An Algorithmic Approach to Heterotic String Phenomenology

With Yang-Hui He, Maximilian Kreuzer, Andre Lukas, Chuang Sun

Seung-Joo Lee
Korea Institute for Advanced Study

IPMU, 6 Dec 2013

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Based mainly on 1309.0223

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Outline

1. **Introduction:**
   Heterotic Calabi-Yau Models
2. **Arena:**
   Specific Calabi-Yau manifolds and bundles
3. **Classifications:**
   An ‘exhaustive’ scan over favourable geometries
4. **Summary and Outlook**

Introduction

String Phenomenology

- The Standard Model

  a Particular 4D Quantum Gauge Field Theory with

  - Gauge group: $\mathbf{G_{std} = SU(3) \times SU(2) \times U(1)}$
  - Particle spectrum:

$$\mathbf{3} \times \left[(\mathbf{3},\mathbf{2})_{\frac{1}{6}} + (\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + (\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} + (\mathbf{1},\mathbf{1})_{\mathbf{1}} + (\mathbf{1},\mathbf{1})_{\mathbf{0}} \right]$$

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- Superstring Phenomenology
  - Find a String Vacuum with the structure of the (SUSY) SM
  - Strings in 10D seen as particles in 4D

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- Compactify Het. E_8 Strings [Candelas, Horowitz, Strominger, Witten '85]

Introduction

Ingredients of the Heterotic Compactification

- Low-Energy Theory

  4D Models, upon compactifying the internal geometry

- Preserving Supersymmetry
  - A CY threefold X
  - A holomorphic, polystable vector bundle V over X

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- Preserving Supersymmetry
  - A CY threefold $\mathbf{X}$
  - A holomorphic, polystable vector bundle $\mathbf{V}$ over $\mathbf{X}$

    s.t. the internal gauge field satisfies the HYM eqns

    $\mathbf{F_{ab} = 0 = F_{\bar{a}\bar{b}}}$ and $\mathbf{g^{a\bar{b}} F_{a\bar{b}} = 0}$

    [Donaldson; Uhlenbeck, Yau]

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Basic Constraints in Model Building

- Gauge group reduction
  - Bundle Structure Group, $\mathcal{G} = \mathbf{SU(4)}, \mathbf{SU(5)}$

$E_8 \rightarrow$ H = $\mathbf{SO(10)}, \mathbf{SU(5)}$ GUT

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- Heterotic anomaly cancellation
  - $\mathbf{c_2(TX) - c_2(V)} \in$ Mori-Cone of $\mathbf{X}$

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- Heterotic anomaly cancellation
  - $\mathbf{c_2(TX) - c_2(V)} \in$ Mori-Cone of $\mathbf{X}$
- Massless spectrum
  - $\mathbf{N_{gen} = -Ind(V) = -\frac{1}{2}\int_X c_3(V)} \rightarrow \mathbf{Ind(V) = -3}$

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Particle Spectrum

- Associated GUT theory
  - Gauge group, H=SU(5)
  - Matter multiplets: $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{1}$

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$\mathcal{G}$	$\mathcal{H}$	Branching of $\mathbf{248}$ under $\mathcal{G}\times\mathcal{H}\subset E_8$
$SU(5)$	$SU(5)$	$(\mathbf{1},\mathbf{24})\oplus(\mathbf{5},\mathbf{10})\oplus(\overline{\mathbf{5}},\overline{\mathbf{10}})\oplus(\mathbf{10},\overline{\mathbf{5}})\oplus(\overline{\mathbf{10}},\mathbf{5})\oplus(\mathbf{24},\mathbf{1})$

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$\mathcal{G}$	$\mathcal{H}$	Particle Spectrum
$SU(5)$	$SU(5)$	$n_{10} = h^1(X,V)$ $n_{\overline{10}} = h^1(X,V^\star) = h^2(V)$ $n_5 = h^1(X,\wedge^2 V^\star)$ $n_{\overline{5}} = h^1(X,\wedge^2 V)$ $n_1 = h^1(X,V\otimes V^\star)$

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- Can lead to SMs upon adding Wilson-line

Arena

Calabi-Yau threefolds

- E.g. the Quintic
  - Zero locus of quintic polynomial in $\mathbb{P}^4$

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- Complete intersection CYs in "multi-proj."
  - Common zero locus $\mathbf{X} = \{p_i = 0\} \subset \mathcal{A}$ of homogeneous polynomials p_i in an ambient space $\mathcal{A} = \otimes_{r=1}^{m} \mathbb{P}^{n_r}$

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  - All of them are simply-connected; Classification of freely-acting discrete symmetries [Braun '10]

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- Hypersurface CYs in "toric"
  - Zero locus $\mathbf{X} = \{p = 0\} \subset \mathcal{A}$ of homogeneous polynomial in a toric ambient space $\mathcal{A} = \mathcal{A}_\Delta$

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- Interesting Subclasses?

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  - Smooth ambient cases, ~100 [He, Lee, Lukas, '10]

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Vector Bundles

- Spectral Cover Construction
  - A natural way only if elliptically fibred
- Extension
  - $0 \to V_1 \to V \to V_2 \to 0$
- Monad
  - $0 \to V \to B \to C \to 0$

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Remark: Bundle Polystability

- Definition
  - *Slope* of a vector bundle V,
  - V is *stable* if . . .
  - V is *polystable* if . . .

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- Polystability test is difficult in general, but . . .
  - Line bundle, $L = \mathcal{O}_X(\mathbf{k})$ is stable
  - Line-bundle sum, $V = \bigoplus_{a=1}^{n} L_a = \bigoplus_{a=1}^{n} \mathcal{O}_X(\mathbf{k}_a)$ is polystable iff

$$\mu(O_X(\mathbf{k}_a)) = 0,\ \forall a$$

Arena

THE GEOMETRICAL ARENA

- A Special Corner of the Heterotic Landscape
  - Singled out the sixteen: $\pi_1(X) \neq \phi$
    Wilson-lines available outright
  - Focus on the favourable fourteen: $H^{1,1}(X) = \mathrm{Span}\{J_i|_X\}$
    Complete control over the line bundles in toric terms

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i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$h^{1,1}(X_i)$	1	2	4	2	3	3	3	3	4	4	4	4	4	3	5_{nf}	5_{nf}
$-\chi(X_i)$	40	54	64	72	80	112	144	144	48	64	64	80	80	112	48	48
$\pi_1(X_i)$	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$

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  - Rank n Line-bundle Sums

$$V = \bigoplus_{a=1}^{n} \mathcal{O}_X(\mathbf{k}_a)\,, \text{ with } \mathbf{k}_a \in \mathbb{Z}^{h^{1,1}}, \text{ n=4 or 5.}$$

Arena

THE PHYSICAL ARENA

- Enhanced Symmetry
  - Structure group reduced, $\mathcal{G} = S(U(1)^5) \subset SU(5) \subset E_8$
  - Low-energy group enhanced, $\mathcal{H} = SU(5) \times S(U(1)^5)$

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  - Matter charges refined, $\mathbf{10_a}$, $\overline{\mathbf{10}}_\mathbf{a}$, $\mathbf{5_{a,b}}$, $\overline{\mathbf{5}}_\mathbf{a,b}$, $\mathbf{1_{a,b}}$

$SU(5) \times S(U(1)^5)$ repr.	associated cohomology	contained in
$\mathbf{10}_a$	$H^1(X, L_a)$	$H^1(X, V)$
$\overline{\mathbf{10}}_a$	$H^1(X, L_a^*)$	$H^1(X, V^*)$
$\overline{\mathbf{5}}_{a,b}$	$H^1(X, L_a \otimes L_b)$	$H^1(X, \wedge^2 V)$
$\mathbf{5}_{a,b}$	$H^1(X, L_a^* \otimes L_b^*)$	$H^1(X, \wedge^2 V^*)$
$\mathbf{1}_{a,b}$	$H^1(X, L_a \otimes L_b^*)$	$H^1(X, V \otimes V^*)$

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$\mathbf{5}_{a,b}$	$H^1(X, L_a^* \otimes L_b^*)$	$H^1(X, \wedge^2 V^*)$
$\mathbf{1}_{a,b}$	$H^1(X, L_a \otimes L_b^*)$	$H^1(X, V \otimes V^*)$

- U(1) Symmetries constrain 4D theory
  - E.g. Possible superpotential terms highly constrained

Classifications

An Algorithmic Approach

- Search for Heterotic SMs
  - SM gauge group times (anomalous) U(1)s; Correct chiral asymmetries; SM singlets; Free of heterotic anomaly.
  - No mirror families; One or more pairs of Higgs doublets.

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- Systematic Model Building
  - CYs characterised by the toric lattice combinatorics
  - Bundles $V = \bigoplus_{a=1}^{5} \mathcal{O}_X(\mathbf{k}_a)$ by an $h^{1,1}(X) \times 5$ matrix $[\mathbf{k}_a] = [k_a^i]$

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  - Bundles $V = \bigoplus_{a=1}^{5} \mathcal{O}_X(\mathbf{k}_a)$ by an $h^{1,1}(X) \times 5$ matrix $[\mathbf{k}_a] = [k_a^i]$
  - A priori no bounds on the entries k_a^i
  - For $k_a^i \in [-k_{\max}, k_{\max}]$, $\sim (2k_{\max}+1)^{4h^{1,1}(X)}$ bundles

Classifications

Physical Constraints Revisited

- Constraints on the Geometry
  - Gauge group ..... $\mathbf{c_1(V) = 0}$ with $\mathbf{n} = 4, 5$
  - Anomaly ..... $\mathbf{c_2(TX) - c_2(V)}$ is effective
  - Supersymmetry ..... $\mu(L_a) = 0, \ \forall a$
  - Three net-generations .... $\mathbf{Ind(V) = -3}$
  - No exotics ..... $\mathbf{-3 \leq Ind(L_a) \leq 0}$

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- Diophantine System
  - With up to cubic constraints
  - May scan over the geometries

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- Constraints on the Geometry
  - Gauge group ..... $\mathbf{c_1(V) = 0}$ with $\mathbf{n} = 4, 5$
  - Anomaly ..... $\mathbf{c_2(TX) - c_2(V)}$ is effective
  - Supersymmetry ..... $\mu(L_a) = 0, \ \forall a$
  - Three net-generations .... $\mathbf{Ind(V) = -3}$
  - No exotics ..... $\mathbf{-3 \leq Ind(L_a) \leq 0}$
- Diophantine System
  - With up to cubic constraints
  - May scan over the geometries
- Is it even Finite?

Classifications

Example - Base Geometry

- Toric Data for X_9

$$\Delta_9 = \begin{pmatrix} -4 & 4 & 0 & 0 & 0 & 0 & 2 & -2 \\ -1 & 2 & 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 & -2 & 1 & -1 \\ 1 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \end{pmatrix}$$

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- Relevant Properties
  - Hodge Numbers: $\mathbf{h^{1,1} = 4;\ h^{2,1} = 28;\ \chi = -48}$

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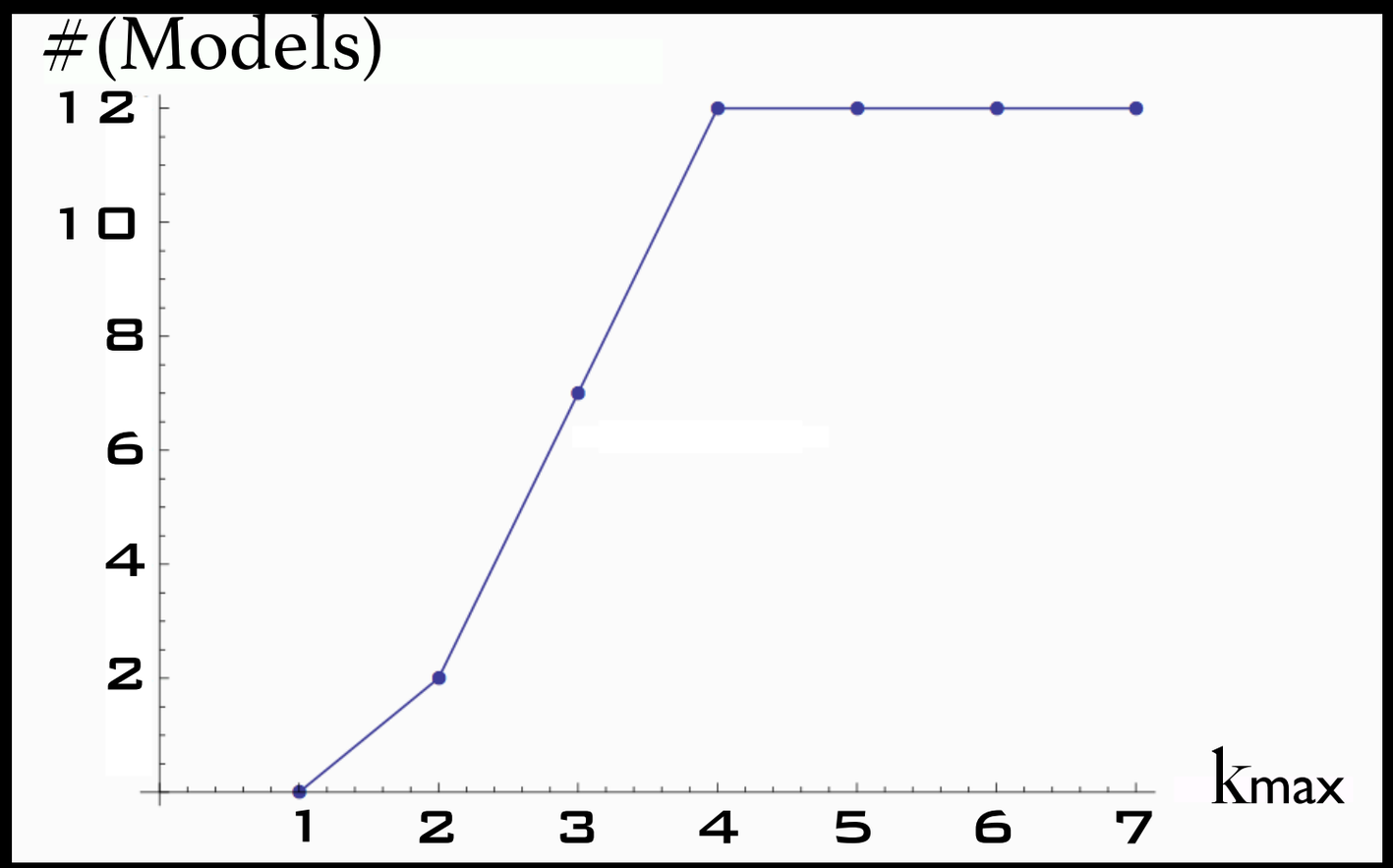
- Relevant Properties
  - Hodge Numbers: $\mathbf{h^{1,1} = 4;\ h^{2,1} = 28;\ \chi = -48}$
  - Kahler Cone: $\mathbf{J = t^i J_i\,;\ t^{i=1,2,4} > 0}$ and $\mathbf{t^3 > 2t^4}$
  - 2nd Chern Class: $\mathbf{c_2(TX) = \{12, 12, 12, 4\}}$
  - Intersection Structure: $\mathbf{J_1J_2J_3 + J_1J_3J_4 + 2J_2J_3J_4 - 2J_1J_4^2 - 4J_2J_4^2 + 2J_3J_4^2 - 8J_4^3}$

Classifications

Example - Bundle Classification

- Finiteness Criterion



- Practically finite, if #(Models) does not increase for three consecutive values for $k_{\max}$

Classifications

Number of Resulting SM Candidates

- # of Consistent GUTs with Correct Indices

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	total
# $SU(5)$	0	0	10	0	0	2	0	0	12	25	54	1	17	1	122
max. $\lvert k_a^r \rvert$	-	-	4	-	-	4	-	-	4	5	5	4	5	4	

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max. $\lvert k_a^r \rvert$	-	-	4	-	-	4	-	-	4	5	5	4	5	4	
# $SO(10)$	0	0	7017 *	0	5	13	0	9	2207	4416 *	8783 *	1109 *	5283 *	28	28870
max. $\lvert k_a^r \rvert$	-	-	17	-	6	7	-	4	15	20	19	21	21	7	

- * means that #(Models) almost converges but have not quite saturated despite the large entries

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- Available at:

http://www-thphys.physics.ox.ac.uk/projects/CalabiYau/toricdata/index.html

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Classifications

Example - SU(5) GUT based model

- Toric Data for X_9

$$\Delta_9 = \begin{pmatrix} -4 & 4 & 0 & 0 & 0 & 0 & 2 & -2 \\ -1 & 2 & 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 & -2 & 1 & -1 \\ 1 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \end{pmatrix}$$

- Gauge Bundle

$$\mathbf{V} = \mathcal{O}_{\mathbf{X}}(-4, 0, 1, 1) \oplus \mathcal{O}_{\mathbf{X}}(1, 3, -1, -1) \oplus \mathcal{O}_{\mathbf{X}}(1, -1, 0, 0)^{\oplus 3}$$

with $\mathbf{Ind(V) = -3}$

- Particle Spectrum

$$\mathbf{10_1, 10_1, 10_1, \bar{5}_{2,3}, \bar{5}_{2,4}, \bar{5}_{2,5}}$$

Summary

An Algorithmic Approach to Heterotic String Phenomenology

- A systematic and algorithmic approach is adequate for heterotic CY model construction, producing a large number of SM candidates.
- Studied in particular line bundle models on the 16 torically-generated CYs with a non-trivial 1st fundamental group.
- Constructed SM candidates based on SU(5), SO(10) GUTs; SUSY, no anomaly, correct chiral asymmetries.
  - For SU(5) GUT - tot. of 122 models
  - For SO(10) GUT - tot. of 28870 models

Outlook

Exploration of the Rich Heterotic Geometry

- Full spectrum of the models can be obtained by figuring out relevant line-bundle cohomologies on the 16 CYs.
- This work on the special corner - the sixteen - can be thought of as the first step towards *the long-term programme: "classification of heterotic SMs over the Kreuzer-Skarke dataset."*
- Classification of freely-acting symmetries on these CYs is another thing we are currently working on.

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THANK YOU