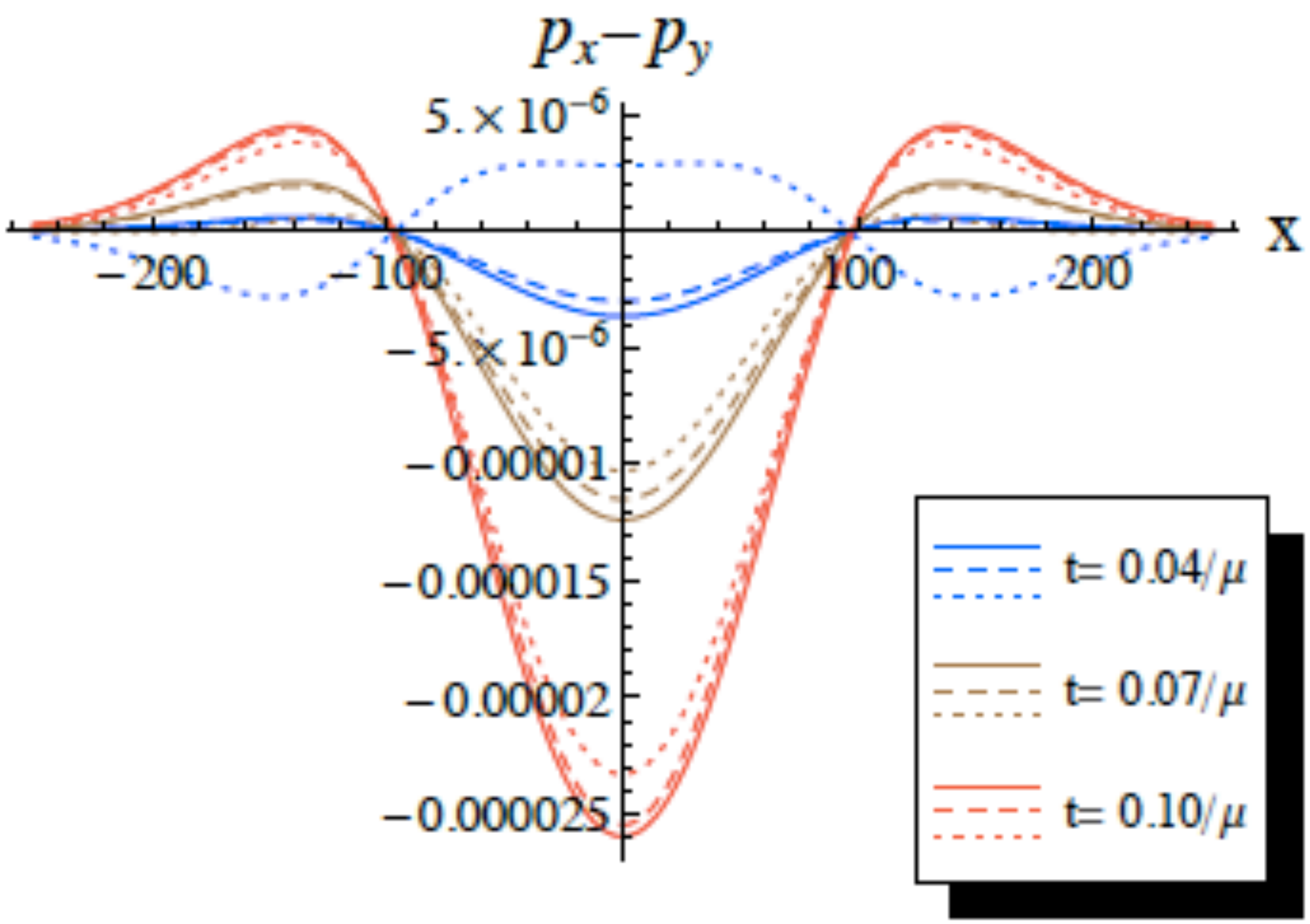
# Results

Time evolution of a large Gaussian density perturbation (only transverse dynamics):



# Comparisons

Comparison with free streaming (dashed) and 2$^{nd}$ order viscous hydro (dotted)




Conclusion: Free streaming works at early times (why?) - 2$^{nd}$ order viscous hydro catches up. [Note: no initial flow field, i.e. no longitudinal flow since $d$ = 2+1.]

# Summary

- The QGP behaves as a nearly "perfect" liquid indicating strong coupling
  - Power spectrum of initial energy density fluctuations is reflected in flow anisotropy Fourier components
  - All data consistent with $\eta/s < 0.15$ (RHIC) and $< 0.25$ (LHC)

- Weak vs. strong coupling
  - Holographic models provide solvable thermalization scenarios at strong coupling
  - Can a weakly coupled turbulent glasma "masquerade" as strongly coupled system?
  - Towards realistic holographic equilibration models
  - What are the main relevant features for thermalization?

Thank you

for your attention!